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A General Methodology for Decoupling Damped Linear Systems

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Abstract

It has long been recognized that coordinate coupling in damped linear systems is a considerable barrier to analysis and design. In the absence of viscous damping, a linear system possesses classical normal modes, which constitute a linear coordinate transformation that decouples the undamped system. This process of decoupling the equation of motion of a dynamical system is a time-honored procedure termed modal analysis. A viscously damped linear system cannot be decoupled by modal analysis unless it also possesses a full set of classical normal modes, in which case the system is said to be classically damped. Rayleigh showed that a system is classically damped if its damping matrix is a linear combination of its inertia and stiffness matrices. Classical damping is routinely assumed in design and computations. Practically speaking, classical damping means that energy dissipation is almost uniformly distributed throughout the system. In general, this condition is not satisfied and thus damped linear systems cannot be decoupled by modal analysis.

The purpose of this presentation is to report on a recently developed methodology to extend classical modal analysis to decouple any damped linear system (no restrictions). This method is based upon a new theory of phase synchronization, which compensates for time drifts caused by viscous damping and external excitation. A fast algorithm for decoupling any damped linear system is also described.

Keywords: Linear systems, modal analysis, coordinate coupling, viscous damping.

1. INTRODUCTION

The equation of motion of an n -degree-of-freedom damped linear system can be written as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}(t), \quad (1)$$

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where for passive systems the mass matrix M , the damping matrix C , and the stiffness matrix K are real, symmetric and positive definite of order n . The Lagrangian coordinate q and the excitation $\mathbf{f}(t)$ are real n -dimensional column vectors. Unless M , C and K are diagonal, Eq. (1) is coupled, i.e., the i th component equation involves not only q_i and its derivatives but also other coordinates and their derivatives as well. The purpose of this paper is to present a general methodology to transform Eq. (1) into

$$\ddot{\mathbf{p}} + \mathbf{D}_1 \dot{\mathbf{p}} + \mathbf{\Omega}_1 \mathbf{p} = \mathbf{g}(t) \tag{2}$$

for which $\mathbf{D}_1, \mathbf{\Omega}_1$ are real diagonal matrices of order n , and \mathbf{p} and $\mathbf{g}(t)$ are also real. This is a well-trodden problem that has attracted the attention of many researchers in the past century. Over the years, various types of decoupling approximation were employed in the analysis of damped systems. Different indices of coupling were also introduced to quantify coordinate coupling. However, a general technique to decouple Eq. (1) has not been reported in the open literature.

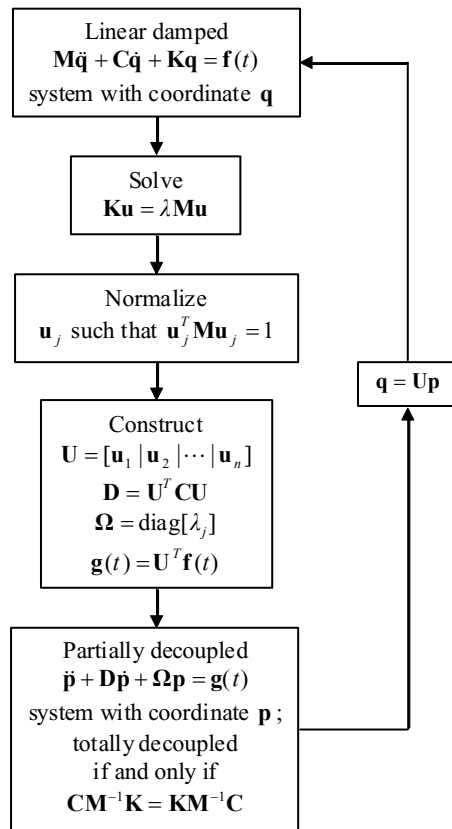


Figure 1: Flowchart for classical modal analysis.

