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# **Adaptive Optimization of Infrastructure Maintenance and Inspection Decisions under Performance Model Uncertainty**

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## **Abstract**

Infrastructure management systems assist agencies in making decisions regarding maintenance, repair, and reconstruction of the facilities under their jurisdiction. The objective in these decision-making tools is to minimize the total expected cost of managing a system of facilities over a given planning horizon. Recent optimization models account for the uncertainty in the selection of facility performance models through an adaptive control approach.

In this paper, we extend the methodology to jointly determine when to inspect and what maintenance activity to perform, while taking into account uncertainty in measuring facility condition. A parametric study is performed to analyze the effect of the initial performance model uncertainty and bias on the expected total cost of managing a facility over a finite horizon. The parametric study shows that reducing model uncertainty leads, as expected, to lower costs. The results also indicate that reducing the initial variance in model uncertainty is more important than reducing the initial bias. In addition, our study shows that cost savings can result from relaxing the constraint of a fixed inspection schedule.

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## 1. Introduction

Infrastructure management is the process through which agencies collect and analyze data about infrastructure systems and make decisions on maintenance, repair, and reconstruction (MR&R) of facilities over a given planning horizon. Bridge maintenance, road improvement, and highway rehabilitation are examples of MR&R activities.

In each period, usually every year, agencies face two types of decisions for each facility in an infrastructure network: whether to inspect or not, and which MR&R action to perform, if any. An agency's objective is to minimize the expected total costs associated with a facility's use and maintenance over a planning horizon. Infrastructure management systems (IMS) support agencies in performing the following functions:

- Data collection;
- Performance modeling and prediction; and
- MR&R decision-making.

MR&R decisions are based on current condition, as well as performance models that predict future condition under MR&R policies. Typically, several competing models can be used to represent deterioration. The multiplicity of models may result from different underlying assumptions about the physical process of deterioration, or they may represent different expert opinions about the future performance of a facility. This leads to a situation where agencies face uncertainty in the choice of model. This type of uncertainty is referred to as performance model uncertainty. Note that this uncertainty is not a result of the underlying randomness in the deterioration process. Indeed, each of the competing models may be probabilistic.

We capture performance model uncertainty by including beliefs about deterioration in the set of information that is used to make MR&R decisions. The beliefs correspond to an agency's assessment about which model can be used to represent the physical deterioration process, i.e., which model governs the process. Adaptive optimization models use observations of condition, obtained during the management of facilities, to update an agency's beliefs. Over time this results in an adequate representation of the physical deterioration process. Adaptive optimization models for MR&R decision-making are introduced in Durango and Madanat (2002) and Durango (2002). In this paper, we present an extension that jointly optimizes MR&R and inspection decisions.

Facility inspections serve two purposes in the context of adaptive MR&R optimization models. First, they provide an assessment of the current facility condition. Second, they provide information that is used to update an agency's beliefs, which in turn determines predictions about future condition. Adaptive MR&R optimization models assume that facilities are inspected in every period and that the inspection process is error-free, i.e., that it reveals the true facility condition. This motivates the development of a methodology that captures both the optimal timing of inspections, as well as the uncertainty that is inherent in measuring facility condition. As argued in Madanat (1993), because the two sets of decisions are linked, inspection policies cannot be optimized in isolation of MR&R policies.

The methodology presented herein can be used for each facility that falls under an agency's jurisdiction. It constitutes a first step in developing a network-level MR&R planning system that captures administrative restrictions that link the facilities that comprise the network. A possible approach to incorporate these restrictions would be to consider formulations such as the one presented in Madanat *et al.* (1999).

## 2. Literature review

The graph below illustrates the chain of events that take place during each time period in the process of managing infrastructure facilities.

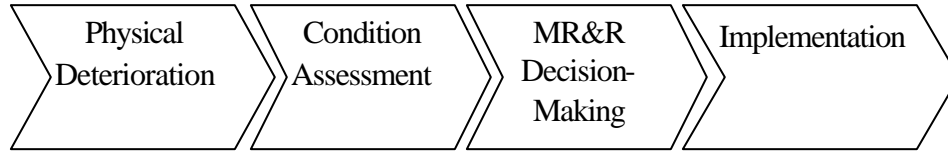


Figure 1. The event chain in infrastructure management

The first event corresponds to facility deterioration during a period. It can be caused by traffic, weather, or aging and results in changes in facility condition. At the end of each period, an agency can choose to inspect a facility to assess its current condition and can use a performance model to predict its future condition. Given the current condition and predictions about future condition agencies make decisions concerning the actions to be applied at the end of the period. The decision rule is to select actions that will minimize the sum of expected discounted costs until the end of the planning horizon. Finally, the last event corresponds to the implementation of the action in the current period.

