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Z-mediated flavor-changing neutral currents and their implications for CP asymmetries in B^0 decays

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In models where the quark sector is extended in a nonsequential way, there are Z-mediated flavor-changing neutral currents at the tree level. These may dominate mixing in neutral-B systems. Unitarity of the three-generation Cabibbo-Kobayashi-Maskawa matrix is violated, and new phases take part in quark mixing. All these effects modify significantly the predictions for CP asymmetries in B^0 decays. The various aspects of new physics can be probed separately by testing specific relations among these CP asymmetries.

I. INTRODUCTION

CP asymmetries in B^0 decays[1] constitute a very useful probe into physics beyond the standard model (SM). They test those aspects of the quark sector which are most likely to show signs of new physics: CP violation, neutral-meson-mixing, and unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Moreover, these CP asymmetries provide us with *separate* tests of the various ingredients of the SM.[2,3] Consequently, inconsistencies with the SM will give precise information as to which of the SM ingredients have to be superceded, and what kind of new physics may be responsible for such inconsistencies.

In a previous work[2] we gave a general discussion of how the various assumptions of the SM can be tested separately. In the present work we analyze in detail a specific model within which relevant ingredients of the SM are modified. The purpose of this analysis is twofold: First, we explicitly show how substantial inconsistencies with the SM may arise; second, we demonstrate how the various aspects of new physics may be separately discovered through appropriately chosen relations among CP asymmetries.

Within the SM, CP asymmetries in B^0 decays measure the three angles of the unitarity triangle. In order to understand how these measurements may be modified with new physics, we looked for a model where unitarity of the three-generation CKM matrix is violated. Models with a sequential fourth generation became less likely after the recent measurements of the invisible width of the Z boson. This led us to choose a model where the quark sector is extended in a nonsequential way. Specifically, we study a model with an $SU(2)_L$-singlet quark of charge $-\frac{1}{3}$. Such an extension of the quark sector arises in several widely studied models, and in particular E_6 grand unified theories and string-inspired models.[4]

The model has several features that make it most interesting for the study of CP asymmetries in B^0 decays. Sizable corrections to the unitarity of the 3×3 CKM matrix are allowed by present experimental constraints. The "unitarity triangle" may turn into a "unitarity quadrangle." The violation of unitarity of the three-generation CKM matrix induces flavor-changing neutral currents (FCNC's) which are mediated, at the tree level, by the Z boson. This allows, in particular, a new mechanism in the mixing of B_d and B_s mesons.[5] There are three independent phases in the full (3×4) mixing matrix and all of these may take part in this mechanism and consequently in the asymmetries. As we shall see, the new mixing mechanism is the most important new ingredient, and its implications for CP asymmetries in B^0 decays could be rather dramatic.

The structure of this paper is as follows: In Sec. II we present the model and explain why it implies FCNC at the tree level. In Sec. III we study the experimental constraints on Z-mediated FCNC's. In Sec. IV we describe the implications of these FCNC's on various aspects of physics which are relevant to CP asymmetries in B^0 decays. We find how large the elements of the neutral-current mixing matrix should be to make interesting modifications of the SM predictions. In Sec. V we make further assumptions on the structure of the mixing matrix and find the range of parameters which is consistent with the constraints of Sec. III and with the requirements of Sec. IV. In Sec. VI we calculate the CP asymmetries in this model, and in Sec. VII we show how they differ from the SM predictions. We give our conclusions in Sec. VIII.

II. THE MODEL

We study a model with an extended quark sector. In addition to the three standard generations of quarks, there is an $SU(2)_L$ singlet of charge $-\frac{1}{3}$. For our purposes, the important feature of this model is that it allows for Z-mediated FCNC's. To understand how these FCNC's arise, its is convenient to work in the basis where the up sector interaction eigenstates are identified with the mass eigenstates. The down sector interaction eigenstates are then related to the mass eigenstates by a 4×4 unitary matrix which we denote by K.

The charged-current interactions are described by

$$\mathcal{L}_{\rm int}^{W}=\frac{g}{\sqrt{2}}(W_\mu^{-1}J^{\mu^+}+W_\mu^{+}J^{\mu^-}) ,$$
$$J^{\mu-}=V_{ij}\bar{u}_{iL}\gamma^\mu d_{jL} . \tag{2.1}$$

The charged-current mixing matrix V is a 3×4 submatrix of K:

$$V_{ij}=K_{ij} \quad \text{for } i=1,\ldots,3,\ \ j=1,\ldots,4 . \tag{2.2}$$

Note that V is parametrized by six real angles and *three* phases, instead of three angles and one phase in the original CKM matrix. As we shall see, all three phases may take part in the determination of CP asymmetries in B^0 decays.

The neutral current interactions are described by

$$\mathcal{L}_{\rm int}^{Z}=\frac{g}{\cos\theta_W}Z_\mu(J^{\mu3}-\sin^2\theta_W J_{\rm EM}^{\mu}) ,$$
$$J^{\mu3}=-\tfrac{1}{2}U_{pq}\bar{d}_{pL}\gamma^\mu d_{qL}+\tfrac{1}{2}\delta_{ij}\bar{u}_{iL}\gamma^\mu u_{jL} . \tag{2.3}$$

The neutral-current mixing matrix for the down sector is $U=V^\dagger V$. As V is not unitary, $U\neq1$. In particular, its nondiagonal elements do not vanish:

$$U_{pq}=-K_{4p}^*K_{4q} \quad \text{for } p\neq q . \tag{2.4}$$

The three elements that are relevant for our study are

$$U_{ds}=V_{ud}^*V_{us}+V_{cd}^*V_{cs}+V_{td}^*V_{ts} ,$$
$$U_{db}=V_{ud}^*V_{ub}+V_{cd}^*V_{cb}+V_{td}^*V_{tb} , \tag{2.5}$$
$$U_{sb}=V_{us}^*V_{ub}+V_{cs}^*V_{cb}+V_{ts}^*V_{tb} .$$

The fact that, unlike the SM, the various U_{pq} do not necessarily vanish, allows FCNC at the tree level. As we shall see, this may substantially modify the analysis of CP asymmetries.

