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MATHEMATICAL CONSIDERATIONS OF DETERMINING NEUTRON SPECTRA FROM ACTIVATION MEASUREMENTS

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Abstract

Several techniques of high-energy neutron spectroscopy, like those using activation detectors, moderating spheres, and nuclear emulsions, require a mathematical unfolding procedure to obtain the neutron energy spectrum from a set of measured data. We discuss the general requirements for a solution method as well as tests to study how well these requirements are met. The solution method has to be able to combine the information contained in the measured data with available prior knowledge of the neutron spectrum. Otherwise complications like nonuniqueness, oscillatory and negative character of the solution, are likely to arise. The commonly used solution techniques are reviewed and their applicability to high-energy neutron spectroscopy is discussed. The resolution obtainable for the neutron spectrum, although limited by the mathematical nature of the problem, is in most cases sufficient to be useful in problems of shielding design and operational safety.

1. Introduction

High-energy accelerators give rise to complex radiation fields which necessitate elaborate shielding around them. The shielding requirements of the machines presently in use, with energies up to 30 GeV, are largely dictated by the high-energy neutron component of the radiation field. Both design economy and operational safety require a good understanding of the radiation fields and reliable experimental techniques to measure them.

When high-energy particles escaping from the accelerator vacuum chamber strike the surrounding material large numbers of secondary particles are produced. Of these particles neutrons with energies above 150 MeV have the longest attenuation lengths, and thus dominate the shielding requirements. The slowing down of the cascade and evaporation neutrons through elastic and inelastic scattering gives rise to neutron spectra whose energy scale extends from thermal regions to the primary particle energies. A comprehensive report[1] of a large shielding study made at the CERN proton synchrotron, in cooperation by the CERN, LRL, and RHEL laboratories, discusses in detail the current understanding of the neutron fields inside the shield. Figure 1, which is taken from that report, shows some characteristic high-energy neutron spectra.

The neutron spectrum to be studied is characterized by energy and intensity ranges extending over several orders of magnitude, by very small flux values beyond any considerable thickness of shielding, and by omnidirectionality. The use of activation detectors has proven to be one of the best techniques to measure such neutron fields. The relative merits and experimental aspects of such detectors, including threshold detectors, moderating spheres, and nuclear emulsions, have been adequately discussed in earlier studies, many of which were presented in the First Symposium on Accelerator Radiation Dosimetry and Experience.[2]

The use of activation detectors does not, however, yield directly the neutron energy spectrum. A mathematical unfolding procedure is required to obtain the spectrum from the set of measured data.

Several numerical methods have been proposed for the solution of neutron spectra from activation measurements. The application of many of these techniques to high-energy neutron spectroscopy has met with considerable difficulty. Some of the problems arise from the mathematical characteristics of the equations to be solved, others are related to specific solution methods. These problems are often compounded by large uncertainties in the response functions of the detectors and in the measured data. However, a critical use of an appropriate solution technique can yield reliable neutron spectra, the resolution of which, although limited, is quite adequate to aid shielding design and operational safety.

In this paper we discuss the mathematical aspects of determining neutron spectra from activation measurements. We specify general requirements which an appropriate solution method has to meet and discuss procedures to test how well these requirements are met. We briefly review the commonly used solution techniques and discuss their applicability to high-energy neutron spectroscopy.

2. Formulation of the Problem

The measurement of radioactivity induced by neutrons provides information on the flux. The study of several activation reactions with different known energy-dependent response functions, or cross sections, enables us to obtain knowledge also of the energy distribution of the neutron flux. Specifically, in activation-detector spectroscopy we search for a solution for a neutron spectrum $\phi(E)$ from a set of activation equations of the form

$$A_j = C_j \int_{E_{min}}^{E_{max}} \sigma_j(E)\, \phi(E)\, dE, \quad \text{for } j = 1, \cdots, m. \tag{1}$$

Here A_j is the saturation activity of jth detector, $\sigma_j(E)$ is the corresponding response function, and C_j is a normalizing constant between count-rate and neutron flux units. The normalizing constants--sometimes difficult to specify--are taken to be equal to unity in the following. E_{min} and E_{max} define the energy range of the neutron spectrum, and m gives the number of detectors, normally between 4 and 15. Equation (1) is a degenerate case of a Fredholm integral equation of the first kind,

$$A(E') = \int_{E_{min}}^{E_{max}} K(E', E)\, \phi(E)\, dE, \qquad (2)$$

which arises in several unfolding problems.

The composition of the kernel of this integral equation is of great importance in several solution methods. In practical applications accurate knowledge of the response functions is greatly desired, and experimentally verified numbers should be preferred. In many cases calculated response functions are more readily available, and are used when there are not complete enough experimental data. In the testing procedures discussed later the calculated response functions may also be equally well used.

In threshold-detector spectroscopy the kernel is composed of the cross sections of different reactions. Figure 2 shows the experimental cross sections for a Berkeley detector system, in the development of which great emphasis has been placed on achieving maximum sensitivity. Fluxes as small as $1n/cm^2 sec$ can be measured with most of these detectors. [1]

The detection of spallation products in medium-heavy targets offers some interesting possibilities for making a new type of high-energy threshold detector system. Our preliminary studies[3] indicate the feasibility of direct γ-spectroscopic measurement of large numbers of reactions. For instance, in a copper target irradiated in high-energy particle flux, we can see more than 20 reaction products by using instrumental gamma spectroscopy with high-resolution Ge(Li) detectors. Figure 3 shows cross sections for some reaction products from elemental copper target, which we have calculated by using Rudstam's formalism.[4] The small values of the cross sections and the low detection efficiency limit these studies to areas of high fluxes.

Figure 4 shows the response functions up to about 100 MeV of moderating or Bonner spheres, that is, detectors having a thermal neutron detector inside moderating spheres of different diameters. These values have been calculated by Hansen and Sandmeier[5] and reported by Awschalom.[6]

The detection of protons scattered by fast neutrons forms the basis of a number of methods for measuring neutron spectra. In this study we consider one such technique, proton-recoil spectroscopy with nuclear emulsions. The kernel of the integral equation calculated from an analytical formula given by Gammel[7] is shown in Fig. 5. The formula, which is applicable up to 42 MeV, includes a correction term for nonisotropy; this becomes significant above 10 MeV. If, as usual, the proton recoil spectrum is determined for a limited number of track length bins, then the mathematical formalism is identical to that of a threshold-detector system. In Fig. 5 the cross section for producing a scattered proton of a specified energy is given by a curve which has a threshold at this energy.

3. Problems and Requirements for Solution Methods

Several solution methods are available for obtaining a formal solution to the first-order Fredholm integral equation. The formal methods, however, are not applicable in the case in which neither the measured distribution A(E') nor

the kernel $K(E', E)$ is known analytically; rather, each is known as a set of discrete points. The solution of such a system may be obtained through numerical techniques. In most of these the integral equation is approximated by a system of linear equations of sufficiently high order, and the methods known for the solution of such a system are applied. Activation-detector spectroscopy represents a difficult case of this problem, and special techniques are required in the solution.

Nonuniqueness of the Solution

The activation equation for a single detector can be matched by a spectrum of any shape, when properly normalized. In many cases the number of activation detectors and thus the number of activation equations is smaller than the number of points that specify the neutron spectrum. The solution of such a system is not unique. If no restrictions are placed on the shape of the solution, the homogeneous system, the system with zero responses, has also nonzero solutions. Such solutions may appear as unwanted oscillations in the solution of the nonhomogeneous system. This is exemplified by a calculation by Burrus,[8] which shows that for any integrable kernel the attenuation of a sinusoidal solution distribution increases without limit when the frequency of the sinusoid increases. In any practical measurement neither the responses nor the kernel are known exactly. These uncertainties add to the uncertainty of the solution.

Solution Classes

The terms exact, approximate, and appropriate solution are often used to characterize the solution obtained. An exact solution satisfies accurately the activation equations but often has unacceptable oscillatory character. An approximate solution matches the responses only within reasonable error limits. Selection of a physically acceptable approximate solution yields an appropriate solution, which is generally not unique. The remaining ambiguity reflects the accuracy and the number of the responses and the composition and the accuracy of the kernel.

