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Becoming a Mathematics Professor: The Identities and Narratives of Mathematics Graduate
Students

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy
in Education

by

T. Royce Olarte

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June 2026

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Students

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by

T. Royce Olarte

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To my friends and family who have supported me not only throughout this PhD journey but through all the highs and lows of life: I would not be where I am today without you. The love, kindness, and generosity you have shown me have shaped not only the person I have become but also the scholar I strive to be. I hope to carry that same spirit of care into my own work and relationships. Thank you for believing in me and walking alongside me every step of the way.

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ABSTRACT

Becoming a Mathematics Professor: The Identities and Narratives of Mathematics Graduate Students

by

T. Royce Olarte

Graduate education is a critical period for socialization and professional identity development, particularly for students aspiring to become professors. However, our understanding of discipline-specific pathways into the professoriate and the development of professional identities within those pathways remains limited. Mathematics carries distinctive cultures, values, and discourses that shape how graduate students come to understand themselves as professionals. To better understand how disciplinary and institutional contexts influence this socialization process, I qualitatively examined the narratives of mathematics graduate students at a research-intensive university as they navigated their training and developed identities as prospective faculty. This three-paper dissertation explores an overarching question from multiple perspectives: What does it mean to become a mathematics professor? Across the papers, I draw on methodologies such as case study design and narrative inquiry, as well as theoretical perspectives including socialization theories and social identity theories, to foreground participants' experiences navigating graduate education and pursuing the professoriate.

First, I explored a model of professional identity development that conceptualizes the enactment of role identities as mediating one's broader professional identity development. I found that graduate students viewed the role identities of *teacher* and *researcher* as most salient. Other role identities included *mathematician*, *mentor*, and *peer and colleague*. Participants attached varied meanings to these identities (e.g., teaching as a way to disrupt exclusionary narratives in mathematics), and dimensions of their intersectional identities (e.g., gender) shaped how they interpreted and enacted these roles.

Next, I focused on the experiences of two women-identifying graduate students to further examine how gender identity intersects with the cultures, norms, and practices of graduate mathematics education. Using narrative inquiry, I identified how participants exercised power and encountered and resisted masculine discourses. This work highlights how masculinity operates within graduate mathematics education while offering actionable insights for reimagining and strengthening support structures for graduate students.

Lastly, I drew on perspectives of mental models of teaching to better understand how participants made sense of their pedagogical development and instructional approaches. Analysis of their reflections on teaching revealed that their mental models encompassed beliefs about student learning, teaching, and awareness of broader discourses within mathematics. I illustrate connections between cognition and practice while offering implications for better supporting graduate students' development as future educators.

Taken together, this dissertation deepens our understanding of the complex process of graduate student socialization and offers new insights into the disciplinary and institutional influences that shape how mathematics graduate students come to see themselves as professors.

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CHAPTER ONE

Hiking the Trail to the Professoriate

Statement of Problem

A long-standing mission of science, technology, engineering, and mathematics (STEM) graduate education has been to prepare the next generation of faculty members (Gemme & Gingras, 2012; Oleson & Hora, 2014; Wood et al., 2011). However, given the current landscape and pressures on higher education institutions, we can no longer assume that students enrolled in doctoral programs will continue to pursue careers in academia (Curtin et al., 2016). For example, the growing influence of industry and government sectors, through their financial presence and expanded career opportunities, has increasingly shaped the career choices and preparation of graduate students (O'Meara et al., 2014). At the same time, higher education institutions have been called to rethink historically held views, including adopting new teaching practices, which has added to the pressures that both faculty and graduate students experience.

Amid these challenges, the mission of preparing future faculty requires the field to broaden access to and increase the participation for all graduate students, with special attention to students historically underrepresented in higher education, including women, LGBTQ+ individuals, and students of color (Russell et al., 2018). Despite growth in graduate enrollment from these underrepresented communities in recent decades, the professoriate remains largely homogeneous, with white, male, heterosexual individuals continuing to represent the majority of STEM faculty (Haley et al., 2014). We risk simultaneously losing the invaluable knowledge, contributions, cultural wealth, and perspectives that historically

underrepresented students bring to our higher education institutions (Hill et al., 2011; MacLachlan, 2006).

Although supporting all graduate students across disciplines is imperative, it is important to examine each discipline, such as biology, mathematics, and other STEM disciplines, as unique cases, because each field is governed by its own practices, norms, and values that shape graduate students' experiences. Clarke et al. (2013) described discipline-specific cultures as "the primary source of faculty members' identity and expertise" because of the ways disciplines may uniquely define "how tasks [are] to be performed, standards for effective performance, patterns of publication, professional interaction and social and political status" (p. 7). Even among STEM fields, mathematics exhibits differences, such as in the lack of laboratory work, an emphasis on abstract knowledge, and a culture of procedural correctness (Mendick, 2006). Furthermore, mathematics has historically upheld and perpetuated exclusionary discourses, such as: mathematics is patriarchal and masculine (Leyva, 2021; Mendick, 2006), mathematics achievement is all about grit (Tefera et al., 2019), mathematics perpetuates Whiteness (Garcia-Olp et al., 2022; Gutiérrez, 2017), mathematics is anti-Black (Martin, 2009), mathematics is cisheteronormative (Voigt & Reinholz, 2020), and mathematics is neutral and objective (Gutiérrez, 2016). These discourses and long-held ideologies may permeate through the practices and policies of mathematics graduate education, but our understanding of how these narratives impact the everyday experiences and professional identity development of aspiring faculty is still limited.

Existing scholarship on graduate students' experiences and the pathways to the professoriate has illustrated that it is through experiences during graduate education that

students are socialized into the profession of higher education faculty (Gardner, 2008; Oleson & Hora, 2014; Tierney & Rhoads, 1993; Weidman et al., 2003). This perspective acknowledges that graduate students are not isolated, passive individuals navigating higher education; instead, they are shaped and influenced by the dynamic interplay of their individual identities and the socio-historic and cultural discourses associated with their discipline and profession. Moreover, higher education institutions are vital to students' career trajectories. Unfortunately, universities have not always provided the appropriate, equitable support and guidance that graduate students need to pursue and succeed in academic careers (Curtin et al., 2016), and this barrier to support may be more negatively felt by underrepresented graduate students (MacLachlan, 2006; Nyquist et al., 1999). While efforts to better prepare postsecondary students have been studied (e.g., Foote 2010; Herzig 2004a; Kersey & Voigt, 2020), and results from this work have been implemented (e.g., providing professional development interventions and mentorship), the pathways from graduate studies in mathematics to the mathematics professoriate is still relatively understudied, likely contributing to why systemic support and guidance is "not the norm" (Austin & McDaniels, 2006, p. 432). It is vital that we understand students' lived experiences to better support their transition to the professoriate.

In this dissertation, I leverage various perspectives of identity development (e.g., Zhang & Wang, 2024) and mathematics-specific discourses and cultures (Herzig, 2004a, 2004b; McBride, 1989) to examine aspects of graduate students' experiences in doctoral programs. Centering students' identities, voices, and experiences provides the field with actionable insights into the structural inequities and sites of supports that are most impactful to students' progressions through graduate school and to their pursuit of faculty positions. A

deeper understanding of *how* graduate students navigate the complexities of graduate education and *how* they develop their identities as mathematics professors positions us to better develop equity-centered career preparation for mathematics doctoral students. This dissertation serves as a foundation for my scholarly agenda on graduate students' identities and experiences and dismantling inequities within graduate education. I explore various contours of the complex process of *becoming* a mathematics professor by amplifying the identities and narratives of doctoral students navigating graduate school and pursuing faculty positions. In short, I ask: What does it mean to become a mathematics professor?

Positionality Statement

To better understand my relationship to this dissertation, I describe my research orientations and the experiences that led me to this work. My research approaches are aligned with social constructivist paradigms (e.g., Denzin & Lincoln, 2011), which consider phenomena to be the product of complex viewpoints, involve subjective meanings formed through interactions, and are shaped by historical and cultural norms (Creswell & Poth, 2016). As a researcher, my role is to make sense of these meanings with an awareness that my interpretations are shaped by my own personal, cultural, and historical experiences. A key motive driving my scholarship is that I seek to use my position as a mathematics education researcher to illuminate the voices and experiences of those who have historically been othered or marginalized in mathematics spaces. This has significantly guided my worldview, the research questions I aim to answer, the participants I hope to work with, and the ways I consider my research efforts to impact and contribute to the field and society more broadly. In line with the social constructivist paradigm, I recognize how my prior

experiences and intersecting identities have informed my dissertation and have guided my approaches and interpretations throughout this body of work.

My professional experiences as a graduate student and an adjunct mathematics faculty have motivated my research interests and inform what I consider to be salient dimensions of graduate school. In my own transition from graduate school to an adjunct mathematics faculty position, I received little formal pedagogical training and felt that my professional practices were an amalgamation of my experiences as a student and my limited experience as an instructor of record. As I have progressed through the PhD program and have engaged in various research projects related to mathematics graduate students – such as examining graduate students’ roles (OlarTE et al., 2024), and their pedagogical values (OlarTE & Street, 2025) – I observed that a largely understudied, and arguably one of the most important dimensions of their transition to the professoriate, is their professional identity development as higher education mathematics educators or university faculty. It became apparent that I have had very few opportunities during my own graduate education to reflect on my professional identity development. Moreover, there have been critical experiences in my own graduate studies that supported or hindered my pursuit of a career as a professor. My academic and professional experiences influence the ways I interpret how other students navigate graduate education and how they develop their professional identities as mathematics educators and university faculty.

Most importantly, I embrace that I am an “outsider within” (Dubbs, 2016, p. 1042) with regards to my dissertation and broader research agenda. My identities as an underrepresented graduate student in mathematics education have inspired me to anchor my research on supporting graduate students who are not part of the dominant cultures and

demographics in mathematics. This deeply influences my approach to research, such as in the frameworks and methodologies on which I draw. Although my male, cis-gendered identities are prevalent in mathematics spaces, my identities as a gay, Filipino-American, immigrant graduate student deeply impact the ways I navigate and progress through graduate school. For example, I have been acutely aware of the lack of other faculty and graduate students who share my social and cultural identities and have felt the pressure to academically excel considering the hardships and sacrifices related to immigrating from the Philippines or living up to the model minority myth. Confronting and persevering through my own challenges have ultimately inspired me to make the pathways from graduate education to the mathematics professoriate the cornerstone of my dissertation and future research agenda. As I examined this transition for graduate students, my own identities and experiences influenced how I interpreted and made sense of their narratives. I was particularly cognizant of any tendencies to use my personal narrative as an analytic center for examining and comparing participants' lived experiences, yet I see the alignment and connection to my lived experiences as a strength that I bring to this work.

Acknowledgement of Participants

This dissertation would not have been possible without Connor, Morgan, Landon, Kayla, Martin, and Kendall (pseudonyms are used for all names). I sought to and continue to hold care for the lived experiences and stories that they graciously shared with me. I am grateful for their time, willingness to participate, and most importantly for their honesty and authenticity in sharing their experiences. To deeply understand the nuances of their narratives and how they experienced the process of *becoming* mathematics professors, I invited them to share their stories in different ways, such as through interviews, writing "Dear Math..."

letters, and graphing visual representations of their graduate school journeys. In these moments of purposeful reflections on their experiences, the graduate students were vulnerable and described incredible successes and difficult hardships they experienced as they navigated graduate school. I honor the varied ways they shared their stories and hold space for the truths they shared with me throughout this process. Notably, Morgan, Kayla, and Kendall invited me into their worlds and lived experiences of navigating graduate mathematics education as women, an identity that I do not share. Despite this misalignment, they were vulnerable in sharing deep reflections and stories – for which I will always be grateful, and I am committed to honoring and protecting. Throughout the development of this dissertation, I tried my best to be critically aware of how my own intersectional identities resonated with or diverged from the experiences of Connor, Morgan, Landon, Kayla, Martin, and Kendall, and how this might have shaped the ways that I made sense of their experiences. This dissertation is an effort to amplify their voices, identities, and narratives to illuminate and dismantle inequities in graduate mathematics education.

The Overall Story

The overall story across this dissertation is centered around the process of *becoming* a mathematics professor, particularly through the lens of graduate students' experiences and identity development. I examined different contours of what it means to become a mathematics professor, where each chapter speaks to an aspect of the nuanced and complex process of developing as a mathematics professor. Although I drew on different theories and perspectives within each chapter, two theories underscore this body of work: *socialization theory* (Gardner & Mendoza, 2023; Tierney & Rhoads, 1993; Weidman & Stein, 2011) and *social identity theory* (Tajfel & Turner, 1986; Trepte, 2013).

Socialization refers to the process through which graduate students come to understand what it means to be a higher education mathematics faculty, acquire the knowledge and skills needed to participate in the profession, internalize and identify with the norms, beliefs, and values associated with the profession, and ultimately develop their professional identities as mathematics educators (Haley et al., 2014; Van Lankveld et al., 2017). Aligned with sociocultural perspectives, socialization theory recognizes the weight and consequences of the context and social interactions in the transmission of culture and acquisition of the knowledge necessary to perform and function in the profession (Clarke et al., 2013; Solari & Ortega, 2020). This can involve observing more expert others, learning by doing, interacting with others, and reconciling personal ideologies and beliefs about the profession with the everyday realities (Pillen et al., 2013; Tateo, 2012). Additionally, the other members of one's profession and context attach and attribute meanings to the individual, thus the notion of becoming a professional is considered "socially legitimated" (Lamote & Engels, 2010, p. 5).

However, a significant, often overlooked, limitation to socialization theory is the purposeful acknowledgement of the preparation, recruitment, training, and hiring of minority populations to faculty positions. As Haley et al. (2014) noted, "previous research has used models of professional socialization that may not take into consideration the unique cultural characteristics of underrepresented minority graduate students" (p. 102). Socialization theory may not be enough to fully explain the lived experiences of underrepresented, marginalized students. Thus, I also center social identity theory (Haley et al., 2014; Tajfel & Turner, 1986; Trepte, 2013) to explain how students' individual social and cultural identities shape how

they are socialized into the professoriate. Ostrove et al. (2011) described the importance of social identity in higher education:

Social identity matters, and its consequences are most destructive for those whose social identities are not only least compatible with the identities that populate most institutions of higher education but also have material consequences for individuals' experiences in higher education. (p. 769)

Illustrating the importance of social and cultural identities within mathematics contexts, Herzig (2004a) argued, "Few women, and even fewer African Americans, Latinos, and Native Americans complete doctoral degrees in mathematics in the United States" (p. 171). As a result, we have seen a growing number of scholars focused on addressing the attrition of underrepresented students, such as women, people of color, Indigenous peoples, and members of the LGBTQ+ community. Austin and McDaniels (2006) added, "The academy needs to honor their [underrepresented students'] contributions in ways that enable their presence to change the academy" (p. 443). In this dissertation, I particularly emphasize the experiences of women-identifying graduate students (i.e., the focus of Chapter 3), because gender-identity became the most influential identity that shaped participants' experiences and one that frequently emerged in their stories and reflections. I argue that critically assessing our perspectives on mathematics graduate education, seeking to better understand the experiences of graduate students, and consequently restructuring systems can better support graduate students' transition to the mathematics professoriate.

Grounding Metaphor: Hiking the Trail to the Mathematics Professoriate

To bring the theories of socialization and social identity together, I conceptualized a grounding metaphor for illustrating the complex story and process of becoming a

mathematics professor. I was particularly inspired to reframe prior metaphors of progression through higher education that historically positioned students as passive participants (e.g., the leaky pipeline metaphor, Alper, 1993; Pell, 1996; White, 2004), Herzig (2004a) suggested conceptualizing graduate school as a cross-country track event. I build on Herzig's cross-country metaphor, but I purposefully diverge from possible messages and connotations that can be connected to colonialist, ableist, and capitalist ideologies. For example, the cross-country track metaphor implies that there may only be a few winners with many losers and that the path is linear and one-directional. I attempt a reimagination of the cross-country race metaphor to mediate some of these implicit messages. I suggest that we can reconceptualize the pathways to the professoriate as *hiking a trail* (see Figure 1). This metaphor does not suggest a competitive endeavor (i.e., winners and losers), it recognizes that there may be multiple trailheads (i.e., starting points) to reach the destination, and it considers that barriers and obstacles may exist at various points throughout the trail (i.e., the complex terrain). These aspects of hiking a trail serve as metaphors for the varied experiences of graduate education that contribute to the process of becoming a mathematics professor.

Figure 1

Hiking the Trail to the Mathematics Professoriate



Within the cross-country race metaphor, I conceptualize graduate students as hikers and active participants on their professional journeys. Graduate students, like hikers, are equipped with a range of abilities, skills, experiences, and resources as they begin the hike. Some hikers may be more skilled at hiking long distances or high elevations, similar to how some graduate students may have varying strengths in conducting research, teaching, writing, and expertise of specific mathematics content areas. Additionally, there may be multiple trailheads (or pathways) to the destination, and hikers can choose when and where to begin their trail. Graduate students can similarly pursue various pathways to the mathematics professoriate, such as selecting which institutions to attend to pursue their graduate degrees, selecting the type of degree, and/or selecting their focal mathematics content areas.

This metaphor also recognizes that there may be challenges and obstacles throughout the hike (e.g., altitudes, requiring to cross rivers, steep ascensions). Applying the idea of a challenging terrain to the graduate school experience, it is well documented that graduate students encounter a plethora of barriers and challenges, including exclusionary discourses in higher education (Alvarado, 2010; Liu, 2011; McNamee & Miller, 2009) and in mathematics (Gutiérrez, 2016, 2017; Leyva, 2021; Tefera et al., 2019). Lastly, social identity theory is reflected in this metaphor because each hiker arrives at the trail with different backgrounds and experiences, shaping how they navigate and experience the journey. Similarly, graduate students move through higher education with varying strengths, experiences, and social and cultural identities. The hiking the trail metaphor grounds the broader story of this dissertation, helping illustrate the different ways students navigate graduate education and develop their professional identities as future faculty members.

Overview of the Findings Chapters

In the subsequent chapters, I explore what hiking the trail entails for mathematics graduate students (i.e., the process of professional socialization and identity development). Each chapter represents a different dimension of how graduate students become mathematics professors as they navigate the hike to the professoriate.

Chapter 2: Mathematics Graduate Students' Professional Identity Development

In Chapter 2, I explore how mathematics graduate students develop their professional identities as prospective faculty from the perspective of role identities (Colbeck, 2008; Zhang & Wang, 2024). As graduate students hike the trail to the professoriate, they are prompted to perform a range of role identities. One perspective of professional identity development posits that sustained role identity enactments stabilize into one's professional identity (McCall, 2003; Stryker & Burke, 2000). Employing an inductive, thematic analysis (Miles et al., 2020) of participants' interviews, I identified five role identities that participants frequently performed during graduate school: *teacher*, *researcher*, *mathematician*, *mentor*, and *peer and colleague*. Teacher and researcher consistently emerged as the most central dimensions of participants' professional identities and their understanding of the mathematics professoriate, suggesting that these role identities occupied higher positions in their salience hierarchies (Zhang & Wang, 2024). This also reflects how the doctoral program and departmental context may have valued and advanced these role identities. Along the hiking the trail metaphor, teaching and research signified the most common roles participants performed during their journeys.