State-of-the-art IMS assume that facility deterioration is both stationary and Markovian. In the remainder of this section we provide a review of such systems. At this point, we wish to emphasize that our contribution is general because uncertainties in selecting performance models or in measuring facility condition are also present in systems that do not rely on these assumptions.

### Markov Decision Process (MDP) Formulations

MDP formulations take into account the inherent randomness in facility deterioration. A finite set of states is used to represent facility condition, and the deterioration process is represented by transition probabilities as defined below:

$$\mathbf{p}_{ij}^t(a) = \Pr(x_{t+1} = j \mid x_t = i, a_t = a) \quad (1)$$

where:

- $x_{t+1}$  is the state of the facility at the beginning of period t+1,
- $x_t$  is the state of the facility at the beginning of period t, and
- $a_t$  is the action taken in period t.
- $i, j$  are elements of a set of states
- $a$  is an element of a (finite) set of actions

The Markovian assumption implies that the probability of a transition between any pair of states during a period only depends on the state at the start of the period and the action applied during the period.

The assumption that deterioration is stationary/time-homogeneous implies that the transition probabilities are constant over time. Among other things, this means that deterioration is independent of facility age. In this case,

$$\mathbf{p}_{ij}^t(a) = \mathbf{p}_{ij}(a), \forall t \quad (2)$$

The transition probabilities can be arranged in a set of matrices (one for each MR&R activity). The transition probabilities can be derived from empirical data or from expert opinions. Several approaches to estimate the probabilities are reported in the literature. Statistical estimation and time series approaches are discussed in Carnahan *et al.* (1987) and Olsonen (1988). Another approach based on a performance model and the properties of Markov Chains is proposed in Madanat (1991). Madanat and Wan Ibrahim (1995) describe how Poisson regression and, more generally, negative binomial regression can be used to estimate the probabilities. These methods are statistically sound and recognize the discrete representation of condition. Finally, Mishalani and Madanat (2001) develop a stochastic duration-based method to estimate the probabilities, which specifically takes into account the effect of causal variables, and recognizes the correlation between successive observations.

The MDP for the problem of finding optimal MR&R policies for infrastructure facilities is usually formulated as a dynamic program. The value function is defined as the expected, discounted cost until the end of the horizon. The cost incurred during each period includes both the user costs and the cost of applying MR&R actions. The optimal policy corresponds to a list of actions for each period and every possible state of the facility.

MDP formulations can be extended to the network-level with a linear programming formulation that is equivalent to the formulation described in the preceding paragraph. The state and action spaces are approximated by continuous spaces. This approximation is justified because of the large number of facilities that comprise infrastructure networks. The optimal policy specifies the joint fraction of the network in a given state that receives an action in a given period. The formulations can account for constraints that agencies face such as budget restrictions. Arizona's Pavement Management System was the first successful implementation of this type of formulation (Golabi *et al.* 1982).

Two important limitations of these models are related to the implicit assumptions that the true condition of the facility or network is revealed in every period, and that agencies can select (without uncertainty) a deterioration model that provides a perfect representation of the physical deterioration process.

The incorporation of joint decisions that include inspections and MR&R actions is relatively straightforward. Klein (1962) and Mine and Kawai (1982) among others present formulations that include inspection decisions. These models, referred to as joint models, do not account for

uncertainty in the process of measuring facility condition. Among other things, this uncertainty is inherent to measuring technologies. Madanat (1993) compares various formulations of the joint MR&R and inspection optimization problem and presents a Latent MDP formulation for the problem that accounts for measurement errors. Durango and Madanat (2002) present an adaptive control formulation that accounts for the uncertainty in choice or specification of performance models to represent facility deterioration. The formulation introduced in this paper relaxes both assumptions simultaneously by combining the Latent MDP formulation and the adaptive control formulations. In the remainder of this section, we discuss Latent MDP formulations and adaptive control formulations in more detail.

### Latent MDP Formulations

Research by Humplick (1992) has shown that there are significant measurement errors in existing inspection technologies. Measurement errors can lead to the selection of inappropriate actions when a policy specifies different actions for the true condition and the measured condition.

Latent MDP formulations are extensions of MDP formulations that include the inspection decision and account for uncertainty in the inspection process, i.e., measurement error. The possibility of a flexible inspection schedule and the presence of measurement errors constitute a violation of the basic premise that the true facility condition is revealed at the end of each period. Madanat and Ben Akiva (1994) present a Latent MDP formulation where the state-space of the problem is augmented as described in Bertsekas (1987). This technique takes advantage of the fact that the decision-maker knows the history of the past transitions and actions, and can capture this information in what is referred to as a sufficient statistic.