Prior Information on the Solution

The selection of an appropriate solution among the nonunique solutions requires the use of prior information on the solution. Such information is almost always available on physical grounds. In neutron spectroscopy the solution is known to be nonnegative, and zero beyond certain maximum energy. Beyond some thickness of shielding the neutron spectrum can also be assumed to be relatively smooth. Additional information may be available on its intensity or shape at some energies.

In the solution technique it is important to properly weigh the prior knowledge and the information contained in the measured responses. The additional constraints applied to the solution should not prevent it from matching the measured responses, nor should they prevent the solution from assuming any physically acceptable shape.

Requirements for a Solution Method

Any appropriate solution method for the determination of a neutron spectrum from activation measurements has to meet two basic requirements. The

first requirement is that the neutron spectrum found be a solution to the activation equations if such a solution exists. This means that the method has to be able to find a solution which accurately matches the responses due to any reasonable spectrum.

The second requirement is that if many solutions to the activation equations exist, then an appropriate solution must be found. In other words there must be a flexible way to apply physical prior information on the solution, such as nonnegativity conditions and requirements of smoothness and general shape of the solution.

In determining neutron spectra from measured data some difficulties are likely to arise. Because of measurement errors and large uncertainties in the response functions one often encounters inconsistent sets of responses, that is, responses for which there does not exist any appropriate solution. In such a case a compromise has to be made between the requirement of matching measured responses and satisfying the prior information on the solution. But here again, with such cases, we can have confidence only if the solution method is known to be able to find a reasonable solution if such exists. The flexibility in applying the constraining information to the solution is also of major importance with these cases.

4. Testing Solution Methods

It is important to make sure that the solution method employed meets the requirements discussed above. This can be done conveniently by simulating the experiment by specifying test spectra and computing the responses of different detectors for these given test spectra. Uncertainties in the response functions and in the measurement are simulated by introducing random errors in the synthetized responses or in the response functions. The solution is then obtained from the synthetized responses without using any information directly.

The testing procedures are much easier to perform in this manner than with actual measurement. Furthermore we will know "the true solution" and can compare it to the spectrum obtained.

To illustrate the testing procedure we use a block diagram shown in Fig. 6. We first specify a test spectrum ϕ test which has a reasonable shape for a high-energy neutron spectrum. The responses due to this are obtained by simply integrating the activation equations. The errors and uncertainties in the measurement and in the cross sections can be simulated by perturbing these responses by random deviations. We thus get the input responses A^{input}. In the case of a measurement we of course obtain these input responses without knowledge of the true spectrum and true errors.

In determining the solution spectrum ϕ^{sol} one combines the information contained in the input responses with the prior information. And finally, or usually in the course of finding the solution ϕ^{sol}, we also compute the responses A^{sol} corresponding to ϕ^{sol}.

The requirement about the ability to satisfy the activation equations can be restated now in the following words. If we start with a reasonable test spectrum and do not use any perturbation--that is, $A^{input} = A^{test}$--then the method employed has to find such a solution that $A^{sol} = A^{input} = A^{test}$. For this to be

true it is generally not necessary that $\phi^{sol} = \phi^{test}$, although such a condition would also satisfy the requirement.

It is very instructive to use the test procedures in studying the importance of the a priori conditions as well. With many response kernels the synthesized responses may be easily matched with an appropriate solution that may be quite different from the test spectrum. In such cases we need to estimate the amount of prior information required for a close match between the two spectra. This is closely related to what could be called the inherent resolution of the kernel. That determines how exactly the solution is defined without using any prior information on the solution. In many cases specific prior knowledge of the solution must be applied to obtain an appropriate solution that is less ambiguous.

To estimate the success obtained in a test case we check the match between the test and the solution responses and the closeness of the solution to the test spectrum. We also evaluate the agreement of integral quantities such as the flux, the dose rate, and the mean energy.

5. Review of Existing Solution Methods

Several numerical techniques have been applied to the solution of neutron spectra from activation-detector measurements. Most of such studies have been directed towards the determination of epithermal and fast neutron spectra in nuclear reactors. The extension of these techniques to high-energy neutron spectroscopy has not always been successful, because of lack of suitable detectors and reliable cross-section data, inadequate prior knowledge of the solution, and the wide energy and intensity ranges encountered.

The utilization of prior information on the solution is essential to obtain a physically acceptable solution. This is done either by smoothing procedures, by nonnegativity constraints, or by a choice of suitable expansion functions. In the following some of the methods employed for fast-neutron spectroscopy and their applicability to high-energy neutron spectrometry are discussed.

Parametric Representation

If there is available a functional representation of the neutron spectrum based either on theoretical considerations or previous experimental results, then the parameters in such a representation can be determined by matching the measured responses. For instance in reactor experiments both thermal-neutron and fission spectra can be approximated by such formulae. Functional representation of neutron spectra has been applied to high-energy neutron spectroscopy as well.[9-11] The formula often used assumes a spectral shape of E^{-n} form, or a spectrum composed of several such sections of different slopes on a logarithmic scale, with possibly a smooth extension to zero at cutoff energy. The slope of the spectrum, the parameter n, and possible other parameters can be easily determined by matching the measured responses in the least-squares sense.

Parametric representations of this kind severely restrict the form which the neutron spectrum may assume. Consequently they should be used only when such restrictions are well founded, or more often, when not enough experimental information is available for other approaches. On the other hand this approach avoids most mathematical complications peculiar to other methods, and in some

cases allows the determination of neutron spectra directly from calculated tables, as described by Patterson et al.[11]

Orthonormal Expansions

Several numerical techniques used for the solution of neutron spectra from the activation equations can be classified as series-expansion methods. The neutron spectrum is expressed as a sum of linearly independent functions $\psi_k(E)$,

$$\phi(E) = W(E) \sum_{k=1}^{m} \beta_k \psi_k(E) , \tag{3}$$

where $W(\gamma)$ is a weighting function and m, the number of terms, is equal to the number of response functions. In the selection of the expansion functions one may try to satisfy boundary conditions of the solution, and use orthonormal functions to simplify the calculations. Orthogonal functions can be obtained also from the cross sections through the Gram-Schmidt procedure. The formalism of the orthonormalization and the determination of the coefficients β_k in the expansion through known technique of linear algebra have been discussed in detail by Ringle[12] and Di Cola et al.[13] With such techniques the linear independence of the response functions is of great importance; this requirement often limits the choice of activation detectors.

The application of orthonormal expansion techniques to neutron spectroscopy have been studied by several authors. Ringle[12] investigated their use with threshold detectors in the energy range of 2.5 to 30 MeV, and Gold[14] and Di Cola and Rota[13] in the determination of reactor fast-neutron spectra with activation foils. Severe limitations in the reliability and accuracy of the method were found in the studies. The convergence rate of the expansion is often not adequate to provide good accuracy and physically acceptable boundary conditions in the solution with a limited number of terms. Proper choice of the functions can improve the convergence; the necessity of such choice limits the flexibility of the method. Unfortunate choice of detectors may result in an ill-conditioned system in which small changes in known terms result in large variations in the solution. The solution often assumes negative values and it is not possible to easily use nonnegativity or other prior information on the solution. The deficiencies of the expansion methods are likely to be amplified when a larger energy range is covered by few detectors.

Least-Squares Expansion Methods

In the least-squares expansion or relative-deviation-minimization method the neutron spectrum is again expressed as a sum of expansion functions, as in Eq. (3). The coefficients β_k are determined by minimizing the quadratic form

$$Q = \sum_{j=1}^{m} \left[\frac{A_j - \sum_{k=1}^{n} \beta_k \int_0^{E_{max}} W(E)\, \psi_k(E)\, \sigma_j(E)\, dE}{A_j} \right]^2 \tag{4}$$

with respect to β_k.

This minimization can be performed for $1 \leq n \leq m$. The optimal value of n corresponds to smallest Q and a physically acceptable solution; in most cases this is found when $n < m$. The case $n = m$ is equivalent to the formal expansion method discussed in the preceding section. The details of the procedures are discussed by Di Cola et al.[13]

The success in the least-squares expansion method depends strongly on the choice of the basis functions. A proper choice gives an opportunity to satisfy the boundary conditions and reflect the expected behavior of the solution.

