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# Three-generation neutrino mixing and LSND dark matter neutrinos

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The reported signal at the LSND experiment, when interpreted as neutrino mixing with $\delta m^2 = 6$ eV$^2$, provides evidence for neutrinos with a cosmologically significant mass. However, attempts to reconcile this interpretation of the experiment with other hints about neutrino properties require a (sterile) fourth neutrino and/or an "inverted" neutrino mass hierarchy. An interpretation of the LSND experiment employing $\delta m^2 = 0.3$ eV$^2$, with three-generation mixing and a "normal" neutrino mass hierarchy, can just barely be reconciled with the negative results of other laboratory neutrino oscillation experiments and the positive hints of neutrino oscillation from the solar and atmospheric neutrino problems. Though subject to test by by future experiments, such a solution allows (but does not demand) neutrino masses relevant for dark matter.

## 1. ACCOMODATING LSND DARK MATTER NEUTRINOS: "UNNATURAL" SCHEMES

Based on the first year of data, a possible signal in the LSND experiment at Los Alamos has been reported [1]. Results from the second year of data confirm this signal [2]. An early and oft-quoted interpretation of this signal was that it represents $\overline{\nu}_e \leftrightarrow \overline{\nu}_\mu$ mixing, with $\delta m^2 \approx 6$ eV$^2$ and $\sin^2 2\theta \approx 5\times 10^{-3}$. The motivations for the selection of these mixing parameters are twofold: (1) compatibility with the negative results of other accelerator or reactor experiments (see Fig. 3 of Ref. [1]); and (2) the cosmologically interesting neutrino mass of 2.5 eV, which fits well into cold+hot dark matter models that seem to satisfy many observational constraints on large-scale structure formation [3].

Many attempts to reconcile the LSND result with neutrino oscillation solutions to the solar and atmospheric neutrino problems have been made [4]. These have at least one of two "unnatural" features: a fourth neutrino, and/or an inverted neutrino mass hierarchy.

A fourth neutrino is required when three different independent mass differences are used, *e.g.*, $\delta m^2_{\rm LSND} \approx 6$ eV$^2$, $\delta m^2_{\rm atmos} \approx 10^{-2}$ eV$^2$, and $\delta m^2_{\rm solar} \approx 10^{-5}$ eV$^2$ or $10^{-10}$ eV$^2$. From measurements of the width of the $Z_0$ boson, it is known that only three neutrino species participate in electroweak interactions [5]. Therefore, a fourth neutrino would have to be "sterile," having no standard model interactions. Since a $\nu_\mu \leftrightarrow \nu_s$ solution to the atmospheric neutrino problem is restricted by big-bang nucleosynthesis considerations [6], the sterile neutrino typically makes its appearance in a $\nu_e \leftrightarrow \nu_s$ small-angle MSW solution to the solar neutrino problem.

An inverted neutrino mass hierarchy is one in which the neutrino mass eigenvalue most closely associated with the $\nu_e$ is heavier than those associated with the $\nu_\mu$ or $\nu_\tau$, or the mass eigenvalue associated with the $\nu_\mu$ is heavier than that associated with the $\nu_\tau$. The requirement of an inverted hierarchy arises from considerations of supernova $r$-process nucleosynthesis [7]. For the typical mixing parameters associated with the LSND experiment cited above, an MSW resonant transformation increases the average energy of the $\nu_e$ population in the post core-bounce supernova environment. This drives the material around the supernova remnant proton-rich, precluding synthesis of the $r$-process elements. This problem does not occur with an inverted mass hierarchy in which the $\nu_e$ is the heaviest neutrino. In this case, an MSW resonant transformation increases the average energy of the $\overline{\nu}_e$ population, enhancing the neutron-rich conditions required for successful $r$-process nucleosynthesis.

Should the existence of sterile neutrinos or an inverted neutrino mass hierarchy be designated


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"unnatural"? After all, we expect the existence of particles beyond the standard model, and we have no solid reason to assume that the generational mass pattern of the neutrinos should follow that of the corresponding charged leptons. The relevant point is that before assuming that the positive hints of neutrino mixing—LSND and the solar and atmospheric neutrino problems—point to things significantly different than what is expected from the standard model, schemes more in harmony with our experience from the standard model should be pushed to their limits first.

## 2. ACCOMODATING LSND NEUTRINOS: A "NATURAL" SCHEME

We seek to construct a "natural" scheme of neutrino mass and mixing that reconciles all known hints about neutrino properties. Such a "natural" scheme would involve only three neutrino flavors, and would not have an inverted neutrino mass hierarchy. We would expect such a solution to take advantage of the possibilities of three-generation mixing, rather than simply consisting of a set of two-flavor neutrino mixings that have been "stitched together." In particular, the atmospheric neutrino problem could involve *both* $\nu_\mu \leftrightarrow \nu_e$ and $\nu_\mu \leftrightarrow \nu_\tau$ mixing.