Under the state augmentation technique, the state of the system at stage  $t$  takes into account all the information available to the decision maker, since the beginning of the planning horizon, and that is relevant for decision-making. This is summarized by the information sets  $I_t$ , " $t$ ". At the end of period  $t$ , the set can be represented as follows:

$$I_t = \{I_0, a_0, \hat{x}_0, a_1, \dots, \hat{x}_{t-1}, a_{t-1}, \hat{x}_t\}, \quad (3)$$

where  $I_0$  represents the initial information available at the start of the planning horizon, and  $\hat{x}_t$  is the measured state during period  $t$ .

The set can also be defined recursively as  $I_t = \{I_{t-1}, a_{t-1}, \hat{x}_t\}$ .

Formulating a dynamic program with a state-space that corresponds to the information set constitutes a natural extension of the framework described earlier. The MDP formulations are adjusted by considering the conditional distribution of states for the given information set. The probability that the facility is in state  $x_t$  given the information set  $I_t$  is denoted  $\Pr(x_t | I_t)$ . The vector of probabilities for each state is denoted  $\underline{P}_t | I_t$ . This vector is referred to as the information vector or as the sufficient statistic. Similarly, the expected cost incurred during period  $t$  is defined as  $\tilde{g}(I_t, a_t) = E_{x_t} [g(x_t, a_t) | I_t]$ .

Due to the presence of measurement errors, the measured facility condition is now only probabilistically related to its true condition. We assume that the distribution of the measurement relative to the true state is known and depends on the technology used. The relationship between measured and true states is given by measurement probabilities:

$$\mathbf{e}_{jk}^r = \Pr(\hat{x}_t = k \mid x_t = j, R_t = r) \quad (4)$$

where:

- $\hat{x}_t$  and  $x_t$  are respectively the observed and true condition state of the facility, and
- $R_t$  is the technology used for measurement.

Measurement probabilities can be derived from empirical measurement error models as discussed in Humplick (1992).

### Adaptive Control Formulations

The models described in the preceding sections all refer to one deterioration model that can be specified with a single set of transition probabilities. This stems from the assumption that agencies can choose and specify (without uncertainty) a perfect, albeit probabilistic, representation of the physical deterioration process. Performance model uncertainty was identified early on as an important consideration in developing MR&R policies. For example, Carnahan (1988) states that MR&R policies are sensitive to the transition probabilities and that care should be taken in the choice and specification of a deterioration model.

Durango and Madanat (2002) present two adaptive control formulations, an *open-loop-optimal feedback* control formulation and a *closed-loop* control formulation, for the facility-level MR&R problem under performance model uncertainty. Model uncertainty is captured with a probability mass function over a finite set of models. It is denoted  $\underline{Q}_t = \{Q_t^1, Q_t^2, \dots, Q_t^D\}$  where D is the number of candidate models. The elements of the vector represent the probability assigned to the event that deterioration is governed by each of the models, i.e.,

$$\forall d \in [1, \dots, D], \quad Q_t^d = \Pr_t(\tilde{d} = d \mid I_t) \quad (5)$$

where  $\tilde{d}$  is a random variable that represents the true deterioration model.

The adaptive control formulations do not allow for flexible inspection schedules and do not account for uncertainty in the measurement process. In the next section we introduce a *closed-loop* control formulation for the facility-level problem that simultaneously accounts for uncertainties in measurements and in the choice or specification of performance models.

Finally, we summarize the contribution of the model we have developed by listing the assumptions used in the formulations presented in the literature. The summary appears in Table 1.

Model	Choice and specification of performance model	Condition assessment	Decisions
MDP	Deterministic	True state	MR&R
Joint MDP	Deterministic	True state	MR&R Inspection
Latent MDP	Deterministic	Measurement errors	MR&R Inspection
Adaptive Control	Probabilistic	True state	MR&R
Proposed model	Probabilistic	Measurement errors	MR&R Inspection

Table 1. Features of MR&R decision-making models

### 3. Proposed Model

Prior to presenting the formulation we introduce the following notation:

- N Number of possible states;
- A Number of possible actions; and
- $\alpha$  Discount factor.

Information sets are defined as in Equation (3) as follows:

$$I_t = \{I_0, a_0, \hat{x}_0, R_0, a_1, \dots, \hat{x}_{t-1}, a_{t-1}, \hat{x}_t, R_t\} \quad (6)$$

The information sets include the measured state and the measuring technology used. Alternatively, the sets can be written recursively as:

$$I_t = \{a_{t-1}, \hat{x}_t, R_t, I_{t-1}\} \quad (7)$$

The elements of the set of beliefs about deterioration are adjusted to fit this new information structure. They are denoted  $Q_t^d(R_t, \hat{x}_t, P_{t-1} | I_{t-1}, Q_{t-1}, a_{t-1})$ , which we write as  $Q_t^d$ .

Decision-making involves the choice of action to perform during time period  $t$ , as well as whether to inspect or not at the beginning of the next period. In this context, inspections provide information that reveals information about the current condition of a facility, that reveals information about the deterioration process, and that reveals information about measurement errors associated with the technology.



