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### Publication Date

1999-08-22

Peer reviewed

# $|V_{cb}|$ and $|V_{ub}|$ from $B$ Decays: Recent Progress and Limitations\*

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## Abstract

The determination of  $|V_{cb}|$  and  $|V_{ub}|$  from semileptonic  $B$  decay is reviewed with a critical discussion of the theoretical uncertainties. Future prospects and limitations are also discussed.

## 1 Introduction

The purpose of  $K$  and  $B$  physics in the near future is testing the Cabibbo–Kobayashi–Maskawa (CKM) picture of quark mixing and  $CP$  violation. The goal is to overconstrain the unitarity triangle by directly measuring the sides and (some) angles in several decay modes. If the value of  $\sin 2\beta$ , the  $CP$  asymmetry in  $B \rightarrow J/\psi K_S$ , is near the CDF central value [1], then searching for new physics will require a combination of precision measurements. This talk concentrates on  $|V_{cb}|$  and  $|V_{ub}|$ ; the latter is particularly important since it largely controls the experimentally allowed range for  $\sin 2\beta$  in the standard model.

## 2 Exclusive decays

In mesons composed of a heavy quark and a light antiquark (plus gluons and  $q\bar{q}$  pairs), the energy scale of strong processes is small compared to the heavy quark mass. The heavy quark acts as a static point-like color source with fixed four-velocity, since the soft gluons responsible for confinement cannot resolve structures much smaller than  $\Lambda_{\text{QCD}}$ , such as the heavy quark's Compton wavelength. Thus the configuration of the light degrees of freedom become insensitive to the spin and flavor (mass) of the heavy quark, resulting in a  $SU(2n)$  spin-flavor symmetry [2] ( $n$  is the number of

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\*Invited talk at the Chicago Conference on Kaon Physics (Kaon'99), June 21–26, 1999, Chicago, IL. [FERMILAB-Conf-99/213-T]

heavy quark flavors). Heavy quark symmetry (HQS) helps understanding the spectroscopy and decays of heavy hadrons from first principles.

The predictions of HQS are particularly restrictive for  $\bar{B} \rightarrow D^{(*)}\ell\bar{\nu}$  decays. In the infinite mass limit all form factors are proportional to a universal Isgur-Wise function,  $\xi(v \cdot v')$ , satisfying  $\xi(1) = 1$  [2]. The symmetry breaking corrections can be organized in a simultaneous expansion in  $\alpha_s$  and  $\Lambda_{\text{QCD}}/m_Q$  ( $Q = c, b$ ). The  $\bar{B} \rightarrow D^{(*)}\ell\bar{\nu}$  decay rates are given by

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D^*\ell\bar{\nu})}{dw} &= \frac{G_F^2 m_B^5}{48\pi^3} r_*^3 (1 - r_*)^2 \sqrt{w^2 - 1} (w + 1)^2 \\ &\quad \times \left[ 1 + \frac{4w}{1+w} \frac{1 - 2wr_* + r_*^2}{(1 - r_*)^2} \right] |V_{cb}|^2 \mathcal{F}_{D^*}^2(w), \\ \frac{d\Gamma(\bar{B} \rightarrow D\ell\bar{\nu})}{dw} &= \frac{G_F^2 m_B^5}{48\pi^3} r^3 (1 + r)^2 (w^2 - 1)^{3/2} |V_{cb}|^2 \mathcal{F}_D^2(w), \end{aligned} \quad (1)$$

where  $w = v \cdot v'$  and  $r_{(*)} = m_{D^{(*)}}/m_B$ .  $\mathcal{F}_{D^{(*)}}(w)$  is equal to the Isgur-Wise function in the  $m_Q \rightarrow \infty$  limit, and in particular  $\mathcal{F}_{D^{(*)}}(1) = 1$ , allowing for a model independent determination of  $|V_{cb}|$ . Including symmetry breaking corrections one finds

$$\begin{aligned} \mathcal{F}_{D^*}(1) &= 1 + c_A(\alpha_s) + \frac{0}{m_Q} + \frac{(\dots)}{m_Q^2} + \dots, \\ \mathcal{F}_D(1) &= 1 + c_V(\alpha_s) + \frac{(\dots)}{m_Q} + \frac{(\dots)}{m_Q^2} + \dots. \end{aligned} \quad (2)$$

