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Proportional representation primer

Marek M. Kaminski¹ 

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Abstract

This primer provides a systematic introduction to proportional representation (PR) electoral systems for scholars and students unfamiliar with their mechanics and political consequences. PR is now used for legislative elections in approximately 118 countries, making it the world's most common family of electoral systems. However, its formal structure and political effects are often poorly understood outside specialist communities. The primer covers the main families of apportionment algorithms (quota, divisor, and ranked-choice methods), the principal types of PR electoral systems (list PR, STV, mixed, and semi-proportional), and their key structural components, including district magnitude, thresholds, measures of disproportionality, volatility, and the effective number of parties. It surveys major empirical regularities, including Duverger's Law, the impact of district magnitude on party fragmentation, women's and minority representation, voter turnout, and coalition formation, before turning to paradoxes and problems, including apportionment paradoxes and the Balinski–Young impossibility theorem, STV Non-Monotonicity and the No-Show paradox, spoilers and ballot-position primacy effects in list PR, and overhang seats and decoy parties in mixed compensatory systems. The primer concludes that every apportionment method is formally flawed, that institutional parameters interact with strategic behavior in ways that frequently produce unexpected outcomes, and that the empirical regularities anchoring the field rest on identification strategies that are vulnerable to the endogeneity of electoral institutions. A Glossary of key concepts is included as an Appendix.

Keywords Proportional representation · Electoral systems · Mixed electoral systems · Single Transferable Vote (STV) · Apportionment · Duverger's Law

JEL Classifications D71 · D72

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1 Introduction

This primer provides a systematic introduction to proportional representation (PR) electoral systems, focusing on legislative elections. It is intended for scholars and students seeking a clear account of how PR systems operate and how their institutional features shape political outcomes.

The prominence of proportionality has its roots in Aristotelian distributive justice, in which goods are allocated in proportion to merit (Aristotle [ca. 350 BCE] 2016). Modern electoral proportionality, however, reflects a different, egalitarian logic: representation is distributed according to votes among formally equal citizens. A PR system allocates representatives in proportion to votes—for parties in national legislatures, for member states in supranational bodies such as the European Parliament, or for shareholders on company boards. Ranked systems such as the Single Transferable Vote (STV) abandon this requirement. They dispense with predefined groups altogether and produce proportionality across the de facto coalitions of preferences that voters reveal through their rankings.

PR has been implemented through multiple voting methods, electoral systems, and apportionment algorithms, each with its own normative or organizational logic. The PR family encompasses three primary algorithm types—quota, divisor, and ranked-choice methods—and three main system types: party list, STV-based, and mixed systems. A few additional systems are termed semi-proportional. Although they do not aim to achieve proportionality, they are sometimes adopted as a compromise to improve proportionality over majoritarian systems. The study of PR belongs to the broader field of *electoral institutions*, also called *the political consequences of electoral systems*. The field was established by such classic works as Duverger (1951, 1954), Rae (1967), Lijphart (1984), Taagepera and Shugart (1989), and Cox (1997).

A central challenge in understanding PR is that proportionality is rarely achieved directly at the national level. Votes are translated into seats through a sequence of steps, including district-level allocation, threshold application, and sometimes compensatory mechanisms across tiers. Therefore, identical vote shares may yield different seat distributions depending on district magnitude, thresholds, and national-level corrections. Understanding this multistage process is essential for interpreting empirical outcomes.

The main competitors of PR are majoritarian (winner-take-all) systems in which voters select one candidate with one vote. First-past-the-post (FPTP, plurality) is the most prominent. The second, the two-round system (runoff), used in most presidential elections worldwide, typically holds a second round between the top two candidates if no candidate wins an outright majority. Finally, in a family of ranked systems, such as the Alternative Vote (Instant-Runoff Voting) or Supplementary Vote (Contingent Vote), voters rank two or more candidates; the weakest candidates are eliminated sequentially, and their votes are transferred until one candidate attains a majority.

The main institutional developments in electoral systems took place in the United States and Europe. Electoral system design was debated by the American Founding Fathers before and during the adoption of the Constitution (ratified 1788) (Balinski et al. 1982; Dahl 2003). Another major step took place in Europe with the first explicit institutional choice between majoritarian and proportional systems in Belgium in 1899, when PR was adopted for parliamentary elections (Emmenegger et al. 2019). Its rapid spread across continental Europe in the early twentieth century was politically driven. Rokkan (1970) argued that established

conservative and liberal parties adopted PR strategically: as newly mobilized socialist and labor movements gained strength under expanding suffrage, incumbents preferred proportional rules that would preserve their seat shares over majoritarian, winner-take-all outcomes. This “insurance” logic is supported by comparative historical research (Boix 1999; Cusack et al. 2007).

The persistence of majoritarian systems in the United Kingdom, Canada, the United States, and other former British territories reflects a different trajectory in which suffrage expanded before strong labor parties formed, leaving incumbents with little incentive to abandon FPTP systems that had already secured their advantages (Blais et al. 1997; Colomer 2016).

Since the American constitutional debates of the late eighteenth century, electoral system choice has been among the most fundamental constitutional decisions, often explicitly entrenched in constitutions (Buchanan et al. 1962). Neither system is uniformly superior, and the choice involves well-recognized trade-offs.

The primary advantage of majoritarian systems is greater governability. By facilitating two-party systems or at least constraining fragmentation, they tend to produce strong single-party cabinets, thereby avoiding the coalition negotiations that are common after PR elections. The ruling party typically bears clear accountability for its policy decisions. Majoritarian voting is also simpler for voters and administrators, often presenting only two viable options, and voters typically know and can directly contact their representatives (Lijphart 1999; Powell 2000).

Several features of PR that critics frame as disadvantages relative to majoritarian systems are viewed as desirable by proponents. By lowering barriers to entry, PR systems allow more parties to enter parliament and increase representation, reduce wasted votes, and facilitate the inclusion of women and minorities. They are also associated with higher turnout (Lijphart 1999; Blais et al. 2006). Administering PR elections imposes distinct challenges such as counting ranked ballots (STV) or managing large multi-party ballots. The greater organizational complexity of open-list systems compared to FPTP may substantially reduce ballot quality by raising the share of invalid votes or amplifying the impact of primacy effects (Flis et al. 2025a).

Electoral reforms often arise from politically neutral triggers, such as defeat in war, democratic transition, administrative overhaul, constitutional change, or international diffusion, but the proximate motives are typically partisan and often reflect the interaction between technical experts and partisan actors (Taagepera and Kaminski 2025). Both systems are vulnerable to manipulation, often called “electoral engineering” (Norris 2004; Evci and Kaminski 2025), although the forms differ. In majoritarian systems, especially in the United States, because precinct-level data make seat outcomes highly predictable, gerrymandering—the manipulation of district boundaries—is the dominant tool. In PR systems, classic gerrymandering is virtually impossible in larger districts, given that seat allocation approximates vote shares and is largely insensitive to boundary manipulation, but manipulation via district magnitudes, thresholds, or apportionment algorithms is common and more complex (Gallagher et al. 2005).

PR has its most consequential effects in lower-house legislative elections. With approximately 118 countries using PR or near-PR versus approximately 60 using FPTP or other majoritarian methods, PR is by far the most popular family of electoral systems (ACE 2026). Majoritarian methods remain more common in upper-house elections (Lijphart 1999;

ACE 2026). Yet the most frequent use of PR occurs in local elections, with thousands held every four or five years, an application that remains under-researched despite growing interest (Farrell 2011; Gallagher et al. 2005). Compared to national PR, local systems typically feature lower thresholds, smaller district magnitudes, and varied ballot structures, which can intensify personalization, alter party strategies, and affect representation patterns (Flis and Kaminski 2022; Pierzgański et al. 2020).

The remainder of the primer is organized as follows. Sections 2 and 3 introduce, respectively, the three apportionment families—quota, divisor, and STV—and the principal PR system types: list, ranked-choice, mixed, and semi-proportional. Section 4 examines structural components common to PR systems, and Sect. 5 reviews the central empirical regularities. Section 6 turns to paradoxes and pathologies across each family, and Sect. 7 concludes. A Glossary of key concepts is included as an Appendix.

2 Apportionment

The apportionment problem was first formally posed in the United States Constitution, which declared the following:

Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons.... [E]ach State shall have at Least one Representative.

In general terms, the problem is how to divide M seats among n competing units in proportion to their votes or populations. In the constitutional context, “population” means all persons residing in a state. Hereafter, unless American apportionment is the focus, the terms “parties” and “votes” are used.

Three types of apportionment problems are most important in politics. First, American apportionment assigns House seats and electoral votes to states; the algorithm together with the constitutional minimum of one seat per state introduces some disproportionality. Second, in any PR system with multiple districts, seats are allocated to districts. Third, within each district, seats are allocated among competing parties. In hierarchical systems, such as Uruguay’s, the algorithm may be applied several times. Apportionment problems also arise in professional societies, company boards, music contests, and other contexts.

The distribution of M seats among the parties is denoted $s_1, \dots, s_n$. Three principal methods translate votes into seats. *Quota* and *divisor* methods require a single vote per party; party votes are denoted $V_1, \dots, V_n$, with proportions of votes denoted $v_1, \dots, v_n$. Both families were first discovered in the United States and later rediscovered independently in Europe (Balinski et al. 1982). The *Single Transferable Vote* (STV) requires voters to provide a full or partial ranking of candidates, who may have no party affiliation. Votes are denoted $R_1, \dots, R_n$, where R_i represents the ranking of voter i and takes the form of $ABCDE$, with A ranked first and so on. Throughout, tiebreakers, which are rare in legislative elections but often complex and would obscure the main argument, are omitted. Quota and divisor methods form the *simple* family; STV is the *ranked* family. When the contrast matters, they are referred to as *simple* and *ranked apportionment*, respectively.

2.1 Quota methods

The discussion begins with the most intuitive family, quota methods. Such methods, or slight modifications of them, are used for legislative elections in approximately 15 countries (Gallagher et al. 2005; Pukelsheim 2017). To obtain party *quotas*, the proportions of votes are multiplied by the number of available seats M , or, in variants, by larger multipliers equal to $M + 1$ or $M + 2$. Seats are then allocated according to each party's quota, rounded down; any remaining seats are allocated to parties with the largest fractional remainders. In the United States, this method is named after its discoverer, Hamilton; elsewhere it is called Hare-Niemeyer or Largest Remainder. Hamilton-Hare-Niemeyer (HHN) quotas correspond to the exactly proportional seat allocations that would arise if fractional seats were permitted. Note that in other contexts, "quota" refers to the minimum number of votes needed to win a seat, which is a distinct usage.

$$\text{HHN quota : For } i = 1, \dots, n, q_i^{\text{HHN}}(v_i, M) = v_i M$$

The procedure is as follows: Party i initially receives $\lfloor v_i M \rfloor$ seats, i.e., the $v_i M$ number rounded down. If $\sum_{i=1}^n \lfloor v_i M \rfloor < M$, unassigned seats remain and are consecutively awarded to parties with the largest remainders, defined as $v_i M - \lfloor v_i M \rfloor$.

In Table 1, the three parties compete for ten seats under the HHN rule. Rounding down the quotas allocates nine seats; the last seat goes to Libs, which has the largest remainder (0.5). The final distribution, adding quota and remainder seats, gives Center six seats and two seats each to Cons and Libs.

The same procedure, allocating rounded-down Droop quota seats followed by remainder seats, applies to the second most popular quota method, Droop (sometimes called Hagenbach-Bischoff or DHB). DHB uses larger quotas than HHN and is slightly more favorable to larger parties.

$$\text{DHB quota : For } i = 1, \dots, n, q_i^{\text{DHB}}(v_i, M) = v_i (M + 1 - \epsilon),$$

where ϵ is a small correction, defined slightly differently across DHB variants, introduced to prevent $M + 1$ parties with identical claims to M seats. In calculations, it is set $\epsilon = 0$. Both the HHN and DHB methods guarantee that there will be enough seats to satisfy all the claims.

The third commonly used method employs the Imperiali quota of $v_i (M + 2)$, and is the most favorable toward larger parties. Unlike HHN and DHB, Imperiali can generate more total claims than available seats, requiring tiebreaking rules and, ultimately, random assignment when other rules fail. In extreme cases, there may be identical claims to $M + 2$ seats; tied claims to $M + 1$ seats are even more likely.

Table 1 Example: application of the HHN quota method

Party	Votes	Proportion v_i	Quota $10 v_i$	Quota seats	Remainder seats	Total seats
Libs	300	0.150	1.50	1	1	2
Center	1210	0.605	6.05	6	0	6
Cons	490	0.245	2.45	2	0	2

Ten seats are allocated among three parties with a total of 2000 votes

When applied to Table 1, both DHB and Imperiali favor larger parties. Center, Cons, and Libs would receive six, three, and one seat under DHB, and seven, two, and one under Imperiali. Larger multipliers scale every party's quota proportionally, but in absolute terms the excess over $\lfloor v_i M \rfloor$ grows faster for larger parties, enabling them to capture additional whole seats and stronger remainder claims.

In general, larger multipliers favor larger parties. Quota methods frequently produce identical results. When they differ, moving from HHN to DHB (or Imperiali) is equivalent to shifting one or more seats from smaller to larger parties.

2.2 Divisor methods

Divisor methods are based on a different logic and are defined by a series of *divisors*, an increasing sequence of positive numbers (d_j). At most the first M divisors are used. For each party i , a sequence of *priority numbers* is computed (v_i/d_j). Seats are assigned in descending order of priority numbers; once a priority number has been used to award a seat, it is removed from consideration.

As with HHN, the two most popular divisor methods were discovered independently in the United States and Europe and bear dual names. These are based on the following divisor sequences:

Jefferson-D'Hondt: $d_i = i$ (consecutive positive integers).

Webster-Sainte-Laguë: $d_i = 2i - 1$ (consecutive positive odd integers).

The Jefferson-D'Hondt method is the most widely used algorithm, employed in legislative elections in approximately 50 countries; it slightly favors larger parties. Webster-Sainte-Laguë is used in about ten countries (Gallagher et al. 2005; Pukelsheim 2017).

Jefferson-D'Hondt is applied to Table 2 (same vote totals as Table 1); for compactness the table starts at $d = 2$; the $d = 1$ priority for each party simply equals its vote total. The first ten priority numbers in descending order are: 1210, 605, 490, 403, 302, 300, 245, 242, 202, 173. Awarding seats in this order gives Center 7 seats, Cons 2 seats, and Libs 1 seat.

Webster-Sainte-Laguë shifts one seat from Center to Libs, giving Center 6, Cons 2, and Libs 2, illustrating that Jefferson-D'Hondt favors larger parties relative to Webster-Sainte-Laguë, just as DHB does relative to HHN.

Less widely used methods include the modified Webster-Sainte-Laguë method, which is used in Norway, Sweden, and a small number of countries, and is slightly more favorable to larger parties than the original. The Huntington-Hill method is currently used in the United States (Balinski et al. 1982).