Prior to presenting the formulation we describe assumptions related to costs, performance models, and measurement errors.

### Model Specification

#### *Transition Probabilities*

A set of transition probabilities is specified for each of the deterioration models. They are denoted as:

$$\mathbf{p}_{ij}^d(a_t) = \Pr(x_{t+1} = j \mid \tilde{d} = d, x_t = i, a_t) \quad (8)$$

#### *Measurement error*

The notation for modeling measurement error is the same as the one presented for the Latent MDP formulation. The inspection decision is represented by a choice between two classes: one with the measurement precision associated with each inspection technology, and the other with a measurement error of infinite variance. The model can accommodate a set of different technologies. However, in the computational study we reduce the choice to a binary decision.  $R_t=1$  denotes an inspection and  $R_t=0$  for no inspection.

The “no inspection” decision ( $R_t=0$ ) refers to a technology where for each state the probability of measuring any state is uniformly distributed:

$$\forall j, k \in [1, \dots, N], \quad \mathbf{e}_{jk}^0 = \Pr(\hat{x}_t = k \mid x_t = j, R_t = 0) = \frac{1}{N} \quad (9)$$

This case, where every condition state is equally likely to be observed regardless of the true state, is shown to be equivalent to not inspecting in Madanat and Ben Akiva (1994). The associated cost is set to zero.

#### *Cost*

In the remainder of this section we will use the notation  $g(x_t, a_t, R_{t+1})$  for the generic cost incurred during period  $t$  associated with activity  $a_t$  on a facility in state  $x_t$  and choosing to use inspection technology  $R_{t+1}$  at the beginning of next period. The cost per period consists of:

User cost:	Assumed function of the state of the facility.
Inspection cost:	Assumed constant.
MR&R cost:	Assumed function of the action and of the state.
Salvage cost:	Assumed function of terminal state at the end of the planning horizon.

### Dynamic Programming Formulation

The formulation of the facility-level joint inspection and MR&R problem consists of the objective value function definition, the recursive relation, and a set of boundary conditions. The information available at the start of each period consists of  $\underline{P}_t \mid I_t$  and  $\underline{Q}_t$ . The first vector summarizes the information about the current facility condition. The second vector captures the

beliefs about deterioration which in turn determine an agency's predictions about future condition. The formulation is presented below.

### *Recursive relation*

At the start of each period, the minimum expected, discounted cost until the end of the planning horizon is defined as  $f_t(\underline{P}_t | I_t, \underline{Q}_t)$  and it can be written as:

$$f_t(\underline{P}_t | I_t, \underline{Q}_t) = \underset{a_t, R_{t+1}}{\text{Min}} \left[ \sum_{i=1}^N \Pr_t(x_t = i | I_t) \cdot \left( g(x_t, a_t, R_{t+1}) + \mathbf{a} \cdot \sum_{d=1}^D Q_t^d \cdot \sum_{j=1}^N \mathbf{p}_{ij}^d(a_t) \cdot \sum_{k=1}^N \mathbf{e}_{jk}^{R_{t+1}} \cdot f_{t+1}(\underline{P}_{t+1} | I_{t+1}, \underline{Q}_{t+1}) \right) \right] \quad (10)$$

Note that this expression explains why the inspection decision for period t+1 is made in period t: the inspection in t+1 directly influences the information vector  $\underline{P}_{t+1} | I_{t+1}$ , which is used for the recursive computation of the objective function in period t and the measurement probabilities  $\mathbf{e}_{jk}^{R_{t+1}}$ . The recursive relation is defined for every decision-making period and every possible state of the facility. For computational reasons the continuous spaces of the state vectors,  $\underline{P}_t | I_t$ ,  $\underline{Q}_t$ , are discretized.

Expression (10) is for the minimum expected total discounted costs given the information set. The expectation is taken over the current state whose probability mass function is specified by the information set. Thus, the expression can be rewritten as:

$$f_t(\underline{P}_t | I_t, \underline{Q}_t) = \underset{a_t, R_{t+1}}{\text{Min}} E_{x_t | I_t, \hat{x}_{t+1} | I_t} \left[ g(x_t, a_t, R_{t+1}) + \mathbf{a} \cdot f_{t+1}(\underline{P}_{t+1} | I_{t+1}, \underline{Q}_{t+1}) \right] \quad (11)$$

### *Boundary conditions*

The boundary conditions for the problem are presented below. They are used to assign the salvage cost for the facility at the end of the planning horizon.

$$f_T(\underline{P}_T | I_T, \underline{Q}_T) = E_{x_T | I_T} [s(x_T)] = \sum_{i=1}^N \Pr_T(x_T = i | I_T) \cdot s(i) \quad (12)$$

The boundary conditions are defined for every possible terminal state of the facility.

