

Lawrence Berkeley National Laboratory

Recent Work

Title

PHASE SHIFT and ZEROS IN $K^+ p$

Permalink

<https://escholarship.org/uc/item/2gz0h1bw>

Author

Urban, Marcel

Publication Date

1975-05-01

0 0 0 4 3 0 3 8 7 2

Submitted to Physics Letters

LBL-3842
Preprint c. 1

PHASE SHIFT AND ZEROS IN K^+p

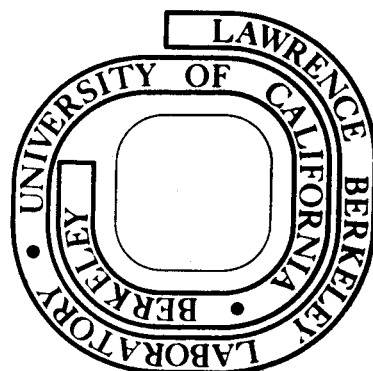
Marcel Urban

May 1975

Prepared for the U. S. Energy Research and
Development Administration under Contract W-7405-ENG-48

For Reference

Not to be taken from this room



LBL-3842
c. 1

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LBL-3842

Phase Shift and Zeros in K^+p *

Marcel Urban**

Lawrence Berkeley Laboratory
University of California
Berkeley, California

ABSTRACT

A specific example-- K^+p elastic scattering--is used to show some drawbacks of the classical phase shift analysis (PSA). We use the recently introduced method of zeros to obtain new results concerning K^+p elastic and to prove that PSA are not well fit to the study of this interaction.

*Work performed under the auspices of the U. S. Energy Research and Development Administration.

**On leave of absence from Ecole Polytechnique, Paris

Introduction

K^+p elastic is among the reactions where the amplitude analysis is the most contradictory. This does not necessarily come from the quality of the experimental data but also from the method of analysis. C. Schmid⁽¹⁾ has pointed out that K^+p elastic shows no structures and that no helpful nearby* zeros should be expected in exotic channels.

In section I we show some faults common to many phase shift analyses and in most of the K^+p ones. In section II we interpret these failures in terms of Barrelet's "zerology"^(2,3) and discover hidden weaknesses of PSA. In section III, a direct search for zeros is applied to K^+p experimental data, and it shows no evidence for the existence of any Barrelet zeros of that amplitude. Hence the presence of nearby zeros in the PSA results invalidates all the PSA quoted in this paper.

I. Abnormal Behavior of Different Sets of K^+p Phase Shift Analysis

We have examined the results of seven PSA's published by different authors. (4,5,6,7,8,9,10)** Differential cross sections and polarizations were reconstructed by means of the amplitudes, and we observed some systematic discrepancies between the smooth behavior of the experimental data and the PSA's results.

*A nearby zero is a zero whose confidence domain is located in the convergence ellipse given by the ρ pole.

**In Ref. 11 we have studied systematically these different sets of PSA. We will here only take a few examples for the sake of clarity.

- 1) Polarization often shows peaks near +1 or -1, with no counterpart in the experimental data. (Fig. 1a)
- 2) An energy variation of a few percent gives a radical change in some fits (Fig. 1b). The experimental data and other PSA's lead us to question such fast change with energy.
- 3) At momenta around and above 2 GeV/c some fits show small oscillations around a good average fit (Fig. 1c). The statistics are too poor to make such microstructures credible. Furthermore these oscillations are eliminated by using Barrelet zeros, then we resent them as an unnecessary complication of the scattering amplitudes.

The above criticisms apply "more or less" to all the phase shift solutions that we have analyzed, but we understand that it is very difficult to get rid of these errors. As we will see now, the advantage of the method of zeros is to explain these difficulties and to overcome them.

II. The Method of Zeros and the PSA in K^+p

A. Summary of the method of zeros

In parity conserving reactions: $0 + 1/2 \rightarrow 0 + 1/2$, E. Barrelet shows that one complex function A sums up all the T matrix and that the measurements of the differential cross section and of the polarization give access only to the modulus of this function. By a Joukosky mapping he transforms the $\cos\theta$ plane* into the $e^{i\theta}$ plane.