Least-squares techniques have been applied to the study of reactor fast-neutron spectra with activation detectors. Chebyshev and Laguerre polynomials have been used as expansion functions, both of which were found to give physically acceptable results.[12,13] The method has generally been found superior to the orthonormal expansion method. Di Cola et al. found the method to be more sensitive to the effects of experimental errors, but the results were still better than those from orthonormal expansions.

The minimum-relative-deviation method has been applied to high-energy neutron spectroscopy in the range of 2.5 to 30 MeV by Kohler.[15] Step-function and polygonal approximations were used for the solution. An iterative technique was employed to minimize the sum of the squares of the deviations with respect to parameters defining the amplitude of each step. These parameters were squared to impose the nonnegativity.

Although least-squares expansion methods have shown good success in the determination of the reactor fast-neutron spectrum, their use is less profitable with high-energy spectra. Since both the shape and the energy range of the spectrum may vary widely, it is difficult to find generally applicable basis functions. The step-function and polygonal approximations provide flexibility in this respect; however, the resolution, which is dictated by the small number of the expansion terms, remains very poor. Furthermore it is not possible to use prior information on the neutron spectrum in a flexible manner.

Iterative Unfolding Method for Response Matrices

An iterative unfolding method has been described by Scofield[16] and Gold.[17] The method finds nonnegative solution by minimizing through an iterative procedure the deviation between the measured and computed responses. The procedure is terminated after a certain number of iterations or when the deviations pass through a minimum.

This iterative method has been applied to proton-recoil spectroscopy by O'Brien et al.[18] in the study of high-energy accelerator leakage spectra, to multisphere spectroscopy by Awschalom,[6] and to multisphere and threshold-detector spectroscopy by Stevenson.[19] The procedure was compared to least-squares techniques by Su.[20]

The studies indicate that the method compares favorably to the others discussed above. The studies of multisphere technique by Awschalom and Stevenson indicated good success in the computation of integral quantities, such as flux and dose, in unfolding given test spectra. The determination of differential spectra indicated larger deviations from test spectra. In application to threshold detectors the method failed to match some of the responses, and consequently there was significant discrepancy between the solution and the test spectra.[19] The

method imposes a nonnegativity condition on the solution, and it is possible to use also smoothness constraints. The application of specific prior knowledge, such as cutoff energy or preferred spectral shape, has not been incorporated to the method.

Iterative Perturbation Methods

An iterative technique which employs the on-line facilities of the CDC-6600 computer has been developed at LRL and used for the analysis of high-energy neutron spectroscopy with few threshold detectors.[1] A cathode-ray-tube display is used with light-pen input. The user draws a spectrum with the light-pen on the screen after which the responses are computed for each detector. The solution is then perturbed in order to get a better match between the computed and the measured responses. After a number of trials the responses are matched, with an accuracy reflecting the experimental errors. The procedure also allows the user to apply any prior knowledge of the solution.

With an increasing number of detectors with overlapping response curves it becomes increasingly difficult to make decisions on the direction of the next iteration. This and the slow speed restrict the applicability of this method to the study of systems with relatively few response functions. In such cases, however, it performs quite well and avoids all the numerical difficulties which are common with all the other methods mentioned.

An iterative method in which the subsequent perturbations to the initial trial spectrum are automatically computed by using energy-dependent sensitivity functions has been reported by McElroy et al.[21] This method has been successfully applied to the determination of neutron spectra, mainly reactor spectra in the energy range 10^{-10} to 18 MeV. Hargreaves and Stevenson[22] have employed a simpler iterative technique based on regions of maximum response defined for each detector. The results reported from such calculations applied to high-energy neutron spectroscopy are still somewhat inconclusive. The iterative procedure used imposes the nonnegativity condition, but ill-conditioned cases may still result in diverging solutions. The results obtained with the simpler method, however, indicate that as good results can be expected as with the more complex procedures mentioned.[22]

Constrained Least-Squares Methods with Matrix Inversion

A numerical technique for the solution of first-order Fredholm integral equations incorporating a controlled degree of smoothness or closeness to a given approximative solution has been proposed by Phillips[23] and further developed by Twomey.[24] More recently extensions of these techniques have been reported by Greer et al.[25] and Strand and Westwater.[26] A generalized formalism was introduced by Routti[3] and is discussed below.

The integral equation

$$\int_{E_{min}}^{E_{max}} K(E', E)\, \phi(E)\, dE = A(E') + \epsilon(E'), \qquad (5)$$

where $\epsilon(E')$ reflects the uncertainties and error, is first replaced by a quadrature form

$$K\phi = A + \epsilon \,. \tag{6}$$

Here A is the measured spectrum with components A_j and errors ϵ_j, $j = 1, \cdots, m$, ϕ is the solution vector with components ϕ_i, $i = 1, \cdots n$, and K is the response matrix of dimensions $n \times m$. In the derivation of the quadrature form we approximate the solution by a piecewise linear continuous function. With an adequate number of steps this approximation provides an arbitrary closeness to any real continuous function without prescribing the shape of the solution.

The solution of the integral equation is obtained by minimizing the quadratic form

$$Q = Q_0 + \gamma(W_1 Q_1 + W_2 Q_2), \tag{7}$$

where

$$Q_0 = \sum_{j=1}^{m} r_j^{\epsilon} \epsilon_j^2 \,,$$

$$Q_1 = \sum_{i=1}^{n} r_i^{\phi^0} (\phi_i - \phi_i^0)^2 ,$$

$$Q_2 = \sum_{i=2}^{n-1} r_i^{d} (\phi_{i-1} - 2\phi_i + \phi_{i+1})^2 \,.$$

The term Q_0 is related to the matching of the responses, which can be weighted by r_j^{ϵ}. The term Q_1 requires closeness to a given approximate solution ϕ^0; this criterion may be weighted with an energy-dependent function specified by weights $r_i^{\phi^0}$. The term Q_2 imposes a smoothness requirement by including the numerical second derivative of the solution in the sum to be minimized; this also can be weighted with energy-dependent terms r_i^d.

The auxiliary conditions included in terms Q_1 and Q_2 are weighted relatively by W_1 and W_2, and finally γ specifies the overall importance of the a priori conditions. The solution is obtained by minimizing Q with respect to ϕ_i by setting

$$\frac{\partial Q}{\partial \phi_i} = 0, \quad \text{for } i = 1, \cdots, n. \tag{8}$$

The resulting equations can be written in matrix form and solved in a single matrix inversion.[3]

Somewhat similar techniques are used in the method reported by Tihonov.[27] In this case the sum to be minimized includes the norm of the solution and its first derivative. The application of that method to the multisphere spectroscopy has been proposed by Buxerolle et al.[28]

The statistical aspects of the numerical solution techniques have been discussed by Burrus[8] and Strand and Westwater.[26] The latter treat the problem where

the covariance matrices of both the observed vector A and the solution ϕ are known, and derive an optimal smoothing criterion based on maximum-likelihood method.

Greer et al.[25] have discussed in detail the case in which the function to be minimized may be written as

$$Q = \sum_{j=1}^{m} e_j^2 + \sum_{i=1}^{n} \left(\frac{\phi_i - \phi_i^0}{\phi_i^0} \right)^2 . \tag{9}$$

An iterative procedure was derived in which the problem is solved in several steps by replacing the approximate solution ϕ^0 by the solution ϕ of the previous step. The limiting solution, which except for numerical difficulties may be obtained directly, is shown to converge to the solution that is closest to the original trial solution in the least-squares sense. The iterative procedure has been applied by Greer et al. to the determination of reactor fast-neutron spectra from activation-detector measurements. Generally a fission-neutron spectrum was used as initial trial solution. Good results of both integral and differential quantities were obtained in test cases and with actual data.