Finally, we describe how the beliefs about deterioration,  $\underline{Q}_t$ , and the information set,  $\underline{P}_t | I_t$ , are updated in each period. The updates reflect how an agency's beliefs about deterioration change to account for periodic measurements of a facility's condition. They also reflect how information from successive measurements can be used to reduce the variance in the inspection process.

### Updating the beliefs about deterioration

Updating the beliefs about deterioration requires the following:

- The inspection technology  $R_t$ ;
- The measured state  $\hat{x}_t = k$ ;
- The *a priori* beliefs about deterioration and the state, i.e.  $\underline{P}_{t-1} | I_{t-1}$  and  $\underline{Q}_{t-1}$ ; and
- The action applied  $a_{t-1}$ .

After the observation phase in period  $t$ , the beliefs about deterioration are updated as follows:

$$\begin{aligned} Q_t^d &= \Pr(\tilde{d} = d | I_t) = \Pr(\tilde{d} = d | \hat{x}_t, a_{t-1}, R_t, I_{t-1}) \\ &= \frac{\Pr(\hat{x}_t | \tilde{d}, a_{t-1}, R_t, I_{t-1}) \cdot \Pr(\tilde{d} = d | a_{t-1}, R_t, I_{t-1})}{\sum_{d'=1}^D \Pr(\hat{x}_t | \tilde{d}, a_{t-1}, R_t, I_{t-1}) \cdot \Pr(\tilde{d} = d' | a_{t-1}, R_t, I_{t-1})} \end{aligned}$$

As  $\Pr(\tilde{d} = d | a_{t-1}, R_t, I_{t-1}) = \Pr(\tilde{d} = d | I_{t-1}) = Q_{t-1}^d$ , we can write:

$$Q_t^d = \frac{\Pr(\hat{x}_t | \tilde{d}, a_{t-1}, R_t, I_{t-1}) \cdot Q_{t-1}^d}{\sum_{d'=1}^D \Pr(\hat{x}_t | \tilde{d}, a_{t-1}, R_t, I_{t-1}) \cdot Q_{t-1}^{d'}}$$

If we observe that:

$$\begin{aligned} \Pr(\hat{x}_t = k | \tilde{d}, a_{t-1}, R_t, I_{t-1}) &= \sum_{j=1}^N \Pr(\hat{x}_t = k | x_t, \tilde{d}, a_{t-1}, R_t, I_{t-1}) \cdot \Pr(x_t = j | \tilde{d}, a_{t-1}, R_t, I_{t-1}) \\ &= \sum_{j=1}^N \mathbf{e}_{jk}^{R_t} \cdot \sum_{i=1}^N \Pr(x_t = j | \tilde{d}, x_{t-1} = i, a_{t-1}) \cdot \Pr(x_{t-1} = i | I_{t-1}) \end{aligned}$$

It follows that:

$$\Pr(\hat{x}_t = k | \tilde{d}, a_{t-1}, R_t, I_{t-1}) = \sum_{i,j} \mathbf{e}_{jk}^{R_t} \mathbf{p}_{ij}^d(a_{t-1}) \cdot \Pr_{t-1}(x_{t-1} = i | I_{t-1}) \quad (13)$$

As  $\Pr_{t-1}(x_{t-1} = j | I_{t-1})$  is a component of the state vector from the previous stage, we have a recursive expression for  $Q_t^d$ .

$$Q_t^d = \frac{\sum_{i,j} \mathbf{e}_{jk}^{R_t} \mathbf{p}_{ij}^d(a_{t-1}) \cdot \Pr_{t-1}(x_{t-1} = i | I_{t-1}) \cdot Q_{t-1}^d}{\sum_{d'=1}^D \sum_{i,j} \mathbf{e}_{jk}^{R_t} \mathbf{p}_{ij}^{d'}(a_{t-1}) \cdot \Pr_{t-1}(x_{t-1} = i | I_{t-1}) \cdot Q_{t-1}^{d'}} \quad (14)$$

where  $k = \hat{x}_t$ .

Note that when the decision is not to inspect, the beliefs about deterioration are not updated. Indeed, no additional information is available to the decision-maker.

### Updating the beliefs about the facility state

Given the updated beliefs about deterioration, the decision-maker then revises her beliefs about the new state of the facility condition, taking into account:

- The current inspection decision  $R_t$  and observed state  $\hat{x}_t = k$  ;
- The new beliefs about the deterioration  $Q_t$  ;
- The past beliefs about the facility state  $P_{t-1} | I_{t-1}$  ; and
- The last action taken  $a_{t-1}$ .

