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ABSTRACT

A phenomenological method is applied to the ρ trajectory. It is argued that once the intercept is known, the method is expected to give reliable information about the trajectory in the region of interest for high-energy scattering. Solutions for different values of the intercept ranging from 0.3 to 0.8 are given. From certain general requirements the higher intercepts seem to be favored.

A PHENOMENOLOGICAL TREATMENT OF THE ρ TRAJECTORY*

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In a previous paper, we presented a phenomenological method for calculating Regge trajectories.[1] The approach was based on the real-analyticity and threshold behavior as well as on the available experimental information, and the first application was to the Pomeranchuk trajectory. It is our purpose in this paper to obtain by the same approach the approximate form for the ρ trajectory.

Our starting point is a four-parameter ansatz for the imaginary part of the trajectory function $\alpha(t)$, namely

$$\mathrm{Im}\,\alpha(t) = \frac{C\nu^{\lambda}}{C_1 + (C_2 - \nu)^2} \quad \text{for} \quad \nu > 0 ,$$

and (1)

$$\mathrm{Im}\,\alpha(t) = 0 \quad \text{for} \quad \nu < 0 ,$$

where $\nu = \frac{1}{4}(t - t_0)$, and $t_0 = 4m_\pi^2 = 4$. Using the usual dispersion relation for α, we have

$$\mathrm{Re}\,\alpha(\nu) = \alpha(\nu = -1) + \frac{C(\nu + 1)}{\pi} P \int_0^\infty \frac{\nu'^{\lambda}\, d\nu'}{(\nu' - \nu)[C_1 + (C_2 - \nu)^2](\nu' + 1)} , \tag{2}$$

where for convenience we have made the subtraction at $\nu = -1$, the point corresponding to the forward direction in the crossed channel. This integral can be evaluated by applying Cauchy's theorem to Eq. (1). The result is

$$\alpha(\nu) = \alpha(\nu=-1) - \frac{\nu+1}{\sin\pi\lambda}\left\{\frac{\nu^{\lambda}\,e^{-i\pi\lambda}}{(\nu+1)[C_1+(C_2-\nu)^2]} - \frac{a^{\lambda}\,e^{-i\pi\lambda}}{(a-b)(a+1)(\nu-a)} - \frac{b^{\lambda}\,e^{-i\pi\lambda}}{(b-a)(b+1)(\nu-b)} - \frac{1}{(\nu+1)[C_1+(C_2+1)^2]}\right\}, \quad (3)$$

where

$$a = -C_2 + i\,(C_1)^{\frac{1}{2}}, \quad \text{and} \quad b = -C_2 - i\,(C_1)^{\frac{1}{2}} .$$

In Eq. (3), the four parameters λ, C, C_1, and C_2 are determined with the help of the four conditions

(i) $\alpha(\infty) = -1$,

(ii) $\Gamma_\rho = \left(\frac{\operatorname{Im}\alpha(t)}{\frac{d\,\operatorname{Re}\alpha(t)}{dt}\,(t)^{\frac{1}{2}}}\right)_{t=m_\rho^2} \approx 100$ MeV , (see reference 2) ,

(iii) $\lambda = \alpha(\nu=0) + \frac{1}{2}$, (see reference 3), and

(iv) $\operatorname{Re}\alpha(t=m_\rho^2) = 1$.

These conditions, when imposed on Eq. (3), and when $\alpha(\nu=-1)$ is known, determine the four parameters. The solution was found with the help of the IBM 7094 computer of the Lawrence Radiation Laboratory. Since the intercept value, $\alpha(\nu=-1)$, is not well known, we take a range of values from 0.3 to 0.8 and attempt to choose the most plausible solution among them. We should also mention that condition (i) is questionable. All that is known is that $\alpha(\infty) < 1$. However, taking $\alpha(\infty) = 0$, for example, makes not too significant a change in the general features of trajectory in the region $-10\,m_\pi^2 < \nu < 10m_\pi^2$. Figure 1 shows Re α vs ν for intercepts of 0.3 to 0.8, and for the sake of comparison, Re α for the Pomeranchuk trajectory of reference 1 is also given.

Figure 2 is a plot of the corresponding Im α. It is seen from these curves that for smaller intercepts the imaginary part of α has a sharper maximum and Re α reaches a higher maximum value. In particular, for the small intercepts, the solution gives rise to a spin-3 recurrence of a width narrow enough to be experimentally observable. These features are examined in Figs. 3, 3(a) showing the relation between the intercept and the width Γ_R of the spin-3 recurrence, and 3(b) showing the relation between the mass and the width of the spin-3 recurrence. It is clear from Fig. 3(b) that if such a recurrence exists at all, its mass should be smaller than 2 BeV, so an experimental search may be correspondingly restricted. Once the mass is experimentally known, the width is predicted by this curve.