*when θ is real, it is the center of mass scattering angle.

$$\cos\theta = \frac{1}{2}\left(w + \frac{1}{w}\right); \quad w = e^{i\theta}$$

In the w plane the physical region is the entire unit circle. The amplitude, defined on this new plane, is noted: $A(w,s)$. s is the center of mass energy squared. If we call transverse cross sections the two quantities:

$$\sum_{(0<\theta<\pi)}^+ = \frac{d\sigma}{d\cos\theta} [1 + P(\cos\theta)]; \quad \sum_{(\pi<\theta<2\pi)}^- = \frac{d\sigma}{d\cos\theta} [1 - P(\cos\theta)]$$

where $d\sigma/d\cos\theta$ is the differential cross section, P is the polarization, the relation between the transverse cross section and the amplitude A is:

$$\Sigma(w) = A(w) \bar{A}(\bar{w}^{-1}); \quad \text{where } \bar{A} \text{ and } \bar{w} \text{ mean complex conjugate of } A \text{ and } w.$$

The local structure of $d\sigma/d\cos$ and of $P(\cos\theta)$ follows directly from the location of the Σ 's zeros. (See Figs. 3, 4 in Ref. 3.) In particular the local maxima or minima of the polarization curves are associated with nearby zeros (Fig. 1).

B. Origins of the inconsistencies described in section I

Fitting the data by a truncated partial wave expansion is equivalent to fitting $\Sigma(w)$ by a polynomial and consequently by a given number of zeros. Fig. 1 displays the partial wave fits to the data with the position in the w plane of the zeros which have been extracted from this fit plotted below each fit.

- 1) Polarization peaks towards +1 or -1: Fig. 1a shows that they come from zeros touching the unit circle.
- 2) Rapid variation of the fits with the energy: These variations are due to the implicit use of false zeros together with the true ones.

- 3) Oscillations: In order to reproduce the forward peak, it has been necessary to make the degree of the polynomial so high that the fits oscillate randomly for large angles where the cross section is very small. This effect introduced some nearby zeros called "statistical zeros."

C. A further study leads to the discovery of other inconsistencies

We expect that when the energy varies zeros follow regular trajectories for the following reasons:

- 1) In every reaction where zeros have been extracted, they lie on smooth trajectories. An example would be π^+p , where the data are very accurate.⁽³⁾
- 2) In the energy-dependent analysis of the BGRT group,⁽⁴⁾ the regularity criterion is automatically satisfied. Fig. 2a shows how the BGRT zeros move in the complex $\cos\theta$ plane when energy varies.

The PSA of BGRT does not carry all of the errors mentioned previously, mainly because of their smooth energy dependence, but Fig. 2a shows that they have a zero near the physical region. Such a zero gives a definite shape to the polarization which does not follow the real one, particularly in the forward region (a feature which is not shown in the figures of this article.)

Keeping this in mind one can find new errors: discontinuities appear in zero trajectories at the energies where new partial waves are introduced

in the fits;⁽¹¹⁾ the structure of zero trajectories is often over-complicated (see Fig. 2b); due to the so-called Barrelet ambiguity, zeros are easily confused with their geometrical inverse in the $e^{i\theta}$ plane, but the associated discontinuities are distinguishable from those mentioned above.

III. The Direct Search for Zeros

We have developed an original method permitting a direct determination of the zeros from the experimental data. Its main features are the following:

- 1) We use the method of moments and, instead of fitting $d\sigma/d\cos\theta$ to a truncated Legendre polynomial series and $P(\cos\theta)d\sigma/d\cos\theta$ to an associated Legendre polynomial series, that is to take a weight function constant over $\cos\theta$ within $+1$ and -1 , we vary this weight function in order to improve the fits. This takes care of the oscillation phenomenon previously seen (II-B). (It is not hard to construct polynomials, starting from a constant as the lowest, that are orthogonal with respect to a given weight function, perhaps proportional to $[d\sigma/d\cos\theta]^{-1}$.)
- 2) Once we have the coefficients of our polynomial expansion (the location of the zeros and one overall coefficient) we are faced with the question: How much confidence may we place in the location of the zeros and are there biases implicit in the method? Since the coordinates of the zeros are not linear functions of the coefficients, the answer is not trivial to give. We have solved this problem,⁽¹¹⁾ though description of the analysis will be left to another (later) publication, and found that there are indeed some biases, but they may be analyzed and corrected for.