2. DECOUPLING BY MODAL ANALYSIS

When $\mathbf{C} = \mathbf{0}$, Eq. (1) can be readily decoupled by classical modal analysis, which utilizes a real congruence transformation to diagonalize M and K simultaneously. If $\mathbf{C} \neq \mathbf{0}$, the system is said to be classically damped if it can still be decoupled by modal analysis. Rayleigh (1894) showed that

proportional damping, for which $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$, is a particular case of classical damping. Subsequently, Caughey and O’Kelly (1965) established that $\mathbf{CM}^{-1}\mathbf{K} = \mathbf{KM}^{-1}\mathbf{C}$ is a necessary and sufficient condition under which a system is classically damped. There is, of course, no particular reason why this condition should be satisfied. The situation is depicted in Fig. 1.

To be sure, Eq. (1) can always be recast as a first-order equation of dimension $2n$ in state space. If the eigenvalue problem associated with the resulting state equation is non-defective, the state equation can be decoupled by complex modal analysis. Upon decoupling, however, the (complex) state variables can no longer be identified as displacements and velocities. Physical insight is thus greatly diminished. This is an important reason why configuration-space decoupling, if possible, is truly preferred.

3. DECOUPLING BY PHASE SYNCHRONIZATION

Based upon a consideration of the physics of damping, classical modal analysis has been extended to decouple any damped linear systems in configuration space. The extension was expounded in two recent papers by Ma, Morzfeld and Imam (2009; 2010). It utilizes a new and intuitive theory of phase synchronization, which shifts the phase angles in each non-classically damped mode of vibration so as to transform it into a classical mode. The overall decoupling transformation is real, invertible, nonlinear and time-dependent in configuration space. In free or forced vibration, all parameters required for the construction of decoupled systems and decoupling transformations are obtained through the solution of the quadratic eigenvalue problem (Lancaster 1966; Gohberg *et al.* 1982; Tisseur and Meerbergen 2001)

$$(\mathbf{M}\lambda^2 + \mathbf{C}\lambda + \mathbf{K})\mathbf{v} = \mathbf{0}. \quad (3)$$

There are competing methods for solving the above equation, and many software packages offer built-in functions to tackle Eq. (3). For example, MATLAB provides the “polyeig” function for solving polynomial eigenvalue problems. Upon solution of the quadratic eigenvalue problem (3), the diagonal matrices \mathbf{D}_1 , $\mathbf{\Omega}_1$ in Eq. (2) can be expressed as simple combinations of the eigenvalues of Eq. (3). If each repeated eigenvalue possesses a full complement of independent eigenvectors, Eq. (3) is non-defective. Under this condition,

$$\mathbf{g}(t) = \mathbf{T}_1^T \mathbf{f}(t) + \mathbf{T}_2^T \dot{\mathbf{f}}(t), \quad (4)$$

$$\mathbf{q}(t) = \mathbf{T}_1 \mathbf{p}(t) + \mathbf{T}_2 \dot{\mathbf{p}}(t) - \mathbf{T}_2 \mathbf{T}_2^T \mathbf{f}(t), \quad (5)$$

where \mathbf{T}_1 , \mathbf{T}_2 are real square matrices whose elements are simple combinations of the eigenvalues and corresponding eigenvectors of Eq. (3). A flowchart for fast decoupling is presented in Fig. 2.

As given in detail in two recent papers (Ma *et al.* 2009; 2010), the initial conditions of the original Eq. (1) and the decoupled Eq. (2) are connected by

$$\begin{bmatrix} \mathbf{p}(0) \\ \dot{\mathbf{p}}(0) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{\Lambda} & \overline{\mathbf{\Lambda}} \end{bmatrix} \begin{bmatrix} \mathbf{V} & \overline{\mathbf{V}} \\ \mathbf{V}\mathbf{\Lambda} & \overline{\mathbf{V}\mathbf{\Lambda}} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{q}(0) \\ \dot{\mathbf{q}}(0) \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{T}_2^T \mathbf{f}(0) \end{bmatrix}. \quad (6)$$

In the configuration space, the decoupling transformation (5) is a real, nonlinear, time-dependent transformation. This contrasts clearly with the time-invariant modal transformation in classical modal analysis. However, the above decoupling process and associated equations reduce to classical modal analysis if Eq. (1) is undamped or classically damped.