Figure 3(c) gives the slope of the trajectory at $t = 0$ vs the intercept value. The position and the width of the spin-3 recurrence are given as running parameters. If the fact that no recurrence has been found is interpreted to mean that the width is too large to have been observed, then the higher intercepts of 0.7 to 0.8 are favored. We should mention that the solution with intercept of 0.7 and slope $\frac{d\alpha}{dt}(t=0) \approx 0.44\ \text{BeV}^{-2}$ is in fair agreement with the results of Scotti and Wong[4] concerning the low energy nucleon-nucleon scattering and with the results of Brandsen et al.[5] concerning the strip approximation in π-π scattering. Also this solution gives essentially the same slope as the Pomeranchuk trajectory (see Fig. 1). Finally, Table 1 gives the values ofthe parameters C, C_1, C_2, and λ .

In conclusion, we should like to make a few remarks about the sensitivity of our results to the conditions (i) and (ii). If we replace condition (i) by $\alpha(\infty) = -2$, the solution for Re α in the region $-10m_\pi^2 < \nu < 10m_\pi^2$ changes very little; however, Γ_R, M_R and Re α_{max} change considerably (Γ_R gets smaller, M_R and Re α_{max} get bigger). In condition (ii), Γ_ρ is not experimentally known very accurately. If we choose $\Gamma_\rho = 80$ MeV , say, then again Γ_R, M_R, and Re α_{max} change considerably (Γ_R gets smaller, M_R and Re α_{max} get bigger). Here again, Re α and its derivative change very little in the region $-10m_\pi^2 < \nu < 10m_\pi^2$. Consequently, once the intercept is known, our treatment is expected to give reliable information about the trajectory in the region $-10_\pi^2 < \nu < 10m_\pi^2$. This is the region of interest in high-energy scattering.

From the present calculation based on conditions (i) and (ii), we have seen that higher intercepts are favored. On the other hand, as pointed out by Phillips,[6] if the ρ exchange is to play a dominant role in np charge exchange scattering then the experiment of Palevsky et al.,[7] who measured the energy dependence at $t = 0$, would require an intercept of about 0.3. This value is in sharp conflict with the higher intercepts of 0.7 and 0.8, which we have favored here.

Recently, Abolins et al.[8] have reported evidence for a resonance at 1.22 BeV that may have the same spin and quantum numbers as the ρ. If so, there would be a second ρ trajectory with a smaller intercept at $t = 0$ than the first. The combined result might be an "average" intercept of about 0.3 as required by the experiment of Palevsky et al.[7] (If the resonance at 1.22 BeV has spin 3 and is the recurrence of the ρ, it would roughly fit into our solution with the intercept of 0.5.)

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FOOTNOTES AND REFERENCES

* Work done under the asupices of the U. S. Atomic Energy Commission

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TABLE I. Values of C_1, C_2, λ, and C.

$\alpha(\nu=-1)$	C_1	C_2	λ	C
0.3	6.798	20.26	0.8750	5.443
0.4	11.99	24.04	0.9670	5.928
0.5	23.38	29.29	1.058	6.550
0.6	67.98	37.06	1.148	8.478
0.7	173.8	49.53	1.2373	9.669
0.8	766.9	72.73	1.3256	13.31
Pomeranchuk				
1.0	259.7	55.24	1.533	3.79