Additionally, examining the meanings and values they attached to these roles provided insight into how mathematics as a discipline and the departmental environment shaped the

ways graduate students understood themselves and their futures as prospective professors. For instance, many participants described research as a collaborative endeavor and viewed teaching as extending beyond supporting mathematical thinking to also caring for students' overall wellbeing. Further, the findings suggest that the department privileged the development of the teacher identity, while structural features of the program positioned more senior graduate students to identify more strongly as researchers. In all, I identify different role identities that graduate students enacted, describe the meanings and values that attribute to these role identities, and illustrate how the terrain and the trail (i.e., the disciplinary culture of mathematics and departmental context) influenced these roles. This work offers insights into how the varied experiences of graduate education shape students' different professional identity dimensions.

Chapter 3: Masculine Discourses in Graduate Mathematics Education

In Chapter 3, I examine how masculine discourses manifest and are experienced within graduate mathematics education. Discourses are thought of as contextual manifestations of power (Leyva, 2016; McBride, 1989) and can create environments that marginalize or devalue the experiences of women and other underrepresented groups. I consider exclusionary discourses in mathematics (e.g., mathematics ability is innate) as barriers and obstacles that graduate students might encounter on their hike to the professoriate. Interrogating these discourses is critical to understanding how systemic inequities are perpetuated within graduate mathematics education. Adopting a multiple-case study (Yin, 2016) and narrative inquiry approach (Creswell & Poth, 2016), I highlight the stories of two women-identifying graduate students – Kayla and Morgan – to explore how they encountered masculine discourses through the program structures, policies, and

everyday practices and interactions. I focused on how Kayla and Morgan's gender identity influenced how they navigated the hike to the professoriate.

By examining the different ways Kayla and Morgan were positioned to exercise, or not to exercise power, I identified and characterized how masculine discourses were manifested in the discursive practices (McBride, 1989) of mathematics graduate education. I utilized a (re)storying approach (Creswell & Poth, 2016; Nardi, 2016) to make sense of the notable experiences, events, and actors in Kayla's and Morgan's stories. Then, I conducted a cross-case analysis of their narratives (Miles et al., 2020) to reveal three prevalent masculine discourses: (1) women are positioned as not good at mathematics, (2) women must constantly prove themselves, and (3) supportive and affirming connections are rare yet impactful. Despite encountering masculine discourses that questioned their mathematical abilities, demanded constant proof of their competence, and isolated them from support networks, Kayla and Morgan exhibited remarkable resilience and resistance to these varying forms of marginalization. Their stories highlighted not just the obstacles women face in graduate mathematics education but also the strength and agency they exercised in pushing back against exclusionary norms. The work within this chapter seeks to broaden the field's understanding of how masculine discourses may operate at the graduate level, while also offering insights into programmatic and structural reforms that can make graduate mathematics more inclusive and accessible.

Chapter 4: Mental Models of Mathematics Teaching and Learning

Lastly, in Chapter 4, I focus on the underpinnings of graduate students' approaches to mathematics pedagogy and instruction. I examine how mathematics graduate students come to learn about and develop their understanding of mathematics teaching as they navigate the

various experiences of the hike (e.g., serving as a teaching assistant, being students themselves). In other words, I sought to identify the underlying beliefs, values, and experiences that shaped how graduate students engaged in undergraduate mathematics teaching (Friedberg, 2005). There is still limited formal, sustained pedagogical training in mathematics graduate programs, thus the complexity of how graduate students learn how to teach mathematics is still largely understudied. I extend existing perspectives of mental models (e.g., Greca and Moreira, 2000; Jones et al., 2011) to examine how participants reflected on their teaching practices. The primary focus of this chapter was to unpack the experiences and beliefs that graduate students considered central to their mental models of mathematics teaching.

I adopted an inductive analytic approach (Saldaña, 2021) to characterize the dimensions and experiences that most informed graduate students' understanding of undergraduate mathematics teaching. I found that the dimensions of their mental models of teaching that emerged as most central were: (1) beliefs about student learning, (2) beliefs about teaching, (3) beliefs about mathematics, (4) prior experiences as students, and (5) prior teaching experiences. Participants revealed how these dimensions informed how they approached mathematics instruction, engaged with students, and made pedagogical decisions – reflective of how prior scholars have conceptualized the formation and function of mental models (Bransford et al., 2000; Jones et al., 2011). The findings highlight the connection between cognition and practice within the realm of undergraduate mathematics teaching.

Interestingly, *awareness of discourses in mathematics* emerged as another significant dimension of their mental models, highlighting graduate students' recognition of often negative perceptions of mathematics and mathematicians and their commitments to engage in

teaching practices that combat and reframe these discourses. Understanding how graduate students make sense of their own teaching practices and then leveraging the knowledge of the dimensions of their mental models of teaching positions higher education institutions to better support them in their current teaching roles and prepare them for positions as mathematics faculty. For example, professional development activities like TA training can explicitly discuss discourses and inequities in mathematics and link those to recognized teaching approaches (e.g., Mesa et al., 2014) in undergraduate mathematics education.

Conclusion

Lastly, in the conclusion of my dissertation, I summarize the theoretical, methodological, and practical implications that came from the insights of the three separate studies. Additionally, I share a reflection on the future of the mathematics professoriate and outline the new questions and areas for potential continued research that emerged from my analysis. By centering the identities and lived experiences of graduate students, we can make impactful change to how higher education institutions structure mathematics doctoral programs to broaden access and participation for all students.

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CHAPTER TWO

Enacting Role Identities During the Hike

Many mathematics graduate students go on to become faculty members; however, our understanding of how they are socialized into the professoriate (Gardner & Mendoza, 2023; Tierney & Rhoads, 1993; Weidman & Stein, 2011) and how they develop their professional identities, or their sense-of-self, as faculty within this disciplinary context remains largely understudied. This limited understanding likely contributes to the minimal formal and institutionalized professional development efforts that intentionally support the development of positive professional identities in higher education (Austin & McDaniels, 2006; Friedberg, 2005; Jensen, 2011). To address this gap, we must first understand how mathematics graduate students come to see themselves as future faculty, how they navigate their varied responsibilities in graduate school, and how these experiences shape who they believe they are and want to become in the profession. Identity theory (e.g., Burke, 2003; Stryker, 1968; Zhang & Wang, 2024) and role identity theory (e.g., Colbeck, 2008; McCall & Simmons, 1978; Jazvac-Martek, 2009) help us understand that the construction of oneself occurs through the process of occupying roles and internalizing the meanings associated with those roles. Leveraging these perspectives, this study views professional identity as emerging through the enactment of socially situated role identities, such as teacher, researcher, and mentor, taken up during graduate education (Jazvac-Martek, 2009).

I conceptualize role identities as dimensions of graduate students' professional identity. These role identities are informed by institutional structures, interpersonal relationships, disciplinary cultures and norms, and broader academic expectations (Clarke et al., 2013; Solari & Ortega, 2020). When affirmed or positively reinforced, role identities may

stabilize into lasting aspects of one's professional identity; conversely when met with conflict or dissonance, they can result in identity tensions, role strain and misalignments (Olarie, Swartz et al., 2024; Tran et al., 2011; Winstone & Moore, 2017).

Scholars of graduate students' socialization and professional identity development have identified the importance of attending to discipline-specific cultures and departmental contexts (e.g., Gardner, 2008; Solari & Ortega, 2020; Tran et al., 2011) as graduate students are considered content experts, with deep immersion in their fields spanning years of study (Clarke et al., 2013). Their years of academic socialization and enculturation (Tierney & Rhoads, 1993; Weidman & Stein, 2001) shape how they view the purpose and meaning of academic work, in ways that can vary across disciplines and departments (Tran et al., 2011). Clarke et al. (2013) posited that "discipline-based cultures are the primary source of faculty members' identity" (p. 7). Additionally, each discipline carries "assumptions about what is to be known and how tasks [are] to be performed, standards for effective performance, patterns of publication, professional interaction and social and political status" (p.7). Thus, the roles and responsibilities that graduate students enact are shaped by the characteristics, norms, cultures, and expectations of higher education mathematics, such as the lack of laboratory work and the emphases on abstraction, correctness, and rigor (Gutiérrez, 2017; Herzig, 2004a). These features can inform how graduate students come to see themselves and their professional work.

Moreover, while some features of graduate education have become commonplace (e.g., serving as teaching assistants, engaging in research), variations to these experiences may reflect the unique departmental contexts, sociohistorical/political contexts, and the individual graduate students' personal commitments, beliefs, and values. Additionally, given the

fragmentation of higher education and the ever-evolving expectations placed on institutions of learning (Nixon, 1996), the ways that graduate students occupy their many role identities, and the meanings and associations they attach to these role identities, may be diverging from how the field has historically conceptualized the mathematics professoriate and in turn, how graduate programs have structured their efforts to prepare students to enter the professoriate. I argue that advancing efforts of career preparation and professional development of graduate students pursuing faculty positions should emerge from an understanding of the cultural and identity-related contexts in which mathematics graduate students perform role identities (Austin & McDaniels 2006; Colbeck, 2008; Jazvac-Martek, 2009), ultimately constructing their professional identities.

In this study, I use a role identity perspective to examine how mathematics graduate students reflected on their professional identity development by focusing on how they performed and made sense of a multitude of role identities. Additionally, I explore how role identities became more salient and anchored (Stryker, 1968; Zhang & Wang, 2024), and how the disciplinary influences of mathematics and the departmental context influenced the enactment of role identities. The research questions that guided this study were:

1. How did mathematics graduate students enact and make sense of various role identities during graduate school?
2. What role identities were most salient and anchored?

Conceptual Framework

I draw on the perspective that graduate students' professional identity development is mediated by and comprised of a multitude of role identities, such that these unique role identities (e.g., teacher, researcher) are dimensions of one's professional identity.

Professional identity development is a dynamic and ongoing process through which individuals come to understand themselves in relation to the work, norms, and expectations of a particular profession (Clarke et al., 2013; Colbeck, 2008; Jensen, 2011; Trede et al., 2012). For graduate students, this development often unfolds through participation in myriad role identities embedded within graduate education, such as student, teaching assistant, researcher, mentor, and emerging scholar (Colbeck, 2008; Winstone & Moore, 2017). Colbeck (2008) characterized the process of professional socialization in graduate education as students' efforts and attempts with trying various professional roles. These role identities are not simply tasks or titles; rather, they are most importantly sites of identity construction, where students enact, negotiate, and internalize aspects of professional life in postsecondary mathematics (Colbeck 2008; Olarte, Swartz et al., 2024).

Identity Theories

I bring together *identity theory* (e.g., Burke, 2003; Stryker, 1968; Zhang & Wang, 2024) and *role identity theory* (e.g., Colbeck, 2008; McCall & Simmons, 1978, Jazvac-Martek, 2009) as complementary, but distinct, perspectives to better understand graduate students' professional identity development. Aligning with broader sociocultural perspectives of identity development (Vygotsky, 1978), identity theory and role identity theory are utilized to recognize the importance of others and the context in one's identity formation. In identity theory, the construction of the "self" is done through performing activities and behaviors (Zhang & Wang, 2024). This contrasts with social identity theory (e.g., Trepte, 2013), which emphasizes identity as the evolving relationships and negotiations between an individual and the group (e.g., sense of belonging, membership classification). Zhang and Wang (2024) clarified this distinction, "In identity theory, the core of an identity is the

categorization of the self as an occupant of a role, and the incorporation, into the self, of the meanings and expectations associated with that role and its performance” (p. 353). This is a particularly well-aligned perspective for studying graduate students’ professional identity development as faculty, because they perform a plethora of roles, such as teacher and researcher, which give them insight into their prospective careers as mathematics faculty.

Multiple Role Identities

Role identities also include the internalized meanings graduate students attach to the roles they perform and enact, and they are shaped by social interactions, feedback, and institutional contexts (McCall, 2003; Stryker & Burke, 2000). To describe the development that occurs during doctoral education, Colbeck (2008) noted: “Individuals’ identities are often associated with labels for social positions or roles. Role labels convey meanings and expectations for behavior that have evolved from countless interactions among people in a social setting” (p. 10). Although role identities are influenced by others’ expectations, individuals are agentic and define their own identities, as they accept or reject these social roles as part of their professional identities as prospective faculty (Tran et al., 2011). When graduate students’ performances of these role identities are affirmed by advisors, peers, or institutional structures, the role identities can stabilize and become more central to their sense of professional self (Colbeck, 2008; Zhang & Wang, 2024). However, when affirmation is absent or when the performance of roles create conflicting demands or are in tension with students’ values and commitments, graduate students may experience identity tensions that disrupt their professional identity development (Austin & McDaniels, 2006; Herzig, 2004b, 2010).

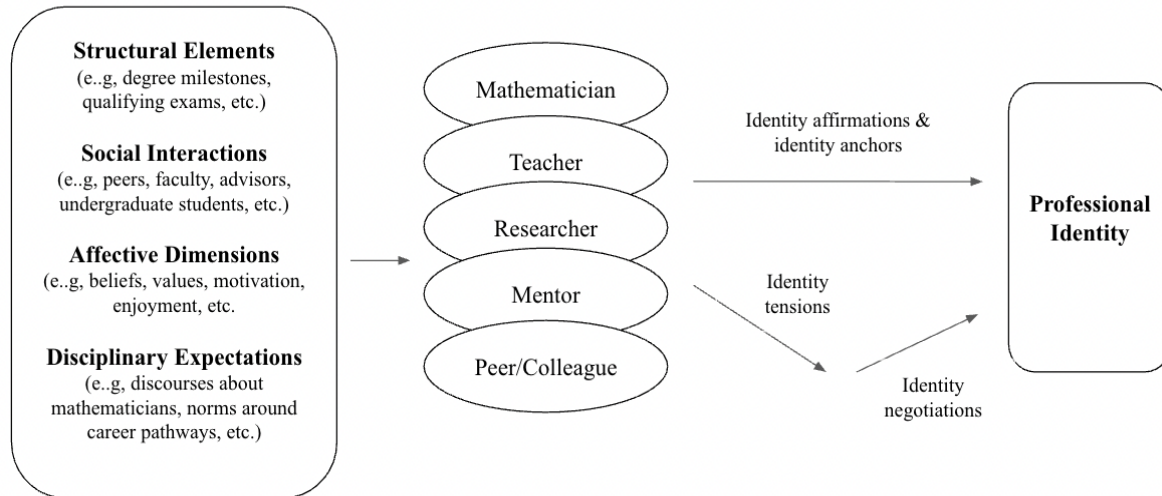
Another important feature of role identity theory is the recognition that graduate students enact *multiple* roles (Jazvac-Martek, 2009; Winstone & Moore, 2017; Olarte, Swartz et al., 2024). For example, serving as a teaching assistant prompts their role identities as teachers and educators, while working with their doctoral advisors may prompt their role identity as a researcher. A result of enacting and negotiating multiple roles is the establishment of a salience hierarchy (Stryker, 1968) amongst the role identities. Zhang and Wang (2024) explained: “Identity is organized into a salience hierarchy, and those identities higher in the hierarchy are more likely to be evoked than those lower in the hierarchy, which are more closely connected to an individual’s actions” (p. 354). Other scholars have described the similar construct of *identity anchors* as the dimensions of one’s identity that are more stable and foregrounded during varying stages of one’s career. For example, Mockler (2010) found that teachers anchored their professional identities onto different dimensions including specific instructional approaches (e.g., content-centered vs student-centered), to more clear roles and responsibilities, such as being advocates for students or being established members of their professional communities. Personal experiences, professional contexts, and even the external political environment contributed to teachers’ identity anchors, and this perspective is aligned with illustrating how the complex academic landscape of higher education influences how graduate students’ role identities become anchored.

In all, graduate students' professional identities are not fixed endpoints but are continually constructed and contested through the situated enactment of multiple, and sometimes conflicting, role identities during graduate education (Olarte, Swartz et al., 2024; Winstone & Moore, 2017). I situate identity theory and role identity theory within

mathematics graduate education to suggest a model of professional identity development. In Figure 1, I offer a framework for mathematics graduate students' professional identity development through the performance and negotiations of multiple role identities. A confluence of internal and external factors, such as structural elements (e.g., program milestones), social interactions (e.g., relationships with faculty), affective dimensions (e.g., beliefs, values), and disciplinary expectations (e.g., discourses about mathematicians, career pathways) may initiate a range of roles identities, such as *mathematician*, *teacher*, and *researcher*. If these role identities are repeatedly performed and affirmed over time, they can stabilize and anchor as more salient dimensions of their professional identities. However, identity tensions may also occur, which prompt graduate students to negotiate and make sense of those conflicts before they place more importance and commitment to these roles as dimensions of their professional identities. For example, Colbeck (2008) found that “doctoral students who are more highly committed to their researcher identities than teacher identities may resent time they must devote to teaching assistantships” (p. 12), illustrating an example of a dissonance in commitments to certain roles. Understanding the multiple role identities graduate students occupy during graduate school can support our efforts in developing shared meaning and importance across the role identities to better support their preparation as faculty.

Figure 2

Role Identity Development in Mathematics Graduate Education



Note. The model situates existing theories of professional identity development and role identities to the specific context of mathematics graduate students.

Literature Review

I review two key bodies of literature: (1) professional identity development, and (2) graduate students' varying role identities throughout graduate education. I briefly describe how prior scholars have conceptualized professional identity development and how I extend this work through examining mathematics graduate students' role identities.

Professional Identity Development

Scholars have approached professional identity from various perspectives, resulting in a wide range of conceptualizations of what constitutes this construct. In their review of higher education literature, Trede et al. (2012) noted professional identity is often loosely defined but broadly understood as an individual's "way of being and a lens to evaluate, learn and make sense of practice" (p. 374). Beijaard, Verloop et al. (2000) similarly defined K-12 teachers' professional identity as "how they perceive themselves as teachers and what factors

contribute to these perceptions” (p. 751), emphasizing self-concept and self-image. These perceptions are shaped not only by personal interpretations but also by how others in the profession assign meaning, making identity both socially situated and legitimized (Beijaard, Verloop et al., 2000; Beijaard, Meyer et al., 2004; Lamote & Engels, 2010).

Socialization theory (Gardner, 2008; Tierney & Rhoads, 1993; Weidman et al., 2001) is often used to describe the process of professional identity formation in higher education, complementing identity theory (e.g., Zhang & Wang, 2024) and role identity theory (e.g., Colbeck, 2008). Socialization describes the transmission of professional culture and acquisition of the knowledge, dispositions, and skills needed to function in a field, shaped through social interactions in specific contexts (Clarke et al., 2013; Solari & Ortega, 2020; Van Lankveld et al., 2017). This includes observing expert others, learning by doing, engaging with peers, and reconciling personal beliefs with professional realities (Pillen et al., 2013; Tateo, 2012). In sum, professional identity reflects how individuals see themselves in their roles and how they are recognized within their communities (Berger & Lê Van, 2019).

Despite variation in definitions, several characteristics of professional identity are widely accepted. As a construct, professional identity is complex and multidimensional. Mockler (2010) described teachers’ identities as a “plurality of identities” (p. 125), negotiated across personal and professional domains, like race, gender, and role. Trede et al. (2012) also emphasized the challenge of navigating multiple communities, leading to non-linear identity development. This often results in tensions between self-concept and external expectations (Pillen et al., 2013; Sachs, 2001), which can shape graduate students’ emerging professional selves. For example, Colbeck (2008) found that graduate students may

encounter competing expectations and commitments between their researcher and teacher identities, resulting in one role identity being more salient and prioritized than the other.

Professional identity is also dynamic and continually constructed through reflection on past, present, and future selves (Beijaard, Meyer et al., 2004; Clarke et al., 2013).

