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ISO -- A FORTRAN PROGRAM FOR UTILIZING THE STRONG-FOCUSING PRINCIPLE IN BETA-RAY SPECTROMETER DESIGN

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ISO--A FORTRAN PROGRAM FOR UTILIZING THE STRONG-FOCUSING PRINCIPLE IN BETA-RAY SPECTROMETER DESIGN*

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Abstract

An IBM FORTRAN II code to calculate beta-ray spectrometer characteristics of aberration, dispersion, transmission, and resolution, using azimuthally varying magnetic fields, is described.

1. Introduction

The program ISO was written at the suggestion of Dr. Andrew M. Sessler as a first step toward investigating the feasibility of using the strong-focusing principle in building a high-transmission high-resolution beta-ray spectrometer. The present program provides a means of extending to sixth order the second-order calculations performed by Dr. Sessler.[1] This work was done under the guidance and financial support of the Nuclear Chemistry Division with the encouragement of Dr. J. M. Hollander.

2. Mathematics

The equations of motion of a particle having momentum P are

$$x'' = \frac{Q^{1/2}}{(1+\epsilon)(1+x)} \left[(1+x)y' b_\theta + x'y'b_r - (Q-y'^2)b_y\right] + \frac{x'^2-y'^2+Q}{(1+x)},$$

$$y'' = \frac{Q^{1/2}}{(1+\epsilon)(1+x)} \left[-(1+x)x' b_\theta - x'y'b_y + (Q-x'^2)b_r\right] + \frac{2x'y'}{1+x},$$

where $x = \dfrac{R-R_0}{R_0}$, $y = \dfrac{z}{R_0}$, R is the particle radius, z is the axial displacement, R_0 is the optic circle, b_z, b_n, and b_θ are the normalized components of the magnetic field as described in Section 5, $\epsilon = \dfrac{P-P_0}{P_0}$,

$P_0 = \dfrac{eR_0B_0}{c}$, $Q = x'^2+y'^2+(1+x)^2$, and primes denote derivatives with respect to θ.

These equations agree with those given by Judd.[2]

3. Input

The program is written in FORTRAN II language, thus, the input parameters must be preceded by a system control card with an asterisk in Column 1 followed by the word DATA beginning in Column 7.

The input is divided into two blocks: Block 1 sets up the initial conditions for the particles to be tracked, and Block 2 defines the field coefficients.

Block 1 may contain any number of pairs of cards using the format (7F10.6). The first card has the parameters θ_i, x_i, y_i, dummy, x_i', y_i', dummy, and the second card, θ_f, Δx_i, Δy_i, dummy, $\Delta x_i'$, $\Delta y_i'$, n, where θ_i is the initial azimuth (degrees), x_i the initial value of x, y_i the initial value of y, x_i' the initial value of x', y_i' the initial value of y', θ_f the final azimuth, Δx_i the increment for x, Δy_i the increment for y, $\Delta x_i'$ the increment for x_i', $\Delta y_i'$ the increment for y_i', and n the number of initial conditions to be defined by this pair of cards. The n sets of initial conditions are generated by the program according to the scheme x_i, $x_i + \Delta x_i$, $\cdots$, $x_i + (n-1)\,\Delta x_i$; y_i, $y_i + \Delta y_i$, $\cdots$, $y_i + (n-1)\,\Delta y_i$; x_i', $x_i' + \Delta x_i'$, $\cdots$, $x_i' + (n-1)\,\Delta x_i'$; y_i', $y_i' + \Delta y_i'$, $\cdots$, $y_i' + (n-1)\,\Delta y_i'$.

The signal to end the first block of parameters is a card with 100. punched in Columns 1 through 10. This card is also used to set up the following options:
Option 1. If Columns 41 through 50 are nonzero, θ, x, and y for each orbit will be printed at each Runge-Kutta step.
Option 2. If Columns 51 through 60 are nonzero, the magnetic fields the particle sees will be output at each Runge-Kutta step.
Option 3. The value of the parameter ϵ in the formula $P = P_0(1+\epsilon)$ is input in Columns 61 through 70.

The parameters for this signal card are input according to the format (7F10.6).

The second block gives the magnetic field information for the fields as described in section 5, Eq. (1). Each of the seven cards in this block contains A_{i0} to A_{i6} to define the a_i for one field level. The parameters for orders one through six are input on the first six cards in this block, while the parameters defining the zeroth order are input on card seven, for historical reasons. All parameters in this block are input according to the format (7F10.6), and all seven cards must be present, even if blank.