The state vector defines the beliefs about the facility state  $P_t | I_t$  :

$$\begin{aligned} \Pr_t(x_t = j | I_t) &= \sum_{d=1}^D \Pr(x_t = j | \tilde{D} = d, I_t) \cdot \Pr(\tilde{D} = d | I_t) \\ &= \sum_d Q_t^d \frac{\Pr(x_t = j | \tilde{D} = d, a_{t-1}, R_t, I_{t-1}) \cdot \mathbf{e}_{jk}^{R_t}}{\sum_j \Pr(x_t = j | \tilde{D} = d, a_{t-1}, R_t, I_{t-1}) \cdot \mathbf{e}_{j'k}^{R_t}} \end{aligned}$$

In the same fashion as earlier, we write:

$$\begin{aligned} \Pr(x_t = j | \tilde{D} = d, a_{t-1}, R_t, I_{t-1}) &= \sum_{i=1}^N \Pr(x_t = j | x_{t-1} = i, \tilde{D} = d, a_{t-1}, R_t, I_{t-1}) \cdot \Pr(x_{t-1} = i | I_{t-1}) \\ &= \sum_{i=1}^N \mathbf{p}_{ij}^d(a_{t-1}) \Pr_{t-1}(x_{t-1} = i | I_{t-1}) \end{aligned} \quad (15)$$

Again, we notice that  $\Pr_{t-1}(x_{t-1} = j | I_{t-1})$  has already been determined, so:

$$\Pr_t(x_t = j | I_t) = \sum_d Q_t^d \frac{\sum_i \mathbf{p}_{ij}^d(a_{t-1}) \Pr_{t-1}(x_{t-1} = i | I_{t-1}) \cdot \mathbf{e}_{jk}^{R_t}}{\sum_{j',i} \mathbf{p}_{ij'}^d(a_{t-1}) \Pr_{t-1}(x_{t-1} = i | I_{t-1}) \cdot \mathbf{e}_{j'k}^{R_t}} \quad (16)$$

In the case where no inspection has been performed at the beginning of the period, the state vector is updated such that the new probabilities are the weighted transition probabilities. Although no new information is available, the decision-maker updates her beliefs using the performance models.

## 4. Computational study

We present a computational study in the context of pavement management with a planning horizon of 15 years and a discount rate  $r=5\%$ , where  $\alpha = 1/(1-r)$ .

As in Carnahan (1987), we assume that pavement condition is represented by 8 states, each representing 12.5 points on the PCI scale of 100. The agency can choose from the following MR&R actions: (1) do-nothing, (2) routine maintenance, (3) 1-in overlay, (4) 2-in overlay, (5) 4-in overlay, (6) 6-in overlay, and (7) reconstruction.

Three possible deterioration models are considered: slow, medium and fast. With each model being characterized by a set of 7 of transition probability matrices (one for each action). The models are taken from Durango and Madanat (2002) and are such that:

- The effect of MR&R actions on transitions is assumed to follow a truncated normal distribution with the mean depending on the action and the model and the variance depending on the model;
- Actions are less effective in improving pavement condition under faster deterioration models; and
- Faster deterioration models have higher variance in forecasting.

The means and standard deviations of the effects of actions are presented in Table 2.

		Deterioration Model		
		Slow	Medium	Fast
Action		Mean effect		
1		-0.50	-0.75	-1.75
2		0.50	0.00	-0.50
3		1.75	1.00	0.25
4		3.00	2.00	1.00
5		4.25	3.00	1.75
6		5.50	4.00	2.50
7		6.00	6.00	4.00
Std. Dev.		0.30	0.50	0.70

Table 2. Mean effect of the MR&R activities

The measurement error is assumed to be zero i.e.  $e_{jk}^1 = 1$  if  $k=j$ , 0 otherwise. If an inspection is performed, the agency is said to have “perfect state information”. This assumption was made to reduce the number of parameters and simplify the interpretation of the results.

The total cost includes the cost of inspection, the user cost and the cost of applying MR&R actions. The user cost is set to restrict a condition of at least state 2. This is done by setting the cost of reaching state one to infinity and is shown in Table 3. Furthermore, in order to prevent the facility from deteriorating too far at the end of the planning horizon, we set the salvage cost to be infinite for any final state worse than 5. Table 3 summarizes all of the costs considered.

Condition state	MR&R Activity							User Cost	Salvage Cost
	1	2	3	4	5	6	7		
1	0.00	6.90	19.90	21.81	25.61	29.42	27.97	∞	∞
2	0.00	2.00	10.40	12.31	16.11	19.92	25.97	25.00	∞
3	0.00	1.10	8.78	10.69	14.49	18.30	25.97	22.00	∞
4	0.00	0.83	7.15	9.06	12.86	16.67	25.97	14.00	∞
5	0.00	0.65	4.73	6.64	10.43	14.25	25.97	8.00	0.00
6	0.00	0.31	2.20	4.11	7.91	11.72	25.97	4.00	0.00
7	0.00	0.10	2.00	3.91	7.71	11.52	25.97	2.00	0.00
8	0.00	0.04	1.90	3.81	7.61	11.42	25.97	0.00	0.00

Table 3. Input costs

As in Madanat and Ben Akiva (1994), the inspection cost is assumed to be \$0.065/lane-yard.