Modified Webster-Sainte-Laguë: $d_i = \begin{cases} 1.4 & \text{for } i = 1 \\ 2i - 1 & \text{for } i > 1 \end{cases}$

Huntington-Hill: $d_i = \sqrt{i(i+1)}$

Table 2 Example: application of Jefferson-D'Hondt divisor method

Party	Votes	Consecutive priorities (rounded)									Seats
		2	3	4	5	6	7	8	9	10	
Libs	300	150	100								1
Center	1210	605	403	302	242	202	173	151			7
Cons	490	245	163	122							2

Ten seats are allocated among three parties with a total of 2000 votes

Other methods that have never been used but have been considered for the American House include the Dean and Adams methods.

The choice of algorithm matters, as our comparison of three quota and two divisor methods demonstrates.

2.3 Single transferable vote (STV)

The third principal method, the Single Transferable Vote (STV, a form of preferential voting), requires voters to rank candidates rather than cast a single vote for a party. Although STV is often treated as an electoral system, it can also be analyzed as a ranked apportionment method.

The components of an STV problem include the number of seats M , the profile of individual preferences (rankings), $(R_1, \dots, R_n)$, and the quota Q . Some variants allow partial rankings; however, full rankings are assumed throughout. The STV quota is defined differently from quota methods: it is the number of first-preference votes required to win a seat. The Droop quota is the most common:

$$Q = \left\lfloor \frac{n}{M+1} \right\rfloor + 1$$

The following version of ranked apportionment approximates the Weighted Inclusive Gregory variant. Its main steps are electing candidates who reach the quota, transferring surpluses to next-preferred candidates, eliminating the weakest candidates, and transferring their votes likewise. Tie-breaking rules are omitted throughout.

1. *Ballots*: At any stage, each ballot is assigned (possibly fractionally) to the highest-ranked active candidate, that is, a candidate who is neither elected nor eliminated. The total vote of a candidate is the sum of the weights of the ballots currently assigned to that candidate.
2. *Election*: If one or more active candidates have totals at least equal to Q , each such candidate is elected. For each elected candidate i with total $T_i > Q$, define the surplus:

$$\sigma_i = T_i - Q.$$

The transfer value is equal to

$$t_i = \frac{\sigma_i}{T_i}.$$

Each ballot currently assigned to candidate i transfers a fraction of its weight equal to its current weight multiplied by t_i , which is then reassigned to the next-ranked active candidate on that ballot. The remaining fraction stays with candidate i , leaving them with total exactly Q .

After this operation, i is removed from the set of active candidates while remaining elected.

If multiple candidates exceed the quota simultaneously, their surpluses are transferred sequentially in the descending order of their totals. Ties are resolved according to predefined tie-breaking rules.

After each surplus transfer, the totals are recomputed, and Step 2 is repeated as long as any active candidate meets the quota.

3. *Elimination*: The active candidate with the smallest total is eliminated from the list. All ballots assigned to the eliminated candidate are transferred at their full current weight to the next-ranked active candidate on each ballot.
4. *Termination*: The procedure stops when either: exactly M candidates have been elected; or the number of active candidates equals the number of remaining unfilled seats, in which case all active candidates are declared elected.

Otherwise, the procedure returns to Step 2.

The algorithm's institutional applications, a worked numerical example, and a discussion of STV variants are treated in Sect. 3.2.

3 Types of PR electoral systems

Electoral systems are richer constructs than apportionment methods alone: while apportionment rules translate votes into seats, electoral systems embed these rules within institutional structures such as districts, ballots, and tiers. In most cases, apportionment operates separately within multiple districts. A few exceptions, including the Netherlands, Israel, Slovakia, and Serbia, use a single nationwide district and can be treated as pure apportionment systems (Lijphart 1994; Gallagher et al. 2005).

Once multiple districts are introduced, outcomes depend on more than aggregate vote shares. For example, redistributing votes for a party across districts without changing its national vote total can still alter its seat allocation. Understanding PR therefore requires attention not only to allocation formulas but also to institutional structure.

3.1 List systems

Party-list PR is the most widely used form of proportional representation. Parties present ranked lists of candidates in electoral districts, and voters cast ballots that determine how many seats each party receives, and, in some cases, which candidates are elected. Exceptions to pre-ranked lists include Denmark, where candidate order is determined locally by nomination district, and Finland, where voters write the candidate's unique number on a blank ballot.

The central distinction concerns how much control voters exercise over candidate selection. In Closed-List PR (CLPR), voters choose only among parties. Seats won by a party are allocated to candidates according to the predetermined order of the party list. This system, used in countries such as Spain, Portugal, and South Africa, concentrates power in party leadership and tends to produce disciplined, programmatic parties (Carey et al. 1995). Approximately 30 to 35 countries use CLPR (ACE 2026).

In Open-List PR (OLPR), voters can influence which candidates are elected. The degree of voter control varies. In some systems, voters cast a single preference vote within a party list (Brazil, Poland, Finland). In others, voters cast multiple preference votes (Czechia, Slovakia—up to four). Switzerland and Luxembourg permit *panachage*—distributing votes across candidates from different lists, with *cumulation* allowed on a single candidate. Flexible-list systems, such as Austria, allow voters to support a party while optionally indicating preferences for specific candidates (see Fig. 1). Although OLPR increases voter choice, its practical impact is often limited (Passarelli 2020) as only a minority of candidates who would not be elected under closed lists win seats through preference votes (Gendźwiłł et al. 2014).

Candidate selection is therefore central to how list PR operates. These differences can be systematized using the framework of personal vote incentives (Carey et al. 1995). Electoral systems vary in the extent to which candidates benefit from cultivating a personal reputation independent of their party. CLPR minimizes personal vote incentives by placing candidate ranking under party control, encouraging programmatic competition and strong party discipline. OLPR and STV systems, by contrast, create incentives for candidates to differentiate themselves from co-partisans, often through constituency service, local ties, or targeted campaigning. While this can strengthen voter–representative linkages, it may also weaken party cohesion and shift effort away from collective platforms toward intra-party competition. The adverse consequences of intra-party competition motivate electoral reform proposals (Flis, Kaminski, and Salamon 2025c).

List systems can also be cognitively complex, with many levels of decision-making. In Uruguay, voters select a party, then a faction within the party, and then a candidate list

[illegible]

Fig. 1 Ballot for flexible-list OLPR elections in Austria. *Note:* Voters must select a party and, optionally, a candidate within that party. *Source:* Wikimedia Commons. CC BY-SA 4.0

within that faction, with apportionment applied at each stage. Other countries, such as Austria, Denmark, Norway, and Sweden, employ multi-tier structures in which seats are first allocated at the district level and then adjusted at higher levels to improve proportionality (see Fig. 1).

3.2 Ranked-choice systems

Ranked-choice systems that aim at proportionality operate on different principles from list PR. Usually implemented as the Single Transferable Vote (STV), these systems dispense with party lists and instead ask voters to rank individual candidates directly, although party affiliations may appear on ballots. Candidates who reach the Droop quota (defined in Sect. 2.3) are elected; surpluses are redistributed down the preference list; the weakest remaining candidates are eliminated sequentially, and their votes are likewise transferred.

Because STV does not require parties, it is a common choice for elections where party structures are absent or undesirable: company boards, professional societies, academic organizations such as the American Statistical Association and the Royal Society, and award selection contests, including the Academy Awards Best Picture selection and short-list nominations (Tideman 1995; Farrell 2011). In legislative elections, STV is used in Ireland and Malta, for Senate elections in Australia, and for local elections in Northern Ireland and Scotland (see Fig. 2).

The following example illustrates two core features of STV: surplus transfer after election and full-weight transfer after elimination. For expositional clarity, the example aggre-

South Australia
Ballot Paper for the Election of 11 Legislative Councillors
 You are not legally obliged to mark the ballot paper

YOU MAY VOTE BY

EITHER → Placing the number 1 in the square next to the group of your choice. You can show more choices if you want to by placing numbers in the other squares starting with the number 2

A	B	C	D	E
☐ GROUP A	☐ GROUP B	☐ GROUP C	☐ GROUP D	☐ GROUP E

OR →

Numbering at least 12 of these squares in the order of your choice

A	B	C	D	E
☐ Candidate	☐ Candidate	☐ Candidate	☐ Candidate	☐ Candidate
☐ Candidate	☐ Candidate	☐ Candidate	☐ Candidate	☐ Candidate
☐ Candidate	☐ Candidate	☐ Candidate		☐ Candidate
☐ Candidate	☐ Candidate	☐ Candidate		
	☐ Candidate	☐ Candidate		
	☐ Candidate			

Issuing Officer Initials

After voting, fold the ballot paper and place it in the ballot box or declaration envelope

Fig. 2 Ballot for STV elections for South Australian Legislative Council. *Note:* Voters rank options by inserting consecutive numbers, either for parties (at least one) or candidates (at least 12). Wikimedia Commons, CC BY-SA 4.0

gates identical rankings rather than listing individual ballots. The numbers were chosen to minimize fractional ballot complications.

Example (3 seats, 5 candidates, 100 voters). 100 voters have the following preferences.

Ballots:

30 voters: ABCDE

25 voters: BCDEA

18 voters: CBDEA

15 voters: DCBEA

12 voters: EDCBA

The quota is:

$$Q = \left\lfloor \frac{n}{M+1} \right\rfloor + 1 = \left\lfloor \frac{100}{3+1} \right\rfloor + 1 = 26$$

Election Candidate A receives 30 first-preference votes and exceeds the quota. A is elected.

The surplus is $\sigma_A = 30 - 26 = 4$, and the transfer value is $t_A = \frac{4}{30}$.

All ballots assigned to A have B as their next available preference; therefore, the entire surplus is transferred to B. After the transfer, the weighted total for B is 29: 25 ballots

originally headed BCDEA plus 30 ballots now carrying weight $\frac{4}{30}$. Henceforth the voter profiles report weighted totals.

After removing A, the new updated profile is.

29 voters: BCDE

18 voters: CBDE

15 voters: DCBE

12 voters: EDCB

Candidate B exceeds the quota and is elected. The surplus is $29 - 26 = 3$, and the transfer value is $\frac{3}{29}$. Because all ballots assigned to B have C as the next available preference, C receives all three votes.

The updated profile is:

21 voters: CDE

15 voters: DCE

12 voters: EDC

No candidate meets the quota, and the procedure advances to the elimination step.

Elimination Candidate E, with 12 votes, is eliminated, and all E ballots are transferred to their next available preference, which is D.

Updated totals of the first votes:

C: 21

D: 27

Election D meets the quota and is elected.

Termination All three seats are filled; thus, the procedure terminates. A, B, and D were elected.

STV personalizes representation and allows for intra-party and inter-party transfers. It reduces wasted votes differently from list PR, and rewards both broad acceptability and first preferences.

STV variants differ in several respects: how the quota is defined; how surplus votes are transferred (from all ballots assigned to the elected candidate, or only from those received in the most recent count); whether integer rounding or exact fractions are used; whether partial rankings are permitted; and how the “weakest” candidate is identified (e.g., fewest first-preference votes or most last-place votes). Some systems incorporate random sampling to reduce the computational burden of determining the winners.

3.3 Mixed systems

Whereas STV seeks proportionality through ranked candidate choice within a single counting process, mixed-member (or simply mixed) systems pursue it by combining two distinct allocation logics. They evolved from early theoretical attempts to combine different voting rules (McLean 2025). Later, they were tested and temporarily abandoned in Denmark, Iceland, France, Italy, and Romania. Germany adopted a mixed system in 1949, but it was not until the 1990s that it became popular (Michalak 2013).

In contemporary variants, PR is combined with majoritarian voting in two tiers. Typically, similar numbers of seats are assigned in single-member districts (SMD), and the remainder through a PR tier. The two main types of mixed systems differ in how the two tiers are related. Mixed-Member Proportional (MMP) uses the PR tier to compensate for disproportionality caused by district contests. Depending on classification, MMP is currently used in roughly seven to nine countries (ACE 2026). Fully compensatory systems (e.g., Germany, New Zealand) adjust total seat allocations to match party vote shares, whereas partially compensatory systems apply corrections only within regions or with limited scope.

In the pioneering German Bundestag system, which initially allocated 50% of seats through PR before increasing the PR share to 52.5% in the 2023 reform, voters cast two votes: one for a constituency candidate and one for a party. Each party is assigned a target seat share based on its party vote. If it wins fewer SMD seats than its target, it receives compensatory PR seats, and if it wins more, it receives no PR seats. Cases in which parties win more constituency seats than their PR-vote entitlement would award created the overhang problems and subsequent reforms discussed in Sect. 6.5.

The more common MMM (parallel) systems, used in 20–30 countries, are simpler than MMP. MMM allocates majoritarian and proportional seats independently, often producing substantial disproportionality. Voters cast two votes that may be split; votes in one tier have no formal effect on the allocation in the other.

MMM systems are used in Japan, Russia, the Philippines, Morocco, and elsewhere (ACE 2026; Maeda 2025). The majoritarian tier may exceed 50% of seats and is often much larger: it reaches 80% in the Philippines, and, in the extreme case of South Korea, which introduced in 2019 a hybrid system with limited compensation features, 84–85%

Fig. 3 Ballot for 2005 MMP German Bundestag elections.
 Source: Wikimedia Commons, CC BY-SA 4.0

Stimmzettel
 für die Wahl zum Deutschen Bundestag
 im Wahlkreis 5 Kiel
 am 18. September 2005

Sie haben 2 Stimmen

hier 1 Stimme
 für die Wahl
 eines
 Wahlkreisabgeordneten

Erststimme

hier 1 Stimme
 für die Wahl
 einer Landesliste (Partei)
 - maßgebende Stimme für die
 Verteilung der Sitze insgesamt
 auf die einzelnen Parteien -

Zweitstimme

1	Dr. Bartels, Hans-Peter CDU Wahlkreisabgeordneter	○	○	SPD	○	1
2	Dr. Mummert, Philipp CDU Wahlkreisabgeordneter	○	○	CDU	○	2
3	Müller, Klaus CDU Wahlkreisabgeordneter	○	○	GRÜNE	○	3
4	Blumenthal, Sebastian FDP Wahlkreisabgeordneter	○	○	FDP	○	4
5	Thorne, Björn DIE LINKE Wahlkreisabgeordneter	○	○	DIE LINKE	○	5
6	Gutsche, Hermann NPD Wahlkreisabgeordneter	○	○	NPD	○	6
7	Mrowinski, Oliver FAMILIE Wahlkreisabgeordneter	○	○	FAMILIE	○	7
		○	○	MLPD	○	8

(Moon et al. 2025). Larger majoritarian tiers encourage consolidation and typically produce two-bloc competition.

The distinction between MMP and MMM carries substantial and predictable implications for electoral outcomes. MMM systems show greater disproportionality and fewer effective parties than MMP systems in otherwise comparable settings—evidence that a PR tier is necessary but not sufficient for proportional representation.

Mixed system classifications can be further refined to distinguish less common variants by how the two tiers are linked and by the number of votes cast (Shugart et al. 2001b; Michalak 2016; Flis et al. 2025b). In mixed compensatory systems (MCS) (e.g., Hungary in 1990–2010 and Italy in 1994–2001), the correction is done not by seat transfers, as in MMP, but through the transfer of votes between tiers. In mixed combined (MCB) systems, all votes—both candidate and list votes—are used to determine overall electoral outcomes using either a single integrated vote or two separate votes (Salamon, Kaminski, and Flis 2026).