The methods described provide convenient means to apply prior information on both the smoothness and the shape of the solution. However, the nonnegativity of the solution is not guaranteed. This leads into difficulties with large uncertainties in the measured responses and the cross sections, where a compromise must be made between matching the responses and satisfying the prior information. It is also difficult to properly weigh the auxiliary conditions in cases in which the neutron spectrum extends over very many orders of magnitude. On the other hand, the computation is quite fast even in cases with many response functions, such as proton recoil spectroscopy.

Generalized Least-Squares Method with Nonnegative Solution

To overcome the difficulties of the above matrix inversion methods we have developed a formalism in which the solution is forced to be nonnegative and the auxiliary conditions can be used on several different scales.[3] The neutron spectrum is again approximated by a piecewise linear, continuous function defined at energy points E_i by intensity values ϕ_i, which are taken to be squares of real numbers, $\phi_i = X_i^2$, to eliminate negative values. The requirements of matching the measured responses as well as satisfying the a priori conditions are combined by defining a quadratic form as in Eq. (7). Because of computation-economy requirements the neutron spectrum may be defined at fewer points than the cross sections. The constraints about the smoothness and approximative shape of the spectrum are now expressed either on a linear, a relative, or a logarithmic scale--for instance on a logarithmic scale as

$$Q_1^{\log} = \sum_{i=1}^{n} r_i \left(\log X_i^2 - \log \phi_i^0 \right)^2 , \tag{10}$$

$$Q_2^{\log} = \sum_{i=2}^{n-1} r_i^d \left(\log X_{i-1}^2 - 2 \log X_i^2 + \log X_{i+1}^2 \right)^2 .$$

The solution can no longer be obtained through matrix inversion, but rather by minimizing Q with respect to the parameters X_i through iterative techniques. We found a gradient minimization technique with variable metric to be well suited for this computation. [3]

The formalism described above allows us to combine prior information on the neutron spectrum with the information contained in the measurement of the responses in a very flexible form. The method and the computer program LOUHI, [3] written to perform the analysis, have been subjected to mathematical tests discussed in Section 4. These results indicate that the method meets all the requirements set for a solution method in Section 4. The technique is best suited for a large computer. In most cases the solution obtained is not a sensitive function of the weighting parameters used in the expression of Q. However, when largely perturbed test responses or inconsistent sets of measured data are used, the analysis benefits greatly from the possibility of running LOUHI on-line with cathode-ray-tube display of intermediate results and the option of choosing optimal weighting parameters while solving the problem.

6. Examples and Comments

To illustrate some of the points made of testing and analysis procedures we next consider a few examples. These computations have been performed with the program LOUHI. [3] All the cases discussed have been run by using a uniform logarithmic smoothing criterion and no other prior information on the solution.

Figure 7 shows a test case run with a simulated neutron spectrum with a 14-MeV peak and the emulsion kernel. Excellent agreement is obtained between the two proton recoil spectra due to the test spectrum and the solution spectrum, as well as the two neutron spectra. In this case the very good agreement between the solution and the test spectra is due to an unrealistically close match between the two proton spectra. If the proton spectrum had realistic uncertainties, then such a close match would result in an oscillatory solution spectrum. Figure 8 shows a case in which such oscillations have been avoided by using the smoothing criteria; however, the agreement between the test and solution spectrum is no longer so good.

With kernels of lower resolution than that of the emulsion kernel the statistical uncertainties of the responses often are not recognizable. In such cases even the perturbed responses may be matched arbitrarily accurately without introducing unacceptable oscillations in the solution spectrum. And often it is difficult to say whether the structure in the solution spectrum necessitated by a close match of the input responses is due to errors in the data or real structure of the neutron spectrum. For example, in Fig. 9 we show a neutron spectrum obtained from a set of detectors exposed inside the beam tunnel of the CERN PS. [1] Requiring a 5% match between the measured and computed responses necessitates the structure shown in the spectrum. If only four of the seven detectors exposed were used, or if 50% discrepancies between the measured and the computed of the additional three responses were accepted, then the smoother solution shown could be obtained. In this case it is difficult to know whether the structure is real or only a reflection of experimental uncertainties.

Kernels of lower resolution often leave some ambiguity in the results even when there are no errors in the input data. For example, in Fig. 10 there is considerable discrepancy at low energies between the test and the solution spectra despite an excellent match between the unperturbed input responses and the solution responses. In this case it is obviously caused by the lack of any response of the detectors at low energies. But often such discrepancy is more subtle, and can be

best explained by the limited inherent resolution of the kernel. This quantity unfortunately can not be easily characterized by any single number, but rather has to be determined in each case by using the testing procedures.

The limited inherent resolution also explains the apparent inconsistency that different solution spectra are obtained when different initial guesses are used in iterative methods. Once the responses have been matched accurately then the remaining ambiguity of the neutron spectrum depends on the a priori information specified or built into the program, rather than indicates any inadequacy of the method. For the same reason it is quite difficult to obtain reliable estimates of the accuracy of the spectral shape, or confidence limits of the solution, from a single computation. Rather this has to be estimated through multiple trials with different starting values and perturbed input responses.

Because of the mathematical nature of the problem the resolution obtained in neutron spectroscopy with activation detectors remains limited even in ideal cases. The detailed structure introduced in the test spectra or possibly existing in the real spectrum is difficult or impossible to recover. For the studies of the accelerator shields the resolution obtained is, however, quite adequate. And although spectral shapes may be sensitive functions of errors in the input data, these errors affect integrated quantities, such as flux and dose rates, much less severely. Knowledge of the energy distribution of the neutrons can also be used to obtain appropriate conversion factors to justify the use of a single or few detectors to measure such integrated quantities.[29] This information can also be used to justify simplified models in shielding calculations and dose estimations.[30]

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FIGURE CAPTIONS

Fig. 1. Typical high-energy neutron spectra. "PS bridge" and "ring top" refer to the CERN 30-GeV proton synchrotron. Relative intensities of different spectra are arbitrary.

Fig. 2. Response functions of high-energy neutron detectors.

Fig. 3. Spallation yields for different reaction products from Cu target calculated from Rudstam formula.

Fig. 4. The response functions of Bonner spheres of different diameters.

Fig. 5. The cross section as a function of the neutron energy for producing a scattered proton of a specified energy is given by a curve which has a threshold at this energy. The small deviations between different curves above their threshold energies are due to the nonisotropy correction in the distribution of scattered protons.