Brubaker and Cooper (2000) therefore referred to identity as a soft construct, subject to instability. Finally, professional identity is highly relational, formed through interactions with colleagues, students, and the broader professional community (Clarke et al., 2013; Nixon, 1996; Solari & Ortega, 2020). This study builds on these characterizations by examining how mathematics graduate students reflected on everyday interactions and aspirations as faculty. I explore how students enact and negotiate intersecting role identities during a formative and transitional period of their careers.

Recognized Role Identities of Graduate Students

Graduate students are commonly expected to enact a variety of role identities, such as researcher, teacher, peer, and mentor (Austin & McDaniels, 2006; Gardner & Barnes, 2007; Kajfez & McNair, 2014). These expectations are situated within departmental structures, program milestones, disciplinary norms, and broader institutional cultures that frame what it means to become a faculty member (Jazvac-Martek, 2009). Research has shown that graduate students' engagement in these role identities not only supports the development of specific skills but also facilitates the formation of their professional identities (Sweitzer, 2009; Weidman et al., 2001; Winstone & Moore, 2017). Studies in doctoral socialization emphasized that graduate students are socialized into the academic profession by undertaking roles and responsibilities associated with research (e.g., Davis & Lester, 2016), teaching (e.g., Beiseigel & Simmt, 2012; Deshler et al., 2015; Olarte, Swartz et al., 2024; Winstone &

Moore, 2017), mentorship (e.g., Brown & Sheerin, 2018; McAlister et al., 2022) and service (e.g., Gardner & Mendoza, 2023). These experiences have been explored, but limited work has taken up the perspective of role identities not only to illustrate how graduate students enact these role identities, but to make sense of the meanings, beliefs, and values they attach to these roles.

Scholars have illustrated that graduate students, depending on their discipline, may identify more strongly with certain role identities. Kajfez and McNair (2014) focused on graduate students in education, engineering, and engineering education and explored their perceptions of their role identities as teachers, researchers, and lifelong learners. In their work, the authors suggested that differences in the social contexts and the cultural values between the disciplines pointed to the differences in how strongly participants identified with these dimensions of identity (e.g., education students identifying more strongly as teachers, engineering students identifying more strongly as researchers). In addition to disciplinary influences, the everyday interactions and key actors within a departmental context also inform how role identities are taken up. In an earlier work (Olarte, Swartz et al., 2024), I specifically explored the role identity of teacher and found nuances in how graduate TAs enacted this role identity based on the interactions they had with members of the mathematics community. For example, graduate TAs' understanding of being a teacher involved being a content instructor and mentor to students and being a supporter to fellow TAs. Aligned with scholars who emphasized the importance of the disciplinary context (e.g., Tran et al., 2011), we also observed that norms and disciplinary expectations within mathematics shaped how graduate students enacted their role identity as teachers.

Furthermore, part of enacting role identities also involves the different meanings graduate students attach to these roles (Zhang & Wang, 2024). In their work on graduate students as researchers, Davis and Lester (2016) found that graduate students framed their researcher role identities around particular paradigms (e.g., positivist, constructivist) and drew on their personal and professional experiences to articulate their researcher role identities. Here, the meanings participants attached to their role identity as researcher involved recognized research orientations. A common theme throughout the body of literature examining graduate students' role identities is the influence of the social context, personal dimensions and experiences, and the broader disciplinary expectations and norms in how these role identities are enacted.

Overall, the transitional space of graduate education is often characterized by the concept of liminality (Bale & Anderson, 2022; Winstone & Moore, 2017), in which graduate students occupy an in-between status that is neither fully student nor fully professional (Austin & McDaniels, 2006; Sweitzer, 2009). The liminal nature of graduate education often means that students are prompted to take on role identities expected of faculty, such as conducting independent research, teaching undergraduate courses, serving as mentors to peers or undergraduate students, and contributing to their academic communities (Gardner, 2008; Jazvac-Martek, 2009; Olarte, Swartz et al., 2024). Encountering conflicting role expectations can result in identity tensions as graduate students attempt to reconcile their self-perceptions of these role identities (Austin & McDaniels, 2006; Colbeck, 2008) with outside expectations and definitions of competence and professionalism (Gardner & Barnes, 2007; Gardner & Mendoza, 2023). I extend this prior work by examining how graduate

students enact and perform multiple role identities and illustrating the meanings they attach to these role identities.

Method

I adopted a thematic analysis approach (Miles et al., 2020) that brought together both inductive and deductive approaches to make sense of participants' multiple role identities.

Context

This study was conducted at Spruce University (pseudonyms are used for proper names), a large, research-intensive university in the West Coast of the United States. The mathematics department at Spruce University confers graduate degrees (i.e., Master's and PhD) in applied and theoretical mathematics, and completion of the doctoral program is expected to take about 5-6 years. The first two years of the program involve completing program requirements and graduate-level sequences for three content areas (e.g., Algebra, Analysis). Students take comprehensive exams (i.e., qualifying exams) for each content area after the required coursework. After fulfilling these three content area requirements, doctoral students select a focal research area, secure a dissertation advisor, and take an oral examination given by their respective doctoral committees to discern if the student is able to reasonably complete a dissertation on their proposed topic. This marks advancement to candidacy. At Spruce University, doctoral students are expected to advance to candidacy within four years of starting their graduate studies. The final milestone is successfully conducting research, writing, and defending their dissertation.

Participants

I employed a convenience sampling (Miles et al., 2020) to recruit graduate students in the mathematics department at Spruce University in Spring 2023. Sixteen doctoral students

completed a survey that elicited academic information (e.g., year in program, prior education), inclusive demographic information (e.g., gender, race), and career aspirations information. Then, purposeful sampling (Yin, 2016) was used to recruit students who indicated that they had considered or were actively pursuing careers as mathematics faculty. Six doctoral students were then selected for an interview. This study focuses on these six students. Full participant profiles are provided in Table 1.

Table 1

Participant Profiles

Participant	Year	Completed prior graduate degree	Gender	Race/Ethnicity	Other Self- Identified Identity	Research Focus
Connor	3 rd	No	Male/Masculine	White/Caucasian	First-generation student	Data-driven methods of learning dynamical systems
Kayla	1 st	MS Mathematics	Female/Feminine	Latinx/Chicanx	Underrepresented student; Christian	Geometric group theory, combinatorial group theory
Kendall	5 th	No	Female/Feminine	White/Caucasian	N/A	Topology
Landon	2 nd	No	Male/Masculine	White/Caucasian	N/A	Quantum topology
Martin	1 st	No	Genderqueer/ Genderfluid	Latinx/Chicanx	Underrepresented student; Person of Color	Applied mathematics and math bio
Morgan	5 th	MS Mathematics	Female/Feminine	White/Caucasian	Underrepresented student	Low-dimensional topology

Note. All demographic data were self-selected, self-identified, or self-described by the participants.

Data Collection

Data collection occurred within a 90-minute interview conducted over Zoom, which involved three narrative, self-reflective activities. In the first hour of the interview, graduate students participated in a semi-structured interview (Rubin & Rubin, 2011) about their experiences in graduate school, their understanding of the mathematics professoriate, the kind of faculty they hoped to be in the future, and other key features of doctoral education including courses for which they had served as teaching assistants and their relationships with their advisors and peers. Some questions from this semi-structured interview included: (a) In what ways have you seen yourself as a mathematics professor or faculty while in graduate school?; (b) What aspects of your graduate education have most contributed to your development as a professional in higher education mathematics?; and (c) Tell me about your relationships with your peers in graduate school.

Additionally, in the last 30 minutes of the interview, participants were invited to write a letter to mathematics, in the form of a “Dear Math...” letter, where they were prompted to personify the mathematics discipline and to describe how they felt to mathematics. Participants also created accompanying visual representations of their graduate school experiences by creating annotated graphs that showed affective dimensions, such as their perceived support from faculty and their fluctuating levels of confidence. During the activities of the letter-writing and graphing, participants remained on the Zoom meeting, but I turned off my camera and allowed them the time and space to engage in these different types of narrative activities without researcher influence. For the purposes of the present study, I drew on the transcriptions of the interviews as the primary source of data and then

utilized the letters and the graphs to triangulate aspects of participants' reflections of their role identities.

Data Analysis

I employed a thematic analysis approach (Miles et al., 2020) that allowed for both inductive and deductive examination of the role identities that emerged within participants' reflections of their graduate education experiences. The analysis process encompassed multiple rounds of coding, guided by Saldaña's (2016) system of open coding. This approach involved a close reading of the transcripts, letters, and graphs to identify emergent themes, actions, and meanings that participants associated with their role identities.

In the first round of coding, I identified the actions and responsibilities that participants reported enacting as part of their day-to-day activities and progression through the doctoral program milestones. Some of these actions and practices included serving as teaching assistants, supporting undergraduate students, researching with faculty, and taking coursework. While the coding process was grounded in open-ended engagement with the data, it was also informed by existing literature on recognized graduate students' roles and activities, such as teacher, researcher, and mentor (e.g., Gardner, 2008; Jazvac-Martek, 2009; Olarte, Swartz et al., 2024). These prior characterizations served as sensitizing concepts that guided my attention without constraining the emergence of new actions and responsibilities, especially those situated within the context of graduate mathematics education.

After the first round of coding, I organized and collapsed coded actions and responsibilities into broader categories that reflected role identities, which included an attention to the contexts and experiences through which these actions and practices were enacted *and* the meanings and values participants attached to these roles (See Table 2 for

codebook). Once the data were coded with the different role identities, I engaged in constant comparison (Miles et al., 2020) across the participants' role identities to discern similarities and differences in *how* participants enacted these role identities (e.g., the different contexts these roles were enacted) and the different meanings they attached to these roles (e.g., being a teacher also meant supporting students beyond the mathematics content). This iterative and recursive process culminated in thematic findings that captured the ways participants made meaning of their present role identities and their professional trajectories as aspiring mathematics faculty. This process answered my first research question.

To address the second research question, I organized the coded data into code charts via matrices (Saldaña, 2021). This qualitative technique allowed for a visual structure through which I could better understand the frequency of role identities and depth of participants' reflections on their roles. The code charts allowed for comparisons across the role identities, which I noted through analytic memos (Saldaña, 2021), and I found that the role identities of teacher and researcher were not only the most frequently cited roles but were the dimensions that participants described in the most depth and explicitly described as most salient to their professional identities as prospective faculty. In the subsequent findings section, I unpack these two role identities in more detail, including identifying the experiences that prompted students to enact these roles and the ways these dimensions of their professional identities became more anchored.

Table 2*Role Identity Codebook*

Role Identity	Description	Example from Transcripts
Teacher	The role identity of <i>teacher</i> refers to how graduate students teach and support undergraduate students' learning of mathematics. This includes planning instruction, facilitating discussions or office hours, and reflecting on their teaching practices and effectiveness in the classroom.	"I think for me, [being] a math faculty, it does mean that you're teaching mathematics, and you get to teach some of the things that you enjoy in math. I think I've always kind of seen a math faculty as just teaching and stuff." – Kayla
Researcher	The role identity of <i>researcher</i> reflects how graduate students engage in mathematical research, including exploring new problems, writing papers, and contributing to the discipline's body of knowledge. This role often emerges through collaborations with advisors, attending conferences, and meeting program milestones such as theses or dissertations.	"[I] contribute to the field of mathematics by researching and publishing papers and stuff like that." – Morgan
Mathematician	The role identity of <i>mathematician</i> encompasses how graduate students see themselves as doers and learners of mathematics beyond research or teaching contexts. It may be enacted through coursework, problem solving with peers, and developing persistence and curiosity in mathematical thinking.	"I guess doing math seems like a really enjoyable way to use my brain's processing power, and I guess I kind of fit in, in that sense." – Landon
Mentor	The role identity of <i>mentor</i> refers to how graduate students support the growth and development of more junior students, including undergraduates or peers in earlier stages of the program. This role can include offering academic advice, modeling professional behaviors, and providing socioemotional support.	"Being a mentor to undergrads has always been a lot of fun ... You basically get to be like an advisor to them in some sense." – Kendall
Peer/Colleague	The role identity of <i>peer/colleague</i> captures how graduate students engage in reciprocal relationships with other graduate students to foster community, share resources, and navigate graduate school. It includes both formal and informal interactions that build solidarity and collegiality within the program.	"It's more of an emotional support. It's nice to have someone to sort of vent to about research ... It's really nice to have people who understand ... It's a nice give and take there." – Landon

Findings

I highlight the role identities that emerged as participants reflected on their experiences navigating graduate school and aspiring to faculty roles. Beyond identifying the actions and behaviors associated with each role identity, I share the meanings and expectations participants attached to performing each role identity. I begin with the two role identities they described as most central to becoming a mathematics professor: teacher and researcher. The prominence of these two role identities suggests they rank high in participants' salience hierarchy (Stryker, 1968; Zhang & Wang, 2024), indicating their centrality in graduate students' developing professional identities as future faculty.

Teacher

Among the various role identities participants performed, the teacher role emerged as both the most consistently enacted and, for many, the most meaningful part of their graduate experience. This identity was expressed through teaching assistantships, instructor-of-record positions, and, importantly, through the relationships they cultivated with undergraduate students. For several participants, this commitment to teaching predated graduate school and remained a steady anchor throughout their journey (Mockler, 2010). As Kayla shared, "I think I've always known that I've wanted to teach, like that has been an unquestionable thing."

Developing as Teachers at Spruce University

At Spruce University, all participants served as teaching assistants (TAs) for undergraduate courses, and some participants, like Kayla and Morgan, also served as graduate instructors of record, where they had sole responsibility for lesson planning, lecturing, and grading at the institutions where they received their master's degrees.

Graduate students at Spruce were offered TA positions as part of the program funding structure, thus they performed this role identity early in their graduate education journeys.

Kendall reflected,

In my first year, I did really enjoy teaching, so I thought, oh, you know, I'll have six years of teaching, so it'll be great, and that might be all I need. But I still really love it, and I don't want to stop. That's part of the reason why I want to pursue a career as a professor.

All participants viewed teaching as a central responsibility of mathematics faculty. Kayla explained, “I think for me, [being] a math faculty, it does mean that you're teaching mathematics ... I've always kind of seen a math faculty as just teaching and stuff.” Many linked this role identity to their undergraduate experiences, where exposure to faculty who taught mathematics, often more than they conducted research, inspired their own academic and career paths. As Connor shared, “Meeting my math professors in my undergraduate ... they do a lot of teaching ... and it really seemed like something I would enjoy doing.” This desire to teach motivated participants to pursue careers as mathematics professors and demonstrated the impact of positive experiences as undergraduate mathematics students.

Positive teaching moments and a sense of impact helped anchor graduate students' role identity as teachers. Landon recalled, “Some interactions ... I've sort of walked away from feeling like something good has been accomplished,” which deepened his commitment to becoming a professor. Kayla also emphasized student relationships and the role of teaching in challenging dominant discourses in mathematics (e.g., Leyva, 2016; Mendick, 2006). Her reflections underscored the affective dimensions (i.e., beliefs, values, and convictions) that shaped her teacher role identity. She said,

I want to be a faculty [member], where students are like, “Oh, I enjoy going to Kayla's office hours” or just stuff like that and not being closed off. Definitely trying to make mathematics a more welcoming community to a lot of people and just kind of breaking the stereotype of like, “Oh, you have to be like this math person or you have to be super scared of your professors or messing up.” I've had professors get upset because you don't know something. And I'm like, yeah, no, I'm not going to be that kind of person. I also just think I would want to be a faculty member [where] I'm not only teaching math, but I'm also making a difference in someone's life or being a reason why they're like, “Oh, I do enjoy coming to college,” and stuff like that. I think it's important to be a good person, too, in addition to teaching math.

Graduate Students' Understanding of Their Teacher Identities

The meanings that Kayla, like others, attached to the role identity of teacher involved beliefs about professors' responsibilities and commitments to humanize mathematics in myriad ways (Gutiérrez, 2016). This was echoed by other participants who understood teaching to encapsulate supporting students' mathematical learning *and* providing socioemotional support beyond the mathematics content (Mesa et al., 2014). For example, Connor shared that as a teacher, being “approachable is my main goal” and Kendall said, “I do feel like our role is to guide them along in more of an emotional sense and a math sense.” Participants valued the opportunities to teach, interact with, and support their students, with many citing these moments as the experiences where they felt most connected to the profession and where they were able to envision themselves as future mathematics professors.

Interestingly, participants distinguished the TA position from the instructor-of-record position, in that although they enjoyed serving as a TA, the agency afforded an instructor of record, where they are solely responsible, helped to stabilize and anchor the teacher role identity. Morgan explained, “I think TAing the classes, like sections, and sometimes over the summer, you're able to teach your own class [as an instructor of record], that probably helps the most of preparing me to be a math faculty member.” Kayla further characterized these instructor-of-record opportunities, saying, “I just loved that environment, you feel content, satisfied. It's very fulfilling.” Having both served as instructors of record in their previous institutions prior to attending Spruce, these experiences seemed especially to solidify Kayla and Morgan’s teacher role identity much higher in their salience hierarchy (Stryker, 1968; Zhang & Wang, 2024) and how they saw themselves as prospective faculty.

Overall, the role identity of teacher was one that participants most enjoyed and also had more opportunities to enact throughout graduate school. Furthermore, meaningful interactions with students led participants to internalize a more nuanced understanding of the role identity of teacher to include providing support with mathematical learning and beyond.

Researcher

The other most salient role identity participants associated with the mathematics professoriate was that of a *researcher* (Davis & Lester, 2016; Kajfez & McNair, 2014). Many viewed the faculty position as a dual responsibility: teaching *and* contributing to the discipline through research. Morgan captured this duality, noting that being a mathematics professor “means that you get to teach students pretty much during the whole school year,” while also contributing through “researching and publishing papers.” In other words, these were the “two aspects of being a math professor.”

Developing as a Researcher at Spruce University

To better understand *how* participants developed their researcher identities, it is important to consider the program structure at Spruce University, a research-intensive institution. Unlike teaching, which began with early TA assignments, students were not expected to choose a research focus or advisor until after passing their qualifying exams. While some mathematics graduate students informally connected with faculty through reading groups or early collaborations, these opportunities were not consistently structured or supported. As a result, engagement in research varied widely depending on students' program stage. Those students who were further along in their programs, like Morgan, Kendall, and Landon, reported more consistent enactment of their researcher role identities. Kendall, a fifth-year student, reflected, "In my third year and thereafter, I've been doing more research." In contrast, Kayla, a first-year student, explained, "We don't start doing research until we pass our classes and qualifying exams." These differences revealed a key tension: Although participants saw research and teaching as equally central to the professoriate, institutional structures offered more formalized and accessible paths into teaching than research. In many cases, students had to take the initiative to pursue research, highlighting the more fragmented and individualized nature of this role identity development.

Their role identities as researchers were most prompted by engaging with faculty and their PhD advisors. Participants especially appreciated the supportive faculty who were willing to read with and introduce students to their research areas *prior* to the completion of qualifying exams. Connor shared an experience about how he came to work with his current advisor. He gave a presentation in a course that she was teaching and an interaction led to them officially working together as advisor and advisee. He reflected,

But when I did this talk, afterwards, she said, “This is my specific research area. We should talk more about this.” Talking more about that turned into doing a project with her for the summer. And then working with her that summer, it sort of was everything that I was missing from my other project experiences. That made me think that this is the one that was going to work out, so then we ended up being advisor and advisee. It naturally came from the happy accident of me picking the right presentation topic for her course.