- 3) We use conformal mapping but not in the usual way. The differences are that we apply the mapping directly to the $\cos\theta$ values, and that the leading singularity is taken at the ρ pole instead of the dipion. This was dictated by the behavior of the fits when the locations of the singularities were chosen.

In order to show clearly the different improvements brought by all these differences in fitting procedure, we keep only four zeros in the $A(w,s)$ amplitude at 1.74 GeV/c out of six or eight zeros at this energy in the PSA.

Figs. 3abc show the successive improvements, and Fig. 3d displays the corresponding zeros in the w plane. The most striking feature is that the fit improves as the zeros become further from the physical region.

Applying this direct method to the other experimental data yields the following results: at energies below 2 GeV/c the data are compatible with the absence of zeros inside an ellipse given by the ρ pole; this configuration of zeros gives evidence that the data can be fitted as well with effective* poles as with these unstable zeros.⁽¹¹⁾ Consequently we claim that it is unwise to parameterize the K^+p amplitude by a polynomial as PSA does.

Conclusion

A non-exhaustive comparison between PSA and the method of zeros is given in Table 1.

*not necessarily on the real axis

Table 1

PHASE SHIFT	ZEROS
Obtained with the no-singularity model	Requires no model
Introduces unknown errors so that estimators are biased and information is partly lost	Errors are known so that estimators are unbiased and more information is kept
Not local, so that all $[-1, +1]$ must be covered by measurements	Local, so that only parts of $[-1, +1]$ need to be measured
Nearby and unstable zeros are implicitly mixed	Nearby and unstable zeros are separated
The relationship between parameters (phase shifts, etc.) and observed quantities is complicated and not particularly direct	The positions of zeros have direct relationships to the observables that are readily visualized
Data must be taken at energies that are close together because interpolation is tricky	Data can be taken at energies that are far apart with very high statistics

The method of zeros explains PSA's errors, easily seen on the fits, but reveals new ones. The use of this new method on K^+p 's experimental data shows that there is no evidence for nearby zeros below 2 GeV/c. This fact leads to two consequences: PSA are unwise to parameterize the K^+p amplitude by a polynomial; there is now a new argument against the formation of an exotic resonance in K^+p with spin greater than $1/2$, for in that case we would see $2(J - 1/2)$ characteristic nearby zeros (J : spin of this resonance).

The author wishes to express his appreciation to Owen Chamberlain, E. Barrelet, and D. Chew for helpful discussions and suggestions concerning this work.