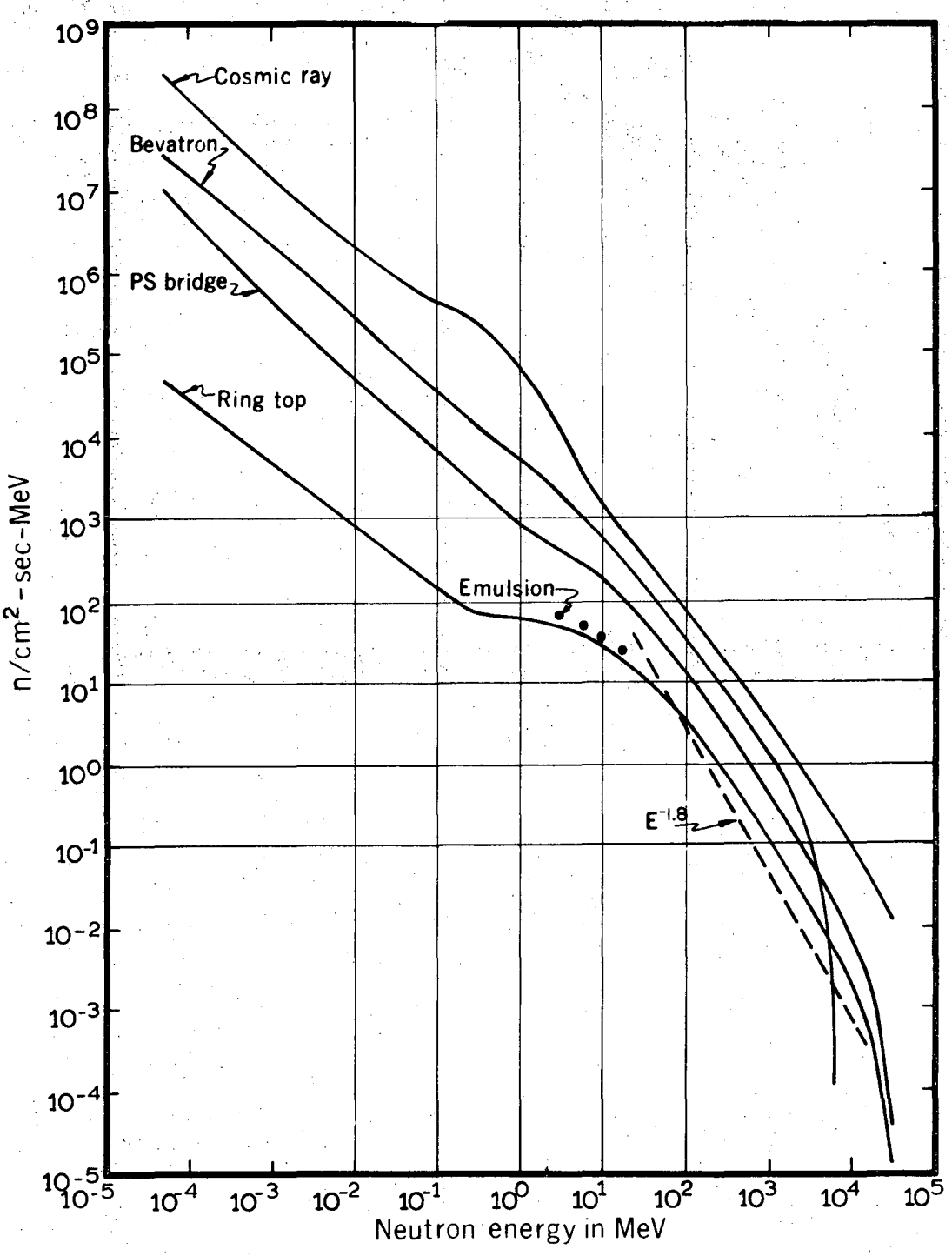
Fig. 6. Block diagram of the procedures used in testing the solution methods and analyzing measured data.

Fig. 7. Results from a test case with the proton-recoil scattering kernel.

Fig. 8. Results from a test case with the proton-recoil scattering kernel.

Fig. 9. Neutron spectra unfolded from measurements with four detectors (BF_3, Al $\rightarrow {}^{24}Na$, C $\rightarrow {}^{11}C$, Bi-fission) and with seven (the above plus S $\rightarrow {}^{32}P$, C $\rightarrow {}^{7}Be$, Hg $\rightarrow {}^{149}Tb$) exposed in the beam tunnel of the CERN 28-GeV proton synchrotron.

Fig. 10. Results from a test case with Cu-spallation kernel and program LOUHI. Excellent agreement is obtained between the unperturbed input responses and the calculated responses. The deviation of the solution from the test spectrum No. 7 can be explained by the absence of all response functions at low energies.



XBL 682 4493

Fig. 1

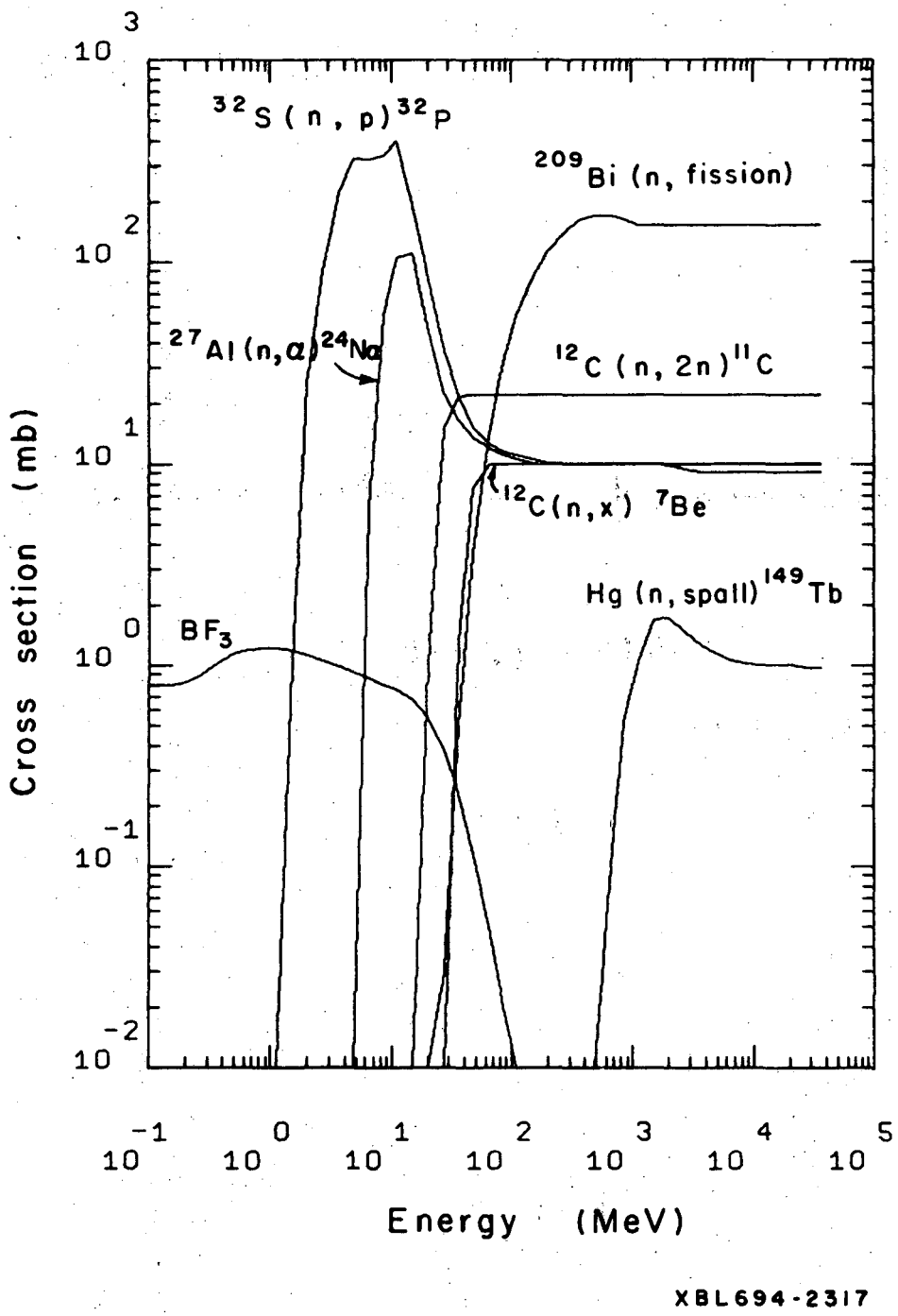

$^{32}S(n,p)^{32}P$
^{209}Bi (n, fission)
$^{27}Al(n,\alpha)^{24}Na$
$^{12}C(n,2n)^{11}C$
$^{12}C(n,x)\ ^{7}Be$
Hg (n, spall) ^{149}Tb
BF_3
Cross section (mb)
Energy (MeV)
XBL694-2317