III. EXPERIMENTAL CONSTRAINTS ON THE U_{pq} ELEMENTS

There are several experimental results that give useful constraints[5,6] on the flavor-changing couplings of the Z boson (U_{pq}):

(i) ΔM_K, the mass difference between the neutral kaons; (ii) ϵ, the CP-violating parameter in the K system; (iii) $B(K_L\to\mu^+\mu^-)$; (iv) $B(B\to l^+l^-X)$; and (v) x_d, the mixing parameter in the B system.

In this section, we calcualte the bounds from these measurements. Our assumption is that there are no large cancellations among various contributions. Thus, rather than attempting to subtract other known contributions [such as long-distance contributions to ΔM_K or $B(K_L\to\mu^+\mu^-)$], we only require that the contribution from the Z-mediated FCNC is not larger than the experimental value (where we allow a one sigma error). For each measurement we quote values of relevant parameters that are either poorly determined or only recently measured. The values that we use for all other parameters are as given in Ref. 7.

A. K-K mixing

The contribution from the Z-mediated tree-level diagram to the mass difference between the neutral kaons is

$$(\Delta M_K)_Z=\frac{\sqrt{2}G_FB_Kf_K^2M_K\eta_1}{6}|\mathrm{Re}[(U_{ds})^2]| . \tag{3.1}$$

The uncertainty in the vacuum-insertion approximation is given by

$$\tfrac{1}{3}\le B_K\le1 . \tag{3.2}$$

Requiring $(\Delta M_K)_Z\le\Delta M_K$, we get

$$|\mathrm{Re}[(U_{ds})^2]|\le4.1\times10^{-7} . \tag{3.3}$$

B. The ϵ parameter

The contribution of the Z-mediated tree-level diagram to $|\epsilon|$ is

$$|\epsilon|_Z=\frac{G_FB_Kf_K^2M_K\eta_1}{12\Delta M_K}|\mathrm{Im}[(U_{ds})^2]| . \tag{3.4}$$

The only uncertain parameter here is B_K, given in Eq. (3.2). Requiring $|\epsilon|_Z\le|\epsilon|$, we get

$$|\mathrm{Im}[(U_{ds})^2]|\le2.6\times10^{-9} . \tag{3.5}$$

C. $K_L\to\mu^+\mu^-$

The contribution of the Z-mediated tree-level diagram to the branching ratio of this mode, normalized to that of the W-mediated $K^+\to\mu^+\nu$ decay, is given by

$$\frac{B(K_L\to\mu^+\mu^-)_Z}{B(K^+\to\mu^+\nu)}=2\left[\frac{\tau(K_L)}{\tau(K^+)}\right][(\tfrac{1}{2}-\sin^2\theta_W)^2+(\sin^2\theta_W)^2]\left[\frac{(\mathrm{Re}U_{ds})^2}{|V_{us}|^2}\right] . \tag{3.6}$$

We use[8]

$$B(K_L\to\mu^+\mu^-)=(7.3\pm0.7)\times10^{-9} . \tag{3.7}$$

Requiring

$$B(K_L\to\mu^+\mu^-)_Z\le B(K_L\to\mu^+\mu^-) ,$$

we get

$$|\mathrm{Re}U_{ds}|\le2.4\times10^{-5} . \tag{3.8}$$

D. $B\to l^+l^-X$

The contribution of the Z-mediated tree-level diagrams to the branching ratio of this mode, normalized to that of the W-mediated semileptonic decay, is given by

$$\frac{B(B\to l^+l^-X)}{B(B\to l\nu X)}=[(\tfrac{1}{2}-\sin^2\theta_W)^2+(\sin^2\theta_W)^2]\times\left[\frac{|U_{db}|^2+|U_{sb}|^2}{|V_{ub}|^2+F_{\rm PS}|V_{cb}|^2}\right], \tag{3.9}$$

where $F_{\rm PS}\approx 0.5$ is a phase-space factor. Both $b\to s$ and $b\to d$ transitions contribute to the Z-mediated decay. Experimentally[9]

$$B(B\to l^+l^-X)_{l=e,\mu}\le 2.4\times 10^{-3}\ . \tag{3.10}$$

Requiring that the Z contribution will not exceed this upper bound gives

$$\begin{aligned}&|U_{db}/V_{cb}|\le 0.2\ ,\\ &|U_{sb}/V_{cb}|\le 0.2\ .\end{aligned} \tag{3.11}$$