Figure 3 shows a typical ballot for the 2005 Bundestag elections for constituency 5 in Schleswig-Holstein. On the left-hand side, there is a list of candidates in single-member districts. On the right-hand side is a list of parties in the PR tier. Voters can cast both votes independently, that is, with no requirement that the candidate vote and party vote go to the same party.

3.4 Semi-proportional systems

Semi-proportional systems may yield results closer to proportionality than majoritarian systems but were not designed to produce seat distributions proportional to votes. Proportionality is at most an incidental byproduct. Elections are held in multi-member districts with magnitude M . Several ballot types are possible; in all cases, the M candidates with the greatest number of votes are elected. Semi-proportional systems differ mainly in ballot structure: how many votes each voter holds and whether votes may be cumulated on a single candidate.

Single Non-Transferable Vote (SNTV): Voters cast a single vote. SNTV was used in legislative elections in Jordan (1993–2016), Taiwan (until 2008), Japan (before 1994), and elsewhere (Cox 1997; Farrell 2011).

Limited Vote (LV): Voters have k votes, where $k < M$, and must use them for different candidates. LV is used for most seats in the Spanish Senate (Lijphart 1994).

Cumulative Voting (CV): Voters have k votes, where $k \leq M$, and can use them for different candidates or give some candidate more than one vote. CV is used in some corporate board elections and formerly in the U.S. Illinois House of Representatives (1870–1980) (Bowler et al. 2003).

Multiple Non-Transferable Vote (MNTV): Voters have M votes that must be used for different candidates. MNTV was used in Polish Senate elections with $M \in \{2, 3\}$: with a plurality runoff in 1989 or plurality-at-large (Bloc Vote) in 1991–2007 (Flis et al. 2025b).

Whether semi-proportional rules produce proportional outcomes is a function of ballot structure, district magnitude, and strategic coordination. SNTV ($k = 1$) offers the great-

est proportional potential, with a natural threshold of approximately $\frac{1}{M+1}$, matching the Jefferson-D'Hondt method. Under SNTV, disciplined parties that optimize candidate numbers and equalize vote distribution can achieve high seat fidelity (Cox 1997). However, the coordination risks are high: over-nomination wastes votes, under-nomination forfeits seats, and uneven distribution creates “surplus” votes that fail to help trailing co-partisans.

At the opposite extreme, MNTV ($k = M$) rewards the largest bloc, where a simple plurality often sweeps the district, as seen in Polish Senate elections from 1991 to 2007 (Flis et al. 2025b). LV and CV fall between these poles, with proportionality depending on a minority's ability to concentrate or cumulate votes against a divided majority (Bowler et al. 2003). Thus, “semi-proportional” describes a system's capacity rather than its default; proportionality is a product of coordination, not a structural certainty.

3.5 Examples of legislative PR systems

The system types described in Sects. 3.1, 3.2, 3.3 and 3.4 interact with differences in district magnitude, thresholds, ballot structure, and tier linkages to produce a large space of institutional configurations. As a result, nearly every legislative PR system displays local quirks, and country cases rarely fit cleanly into a single category. Table 3 summarizes the principal types together with representative countries, providing a reference point for the parameter-by-parameter comparisons developed in Sect. 4.

4 Components and descriptions of electoral systems

The preceding sections described how PR methods translate votes into seats and how real-world electoral systems combine these methods with party lists, ranked ballots, or mixed tiers. As Table 3 shows, systems that share a label can differ sharply in district magnitude, threshold rules, and ballot structure—parameters that often matter at least as much for political outcomes as the choice of apportionment algorithm. Therefore, classifying systems by type is only the first step. The following sections introduce the key institutional parameters and summary measures—disproportionality indices, volatility,

Table 3 Example: countries of main types of PR or semi-PR systems

System type	Ballot	Algorithm	District	Example countries
CLPR	Party	Quota / Divisor	Multi-member	Spain, South Africa
OLPR	Party + Candidate	Quota / Divisor	Multi-member	Poland, Brazil, Finland
STV	Ranked candidates	Ranked-choice	Multi-member	Ireland, Malta, Australia (Senate)
MMP	Two votes	Divisor + compensatory	SMD + national	Germany, New Zealand
MMM	Two votes	Divisor (parallel)	SMD + multi	Japan, Russia
SNTV	Single candidate	Plurality	Multi-member	Jordan (1993–2016), Japan (pre-1994)
MNTV	2 or 3 votes	Plurality runoff or plurality-at-large	2–3	Poland Senate (1989–2007)

and effective party counts—used to compare systems that differ along multiple dimensions simultaneously.

4.1 Districts

Districts matter in two distinct ways: by determining how many seats are allocated per district, and by determining how populations are distributed across districts.

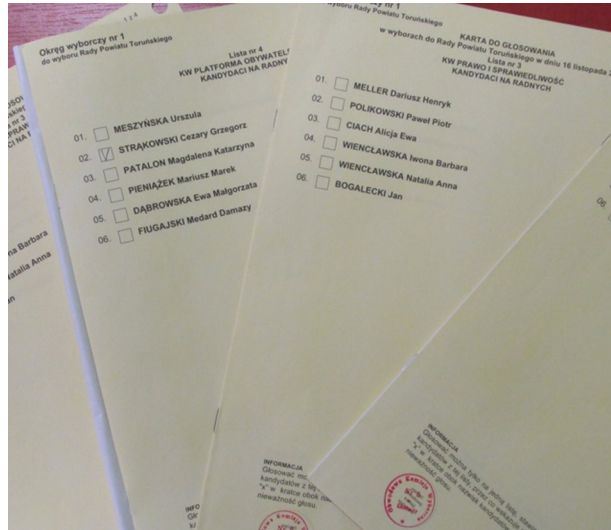
District magnitude (M) is the number of seats allocated in a given electoral district. In a single-member district system—such as the US House of Representatives or British House of Commons— $M = 1$. In proportional representation systems, districts return multiple seats, so M ranges from as few as 2 (small-district PR systems such as Chile under the binomial system from 1989 until the 2015–17 reform) to the full legislature (single-constituency systems such as Serbia, Israel, Slovakia, or the Netherlands). The political consequences of these differences—above all, the strong relationship between magnitude and party system fragmentation—are discussed in the sections on Duverger’s Law and the impact of district magnitude.

A separate consideration concerns how districts are *drawn*: the geographic distribution of districts can introduce systematic disproportionality, even within PR systems. Gerrymandering (manipulating district shapes) is primarily a majoritarian concern, but malapportionment (unequal district populations) is a distinct phenomenon that applies to PR systems as well (Samuels et al. 2001). Malapportionment arises within PR systems through constitutional minima (the antithresholds discussed in Sect. 4.3), failure to update boundaries as populations shift, or deliberate over-representation of rural areas. In many PR countries (Spain, Argentina, and Brazil), malapportionment is quantitatively large and systematically favors conservative rural parties.

4.2 Ballot structure and runoffs

Because PR allocates multiple seats, legislative ballots are typically more complex than majoritarian ballots. Voters may cast a single candidate vote (Poland), up to four votes (Czechia, Slovakia), or separate party and candidate votes (Austria). The key distinction is between *preferential* ballots, where voters rank or express their ordered preferences, and *categorical* ballots, where voters make one or a few choices. The distinction matters:

Fig. 4 Ballot booklet from the 2014 Polish local elections.
Source: Photo by Bartłomiej Michalak



preferential systems (STV, ranked OLPR) place substantially higher cognitive demands on voters and produce more invalid ballots, particularly among less educated voters and in contests with large candidate pools (Reynolds and Steenbergen 2006).

In many mixed systems, such as Germany's, voters cast one vote for the party and one for a candidate. In others, such as Baden-Württemberg's, voters have only one vote (Salamon, Kaminski, and Flis 2026).

Ballots are often single large sheets but sometimes appear as booklets (see Fig. 4). Majoritarian ballots can also be long, although that length reflects the number of offices being filled rather than the complexity of any single voting act.

Despite their complexity, list-PR elections typically allocate seats directly from a single ballot, while preferential systems such as STV use rankings to handle elimination internally without requiring a second round. A notable exception is Armenia, which requires a stable legislative majority of 54% of seats; if no party achieves this, the two strongest parties contest a runoff (Republic of Armenia 2016). Mixed systems or semi-proportional methods may include district-level runoffs, typically held two weeks after the first round (e.g., Hungary before 2014).

4.3 Thresholds and antithresholds

Most PR systems include electoral thresholds that limit parliamentary representation to parties with sufficient nationwide support (or, less frequently, district-level support). They may include legal thresholds for parties or coalitions, “effective” thresholds connected to district magnitudes, and “antithresholds,” the rules guaranteeing some seats regardless of votes or populations.

Although thresholds generally reduce proportionality, they guard against party fragmentation, which complicates coalition negotiations. Nationwide thresholds can also exclude small but territorially concentrated parties. When coalition thresholds are set well above single-party thresholds, they incentivize merger over alliance: small parties face stronger pressure to consolidate into a single party rather than a formal electoral coalition. Majoritar-

ian systems typically lack legal thresholds because winning a majority or large plurality in a district already imposes a high effective barrier to smaller parties.

Legal thresholds vary widely. Some countries have none (Finland, the Netherlands, and Portugal). Others set low bars, such as Denmark (2%) and Mexico (3%). Israel's threshold rose gradually from 1% before 1988 to 3.25% after 2014. The most common threshold for single parties is 5%, as in Germany, Hungary, Czechia, and Poland. Turkey had the highest threshold at 10%, which was lowered in 2022 to a still elevated 7% (Lijphart 1994; Gallagher et al. 2005; ACE 2026).

Some systems also use district-level thresholds, such as Argentina's 3%. However, their independent effect is often limited because small districts already generate high "natural thresholds." Coalition thresholds may be moderately higher (8% in Poland) or steeply graduated, as in Hungary (10% for two-party coalitions and 15% for three or more).

Many systems allow threshold bypasses for recognized minorities. Poland's exemption from its 5% threshold has benefited the German minority, while Serbia's exemption from its 3% threshold mainly benefits Hungarian and Bosnian minorities. Germany's more restrictive rules cover only small, historically rooted, territorially concentrated legacy minorities, such as the Danish community, effectively leaving large Turkish, Syrian, and other immigrant minorities without exemptions.

In mixed-member systems, an alternative route past the threshold is winning a minimum number of SMD seats: five in the older South Korean MMM system, three in Germany (abolished in 2023 and reinstated by the Federal Constitutional Court in 2024) and just one in New Zealand. The low barrier in New Zealand creates strong incentives for larger parties to help smaller coalition partners clear the threshold in a single district, generating so-called trigger electorates for parties that poll 1–3% nationally but command strong pluralities in one constituency (Salamon 2024).

Even in the absence of legal thresholds, electoral systems generate implicit barriers to entry. The effective threshold depends primarily on district magnitude and can be approxi-

mated as the proportion of votes in a district equal to $t \approx \frac{1}{M+1}$ (Lijphart 1994; Taagepera 2002). Small districts effectively block smaller parties; systems with low magnitudes may thus behave much like majoritarian ones despite their formal PR labels.

At the opposite extreme are guaranteed-seat rules, which function as *antithresholds* by ensuring representation regardless of population size or vote share. The best-known examples are American states, each of which is constitutionally guaranteed at least one House seat regardless of population. Hungary and Italy similarly reserve seats for designated minorities. Romania guarantees one seat for minorities that reach a minimal-vote threshold.

Threshold effects can be dramatic, especially in emerging democracies after electoral reforms. In Poland's 1993 elections, fragmented post-Solidarity conservative parties mostly failed to clear the 5% or 8% thresholds for parties and coalitions, respectively. The two post-Communist parties—Sojusz Lewicy Demokratycznej (SLD) and Polskie Stronnictwo Ludowe (PSL)—converted 35.8% of votes into 65.9% of seats. Their four-year rule blocked lustration and produced an improvised and flawed constitution (Kaminski et al. 1998).

4.4 Disproportionality

Thresholds matter not only because they exclude parties but also because they are one of the main institutional sources of disproportionality. While “proportional” methods aim for seat shares that match vote shares, exact proportionality is practically unattainable even without thresholds due to unavoidable rounding.

Malapportionment and disproportionality are distinct concepts. Malapportionment refers to unequal voter representation across districts due to differences in district population sizes. Disproportionality refers to a mismatch between parties’ vote and seat shares. A system can translate votes into seats proportionally within districts yet still malapportion across districts—and vice versa.

A nominally proportional system can still deviate substantially from exact proportionality. Beyond the allocation algorithm, the main drivers of disproportionality are district magnitude and thresholds. Disproportionality is measured by comparing the vote and seat distributions numerically.

Typical measures of disproportionality are based on distance functions comparing two distributions (points) in an n -dimensional Cartesian space. A distance is zero if and only if the two distributions are identical, is symmetric, and satisfies the triangle inequality. The two principal measures use different metrics.

Recall that the distributions of vote and seat shares are denoted $V = (v_1, \dots, v_n)$, and $S = (s_1, \dots, s_n)$, respectively. Geometrically, all possible vectors of votes or seats form a fine grid on an $(n - 1)$ -dimensional unit simplex (the higher-dimensional analogue of an equilateral triangle). Gallagher’s (1991) index uses the standard Euclidean distance:

$$G(V, S) = \sqrt{\frac{1}{2} \sum_{i=1}^n (v_i - s_i)^2}$$

By definition, G ranges from 0 (exact proportionality) to 1 (total disproportionality). Maximum disproportionality occurs for $n = 2$ and $v_1 = s_2 = 1$ or $v_2 = s_1 = 1$; exact proportionality requires all terms $(v_i - s_i)$ to be zero.

The second popular index is that of Loosemore and Hanby (1971), which is normalized and based on the city-block metric. It returns full proportionality when the vote and seat shares match exactly, but reaches total disproportionality more readily than Gallagher’s index, specifically when no party simultaneously has positive vote and seat shares. In practice, neither index reaches 1 in real elections.

$$LH(V, S) = \frac{1}{2} \sum_{i=1}^n |v_i - s_i|$$

Both indices are often multiplied by 100 and reported as percentages. Karpov (2008) surveys 19 disproportionality indices, though the two above remain by far the most widely used.

4.5 Volatility

Measuring disproportionality is closely related to measuring electoral volatility across consecutive elections. The Pedersen (1979) index PED is structurally identical to Loosemore et al. (1971) index, substituting seat (or vote) shares in two successive elections for vote and seat shares.

$$PED(S_{t_1}, S_{t_2}) = \frac{1}{2} \sum_{i=1}^n |s_i^{t_1} - s_i^{t_2}|$$

The index can be computed for either seats or votes. It ranges over $[0,1]$, measuring how much the seat results changed between two consecutive elections: 0 indicates no change, and 1 indicates a completely different set of parties winning seats. The latter extreme occurred in Moldova between the 1994 and 1998 elections, if a new coalition CDM is counted as a new entrant (Shvetsova 2003).

High volatility indicates an unstable party system or rapidly shifting electoral alignments; low volatility suggests stable partisan attachments. Pedersen's index has been used to characterize emerging party systems, often highly volatile, relative to mature democracies (Shvetsova 2003). Grouping parties into broader blocs allows us to capture shifting bloc support rather than individual party changes caused by mergers, new entrants, or party exits between elections.