Fig. 2

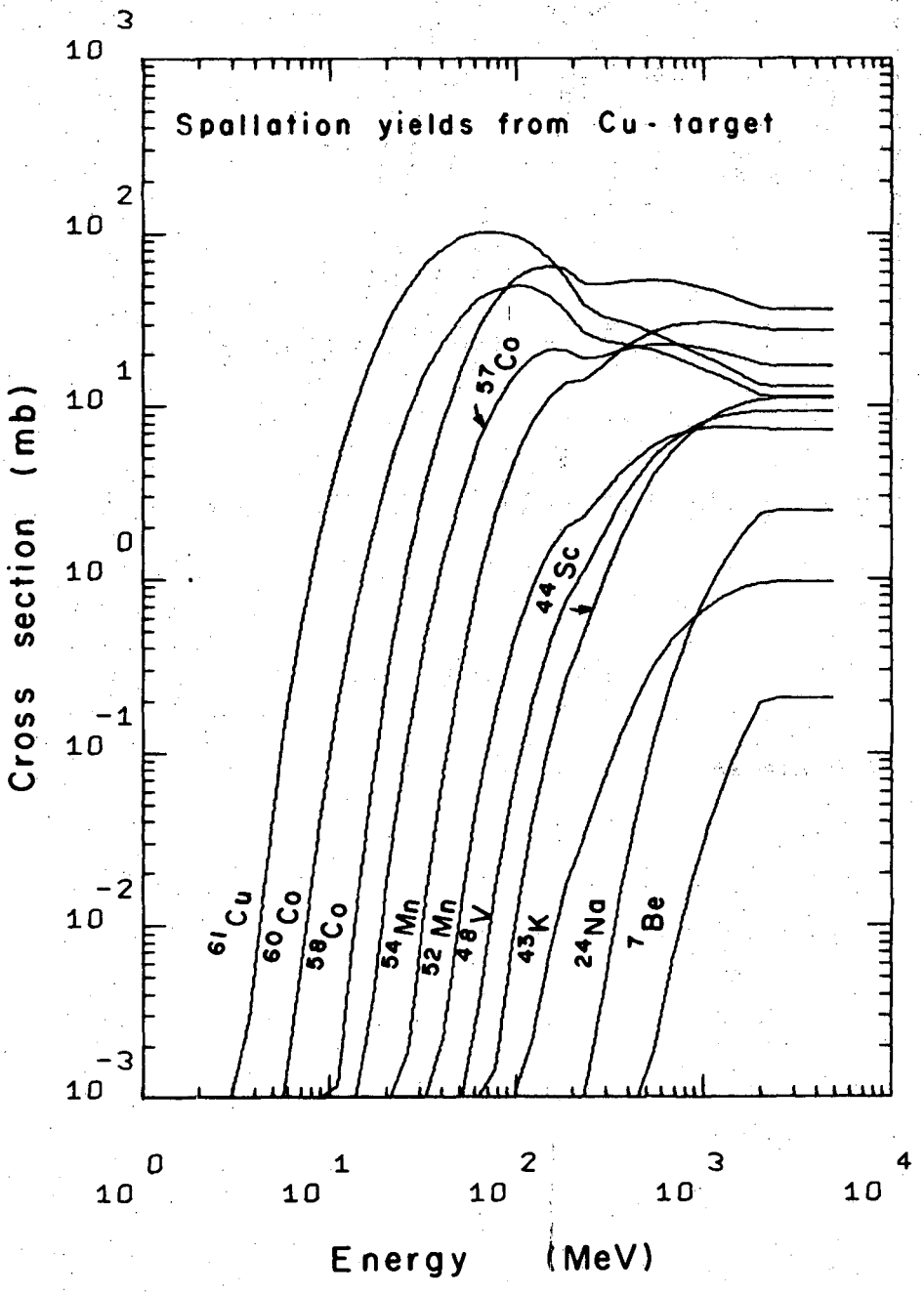

Spallation yields from Cu-target
Cross section (mb)
Energy (MeV)
61Cu
60Co
58Co
54Mn
52Mn
48V
57Co
44Sc
43K
24Na
7Be
XBL694-2320

Fig. 3

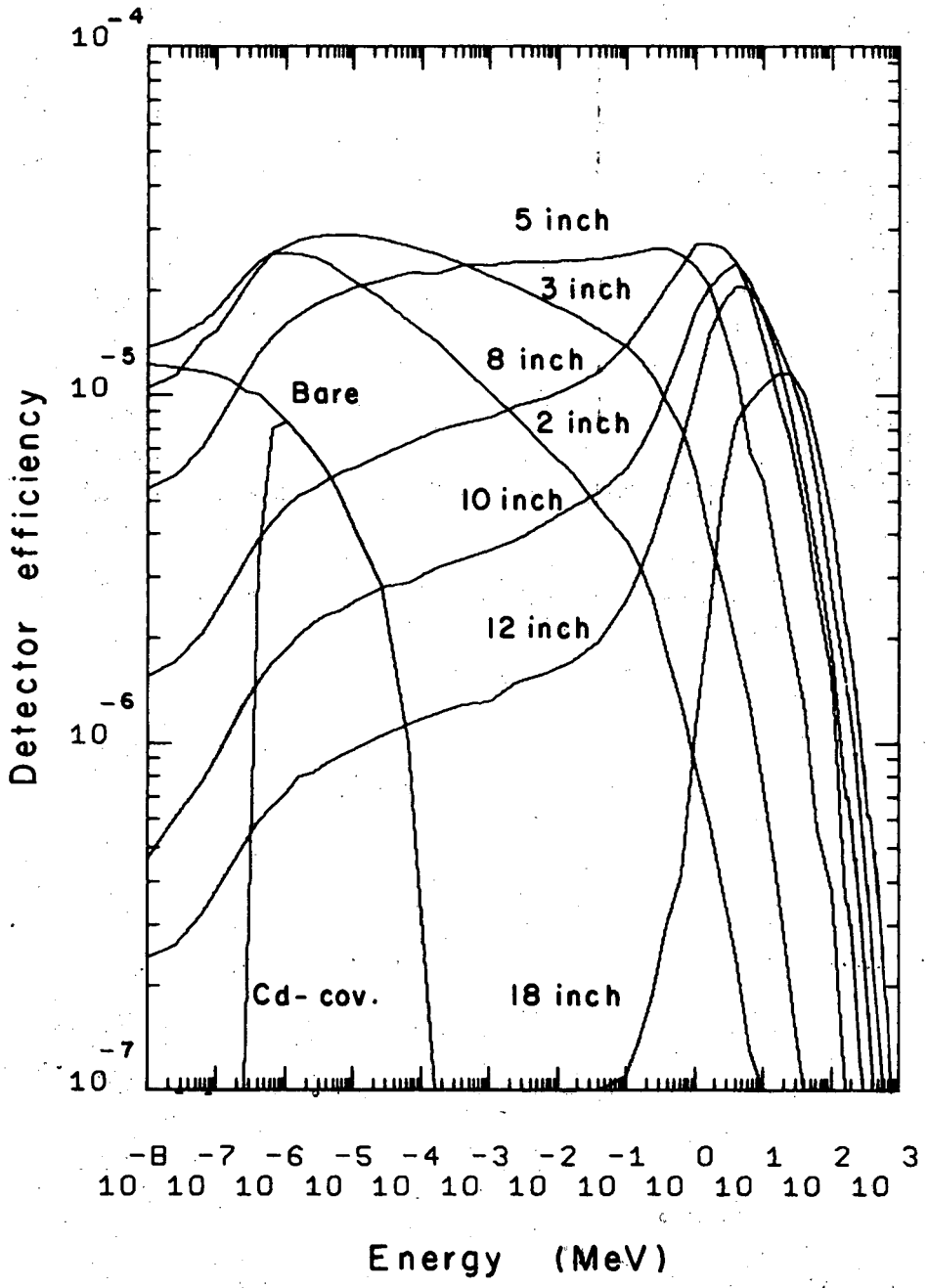

Detector efficiency
10^-4
10^-5
10^-6
10^-7
5 inch
3 inch
8 inch
Bare
2 inch
10 inch
12 inch
Cd-cov.
18 inch
10^-8 10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 10^0 10^1 10^2 10^3
Energy (MeV)
XBL694-2321