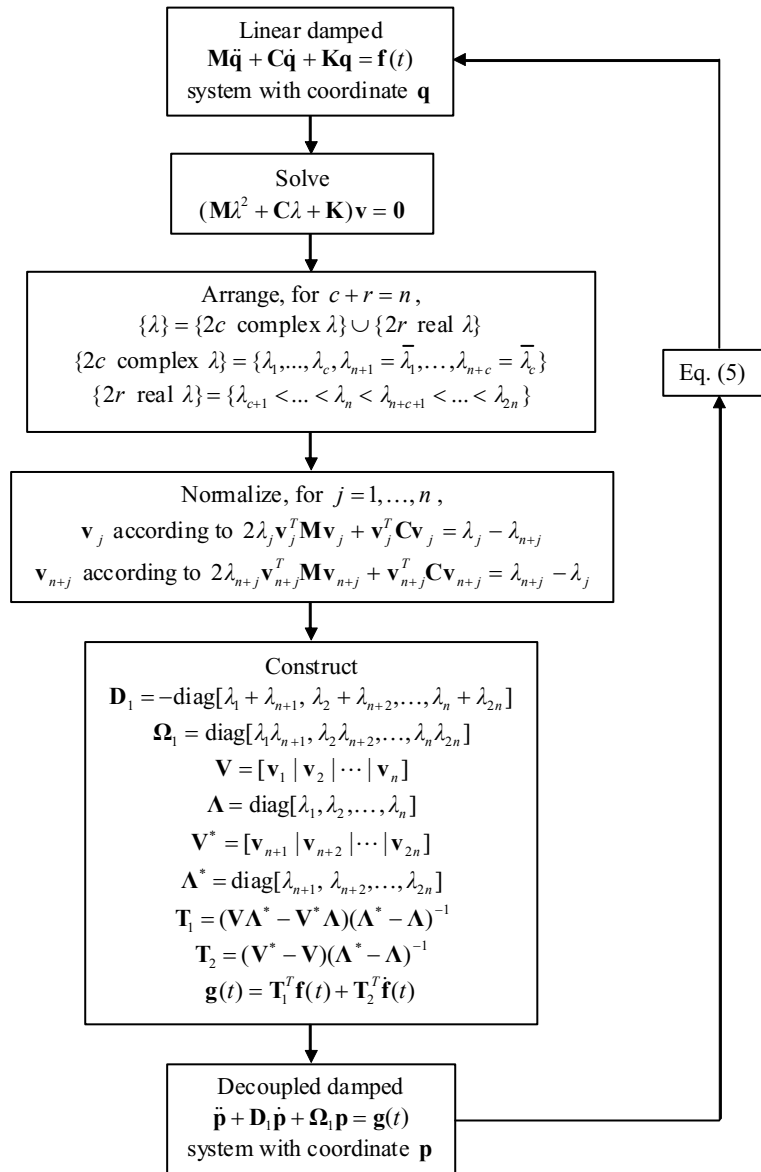


Figure 2: Algorithm for decoupling any damped linear systems when the eigenvalues of Eq. (3) are distinct.

4. CONCLUSIONS

Several observations about the decoupling of damped linear systems by the new technique of phase synchronization are summarized in the following remarks.

1. In free or forced vibration, all parameters required for the decoupling of a linear system are obtained through the solution of a quadratic eigenvalue problem.

2. Any viscously damped linear system (no restrictions) can be completely decoupled by phase synchronization. A flowchart for fast decoupling is given in Fig. 2. The assumption that the eigenvalues of Eq. (3) are distinct is made to streamline the flowchart and can be readily relaxed.
3. Under classical damping, the decoupling methodology presented herein reduces to classical modal analysis. Similar to modal analysis, decoupling by phase synchronization possesses ample physical insight and it also lends itself to efficient numerical computations.

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