### Results

Figure 3 (4) shows a comparison of the expected cost when the physical process corresponds to the slow (fast) model. For these figures, the initial beliefs about the state are  $P_0 | I_0 = (0, 0.1, 0.1, 0.2, 0.4, 0.2, 0, 0)$ . By “slow” beliefs, we mean that the initial beliefs about deterioration are such that  $\underline{Q}_0 = (0.8, 0.1, 0.1)$ . That is, a probability of 0.8 is assigned to the event that the physical process is governed by the slow model, 0.1 to the medium, and 0.1 to the fast model. Similarly, “fast” beliefs indicate that the initial belief vector is  $\underline{Q}_0 = (0.1, 0.1, 0.8)$ . We also consider an initial belief vector that corresponds to a case of high uncertainty. This vector is labeled “no” which stands for the non-informative initial beliefs i.e.  $\underline{Q}_0 = (0.3, 0.4, 0.3)$ . In computing the expected cost, we assume that the pavement is in state 5 at the start of the planning horizon.

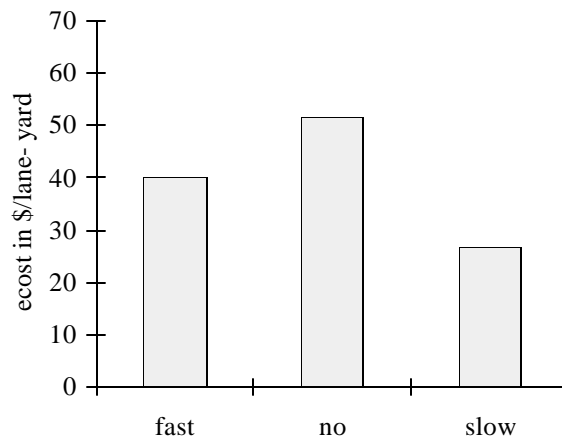


Figure 3. Expected cost: Physical deterioration = Slow

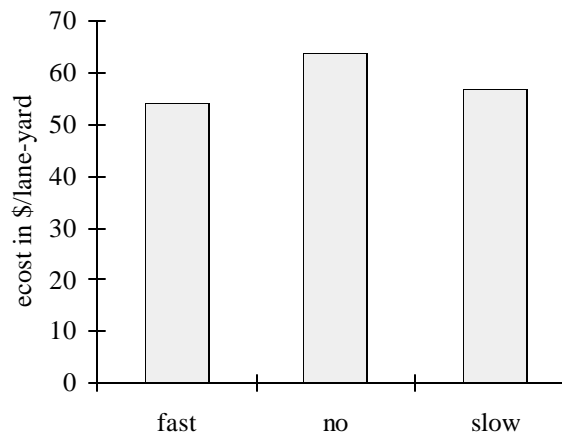


Figure 4. Expected cost: Physical deterioration = Fast

As expected, in both instances, whether the deterioration is slow or fast, when the initial beliefs are close to the physical process, the expected cost is the lowest. The expected costs are higher in Figure 4 than in Figure 3 because it is costlier to maintain a pavement that deteriorates faster. A noteworthy result is that the non-informative initial beliefs are the worst in both instances. This result seems to indicate that biased, but precise, beliefs about the deterioration model are preferred to less biased beliefs of higher variance.

To understand this strange result, we conducted a simulation study whose results are presented in Figure 5. That is instead of computing the expected costs analytically we generate instances for the two cases described above. The physical process corresponds to the fast model. The beliefs about the model in each period are averaged over the 1,000 simulations. We plot the trajectory of the average  $Q_t^3 = \Pr(d = \text{Fast} \mid \text{information})$ .