A recurring theme in participants’ reflections on research was the vital role of faculty support in affirming and stabilizing their identities as researchers. Participants valued faculty who encouraged their interests and helped shape their research questions—interactions that were pivotal in their early identification with the researcher role. Kendall, for example, appreciated how her research advisor “always has these new questions we can address” and regularly fostered her intellectual curiosity. She said, “He’s just really good at coming up with questions and asking questions to make me rethink things and be like, ‘Wait, I didn’t think about it like that.’” Connor similarly acknowledged his advisor’s knowledge of the field and ability to direct his research when he encounters challenges, including giving him “some suggestion of something to read, or something to look at, or to see what other people have done.” He summed this up saying, “Her knowledge of the wider field has been really helpful in terms of targeting me correctly in my research.” Faculty mentorship, along with opportunities to present and engage at conferences, helped participants build confidence, deepen their sense of belonging, and refine their understanding of what it means to do mathematics research as a faculty member.

Graduate Students' Understanding of Their Researcher Identities

Regarding their understanding of the primary roles and responsibilities of being a researcher, many participants articulated their goals of making intellectual contributions to the fields of pure and applied mathematics. For example, Connor expressed “wanting to research for the sake of research and a sense of wanting to push forward education.” He critiqued that “math can turn a blind eye to certain things,” in reference to the perception that mathematics is detached from everyday applications. He understood his research in applied mathematics as a way of bridging this disconnect. Landon also reflected on his future research as faculty, and he shared that “the overarching goal is to learn new things about math. And that's it.” Landon distinguished research in mathematics as different from other disciplines, which perhaps engage in research endeavors “to deliver something to customers.” Landon believed that the researcher role within mathematics is performed differently as compared to research within other disciplines. The researcher role identity primarily centered around participants’ efforts and commitments to offer new knowledge and insights related to mathematics and is performed with special attention to the field and community of mathematicians.

Beyond the act of conducting research and publishing papers, participants’ reflections revealed a range of meanings and values they attached to this role identity. They described mathematics research as collaborative, flexible, exploratory, and intellectually rewarding. For many, the researcher role identity encompassed values like curiosity, agency, and independence. Landon shared,

I really liked the flexibility [of being a researcher]. I get excited about some idea, and then I spend eight hours on some day working on something. Other days, I will

spend zero hours doing math. It's nice that I can make my own schedule. Also, for the first time, I can just follow things that I'm interested in and see where it takes me, whereas everything before was a lot more structured. I think it's just the freedom to manage my own schedule and what I do.

He acknowledged the unpredictability of progress, yet this served to deepen his commitment to conducting research. Additionally, participants understood their role identities as researchers as contributing to a sense of community and a feeling of belonging with other scholars who shared their interests. Kendall described research as “really hard, but super fun,” offering “this niche world to live in that only a few other people can even talk to you about.” Echoing the importance of the mathematics community in relation to their researcher identity, Martin said, “I had the chance to go to a couple of conferences and stuff and present my work. And so, that's been real.” Research allowed them to feel connected to scholars who shared their interests and to feel affirmed and recognized by the broader mathematics community.

Some participants' role identities as researchers were also connected to their gender-identities (Mockler, 2010). The women-identifying participants described an alignment and connection with the subfield of topology, because they felt that this field of mathematics has more representation of women and other underrepresented scholars. Kendall shared that a notable number of women-identifying students who finished the doctoral program at Spruce University all researched topology. She added, “...even outside of our school, women in math do topology more than other topics. That was also something that helped me decide what I wanted to do. I didn't want to go to a conference and be the only girl.” Kendall explicitly connected her identity as a woman to her researcher identity, such that her gender

identity helped to anchor her researcher identity. She summed this up, saying, “I guess my identity has helped me sort of pick a research area.” Although Kendall reflected on this alignment as a positive experience, it is important to recognize that her decision may reflect structural constraints within mathematics. Given the historic underrepresentation of women in mathematics, coupled with Kendall’s observation that there are more women in topology, she might have felt drawn to this subfield, thus limiting her choices for a research area.

Mathematician

Although participants emphasized the role identities of teacher and researcher as most central to mathematics professors’ professional identities, their reflections also revealed another critical identity specific to the discipline: that of a mathematician. While connected to teaching and research, this role identity carried distinct meanings, often tied more to the process of learning mathematics and engaging with mathematics out of personal enjoyment. What most differentiated the role identities of researcher and mathematician was participants’ attention to the field and broader mathematics community. As researchers, the graduate students saw their efforts and mathematical competencies serving the community through publishing, presenting, and growing the knowledge base. On the other hand, as mathematicians, graduate students reflected on themselves as learners and practitioners of mathematics, often out of enjoyment and genuine interest. This role identity more closely aligns with prior conceptualizations of mathematics identity that involves students *doing* mathematics or participating in mathematics activities (e.g., Cribbs et al., 2022; Olarte, Street et al., 2024).

Participants described being a mathematician as involving deep thinking, intellectual exploration, and a love of learning mathematics for its own sake. Notable activities and

responsibilities included learning mathematics in their courses, demonstrating their content expertise through qualifying exams or doing mathematics for fun. Landon expressed this sentiment, saying, “I guess doing math seems like a really enjoyable way to use my brain’s processing power, and I guess I kind of fit in, in that sense.” Similarly, Connor shared, “I really like math, and I really like learning math.” Many participants said they saw themselves as “math persons” even before graduate school. Successes during their undergraduate years, especially in abstract mathematics, sparked curiosity and enjoyment that motivated them to pursue graduate study. Landon reflected, “There was never any doubt in my mind that I was going to study math, and taking college level math classes certainly didn't do anything to dissuade me from studying math.” Participants also associated being a mathematician with qualities like persistence, curiosity, and determination, particularly in facing difficult problems without immediate solutions. Kendall noted,

I guess I don't give up very easily. That helps with being a mathematician. You have to try to answer the same questions and get stuck, and you have to try a new thing, or come up with a new question altogether if one doesn't work out. A lot of it is determination and resilience.

Additionally, collaboration was also a defining feature of this role identity. Participants frequently shared how working with peers supported their learning and strengthened their sense of being mathematicians. Kayla said, “I think the thing that contributed most to me as a mathematician was just the community, or like my cohort, or being able to do math with other people, because that's when I feel like you learn the most math.” Kendall echoed this sentiment: “I really want to be the type of mathematician that people want to work with. I think part of math is collaboration.” For participants, doing

mathematics in community not only supported their academic development but also shaped the values they hoped to carry into faculty roles, echoing the idea that mathematics identity involves recognition from others (e.g., Cribbs et al., 2022). They came to view mathematicians as those who persist through challenges, stay motivated by a desire to understand, and thrive in collaborative learning environments.

The mathematician role identity was most often enacted as participants were taking coursework, grappling with advanced concepts, and navigating qualifying exams—experiences that both initiated and tested their identities as mathematicians. The qualifying exams, in particular, surfaced significant challenges and self-doubt. Several participants described this period as the most difficult of their graduate careers. Morgan recalled, “It was definitely the qualifying exams. That’s the only time where I’ve slightly wavered on, ‘Is this right for me?’ That was the only time.” These moments of uncertainty created tensions with their mathematician identities, which were not grounded in research contributions but in a connection to the discipline through learning, struggle, and shared participation in mathematical practices.

Mentor

The role identity of *mentor* captured how participants saw themselves primarily in relation to undergraduate students (Brown & Sheerin, 2018; McAlister et al., 2022). Kendall shared, “Being a mentor to undergrads has always been a lot of fun... you basically get to be an advisor to them in some sense.” Like others, she found fulfillment in supporting students beyond classroom instruction. While participants often linked mentorship to their teaching and research identities, they also recognized it as a distinct role involving responsibilities such as providing socioemotional support outside the classroom, offering guidance about

graduate school, and challenging narrow narratives about who can be a mathematician (Herzig, 2004a; Mendick, 2006). Landon described mentorship as a way to give back to the community or “a great way to pay that forward in some sense and make that experience good for other people.” Having faced challenges during his own transition into graduate school, he saw mentorship of undergraduate students as an opportunity to ease that path for future graduate students.

Participants’ approaches to mentorship were notably shaped by their own experiences as undergraduate students. They sought to provide undergraduates with the kind of support they had benefited from, or wished they had, hoping to dismantle barriers within undergraduate mathematics education. Mentorship frequently took place in structured contexts such as the Directed Reading Program (DRP) and Research Experiences for Undergraduates (REUs). In these settings, participants helped students learn to read scholarly work and to explore mathematical research while also offering insights into the discipline and their own academic journeys. Landon described mentoring undergraduates in an REU project as his most meaningful experience in graduate school. He reflected: “The other thing is research with undergrads. The REU experience I had, still to this day, is my favorite thing to ever happen in math.” For him, supporting undergraduate students through the REU program was central to anchoring his mentor role identity, also shaping the kind of professor he hopes to be. Similarly, Morgan, who mentored students through both DRP and REU, found these roles deeply affirming. She said, “I was also a mentor for a REU program. So that was more research focused, and I really enjoyed that as well, which kind of made me think, like, I don't know, that being a professor would be good.” She added, “I just read a book with my students and it's more the mentoring aspect. I feel like that kind of makes me

feel like, ‘Oh, I’d really want this collaborative research environment or to be an advisor someday.’” These structured mentorship experiences anchored their evolving professional identities and inspired continued interest in the professoriate.

For women-identifying participants, mentorship also intersected with their gender identities. Kayla, reflecting on her own experiences as a woman in mathematics, described a disconnect with some of her past professors and emphasized the importance of demonstrating care and compassion as a mentor. She viewed mentorship as a way to challenge the stereotype of mathematicians as cold or distant and to offer more humanizing, supportive interactions (Gutiérrez, 2017). Kendall also spoke to the importance of gender representation in mathematics: “That’s something I want to do. I want to be a woman in a field where I can make a trail for future girls ... I’m sure it was in the back of my mind for a lot of the time.” These reflections demonstrated how participants’ mentor identities were shaped by and connected to their social and cultural identities. Finally, participants saw informal spaces, like office hours, as key opportunities to act as mentors. Beyond helping with coursework, they used these interactions to offer emotional support and share aspects of their professional and personal lives. Kayla said, “I want to be a faculty where students are like, ‘Oh, I enjoy going to Kayla’s office hours,’ or just stuff like that and not being closed off.” Some described conversing with students about life beyond math, simply to ensure that students felt heard. These practices of trying to connect with students beyond math reflected participants’ holistic understanding of mentorship and their commitment to supporting students both academically and personally.

Peer and Colleague

Participants enacted the role identity of *peer and colleague* primarily through relationships with fellow graduate students, centering this identity on mutual support, peer mentorship, and advocacy. Grounded in values of collegiality and community, this role identity was often the most significant source of belonging in graduate school. Kendall summarized this powerfully, saying, “I feel a sense of belonging in the field, but only because of the other graduate students. I feel a sense of belonging in my [community of] graduate students at Spruce.” Rather than emerging from formal institutional structures, participants described informal spaces and social activities as essential to cultivating academic and emotional support. For example, Connor shared, “Things like game night that we run have been really helpful in actually meeting friends in the math department. I was invited to go climbing with a couple of the math department people.” These relationships in the department made it easier to ask for academic help, vent about challenges, and feel connected. For some, community was also cultivated through shared identities and interests. Morgan, for example, described bonding with a close group of women-identifying graduate students, while Landon reflected on early social moments like playing video games and getting pizza with peers that helped ease his transition into graduate school.

Peer support across cohorts also played a critical role in shaping this identity. Kendall noted how observing senior students navigate the job market helped her understand and anticipate future steps in her career. She explained, “Watching them, like older grad students, decide what to do and watch them apply to postdocs ... Talking to older graduate students, talking to graduate students in my year and below, because you're all navigating the same career trajectories.” This exchange of knowledge, support, and empathy was reciprocal and

sustaining. Connor emphasized the emotional care such a peer network provided. He shared, “It’s more of an emotional support. It’s nice to have someone to sort of vent to about research. It’s really nice to have people who understand ... and it’s a nice give and take there.” Landon noted that virtual platforms, like their department’s Discord server, helped facilitate community and information-sharing. He said, “There’s a math grad students Discord, which is pretty active. That’s great because sometimes people ask questions, and then everyone else gets to see the answer, it’s good information to see.”

For women-identifying students, peer support was particularly tied to identity, representation, and advocacy. Morgan, Kendall, and Kayla all spoke to the importance of creating and sustaining a sense of community among women in a male-dominated department. Morgan shared, “There’s way less women in the department, but the women, I feel are treated a lot better. And there’s even more bonding between the women in the department.” In response to the lack of representation, Morgan founded a chapter of the Association for Women in Mathematics (AWM) at Spruce University, creating a space for solidarity and mentorship. Kendall recalled how this space was sustained over time:

We met weekly for a while, and it was organized by one of the older grad students (Morgan) when I was a first year. Then my friends and I started organizing when I was a second year, so we kind of kept that going. It’s been really good. It’s nice to be a part of that community, even though it’s small within our math grad program. I think that’s been a good support.

Morgan’s leadership laid the foundation for others, like Kendall, to take on organizing roles, continuing a tradition of peer support and advocacy. Across participants, the peer and colleague role identity reflected deep commitments to collective care, shared

learning, and inclusive community building. These relationships helped participants navigate the challenges of graduate school and modeled the kinds of collegial, supportive professional environments they hoped to cultivate as future faculty.

Discussion

Throughout graduate school, students engage in learning, teaching, research, and academic life, concurrently developing an understanding of themselves and the profession's values, norms, and practices (Clarke et al., 2013; Gardner, 2008; Garnder & Mendoza 2023; Trede et al., 2012). As participants shared reflections of their experiences in their doctoral program, they revealed the performances of role identities that were informed by their program's structure, their social interactions with others, their personal commitments and values, and the broader disciplinary expectations associated with higher education mathematics. By leveraging sociocultural perspectives of professional identities and role identity theories, I identified five main role identities that mathematics graduate students enacted: *teacher*, *researcher*, *mathematician*, *mentor*, and *peer and colleague*.

While existing literature has studied graduate students' roles and responsibilities, such as that of a teacher (e.g., Kajfez & McNair, 2017; Winstone & Moore, 2017), researcher (Davis & Lester, 2016) and mentor (e.g., Brown & Sheerin, 2018; McAlister et al., 2022), my analysis offered new insights into identity development situated within the discipline of mathematics and a unique departmental context, and into the different meanings that mathematics graduate students associated with these five role identities. For example, some participants attributed a sense of belonging to mathematics through their researcher identity, and understanding being a peer and colleague to involve advocacy and creating safe spaces for each other beyond the mathematics content. These nuances broaden our understanding of

how graduate students navigate their everyday realities in graduate education, while providing insights into how their professional identities as aspiring faculty are developing.

I found that the role identities of teacher and researcher were most central to what it means to be a mathematics professor – suggesting these two dimensions of their professional identities are more strongly anchored (Mockler, 2010) and higher in participants’ salience hierarchies (Stryker, 1968; Zhang & Wang, 2024). Morgan referred to the roles of teacher and researcher as the “two aspects of being a math professor.” The prevalence of these dimensions suggests that participants have been enculturated to understand the professoriate to primarily involve these roles, and that the mathematics department at Spruce University further confirmed these roles as most important to doctoral students. The context and program structure of Spruce University significantly impacted the ways the role identities of teacher and researcher were enacted because of the norms around progressing through milestones – mainly that graduate students were afforded more formal opportunities to be teachers and less structured opportunities to be researchers. This illuminated an imbalance in preparation for what is communicated as two salient roles necessary as mathematics faculty.

Additionally, as participants reflected on their social interactions, beliefs and values, and broader disciplinary expectations, I identified the other commonly enacted role identities of mathematician, mentor, and peer and colleague. These role identities often meant learning and doing mathematics as mathematicians, serving as advisors to undergraduate students beyond teaching mathematics content, and supporting peers through building community and collegiality. Although features of graduate students’ everyday activities and responsibilities may be similar across other graduate programs and disciplines (e.g., taking up teaching assistantships), this work began to shed light on the complexities of how a specific

departmental context and the mathematics discipline can influence how role identities as performed, ultimately shaping graduate students' professional identity development.

The Influence of Mathematics

Clarke et al. (2013) referred to discipline-based cultures as “the primary source of faculty members’ identity and expertise and include assumptions about what is to be known and how tasks [are] to be performed, standards for effective performance, patterns of publication, professional interaction and social and political status” (p. 7). I applied this perspective to unpack how mathematics informed the role identities that participants performed and enacted. The role identity most explicitly influenced by the discipline is graduate students as practitioners of mathematics (i.e., the mathematician role identity). This identity dimension commonly emerged as participants reflected on taking coursework and completing qualifying exams; interestingly they characterized these experiences as moments when they felt this role identity most questioned and challenged. As mathematicians, participants often described themselves as learners and doers of mathematics, resonating with literature on undergraduate students’ mathematics identity (e.g., Cribbs et al., 2022). However, participants revealed additional meanings and attributes that underscored their understanding of being mathematicians, particularly that mathematicians are creative, perseverant, and collaborative. In many ways, the meanings they associated with the role counters dominant discourses about mathematicians (e.g., Mendick, 2006) and echoes many of the participants’ commitment to humanizing mathematics as part of their professional commitments.

Moreover, the discipline also influenced the ways graduate students negotiated tensions among role identities (Colbeck, 2008) and how certain role identities became more

anchored (Mockler, 2010). For example, this work revealed differences in participants' understanding of what it means to be a mathematician and a researcher. Being a researcher meant participating in mathematical activities in service of the discipline and the broader community, while being a mathematician often meant learning and doing mathematics for fun. Participants negotiated these meanings and distinctly characterized these two role identities. Moreover, I found intersections between disciplinary norms and participants' gender identity to shape how certain role identities came to be anchored. Notably, Kendall described that the subfield of topology was more affirming to women and that solidified her interests and commitment to this research area, supporting her researcher identity development. However, this could also reflect structural constraints with mathematics.

Other disciplinary influences informed their role identities as mentors and peers. As mentors, participants were aware of exclusionary discourses in mathematics (Austin & McDaniels, 2006; Herzig, 2004a, 2010) and they described commitments to supporting undergraduate students in ways that countered these narratives. For example, to combat the underrepresentation of women in mathematics, Kendall said, "I want to be a woman in a field where I can make a trail for future girls" as she discussed her mentorship efforts and engagement with undergraduate students. Similarly, as peers and colleagues, the women-identifying participants were deeply committed to creating safe spaces (e.g., AWM chapters) for each other as they navigated the male dominated field of mathematics. Overall, some of the meanings that participants attached to their role identities were shaped by their understanding of the cultures and norms of mathematics, demonstrating how disciplinary contexts play a crucial part in the professional identity development of graduate students.

The Influence of the Departmental Context

Contextual features of Spruce University's mathematics department, primarily norms around progressing through the doctoral program milestones, also impacted participants' opportunities to engage in roles and responsibilities. Comparing the two most salient role identities, I found differences in participants' identifications as teachers and researchers reflective of their progression in the program and formalized support structures. Early in the doctoral program at Spruce University, participants are afforded more opportunities to teach – such as employment as TAs, institutionalized TA trainings, and tutoring undergraduate students. However, opportunities to develop as researchers are not formalized until after completing qualifying exams, and students can have vastly different exposures and support for their development as researchers that seemed to be dependent on faculty's willingness and availability. Interestingly, many of the participants cited chance, positive experiences with faculty who disrupted these norms and positioned students as researchers even though they may not have completed certain milestones. Some faculty exhibited commitment to supporting graduate students as researchers beyond departmental norms like only establishing an advisee-advisor research relationship after completing qualifying exams.