The perturbative corrections,  $c_A = -0.04$  and  $c_V = 0.02$ , have been computed to order  $\alpha_s^2$  [3], and the unknown higher order corrections should affect  $|V_{cb}|$  at below the 1% level. The vanishing of the order  $1/m_Q$  corrections to  $\mathcal{F}_{D^*}(1)$  is known as Luke's theorem [4]. The terms indicated by (...) are only known using phenomenological models at present. Thus the determination of  $|V_{cb}|$  from  $\bar{B} \rightarrow D^*\ell\bar{\nu}$  is theoretically more reliable than that from  $\bar{B} \rightarrow D\ell\bar{\nu}$  (unless using lattice QCD for  $\mathcal{F}_{D^{(*)}}(1)$  — see below), although for example QCD sum rules predict that the order  $1/m_Q$  correction to  $\mathcal{F}_D(1)$  is small [5]. Due to the extra  $w^2 - 1$  suppression near zero recoil,  $\bar{B} \rightarrow D\ell\bar{\nu}$  is also harder experimentally.

The main uncertainty in this determination of  $|V_{cb}|$  comes from the estimate of nonperturbative corrections at zero recoil. In the case of  $\bar{B} \rightarrow D^*\ell\bar{\nu}$ , model calculations [6] and sum rule estimates [7] suggest about  $-5\%$ . Assigning a 100% uncertainty to this estimate, I will use

$$\mathcal{F}_{D^*}(1) = 0.91 \pm 0.05, \quad \mathcal{F}_D(1) = 1.02 \pm 0.08. \quad (3)$$

The most promising way to reduce these uncertainties may be calculating directly the deviation of the form factor from unity,  $\mathcal{F}_{D^{(*)}}(1) - 1$ , in lattice QCD from certain double ratios of correlation functions [8]. Recent

quenched calculations give  $\mathcal{F}_D(1) = 1.06 \pm 0.02$  and  $\mathcal{F}_{D^*}(1) = 0.935 \pm 0.03$  [8], in agreement with Eq. (3) but with smaller errors.

Another uncertainty comes from extrapolating the experimentally measured quantity,  $|V_{cb}|\mathcal{F}_{D^{(*)}}(w)$ , to zero recoil. Recent theoretical developments largely reduce this uncertainty by establishing a model independent relationship between the slope and curvature of  $\mathcal{F}_{D^{(*)}}(w)$  [9]. This may also become less of an experimental problem at asymmetric  $B$  factories, where the efficiency may fall less rapidly near zero recoil.

Eq. (3) and the experimental average,  $|V_{cb}|\mathcal{F}_{D^*}(1) = 0.0347 \pm 0.0015$  [10], obtained using the constraints on the shape of  $\mathcal{F}_{D^*}(w)$  yield

$$|V_{cb}| = (38.1 \pm 1.7_{\text{exp}} \pm 2.0_{\text{th}}) \times 10^{-3}. \quad (4)$$

The value obtained from  $\bar{B} \rightarrow D\ell\bar{\nu}$  is consistent with this, but the experimental uncertainties are significantly larger.

For the determination of  $|V_{ub}|$  from exclusive heavy to light decays, heavy quark symmetry is less predictive. It neither reduces the number of form factors parameterizing these decays and nor determines the value of any form factor. Still, there are model independent relations between  $B$  and  $D$  decay form factors, e.g., the form factors which occur in  $D \rightarrow K^*\bar{\ell}\nu$  can be related to those in  $\bar{B} \rightarrow \rho\ell\bar{\nu}$  using heavy quark and chiral symmetry [11]. These relations apply for the same value of  $v \cdot v'$  in the two processes, i.e., from the measured  $D \rightarrow K^*\bar{\ell}\nu$  form factors one can predict the  $\bar{B} \rightarrow \rho\ell\bar{\nu}$  rate in the large  $q^2$  region [12]. Such a prediction has first order heavy quark and chiral symmetry breaking corrections, each of which can be 15 – 20%. Lattice QCD also works best for large  $q^2$ , but the existing calculations are still all quenched. Light cone sum rules [13] are claimed to yield predictions for the form factors with small model dependence in the small  $q^2$  region. Recently CLEO made the first attempt at concentrating at the large  $q^2$  region to reduce the model dependence, and obtained [14]

$$|V_{ub}| = (3.25 \pm 0.14_{-0.29}^{+0.21} \pm 0.55) \times 10^{-3}. \quad (5)$$