4.6 Effective number of parties

The number of parties is deceptively simple to define. Raw counts yield misleading results when many small parties exist. Singapore's 2011 legislative elections illustrate the problem: the People's Action Party (PAP) won 81 of 90 seats, while the other two parties received 8 and 1, respectively (totals include three non-constituency MP seats awarded to opposition parties). This is a dominant-party system that a simple count of three parties misrepresents.

The concept of the *effective number of parties* (Laakso and Taagepera 1979) is defined on vectors of proportions of votes or seats $(p_1, \dots, p_n)$, points on the unit simplex, to provide a better measure. It quantifies the number of parties on the continuum $[1, \infty)$, weighting parties by size. It is the reciprocal of the Herfindahl index of economic concentration. Vote ENP captures fragmentation in vote support, while seat ENP captures fragmentation in parliamentary representation.

$$ENP(p_1, \dots, p_n) = \frac{1}{\sum_{i=1}^n p_i^2}$$

The index has intuitive properties. With $n > 1$ parties, it is equal to n only when there are exactly n identical parties and always lies in $(1, n]$. With one dominant party and several small parties, it approaches 1. In Singapore's 2011 elections, ENP equals 1.22, a far more informative description than the raw count of three parties.

Party system fragmentation, operationalized by ENP, is a central variable for testing hypotheses about how district magnitudes, thresholds, and apportionment algorithms shape party systems. ENP is also used in predictive models of coalition likelihood and cabinet durability.

5 Key empirical regularities

Simple apportionment problems are well-defined mathematical entities that are amenable to axiomatic studies. Empirical electoral systems are messier: configurations of rules whose parameters change over time, and which can be studied primarily through empirical regularities drawn from typically sparse samples. The standard analytical approach broadly compares majoritarian and PR systems.

5.1 Duverger's Law

Duverger's Law encapsulates Maurice Duverger's claim (1954, 217) that "[the] simple-majority single-ballot system favors the two-party system." A closely related claim is the *Duverger hypothesis*, which holds that PR is associated with multipartyism. Although the empirical relationship is robust at the district level, Riker (1982a) noted that it does not always aggregate to a national two-party system: the United Kingdom and India remain multi-party despite using SMD plurality.

Another line of criticism concerns the implicit direction of causality. Benoit (2002) and Colomer (2005) argue for reverse causation between party system fragmentation and electoral system choice, while Kaminski (2006) models the relationship as a general equilibrium with indeterminate causality. The contemporary view treats electoral institutions as endogenous to political conditions, including party system structure, social cleavages, and anticipated electoral advantages. A second layer of endogeneity arises from strategic adaptation: voters and parties adjust their behavior in response to institutional incentives.

Duvergerian dynamics operate through three effects, the first two identified by Duverger (1951).

The *mechanical effect* refers to how electoral rules translate votes into the number of seats. When $M = 1$, only one party wins and all other votes are wasted. As the magnitude rises, the effective threshold—the vote share needed to win a seat—falls (see Sect. 4.3). A ten-seat district has an effective threshold of approximately 9%, while a single-seat district has one of 50% (under plurality, parties can in practice win at lower vote shares with a fragmented opposition). Lower thresholds allow smaller parties to survive.

The *psychological effect* compounds this. Voters expecting a small party to fall short will strategically desert it for a larger, less preferred but viable party. Candidates and party leaders also avoid investing in small parties with limited prospects. The two effects reinforce each other: low magnitude drives strategic concentration, while high magnitude removes those incentives and allows smaller parties to persist.

Finally, the *strategic* or *coalitional* effect captures parties' greater incentive to unite. Rather than competing independently with low odds, parties merge or consolidate to create entities with much better chances of winning single-member districts (Kaminski 1994; Clark and Golder 2006).

Table 4 Typical party systems corresponding to different district magnitudes and threshold levels

District magnitude	System type	Typical outcome
$M = 1$	SMD / Plurality	Two parties dominate
$M \in \{3, 4, 5\}$	Small-district PR	3–5 parties viable
$M \in [10, 20]$	Mid-size PR district	5–8 parties viable
$M = L$	National-list PR	Many parties compete

Outcomes reflect tendencies under typical conditions and vary with thresholds, social cleavages, and other institutional factors. L is the legislature's size.

Improved polling and computation tools have enabled a related but distinct strategy: coordinated candidate withdrawal. Rather than merging, allied parties agree that A withdraws in districts where B is stronger and B reciprocates elsewhere. Both parties gain against a third competitor who would otherwise win both seats. This tactic helped the Ensemble and the New Popular Front defeat the favored National Rally in France's 2024 legislative elections (Flis and Kaminski 2026). Notwithstanding such innovations, PR systems remain strongly correlated with multipartyism, while majoritarian systems favor two-party competition.

5.2 The impact of district magnitude

The three Duvergerian effects operate not only across the majoritarian–PR divide but also *within* PR: an increasing district magnitude consistently produces greater partisan fragmentation (Rae 1967; Taagepera and Shugart 1989; Blais et al. 2006).

Rae (1967) first systematically linked district magnitude to party system fragmentation across democracies. Taagepera and Shugart (1989, 1993) formalized this relationship quantitatively, finding it curvilinear: each additional unit of magnitude matters more at low values than at high ones. Going from $M = 1$ to $M = 5$ has a dramatic effect on party viability; going from $M = 15$ to $M = 20$ has a relatively modest effect (see Table 4). Taagepera (1999) later derived a predictive equation linking ENP to both the district magnitude and the number of political issue dimensions.

5.3 Women and minority representation

PR systems tend to increase the inclusion of women and minorities in politics. Paxton et al. (2010) find that PR countries consistently had about 2.5% points more women in parliament than majoritarian ones, with this advantage remaining stable between 1975 and 2000 rather than growing over time, as some theories had predicted. Högström (2025) confirms this for countries that switched from majoritarian to PR and finds no meaningful difference for those that switched from mixed to PR.

Similar patterns are observed for minority inclusion. Using cross-national data, Moser (2008) shows that proportional and mixed systems are generally more effective at translating minority votes into legislative seats than majoritarian systems.

Parties also influence each other in PR. Matland et al. (1996) found that in Norway, when smaller parties added more women to their lists, larger parties followed suit; in some districts, Labor Party lists carried 8.7–17.4% more women as a result. By contrast, this “contagion effect” did not appear in Canada's SMD system.

The explanation centers on party strategy. In list PR, parties have an incentive to balance lists by gender, region, or ethnicity to broaden their appeal. PR systems also reduce the political risk to incumbents of accommodating new groups, since multi-member districts allow inclusion without displacing sitting candidates.

Many countries have gone beyond party incentives by introducing statutory gender quotas. Candidate list quotas require parties to place a minimum share of women (typically 30–50%) on their candidate lists. Zipper rules mandate alternation between male and female candidates, so women are not clustered at the bottom, where they are unlikely to win. Reserved seats, a distinct mechanism more common in majoritarian systems, guarantee specific parliamentary seats for women or minorities regardless of vote results. Countries that combine statutory quotas with PR experienced faster growth in women's legislative representation than those that rely on voluntary party measures (Krook 2010; Dahlerup 2013).

A persistent criticism in the PR literature focuses on who gets elected without asking what policy difference this makes: descriptive representation (who holds office) does not automatically translate into substantive representation (what policies are adopted). A growing body of research supports this cautious view.

Franceschet et al. (2008) illustrate this ambiguity: in Argentina, quotas increased women's presence but had uneven effects on legislation and agenda-setting, with outcomes depending heavily on party dynamics and institutional context. Hessami et al. (2020) review finds that in developed countries higher women's representation often does not significantly alter public spending, though in developing countries it is associated with greater provision of public goods such as education. Kläy et al. (2025) find that once party positions are controlled for, female legislators do not systematically differ from male legislators in representing women's preferences in the legislature.

Collectively, these studies support a qualified view: PR and quotas reliably increase descriptive representation, but their link to substantive representation is weaker and conditional. Party discipline, coalition bargaining, and platform constraints often shape policy outcomes more than the personal characteristics of legislators, meaning that even systems with fewer women or minority representatives may produce strong policy responsiveness if parties directly target those voter blocs.

5.4 Voter turnout

Voter turnout is one of the fundamental parameters characterizing elections (Downs 1957; Riker and Ordeshook 1968). PR systems are generally associated with higher turnout. Karp and Banducci (2008) estimate the PR advantage at about 5% points over plurality or majoritarian systems. The basic mechanism is intuitive: proportional allocation makes voters more likely to feel that their vote counts, primarily strengthening external efficacy (the belief that the political system is responsive to one's vote), which in turn encourages turnout.

Party attachments reinforce this: PR typically sustains a wider range of parties, giving voters more meaningful choices and a better chance of finding a party that genuinely reflects their preferences.

The before-and-after evidence is compelling. Gathmann (2019) exploits the staggered adoption of PR across 18 of 25 Swiss cantons between 1890 and 1950 as a natural experiment, finding that PR adoption raised cantonal parliamentary turnout by an average of 14% points.

The turnout advantage is not automatic. Post-election dynamics matter: broad governing coalitions can make voters feel that their choices were blurred or diluted, dampening their motivation to vote in future elections. Brockington (2004) shows that oversized coalitions—those exceeding the minimum needed to govern—depress turnout, particularly among supporters of major parties. Moreover, in MMM systems, turnout may be depressed in larger districts (Keena 2025).

A methodological caveat: Several high-turnout PR countries also mandate voting. Belgium, Luxembourg, and Australia (for Senate elections) combine PR with compulsory voting, which independently raises turnout by 10–15% points (Lijphart 1997). Cross-national comparisons should disentangle the electoral system's effect from the compulsory voting effect.

A further complication concerns vote quality, not just quantity. Flis and Kaminski (2025a) argue that OLPR imposes greater cognitive demands than SMD systems, producing three categories of “faulty” votes: invalid ballots, votes for the first-listed party due solely to the list position, and votes for the top candidate on the list due solely to that privileged position. Using simultaneous Polish local elections held under different electoral rules, they estimate that roughly 13% points more eligible voters cast a faulty vote under OLPR than under SMD, a gap comparable in magnitude to the PR turnout boost but working in the opposite direction.

On balance, PR raises participation by making more voters feel that their votes matter, but this advantage can be offset by coalition dynamics that blur accountability and by the cognitive complexity of open-list ballots. The net result is that PR can inflate raw participation while degrading vote quality.

5.5 Coalition formation and governability

A key consequence of PR is that it typically produces multiparty parliaments in which no single party holds a majority, making government formation a matter of negotiation. Scholars distinguish pre-election coalitions, where parties publicly align before voting (Kaminski 2001; Golder 2006), from post-election bargaining once election results are known (Lijphart 1984). Both occur in PR systems, but post-election bargaining is more prevalent. The head of state appoints a *formateur*, usually the largest party leader, to assemble a coalition capable of winning parliamentary investiture (Laver and Shepsle 1996; Laver et al. 1998).

Coalition research has identified several recurring patterns. Minimal winning coalitions (von Neumann and Morgenstern 1944; Riker 1962; Axelrod 1970)—those that would lose their majority if any member exited—are theoretically efficient but are less common in practice than expected. Oversized coalitions form when parties prioritize stability or broad legitimacy over policy coherence. Minority governments, common in Scandinavian PR systems, survive by securing legislative support from outside the cabinet on a case-by-case basis. Cabinet durability is negatively related to the number of partners, their ideological distance, and parliamentary fragmentation—all factors that PR systems tend to amplify by supporting more parties (Laver and Shepsle 1990; Müller et al. 2003).

PR can produce broad and fractious coalitions. Voters may not know in advance how their votes will translate into governing parties or policies, and the transaction costs of coalition formation can be especially high in countries such as Israel, the Netherlands, and Bulgaria, which have seen long runs of inconclusive elections producing untenable coalitions.

Majoritarian systems, typically with just two parties, tend to produce more stable cabinets and clearer voter accountability (Powell and Whitten 1993; Powell 2000). In single-party governments, accountability is straightforward; in coalition governments, shared responsibility blurs lines of credit and blame. This tension between proportionality and accountability is central to normative debates about electoral system design.

Accountability and stability are only indirectly determined by the electoral system; the more direct influence is through the number of parties. A PR system with high enough thresholds and low enough district magnitudes can reach a “sweet spot” of moderate consolidation that captures some governability benefits of majoritarian systems (Carey et al. 2011).

5.6 Strategic voting

Strategic voting is a well-documented feature in electoral behavior. Although it is most commonly associated with majoritarian systems, it also arises under PR, contributing to Duvergerian psychological effects (Cox 1997). These pressures are typically weaker under PR and operate locally—at the district or threshold level—rather than generating system-wide two-party consolidation. Their prevalence depends on the availability of information about party viability, including polls, past results, and elite signals (Cox 1997; Blais 2000).

Under PR, strategic voting takes more complex forms and depends heavily on institutional details. Even with a single vote, voters may support not their most preferred party but a viable ally that needs additional votes to clear a legal threshold. Three main patterns emerge: threshold insurance voting, coalition-oriented voting, and pivotality-based voting, whereby voters desert parties unlikely to clear thresholds, coordinate around viable coalition partners, or back parties expected to be pivotal in government formation (Gschwend 2007; Bargsted et al. 2009). These incentives intensify in systems with low district magnitudes and high thresholds.

In STV, strategic coordination operates both within parties, balancing first-preference votes across co-partisan candidates, and across parties by facilitating favorable transfer flows between allies. Parties typically advise supporters on how to rank their lower preferences to ensure efficient transfers to co-partisans or allied candidates. These practices, commonly referred to as vote management, transfer management, or vote pooling (Farrell and McAllister 2006; Mitchell 2014), aim to maximize seat yields by coordinating first-preference distributions and subsequent transfers. Surplus votes from a party that has already secured its seats or transfers from an eliminated smaller party can thus be directed toward allied candidates.

In dual-ballot systems such as MMP or MMM, strategic behavior frequently takes the form of split-ticket voting, whereby voters support different parties in the candidate and list tiers (Herron et al. 2001). This can be interpreted as a rational, risk-mitigating response to uncertainty regarding electoral viability and government formation (Shikano and Herron 2025). Institutional features, such as trigger electorates and decoy parties (discussed in Sect. 4.3 and 6.5), create additional structured opportunities for strategic coordination by voters and parties alike. Split-ticket voting may either reinforce or undermine the compensatory logic of mixed systems, contributing to the contamination effects between tiers.

5.7 PR in divided societies

A substantial comparative literature identifies PR as a key institutional tool for managing ethnic and religious conflict in societies with deep, politically salient cleavages. In consociational frameworks, PR facilitates power-sharing: by securing representation for segmental groups, it reduces the cost of electoral defeat and supports complementary arrangements such as grand coalitions, mutual vetoes, proportional resource allocation, and segmental autonomy (Lijphart 1977, 1999). The underlying logic is probabilistic inclusion. More proportional seat allocation lowers the risk of permanent minority exclusion, thereby weakening incentives for extra-institutional mobilization. This rationale shaped institutional design in Belgium, Northern Ireland, Bosnia-Herzegovina, and South Africa, where electoral rules were deliberately chosen to reassure historically marginalized communities.