Lastly, an important feature of the departmental context was the unique sense of community fostered amongst the graduate students. All participants reported that as a peers and colleagues, they found critical resources emerging from other graduate students (e.g., advice on career preparation, socioemotional support). Morgan, who was previously enrolled in another PhD program, was particularly insightful in characterizing the supportive and affirmative environment at Spruce. Despite having fewer women-identifying graduate students within the program at Spruce versus her previous institution, Morgan felt a stronger

sense of belonging in the department and especially amongst the other women. This culture of support shaped how graduate students particularly took up roles as peers and colleagues. The narratives surrounding the AWM chapter demonstrated this affirming community as Morgan founded the AWM chapter and Kendall saw the value in this space and decided to take up leadership positions to continue the chapter in years to come. Kendall reflected, “Then my friends and I started organizing [the AWM chapter] when I was a second year, so we kind of kept that going.” Experiencing the impact of the strong sense of community amongst the graduate students in the department reaffirmed participants’ commitments to being supportive peers and colleagues; demonstrating a transmission of culture, beliefs, and values that ultimately shape students’ professional identity development.

Implications

This study offers theoretical and practical implications for understanding professional identity development and preparing graduate students for faculty careers within mathematics. By examining how graduate students enacted and constructed meanings around five distinct, yet connected, role identities—teacher, researcher, mathematician, mentor, and peer/colleague—I sought to contribute to broader conversations about the multifaceted and dynamic nature of professional identity formation in graduate school (Clarke et al., 2013; Solari & Ortega, 2020). The findings highlighted that identity development is not merely an individual psychological process but one that is socially mediated and shaped by contextual affordances and constraints (Zhang & Wang, 2024). Theoretically, this study underscores the utility of role identity theory in examining how aspiring faculty navigate multiple, and sometimes conflicting, roles in a liminal space between graduate student and professor (Bale & Anderson, 2022; Winstone & Moore, 2017). By situating the work within the disciplinary

context of mathematics, I theorized a process of professional identity development through the enactments and stabilization of role identities (See Figure 2). This work demonstrated how a range of factors ranging from social interactions to departmental contexts and cultures, to the mathematics discipline itself, can inform how graduate students come to understand and perform roles throughout their doctoral education.

Practically, the identification of critical experiences that support or hinder role enactments (Jazvac-Martek, 2009) can inform the design, policies, and practices of graduate programs, suggesting the need for more intentional opportunities for mentoring, pedagogical development, and community building. For example, graduate programs can support students' development as researchers early in their doctoral education, such as through avoiding waiting for the completion of qualifying exams. Additionally, departments can invest in other identity-based organizations, like AWMs, to create more safe spaces for students (MacLachlan, 2006). Graduate programs that aim to prepare future faculty should consider how structural elements, such as advising relationships, teaching assignments as instructors of record, and peer networks, either expand or limit students' ability to explore and integrate these role identities in meaningful ways.

Limitations and Future Work

There are three key limitations and areas for future work I have identified. First, data were collected at a single point in time rather than longitudinally, which may have limited the ability to capture shifts in participants' identities and role enactments throughout their graduate education. Although participants in the present study were prompted to reflect on their experiences up to that moment, future work can involve collecting data at multiple stages of graduate school to better capture the evolving nature of role performance and

professional identity development. Second, the study included a small number of participants within a single doctoral program. While this allowed for a deep, contextualized understanding of how departmental norms and cultures at Spruce University shaped participants' identities, further comparative and cross-institutional research is needed to explore how different contexts influence graduate students' roles. Finally, this study focused primarily on identifying and analyzing the critical experiences that anchored participants' teacher and researcher identities. In future work, I will seek to further examine identity tensions, for instance, how qualifying exams challenged students' identities as mathematicians, how sociopolitical events such as a graduate student strike disrupted their identities as peers, and how their social and cultural identities were sometimes in tension with institutional expectations. Exploring both identity anchors and tensions will broaden our understanding of the complex and layered processes through which graduate students begin to see themselves as future mathematics faculty.

Conclusion

This study offers a more nuanced portrait of the multiple role identities that mathematics graduate students navigate and the meanings they attach to these dimensions of their professional identities throughout their graduate school journeys. By identifying the key role identities of teacher, researcher, mathematician, mentor, and peer, and examining the experiences that support or constrain their enactment, this work highlights the layered and situated nature of identity development in graduate education. These findings extend our understanding of how graduate students begin to see themselves as future faculty, shaped by both institutional contexts and personal experiences. Ultimately, attending to graduate

students' role identities provides valuable insight into how we might better support their growth within mathematics departments and the broader academic profession.

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CHAPTER THREE

A Narrative Inquiry of Masculine Discourses Along the Hike

A reality widely documented about graduate students' experiences – particularly students historically underrepresented in higher education – is that they do not feel adequately supported during their graduate education, impacting their persistence (Russell et al., 2018), sense of belonging (Ostrove et al., 2011), self-efficacy (Nyquist et al., 1999), and in some extreme cases, their decision to depart from the academy altogether (Foote, 2010; MacLachlan, 2006). Given the central role of graduate education in shaping the professoriate, it is essential to examine how the process of socialization, or the collective experiences during graduate school (Gardner, 2008; Gardner & Barnes, 2007; Tierney & Rhoads, 1993), unfolds within specific disciplines. Mathematics presents a unique case of graduate education due to its deeply embedded cultural norms and values, which influence how students experience their doctoral training (Ataide Pinheiro, 2021; Herzig, 2004a). The structures and expectations within mathematics graduate programs not only shape students' professional identities but also contribute to historic patterns of inclusion and exclusion.

Herzig (2004a) called for increased attention to doctoral programs in mathematics, emphasizing that the discipline operates within distinct cultures and norms that perpetuate the marginalization of various identities and communities. These norms are often influenced by dominant masculine discourses, which can create environments that marginalize or devalue the experiences of women and other underrepresented groups (Austin & McDaniels, 2006; Leyva, 2017; MacLachlan, 2006). Dominant masculine discourses, as Mendick (2006) described, frame mathematics as a domain associated with qualities historically aligned with masculinity, such as independence, competitiveness, and detachment (Hottinger, 2016;

Leyva, 2016, 2017). Women in graduate mathematics programs often navigate environments where these discourses contribute to stereotype threat, implicit bias, and tensions in balancing academic and personal responsibilities (e.g., Ataide Pinheiro, 2021; Blickenstaff, 2005; Cech & Waidzunus, 2011; Glass et al., 2013), and diminish alternative ways of engaging with mathematics, such as collaboration or relational approaches to teaching and learning (Herzig, 2010; Olarte & Street, 2025). Striving for a deeper understanding of the individual and collective experiences of women sheds light on how these gendered norms and values shape their opportunities and challenges in academia. Such insights are crucial for developing purposeful interventions and informing the reimagining of graduate programs that address root causes of gender inequities.

Adopting a multiple-case study and narrative inquiry approach (Creswell & Poth, 2016), I share the stories of two women-identifying graduate students, Kayla and Morgan (pseudonyms are used), to better understand how masculine discourses are encountered through the program structures, policies, and everyday practices and interactions within a graduate mathematics program, which may impact the access and participation of women in such programs. The research questions that guided this study were: (1) How did women-identifying graduate students reflect on their experiences and narratives about navigating graduate education and pursuing mathematics faculty positions? (2) How did masculine discourses manifest in the discursive practices and policies of mathematics graduate education?

Conceptual Framework

To frame this study, I adopted a poststructuralist perspective that discourses, produced by and perpetuating power, shape the realities and identities of women in mathematics

(McBride, 1989). Graduate students encounter and engage in these discourses in various ways, often requiring them to perform masculinity (Mendick, 2006) to navigate their graduate education journeys. Illuminating women's narratives allows us to examine how mathematics graduate programs may continue to position women as outsiders in mathematics.

Discourses and the Historic Marginalization of Women in Mathematics

Foucault's (1980) conception of discourse provides a foundational lens for understanding how power operates within and through language, practices, and systems of knowledge to produce and maintain social hierarchies. Discourse, in Foucault's terms, is not merely used for communication but is a means of constructing and regulating social realities and one's identities (Martin, 2009). Importantly, discourse is deeply intertwined with power, which Foucault theorized as relational and diffuse rather than simply hierarchical (McBride, 1989). Power is not something that an individual holds, rather it is something that an individual is able to exercise within certain contexts and based on how they are positioned by others. In institutions like higher education, power is exercised through the establishment and reinforcement of dominant discourses that shape norms, values, and behaviors. Leyva (2016) conceptualized discourses as "contextual manifestations of these power relations" (p. 89) and suggested that institutions, others (e.g., peers, teachers), and broader society uphold and perpetuate these power relations and discourses.

In the present work, I focus on how women exercise power in graduate mathematics education spaces to identify the discourses that shaped their realities and identities. I especially draw on McBride's (1989) conceptualization of *discursive practices*, or experiences and practices of mathematics education (e.g., textbooks, teaching methods,

examinations) where individuals encounter discourses that govern the rules, knowledge, behaviors within that space. McBride posited that discursive practices in mathematics education may “promote gender bias” and have “adopted a masculine perspective and excludes the feminine voice” (p. 42). Mathematics is a field with discourses that are historically grounded in ideals of objectivity, rationality, and universal truth. These discourses privilege characteristics associated with masculinity, such as competition, abstraction, and emotional detachment (Hottinger 2016; Mendick, 2006). Moreover, discourses in mathematics may perpetuate a power dynamic that marginalizes those who do not conform to such ideals. For instance, scholars have observed how women in mathematics may value collaboration, relationality, and pedagogy—values that oppose the competitive and individualistic ideals and discourses historically valued by the field (Leyva, 2017; Mendick, 2006; Olarte & Street, 2025; Stinson, 2013).

Mendick (2006) built on Foucault’s conception of discourse to argue that mathematics is especially imbued with gendered discourses that align with traditional forms of masculinity. For example, the performance of mathematical competence and abilities often becomes synonymous with performing masculinity, reflective of discourses about mathematical success as culturally framed with attributes, like assertiveness, dominance, and individualism (Ataide Pinheiro, 2021; Leyva, 2017). These gendered discourses not only exclude women but also require them to navigate conflicting expectations and to adopt masculine-coded behaviors, such as being competitive and working individually, to succeed in mathematics while simultaneously being penalized for deviating from traditional femininity. Furthermore, women’s experiences and resistance often come at significant emotional and professional costs, compelling them to either suppress their values or to

navigate tensions between their identities and the discipline's expectations (De Welde & Laursen, 2011b; MacLachlan, 2006). Gee's (1999) notion of Discourse (with a capital "D") complements this perspective by emphasizing the ways in which Discourses are embodied and enacted in socially recognizable ways, integrating language, actions, values, and identities. Gee's work highlights how Discourses are not neutral but inherently tied to the distribution of social capital, such as prestige, authority, and legitimacy (Leyva, 2016), which are particularly valuable in graduate school. Women in mathematics are often positioned outside dominant Discourses, meaning their values, ways of being, and ways of knowing might be considered illegitimate or oppositional to the mainstream norms of the discipline.

I leverage these prior perspectives and conceptualize discourses in mathematics as sites where power and resistance intersect. Since power is deeply embedded in the institutional structures and everyday practices and interactions within mathematics spaces, we can observe how women encounter masculine discourses through the ways they are positioned to exercise – or not exercise – power throughout their graduate education journeys. Understanding the interplay of power and discourse in this context allows for illuminating the systemic forces that continue to marginalize women and offers a critical foundation for examining how their values and experiences reshape the discipline of mathematics in higher education.

Literature Review

I review two areas of scholarship to situate the present study: literature on the experiences of women in higher education mathematics and research on masculine discourses in mathematics. I drew on this existing research and highlighted how Kayla and

Morgan's experiences broaden the field's understanding of how women-identifying graduate students encounter and resist masculine discourses in graduate education.

Women in Higher Education Mathematics

The literature on gendered experiences in higher education mathematics reveals a persistent pattern of inequities that women face as they navigate academic and professional spaces within the discipline. Women in mathematics often encounter stereotypes that undermine their competence and belonging, such as the pervasive assumption that men are inherently more skilled in mathematical reasoning or that women in mathematics need to take up more nurturing and emotionally supportive roles than their male counterparts (Herzig, 2004a, 2004b; De Welde & Laursen, 2011a; Mendick, 2006; Nosek et al., 2002; Yang & Carroll, 2018). These stereotypes contribute to microaggressions, including dismissive comments, subtle exclusion from collaborative opportunities, or excessive scrutiny of women's work in the field of mathematics (Ataide Pinheiro, 2021; Herzig, 2010; Kim & Meister, 2023).

Gender identities have also been linked to levels of access to critical resources and supports. For example, Kersey and Voigt (2020) found that LGBTQ+ and gender-minority students experienced more tensions with the mentorship they received from faculty than their male, heterosexual, cisgendered counterparts – likely due to how their gender, sexual orientation, and other identities are rarely discussed or included in the curriculum. Furthermore, research highlights a misalignment between the values that women may prioritize (e.g., collaboration, inclusivity, and real-world applications of mathematics) and those embraced in mathematics, which often emphasizes competition, individual achievement, and abstraction (Herzig, 2004a; Leyva, 2017; Olarte & Street, 2025). Such a

misalignment of values can intensify feelings of isolation and alienation for women in mathematics.

Within education contexts, McBride (1989) posited that navigating institutional and program policies can serve as sites of discursive practices, thus the program policies and the degree milestones may function as barriers for women in higher education. For example, high-stakes examinations (e.g., qualifying exams) are areas where women may encounter compounded challenges. The competitive nature of exams in graduate programs often amplify existing inequities, as women may face doubts about their abilities from faculty, peers, or even themselves due to stereotype threat (Ataide Pinheiro, 2021; Herzig, 2004a; Stage & Maple, 1996). Studies have shown that the pressure to perform in an environment that already questions their legitimacy can lead to higher levels of anxiety and attrition for women compared to their male counterparts (e.g., Glass et al., 2013; Herzig, 2004a, 2004b, 2010; White, 2004; Yang & Carroll, 2018). Additionally, women's relationships with peers and other graduate students also reveal inequities in how support networks are formed and maintained within mathematics graduate programs. Women often report feelings of exclusion from informal networks of collaboration and mentorship, which can be critical for success in graduate school (De Welde & Laursen, 2011a, 2011b; Herzig, 2010). The hyper-competitive nature of mathematics graduate programs can create environments where collaboration is undervalued, further isolating women who may prioritize relational and communal approaches to learning and professional development (Leyva, 2017; Stage & Maple, 1996).

Women's relationships with faculty can further underscore the gendered dynamics of graduate mathematics education. Faculty members often hold significant power in shaping students' academic trajectories, but women in mathematics frequently report less access to

mentorship and advocacy from faculty compared to their male counterparts (De Welde & Laursen, 2011b; Herzig, 2004a, 2004b; MacLachlan, 2006). Women may encounter faculty who hold biased assumptions about their capabilities or who fail to provide the encouragement and sponsorship that are critical for their success (Barthelemy et al., 2016). Furthermore, women may find it difficult to connect with faculty in a discipline where female representation among faculty remains low, limiting their access to role models and mentors who share their identities and experiences (De Welde & Laursen, 2011b; Guy & Boards, 2019; Leyva, 2016). Collectively, challenges across multiple discursive practices can compound the inequities women face in mathematics graduate programs.

Masculine Discourses in Mathematics

Mathematics, as a gendered space (Hottinger, 2016; Leyva, 2017; Voigt & Reinholz, 2020) is imbued with masculine discourses that impacts opportunities for learning and succeeding. Discourses, shaped by and perpetuating power relations, influence how individuals behave, interact, value, think, and believe (Leyva, 2016; McBride, 1989). Many scholars have theorized and identified how masculinity operates in mathematics via discourses, such as framing mathematical ability as innate rather than learned or privileging individual achievement over collaboration (e.g., Mendick, 2006; Nosek et al., 2002; Stinson, 2013). Mendick (2006) examined how mathematics as a discipline is culturally constructed and sustained through norms and practices that privilege masculinity. In this framing, masculinity is not merely a characteristic of individuals, but a set of cultural norms and values entangled in the practices associated with mathematics. Traits, such as competitiveness, independence, and emotional detachment, are often valued within

mathematics classrooms, while collaborative or relational approaches to learning are feminized and consequently undervalued.

Mendick (2006) approached her work on discourses by identifying interrelated binary oppositions (i.e., masculine/feminine) that position mathematics with the masculine side such as: maths people/non-maths people; fast/slow; competitive/collaborative, etc. Leyva (2016) similarly identified and characterized discourses by employing an intersectional analysis of the counterstories of two Latina women in higher education, Lauren and Tracey. In this work, Leyva found that Lauren and Tracey encountered the following discourses: mathematics ability is innate, women and Latinas are not good at mathematics, Latina women are an underrepresented group in STEM, and Latina women become young mothers and wives instead of college students. These discourses impacted how Lauren and Tracey navigated their undergraduate mathematics journeys, and they confronted these discourses through discursive practices, such as interactions with peers and teachers. As these prior scholars highlighted, masculine discourses manifest in a variety of experiences throughout students' mathematics journeys.

In a later work, Leyva (2017) reviewed the research on gender in mathematics education to characterize dominant narratives and discourses that contribute to the male superiority myth and masculinization of mathematics. These discourses often shaped women's sense of belonging, ability, persistence, achievement, and participation. Some prevalent discourses about women in mathematics include: women have to prove themselves to others and themselves (Herzig, 2010; Mendick, 2006), women are invisible (Guy & Boards, 2019), women receive little to no guidance and support from faculty (Herzig, 2004a, 2010), women are not part of the boys' club of mathematics (Ataide Pinheiro, 2021; De

Welde & Laursen, 2011a; Herzig 2004b), women are spotlighted and tokenized (McLoughlin, 2005; Yang & Carroll, 2018), and women are underrepresented in mathematics (Kessel, 2014; Poirier et al., 2009). Specific to women in mathematics doctoral programs, Herzig (2010) reiterated Damarin's (2000) notion of marking women in doctoral programs as "deviant," as they are perceived as "not being 'real' mathematicians because of their gender and not being 'real' women because of their work in mathematics" (p. 194). These narratives about women can manifest in the policies and practices of graduate education, which serve as *discursive practices* (McBride, 1989).

Bringing together the literature reviewed above, I considered the various experiences and features of graduate education (i.e., coursework, qualifying exams, relationships with advisors) as potential discursive practices where women encounter and negotiate masculine discourses.

Method

I employed an exploratory and descriptive multiple case study approach (Stake, 1995; Yin, 2018) and narrative inquiry (Creswell & Poth, 2016) to center the voices and the narratives of two women-identifying graduate students, Kayla and Morgan. I sought to highlight the invaluable knowledge inherent in their lived experiences and share stories of tensions with and resilience to masculine discourses in higher education mathematics.

Context

This study was conducted at Spruce University, a large, research-intensive university on the West Coast of the United States. The mathematics department at Spruce University offers master's and doctoral degrees in pure and applied mathematics, and progression through the doctoral program is expected to take about 5-6 years. According to the publicly

available program overview, the first two years of the program involve completing the requirements and graduate-level sequences for three content areas (e.g., Algebra, Analysis). Students take comprehensive exams (i.e., qualifying exams) for each of the three content areas after completing the required coursework. After fulfilling these three area requirements, doctoral students then select a focal research area, secure a dissertation advisor, and take an oral examination given by their respective doctoral committees to discern if the student is able to reasonably complete a dissertation on their proposed topic, marking advancement to candidacy. At Spruce University, doctoral students are expected to advance to candidacy within four years of starting their graduate studies. The final milestone is to successfully research, write, and defend their dissertation.