In using only three neutrinos, one is immediately faced with the problem mentioned earlier, *i.e.* three independent mass differences. To accomodate a neutrino mixing solution to the solar neutrino problem, a small mass difference is required: $\delta m^2_{\rm solar} \approx 10^{-5}$ eV$^2$ or $10^{-10}$ eV$^2$ for MSW and "just-so" vacuum oscillation solutions, respectively [8]. These scales for the mass difference cannot be altered much. In the case of the MSW solution, a mass level crossing is the basis of the whole effect, and so the mass difference is essentially determined by solar parameters. The "just so" vacuum oscillation solution requires that the earth-sun distance be on the order of the neutrino oscillation length; thus the mass difference for this solution is determined as well.

There may be a little more freedom in the neutrino mass differences used to explain the LSND signal and the atmospheric neutrino problem, however. In particular, we consider the possibility of explaining *both* LSND and the atmospheric neutrino data with a single common mass difference in the range $\delta m^2_{\rm atmos,\,LSND} \approx 0.2 - 0.4$ eV$^2$. This is an order of magnitude *smaller* than the mass difference usually associated with the LSND signal, and an order of magnitude *larger* than that usually associated with atmospheric neutrinos.

Is the use of a common mass difference for LSND and atmospheric neutrinos feasible? Fig. 3 of Ref. [1] shows that $\delta m^2_{\rm atmos,\,LSND} \approx 0.2 - 0.4$ eV$^2$ is compatible with the negative results of other accelerator and reactor experiments. However, the claimed zenith-angle dependance of the atmospheric neutrino data seem to imply a smaller mass difference, $\delta m^2_{\rm atmos} \approx 10^{-2}$ eV$^2$, with a 90% C.L. upper limit of about 0.1 eV$^2$ [9]. We note that the statistical significance of this fit has been questioned [10]. According to another analysis, of the Kamiokande multi-GeV data, $\delta m^2_{\rm atmos} \geq 0.25$ eV$^2$ is excluded at 90% C.L., and $\delta m^2_{\rm atmos} \geq 0.47$ eV$^2$ is excluded at 95% C.L. [11]. Thus $\delta m^2_{\rm atmos,\,LSND} \approx 0.3$ eV$^2$ would be allowed at 95% C.L.

Next we turn to a discussion of the mixing angles. The neutrino flavor eigenstates, $\nu_\alpha$, can be written as a linear combination of mass eigenstates, $\nu_i$:

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i, \tag{1}$$

where $U_{\alpha i}$ are elements of a unitary mixing matrix $U$. We take $U$ to be a standard parametrization of the Cabbibo-Kobayashi-Maskawa (CKM) matrix involving three mixing angles and a CP violating phase [12].

The mass differences indicated above satisfy $\delta m^2_{\rm solar} \ll \delta m^2_{\rm atmos,\,LSND}$. A useful approximation in this case is the "one mass scale dominance" (OMSD) limit, in which we neglect $\delta m^2_{\rm solar}$ relative to $\delta m^2_{\rm atmos,\,LSND}$. The discussion is considerably simplified in this limit, as the results from two-flavor interpretations of neutrino mixing experiments can be directly converted to the three-neutrino OMSD interpretation [13]. For two-neutrino vacuum oscillations, the survival probability $P$ is given by

$$P = 1 - \sin^2 2\theta \sin^2\left(1.27 \frac{\delta m^2 L}{E}\right), \tag{2}$$

where $L$ is the path length (in km) of a neutrino initially in a flavor eigenstate at $L = 0$, $E$ is the neutrino energy in GeV, and $\delta m^2$ ($= \delta m^2_{\rm atmos,\,LSND}$) is in eV. The two-flavor mixing angle is $\theta$. In the OMSD limit, we have the following correspondence between the two-flavor mixing angle and the elements of the three-flavor mixing matrix:

$$\sin^2 2\theta \Leftrightarrow \quad 4|U_{\alpha 3}|^2|U_{\beta 3}|^2 \qquad \text{(app.)}, \tag{3}$$

$$\sin^2 2\theta \Leftrightarrow \quad 4|U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2) \qquad \text{(disapp.)}; \tag{4}$$

for appearance and disappearance experiments, respectively. For the parametrization of $U$ in Ref. [12], we have

$$|U_{e3}|^2 = \quad \sin^2\theta_{13}, \tag{5}$$

$$|U_{\mu 3}|^2 = \quad \cos^2\theta_{13}\sin^2\theta_{23}, \tag{6}$$

$$|U_{\tau 3}|^2 = \quad \cos^2\theta_{13}\cos^2\theta_{23}. \tag{7}$$

Notice that the mixing angle $\theta_{12}$ and the CP-violating phase do not appear in the oscillation probability. Thus $\delta m^2$ and the mixing angles $\theta_{13}$ and $\theta_{23}$ are the only parameters needed to describe three-flavor mixing involving the larger mass difference ($\delta m^2_{\rm atmos,\,LSND}$) in the OMSD case.