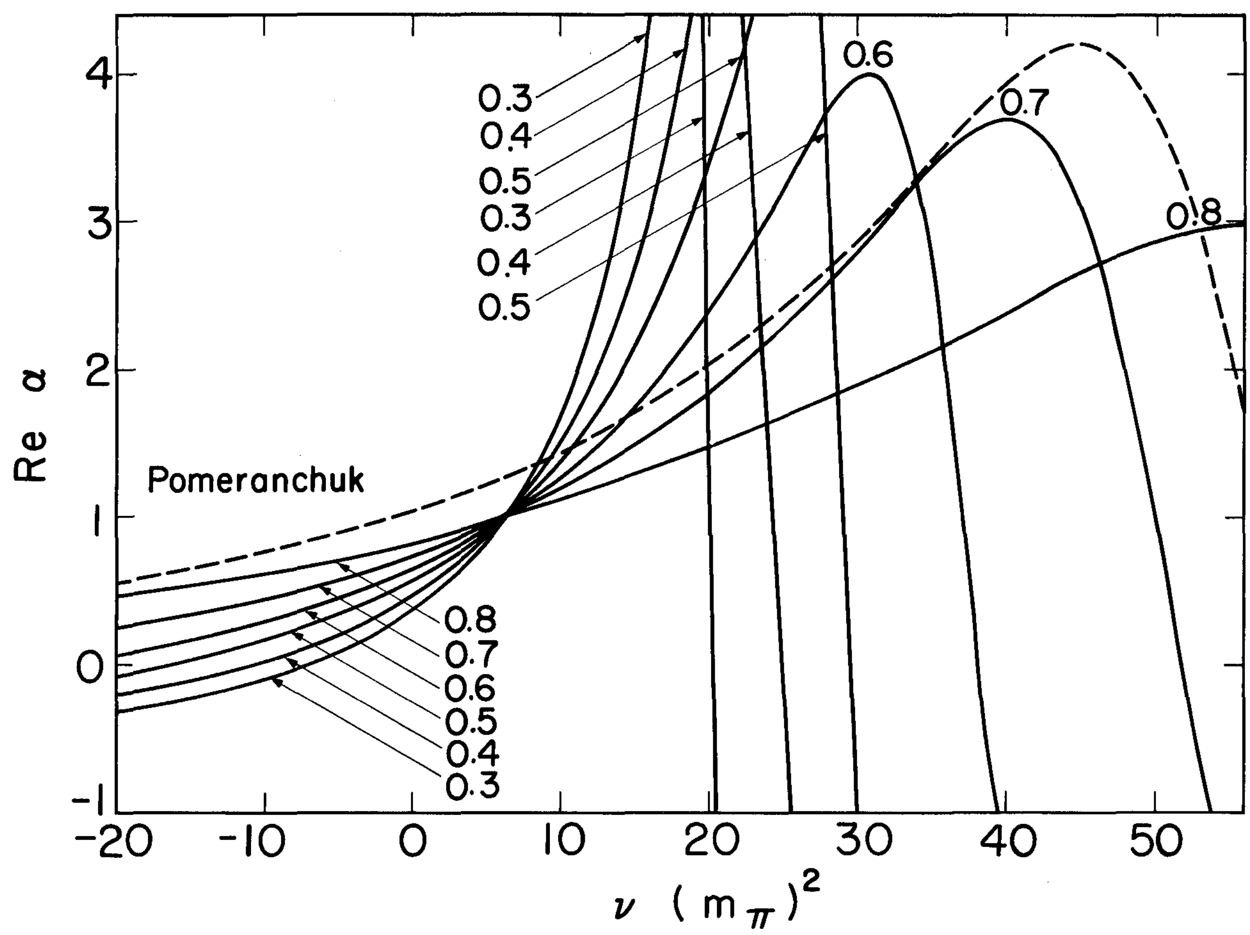

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Fig. 1. Re α vs ν for intercepts of 0.3 to 0.8. The dotted curve is Re α vs ν for the Pomeranchuk trajectory given in reference 1.