A competing tradition, however, questions whether PR stabilizes divided societies or merely freezes their divisions. By lowering entry barriers and rewarding cohesive group voting, PR may encourage ethnically or religiously segmented party systems and reinforce elite incentives to mobilize along communal lines. On this view, PR displaces conflict from violent to electoral arenas without advancing intergroup accommodation. “Centripetal” theorists respond by advocating designs that induce cross-group vote pooling, such as the Alternative Vote, which rewards candidates capable of attracting secondary support beyond their core constituency (Horowitz 2000; Reilly 2001). The empirical record suggests both logics hold under particular conditions: PR manages conflict most effectively when paired with institutions that promote inter-elite cooperation and when group identities are organized but not fully polarized. Otherwise, it risks entrenching segmentation without generating moderation. The choice between proportional and centripetal designs therefore reflects a fundamental trade-off between guaranteed inclusion and incentives for cross-group accommodation.

5.8 Other findings

The empirical literature on PR is too extensive to be fully surveyed here. For example, PR systems are associated with larger welfare states, higher public spending, and more redistributive policies (Persson and Tabellini 2005; Iversen and Soskice 2006; Lijphart 1999), although the causal links remain contested. Acemoglu (2005) highlighted pervasive identification problems, notably institutional endogeneity (electoral systems are not randomly assigned) and omitted-variable bias, which apply broadly across the empirical literature surveyed in this section.

Empirical regularities are not the only basis for evaluating electoral systems. A complementary approach, rooted in social choice theory, asks whether a system’s rules can produce outcomes that violate the properties that the system was ostensibly designed to satisfy. Such violations—commonly called paradoxes—reveal structural vulnerabilities that no amount of empirical data can detect or resolve because they follow from the rules themselves.

6 Paradoxes and problems

The catalogue of known paradoxes is extensive. The discussion includes a small set of examples that illustrate both the counterintuitive properties of voting rules and their practical implications for system design and reform. Many of these results reflect deeper impossibility theorems in social choice theory, which show that no voting rule can simultaneously satisfy even a modest set of fairness and rationality conditions (Arrow 1951) or be fully strategy-proof (Gibbard 1973; Satterthwaite 1975). Black (1958) stands as a notable exception, demonstrating that under the empirically plausible condition of single-peaked preferences, majority rule performs well. The broader implication is that every electoral system is imperfect; the relevant question is not which system is ideal, but which trade-offs are acceptable in a given constitutional context (Riker 1982b; Gallagher 1992; Nurmi 1999).

6.1 Apportionment paradoxes

The analysis begins by listing informally the “seemingly obvious” properties satisfied by all popular quota and divisor methods (Young 1994). Ranked-choice methods have different inputs, so their axioms are formulated separately.

Three fairness conditions are typically imposed on simple apportionment methods. *Symmetry* requires that two parties with identical vote totals receive identical seat totals, ensuring that the method does not discriminate between parties on any basis other than votes. *Scalability* strengthens this by requiring that multiplying all parties’ vote totals by the same positive integer leaves the allocation unchanged, so that only relative—not absolute—vote shares determine outcomes. Finally, *weak monotonicity* requires that a party receiving strictly more votes than another must receive at least as many seats, guaranteeing that casting additional votes can never disadvantage a party relative to one that performed worse.

The three properties are satisfied by all quota and divisor methods. The next one is not.

Population Monotonicity states that when states’ populations are recalculated, a state that gains population cannot lose a seat to a state that loses population.

HHN violates this property (see Table 5).

In this example, A, B, and C initially receive 1, 1, and 2 seats, respectively. When A’s population grows by 1 and C’s falls by 1, A nonetheless loses a seat while C gains one. The culprit is B, which loses a large share of its population, inflating C’s remainder more than A’s and overriding direct population changes.

With divisor methods, a state’s priority numbers move in the same direction as its population; all such methods satisfy Population Monotonicity.

Table 5 Example: HHN quota method violates population monotonicity. *Source:* Young (1994, 140–42)

State	Census T1	Quota $N=4$	Seats	Census T2	Quota $N=4$	Seats
A	41	0.41	1	42	0.47	0
B	120	1.2	1	80	0.89	1
C	239	2.39	2	238	2.64	3

6.2 Balinski–Young impossibility theorem and the Alabama paradox

All PR methods are vulnerable to some paradoxes. The elegantly simple Balinski–Young Theorem captures one universal impossibility: two basic properties, Staying within the Quota and Seat Monotonicity, cannot be satisfied simultaneously.

Staying within the Quota *Each party receives either the floor or the ceiling of its HHN quota.*

The justification for this is straightforward. The HHN quota is the exactly proportional (possibly fractional) seat count. Since fractional seats are infeasible, requiring the allocation to round this ideal number up or down is natural.

Seat monotonicity *When the number of seats to be divided, M , increases, no party loses a seat.*

The Alabama paradox that motivated Balinski and Young was discovered accidentally. In 1880, Census Bureau chief clerk C. W. Seaton observed that under HHN, Alabama would receive eight seats with a House size of 299 but only seven with a House of 300. The ensuing political uproar eventually prompted Congress to abandon HHN in favor of Webster-Sainte-Laguë method in 1911 (later replaced by Huntington-Hill in 1941).

Young (1994) presents the theorem in its most concise form: the two principles are incompatible.

Theorem [Balinski et al., 1982]: No simple apportionment method simultaneously satisfies Staying within the Quota and Seat Monotonicity.

Young (1994) used a slightly weaker version of *Seat Monotonicity* called *House Monotonicity*, where the increase of M is by just one seat. It is relatively easy to prove that all divisor methods satisfy Seat Monotonicity. Therefore, all such methods must violate Staying within the Quota. By contrast, the HHN method satisfies Staying within the Quota by definition. Thus, it must violate Seat Monotonicity, as discovered in 1880.

The consequences of this theorem are far-reaching. Every simple apportionment method is flawed. This guaranteed flaw in every method fuels political manipulation: politicians can always identify a defect in the current method and argue for its replacement, even when their true motivation is partisan. The replacement is, however, equally vulnerable and may eventually face the same challenge.

6.3 STV: Non-Monotonicity and the No-Show paradox

STV is not free of paradoxes (Doron et al. 1977; Fishburn et al. 1983). The most criticized aspect is Non-Monotonicity: a winning candidate can be harmed by a voter upgrading the ranking.

Candidate monotonicity *If a voter moves a winning candidate up in the ranking and nothing else changes, the candidate continues to win.*

Consider the following example with four candidates competing for two seats, which illustrates the problem. Assume that the tiebreaker in case of identical numbers of first votes is the greater number of second votes.

6 voters: xywz

3 voters: wyxz

4 voters: yxzw

3 voters: zwyx

1 voter: zywx

The Droop quota is six. Candidate x has exactly six first-preference votes and wins with no surplus to transfer. Candidate w has only three votes while z and y have four; thus, w is eliminated, and its three votes transfer to y, and y wins the second seat with seven votes.

Now, suppose the last voter moves y above z, presenting the ranking yzwx. Candidates z and w have three first-preference votes. Using the tiebreaker, z has one second vote while w has three. Candidate z is now eliminated, z's three votes transfer to w, and w wins the second seat with six votes, leaving y with only five votes.

Related is the No-Show paradox, where a voter is better off abstaining than voting sincerely (Moulin 1988; Fishburn et al. 1983). Fortunately, both paradoxes arise infrequently in practice.

6.4 List PR: spoilers and primacy effects

PR systems exhibit many paradoxical or problematic effects (Kurrild-Klitgaard 2008, 2013). The two discussed below—spoilers and primacy effects—also occur in majoritarian systems but can be more complex and sometimes more severe under PR.

Spoilers. In single-member districts, a spoiler is straightforward: a non-winning candidate whose entry changes who wins. Under PR, the spoiler phenomenon takes a richer form, although the SMD-style classic spoiler is rare. The closest analog is the Kingmaker: a party whose entry converts another party from a non-majority to a majority winner, especially when the Kingmaker itself wins no seats (Kaminski 2018). Symbols s_j^- and s_j^+ below denote the seats of party P_j competing in the elections without and with P_i competing, respectively.

Kingmaker spoiler Party P_i is a Kingmaker if there is another party P_j such that $s_j^- \leq 50\%$ and $s_j^+ > 50\%$.

A Kingmaker appeared in the 2015 Polish legislative elections in the form of a small leftist party *Razem (Together)*, which entered the race at the last minute and fell short of the 5% threshold with 3.62% of votes. Its entry nonetheless pushed the larger United Left coalition below the 8% coalition threshold, which it failed to clear with only 7.55% of the votes. Law and Justice (PiS) was the main beneficiary of the resulting larger pool of available seats. PiS won 51.09% of seats with *Razem* competing versus the estimated 46.3% without. Across eight Polish legislative elections from 1991 to 2015, Kaminski (2018) identifies 16 spoilers of various types, two spoiler threats, and three last-minute potential spoilers absorbed by larger parties.

Primacy effects: party primacy and list primacy. Two well-documented ballot-position effects distort voter choices in PR systems (Van Erkel and Thijssen 2016; Flis and Kaminski

2022), both of which are substantially larger than their typical majoritarian counterparts. *Party primacy* refers to voters' tendency to select the party that appears first on the ballot, regardless of their policy preferences. The effect is especially pronounced on large multi-party ballots, where cognitive load is high, and can be amplified by poor ballot design, such as a booklet format (Pierzgalski et al. 2020).

List primacy refers to the tendency in open-list systems to favor the top-listed candidate because of that candidate's privileged position. Both effects give parties and electoral administrators significant—and potentially manipulable—influence over outcomes through ballot ordering decisions. Randomized ballot ordering across polling stations has been adopted as a remedy in majoritarian systems but remains uncommon in PR elections.

6.5 MMP: overhang and orphaned seats, decoy parties

Around 2000, electoral reformers hailed MMP systems as “the best of both worlds” (Shugart and Wattenberg 2001a). Greater experience with MMP and other variants has revealed significant shortcomings. The root cause in MMP is the “contamination” of one tier by the other, or the interaction between them, shaping strategies and outcomes in surprising ways (Herron et al. 2001; Vučićević 2025). Among the issues are excessive complexity, “zombie candidates” (electing SMD losers in PR tiers), and uncontrolled growth of parliaments (Salamon, Kaminski, and Flis 2026). The most serious of these shortcomings are the overhang seats and decoy parties (Bochsler 2012).

When a party wins more SMD seats than its PR entitlement, it keeps the surplus — known as *overhang seats* (*Überhangmandate*). To restore proportionality, *leveling seats* must be added for other parties, expanding the legislature.

Consider an example of an MMP system with 10 seats assigned via PR and 10 seats assigned in SMDs. The results of the two parties competing in the elections are presented in Table 6.

Party A won 20% of party votes, entitling it to 4 seats total, but won 6 SMD seats; under strict proportionality, A's compensatory “adjustment” would have to be -2 seats, which is infeasible. Conversely, Party B is entitled to 12 PR seats, but only 10 are available, leaving B two seats short.

The overhang-seat solution is to create two extra seats for B, but the resulting seat shares for A (6) and B (12) are still far from proportionality. Achieving full proportionality requires further upward adjustment to B's total of 12 more seats to maintain proportional ratios of 20:80. New Zealand and other countries have adopted this approach. Germany followed this approach through the 2021 election, by which point the Bundestag had swelled from the default 598 to 736 seats (Behnke 2025). The 2023 reform, effective in 2025, replaced overhang compensation with a rule that effectively cancels district seats when a party wins

Table 6 Example: problem with overhang seats

Party	Party votes	SMD seats	Total seats entitlement (set by PR votes)	Additional PR seats
A	20%	6	4	$4 - 6 = -2$
B	80%	4	16	$16 - 4 = 12$

more than its PR entitlement, creating “orphaned constituencies.” Critics have argued that this replaces one problem with another (Flis et al. 2025a).

Overhang seats are a mechanical problem—an unintended consequence of the gap between a party’s majoritarian success and its proportional entitlement. A more serious vulnerability arises when political actors strategically exploit this gap by creating *decoy parties*. When a party wins so many majoritarian seats that it is entitled to no compensatory PR seats, its PR votes are wasted from a compensation standpoint. The party can then create a shell organization—a decoy—and encourage its supporters to cast their PR votes for it, capturing additional PR seats that the compensatory mechanism was never designed to provide. Alternatively, it can redirect votes to a smaller allied party positioned as a future coalition partner.

The same example illustrates this logic. Party A, entitled to no compensatory PR seats, holds 20% of party votes, all of which are wasted. If it creates decoy party A1 and redirects all PR votes there, A retains its six majoritarian seats while A1 gains extra PR seats—a net gain manufactured entirely through strategic party creation. However, under the “orphaned constituencies” rule, if A moved all its support to A1, such a move would cancel all its majoritarian seats, and A would not receive any seats. This would eliminate A’s decoy incentive but at a steep price: the majoritarian tier would shrink to four seats won by B, as the six majoritarian seats won by A would be converted to PR seats. With 16 PR and 4 majoritarian seats, the system would scarcely qualify as mixed.

Decoy parties have proved not a theoretical curiosity but a recurring practical problem, emerging in Venezuela, Albania, Italy, Lesotho, South Korea, and Hungary. The first four countries responded by abandoning the compensatory design altogether, replacing it with a parallel system (Venezuela), list PR (Albania), idiosyncratic hybrid (Italy), or single-vote rule (Lesotho) (Shugart 2026; Chiaramonte and Paparo 2025; Tanács-Mandák and Horváth 2025; Moon et al. 2025). Japan illustrates a similar but distinct dynamic under MMM, where smaller parties began coordinating SMD candidacies with larger allies, effectively exchanging district support for PR votes (Maeda 2025; Flis and Kaminski 2026).

7 Conclusion

The primer began with the aspiration that legislative seats should reflect the distribution of voter preferences and traced the distance between that aspiration and concrete implementations. Three findings from the preceding sections deserve emphasis because together they define the landscape within which electoral system design necessarily operates.

First, no apportionment method is free from formal defects. The Balinski–Young Theorem guarantees that every method either violates Staying within the Quota (deviating from proportionality) or violates Seat Monotonicity (some state loses a seat when the legislature grows). STV is vulnerable to Non-Monotonicity and No-Show paradoxes. These are not failures of ingenuity; they are mathematical constraints. Because every method is demonstrably flawed, reform debates are permanent and politically motivated: incumbents can always identify a defect in the current system and propose a replacement that favors their party, knowing that the replacement will be equally vulnerable.

Second, the institutional parameters that shape PR outcomes—such as district magnitude, legal thresholds, the choice between open and closed lists, and the linkage between

tiers in mixed systems—do not produce simple, predictable trade-offs between competing values. The same threshold that guards against fragmentation can, as the 1993 Polish elections showed, convert a 35.8% vote share into a nearly two-thirds legislative majority. The same compensatory mechanism that makes mixed systems “the best of both worlds” also creates the incentive structures that produced decoy parties in Italy, Lesotho, Albania, and Venezuela, forcing each of those countries to modify the system. These are not edge cases; they are the normal operation of institutional parameters under conditions of strategic behavior.

Third, the empirical regularities that anchor comparative research—the magnitude–fragmentation relationship, the PR turnout advantage, and the association between PR and women’s representation—are robust as correlations, but the causal claims that depend on them rest on identification strategies that cannot fully resolve the endogeneity of electoral institutions. Electoral systems are chosen by political actors with interests; they are not randomly assigned to countries. This does not invalidate the findings surveyed here, but it means that confident causal claims require the kind of quasi-experimental or institutional variation—such as Swiss cantonal adoption of PR or simultaneous Polish local elections under different rules—that remains scarce.