REFERENCES

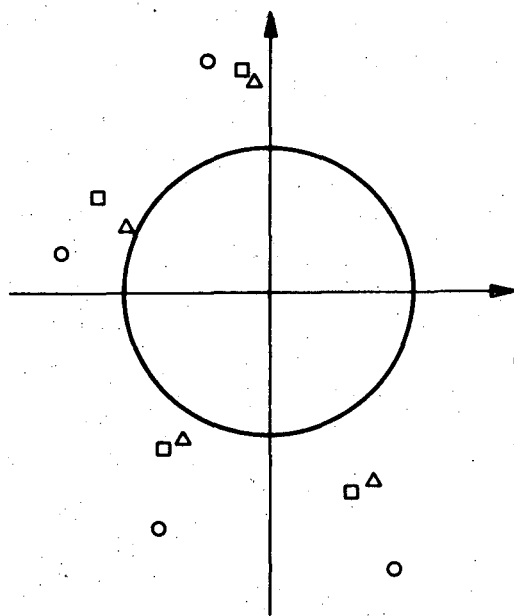
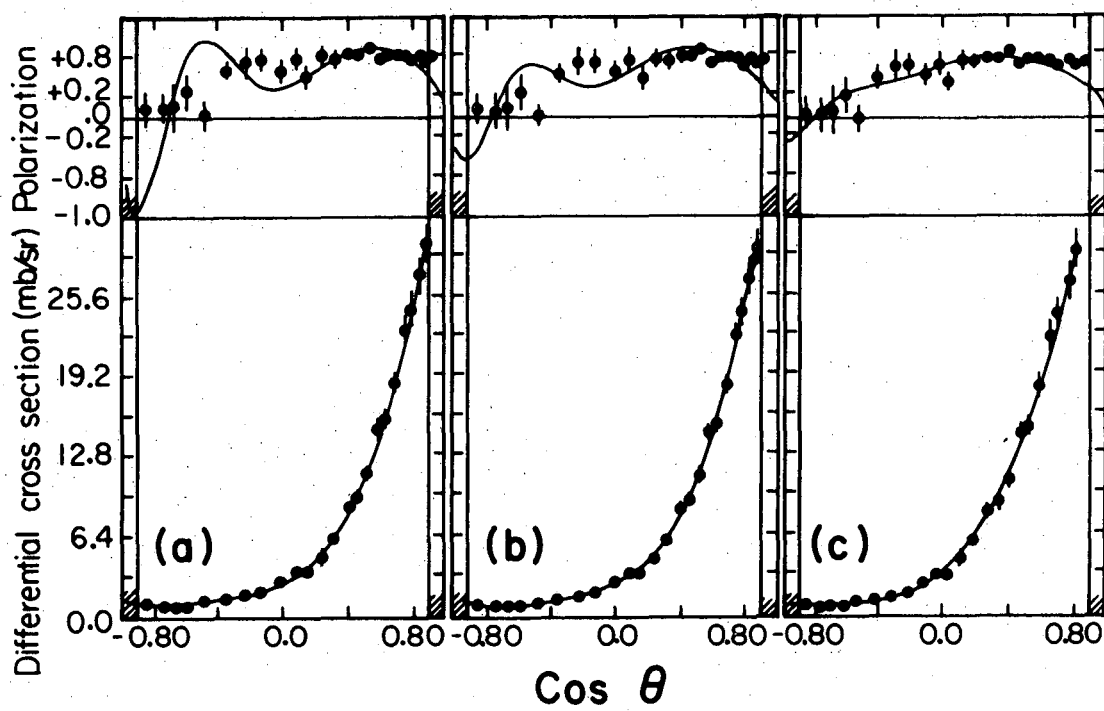
1. C. Schmid at Amsterdam International Conference (1971).
2. E. Barrelet, thesis, Paris 1970.
3. E. Barrelet, Nuovo Cimento, 8A, 331 (1972).
4. Giacomelli et al., Nucl. Phys. 20B, 301 (1970), referred to as BGRT.
5. Cutkosky and Deo, Phys. Rev. 7D, 2547 (1970).
6. Asbury et al., Phys. Rev. Lett. 24, 615 (1970).
7. Rebka et al., Phys. Rev. Lett. 24, 160 (1970).
8. Lovelace and Wagner, preprint, TH 1250, CERN (1970).
9. Albrow et al., Nucl. Phys. B30, 273 (1971).
10. Miller et al., Nucl. Phys. B37, 401 (1971).
11. Urban, thesis, Paris (1973).
12. Ehrlich et al., Phys. Rev. Lett. 26, 927 (1971).
13. Hughes et al., Kiev Conference, 1970.

FIGURE CAPTIONS

- Fig. 1. (a) The fit is the 1B solution of Ref. 8 at $P_{\text{lab}} = 1256 \text{ MeV/c}$, and the data points come from Ref. 13. Below this are shown the zeros of this fit in the w plane. The polarization's peak toward +1 comes from the zero on the unit circle.
- (b) Fits (from solution 1 of Ref. 7) to the polarization at 2.00 GeV (full curve) and 2.02 GeV (dashed curve) in the center of mass with the corresponding zeros in the w plane (■ and ● respectively).
- (c) The fit is the α -solution of Ref. 9 at $P_{\text{lab}} = 2500 \text{ MeV/c}$, and the data points come from Ref. 12. Below this are shown the nearest zeros of this solution in the w plane. The oscillations of the fit come from the implicit use of statistical zeros as true ones.
- Fig. 2. (a) Zeros' trajectories of Ref. 4 in the complex $\cos\theta$ plane. The ellipse corresponds to the ρ pole and to the highest energy of this fit. Arrows go from low to high energies. A cross (dot) means that the zero has a positive (negative) imaginary part in the complex $e^{i\theta}$ plane. Notice the rectilinear character of the trajectories.
- (b) Same as for (a) but the zeros are from Ref. 10.
- Fig. 3. Data points from Ref. 9.
- (a) Fit with a weight function equal to 1 on $\cos\theta$ (Legendre Polynomial for $d\sigma/d\cos\theta$).