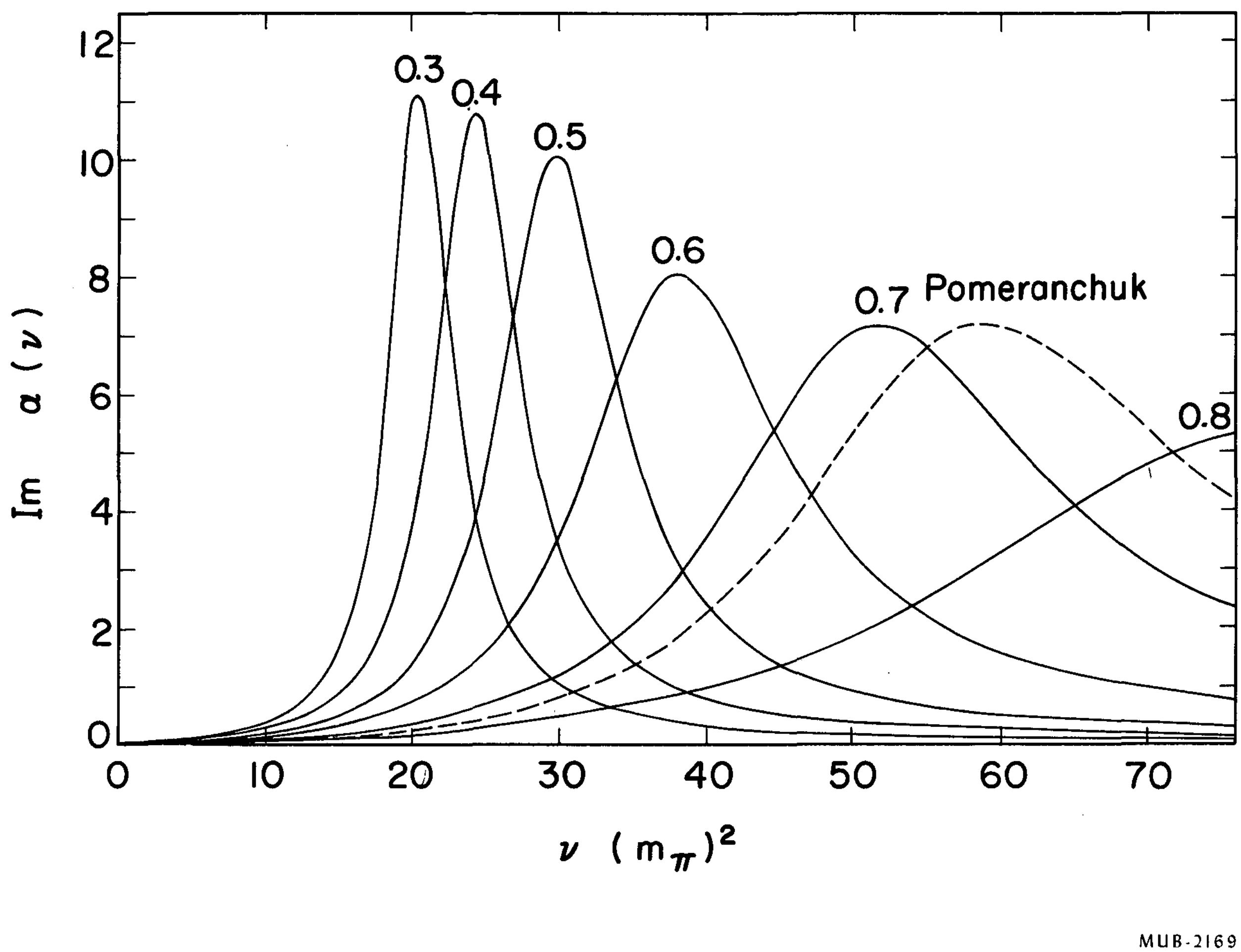

Fig. 2. Im α vs ν corresponding to the curves in Fig. 1. The intercept values are indicated on these curves.

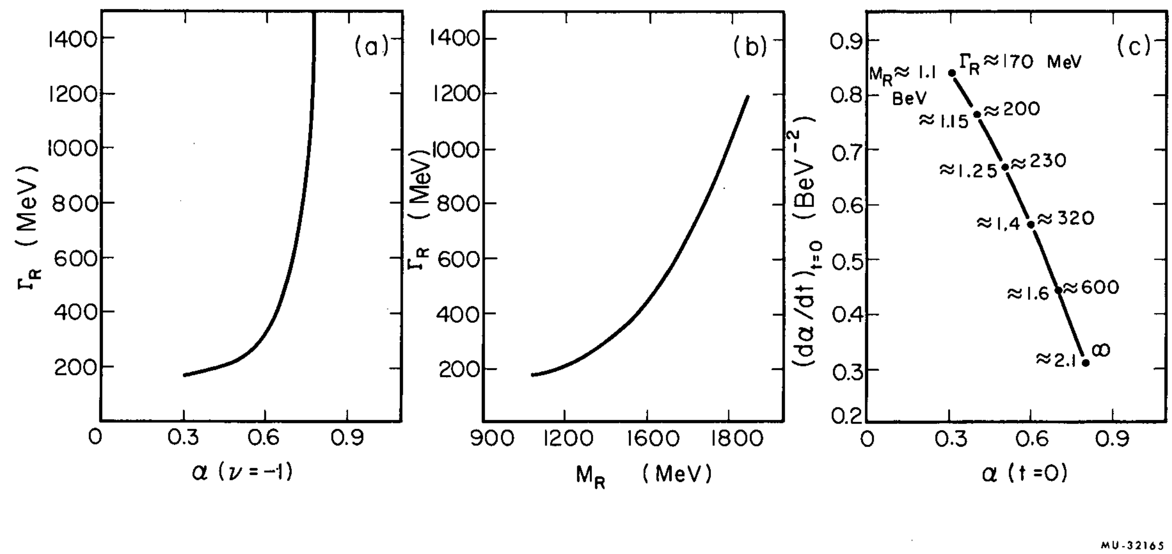

Fig. 3. (a) Γ_R vs $\alpha(\nu = -1)$.

Fig. 3. (b) Γ_R vs M_R .

Fig. 3. (c) $\frac{d\alpha}{dt}$ (t=0) vs α(t=0) .