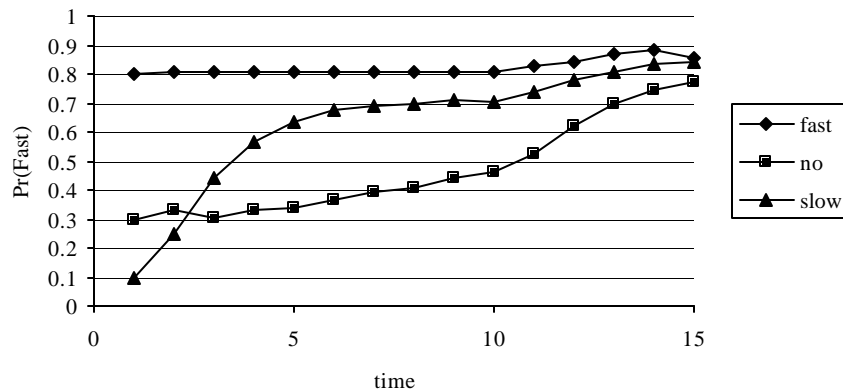


Figure 5. Trajectories of  $Q_t^3$ : Physical deterioration = Fast

As  $Q_t^3$  converges much faster when the initial beliefs are biased (slow), than when they are non-informative, the actions taken in the non-informative situation are not as efficient as those taken when the initial beliefs are wrong. Hence the higher expected cost when the initial beliefs about deterioration have a higher variance attached to them. The faster convergence of the beliefs in the biased case compared to the non-informative case can be explained qualitatively by the contrast between the observations and the expectations. This contrast is augmented by the action taken in both cases: when the initial beliefs are biased, the MR&R actions taken will be mild compared to the non-informative case. Therefore, worse states are more likely to be observed. Such unexpected outcomes provide feedback that leads to drastic and prompt revision of the beliefs in the biased case.

Finally, we compare the formulation we introduce to the *closed-loop* control formulation with fixed (yearly) inspections presented in Durango and Madanat (2002). The results are presented below:

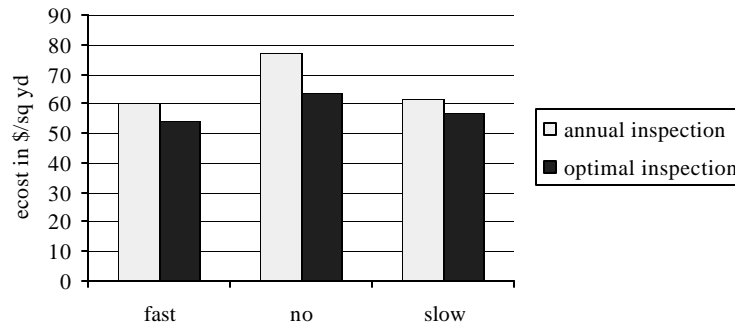


Figure 6. Comparison of expected cost: annual vs. optimal inspection:  
Physical deterioration = Fast

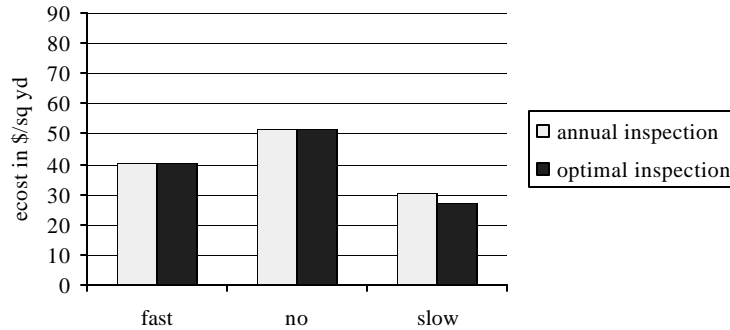


Figure 7. Comparison of expected cost: annual vs. optimal inspection:  
Physical deterioration = Slow

Figures 6 and 7 show that relaxing the constraint of annual inspections leads to a reduction in the expected costs. This is due to the fact that an inspection is performed only when it provides information that will improve future decisions. As a result, the expected number of inspections is reduced.

A noteworthy feature of both figures is that when the variance in the beliefs about the deterioration is low, the larger reduction in expected cost is observed when the initial beliefs are adequate. This indicates that the benefit of a flexible inspection schedule is greater when inspections provide less information, which is an intuitively correct result.

## 5. Conclusions

This paper presents an adaptive optimization model for the problem of finding joint inspection and maintenance policies for infrastructure facilities. The model relaxes the assumption of a fixed inspection schedule while accounting for uncertainties in the choice or specification of a performance model (to represent deterioration), and in the process of measuring facility condition. The methodology we present is referred to as adaptive because the information from measurements of condition is used to obtain an adequate representation of a facility's physical deterioration process over time, i.e., to learn about deterioration. In addition, the formulation



captures the value in inspecting facilities to assess current condition and/or to reduce the measurement error associated with the technology used for inspections

A computational study in pavement management leads to several insights about the problem. The results show that reducing the initial variance in model uncertainty is more important than reducing the initial bias. This means that providing the wrong information is less costly than providing no information about deterioration. The reason for this strange result is that the beliefs about deterioration can be adjusted drastically and quickly in response to unexpected events. Another result is that substantial benefits can be achieved by implementing a flexible inspection schedule when the initial beliefs are adequate. The reason is that inspections are providing very little information and so it is not necessary to perform them as frequently.

The scope of this research was purposely limited to the facility-level of the MR&R problem. An immediate extension is to adapt the formulation to the network-level problem with administrative restrictions. A possible approach to incorporate network-level constraints is to formulate the model developed herein using randomized policies and to solve it using linear programming.

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