- (b) Fit with the same polynomial degree but with a weight function different from 1 on $[-1, +1]$ in $\cos\theta$ plane.
- (c) Fit using conformal mapping on $\cos\theta$ and an appropriate weight function.
- (d) The zeros of the fits in the $e^{i\theta}$ plane
 - a) (Δ), b) ($\square$), c) (o).

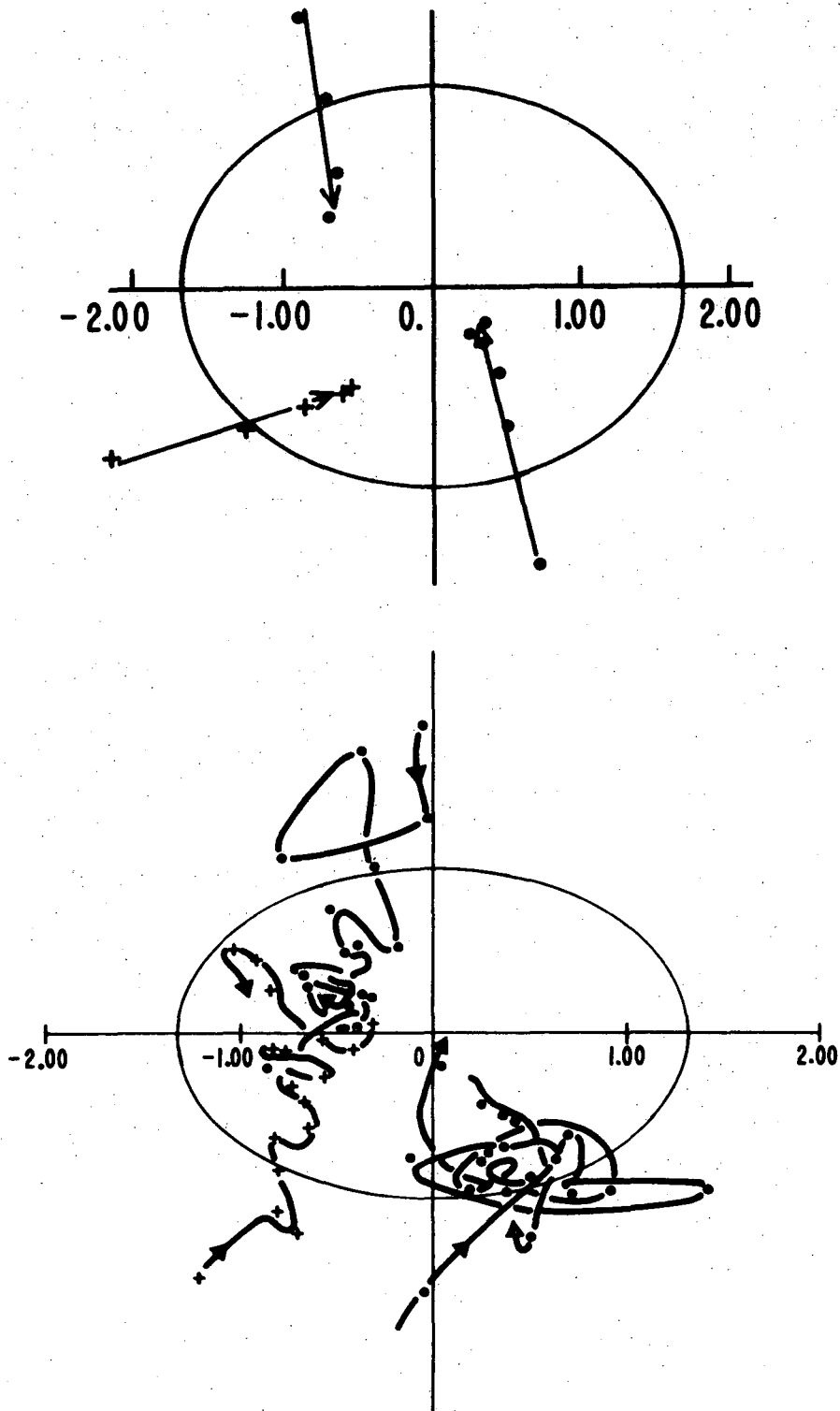
Notice that as the fit improves the corresponding zeros go away from the unit circle.