Successive runs may be stacked to any depth.

4. Output

Normal output consists of the list of input cards from Blocks 1 and 2 followed by a pair of lines for each orbit traced. The first line gives the initial conditions at the starting azimuth and the second line the values of x and y at the final azimuth. The end of the orbits for each run is indicated by a repeat of the Block 2 data for that run.

If Option 1 is exercised, the complete orbit trace for each particle is output between the initial output of Block 2 and the final conditions. If Option 2 is exercised the field values encountered by each particle are output, interspersed with x and y if Option 1 is used.

5. Magnetic Field Specification

If the expressions

$$b_y = \frac{B_y}{B_0} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij}(\theta) x^i y^j ,$$

$$b_r = \frac{B_r}{B_0} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} b_{ij}(\theta) x^i y^j ,$$

and

$$b_\theta = \frac{B_\theta}{B_0} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} c_{ij}(\theta) x^i y^j$$

are required to satisfy Maxwell's equations, $\nabla \cdot B = 0$, $\nabla \times B = 0$, and if the requirements of symmetry about the median plane be adhered to, the above equations may be expressed to sixth order in terms of $a_{i0}(\theta)$. If we leave off the (θ) and the second subscript for convenience, the fields are represented as follows:

$$by = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6$$

$$-(\tfrac{1}{2} a_1 + a_2)y^2 + (\tfrac{1}{2} a_1 - a_2 - 3a_3 - \tfrac{1}{2} a_1'')xy^2$$

$$+(-\tfrac{1}{2} a_1 + a_2 - \tfrac{3}{2} a_3 - 6a_4 + a_1'' - \tfrac{1}{2} a_2'')x^2y^2$$

$$+(\tfrac{1}{24} a_1 - \tfrac{1}{12} a_2 + \tfrac{1}{2} a_3 + a_4 + \tfrac{1}{6} a_2'' - \tfrac{1}{12} a_1'')y^4$$

$$+(\tfrac{1}{2} a_1 - a_2 + \tfrac{3}{2} a_3 - 2a_4 - 10a_5 - \tfrac{1}{2} a_3'' + a_2'' - \tfrac{3}{2} a_1'')x^3y^2$$

$$+\left(-\frac{1}{8}a_1+\frac{1}{4}a_2-\frac{3}{4}a_3+2a_4+5a_5+\frac{1}{2}a_3''-\frac{1}{2}a_2''+\frac{5}{12}a_1''+\frac{1}{24}a_1^{(4)}\right)xy^4$$

$$+\left(-\frac{1}{2}a_1+a_2-\frac{3}{2}a_3+2a_4-\frac{5}{2}a_5-15a_6-\frac{1}{2}a_4''+a_3''-\frac{3}{2}a_2''+2a_1''\right)x^4y^2$$

$$+\left(\frac{1}{4}a_1-\frac{1}{2}a_2+\frac{9}{8}a_3-\frac{5}{2}a_4+5a_5+15a_6+a_4''-\frac{5}{4}a_3''+\frac{7}{6}a_2''-\frac{7}{6}a_1''\right.$$
$$\left.+\frac{1}{24}a_2^{(4)}-\frac{1}{6}a_1^{(4)}\right)x^2y^4$$

$$+\left(-\frac{1}{80}a_1+\frac{1}{40}a_2-\frac{1}{20}a_3+\frac{1}{10}a_4-\frac{1}{2}a_5-a_6-\frac{1}{10}a_4''+\frac{1}{20}a_3''-\frac{7}{120}a_2''\right.$$
$$\left.+\frac{1}{16}a_1''-\frac{1}{120}a_2^{(4)}+\frac{1}{80}a_1^{(4)}\right)y^6$$

$$-\frac{a_0''y^2}{(1+x)^2}+\frac{y^4}{6(1+x)^4}\left(a_0''+\frac{1}{4}a_0^{(4)}\right)-\frac{y^6}{90(1+x)^6}\left(8a_0''+5a_0^{(4)}+\frac{1}{8}a_0^{(5)}\right),$$