E. B-$\bar{B}$ mixing

The contribution from the Z-mediated tree-level diagram to the mixing between the neutral B mesons is

$$(x_d)_Z=\frac{\sqrt{2}G_FB_Bf_B^2M_B\eta\tau_b}{6}|U_{db}|^2\ . \tag{3.12}$$

There are three parameters with large uncertainties:

$$\begin{aligned}&x_d=0.66\pm0.11\ ,\\ &\tau_b|V_{cb}|^2=(3.5\pm0.6)\times10^9\ \mathrm{GeV}^{-1}\ ,\\ &\sqrt{B_B}f_B=0.15\pm0.05\ \mathrm{GeV}\ .\end{aligned} \tag{3.13}$$

Requiring $(x_d)_Z\le x_d$, we get

$$|U_{db}/V_{cb}|\le 0.047\ . \tag{3.14}$$

To summarize, measurements of FCNC's give the following bounds on the nondiagonal Z couplings:

$$\begin{aligned}&|\mathrm{Re}U_{ds}|\le 2.4\times10^{-5}\ ,\\ &|\mathrm{Im}U_{ds}|\le \min\{6.4\times10^{-4},\ 1.3\times10^{-9}/|\mathrm{Re}U_{ds}|\}\ ,\end{aligned} \tag{3.15}$$

where we combined the limits in Eqs. (3.3), (3.5), and (3.8), and

$$|U_{db}/V_{cb}|\le 0.047, |U_{sb}/V_{cb}|\le 0.2\ , \tag{3.16}$$

where we used the limits in Eqs. (3.11) and (3.14).

IV. IMPLICATIONS OF Z-MEDIATED FCNC

If the U_{pq} elements are not negligibly small, they will affect many aspects of physics related to CP asymmetries in B decays:

(i) Contributions to neutral-meson mixing from tree-level diagrams, (ii) violation of the unitarity of the 3×3 CKM matrix, (iii) contributions to B decays from Z-mediated diagrams, and (iv) contributions to the width difference Γ_{12} between the neutral B mesons from Z-mediated decays.

We study the four subjects in turn. The first two effects may give interesting deviations from the SM predictions for CP asymmetries. We will choose the mixing parameters to lie within the range in which such effects indeed occur. The last two effects may spoil the cleanliness of the theoretical interpretation of the experimental results. We will check whether our choice of parameters still allows a clean interpretation.

A. Mixing of neutral mesons

The experimentally measured values of mixing in the K and B systems can be explained by SM processes. Still, the uncertainties in the theoretical calculations (such as in the values of B_K, f_B, or V_{td}) allow a situation where SM processes do not give the dominant contributions to various mixing processes. Instead, it is possible that the dominant mechanism is Z-mediated FCNC's. We will now find how large should the elements of the neutral-current mixing matrix be in order that this would be the case.

If K-$\bar{K}$ mixing is to be accounted for by Z-mediated tree-level diagrams, then Eq. (3.1) and (3.2) give

$$|\mathrm{Re}[(U_{ds})^2]|\ge 1.4\times10^{-7}\ . \tag{4.1}$$

If B_d-$\bar{B}_d$ mixing is to be accounted for by Z-mediated tree-level diagrams, then Eqs. (3.12) and (3.13) give

$$|U_{db}/V_{cb}|\ge 0.017\ . \tag{4.2}$$

The various x_q $(q=d,s)$ also get contributions from SM box diagrams:

$$(x_q)_{\rm box}=\frac{G_F^2M_W^2B_Bf_B^2M_B\eta\tau_b}{6\pi^2}\times[y_tf_2(y_t)|V_{tq}^*V_{tb}|^2+|U_{qb}|^2]\ , \tag{4.3}$$

where $y_t\equiv(m_t^2/M_W^2)$ and the function $f_2(y_t)$ can be found, for example, in Ref. 7. The condition that the tree-level diagram dominates is

$$\frac{(x_q)_Z}{(x_q)_{\rm box}}=\frac{\sqrt{2}\pi^2}{G_FM_W^2}\frac{1}{y_tf_2(y_t)}\frac{|U_{qb}|^2}{|V_{tq}^*V_{tb}|^2}=\frac{185}{y_tf_2(y_t)}\frac{|U_{qb}|^2}{|V_{tq}^*V_{tb}|^2}\ge 1\ . \tag{4.4}$$

For $m_t\sim 100$ GeV, $y_tf_2(y_t)\sim 1$, in which case we find[10]

$$|U_{qb}/(V_{tq}^*V_{tb})|\ge 0.08\ . \tag{4.5}$$

The model has interesting effects on CP asymmetries in B^0 decays if the mechanism for mixing in the B_q systems is Z-mediated FCNC's. Thus, we require

$$\begin{aligned}&|U_{db}/V_{cb}|\ge 0.017\ ,\\ &|U_{db}/(V_{td}^*V_{tb})|\ge 0.08\ ,\\ &|U_{sb}/(V_{ts}^*V_{tb})|\ge 0.08\ .\end{aligned} \tag{4.6}$$

Whether K-$\bar{K}$ mixing is affected by the new physics will follow from these and other assumptions, as will be explained in Sec. V.