A determination of  $|V_{ub}|$  from  $\bar{B} \rightarrow \pi\ell\bar{\nu}$  is more complicated because very near zero recoil “pole contributions” [15] spoil the simple scaling of the form factors with the heavy quark mass. Still, in the future some combination of the soft pion limit, model independent bounds based on dispersion relations and analyticity [16], and lattice results may provide a determination of  $|V_{ub}|$  from this decay with small errors.

If experimental data on the  $D \rightarrow \rho\ell\nu$  and  $\bar{B} \rightarrow K^*\ell\bar{\ell}$  form factors become available in the future, then  $|V_{ub}|$  can be extracted with  $\sim 10\%$  theoretical uncertainty [12] using a “Grinstein-type double ratio” [17], which only deviates from unity due to corrections which violate both heavy quark and chiral symmetries. Such a determination is possible even if only the  $q^2$

spectrum in  $D \rightarrow \rho \bar{\ell} \nu$  and the integrated  $\bar{B} \rightarrow K^* \ell \bar{\ell}$  rate in the large  $q^2$  region are measured [18].

### 3 Inclusive decays

Inclusive  $B$  decay rates can be computed model independently in a series in  $\Lambda_{\text{QCD}}/m_b$  and  $\alpha_s(m_b)$ , using an operator product expansion (OPE) [19, 20, 21]. The  $m_b \rightarrow \infty$  limit is given by  $b$  quark decay, and for most quantities of interest it is known including the dominant part of the order  $\alpha_s^2$  corrections. Observables which do not depend on the four-momentum of the hadronic final state (e.g., total decay rate and lepton spectra) receive no correction at order  $\Lambda_{\text{QCD}}/m_b$  when written in terms of  $m_b$ , whereas differential rates with respect to hadronic variables (e.g., hadronic energy and invariant mass spectra) also depend on  $\bar{\Lambda}/m_b$ , where  $\bar{\Lambda}$  is the  $m_B - m_b$  mass difference in the  $m_b \rightarrow \infty$  limit. At order  $\Lambda_{\text{QCD}}^2/m_b^2$ , the corrections are parameterized by two hadronic matrix elements, usually denoted by  $\lambda_1$  and  $\lambda_2$ . The value  $\lambda_2 \simeq 0.12 \text{ GeV}^2$  is known from the  $B^* - B$  mass splitting. Corrections to the  $m_b \rightarrow \infty$  limit are expected to be under control in parts of the  $b \rightarrow q$  phase space where several hadronic final states are allowed (but not required) to contribute with invariant masses satisfying  $m_{X_q}^2 \gtrsim m_q^2 + (\text{few times}) \Lambda_{\text{QCD}} m_b$ .

The major uncertainty in the predictions for such “sufficiently inclusive” observables is from the values of the quark masses and  $\lambda_1$ , or equivalently, the values of  $\bar{\Lambda}$  and  $\lambda_1$ . These quantities can be extracted, for example, from heavy meson decay spectra. A theoretical subtlety is related to the fact that  $\bar{\Lambda}$  (or the heavy quark pole mass) cannot be defined unambiguously beyond perturbation theory [22], and its value extracted from data using theoretical expressions valid to different orders in the  $\alpha_s$  may vary by order  $\Lambda_{\text{QCD}}$ . These ambiguities cancel [23] when one relates consistently physical observables to one another. One way to make this cancellation manifest is by using short-distance quark mass definitions, but recent determinations of such  $b$  quark masses still have about 50 – 100 MeV uncertainties [24].