$P_{lob} = 1.74 \text{ GeV/c}$
(d)

XBL754-2627

Fig. 1



XBL 754 - 2628

Fig. 2

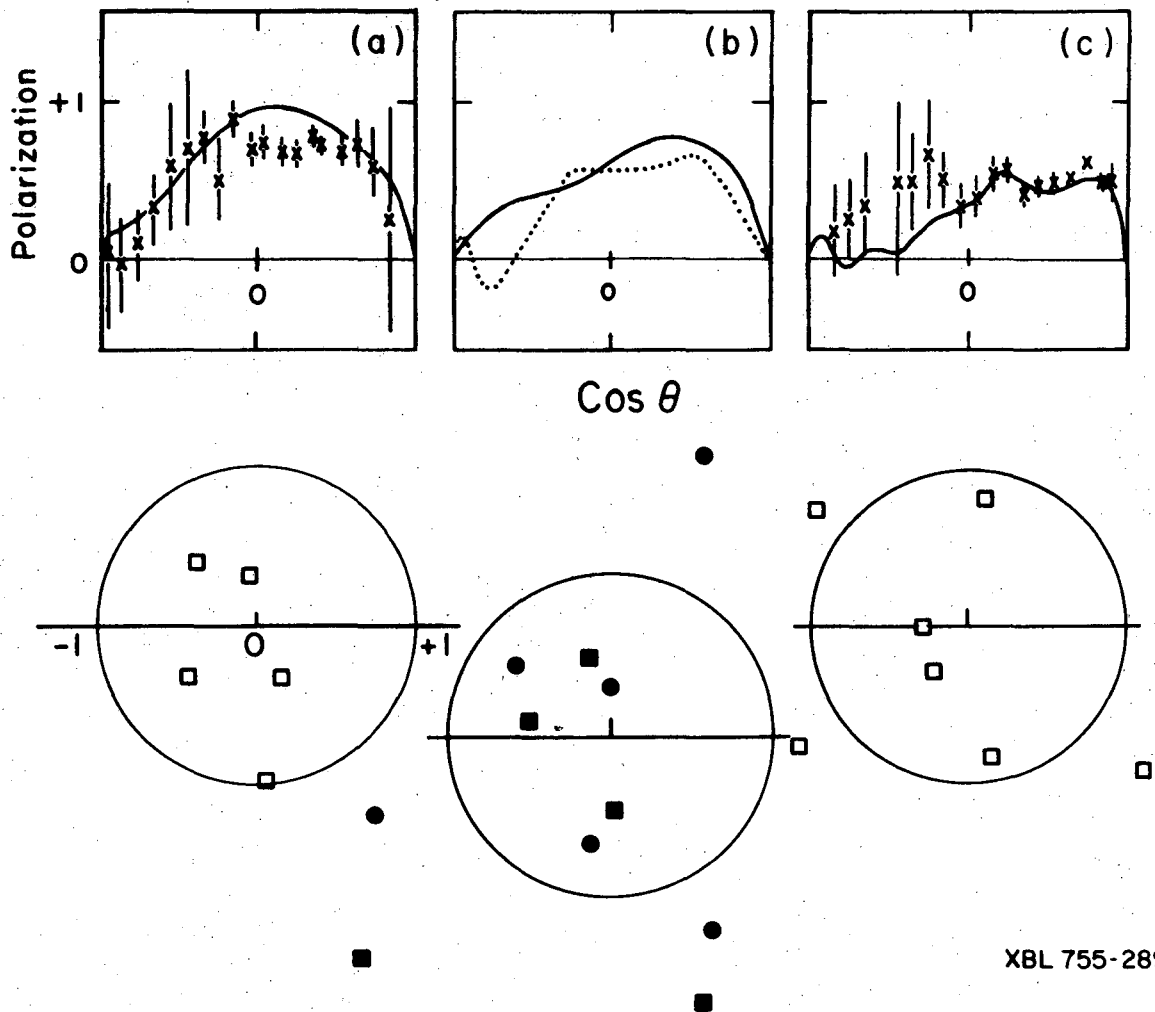


Fig. 3

LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720