The electoral systems literature continues to evolve along several active frontiers. Computational tools and large-scale data now allow researchers to simulate the distributional consequences of electoral rules with a precision unavailable to earlier generations. Once-theoretical questions—such as how sensitive party seat share is to small changes in district magnitude, or how often apportionment paradoxes actually occur—are now empirically tractable problems. New institutional challenges have also emerged: AI-assisted redistricting raises questions about the boundary between technical optimization and partisan manipulation. Digital and ranked ballots test voter comprehension in ways that classic ballot-design research did not anticipate, and new compensatory mechanisms in mixed systems continue to generate both formal paradoxes and practical reform proposals. The canonical texts surveyed in this primer provide the necessary foundations, but they are starting points, not endpoints, in an active and expanding literature.

Appendix: glossary of proportional representation terms

How to use this glossary Terms appear in alphabetical order. Abbreviations are given in parentheses after the term.

A

Alabama paradox. A famous paradox in apportionment, discovered in 1880, in which increasing the total number of seats to be distributed causes a particular state (or party) to lose a seat. The paradox arises under the Hamilton–Hare–Niemeyer quota method and motivated the switch to divisor methods in the United States. See: Balinski–Young impossibility theorem; Hamilton–Hare–Niemeyer method; Seat Monotonicity.

Alternative vote (AV). A ranked majoritarian system (also called Instant-Runoff Voting) in which voters rank candidates and the weakest candidates are eliminated sequentially, their votes redistributed, until one candidate holds a majority. Used in Australian House

elections. Not a PR method, but mentioned as a comparator. See: ranked-choice systems; Single Transferable Vote.

Antithreshold. A constitutional or statutory guarantee of at least one seat regardless of vote share. Well-known examples include the U.S. guarantee of one House seat per state and reserved minority seats in Hungary, Italy, and Romania. The logical opposite of an electoral threshold. See: electoral threshold.

Apportionment. The mathematical problem of dividing a fixed number of seats (M) among competing units—states, parties, or districts—in proportion to their populations or votes. Three principal families of solutions exist: quota methods, divisor methods, and ranked methods (STV). See: quota methods; divisor methods; Single Transferable Vote.

B

Balinski–Young impossibility theorem. A fundamental result (Balinski et al. 1982) proving that no apportionment method can simultaneously satisfy Staying within the Quota and Seat Monotonicity. Because divisor methods always satisfy Seat Monotonicity, they must violate Staying within the Quota; because the Hamilton–Hare–Niemeyer method always stays within the quota, it must violate Seat Monotonicity (as the Alabama paradox illustrates). The theorem implies that every simple apportionment method is flawed in some respect. See: Alabama paradox; Hamilton–Hare–Niemeyer method; divisor methods; quota methods.

Ballot. The physical or electronic form on which voters express their choices. In PR systems, ballots are typically more complex than in single-member district systems, often presenting multiple parties and individual candidates; in open-list systems, voters may cast one or more preference votes for candidates within a chosen party list. See: closed-list PR; open-list PR; primacy effects.

C

Cabinet durability. The length of time a government cabinet remains in office without collapse, replacement, or major reshuffle. Cabinet durability is negatively related to the number of coalition partners, their ideological distance, and overall parliamentary fragmentation (Laver and Shepsle 1990; Müller et al. 2003)—factors that PR systems tend to amplify by sustaining more parties. Single-party cabinets under majoritarian rules are typically more durable than PR coalition cabinets, though Scandinavian minority governments show that durability also depends on parliamentary culture and institutional context. See: minimal winning coalition; oversized coalition; minority government; effective number of parties; formateur.

Candidate monotonicity. A desirable property of a voting method stating that if a voter moves a winning candidate higher in their ranking, that candidate should not thereby be caused to lose. Single Transferable Vote (STV) violates this property, meaning a candidate can paradoxically be harmed by receiving additional support. See: Single Transferable Vote; Monotonicity.

Candidate selection. The intra-party process by which parties decide who appears on their lists or runs as their candidate, and in what order. In closed-list PR, candidate selection effectively determines who enters parliament, since voters cannot reorder lists; in open-list

and STV systems, voters share this power with party leaders. The rules and incentives of candidate selection shape party cohesion, legislator independence, and the prevalence of clientelistic exchange. See: closed-list PR; open-list PR; list PR; primacy effects.

Categorical ballot. A ballot on which voters make one or a small number of discrete choices—a party, a candidate, or both—rather than providing a ranking. Closed-list PR ballots, open-list PR ballots with a single preference vote, and standard FPTP ballots are all categorical. Contrasted with preferential ballots, which require voters to order options and impose higher cognitive demands (Reynolds and Steenbergen 2006). See: preferential ballot; ballot; closed-list PR; open-list PR; First-Past-The-Post.

Centripetal incentives. Institutional arrangements that reward politicians for appealing across ethnic, religious, or other social cleavages rather than mobilizing within them. Associated with Donald Horowitz (1985), who argued that vote-pooling rules such as Alternative Vote in heterogeneous constituencies can dampen ethnic conflict more effectively than the consociational use of PR, which (critics argue) may entrench communal divisions. See: consociational democracy; Alternative Vote.

Closed-list proportional representation (CLPR). A list PR system in which voters cast a single vote for a party, not for individual candidates. Seats won by the party are allocated to candidates in the fixed order determined by the party leadership. Used in Spain, Portugal, and South Africa, among others. Tends to strengthen party discipline and reduce candidate independence. See: open-list PR; flexible-list PR; list PR.

Coalitional effect. One of Duverger's three effects: the tendency of parties facing majoritarian rules to merge or form alliances in order to improve their chances of winning single-member contests. Distinct from the mechanical and psychological effects in that it operates through party organization rather than directly through votes or seats. See: Duverger's Law; mechanical effect; psychological effect.

Coalition-oriented voting. A form of strategic voting under PR in which voters cast their ballot to influence the composition of the future governing coalition rather than only to express a preference for an individual party. Voters may support a smaller coalition partner of their preferred large party, or shift toward a party expected to be a pivotal coalition member (Bargsted et al. 2009). See: strategic voting; threshold insurance voting; pivotality-based voting; pre-election coalition.

Coalition threshold. A higher legal threshold applied to electoral coalitions of two or more parties than to single parties. In Poland, single parties face a 5% threshold while coalitions face 8%. In Hungary, coalitions face a steeply graduated threshold (10% for two-party coalitions, 15% for three or more). Coalition thresholds discourage permanent electoral alliances and can dramatically reshape vote-to-seat translation when coalitions narrowly fail to clear them. See: electoral threshold; effective threshold.

Compensatory mechanism. The general feature of mixed and multi-tier PR systems whereby seats allocated in one tier (typically a list-PR or national tier) correct disproportionalities produced by another tier (typically single-member districts or low-magnitude regions). Fully compensatory systems target overall proportionality (Germany, New Zealand); partially compensatory systems compensate only within sub-national regions or with limited scope. The non-compensatory alternative mixed system uses parallel allocation. See: Mixed-Member Proportional system; Mixed-Member Parallel system; overhang seats.

Compulsory voting. A legal requirement that eligible citizens vote in elections, often enforced by modest fines for non-participation. Used in Belgium, Luxembourg, and (for

Senate elections) Australia, among others. Compulsory voting independently raises turnout by 10–15% points and is a methodological confound in studies of the PR-turnout association: cross-national comparisons must disentangle the electoral system effect from the compulsory voting effect (Lijphart 1997). See: voter turnout.

Consociational democracy. A model of democratic governance, developed by Arend Lijphart, in which deeply divided societies are stabilized through power-sharing rather than majority rule. Its four pillars are grand coalition government, mutual veto rights for segmental groups, segmental autonomy, and proportionality (typically through PR elections). PR plays a central role by ensuring inclusive representation of all major segments and by lowering the stakes of electoral defeat. Examples include Belgium, the Netherlands, Switzerland, Northern Ireland, Bosnia and Herzegovina, and post-apartheid South Africa. Critics argue that consociationalism may entrench communal divisions; centripetal alternatives have been proposed (Horowitz 1985). See: power-sharing; centripetal incentives; proportional representation.

Contamination effects. In mixed electoral systems, the unintended interaction between the majoritarian and PR tiers, in which candidate strategies, party behavior, or voter choices in one tier are shaped by the other tier (Herron et al. 2001). The term captures phenomena such as parties fielding district candidates primarily to mobilize PR votes, supporters of large parties redirecting their list votes through decoy parties, and split-ticket voting that crosses tier boundaries. Contamination is the underlying mechanism behind several pathologies of mixed systems. See: Mixed-Member Proportional system; Mixed-Member Parallel system; decoy party; split-ticket voting.

Coordinated candidate withdrawal. A strategic alternative to party merger under majoritarian rules in which allied parties agree that party A withdraws its candidate in districts where B is stronger and B reciprocates elsewhere, consolidating the anti-opponent vote without the costs of formal unification. Deployed successfully against the National Rally in France's 2024 legislative elections, where Ensemble and the New Popular Front coordinated withdrawals on a district-by-district basis (Flis and Kaminski in press). Differs from the coalitional effect of Duverger's Law in that parties retain separate identities, separate lists, and separate programs. See: coalitional effect; Duverger's Law; pre-election coalition; strategic voting.

Cumulative voting (CV). A semi-proportional system in which each voter holds k votes (where $k \leq M$, the district magnitude) and may allocate multiple votes to the same candidate. Used in some corporate board elections and historically in the Illinois House of Representatives. See: semi-proportional systems; Limited Vote; Single Non-Transferable Vote.

D

Decoy party. In a Mixed-Member Proportional (MMP) system, a satellite party created by or affiliated with a larger party to capture proportional-tier (PR) votes that would otherwise be wasted. Because the larger party wins enough single-member district seats to exhaust its PR entitlement, it encourages supporters to cast their party vote for the decoy, which then wins additional seats. A documented problem in Italy, Lesotho, Albania, Venezuela, and Hungary. See: Mixed-Member Proportional system; overhang seats; wasted votes.

Descriptive representation. Representation in the sense of 'looking like' the electorate—having women, minorities, or other demographic groups present in the legislature in

numbers roughly proportional to their share of the population. Distinct from substantive representation, which concerns whether policies actually reflect those groups' interests.

Disproportionality. The degree to which the distribution of legislative seats deviates from the distribution of votes. Even systems designed to be proportional will exhibit some disproportionality due to rounding, district magnitude effects, and electoral thresholds. Measured by indices such as the Gallagher index and the Loosemore–Hanby index. See: Gallagher index; Loosemore–Hanby index; district magnitude; electoral threshold.

District magnitude (M). The number of seats allocated within a single electoral district. District magnitude is considered the single most powerful institutional determinant of party system fragmentation: larger districts allow more parties to win seats, while smaller districts concentrate competition. Ranges from $M = 1$ (single-member districts) to $M = L$ (the full size of the legislature in national-constituency systems). See: Duverger's Law; effective number of parties; proportional representation.

Divisor methods. A family of apportionment algorithms that assign seats by constructing a series of 'priority numbers' (votes divided by successive divisors) and awarding seats to the parties with the highest priorities. The two most prominent are the Jefferson–D'Hondt method (divisors 1, 2, 3, ...) and the Webster–Sainte-Laguë method (divisors 1, 3, 5, ...). All divisor methods satisfy Seat Monotonicity and hence must sometimes violate quota. See: Jefferson–D'Hondt method; Webster–Sainte-Laguë method; Balinski–Young theorem; quota methods.

Droop quota. A quota used in both quota apportionment methods and STV, defined (approximately) as votes/(seats + 1). Slightly larger than the Hare quota, it is marginally more favorable to larger parties. In STV, it defines the threshold of first-preference votes a candidate must reach to win a seat. See: quota methods; Single Transferable Vote; Hamilton–Hare–Niemeyer method.

Duverger's Law. The proposition, formulated by Duverger (1951), that plurality and majority single-member district systems tend to produce two-party systems, while PR systems tend toward multipartyism. The mechanism operates through the mechanical effect (votes-to-seats translation), the psychological effect (strategic desertion of small parties), and the coalitional effect (party mergers). The 'law' is better understood as a strong empirical tendency than a deterministic rule; it has better empirical accuracy when applied to districts rather than at the national level. See: mechanical effect; psychological effect; coalitional effect; effective number of parties.

E

Effective number of parties (ENP). An index (Laakso and Taagepera 1979) that measures party system fragmentation on a continuous scale, giving more weight to larger parties. Defined as 1 divided by the sum of squared seat (or vote) shares. A parliament dominated by one large party scores near 1; a perfectly balanced two-party system scores 2; a fragmented multi-party system scores higher. A widely used summary statistic in comparative electoral research. See: district magnitude; Duverger's Law; party system fragmentation.

Effective threshold. The de facto vote share a party must reach to win a seat in a given district, even in the absence of any legal threshold. Approximated by Lijphart (1994) and Taagepera (2002) as $t \approx 1/(M + 1)$, where M is district magnitude. A ten-seat district has an effective threshold of about 9%; a single-seat district about 50%. The effective threshold

explains why low-magnitude PR systems can behave much like majoritarian ones despite their formal PR label. See: district magnitude; electoral threshold; mechanical effect.

Electoral engineering. The deliberate manipulation of electoral rules—district boundaries, magnitudes, thresholds, ballot structures, or apportionment algorithms—to favor particular parties or political outcomes (Norris 2004; Evci and Kaminski 2025). The term covers a broad spectrum from gerrymandering in majoritarian systems to district-magnitude manipulation, threshold adjustments, and choice of apportionment algorithm in PR systems. Rule changes are often justified on technical grounds while serving partisan ends. See: gerrymandering; malapportionment; electoral threshold.

Electoral system. The set of rules determining how votes cast in an election are translated into legislative seats. More complex than an apportionment formula alone, an electoral system includes district structure, ballot format, legal thresholds, tier structure (in mixed systems), and apportionment algorithm. See: apportionment; district magnitude; electoral threshold; mixed systems; proportional representation.

Electoral threshold. A minimum vote share a party must achieve—nationwide or within a district—before it is eligible to receive any seats. Thresholds reduce parliamentary fragmentation but increase disproportionality. Common values are 5% (Germany, Poland, Czechia), 3.25% (Israel), and 2% (Denmark). Some systems provide lower or no thresholds for recognized national minorities. See: antithreshold; disproportionality; party system fragmentation; trigger electorate.

Electoral volatility. The degree of change in parties' vote or seat shares between two consecutive elections. Measured by the Pedersen index. High volatility indicates an unstable party system; low volatility indicates stability. PR systems in established democracies typically show moderate volatility; emerging democracies may show extreme volatility. See: Pedersen index; party system fragmentation.

Endogeneity of electoral institutions. The methodological problem that electoral systems are not randomly assigned across countries but are chosen by political actors whose interests and circumstances also shape the very outcomes—party fragmentation, turnout, representation—that electoral rules are supposed to cause (Benoit 2002; Acemoglu 2005). Observed correlations between PR and, say, higher turnout or greater women's representation may therefore reflect selection into PR rather than the causal effect of PR itself. Credible causal claims require quasi-experimental variation, such as the staggered Swiss cantonal PR adoptions (Gathmann 2019) or simultaneous Polish local elections held under different rules. See: electoral engineering; Duverger's Law; voter turnout.