B. Unitarity of the 3×3 CKM matrix

Within the SM, unitarity of the three-generation CKM matrix gives

$$\begin{aligned}
\mathcal{U}_{ds} &\equiv V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0 \ , \\
\mathcal{U}_{db} &\equiv V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0 \ , \\
\mathcal{U}_{sb} &\equiv V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0 \ .
\end{aligned} \tag{4.7}$$

Within the model under study, where the quark sector is extended to include an SU(2) singlet of charge $-\frac{1}{3}$, Eq. (2.5) shows that these constraints are replaced by

$$\mathcal{U}_{ds} = U_{ds}, \quad \mathcal{U}_{db} = U_{db}, \quad \mathcal{U}_{sb} = U_{sb} \ . \tag{4.8}$$

One can gain a quantitative understanding of how badly the SM relations of Eq. (4.7) are violated by comparing the U_{pq}'s to the largest terms in the respective sums:

$$\begin{aligned}
&|U_{ds}|/|V_{ud}^* V_{us}| \le 1.4 \times 10^{-4} \ , \\
&|U_{db}|/|V_{cd}^* V_{cb}| \le 0.22 \ , \\
&|U_{sb}|/|V_{cs}^* V_{cb}| \le 0.2 \ .
\end{aligned} \tag{4.9}$$

These bounds follow from the experimental bounds given in Sec. III. The first of the SM relations is practically maintained. The other two may be violated quite significantly. The unitarity triangle of the SM, representing the $\mathcal{U}_{db}=0$ constraint, should be replaced by a unitarity quadrangle. The $\mathcal{U}_{sb}$ constraint is now represented by a unitarity triangle.[11] A geometrical presentation of the new relations is given in Fig. 1. It should be stressed that, at present, only the *magnitudes* of U_{db} and U_{sb} are experimentally constrained, but not their phases. Each of the angles α', β', and δ' could be anywhere in the range $[0,2\pi]$.

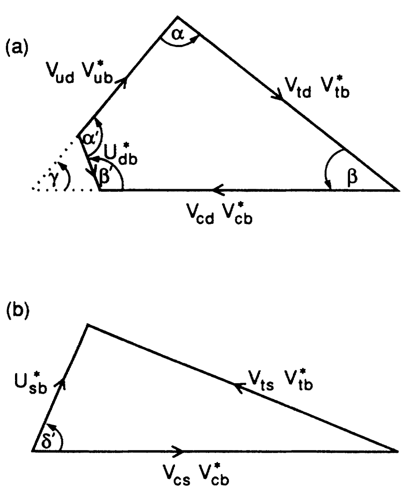

FIG. 1. A geometrical representation of the unitarity constraints in Eq. (4.8). (a) The unitarity quadrangle representing the $\mathcal{U}_{db}$ constraint. (b) The unitarity triangle representing the $\mathcal{U}_{sb}$ constraint.

C. Z-mediated B decays

Our main interest is in hadronic B^0 decays to CP eigenstates, where the quark subprocess is $\bar{b} \to \bar{u}_i u_i \bar{d}_j$, with $u_i = u, c$ and $d_j = d, s$. These processes get additional contributions from Z-mediated FCNC's. The ratio between the magnitudes of the Z-mediated amplitude and the W-mediated amplitude is

$$\left(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W\right) \left| \frac{U_{jb}^*}{V_{ij} V_{ib}^*} \right| \approx \frac{1}{3} \left| \frac{U_{jb}^*}{V_{ij} V_{ib}^*} \right| \ . \tag{4.10}$$

To bound this ratio, we use the experimental constraints in Eq. (3.16), the requirement that mixing of B_q mesons be dominated by Z-mediated FCNC's in Eq. (4.6), and the range $0.06 \le |V_{ub}/V_{cb}| \le 0.16$. We get

$$0.04 \le \frac{1}{3} \left| \frac{U_{db}^*}{V_{ud} V_{ub}^*} \right| \le 0.26 \ , \tag{4.11}$$

$$0.03 \le \frac{1}{3} \left| \frac{U_{sb}^*}{V_{cs} V_{cb}^*} \right| \le 0.07 \ . \tag{4.12}$$

The lower bound in Eq. (4.12) holds only if unitarity is not strongly violated, so that $|V_{ts}| \approx |V_{cb}|$ and $|V_{tb}| \approx 1$. We note that the different color structure between the two amplitudes may further modify the ratio between their contributions, but we will assume that this does not give a significant effect. We conclude that the Z-mediated diagram never dominates the relevant B decays. It can be safely neglected for $b \to s$ transitions (3–7 % effect), but may be significant for $b \to d$ (4–26 %). We will assume that the Z contribution is on the lower side of these ranges, and thus can be ignored.

In Ref. 3, CP asymmetries in B_q decays through $\bar{b} \to \bar{s} s \bar{d}_j$ (with $d_j = d, s$) were studied. These decays cannot proceed via charged-current interactions. Within the SM the direct decay is dominated by penguin diagrams. In the model under study there are contributions from the Z-mediated FCNC. The ratio between the Z-mediated amplitude and the penguin amplitude can be estimated to be

$$\frac{\left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right)|U_{jb}^*|}{(\alpha_s/12\pi)\ln(m_t^2/m_b^2)|V_{tj} V_{tb}^*|} \approx 30 \left| \frac{U_{jb}^*}{V_{tj} V_{tb}^*} \right| \ . \tag{4.13}$$

(To have a rough estimate of the penguin diagram we used $\mathcal{U}_{sb} \sim 0$, $m_t \sim 100$ GeV and $\alpha_s \sim 0.1$.) The bound in Eq. (4.6) implies that if Z-mediated FCNC's are to dominate neutral B mixing, then their contribution to the $\bar{b} \to \bar{s} s \bar{d}_j$ processes is comparable to the penguin diagrams: CP asymmetries in these processes may arise from interference between the two direct decays and, therefore, the tests proposed in Ref. 3 are not applicable in this framework.