The shape of the lepton energy [25, 26, 27] or hadronic invariant mass [28, 29, 27] spectra in  $\bar{B} \rightarrow X_c \ell \bar{\nu}$  decay can be used to determine  $\bar{\Lambda}$  and  $\lambda_1$ . Last year the CLEO Collaboration measured the first two moments of the hadronic invariant mass-squared distribution. Each of these measurements gives an allowed band in the  $\bar{\Lambda} - \lambda_1$  plane, and their intersection gives [30]

$$\bar{\Lambda} = (0.33 \pm 0.08) \text{ GeV}, \quad \lambda_1 = -(0.13 \pm 0.06) \text{ GeV}^2. \quad (6)$$

This result agrees well with the one obtained from an analysis of the lepton energy spectrum in Ref. [25]. CLEO also considered moments of the lepton spectrum, however, without any restriction on the lepton energy, yielding

unlikely central values of  $\bar{\Lambda}$  and  $\lambda_1$ . Since this analysis uses a model dependent extrapolation to  $E_\ell < 0.6$  GeV, I consider the result in Eq. (6) more reliable [31]. The unknown order  $\Lambda_{\text{QCD}}^3/m_b^3$  terms not included in Eq. (6) introduce a sizable uncertainty [27, 29], which could be significantly reduced when more precise data on the photon energy spectrum in  $\bar{B} \rightarrow X_s \gamma$  becomes available [32, 33].

The significance of Eq. (6) is that, taken at face value, it gives  $|V_{cb}| = 0.0415$  from the  $\bar{B} \rightarrow X_c \ell \bar{\nu}$  width with only 3% uncertainty. The theoretical uncertainty hardest to quantify in the inclusive determination of  $|V_{cb}|$  is the size of quark-hadron duality violation [34]. Studying the shapes of these  $\bar{B} \rightarrow X_c \ell \bar{\nu}$  decay distributions may be the best way to constrain this experimentally, since it is unlikely that duality violation would not show up in a comparison of moments of different spectra. Thus, testing our understanding of these spectra is important to assess the reliability of the inclusive determination of  $|V_{cb}|$ , and especially that of  $|V_{ub}|$  (see below).

A new approach to replace the  $b$  quark mass in theoretical predictions with the  $\Upsilon(1S)$  mass was proposed recently [35]. The crucial point of this “upsilon expansion” is that for theoretical consistency one must combine different orders in the  $\alpha_s$  perturbation series in the expression for  $B$  decay rates and  $m_\Upsilon$  in terms of  $m_b$ . As the simplest example, consider schematically the  $\bar{B} \rightarrow X_u \ell \bar{\nu}$  rate, neglecting nonperturbative corrections,

$$\Gamma(\bar{B} \rightarrow X_u \ell \bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} m_b^5 \left[ 1 - (\dots) \frac{\alpha_s}{\pi} \epsilon - (\dots) \frac{\alpha_s^2}{\pi^2} \epsilon^2 - \dots \right]. \quad (7)$$

The coefficients denoted by  $(\dots)$  are known, and the parameter  $\epsilon \equiv 1$  denotes the order in the upsilon expansion. In comparison, the expansion of the  $\Upsilon(1S)$  mass in terms of  $m_b$  has a different structure,

$$m_\Upsilon = 2m_b \left[ 1 - (\dots) \frac{\alpha_s^2}{\pi^2} \epsilon - (\dots) \frac{\alpha_s^3}{\pi^3} \epsilon^2 - \dots \right], \quad (8)$$

In this expansion one must assign to each term one less power of  $\epsilon$  than the power of  $\alpha_s$  [35]. At the scale  $\mu = m_b$  both of these series appear badly behaved, but substituting Eq. (8) into Eq. (7) and collecting terms of a given order in  $\epsilon$  gives [35]

$$\Gamma(\bar{B} \rightarrow X_u \ell \bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \left( \frac{m_\Upsilon}{2} \right)^5 \left[ 1 - 0.115\epsilon - 0.035\epsilon^2 - \dots \right]. \quad (9)$$