Fig. 4

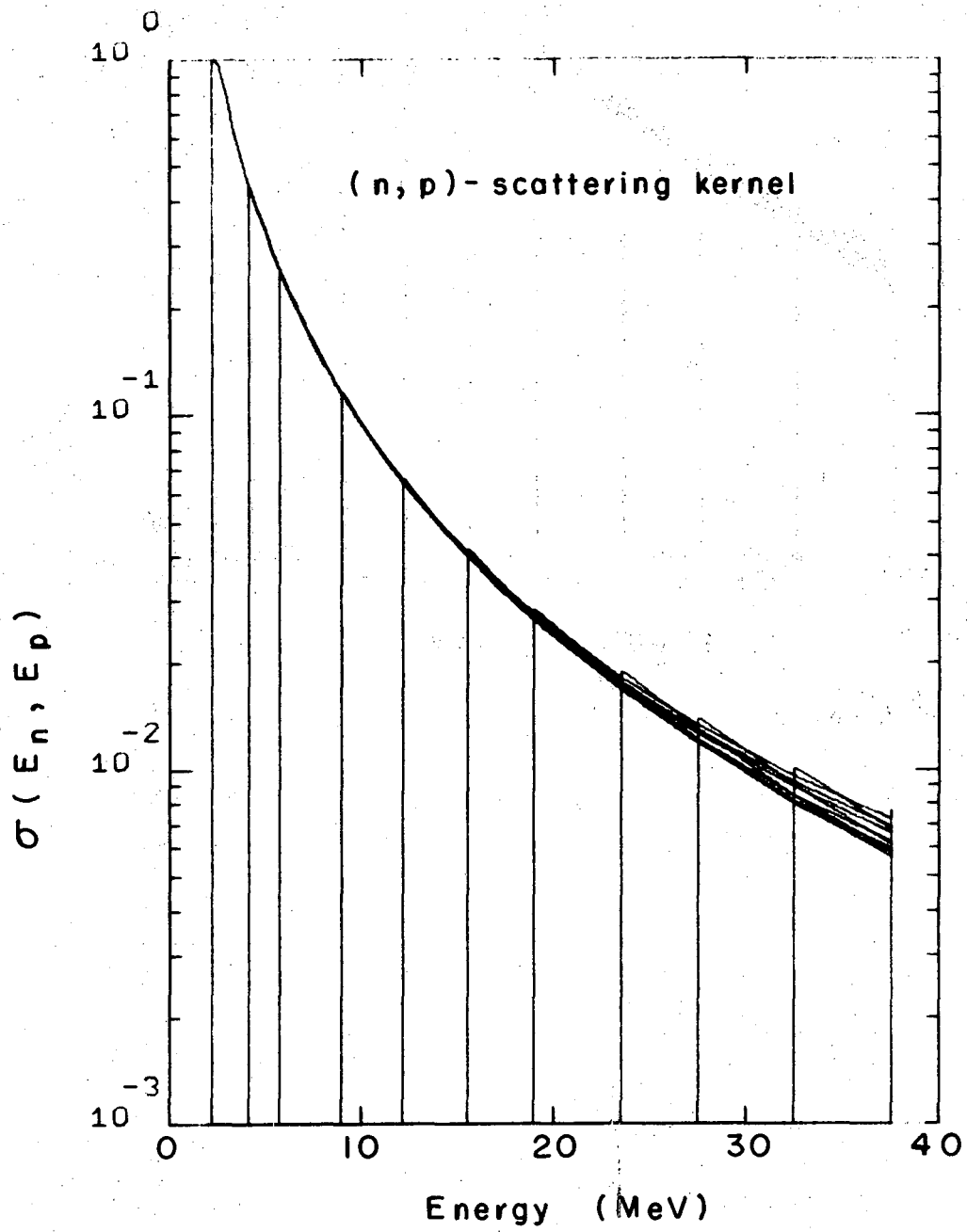

(n, p)- scattering kernel
$\sigma(E_n, E_p)$
10^{0}
10^{-1}
10^{-2}
10^{-3}
0
10
20
30
40
Energy (MeV)
XBL694-2306

Fig. 5

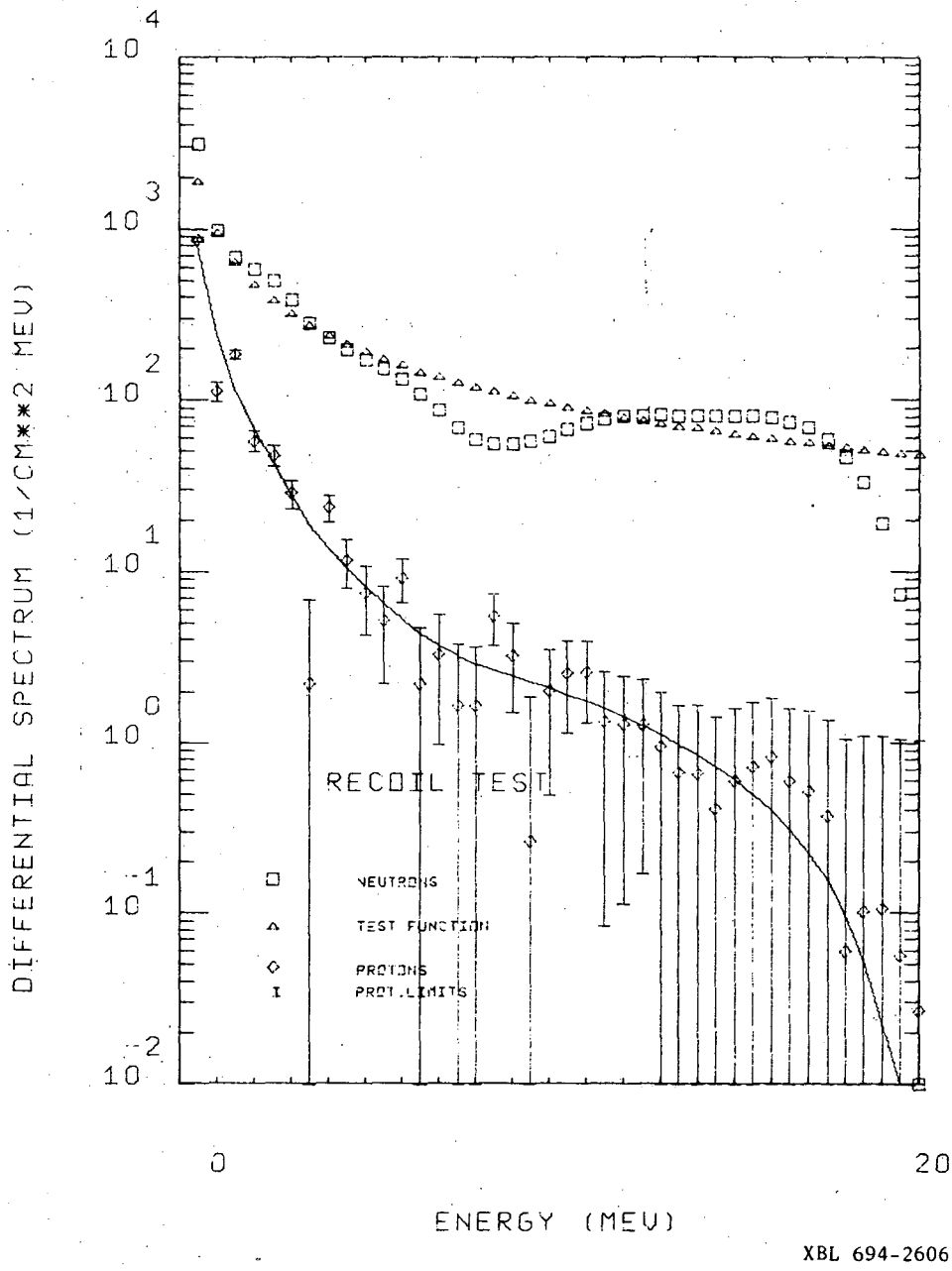

DIFFERENTIAL SPECTRUM (1/CM**2 MEV)
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
RECOIL TEST
NEUTRONS
TEST FUNCTION
PROTONS
PROT.LIMITS
0
20
ENERGY (MEV)
XBL 694-2606