D. New contributions to $\Gamma_{12}(B_q)$

The difference in width comes from decay modes that are common to B_q and $\bar{B}_q$. As discussed above, there are new contributions to such decay modes from Z-mediated FCNC's. It is important to note, however, that while the contributions to the difference in mass M_{12} are from the tree-level diagrams, namely, $O(g^2)$, those to the difference in width Γ_{12} are still of $O(g^4)$. Consequently, no signifiant enhancement of the SM value for Γ_{12} is expected, and the relation

$$\Gamma_{12}(B_q) \ll M_{12}(B_q) \tag{4.14}$$

is maintained.

In summary, the dominant mechanism for mixing in neutral B_q systems could be Z-mediated FCNC's. The conditions for that are given in Eq. (4.6). Mixing in the neutral-K system can be dominated by Z-mediated FCNC's if Eq. (4.1) is satisfied. But the hadronic B decays of relevance are still dominated by SM W-mediated diagrams and $\Gamma_{12}(B_q) \ll M_{12}(B_q)$, so that CP asymmetries can be cleanly interpreted.

V. CHOOSING THE MIXING ANGLES

We find it convenient to use an explicit parametrization for the mixing matrices. We use the parametrization of Ref. 12 (appropriately modified to the 3×4 case). Assuming that all mixing angles θ_{ij} are small,[13] we put $\cos\theta_{ij}=1$. We use the following constraints from SM tree-level processes and from the unitarity of K:

$$\begin{aligned} &s_{12}=0.22, \quad s_{23}\approx 0.04, \quad s_{13}\approx 0.005 , \\ &s_{14}\lesssim 0.07, \quad s_{24}\lesssim 0.5 \end{aligned} \tag{5.1}$$

($s_{ij}\equiv\sin\theta_{ij}$). We further assume that the unmeasured mixing angles satisfy the hierarchy $s_{14}<s_{24}<s_{34}$. More specifically, we assume that

$$\begin{aligned} &q_{24}\equiv s_{24}/(s_{23}s_{34}) , \\ &q_{14}\equiv s_{14}/(s_{12}s_{23}s_{34}) \end{aligned} \tag{5.2}$$

are both ~ 1. We remind the reader that a similar relation for the three-generation mixing angles is experimentally verified:[14]

$$q_{13}\equiv s_{13}/(s_{12}s_{23})=0.50\pm0.25 . \tag{5.3}$$

Thus, V has the approximate form

$$V=\begin{bmatrix} 1 & s_{12} & s_{13}e^{-\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ -s_{12} & 1 & s_{23} & s_{24}e^{-i\delta_{24}} \\ s_{12}s_{23}-s_{13}e^{i\delta_{13}} & -s_{23} & 1 & s_{34} \end{bmatrix} . \tag{5.4}$$

We gave here only the leading term for each element. In our calculations, we use more exact expressions, e.g., $V_{ts}=-s_{23}-s_{12}s_{13}e^{i\delta_{13}}-s_{34}s_{24}e^{i\delta_{24}}$, when relevant. Using

$$\begin{aligned} &K_{41}=-s_{12}s_{23}s_{34}(1-q_{13}e^{i\delta_{13}}-q_{24}e^{i\delta_{24}}+q_{14}e^{i\delta_{14}}) , \\ &K_{42}=s_{23}s_{34}(1-q_{24}e^{i\delta_{24}}) , \\ &K_{43}=-s_{34} \end{aligned} \tag{5.5}$$

we find

$$\begin{aligned} U_{ds}&=s_{12}s_{23}^2s_{34}^2[(1-q_{13}e^{-i\delta_{13}}-q_{24}e^{-i\delta_{24}} \\ &\qquad +q_{14}e^{-i\delta_{14}})(1-q_{24}e^{i\delta_{24}})] , \\ U_{db}&=-s_{12}s_{23}s_{34}^2[1-q_{13}e^{-i\delta_{13}}-q_{24}e^{-i\delta_{24}}+q_{14}e^{-i\delta_{14}}] , \\ U_{sb}&=s_{23}s_{34}^2[1-q_{24}e^{-i\delta_{24}}] . \end{aligned} \tag{5.6}$$

All the experimental constraints in Eqs. (3.15) and (3.16) as well as the requirements in Eq. (4.6) can be satisfied with the terms in square brackets in Eq. (5.6) being ~ 1 and

$$0.08 \lesssim s_{34}^2 \lesssim 0.2 . \tag{5.7}$$

If we take $s_{34}^2 \sim 0.15$, *the dominant mechanism for B_d mixing and B_s mixing will be the Z-mediated FCNC's.* On the other hand, we expect $\mathrm{Im}\,U_{ds}$ to be of the same order of magnitude as $\mathrm{Re}\;U_{ds}$. Consequently, Eq. (4.1) is not satisfied, so that *ΔM_K gets no significant contributions from the Z-mediated FCNC's.*

Equation (5.6) implies that the phases in the mixing of B_d and B_s may depend on phases of the mixing matrix other than the single phase of the SM. This may give CP asymmetries very different from those predicted by the SM.

VI. CP ASYMMETRIES IN B DECAYS

Our study involves six classes of processes for which the direct decay is dominated by the W-mediated tree-level diagram. The asymmetries, given in Table I, are denoted by $\mathrm{Im}\lambda_{iq}$. The subindex $i=1,2,3$ denotes the quark subprocess. The subindex $q=d,s$ denotes the type of decaying meson B_q. The list of hadronic final states gives examples only. Other states may be experimentally more favorable, and several such states may be summed over to improve statistics. We always quote the CP asymmetry for CP even states, regardless of the specific hadronic states listed.