The perturbation series,  $1 - 0.115\epsilon - 0.035\epsilon^2$ , is far better behaved than the series in Eq. (7) in terms of the  $b$  quark pole mass,  $1 - 0.17\epsilon - 0.13\epsilon^2$ , or the series expressed in terms of the  $\overline{\text{MS}}$  mass,  $1 + 0.30\epsilon + 0.19\epsilon^2$ . The uncertainty in the decay rate using Eq. (9) is much smaller than that in Eq. (7), both because the perturbation series is better behaved, and because  $m_\Upsilon$  is better

known (and better defined) than  $m_b$ . The relation between  $|V_{ub}|$  and the  $\bar{B} \rightarrow X_u \ell \bar{\nu}$  rate is [35]

$$|V_{ub}| = (3.06 \pm 0.08 \pm 0.08) \times 10^{-3} \left( \frac{\mathcal{B}(\bar{B} \rightarrow X_u \ell \bar{\nu}) 1.6 \text{ ps}}{0.001 \tau_B} \right)^{1/2}. \quad (10)$$

The epsilon expansion also improves the behavior of the perturbation series for the  $\bar{B} \rightarrow X_c \ell \bar{\nu}$  rate, and yields

$$|V_{cb}| = (41.9 \pm 0.8 \pm 0.5 \pm 0.7) \times 10^{-3} \left( \frac{\mathcal{B}(\bar{B} \rightarrow X_c \ell \bar{\nu}) 1.6 \text{ ps}}{0.105 \tau_B} \right)^{1/2}. \quad (11)$$

These results agree with other estimates [36] within the uncertainties. The first error in Eqs. (10) and (11) come from assigning an uncertainty equal to the size of the  $\epsilon^2$  term, the second is from assuming a 100 MeV uncertainty in Eq. (8), and the third error in Eq. (11) is from a 0.25 GeV<sup>2</sup> error in  $\lambda_1$ . The most important uncertainty is the size of nonperturbative contributions to  $m_\gamma$  other than those which can be absorbed into  $m_b$ , for which we used 100 MeV. By dimensional analysis it is of order  $\Lambda_{\text{QCD}}^4 / (m_b \alpha_s)^3$ , however, quantitative estimates vary in a large range. It is preferable to constrain such effects from data [32, 37].

For the determination of  $|V_{ub}|$ , Eq. (10) is of little use by itself, since  $\mathcal{B}(\bar{B} \rightarrow X_u \ell \bar{\nu})$  cannot be measured without significant cuts on the phase space. The traditional method for extracting  $|V_{ub}|$  involves a study of the electron energy spectrum in the endpoint region  $m_B/2 > E_\ell > (m_B^2 - m_D^2)/2m_B$  (in the  $B$  rest frame), which must arise from  $b \rightarrow u$  transition. Since the width of this region is only 300 MeV (of order  $\Lambda_{\text{QCD}}$ ), an infinite set of terms in the OPE may be important, and at the present time it is not known how to make a model independent prediction for the spectrum in this region. Another possibility for extracting  $|V_{ub}|$  is based on reconstructing the neutrino momentum. The idea is to infer the invariant mass-squared of the hadronic final state,  $s_H = (p_B - p_\ell - p_{\bar{\nu}})^2$ . Semileptonic  $B$  decays satisfying  $s_H < m_D^2$  must come from  $b \rightarrow u$  transition [38, 39, 40].

Both the invariant mass region  $s_H < m_D^2$  and the electron endpoint region  $E_\ell > (m_B^2 - m_D^2)/2m_B$  receive contributions from hadronic final states with invariant masses between  $m_\pi$  and  $m_D$ . However, for the electron endpoint region the contribution of states with masses nearer to  $m_D$  is strongly suppressed kinematically. This region may be dominated by the  $\pi$  and the  $\rho$ , and includes only of order 10% of the total  $\bar{B} \rightarrow X_u \ell \bar{\nu}$  rate. The situation is very different for the low invariant mass region,  $s_H < m_D^2$ , where all such states contribute without any preferential weighting towards the lowest mass ones. In this case the  $\pi$  and the  $\rho$  exclusive modes comprise a smaller fraction, and only of order 10% of the  $\bar{B} \rightarrow X_u \ell \bar{\nu}$  rate is excluded from the  $s_H < m_D$  region. Consequently, it is much more likely that the

first few terms in the OPE provide an accurate description of the decay rate in the region  $s_H < m_D^2$  than in the region  $E_\ell > (m_B^2 - m_D^2)/2m_B$ .