Fig. 8

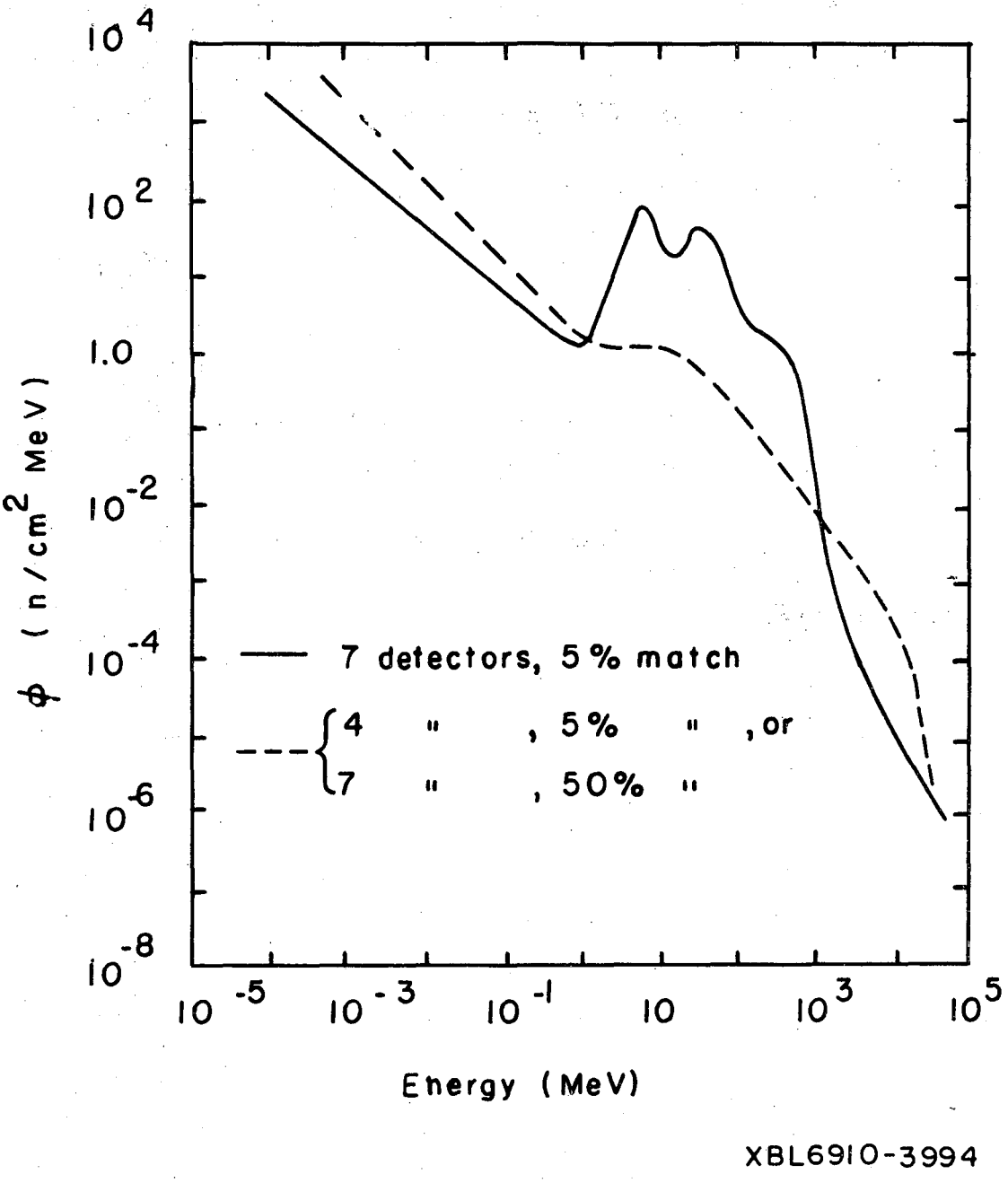

10^4
10^2
1.0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
ϕ (n / cm^2 MeV)
7 detectors, 5 % match
4 " , 5% " , or
7 " , 50% "
10^{-5}
10^{-3}
10^{-1}
10
10^3
10^5
Energy (MeV)
XBL6910-3994

Fig. 9

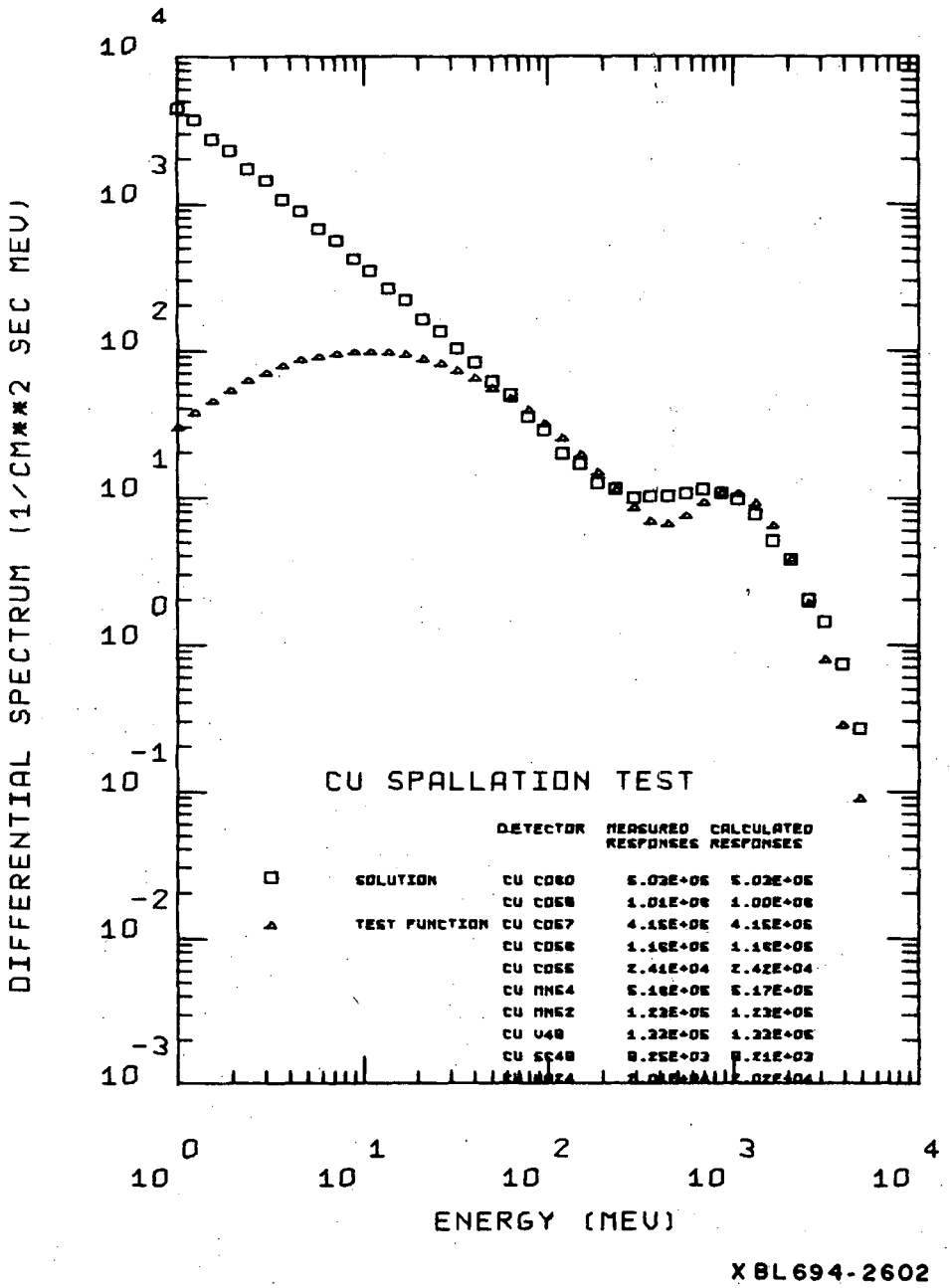

DIFFERENTIAL SPECTRUM (1/CM**2 SEC MEV)
ENERGY (MEV)
CU SPALLATION TEST
SOLUTION
TEST FUNCTION
DETECTOR
MEASURED RESPONSES
CALCULATED RESPONSES
CU C060 5.03E+05 5.03E+05
CU C058 1.01E+06 1.00E+06
CU C057 4.15E+05 4.15E+05
CU C056 1.16E+05 1.16E+05
CU C055 2.41E+04 2.42E+04
CU MN54 5.16E+05 5.17E+05
CU MN52 1.23E+05 1.23E+05
CU V48 1.32E+05 1.32E+05
CU SC48 8.25E+03 8.21E+03
XBL694-2602

Fig. 10

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