As explained in Sec. IV, the two following conditions are satisfied within the model of Z-mediated FCNC's.

(a) For the neutral-B system, $\Gamma_{12} \ll M_{12}$.

(b) The direct decays are dominated by a single combination of CKM parameters. This means that the asymmetries arise dominantly from the interference of ampli-

TABLE I. Classes of CP asymmetries.

Class (iq)	Quark subprocess	Decaying meson	Final state (example)
$1d$	$\bar b\to\bar c c\bar s$	B_d	ψK_S
$2d$	$\bar b\to\bar c c\bar d$	B_d	D^+D^-
$3d$	$\bar b\to\bar u u\bar d$	B_d	$\pi^+\pi^-$
$1s$	$\bar b\to\bar c c\bar s$	B_s	$D_s^+D_s^-$
$2s$	$\bar b\to\bar c c\bar d$	B_s	ψK_S
$3s$	$\bar b\to\bar u u\bar d$	B_s	ρK_S

tudes corresponding to two paths to the same final state, one of which involves B^0-$\bar{B}^0$ mixing.

Consequently, the decay rate of a time-evolved initially pure B_q^0 ($\bar{B}_q^0$) into a CP eigenstate f is (see Ref. 15, and references therein):

$$\begin{aligned}\Gamma(B^0_{q\,\mathrm{phys}}(t)\to f) &\propto e^{-\Gamma t}[1-\mathrm{Im}\lambda_{iq}\sin(\Delta M_q t)]\ ,\\ \Gamma(\bar{B}^0_{q\,\mathrm{phys}}(t)\to f) &\propto e^{-\Gamma t}[1+\mathrm{Im}\lambda_{iq}\sin(\Delta M_q t)]\ ,\end{aligned}\tag{6.1}$$

where ΔM_q is the mass difference in the B_q system and λ_{iq} is of the form ($|\lambda_{iq}|=1$)

$$\lambda_{iq}=\left[\frac{X_i}{X_i^*}\right]\left[\frac{Y_q}{Y_q^*}\right]\left[\frac{Z_{iq}}{Z_{iq}^*}\right]\ .\tag{6.2}$$

The X_i factor depends on the quark-subprocess amplitude. As the quark subprocesses are dominated by the W-mediated tree-level diagrams:

$$\begin{aligned}X_1&\equiv(\bar{b}\to\bar{c}c\bar{s})=V_{cb}V_{cs}^*\ ,\\ X_2&\equiv X(\bar{b}\to\bar{c}c\bar{d})=V_{cb}V_{cd}^*\ ,\\ X_3&\equiv X(\bar{b}\to\bar{u}u\bar{d})=V_{ub}V_{ud}^*\ .\end{aligned}\tag{6.3}$$

The Y_q factor depends on the mixing amplitude of the decaying meson. As the dominant mechanism of mixing in the B_q system is the Z-mediated tree-level diagram:

$$\begin{aligned}Y_d&\equiv Y(B_d)=U_{db}^*\ ,\\ Y_s&\equiv Y(B_s)=U_{sb}^*\ .\end{aligned}\tag{6.4}$$

The Z_{iq} factor depends on the K-$\bar{K}$ mixing amplitude. Z_{iq} can have a nonzero phase only for those asymmetries where there is a single unpaired neutral kaon in the final state, and depends on whether this comes from a K^0 or a $\bar{K}^0$. As K-$\bar{K}$ mixing is dominated by the SM box diagram with virtual c quarks:

$$\begin{aligned}Z_{2d}&=Z_{3d}=Z_{1s}=1\ ,\\ Z_{1d}&=Z_{2s}^*=Z_{3s}^*=V_{cd}^*V_{cs}\ .\end{aligned}\tag{6.5}$$

It is now straightforward to calculate the λ_{iq}'s. We find that the various asymmetries simply measure angles of the unitarity quadrangles shown in Fig. 1:

$$\begin{aligned}&\mathrm{Im}\lambda_{1d}=\mathrm{Im}\lambda_{2d}=\sin2\beta'\ ,\\ &\mathrm{Im}\lambda_{3d}=-\sin2\alpha'\ ,\\ &\mathrm{Im}\lambda_{1s}=\mathrm{Im}\lambda_{2s}=\sin2\delta'\ ,\\ &\mathrm{Im}\lambda_{3s}=\sin2(\delta'-\gamma)\ .\end{aligned}\tag{6.6}$$

The three angles which within the SM constitute the angles of the unitarity triangle are defined by

$$\begin{aligned}&\alpha\equiv\arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right],\quad \beta=\arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right],\\ &\gamma\equiv\arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]\ .\end{aligned}\tag{6.7}$$

The three angles in Eq. (6.6) connected to the U_{qb} elements are defined by

$$\begin{aligned}&\alpha'\equiv\arg\left[\frac{V_{ud}V_{ub}^*}{U_{db}^*}\right],\quad \beta'\equiv\arg\left[\frac{U_{db}^*}{V_{cd}V_{cb}^*}\right],\\ &\delta'\equiv\arg\left[\frac{U_{sb}^*}{V_{cs}V_{cb}^*}\right]\ .\end{aligned}\tag{6.8}$$

A comparison between the predictions of the SM and of the model with Z-mediated FCNC's, is made in Table II. *The important feature is not that the angles α, β, and γ, defined in Eq. (6.7), may have very different values from those predicted by the SM, but rather that the CP asymmetries do not measure these angles any more.*