Since  $m_D^2$  is not much larger than  $\Lambda_{\text{QCD}}m_b$ , one needs to model the non-perturbative effects in both cases. However, assigning a 100% uncertainty to these estimates affects the extracted value of  $|V_{ub}|$  much less from the  $s_H < m_D^2$  than from the  $E_\ell > (m_B^2 - m_D^2)/2m_B$  region. Such estimates suggest that the theoretical uncertainty in  $|V_{ub}|$  determined from the hadronic invariant mass spectrum in the region  $s_H < m_D^2$  is about  $\sim 10\%$ . If experimental constraints force to consider a significantly smaller region, then the uncertainties increase rapidly. The first analyses of LEP data utilizing this idea were performed recently [41], but it is not transparent how they weigh the Dalitz plot, which affects crucially the theoretical uncertainties.

The inclusive nonleptonic decay rate to “wrong sign” charm ( $\bar{B} \rightarrow X_{u\bar{c}s}$ ) may also give a determination of  $|V_{ub}|$  with modest theoretical uncertainties [42], if such a measurement is experimentally feasible.

## 4 Conclusions

The present status of  $|V_{cb}|$  and  $|V_{ub}|$  is approximately

$$|V_{cb}| = 0.040 \pm 0.002, \quad |V_{ub}/V_{cb}| \simeq 0.090 \pm 0.025. \quad (12)$$

The central value and error of  $|V_{cb}|$  comes from first principles, and the uncertainty in both its exclusive and inclusive determination is of order  $1/m_Q^2$ . On the other hand, the above error on  $|V_{ub}|$  is somewhat ad hoc, since it is still estimated relying on phenomenological models.

Within the next 3–5 years, in my opinion, an optimistic scenario is roughly as follows. The theoretical error of  $|V_{cb}|$  might be reduced to 2–3%. This requires better agreement between the inclusive and exclusive determinations, since in the exclusive determination the nonperturbative corrections to  $\mathcal{F}_{D^{(*)}}(1)$  are at the 5% level and model dependent, while in the inclusive determination it is hard to constrain model independently the size of quark-hadron duality violation. It will give confidence in lattice calculations of  $\mathcal{F}_{D^*}(1)$  and  $\mathcal{F}_D(1)$  if they give the same value of  $|V_{cb}|$ , and the deviations of the form factor ratios conventionally denoted by  $R_{1,2}(w)$  from unity can also be predicted precisely. Quark-hadron duality violation in the inclusive determination of  $|V_{cb}|$  can be constrained by comparing the measured shapes of  $\bar{B} \rightarrow X_c \ell \bar{\nu}$  decay spectra in different variables (e.g., lepton energy, hadronic invariant mass, etc.).

At the same time, the theoretical error of  $|V_{ub}|$  might be reduced to about 10%. Again, a better agreement between the inclusive and exclusive determinations is needed. At this level only unquenched lattice calculations will be trusted, and they ought to give consistent values of  $|V_{ub}|$  from

$\bar{B} \rightarrow \pi \ell \bar{\nu}$  and  $\bar{B} \rightarrow \rho \ell \bar{\nu}$ . From exclusive decays a double ratio method discussed in Sec. 2 may give  $|V_{ub}|$  with  $\sim 10\%$  error. In inclusive  $\bar{B} \rightarrow X_u \ell \bar{\nu}$  decay, the hadron invariant mass spectrum should be measured up to a cut as close to  $m_D$  as possible. It would be reassuring as a check if varying this cut in some range leaves  $|V_{ub}|$  unaffected.

I would like to thank Jon Rosner and Bruce Winstein for inviting me and for organizing a very interesting and stimulating workshop. I also thank Adam Falk and Andreas Kronfeld for comments on the manuscript. Fermilab is operated by Universities Research Association, Inc., under DOE contract DE-AC02-76CH03000.

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