F

Faulty votes. A class of voter errors induced by the cognitive demands of open-list PR ballots, identified by Flis and Kaminski (2025a). Three types are distinguished: invalid ballots (formally uncountable), votes cast for the first-listed party driven solely by ballot position rather than policy preference, and votes for the top-listed candidate within a party driven solely by list position. Using simultaneous Polish local elections held under OLPR and SMD rules, the authors estimate that roughly 13% more eligible voters cast a faulty vote under OLPR than under SMD—a gap comparable in magnitude to the PR turnout advantage but working in the opposite direction, degrading vote quality while inflating turnout. See: primacy effects; open-list PR; voter turnout; ballot.

First-Past-The-Post (FPTP). A majoritarian single-member district system in which the candidate with the most votes wins, even without a majority. The most common majoritarian system, used in the United States, United Kingdom, Canada, and India. Contrasted with PR systems throughout the primer. See: Duverger's Law; single-member district; proportional representation.

Flexible-list PR. A variant of open-list PR in which voters cast a party vote and may additionally express a preference for individual candidates within the chosen list. Used in Austria and some other European countries. See: open-list PR; closed-list PR; panachage.

Formateur. The political actor—usually the leader of the largest party, often formally appointed by the head of state—charged with assembling a coalition capable of winning parliamentary investiture after an inconclusive election (Laver et al. 1998). The formateur's bargaining position depends on coalition arithmetic, party ideological distances, and the credibility of alternative coalitions. Formateur theory is central to formal models of government formation in PR systems. See: pre-election coalition; minimal winning coalition; oversized coalition.

G

Gallagher index (G). A widely used measure of electoral disproportionality, defined as the square root of half the sum of squared differences between each party's vote share and seat share. Values range from 0 (perfect proportionality) to 1 (total disproportionality). Based on Euclidean distance in vote–seat space. See: disproportionality; Loosemore–Hanby index.

Gender quotas. Statutory or voluntary rules requiring parties to nominate a minimum share of women candidates. Three main types exist: candidate list quotas (typically requiring 30–50% women on party lists), zipper rules (mandating alternation between male and female candidates so women are not clustered at unwinnable list positions), and reserved seats (a distinct mechanism guaranteeing specific parliamentary seats for women regardless of electoral outcomes). PR systems with statutory quotas have generally seen faster growth in women's legislative representation than those relying on voluntary measures (Krook 2010; Dahlerup 2013). See: descriptive representation; substantive representation; list PR.

Gerrymandering. The manipulation of electoral district boundaries to advantage a particular party or group. Primarily a concern in single-member district systems; practically very difficult to execute in PR systems with large, multi-member districts because the results are less sensitive to boundary placement. See: malapportionment; single-member district.

H

Hamilton–Hare–Niemeyer method (HHN). The most intuitive quota apportionment method, known as Hamilton's method in the United States and Hare–Niemeyer or Largest Remainder in Europe. Each party first receives seats equal to its quota rounded down; remaining seats go to parties with the largest fractional remainders. Satisfies Staying within the Quota but violates Seat Monotonicity (Alabama paradox). See: quota methods; Alabama paradox; Balinski–Young theorem; Droop quota.

House Monotonicity. A weaker version of Seat Monotonicity: when the total number of seats increases by one seat, no party loses a seat. All divisor methods satisfy this property;

the Hamilton–Hare–Niemeyer method does not (Alabama paradox). See: Seat Monotonicity; Alabama paradox; Balinski–Young theorem.

Huntington–Hill method. A divisor apportionment method using divisors equal to the geometric mean of consecutive integers: $\sqrt{i(i+1)}$. Currently used for apportioning seats in the U.S. House of Representatives among states. Tends to favor smaller states/parties compared to Jefferson–D’Hondt. See: divisor methods; Jefferson–D’Hondt method.

I

Imperiali quota. A quota apportionment method using a multiplier of $M + 2$ (rather than M or $M + 1$), making it the most favorable of the three common quota methods toward larger parties. May produce more claims than available seats, requiring tiebreakers. See: quota methods; Droop quota; Hamilton–Hare–Niemeyer method.

J

Jefferson–D’Hondt method. The most widely used divisor apportionment method (about 50 countries), with divisors 1, 2, 3, ... Known as Jefferson’s method in the United States and D’Hondt’s method in Europe, where it was independently discovered. Slightly favors larger parties compared to Webster–Sainte-Laguë. See: divisor methods; Webster–Sainte-Laguë method; apportionment.

K

Kingmaker spoiler. A type of spoiler specific to PR systems: a party whose entry into the election, even if it wins no seats itself (e.g., failing to clear the threshold), causes another party to cross the majority threshold in seats—making it the outright parliamentary majority winner. Distinguished from classic spoilers in single-member districts. See: spoiler; electoral threshold; wasted votes.

L

Limited vote (LV). A semi-proportional system in which voters hold k votes (where $k < M$, the district magnitude) and must distribute them among different candidates; the M candidates with the most votes win. Used for most seats in the Spanish Senate. See: semi-proportional systems; Cumulative Voting; Single Non-Transferable Vote.

List PR. The dominant family of proportional representation, in which parties present lists of candidates in multi-member districts and voters cast ballots that determine both party seat shares and, in some variants, which individual candidates are elected. The principal subtypes are closed-list, open-list, and flexible-list PR; variants such as panachage allow preference votes across lists, and multi-tier systems (Austria, Denmark, Norway, Sweden) adjust district allocations at higher levels for proportionality. See: closed-list PR; open-list PR; flexible-list PR; panachage; apportionment.

Local PR systems. Proportional representation as used in sub-national elections—municipal, county, or regional. Local PR elections vastly outnumber national PR elections (often in the thousands every four or five years) and typically feature lower thresholds, smaller

district magnitudes, and varied ballot structures compared to national systems. These differences can intensify candidate personalization, alter party strategies, and affect representation patterns in ways that distinguish local from national PR dynamics. Despite their prevalence, local PR systems remain comparatively understudied. See: district magnitude; electoral threshold; primacy effects.

Loosemore–Hanby index (LH). A measure of electoral disproportionality defined as half the sum of the absolute differences between each party's vote share and seat share. Based on city-block distance; somewhat more generous than the Gallagher index in assigning high disproportionality values. Also structurally identical to the Pedersen volatility index. See: Gallagher index; disproportionality; Pedersen index.

M

Malapportionment. Unequal district sizes that cause voters in different districts to have unequal influence over election outcomes, even without deliberate boundary manipulation (gerrymandering). Within PR systems, malapportionment can arise through unequal district populations, through constitutional minima (the antithresholds discussed in the primer's § 4.3), or through deliberate over-representation of rural areas. In several PR countries (Spain, Argentina, Brazil), malapportionment is quantitatively large and systematically favors conservative rural parties. See: gerrymandering; district magnitude; antithreshold; apportionment.

Mechanical effect. One of Duverger's three effects: the direct mathematical translation of votes into seats by electoral rules. Under $M = 1$, only one candidate can win, and all votes for losing candidates are wasted; under higher magnitudes, smaller parties can cross the effective threshold and win seats. See: Duverger's Law; psychological effect; coalitional effect; district magnitude; wasted votes.

Minimal winning coalition. A government coalition that would lose its parliamentary majority if any single member party withdrew. Theorized by Riker (1962) following von Neumann and Morgenstern (1944) as the rational equilibrium outcome of coalition bargaining under the size principle: parties have no incentive to share power more broadly than necessary. Empirically, minimal winning coalitions are common but not universal—oversized and minority governments also occur, especially in PR systems where uncertainty, ideological constraints, or external threats favor broader inclusion. See: oversized coalition; minority government; formateur.

Minority government. A government coalition (or single party) holding fewer than half the seats in parliament, surviving by securing case-by-case legislative support from outside the cabinet. Common in Scandinavian PR systems (Denmark, Sweden, Norway), where strong parliamentary committees and consensual political culture make minority cabinets viable. Minority governments trade away guaranteed legislative majorities for greater policy flexibility and reduced coalition transaction costs. See: minimal winning coalition; oversized coalition; formateur.

Mixed-Member Parallel system (MMM). A mixed electoral system in which seats are allocated independently in two tiers—a single-member district (majoritarian) tier and a PR tier—with no formal link between the two. Used in Japan, South Korea, Russia, and the Philippines. Because the tiers are disconnected, the system does not fully compensate

for distortions in the majoritarian tier. See: Mixed-Member Proportional system; mixed systems.

Mixed-Member Proportional system (MMP). A mixed electoral system in which the PR tier compensates for the disproportionality of the majoritarian tier: parties are entitled to a total seat share based on their PR vote, and if they win fewer seats in the single-member tier than this total, they receive ‘top-up’ seats from the PR list. Used in Germany and New Zealand. Vulnerable to overhang seats and decoy party strategies. See: Mixed-Member Parallel system; mixed systems; overhang seats; decoy party.

Mixed systems. Electoral systems that combine single-member district (majoritarian) seats with proportional representation seats in two tiers. The key distinction is whether the two tiers are linked (Mixed-Member Proportional) or parallel and independent (Mixed-Member Parallel). See: Mixed-Member Proportional system; Mixed-Member Parallel system; overhang seats; decoy party.

Monotonicity. A family of properties requiring that, loosely, ‘more support cannot hurt.’ Several variants are discussed in the primer: Weak Monotonicity (more votes $\rightarrow$ at least as many seats), Population Monotonicity (growing population $\rightarrow$ no seat loss), Seat Monotonicity (more total seats $\rightarrow$ no party loses a seat), and Candidate Monotonicity (upgrading a candidate in a ranking cannot cause them to lose). Divisor methods satisfy Seat and Population Monotonicity; STV violates Candidate Monotonicity. See: Candidate Monotonicity; Seat Monotonicity; Population Monotonicity; Balinski–Young theorem.

Multiple Non-Transferable Vote (MNTV). A semi-proportional system in which voters hold M votes (equal to district magnitude) and must distribute them among different candidates; the top M vote-getters win. Used in the landmark June 4, 1989 Polish Senate elections. See: semi-proportional systems; Single Non-Transferable Vote; Limited Vote.

N

Natural threshold. The de facto minimum vote share required to win a seat in a given district, imposed by district magnitude alone in the absence of any legal threshold. Sometimes used interchangeably with “effective threshold,” but strictly denoting the structural barrier arising purely from M rather than the combined effect of magnitude and formal rules (Lijphart 1994; Taagepera 2002). Natural thresholds explain why district-level legal thresholds (e.g., Argentina’s 3%) often have limited independent effect: small districts already impose a substantial bar. A ten-seat district has a natural threshold of roughly 9%; a three-seat district roughly 25%. See: effective threshold; district magnitude; electoral threshold.

No-Show paradox. A counterintuitive failure of certain voting rules in which a voter can obtain a more preferred outcome by abstaining than by voting sincerely. Identified for STV by Fishburn et al. (1983) and shown by Moulin (1988) to follow inevitably from any voting rule satisfying the Condorcet principle. The No-Show paradox is closely related to STV Non-Monotonicity and, like that defect, arises infrequently in practice but reveals a structural vulnerability of ranked methods. See: Single Transferable Vote; Candidate Monotonicity; Monotonicity.

O

Open-list proportional representation (OLPR). A list PR system in which voters can express a preference for individual candidates within their chosen party, influencing which candidates on the list actually receive seats. The degree of voter influence varies from a single preference vote (Poland, Brazil) to multiple votes (Czechia, Slovakia) to full panachage (Switzerland, Luxembourg). See: closed-list PR; flexible-list PR; panachage; list PR; primacy effects.

Orphaned constituencies. Single-member district seats that, under Germany's post-2021 electoral reform, are cancelled rather than awarded to their nominal winners when a party would otherwise overshoot its proportional entitlement. The mechanism replaces overhang seats and decoy-party incentives with a different problem: constituency winners can find themselves disenfranchised despite winning their local race. Most experts regard the reform as having substituted one underlying problem with another (Flis et al. 2025a). See: overhang seats; decoy party; Mixed-Member Proportional system.

Overhang seats. Extra seats added to a legislature in a Mixed-Member Proportional system when a party wins more single-member district seats than its proportional entitlement. To restore overall proportionality, additional 'leveling' seats must be created for other parties as well, increasing the total size of the legislature unpredictably. A significant problem in Germany before the 2025 reform. See: Mixed-Member Proportional system; decoy party.

Oversized coalition. A government coalition containing more parties than strictly required to maintain a parliamentary majority. Oversized coalitions form when parties prioritize stability, broad legitimacy, crisis management, or constitutional supermajority requirements over the size principle's prediction of minimal winning coalitions. Brockington (2004) shows that oversized coalitions can dampen voter turnout by blurring lines of accountability. See: minimal winning coalition; minority government; formateur.

P

Panachage. A feature of some open-list PR systems (notably Switzerland and Luxembourg) allowing voters to distribute their preference votes across candidates from different party lists, rather than being restricted to one party. See: open-list PR; flexible-list PR; list PR.

Party system fragmentation. The degree to which legislative seats (or votes) are spread across many parties rather than concentrated in a few. Measured most commonly by the effective number of parties. Higher under PR, especially with large district magnitudes and low thresholds. See: effective number of parties; district magnitude; electoral threshold; Duverger's Law.

Pedersen index. A measure of electoral volatility between two consecutive elections, defined as half the sum of absolute changes in each party's seat (or vote) share. Structurally identical to the Loosemore–Hanby disproportionality index, but applied across time (between elections) rather than across dimensions (votes vs. seats). See: Loosemore–Hanby index; electoral volatility.

Pivotality-based voting. A form of strategic voting under PR in which voters support parties expected to occupy pivotal positions in coalition formation—particularly small parties whose participation is necessary for any majority government to form. Distinct from threshold insurance voting (which targets the threshold barrier) and from coalition-oriented

voting (which targets the composition of the governing bloc as a whole). See: strategic voting; threshold insurance voting; coalition-oriented voting; formateur.

Population Monotonicity. The property that if one unit's population grows and another's shrinks, the growing unit should not lose a seat to the shrinking one. All divisor methods satisfy this property; the Hamilton–Hare–Niemeyer method does not. See: Monotonicity; Seat Monotonicity; Balinski–Young theorem; Hamilton–Hare–Niemeyer method.

Power-sharing. A constitutional arrangement in which executive authority and key political decisions are jointly exercised by representatives of multiple distinct groups, rather than concentrated in a winning majority. The four main mechanisms identified by Lijphart are grand coalition (executive composed of major parties from all significant segments), mutual veto (each segment can block decisions affecting its vital interests), segmental autonomy (each group governs its own affairs in education, religion, and culture), and proportionality (representation in proportion to group size, typically through PR). Power-sharing is the operational core of consociational democracy and is widely used in deeply divided societies. See: consociational democracy; centripetal incentives; proportional representation.

Pre-election coalition. A formal alliance between two or more parties announced and presented to voters before an election, in which the parties commit to govern together if the alliance wins enough seats. Distinct from post-election bargaining among parties whose alignment was not pre-announced (Lijphart 1984; Golder 2006; Kaminski 2001). Pre-election coalitions reduce voter uncertainty about the policy implications of their vote and enable coordination strategies such as joint lists, mutual stand-down agreements, and lower-preference transfer instructions. See: coalition-oriented voting; formateur; minimal winning coalition.