As there are no experimental constraints on α', β', and δ' so that the full range $[0,2\pi]$ is allowed for each of them, the full range $[-1,+1]$ is possible for each of the asymmetries. This is clearly seen when using the explicit parametrization given in Eqs. (5.4) and (5.6):

$$\begin{aligned}\mathrm{Im}\lambda_{1d}&=\mathrm{Im}\lambda_{2d}\\ &=\mathrm{Im}\left[\frac{1-q_{13}e^{i\delta_{13}}-q_{24}e^{i\delta_{24}}+q_{14}e^{i\delta_{14}}}{1-q_{13}e^{-i\delta_{13}}-q_{24}e^{-i\delta_{24}}+q_{14}e^{-i\delta_{14}}}\right],\\ \mathrm{Im}\lambda_{3d}&=\mathrm{Im}\left[\frac{e^{-i\delta_{13}}(1-q_{13}e^{i\delta_{13}}-q_{24}e^{i\delta_{24}}+q_{14}e^{i\delta_{14}})}{e^{i\delta_{13}}(1-q_{13}e^{-i\delta_{13}}-q_{24}e^{-i\delta_{24}}+q_{14}e^{-i\delta_{14}})}\right],\end{aligned}\tag{6.9}$$

$$\begin{aligned}\mathrm{Im}\lambda_{1s}&=\mathrm{Im}\lambda_{2s}=\mathrm{Im}\left[\frac{1-q_{24}e^{i\delta_{24}}}{1-q_{24}e^{-i\delta_{24}}}\right],\\ \mathrm{Im}\lambda_{3s}&=\mathrm{Im}\left[\frac{e^{-i\delta_{13}}(1-q_{24}e^{i\delta_{24}})}{e^{i\delta_{13}}(1-q_{24}e^{-i\delta_{24}})}\right]\ .\end{aligned}$$

Is is rather obvious that our ignorance of the phases δ_{14} and δ_{24} allows any value for the various asymmetries. Thus, it should be interesting to examine whether any of the SM predictions for these asymmetries are maintained, and whether precise information can be gained from those predictions that are violated. The answer to these questions is given in the next section.

TABLE II. Predictions for CP asymmetries.

Class (iq)	$U_{pq}\neq0$ (FCNC)	$U_{pq}=0$ (SM)
$1d$	$\sin2\beta'$	$-\sin2\beta$
$2d$	$\sin2\beta'$	$-\sin2\beta$
$3d$	$-\sin2\alpha'$	$\sin2\alpha$
$1s$	$\sin2\delta'$	0
$2s$	$\sin2\delta'$	0
$3s$	$\sin2(\delta'-\gamma)$	$-\sin2\gamma$

VII. COMPARISON WITH THE SM PREDICTIONS

Equation (6.2) gives the relations among the various λ_{iq}:

$$\arg\lambda_{1d}-\arg\lambda_{2d}-\arg\lambda_{1s}+\arg\lambda_{2s}=0 ,$$
$$\arg\lambda_{1d}-\arg\lambda_{3d}-\arg\lambda_{1s}+\arg\lambda_{3s}=0 , \quad (7.1)$$
$$\arg\lambda_{2d}-\arg\lambda_{3d}-\arg\lambda_{2s}+\arg\lambda_{3s}=0 .$$

These relations can be experimentally checked. They should hold within both the SM and the framework of an extended quark sector with parameters as chosen in Sec. V. This is because the two conditions sufficient for these relations [namely, conditions (a) and (b), given above Eq. (6.1)] hold in both models. If the experimental results are inconsistent with these relations then, most likely, there are two comparable contributions to the direct decay, namely either the Z-mediated amplitude or the penguin amplitude have their matrix element enhanced, so that they compete with the W-mediated diagram. If this happens in only one of the relevant quark sub-processes then the one relation that is independent of that process will still hold, e.g., the first relation in Eq. (7.1) if only $\bar{b}\to\bar{u}u\bar{d}$ has significant interference between two direct decays.

These two conditions can be tested in other ways as well. If there is a significant interference between two direct decay amplitudes, then the time dependence of the decay rate is different from that given in Eq. (6.1).[16] Also, various hadronic states, corresponding to the same quark subprocess, are likely to exhibit different CP asymmetries.

The dominant mechanisms for direct decays (W-mediated tree-level diagrams) and for K-K mixing (box diagrams with virtual c quarks) are the same in the present model and in the SM. Consequently, Eq. (6.3) for X_i and Eq. (6.5) for Z_{iq} hold in both frameworks. This gives

$$\arg Z_{1q}+\arg X_1=\arg Z_{2q}+\arg X_2 . \quad (7.2)$$

Equation (6.2) then gives

$$\mathrm{Im}\lambda_{1d}=\mathrm{Im}\lambda_{2d},\quad \mathrm{Im}\lambda_{1s}=\mathrm{Im}\lambda_{2s} . \quad (7.3)$$

These relations between the asymmetries can be experimentally checked. As explained in Ref. 2, their validity goes far beyond our specific model, and is practically guaranteed by the constraints from the measurement of the ϵ parameter.

The mechanism for mixing in the B_q systems is different in the model under study from that of the SM. Consequently, several relations among CP asymmetries predicted by the SM will be violated.