Participants

Convenience sampling (Miles et al., 2020) was used to recruit graduate students in Spring 2023. Sixteen doctoral students completed a survey that elicited academic information (e.g., year in program, prior education), inclusive demographic information (e.g., genders, sexual-orientations, race), and career aspirations information. Then, purposeful sampling (Yin, 2016) was used to recruit students who indicated that they had considered or were actively pursuing careers as mathematics faculty. Six doctoral students participated in a 90-minute interview that involved three narrative, self-reflective activities.

Through a multiple case study approach (Yin, 2018), I focused on two participants, Kayla and Morgan. They represented two different stages of the doctoral education journeys but shared similar motivations and career aspirations for pursuing a doctorate, and they reported experiencing similar challenges and barriers as women in mathematics. Elements and themes of their narratives of graduate education converged and suggested aspects of

replication among cases (Yin, 2018). Their experiences also diverged in that Kayla had only completed the first year of the PhD program, while Morgan had completed her fourth year, after previously being enrolled in another PhD program, leaving that program, and starting her current doctoral program. The varied experiences between Kayla and Morgan expand our understanding of women pursuing mathematics faculty positions, and through a case study approach, I emphasize how the institutional context and the mathematics discipline interact with participants' gender identities in similar and dissimilar ways.

I aimed to expand the field's understanding of the process of socialization into faculty positions (Gardner, 2008) by highlighting stories situated within a discipline (i.e., mathematics), an institution (i.e., Spruce University) and sensitive to graduate students' identities as women. My analytical focus on Kayla's and Morgan's experiences is intended neither to homogenize nor to generalize to all women-identifying graduate students in mathematics doctoral programs. Rather, I illustrate the ways that masculine discourses were exemplified in their stories to broaden our understanding of how discourses shape the experiences and identity development of graduate students in mathematics.

Kayla's Background

Kayla was an incoming second-year mathematics PhD student at Spruce University. She was interested in geometric and combinatorial group theory, stemming from her master's thesis in a similar area. Kayla's passion for mathematics emerged early, as she excelled in the subject during high school and gained confidence from helping her peers. Growing up in one of California's farming towns, she became deeply connected to her community and was driven by a desire to serve as a role model for students from similar backgrounds, particularly as the first in her family to pursue a doctorate. Kayla's love for teaching strongly influenced

her decision to pursue a career in academia. She expressed that teaching had always been her primary goal, coming from the joy and fulfillment she derives from it. Her choice of graduate programs was motivated by personal and practical reasons, prioritizing proximity to her family and fiancé, who was also pursuing a graduate degree in California. The established academic pathway between Beech University (2020-2022), where she earned her master's degree, and Spruce University also contributed to her decision to remain in California. While Kayla received acceptance offers from other institutions, her commitment to stay close to her support system solidified her choice to join Spruce University in 2022.

Morgan's Background

Morgan began her postsecondary mathematics journey with a BS in Mathematics from Fir University (2010–2014). Directly after her undergraduate degree, she pursued a PhD at Willow University, motivated by her interest in teaching mathematics. However, after earning a Master's degree in 2017, she left the PhD program at Willow University due to negative experiences. Further, she considered leaving academia and mathematics altogether. Between 2017 and 2019, Morgan worked in industry and taught at a community college, rediscovering her passion for teaching. In 2019, she joined the PhD program at Spruce University, drawn by its strong research presence in her area of interest (quantum algebra and knot theory), its proximity to her California community, and its program structure, which allowed flexibility between studying pure and applied mathematics. At the time of data collection in 2023, Morgan was entering her fifth year in the program, projecting graduation after her sixth year, and she had continued her pursuit of a career that balanced teaching and research in mathematics.

Data Collection

Data collection occurred within a 90-minute interview conducted over Zoom. In the first hour, graduate students participated in a semi-structured interview (Rubin & Rubin, 2011) about their experiences in graduate school, with a focus on supports and barriers and their experiences navigating higher education as an underrepresented student. Questions included: Tell me about the aspects of your graduate education that most supported your career goals; Describe, if any the barriers you have experienced in completing your doctoral education; and How, if at all, have you felt your identities recognized and celebrated during your graduate education? In the last 30 minutes, participants wrote a letter, in the form of a “Dear Math...” letter (Olarie, Street et al., 2024), where they were prompted to tell mathematics how they felt, and they created visual representations of their graduate school experiences in the form of graphs. Participants were asked to use time as the quantity along the x-axis, but they could graph any experience or affective dimension (e.g., levels of confidence) along the y-axis. They were not requested to create a specific number of graphs; Kayla created three graphs, and Morgan created two graphs.

Creswell and Poth (2016) discussed that good data sources for narrative inquiry include interviews as well as “observations, documents, pictures, and other sources of qualitative data” (p. 91). The three data sources – responses to the semi-structured interview, the “Dear Math....” letters, and the graphs that represent their journeys through graduate school and to the professoriate – afforded multiple avenues for participants to share their stories and lived experiences. They remained on the Zoom meeting during the last 30 minutes, but I turned off my camera, providing them the time and space to write their letters

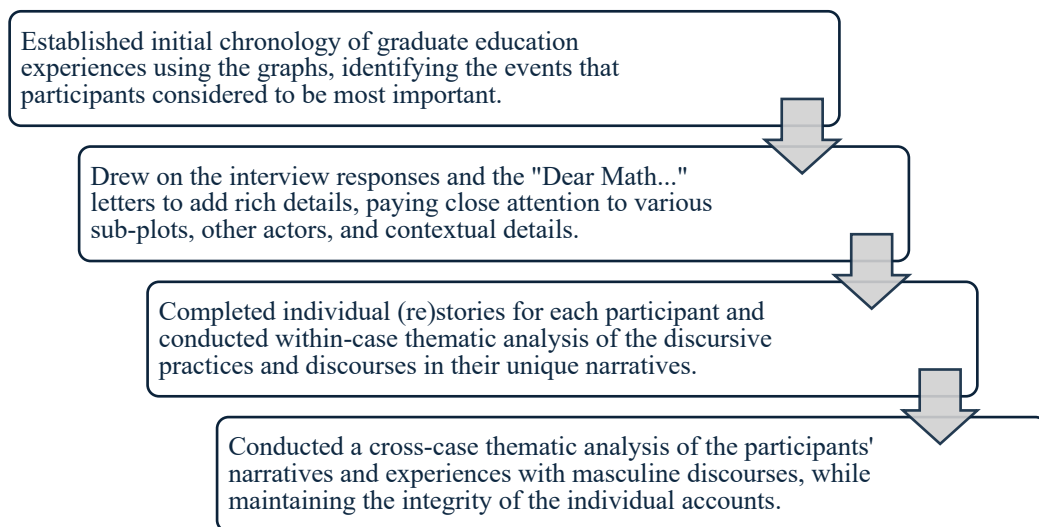
and create their graphs without direct researcher influence. The letters and the graphs were submitted via an online Google Form.

Data Analysis

I employed a narrative inquiry and (re)storying approach (Creswell & Poth, 2016; Nardi, 2015) to construct and examine participants' stories of navigating graduate school and to interrogate the ways that masculine discourses in higher education mathematics shaped their academic journeys. To begin the analysis process, I annotated the graphs to help make sense of the participants' stories and chronologies, and to highlight the aspects of graphs most relevant to their stories. The graphs presented in the findings below include some of these annotations. Additionally, since participants typed their letters, data cleaning was not required to utilize the letters for interpretation or triangulation (Stake, 1995). Figure 3 below outlines the narrative analysis procedures that I developed for this study.

Figure 3

Narrative Analysis Procedures



Note. I developed this procedure by applying prior considerations and examples of narrative inquiry (e.g., Leyva, 2016) and leveraging the specific data sources of this study.

The centrality of chronology sets narrative inquiry apart from other qualitative approaches (Bell, 2003; Connelly & Clandinin, 2012; Creswell & Poth, 2016). I drew on a naturalist narrative approach (McAlpine, 2016) to create and analyze each participant's (re)story, without direct comparison across the cases at the early stages of creating their (re)stories. Doing so was an attempt to preserve the idiosyncrasies of their narratives and honor their unique lived experiences. Leyva (2016) engaged in a similar approach to analyze the counterstories of women in undergraduate mathematics courses, and I modeled my approach on this prior work to first structure Kayla's and Morgan's individual narratives.

I began with using the participants' graphs to establish a chronology of their graduate education experiences. Table 3 illustrates the chronology of events related to their progression through institutions and program milestones. Once an initial chronology was established for each participant, I drew on their interview transcripts and "Dear Math..." letters to gather details and insights into the additional events and experiences that comprised their graduate education journeys. Like Leyva (2016), I structured participants' narratives using shared experiences (e.g., taking qualifying exams) while also describing experiences unique to each participant (e.g., Morgan leaving a previous doctoral program prior to attending Spruce University and Kayla receiving support from the graduate student coordinator). I returned to the graphs, interviews, and letters and refined the preliminary draft of their narratives, leading to the final construction of the (re)stories.

Table 3*Kayla's and Morgan's Chronology: Beginning the (Re)storying Process*

Year	Events in Kayla's Narrative	Events in Morgan's Narrative
2014		Started the PhD program at Willow University
2017		Left Willow University, finished with a Master's in Mathematics
2017-2019		Worked in industry, taught at a community college, applied to other PhD programs.
2019		Started the PhD program at Spruce University, began coursework.
2020	Began Master's in Mathematics program at Beech University.	Took qualifying exams during Year 1
2021	Passed Master's qualifying exams	Took qualifying exams during Year 2
2022	Presented MS thesis and graduated from Beech University. Started the PhD program at Spruce University and began coursework.	Took qualifying exams during Year 3, took advancement exam
2023	Took and passed qualifying exam during Year 1	Passed all exams, officially began research with advisor.

Note. The details in the chronology emerged from the annotations and analysis of Kayla's and Morgan's graphs.

Next, I employed a thematic analysis within each case (i.e., individual (re)story), identifying themes in how Kayla and Morgan encountered the various discursive practices and exercised power in these settings (McAlpine, 2016). I also focused on aspects of their stories where their gender identities were central to these experiences. To address my first research question, I modeled my analysis after the work of Leyva (2016), who first identified shared experiences of the Latina women in undergraduate mathematics courses (e.g., first

year in college, interactions with peers and family). In this present study, the selected shared experiences were the qualifying exams, interactions with peers, and relationships with faculty and staff. These experiences, conceptualized as discursive practices (McBride, 1989), were the most impactful sites that Kayla and Morgan encountered and resisted masculine discourses. To answer my second research question, I engaged in a cross-case thematic analysis (Leyva, 2016; Miles et al., 2020) of the (re)stories to discern how Kayla and Morgan were similarly positioned to exercise or not exercise power, and to identify and characterize the discourses that undergirded those experiences. Leyva's cross-case analysis of Latina women's counterstories revealed that participants institutionally and interpersonally encountered and negotiated discourses that shaped their everyday realities such as: "mathematical ability is innate", "women and Latin@s are not good at mathematics", and "Latin@ women become young mothers and wives instead of college students" (p. 109).

I adopted the same process of identifying similarities and variations in how they navigated the discursive practices. For example, I compared how Kayla and Morgan navigated challenging encounters with peers and faculty who made them question their mathematical competencies, which revealed the discourse that *women are positioned as not good at mathematics* (e.g., Ataide Pinheiro, 2021; Herzig, 2004a). The other two discourses that emerged were: *women have to constantly prove themselves to others*, and *supportive and affirming connections are rare yet impactful for women*. The analytic goal was to highlight how these discourses manifested in graduate mathematics education contexts.

Validity

I employed methodological triangulation (Stake, 1995) to identify the critical features (e.g., events, characters, storylines) of their narratives and experiences with masculine

discourses. Stake wrote that, “the protocols of triangulation have come to be the search for additional interpretations more than the confirmation of a single meaning” (p. 115). The participants’ responses to the interviews, their writings and reflections in the “Dear Math...” letters, and the visual stories they shared by creating the graphs confirmed the occurrence of events (e.g., describing the difficulties of qualifying exams in the interview *and* visually representing this period of low confidence in the graphs). After completing the (re)stories and making sense of their experiences with discursive practices, I conducted a member-checking interview (Miles et al., 2020) with Morgan to corroborate my findings and interpretations. Prior to the interview, I sent her a copy of her (re)story to review, and during the interview, we discussed the extent to which I captured her narratives accurately. At the time, Kayla had recently given birth and was unable to participate in a member-check interview. Instead, I sent Kayla her (re)story and my preliminary analysis, and she provided comments and feedback to my interpretations of her experiences. I synthesized their feedback and made changes accordingly.

Findings

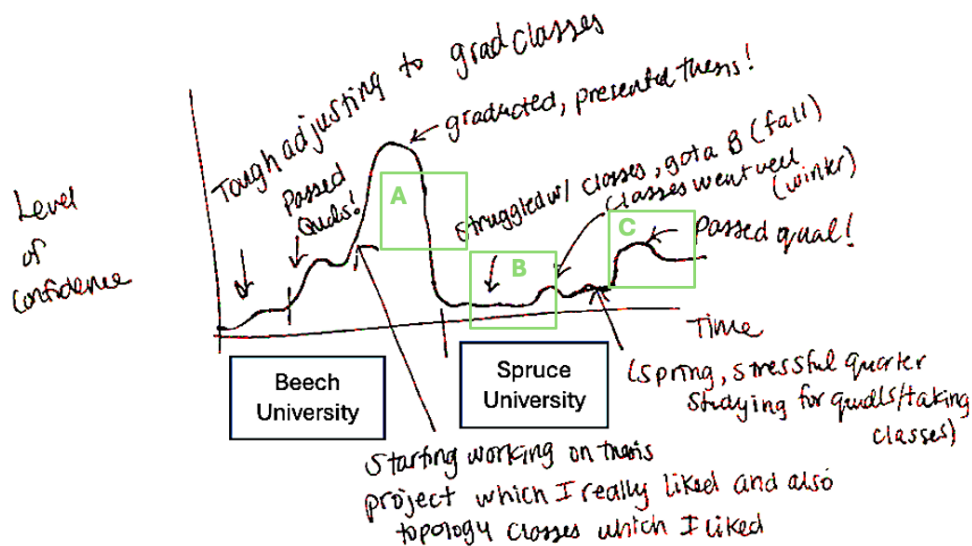
Drawing on the conceptualizations of discourses as constructed and performed in relation to others and broader systems (e.g., Leyva, 2016; McBride, 1989; Mendick, 2006), I provide a brief introduction to each of the participants’ stories and highlight the following discursive practices that Kayla and Morgan encountered and through which they negotiated masculine discourses: (a) qualifying exams, (b) relationships with peers, and (c) relationships with faculty and staff. I then present a cross-case analysis of their stories to illustrate how masculine discourses within graduate mathematics education shaped their everyday realities.

Kayla's Stories

The defining theme in Kayla's story was the shift from an overwhelmingly positive experience as a Master's student at Beech University, to unexpected challenges and difficulties across multiple dimensions as she began the doctoral program at Spruce University. Through her graphs, Kayla shared three dimensions of her graduate education journey that particularly declined during this transition: her levels of confidence, the quality of support she received from peers and faculty, and the quality of support she received from her friends and family. In Figure 4, she graphed her evolving level of confidence, and in Box A, she illustrated the significant drop in confidence she experienced upon starting the PhD program at Spruce.

Figure 4

Kayla's Levels of Confidence



Notes. Box A illustrates the largest drop in levels of confidence upon starting the PhD program at Spruce University, Box B illustrates the lowest levels of confidence during coursework and qualifying exams, and Box C indicates an increase in levels of confidence after passing a qualifying exam.

Kayla explained that she began to question her sense of belonging in mathematics and whether she was adequately prepared to pursue a career in academia. The transition between universities was also marked by personal uncertainty, as she grappled with the demanding time commitments of a PhD program and the challenges of balancing academic goals with other life aspirations, such as marriage and motherhood. She noted that these factors, like balancing academics and life commitments, led to a noticeable decrease in the support from her family and friends. Upon entering Spruce University, Kayla also explained that she felt like she no longer stood out in terms of mathematical competencies, further compounding her feelings of self-doubt. She said,

Now in grad school, it's like, "Oh, everyone is like me." I'm not the math person. It's like, oh, I'm a math person now. It's definitely gotten to where now I feel like a small fish in a big pond because everyone's good at math, and it's just like, you don't feel as special anymore. And I think that's something that I've really struggled with coming here. I still feel like a math person, but that's definitely evolved as I've continued through my education.

Kayla's experiences highlighted a contrast between the largely supportive and affirming environment she encountered at Beech University and the unexpected barriers she faced as a doctoral student at Spruce University. Her lived experience with a supportive graduate program gave her unique insight into the challenges at Spruce, allowing her to critically identify and name the barriers she encountered.

Qualifying Exams & Beginning Coursework

The first set of milestones that Kayla completed in her first year at Spruce was finishing coursework and taking her qualifying exams. Her confidence wavered during her

first year due to balancing the dual demands of these two milestones (See Figure 4, Box B).

She described this period as the most notable barrier saying,

I think probably the qualifying exams has been the biggest [challenge]. It just feels like there's so many things that we're expected to perform at this super, super high level. And, it's just like, there's no room for mistakes, kind of thing. Our first year is kind of, you're taking classes, so we usually have to take three classes each quarter. We have to get A minuses in those sequences, and then we also have the qualifying exams.

This period was extremely challenging because of the exams' proof-based nature coupled with the pressures to perform well in her classes. She shared that it had become a norm within the department that information and details about the qualifying exams, such as which professor was tasked with writing the exam, was withheld from students until the day of the exam. She shared, "You just kind of never know what kind of an exam you're gonna get. They don't tell you who writes it until you show up there the day of and it's like, 'Oh, this professor wrote it.'" Kayla mentioned that she struggled with test anxiety, and the lack of transparency was especially taxing. The exams demanded "such a high level of performance" and Kayla felt like she had to continually prove herself through high-stakes assessment practices. She explained,

I think it's just hard, too, when you've made it this far in your grad studies, and it just feels like you keep having to prove yourself, I guess. It's just like, "Well, I still have to keep passing these exams."

Kayla was critical of the culture of qualifying exams that communicated to students that despite prior demonstrations of mathematical ability and competence, there is a structural

feature that tells them that they are not enough and must still prove a level of worthiness to be accepted in this space. She successfully passed her first qualifying exam in the spring quarter, marking the highest level of confidence she had experienced in her early experiences at Spruce University (see Figure 4, Box C).