$$b_r=a_1y+2a_2xy+3a_3x^2y+4a_4x^3y+5a_5x^4y+6a_6x^5y$$
$$+\left(\frac{1}{6}a_1-\frac{1}{3}a_2-a_3-\frac{1}{6}a_1''\right)y^3+\left(-\frac{1}{3}a_1+\frac{2}{3}a_2-a_3-4a_4-\frac{1}{3}a_2''+\frac{2}{3}a_1''\right)xy^3$$
$$+\left(-\frac{1}{40}a_1+\frac{1}{20}a_2-\frac{3}{20}a_3+\frac{2}{5}a_4+a_5+\frac{1}{10}a_3''-\frac{1}{10}a_2''+\frac{1}{12}a_1''+\frac{1}{120}a_1^{(4)}\right)y^5$$
$$+\left(\frac{1}{2}a_1-a_2+\frac{3}{2}a_3-2a_4-10a_5-\frac{1}{2}a_3''+a_2''-\frac{3}{2}a_1''\right)x^2y^3$$
$$+\left(\frac{1}{10}a_1-\frac{1}{5}a_2+\frac{9}{20}a_3-a_4+2a_5+6a_6+\frac{2}{5}a_4''-\frac{1}{2}a_3''+\frac{7}{15}a_2''-\frac{7}{15}a_1''\right.$$
$$\left.+\frac{1}{60}a_2^{(4)}-\frac{1}{15}a_1^{(4)}\right)xy^5$$
$$+\left(-\frac{2}{3}a_1+\frac{4}{3}a_2-2a_3+\frac{8}{3}a_4-\frac{10}{3}a_5-20a_6-\frac{2}{3}a_4''+\frac{4}{3}a_3''-2a_2''+\frac{8}{3}a_1''\right)x^3y^3$$
$$+\frac{a_0''y^3}{3(1+x)^3}-\frac{y^5}{15(1+x)^5}\left(2a_0''+\frac{1}{2}a_0^{(4)}\right),$$

$$b_\theta=a_1'xy+(-a_1'+a_2')x^2y+\left(-\frac{1}{6}a_1'-\frac{1}{3}a_2'\right)y^3+\left(\frac{1}{3}a_1'-a_3'-\frac{1}{6}a_1'''\right)xy^3$$
$$+(a_1'-a_2'+a_3')x^3y+(-a_1'+a_2'-a_3'+a_4')x^4y$$

$$+(-\frac{1}{2}a'_1+\frac{1}{3}a'_2+\frac{1}{2}a'_3-2a'_4-\frac{1}{6}a'''_2+\frac{1}{2}a'''_1)x^2y^3$$

$$+(\frac{1}{120}a'_1-\frac{1}{60}a'_2+\frac{1}{10}a'_3+\frac{1}{5}a'_4+\frac{1}{30}a'''_2-\frac{1}{60}a'''_1)y^5$$

$$+(-\frac{1}{30}a'_1+\frac{1}{15}a'_2-\frac{1}{4}a'_3+\frac{1}{5}a'_4+a'_5+\frac{1}{10}a'''_3-\frac{2}{15}a'''_2+\frac{1}{10}a'''_1+\frac{1}{120}a^{(5)}_1)xy^5$$

$$+(\frac{2}{3}a'_1-\frac{2}{3}a'_2+\frac{4}{3}a'_4-\frac{10}{3}a'_5-\frac{1}{6}a'''_3+\frac{1}{2}a'''_2-a'''_1)x^3y^3$$

$$+(a'_1-a'_2+a'_3-a'_4+a'_5)x^5y+\frac{a'_0y}{(1+x)}-\frac{a'''_0y^3}{6(1+x)^3}$$

$$+\frac{y^5}{60(1+x)^5}(2a'''_0+\frac{1}{2}a^{(5)}_0).$$

The azimuthal dependence is expressed as

$$a_i = A_{i0}+A_{i1}\sin\theta+A_{i2}\cos\theta+A_{i3}\sin 2\theta+A_{i4}\cos 2\theta$$
$$+A_{i5}\sin 3\theta+A_{i6}\cos 3\theta. \qquad (1)$$

Footnotes and References

*Work done under the auspices of the U. S. Atomic Energy Commission.

1. Andrew M. Sessler, Beta-Ray Spectrometer with Reduced Spherical Aberration, UCRL-10668, March 1963.

2. David L. Judd, A Study of the Injection Process in Betatrons and Synchrotrons (Thesis), California Institute of Technology, 1950, page 124.