Within the SM, $Y_s=V_{ts}V_{tb}^*$ and $\mathcal{U}_{sb}=0$. Assuming the SM tree level decays, these are sufficient conditions for

$$\mathrm{Im}\lambda_{1s}=\mathrm{Im}\lambda_{2s}=0 . \quad (7.4)$$

In the model of an extended quark sector, $Y_s=U_{sb}^*$ and consequently $\mathrm{Im}\lambda_{1s}$ and $\mathrm{Im}\lambda_{2s}$ are as given in Eq. (6.9). By an appropriate choice of q_{24} and δ_{24}, it is possible to get any value within

$$-1\le \mathrm{Im}\lambda_{1s}=\mathrm{Im}\lambda_{2s}\le +1 . \quad (7.5)$$

Instead of the SM prediction of zero asymmetry, we could have, for example, maximal asymmetry (with $q_{24}e^{i\delta_{24}}=1+e^{i\pi/4}$).

We note, however, that if for some reason $\delta_{24}=0$ or $q_{24}\ll 1$, the asymmetries in classes (1s) and (2s) will vanish, as they do in the SM. The reason is that in this case U_{sb}^* and $V_{cs}V_{cb}^*$ carry the same phase (mod π), just as $V_{ts}V_{tb}^*$ and $V_{cs}V_{cb}^*$ do within the SM. The only property of the mixing mechanism probed by CP asymmetries is its phase structure, and thus different mechanisms carrying the same phase cannot be distinguished.

Within the SM, $Y_d=V_{td}V_{tb}^*$ and $\mathcal{U}_{db}=0$. Assuming the SM tree level decays, these are sufficient conditions for

$$\mathrm{Im}\lambda_{3d}=\sin 2\alpha ,$$
$$\mathrm{Im}\lambda_{1d}=\mathrm{Im}\lambda_{2d}=-\sin 2\beta . \quad (7.6)$$

(For a recent study of the SM predictions for α and β, see Ref. 17.) In the model of an extended quark sector, $Y_d=U_{db}^*$ and consequently $\mathrm{Im}\lambda_{id}$ are as given in Eq. (6.9). By an appropriate choice of the various q_{ij}'s and δ_{ij}'s, it is possible to get any value within

$$-1\le \mathrm{Im}\lambda_{id}\le +1 . \quad (7.7)$$

Thus, for example, instead of the SM prediction of negative $\mathrm{Im}\lambda_{1d}$, we could have zero or positive asymmetry.

To summarize, the SM prediction given in Eq. (7.1) is likely to hold within the extended model. It will imply that indeed the direct B decay is dominated by a single combination of CKM parameters. The SM prediction given in Eq. (7.3) is also likely to hold. It will imply that the phase in K-$\bar{K}$ mixing is the same as in the SM. The SM prediction given in Eq. (7.4) is likely to be violated. It will strongly indicate a new mechanism for mixing in the B_s system. The SM predictions given in Eq. (7.6) are also likely to be violated. This will strongly indicate a new mechanism for mixing in the B_d system.

VIII. CONCLUSIONS

In a model where the quark sector is extended in a nonsequential way, the following features arise.

(a) There are new phases in quark mixing, in addition to the single phase of the SM. These give new sources for CP violation.

(b) Unitarity of the 3×3 CKM matrix is violated. The constraint represented by the "unitarity triangle" no longer holds, and new constraints appear.

(c) FCNC's mediated by the Z boson arise at the tree level. These may be the dominant mechanism for mixing in the neutral B systems.

On the other hand, the following ingredients of the SM are likely to be maintained.

(a) Direct B decays are dominated by W-mediated tree-level amplitudes.

(b) $\Gamma_{12}\ll M_{12}$ for both B_d and B_s.

As a result of these properties, predictions for CP asymmetries in B^0 decays are significantly modified from the SM predictions. The results can be cleanly interpreted within the new framework.

Within the SM, the asymmetries measure angles in the complex plane between various combinations of elements of the charged-current mixing matrix, as those determine both b decays and B_q-$\bar{B}_q$ mixing. These angles are calculated within the SM on the basis of direct measurements and unitarity of the CKM matrix. Within the model of extended quark sector, unitarity of the charged-current mixing matrix is lost, but this is not the reason for the asymmetries being modified. The reason is rather that, when B_q-$\bar{B}_q$ mixing is dominated by the Z-mediated FCNC's, the asymmetries measure different quantities, namely, angles between combinations of elements of the charged-current mixing matrix determining b decays and elements of the neutral-current mixing matrix determining B_q-$\bar{B}_q$ mixing.

The CP asymmetries may depend on all three phases that appear in the 3×4 mixing matrix. With current mild bounds on the new mixing angles and no knowledge of the new phases, it is possible to have any value for the various asymmetries. Consequently, CP asymmetries that are dramatically different from the SM predictions may arise. For example, the mode $B_s \to D_s^+ D_s^-$ may exhibit maximal asymmetry instead of the zero asymmetry predicted by the SM.

The richness of experimental results will enable us to disentangle the various ingredients of the new framework. As direct decays are still dominated by a single combination of CKM parameters, certain relations among asymmetries are maintained. Similarly, as K-$\bar{K}$ mixing is dominated by the SM mechanism, certain asymmetries corresponding to different quark subprocesses remain equal. On the other hand, the existence of new mechanisms for B_d and for B_s mixing can be proven separately of each other and independently of whether unitarity is violated.

If the CKM model is not the complete picture of the quark sector, CP asymmetries in B^0 decays will constitute a powerful probe into the nature of its extension.

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