Relationships with Peers

Kayla's experience with peer support at Spruce University fluctuated significantly during her first year. At the beginning of the year, Kayla enjoyed a sense of camaraderie within her cohort. They regularly shared meals, worked on homework together, and lived in close proximity to one another in graduate student housing. She reflected,

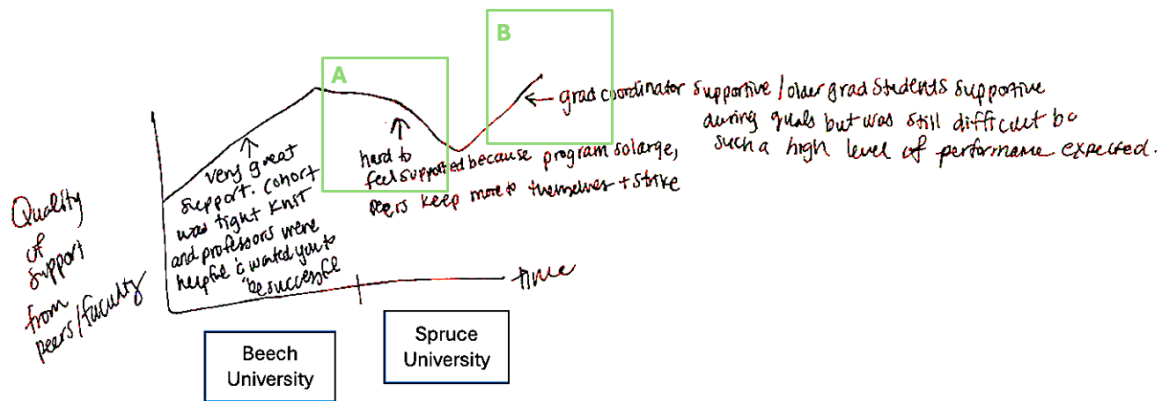
Before the [union TA-graduate student researcher] strike, I felt like everyone was pretty close. We were all in the same classes together ... We'd go to lunch every day as a cohort, because we have time in between classes, so I really liked that.

The strike, however, marked a critical turning point in Kayla's narrative. Tensions emerging from this event with the strike, fractured the sense of community within her department (see Figure 5, Box A). Graduate students were divided between those who chose to participate in the strike and those who did not, creating strain within the cohort. Kayla opted not to strike due to financial constraints, explaining, "I personally couldn't do that just because I'm saving for a wedding. My family can't afford to keep me at school, so I just decided not to." This decision led to a backlash, particularly within her cohort of 11 students, where three students, including Kayla, chose not to participate. She described the resulting divide as "this very big shift or this very big gap," which made sustaining relationships with fellow graduate students particularly difficult in the aftermath. Over time, the relationships

within her cohort did begin to improve. She noted, “It took a little while to heal those wounds from that ... but I think towards the end of the year, it was better relationships.”

Figure 5

Kayla's Support from Peers & Faculty



Notes. Box A illustrates a drop in the perceived quality of support from peers/faculty because of program size; Box B depicts increased perceived quality of support primarily resulting from the graduate coordinator Mabel.

Despite the relational tensions in her first year, Kayla eventually found support and community within the department. She emphasized the helpfulness of upper cohorts, who offered guidance on classes and exams. Within her own cohort, the presence of two other women-identifying students helped her feel a sense of belonging. She shared, “It was great because I never felt like it [her gender identity] was a thing.” Moreover, her male colleagues were also supportive, and she reflected positively on her overall experience within the cohort, saying, “I never felt like it [my gender identity] was a bad thing, or like I was disadvantaged and stuff within the cohort. So, I think that was something that was really positive.” While the strike and its aftermath created significant interpersonal challenges, Kayla’s narrative highlights her ability to rebuild connections and find support within her academic community.

Lastly, Kayla cited that her involvement and participation in the Association for Women in Mathematics (AWM) chapter at Spruce University, which was led by her peers and more senior graduate students, served to make her gender identity feel seen and celebrated within the department. She recalled events held by the AWM, such as meetings focused on the contributions of women in mathematics, as efforts that combatted the underrepresentation of women in mathematics. The culture within her cohort resisted narratives about women in mathematics, as Kayla did not feel her mathematics ability questioned by her peers.

Challenging Relationships with Faculty and Staff

Several key storylines revolved around Kayla's relationships with faculty and staff. She commented that some faculty were neither sensitive to students' experiences nor the academic demands. She said, "They [faculty] don't have a gauge of like, we're on the quarter system, and we're kind of dying in our other classes, too." She felt that this culture cultivated unnecessary pressures to consistently perform at an exceptionally high level and led her to feel overworked and undervalued.

"That was probably the hardest thing of the year." In the spring quarter of her first year, Kayla navigated a difficult course with an unsupportive professor, Dr. Campbell (a pseudonym). She characterized this experience as "the hardest thing of the year" and she shared that he caused her to question her mathematical abilities and made her feel inferior and "dumb." Kayla began her story with Dr. Campbell, saying,

I had this professor that I did not get along with him at all. I think that was the very first time I felt like, "Wow, I think this guy thinks I'm dumb." I would go to his office hours, and he would not help me on homework. He would tell you, "Oh, that's like

asking me to grade something before I have to grade it.” He would roll his eyes at me.

I don’t think I’ve ever felt that level of just being not with it, with the professor

before. So, I think that was probably just the most difficult.

Kayla expressed frustration about Dr. Campbell to other faculty and staff, and it was communicated to her that Dr. Campbell was known to be a difficult grader and that other students had voiced similar critiques.

Kayla recalled “dreading going to office hours” and “walking around eggshells” when it came to asking questions about the homework or the course. She then shared details about the hostile incident during office hours when he directly made her doubt her mathematics abilities and her place in the program:

I think that was, in terms of professors, that was probably the hardest thing of the year. I remember that whole day, I just felt awful about myself, like maybe I don’t know math as well as I think I do. This guy doesn’t think I do, so I don’t know. I remember talking to people about it and they’re like, “You just have to remember it’s not you. It’s literally just him.” So, I think that was kind of the toughest ... I think I’ve experienced a lot of where it’s like, you don’t want to ask dumb questions, even though people say that there aren’t, there definitely is this underlying like, you could definitely ask a dumb question kind of thing. And it comes out worse in higher education stuff. So, it’s like people are like, “Why is she asking that? That’s just so trivial.”

Kayla considered that Dr. Campbell’s responses to her questions reflected broader discourses and narratives in higher education mathematics. His actions reaffirmed the narrative that

students can ask a “dumb question” and Kayla recognized this to be more prevalent in higher education (Ataide Pinheiro, 2021). She detailed,

If I did ask him [about] something I wasn't understanding, he was just upset that I wasn't at his level. He was rolling his eyes at me, like stuff like that. I usually don't like playing the female card with male faculty, but I feel like that's what it felt like. I didn't know if that was a reason, maybe he treated male students like that as well. But when I went, I was always the only female person that went.

Kayla was explicitly aware of how her gender identity may have influenced the relational power she was able to exercise with Dr. Campbell. She felt that she could not ask questions or ask him for support. Although Dr. Campbell's reputation for poorly treating students was well-recognized within the department and despite her peers consoling her by attempting to remove any blame or guilt on Kayla, this relationship was a notable discursive practice where she encountered masculine discourses surrounding her mathematical abilities.

Positive Interactions with Faculty and Staff

Despite the challenging relationship with Dr. Campbell, Kayla overall felt supported by the faculty and staff at Spruce University. She highlighted two significant positive relationships. One was with a woman-identifying faculty member, Dr. Davis, who provided advice and guidance on navigating academia as a woman, wife, and mother. Kayla recalled their initial correspondence:

One of the female math professors [Dr. Davis] is a mom, and I just reached out to her and I was like, “Can I just like, literally have lunch with you? And talk about being a mom?” She was more than happy and she was like, “I was so glad you emailed me that.” She was like, “I remember I was probably where you are now, like, [in my] first

year in the doctoral program, and I was having that crisis, and I met with someone else to talk about it, too.”

Alignment in gender identity and personal commitments with a faculty member significantly contributed to Kayla’s sense of belonging at Spruce University.

Another key actor in Kayla’s narrative at Spruce University was Mabel, the mathematics department’s graduate student coordinator. As the graduate coordinator, Mabel oversaw graduate students’ wellbeing and program milestone progression. Kayla identified Mabel as an important source of support for all graduate students in the department, encompassing and extending beyond advice on degree requirements or academic support:

So, I feel like she's been a great support system to where if I do have issues with a professor, or just basically anything ... I remember last year, I was having trouble with one of the roommates I was getting housed with at [graduate student housing], and I was trying to do some stuff and I reached out to her and she was able to help me with that. So, it wasn't even math related, it was literally just housing. And so, I feel like she's been a good resource for me.

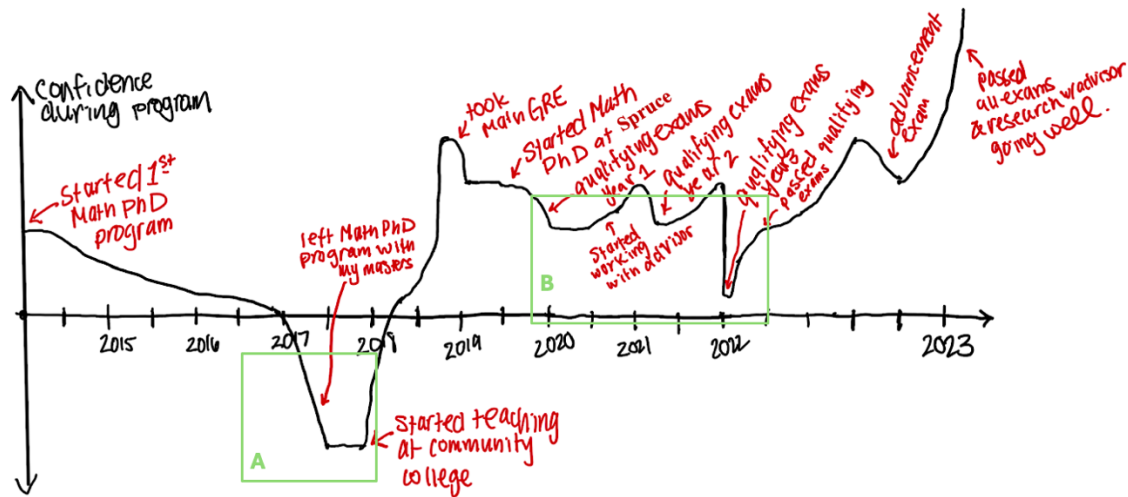
Kayla shared how valuable it was to feel supported beyond the degree milestones, like challenges with housing conditions and roommates. Kayla’s graduate education experiences extended beyond mathematics or her PhD milestones and that she felt that Mabel was a prominent resource for her to confidently navigate these challenges (see Figure 5, Box B).

Morgan's Stories

Contrary to Kayla, the defining theme of Morgan's stories was the dramatic shift from her extremely negative experiences at Willow University to the supportive environment she experienced at Spruce University. After encountering a plethora of negative experiences, ranging from gender-based stereotypes to a lack of financial support at Willow University, she ultimately left with her Master's degree in 2017 (see Figure 6, Box A).

Figure 6

Morgan's Levels of Confidence



Notes. Box A illustrates the largest drop in levels of confidence resulting from leaving the doctoral program at Willow University; Box B depicts the fluctuating levels of confidence during the qualifying exam process.

As Morgan reflected on her collective graduate education experiences, she shared stories from leaving Willow University and resuming her doctoral studies at Spruce University. The sense of empowerment and confidence that Morgan felt because of her perseverance and self-determination navigating the challenges at Willow University equipped her to successfully navigate Spruce University. She shared, “I do feel like because I’ve been

through a PhD program before, it's easier for me to kind of ignore certain things at Spruce University, because I've dealt with Willow University before." With this dichotomy being a central feature of Morgan's narratives, I share relevant stories from both institutions.

To provide additional contextual details of Spruce University, Morgan reflected on her gender identity and the extent to which being a woman was recognized and supported at Spruce University. She explained that out of about 70 total mathematics graduate students in the department at Spruce, there were about 15 women-identifying graduate students, and in her specific cohort, there were three women. Despite this disparity in the number of women-identifying graduate students, Morgan still felt that she was treated better, that the women in the department were more connected, and that, overall, she felt more supported than when she was previously at Willow University. Morgan shared,

I was hoping it wasn't like that everywhere [how it had been at Willow]. Spruce University has, I know it's not perfect, but I feel much more supported. There's way less women in the department, but the women, I feel are treated a lot better. And there's even more bonding between the women in the department.

In all, Morgan reframed the negative experiences at Willow University as strengths that equipped her to successfully navigate the doctoral program at Spruce University. In her words, "If I didn't leave Willow University, I wouldn't be here [at Spruce University]."

Navigating Program Milestones

As an incoming fifth-year doctoral candidate, Morgan had successfully completed coursework, completed qualifying exams, advanced to candidacy, and began the groundwork of her dissertation – marking a notable difference with Kayla's experiences. Morgan began the program at Spruce University in 2019, and she felt that the coursework requirements

were manageable. Her main challenge resulted from being away from academia and not formally studying mathematics for several years. She similarly reported that the qualifying exams were the most significant challenge – directly echoing Kayla’s experiences. Morgan said, “So, I was taking at least one [qualifying exam] every time it was offered from my first year to the end of my third year. So, it was a three-year thing of me taking exams.”

Throughout these years, her levels of confidence were the lowest during her time at Spruce University (see Figure 6, Box B). Morgan expanded, “It had been a long time since I’ve taken certain classes, so I’d say the hardest thing for me was specifically my analysis qualifying exam, but just exams in general.” During the qualifying exam process, she questioned her academic and career pursuits. She recalled, “That’s the only time where I’ve slightly wavered on, ‘Is this [career] right for me?’ That was the only time.” The pressures of passing the qualifying exams served as gatekeepers to her broader career development and contributed to lower levels of confidence, which she visually represented as low points in her graphs.

Relationships with Peers at Willow University

Morgan’s experiences with peers underscored a stark dichotomy between the more alienating environment she endured at Willow University and the collaborative, affirming community she later found at Spruce University. At Willow University, Morgan explained that she encountered pervasive gender-based microaggressions and overt stereotypes from fellow graduate students, which reaffirmed masculine discourses that challenged her place in mathematics (Herzig, 2004a, 2004b). Her peers at Willow University often dismissed her accomplishments, framing her successes as undeserved and solely the result of her gender. She shared a particularly hurtful experience from her first year: “I won an award for best

incoming, or best first year. And then another grad student was like, ‘Oh, you just won it because you’re a woman.’” These comments, paired with the broader dismissive attitude of her peers, created a sense of constant scrutiny and invalidation. Morgan felt as though she had to overperform and prove herself just to be taken seriously, explaining, “I felt like I just wasn't really listened to as much and there wasn't really support.” She endured a competitive, individualistic atmosphere, where some students sought to bolster their own standing by demeaning others. She reflected, “A lot of the people there [Willow University] are very sexist, and I'm sure that there are exceptions to that. But it wasn't just professors, it was grad students, too.”

Negative peer interactions at Willow served as discursive practices, where she confronted masculine discourses that falsely positioned women as outsiders, lacking innate mathematics ability (Herzig, 2004a; Mendick, 2006). The antagonistic behaviors from some of her peers contributed to her decision to leave the program, to step away from graduate studies, and to reevaluate her career path. Despite the harm caused by her experiences at Willow University, Morgan reframed the challenges as a source of resilience. Driven by the desire to teach mathematics in university settings, she returned to academia and enrolled at Spruce University. She found herself in a drastically different environment, one where peer relationships were characterized by collaboration, encouragement, and mutual respect. The peer culture at Spruce University offered a stark contrast, as her interactions with fellow graduate students, particularly other women, helped her reclaim a sense of belonging and reaffirmed her identity as a mathematician and prospective faculty.

Relationships with Peers and Undergraduate Students at Spruce University

At Spruce, seminars and collaborative discussions became spaces where she could engage without fear of judgment from other graduate students. Her peers supported her during moments of uncertainty, illustrating that a culture of intellectual support and affirmation was present at Spruce, which was notably absent at Willow University. She described this contrast, saying,

If I don't give a perfect talk, or if we're doing a reading seminar and I don't fully understand, people are very supportive. I feel like I can just talk to a lot of the grad students here, being like, "Oh, I have a question on this thing. Can you help me?"

The openness and willingness to help each other marked a significant departure from her previous experiences. Beyond academic spaces, the social connections Morgan developed with her peers further strengthened her sense of community. She described her relationships with her cohort and the women in the department as transformative: "Especially a lot of the women in the department, we're very, very close, and they're my friends, too." These friendships not only provided academic support but also a shared understanding of the unique challenges faced by women in mathematics (De Welde & Laursen, 2011a; Herzig, 2004b). Informal gatherings and events further reinforced these relationships, offering moments of connection that helped her navigate the pressures of graduate school.

Additionally, Morgan recognized the underrepresentation of women in the mathematics department and the need for a safe, affirming community. She took it upon herself to establish an Association for Women in Mathematics (AWM) chapter at Spruce University. Morgan was already aware of AWM chapters across the United States and was surprised that Spruce University did not have one. She considered AWM chapters as safe

spaces for women and wanted to create this environment at Spruce University. Although she had positive experiences at Spruce, she acknowledged that this might not be true for other women-identifying graduate students. She believed in the impact that an AWM chapter can have, noting, “So to have a place where, kind of a gathering and know that, where we learn about historical women in math and have this purpose. I thought was really important.” She secured departmental financial support, found a faculty sponsor, and cultivated a space where her peers, especially other women, could gather, share experiences, and celebrate the contributions of women in mathematics. This initiative reflected Morgan’s commitment to fostering inclusivity and ensuring that her women-identifying peers felt supported, even as she navigated her own challenges.

Although Morgan characterized her time at Spruce as drastically more positive than Willow, it is also important to acknowledge that she navigated interpersonal relationships with others that served as discursive practices for masculine discourses. For example, her relationships with peers at Spruce University were not without challenges. She acknowledged occasional feelings of jealousy when observing the achievements of her peers, but she credited her previous experiences at Willow University with teaching her how to manage these emotions. She explained: “Since this isn't my first time in a PhD program, I've learned a little bit more to kind of ignore the bad noise.” Rather than allowing jealousy to discourage her, she reframed it as a motivator to continue persevering. Additionally, she mentioned interacting with undergraduate students during math labs (i.e., open tutoring hours), as a site where she was positioned as less capable at mathematics compared to her male peers. While most undergraduate students were receptive and respectful, a small subset of students seemed to approach her with an implicit bias. Morgan observed that when she explained a

mathematical concept to these students and they struggled to understand, they seemed to question her expertise. She said, “I feel like sometimes the students in math lab think I don’t know what I’m talking about, and they feel like my male counterparts do.” Male-identifying graduate students may have been perceived as more credible, furthering some of these discourses about mathematical ability.

Relationships with Faculty

Overall, Morgan experienced positive relationships with faculty at Spruce University and felt more confident in reaching out for support as she progressed through the program.

This was a notable contrast to interacting with faculty at Willow University. She shared,

As I’ve gotten farther along in the program [at Spruce University], I feel more comfortable talking to professors and a lot of the time they’re very open and willing to talk to grad students and give advice and collaborate with them and everything like that. So, I’ve certainly felt belonging and stuff like that.