Fig. 1 shows the allowed ranges of the mixing angles $\theta_{13}$ and $\theta_{23}$. Each panel corresponds to a different value of $\delta m^2_{\rm atmos,\,LSND}$. This figure shows that a small area of the $\theta_{13}$ and $\theta_{23}$ plane is available for $\delta m^2_{\rm atmos,\,LSND} \approx 0.3 - 0.4$ eV$^2$ which explains the LSND signal, provides a solution to the atmospheric neutrino problem, and does not conflict with the negative results of other accelerator and reactor experiments. For small values of $\theta_{13}$—which are indicated by the solution in Fig. 1—the small angle MSW solution to the solar neutrino problem is essentially unaffected [16].

Thus it appears that there is an essentially unique solution that can simultaneously explain the LSND signal and solve the solar and atmospheric neutrino problems, all without introducing sterile neutrinos. We have shown elsewhere that existing limits on two-flavor mixing parameters based on supernova $r$-process nucleosynthesis can be applied to the three-flavor mixing case in the OMSD limit [17]. We conclude that the above solution has a small enough mass difference that supernova $r$-process production is not adversely affected [7].

Cold+hot dark matter models, with about 20% of the dark matter comprised of two species of ~2.5 eV neutrinos, are reported to fit large-scale structure data well [3]. Choosing $\delta m^2_{\rm LSND} \approx 0.3 - 0.4$ eV$^2$ no longer directly implies the existence of neutrinos with this mass. Since neutrino oscillations are only sensitive to neutrino mass *differences*, however, all of the neutrino masses can be offset from zero to provide a neutrino mass sum ~ 5 eV. In such a scheme the neutrino masses would be nearly degenerate, a possibility considered in Refs. [18].[1]

It should be noted that the putative solution in Fig. 1 is fragile. We have already mentioned that the mass difference $\delta m^2_{\rm atmos,\,LSND} \approx 0.3 - 0.4$ eV$^2$ is allowed at 95% C.L., but not at 90% C.L. A similar situation exists for the mixing angles: the contours in Fig. 1 (except the atmospheric neutrino solution contours) are 95% C.L. contours. At 90% C.L., the LSND detection band shrinks and the exclusion region from other accelerator and reactor experiments expands, and the solution in Fig. 1 virtually disappears.

We have sought a "natural" solution, but in the end it still has what might be considered some "unnatural" features. For example, we have taken $\delta m^2_{\nu_e\nu_\tau} \approx \delta m^2_{\nu_\mu\nu_\tau} \gg \delta m^2_{\nu_e\nu_\mu}$, in analogy with the hierarchy of mass differences of the charged leptons. For the charged leptons, this hierarchy of mass *differences* arises naturally from the hierarchy of the *masses themselves*, i.e. $m_e^2 \ll m_\mu^2 \ll m_\tau^2$. However, the offset of neutrino masses from zero required to make the neutrino masses sum to about 5 eV for cold+hot dark matter models causes the absolute masses to have roughly similar magnitudes, in contrast to the masses of the charged leptons.

Another unnatural feature of this putative "natural" solution is that several of the off-diagonal elements of the mixing matrix $U$ have relatively large magnitudes. This is in contrast

[1]Note that with all three neutrino masses offset from zero, the OMSD limit (which applies to mass *differences*) can still apply, even if the masses themselves are "nearly degenerate."