Preferential ballot. A ballot on which voters rank candidates or parties in order of preference, rather than selecting a single option or small cluster of options. Used in STV, Alternative Vote, Supplementary Vote, and some open-list PR systems (e.g., Czechia and Slovakia allow up to four preference votes). Preferential ballots place substantially higher cognitive demands on voters than categorical ballots and produce more invalid ballots, particularly among less-educated voters and in contests with large candidate pools (Reynolds and Steenbergen 2006). See: categorical ballot; Single Transferable Vote; Alternative Vote; ranked-choice systems; ballot.

Primacy effects. Systematic biases in voter behavior caused by the position of parties or candidates on the ballot. Party primacy refers to excess support for the party listed first on the ballot. List primacy (or list-position effect) refers to excess preference votes going to the candidate listed first within a party, independently of merit. Both effects give parties and electoral administrators de facto influence over outcomes through ballot ordering decisions. See: open-list PR; ballot.

Priority numbers. In divisor apportionment methods, the values obtained by dividing a party's vote total by successive divisors (1, 2, 3, ... for Jefferson–D'Hondt; 1, 3, 5, ... for Webster–Sainte-Laguë). Seats are awarded sequentially to the party with the largest remaining priority number. See: divisor methods; Jefferson–D'Hondt method; Webster–Sainte-Laguë method.

Proportional representation (PR). A family of electoral systems designed to distribute legislative seats among parties (or candidates) in proportion to their share of votes. Operationalized through quota methods, divisor methods, and ranked methods (STV), and implemented in list systems, STV systems, and mixed systems. Used in roughly 118 countries

for lower-house elections. See: apportionment; list PR; Single Transferable Vote; mixed systems; disproportionality.

Psychological effect. One of Duverger's three effects: the tendency of voters who prefer a small party—knowing it is unlikely to win under restrictive rules—to vote 'strategically' for a larger, more viable party instead, in order not to waste their vote. Similarly, potential candidates and donors may abandon small parties they expect to lose. See: Duverger's Law; mechanical effect; coalitional effect; wasted votes.

Q

Quota. In apportionment, the exactly proportional (possibly fractional) number of seats a party would receive if fractional seats were allowed. In STV, it refers instead to the minimum number of votes required to win a seat (Droop quota). The two uses of the word are distinct. See: quota methods; Droop quota; Hamilton–Hare–Niemeyer method; Single Transferable Vote.

Quota methods. Apportionment algorithms that begin by calculating each party's quota (proportional seat entitlement) and assign seats first by rounding quotas down, then by distributing remaining seats according to the largest fractional remainders. The three principal quota methods are Hamilton–Hare–Niemeyer (multiplier M), Droop–Hagenbach-Bischoff (multiplier $M + 1$), and Imperiali (multiplier $M + 2$). See: Hamilton–Hare–Niemeyer method; Droop quota; Imperiali quota; divisor methods; Balinski–Young theorem.

R

Ranked apportionment. The class of apportionment problems in which voters supply rankings of candidates rather than casting a single vote for a party, and seats are allocated through sequential election and elimination procedures. The principal example is the Single Transferable Vote. Contrasted with simple apportionment, ranked apportionment can produce paradoxes—Non-Monotonicity and the No-Show paradox—that simple methods avoid, but it dispenses with the need for party lists and permits both intra-party and inter-party vote transfers. See: Single Transferable Vote; simple apportionment; Candidate Monotonicity; No-Show paradox.

Ranked-choice systems. Voting systems in which voters rank candidates in order of preference rather than selecting a single candidate or party. In PR contexts, the primary ranked method is Single Transferable Vote (STV). In majoritarian contexts, ranked methods include Alternative Vote and Supplementary Vote. See: Single Transferable Vote; Alternative Vote.

S

Scalability. An axiomatic property of apportionment methods: multiplying all parties' vote totals by the same positive integer should not change the seat allocation. Satisfied by all standard quota and divisor methods. See: Symmetry; Weak Monotonicity; apportionment.

Seat Monotonicity. The property that when the total number of seats to be distributed increases, no party should lose a seat. All divisor methods satisfy this; the Hamilton–Hare–Niemeyer method does not (Alabama paradox). Seat Monotonicity and Staying within the Quota cannot be satisfied simultaneously—the content of the Balinski–Young theorem.

See: Balinski–Young theorem; Alabama paradox; House Monotonicity; Staying within the Quota.

Semi-proportional systems. A small family of electoral systems—Single Non-Transferable Vote, Limited Vote, Cumulative Voting, Multiple Non-Transferable Vote—that are not designed to achieve proportionality but may produce results somewhat closer to proportional than purely majoritarian systems. Winning seats for minority groups often requires deliberate voter coordination. See: Single Non-Transferable Vote; Limited Vote; Cumulative Voting; Multiple Non-Transferable Vote.

Simple apportionment. The class of apportionment problems in which each voter casts a single vote for a party and seats are distributed among parties in proportion to their vote totals. Includes quota methods (Hamilton–Hare–Niemeyer, Droop–Hagenbach-Bischoff, Imperiali) and divisor methods (Jefferson–D’Hondt, Webster–Sainte-Laguë, Huntington–Hill, modified Webster–Sainte-Laguë). Contrasted with ranked apportionment. See: quota methods; divisor methods; apportionment; ranked apportionment.

Single-member district (SMD). An electoral district returning exactly one seat (district magnitude $M = 1$). The basic unit of majoritarian systems such as FPTP, two-round systems, and Alternative Vote. Under SMDs, only one party can win, making all other votes wasted in the strict sense. See: district magnitude; First-Past-The-Post; wasted votes; Duverger’s Law.

Single Non-Transferable Vote (SNTV). A semi-proportional system in which each voter casts exactly one vote for an individual candidate in a multi-member district; the M candidates with the most votes win. Used in Japan before 1994 and Jordan 1993–2016. See: semi-proportional systems; Limited Vote; Cumulative Voting.

Single Transferable Vote (STV). A ranked apportionment method in which voters rank individual candidates; seats are filled sequentially by electing candidates who reach a quota, transferring surplus votes to next-ranked candidates, and eliminating the weakest remaining candidate when no one meets the quota. Used in Irish and Maltese legislative elections and the Australian Senate. STV is non-monotonic in votes: upgrading a winning candidate can paradoxically cause them to lose. See: ranked-choice systems; Droop quota; Candidate Monotonicity; list PR.

Social choice theory. The formal study of how individual preferences can be aggregated into collective decisions, especially through voting and apportionment rules. Foundational impossibility results—Arrow (1951) on preference aggregation and Gibbard (1973) and Satterthwaite (1975) on strategy-proofness—establish that no voting rule satisfying a modest set of rationality and fairness axioms can simultaneously meet all of them. These general results imply that every electoral system is formally flawed in some respect, extending the logic of the Balinski–Young impossibility theorem from apportionment to voting rules more broadly. See: Balinski–Young impossibility theorem; Alabama paradox; Candidate Monotonicity; No-Show paradox.

Split-ticket voting. The practice, in dual-ballot mixed systems (MMP, MMM), of casting one’s two votes for different parties—typically a candidate vote for one party in the single-member tier and a list vote for a different party in the PR tier. Split-ticket voting can express genuine preferences across tiers, hedge against electoral uncertainty (Shikano and Herron 2025), or implement strategic coordination such as redirecting list votes to a coalition partner. It is one of the principal channels through which contamination effects operate in mixed

systems. See: contamination effects; Mixed-Member Proportional system; Mixed-Member Parallel system; strategic voting; decoy party.

Spoiler. A candidate or party whose presence in the election changes the winner without winning themselves. In single-member districts, a classic spoiler splits the vote of ideologically similar competitors. In PR systems, spoilers take more varied forms, including the kingmaker type, where a small party's entry makes a non-majority party a majority winner. See: Kingmaker spoiler; wasted votes.

Staying within the Quota. An axiomatic property of apportionment methods requiring that every party receive a number of seats equal to its HHN quota rounded either up or down—never more than one seat away from exact proportionality. The Hamilton–Hare–Niemeyer method satisfies this by construction; all divisor methods may violate it. Cannot be simultaneously satisfied with Seat Monotonicity. See: Balinski–Young theorem; quota; Hamilton–Hare–Niemeyer method; Seat Monotonicity.

Strategic voting. The casting of a ballot for a party or candidate other than the voter's most-preferred option, in order to obtain a more favorable overall outcome. In majoritarian systems, strategic voting typically means deserting a sub-threshold candidate for a viable one (the psychological effect of Duverger's Law). Under PR, strategic voting takes more complex forms and depends heavily on institutional details: voters may engage in threshold insurance voting, coalition-oriented voting, pivotality-based voting, vote management in STV, or split-ticket voting in mixed systems. Strategic voting under PR is generally weaker than under FPTP and operates locally rather than producing system-wide consolidation (Cox 1997; Blais 2000). See: threshold insurance voting; coalition-oriented voting; pivotality-based voting; vote management; split-ticket voting; psychological effect.

Substantive representation. Representation in terms of policy outcomes—whether the interests and preferences of a group (women, minorities) are actually reflected in legislation, independently of how many members of that group hold office. Distinct from descriptive representation, and not guaranteed by it. See: descriptive representation; women and minority representation.

Supplementary Vote (SV). A ranked majoritarian system, also known as the Contingent Vote, used to fill a single seat. Voters mark a first preference and one or more lower preferences; if no candidate wins an outright majority of first preferences, weaker candidates are eliminated and their lower preferences transferred until one candidate has the higher total. Used in English mayoral elections (until 2023) and Sri Lankan presidential elections. Not a PR method, but mentioned as a comparator. See: Alternative Vote; ranked-choice systems; First-Past-The-Post; two-round system; preferential ballot.

Surplus votes. In STV, the votes a candidate receives above the quota after being elected. These surplus votes are redistributed (in fractional form) to the next-ranked active candidates on those ballots, so as not to be wasted. The transfer value equals the surplus divided by the candidate's total vote. See: Single Transferable Vote; Droop quota; transfer value.

Symmetry. An axiomatic property of simple apportionment methods: two parties with identical vote totals must receive identical seat allocations. Satisfied by all standard methods. See: Scalability; Weak Monotonicity; apportionment.

System, electoral. See: electoral system.

T

Threshold, electoral. See: electoral threshold.

Threshold bypass rule. A provision in mixed-member proportional systems that exempts a party from the national PR threshold if it wins a specified minimum number of single-member district seats—3 in Germany, just 1 in New Zealand. The rule is intended to protect small parties with geographically concentrated support, but it also generates strategic incentives: large coalition parties may steer their supporters in targeted districts toward smaller allies, creating trigger electorates that shift the ultimate distribution of PR seats. See: trigger electorate; electoral threshold; Mixed-Member Proportional system; antithreshold.

Threshold insurance voting. A form of strategic voting under PR in which voters cast their ballot for a small coalition partner of their preferred large party in order to ensure the smaller party clears the legal threshold and contributes seats to the governing bloc. Documented by Gschwend (2007) and others in studies of German MMP elections, where supporters of the larger CDU or SPD have at times voted for coalition partners (FDP, Greens) at risk of falling below the 5% threshold. See: strategic voting; electoral threshold; coalition-oriented voting.

Transfer value. In STV, the fraction of each ballot assigned to an elected candidate that is passed on to the next-ranked active candidate. Equals the candidate's surplus votes divided by their total vote. Ensures that no vote is entirely wasted once a candidate is elected. See: Single Transferable Vote; surplus votes; Droop quota.

Trigger electorate. In Mixed-Member Proportional systems with a threshold-bypass rule (e.g., Germany's rule that winning 3 SMD seats exempts a party from the 5% threshold, New Zealand's rule requiring only 1 such seat), the voters in districts where a small allied party has a realistic chance of winning a single-member seat. Larger parties may encourage their supporters in those districts to vote for the smaller ally, 'triggering' its entry into parliament. See: electoral threshold; Mixed-Member Proportional system; antithreshold.

Two-round system. A majoritarian system in which, if no candidate achieves a majority in the first round, a second round is held (typically between the top two candidates). Used in most presidential elections worldwide and in French legislative elections. Not a PR method, but mentioned as a comparator. See: First-Past-The-Post; Alternative Vote; proportional representation.

V

Volatility, electoral. See: electoral volatility.

Vote management. The set of practices by which parties in STV systems coordinate first preferences and lower preferences across their candidates and across allied parties to maximize overall seat yields. Also called transfer management or vote-pooling (Farrell and McAllister 2006; Mitchell 2014). Parties typically issue how-to-vote cards advising supporters how to distribute first preferences across co-partisan candidates and how to rank lower preferences to ensure efficient transfers. Surplus votes from a party that has secured its seats, or transfers from an eliminated allied party, can thus be directed toward strategically valuable candidates. See: Single Transferable Vote; surplus votes; transfer value; strategic voting.

Voter turnout. The share of eligible citizens who cast a ballot in a given election. PR systems are generally associated with higher turnout than majoritarian ones—Karp and Banducci (2008) estimate the PR advantage at about 5% points—commonly attributed to greater external efficacy and a broader range of meaningful party choices. Quasi-experimental evidence from staggered Swiss cantonal PR adoption shows a boost of about 14% points (Gathmann 2019). The advantage is not automatic: oversized coalitions can blur accountability and dampen turnout (Brockington 2004), and compulsory voting independently raises turnout by 10–15% points (Lijphart 1997)—a confound in cross-national comparisons. The increase in turnout can be offset in OLPR by lower vote quality (Flis and Kaminski 2025a). See: compulsory voting; oversized coalition; faulty votes; psychological effect; endogeneity of electoral institutions.

W

Wasted votes. Votes that do not contribute to electing any representative. In single-member district systems, all votes for losing candidates are wasted; in PR systems, votes for parties that fail to clear the electoral threshold are wasted, as are small fractional remainders. PR systems generally produce fewer wasted votes than majoritarian systems. See: electoral threshold; proportional representation; spoiler; mechanical effect.

Weak Monotonicity. An axiomatic property of apportionment methods: a party with strictly more votes than another must receive at least as many seats. Satisfied by all standard quota and divisor methods. See: Monotonicity; Symmetry; Scalability; Population Monotonicity.

Webster–Sainte-Laguë method. A divisor apportionment method using odd-integer divisors (1, 3, 5, ...), independently discovered by Daniel Webster in the United States and Victor Sainte-Laguë in France. More favorable towards small parties than Jefferson–D’Hondt. Used in about ten countries, including Norway and Sweden. See: divisor methods; Jefferson–D’Hondt method; apportionment.

Women and minority representation. The share of women, ethnic, religious, or other historically underrepresented groups elected to legislative office, and the related question of whether their presence shapes policy. PR systems consistently elect more women than majoritarian ones—Paxton, Hughes, and Painter (2010) estimate the gap at about 2.5% points—and Moser (2008) finds similar advantages for minority inclusion. The principal mechanism is party strategy: list PR rewards balanced lists, and multi-member districts allow new groups to be added without displacing incumbents. Statutory gender quotas, zipper rules, and reserved seats reinforce these effects (Krook 2010; Dahlerup 2013). Gains in descriptive representation, however, do not translate automatically into substantive representation (Franceschet et al. 2008; Kläy et al. 2025). See: descriptive representation; substantive representation; gender quotas; list PR; consociational democracy.

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