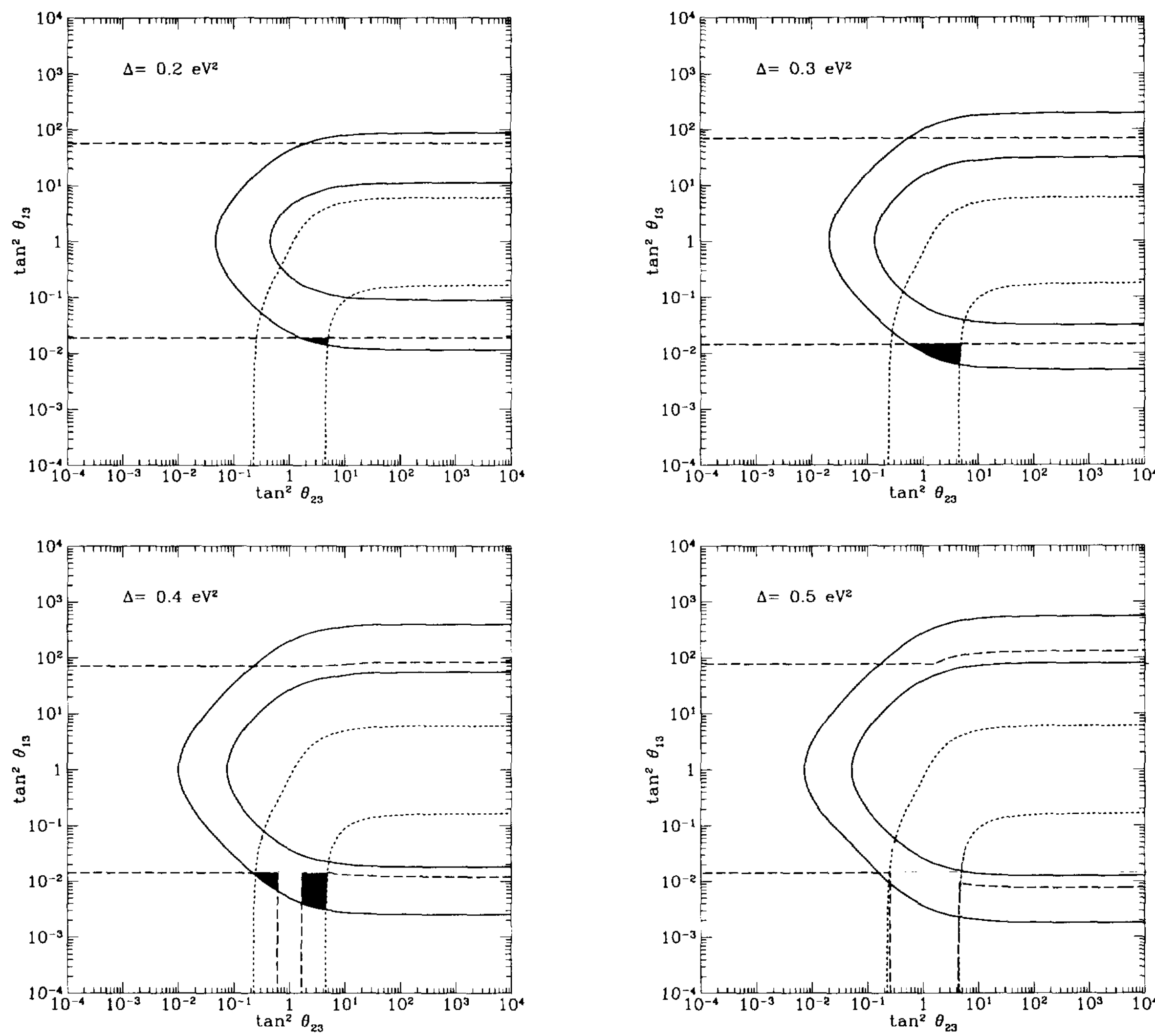


Figure 1. Allowed regions of the mixing angles $\theta_{13}$ and $\theta_{23}$, for four values of the dominant mass difference. The region between the solid lines is the 95% C.L. detection of LSND [14]. The region inside the long dashed lines is excluded by accelerator/reactor 95% C.L. limits [13]. The region enclosed by the short dashed lines represents a solution to the sub-GeV atmospheric neutrino data [15].

to the quark mixing case [12]. It has been recognized previously that a three-neutrino oscillation explanation of the LSND experiment requires this unusual feature [19]. Large off-diagonal terms will generally be present whenever oscillation probabilities are large, and neutrino oscillation explanations of the atmospheric neutrino anomaly and the LSND data both invoke relatively large oscillation probabilities.

## 3. FUTURE TESTS

Fortunately, the prospects are good for testing the last remaining three-neutrino solution that explains the LSND signal and solves the solar and atmospheric neutrino problems. The LSND experiment continues to run. A comparable experiment, KARMEN, is receiving an upgrade that within a few years should provide either a confirmation or refutation of the LSND signal [20]. Super-Kamiokande will provide much better statistics on the claimed zenith-angle dependance of the atmospheric neutrino data [21]. Super-Kamiokande and SNO may also be able to distinguish solar neutrino solutions involving sterile neutrinos from solutions involving only active neutrino flavors [22]. A null result at the San Onofre reactor experiment [23] would not rule out the "natural" solution proposed here; on the other hand, a detection of neutrino oscillations at San Onofre could not be accounted for by our solution. However, proposed long-baseline accelerator experiments will probe the entire region of parameter space favored by the atmospheric neutrino sub-GeV data [13,24] and could thus provide the crucial test.

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