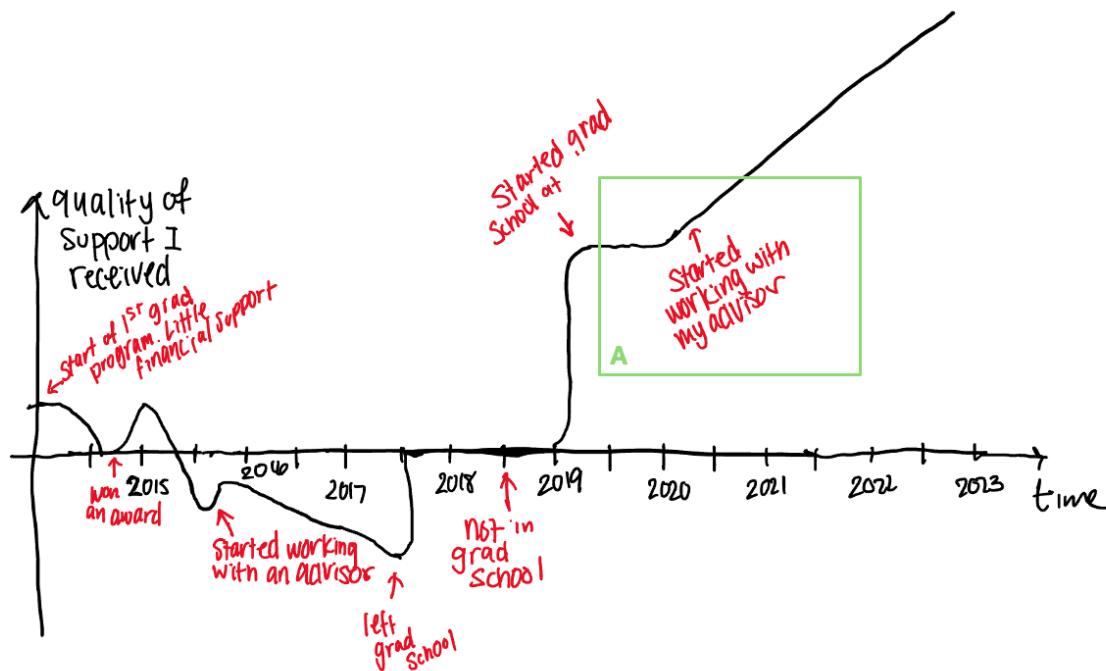
Her accrued experiences in graduate school empowered her to initiate conversations and to advocate for her own needs, which contributed to her sense of belonging. One of the most notable faculty relationships in Morgan’s journey was with her PhD advisor, Dr. Bennett. This discursive practice diverged from Kayla’s experience, because Morgan was further along in her degree milestones and had officially selected an advisor.

Morgan’s advisor, Dr. Bennett, was the cornerstone of her academic and emotional support during her doctoral journey (see Figure 7, Box A). She initially sought him out for his expertise in topology and because of the positive feedback he received from other graduate students. Over time, their advisor-advisee relationship became a critical source of encouragement, countering many of the exclusionary narratives Morgan had previously

encountered in mathematics. Morgan described Dr. Bennett as “super nice and patient,” highlighting his mentorship as a defining positive experience at Spruce.

Figure 7

Morgan's Perception of Quality of Support Received



Notes. Box A illustrates the notable increase of quality of support received as Morgan began working with Dr. Bennett, her PhD advisor.

One pivotal moment in their relationship occurred after Morgan unfortunately did not PhD-level pass a qualifying exam, a setback that left her feeling emotionally defeated. While others may not have considered this an achievement, Dr. Bennett offered validation, reframing her performance as a success at the Master's-level pass. This interaction not only affirmed her mathematical abilities but also established a foundation of respect and care. She shared,

So, I had failed his [qualifying exam] ... Everyone else had PhD passed it, and I had Master's passed it. So, I had met with [Dr. Bennett] to go over the [exam], and I had not been doing well emotionally because of this. I've never really talked to him

before. He was like, “You seem really sad that you got, about your score, but you should be happy you master’s passed. That’s great!” And everyone else I had talked to was like, “Oh, you didn’t pass.” He was just really, really kind. And I knew he was in the area that I wanted to be in ... So, I started reading with him and it worked out really well.

Dr. Bennett’s support extended beyond academic assessments to moments of professional and personal challenge. This pattern of empathy and understanding was a recurring theme in their interactions, as he consistently provided Morgan with a safe space to express doubts and seek guidance.

Dr. Bennett was also responsive to Morgan’s emergent professional and academic needs. Whether writing last-minute recommendation letters for conferences and workshops or providing timely feedback, he prioritized her success despite his demanding schedule. Morgan appreciated how he made her feel valued despite his many roles, such as serving as the department chair or being a father to a newborn. She explained, “It doesn’t feel like when I’m with him that he’s this super busy person. He gives me that time of day.” Overall, Dr. Bennett’s support offered a stark contrast to some of the dismissive and condescending attitudes Morgan had previously experienced in academia. By treating her with respect, validating her efforts, and fostering a collaborative relationship, he disrupted masculine discourses that often marginalize women in mathematics (Leyva, 2016). His guidance not only bolstered Morgan’s confidence but also affirmed her place in the field, exemplifying the transformative power of supportive faculty mentorship (e.g., De Welde & Laursen, 2011b).

Cross-Case Analyses of Kayla's and Morgan's (Re)stories

I conducted a cross-case analysis of how Kayla and Morgan engaged and negotiated masculine discourses related to mathematics ability and being a woman in graduate mathematics education modeled on the work of Leyva (2016) and Mendick (2006). Kayla and Morgan encountered these discourses through the discursive practices of qualifying exams and interactions with peers and faculty. I identified the discourses by comparing their stories and lived experiences, paying special attention to how Kayla and Morgan were positioned *and* positioned themselves to exercise power within the mathematics discipline and the departmental context of Spruce University. I share three prevalent discourses that emerged from their narratives: (a) women are positioned as not good at mathematics; (b) women have to constantly prove themselves to others; and (c) supportive and affirming connections are rare yet impactful for women in graduate mathematics education.

Women Are Positioned as Not Good at Mathematics

The pervasive discourse that *women are positioned as not good at mathematics* continues to shape the experiences of women in graduate mathematics programs, influencing how they navigate their academic environments and construct their identities (e.g., De Welde & Laursen, 2011a; Leyva, 2016, 2017; Mendick, 2006). This discourse reinforces stereotypes that undermine women's confidence and positions them as less-than in the male-dominated space of higher education mathematics, often through stereotypes and microaggressions that scrutinize their competence and mathematics ability (Kim & Meister, 2022; McLoughlin, 2005; Yang & Carroll, 2018). Despite their extensive prior successes (e.g., completing undergraduate and graduate degrees in mathematics), Kayla and Morgan still encountered

this discourse primarily through their interpersonal relationships with members of the mathematics community (e.g., faculty, peers, undergraduate students).

Both students cited interactions where others positioned them as not good at mathematics. For example, Morgan was tokenized at Willow University (De Welde & Laursen, 2011a) as her peers suggested that she only received the first-year award because of her identity as a woman – rather than for her mathematical abilities and hard work. This led her to question her competence and wrestle with the false narrative that she cannot succeed in the PhD program. She also felt scrutinized for her mathematics ability by undergraduate students and found herself feeling less intelligent than her male-identifying peers during these moments in the math lab. In Morgan's narratives, interactions with peers and students were the primary discursive practices through which she encountered this discourse.

Kayla primarily encountered this discourse through her interactions with faculty. Dr. Campbell's dismissive behavior, such as refusing to help during office hours and rolling his eyes at her questions, conveyed disregard for her seeking support and undermined her confidence, reinforcing the stereotype that her intellectual contributions were less valid than her male-identifying peers. Kayla linked this experience to broader narratives in higher education mathematics that discourage "dumb questions" and privilege those who meet traditional, masculine-coded standards of excellence (Ataide Pinheiro, 2021; Leyva, 2017). Kayla questioned whether her gender played a role in his condescension, further upholding the gendered power dynamics in their interactions. This experience, compounded by the department's passive acceptance of Dr. Campbell's behavior, led Kayla to doubt her abilities and feel isolated.

Mendick (2006) argued that mathematics is often framed as a domain for the naturally gifted, aligning with cultural archetypes of the solitary, competitive, and rational male genius. Her work highlighted many instances of young girls being positioned as not good at mathematics. Similarly, Leyva (2016) highlighted the counterstories of two Latina undergraduate women and found that the discourse of women not being good at mathematics was exemplified in feeling more disconnected from teachers than their male peers. Kayla's and Morgan's experiences echo these findings, illustrating that women in PhD programs, who have already demonstrated remarkable success in mathematics, continue to face discursive practices that undermine their abilities and position them within a conflicting narrative of success.

Women Have to Constantly Prove Themselves to Others

The masculine discourse that *women have to constantly prove themselves to others* underscores the persistent pressure for women in mathematics to demonstrate their competence and value (Guy & Boards, 2019; Herzig, 2004a). This discourse not only demands sustained high performance but also contributes to feelings of invisibility, as efforts and achievements often go unrecognized or undervalued, and women's access to crucial resources and support seem to be withheld until they have proven themselves. Herzig (2004a) examined the experiences of women in doctoral mathematics programs and found that "some of the women believed that the professors do not pay attention to graduate students until they prove themselves by passing their qualifying exams" (p. 384). Kayla's and Morgan's narratives revealed that their program's structure and norms around progressing through milestones were the primary discursive practices for encountering this discourse. Their experience also highlighted how qualifying exams functioned as a gatekeeping mechanism

that rendered them largely invisible within the department until they proved their legitimacy. This invisibility limited access to mentorship, networking, and professional development, delaying their ability to envision and prepare for faculty careers. As a result, the constant need to prove oneself not only restricted access to opportunities but also imposed an additional emotional burden on women navigating graduate mathematics.

Amplified by the underrepresentation of women in mathematics and the discourse that women are positioned as not good at mathematics, Morgan and Kayla described that they had to perform exceptionally well on the qualifying exams to be seen as intelligent and deserving to be in the program. Kayla critiqued the exam process and expressed disdain towards the norm that despite having successfully completed two degrees in mathematics, “It just feels like you keep having to prove yourself, I guess. It's just like, ‘Well, I still have to keep passing these exams.’” Given this immense pressure and expectation, they felt especially defeated and demoralized when they had to retake exams or only passed at the Master’s level (rather than the PhD level), exhibited by the lowest points of confidence in their graphs. As Mendick (2006) suggested, participating in norms and navigating mathematics spaces often necessitates women to *perform* masculinity to be “successful,” and Kayla and Morgan needed to constantly prove themselves to others in order to “successfully” progress through the PhD program in the eyes of the department, institution, and discipline.

Supportive and Affirming Connections are Rare yet Impactful for Women

Kayla and Morgan understood the underrepresentation of women in doctoral programs (Kessel, 2014; Leyva, 2016; Yang & Carroll, 2018) and their experiences confirmed the discourse that *supportive and affirming connections are rare yet impactful for women*. Both women explicitly shared stories of feeling like outsiders, being different, and

experiencing a poor sense of belonging. Kayla and Morgan connected the underrepresentation of women in mathematics to the ways that their beliefs and values were often dissonant with the masculine values of mathematics (Mendick, 2006). For example, Kayla questioned whether a career as a faculty member would be “compatible with my life choices, or wanting to be a mom and wanting to be married and be there like for my kids as well” – which she perceived as neither privileged nor celebrated in mathematics. As a result, Kayla especially appreciated her relationship with Dr. Davis, with whom she discussed navigating multiple roles and identities, such as being a wife, mother, and mathematics faculty. Kayla felt more connected to the discipline and the profession after connecting with someone whose life priorities resonated with hers. Morgan’s advisor, Dr. Bennett, similarly exemplified how powerful encouragement and affirmations are for students encountering the difficulties of the PhD program. Treating Morgan like a colleague, not talking down to her, and supporting her career pursuits positively contributed to Morgan’s confidence and sense of belonging in mathematics.

De Welde and Laursen (2011a) discussed how the lack of role models contributes to women feeling isolated and disconnected in the “old boys’ club” (p. 577) of STEM fields, further contributing to exclusionary environments and the positioning of women as outsiders. For Kayla and Morgan, meaningful support from faculty and staff stood out as transformative, countering the cultures of competition and skepticism and representing the impact of receiving support from those in positions of power within mathematics departments. Herzig (2004a) noted that these forms of moral support and encouragement from faculty are “the exception, rather than the rule” (p. 387). While the impact of faculty is well recognized (e.g., De Welde & Laursen, 2011b; Guy & Boards, 2019; Herzig, 2004b;

Leyva, 2017), Kayla's and Morgan's experiences with faculty and staff further highlighted how transformative receiving the right type of support during difficult moments can be for women in doctoral programs.

Lastly, affirming peer networks were an important source of support for both students. Morgan mentioned that despite Spruce University having fewer women-identifying graduate students in the program than Willow University, she felt more supported and did not believe that her identity as a woman was used to provide rationale for her successes or hardships. Kayla similarly shared that sentiment, explaining, "I never felt like [my gender identity] was a bad thing or like I was disadvantaged and stuff within the cohort. So, I think that was something that was really positive."

The intersection of their experiences through the AWM chapter also confirmed the value of supportive and affirming spaces for women in graduate mathematics education. Morgan's initiative to establish the AWM chapter and Kayla's meaningful engagement within it exemplified the discourse and illustrated the profound impact of spaces that nurture, protect, and validate women in mathematics (Greenwald et al., 2015). Recognizing that connections amongst women are rare yet impactful, their experiences highlight how women exercise power and build empowering relationships and inclusive communities.

Discussion

Employing an in-depth narrative inquiry and aligning with a poststructuralist perspective of discourses in mathematics (Leyva, 2016; McBride, 1989; Mendick, 2006), I explored how Kayla and Morgan encountered and negotiated masculine discourses by examining how they navigated the "contextual manifestations of the varying power relations" (Leyva, 2016, p. 89). Masculine discourses positioned Kayla and Morgan as outsiders of

mathematics (i.e., as individuals who were seen as less capable of exercising power in this space) through diminishing their sense of belonging, negatively affecting their levels of confidence, and scrutinizing their mathematical abilities and competence. Although the field has identified ways that masculinity operates and is performed in mathematics spaces (e.g., Berry, 2008; Kersey & Voigt, 2020; Leyva, 2016, 2017; Mendick, 2006), this work yielded insights into experiences and features that nuances our understanding of *graduate* mathematics education as a gendered space. Kayla and Morgan encountered and negotiated dominant masculine discourses through three notable discursive practices: the qualifying exam process, relationships with peers, and relationships with faculty and staff. In these discursive practices, they wrestled with the following masculine discourses: (a) *women are positioned as not good at mathematics*, (b) *women have to constantly prove themselves to others*, and (c) *supportive and affirming connections are rare yet impactful*.

First, Kayla and Morgan performed masculine-aligned behaviors, such as proving themselves worthy of resources and support through qualifying exams or adopting a thick skin to navigate challenges and inequities. This echoes the work of Mendick (2006) and Ataide Pinheiro (2021), who illustrated how girls in secondary contexts and women-identifying doctoral students performed masculinity to successfully navigate their mathematics learning. The discourses highlighted above minimized Kayla's and Morgan's ability to exercise power and continued to limit their participation in mathematical practices, previously described as contributing factors to why women leave mathematics doctoral programs (e.g., Herzig, 2004a, 2004b; MacLachlan, 2006; Stage & Maple, 1996). Mathematics graduate education is already a complex experience, with students expected to reconcile multiple identities and roles (e.g., as students, as instructors, as prospective

faculty), and for women, their gender-identities are often in tension with the dominant masculine discourses (Ataide Pinheiro, 2021; Voigt & Reinholz, 2020). Yet, Kayla and Morgan demonstrated resistance to these masculine discourses and exerted power in ways that aligned with their personal beliefs and values. For example, Kayla shared that she wanted to become a mathematics professor to increase the representation of individuals who share her identities (e.g., gender, geographic background, socioeconomic status). Morgan resisted the oppressive masculine discourse that women are not good at mathematics as she exhibited immense perseverance and resilience leaving Willow University and returning to academia to pursue the PhD at Spruce University. Lastly, their intertwined experience with the AWM chapter illustrated their resistance to narratives about women's isolation and disconnectedness in graduate mathematics (Herzig, 2004a, 2004b).

Second, their narratives also illustrate how relational power can be in flux as students engage in different discursive practices (McBride, 1989). For example, for both women, navigating the discursive practice of qualifying exams resulted in feeling defeated and less confident in their mathematical abilities, positioning them as individuals with less power in this space. However, for Morgan, Dr. Bennett's support and encouragement exemplified how faculty, representing individuals in the mathematics community historically able to exercise more power can leverage their positions to enable students to exercise power themselves (e.g., De Welde & Laursen, 2011b; MacLachlan, 2006). As a result of their positive advisor-advisee relationship, Morgan felt empowered to advocate for herself and request support and resources related to her professional development. Similarly, Kayla's relationship with Dr. Davis enabled her to embrace her identities as a first-generation, Latina woman in mathematics as a strength, reinforcing the counternormative discourse that being different is

good in mathematics (e.g., Leyva, 2021; Voigt, 2024). The perspective that discourses shape and maintain social roles and hierarchies of power was a useful lens for analyzing how masculinity operates in graduate mathematics education contexts. Understanding these discourses and their impact is essential for disrupting exclusionary practices and creating a more equitable and inclusive mathematics culture.

Implications and Future Research

The present study sheds light on actionable insights that can disrupt inequities that diminish women's access and participation in graduate mathematics education. First, critically interrogating the qualifying exams process is one avenue for mathematics departments to identify and disrupt the motivations and underpinnings that perpetuate masculine discourses. Prior scholars have highlighted the challenges with this milestone (e.g., Ataide Pinheiro, 2021; Herzig, 2004a; Stage & Maple, 1996), and I found that this experience prompted Kayla and Morgan to perform masculinity to exercise power and combat invisibility related to access to resources. While I do not suggest completely disregarding this milestone, I implore mathematics departments and faculty to assess how norms, such as omitting details about the exam, employing tiered grading practices, and inhibiting research support and mentorship until passing these exams, serve the graduate students, especially women-identifying graduate students. The field has historically valued competition, rigor, individualism, and elitism – evident in how this milestone is traditionally structured and assessed. Drawing on Kayla's and Morgan's experiences, I suggest that being more transparent about exam details, facilitating supportive peer and faculty networks for students in their preparation before and after the exams, and no longer requiring students to pass the qualifying exams before they can access various professional development resources

(e.g., research support) are ways that departments can address the challenges and low self-confidence that students report as they navigate this milestone.

Secondly, Kayla's and Morgan's experiences with faculty highlighted the critical need for mathematics departments to create meaningful professional learning experiences for faculty and staff to better support and mentor graduate students. Their narratives revealed how faculty interactions can either perpetuate harmful discourses or foster affirming and supportive environments. For instance, Morgan's ability to secure departmental support for the AWM chapter demonstrated the potential for faculty to act as allies when they recognize and validate the unique challenges women in mathematics face. Conversely, Kayla's experiences with a lack of recognition of her identities suggest a gap in faculty sensitivity to the diverse and emergent needs of their students. These experiences underscore the importance of professional learning programs that equip faculty and staff with tools to engage in equitable mentorship, foster inclusive departmental cultures, and actively address the systemic barriers that underrepresented students face (Voigt, 2024). By investing in such initiatives, mathematics departments can ensure that faculty are prepared to provide the holistic and adaptive support graduate students need to thrive academically and professionally.

Lastly, this work provides a methodological foundation for continued research on graduate students' experiences and identity development. The narrative inquiry approach, coupled with data sources not commonly utilized in graduate mathematics education research (e.g., the graphs of graduate school) provides a possible starting point for continued research on graduate students' experiences and identities. In addition to its practical implications, this research has significant theoretical value. It contributes to the broader scholarship on gender

and STEM by providing empirical, narrative evidence of how structural and cultural factors intersect to shape women's academic trajectories. This narrative inquiry recognizes that a multitude of masculine discourses influence women's experiences, offering a more nuanced understanding of gender inequity in mathematics (Crenshaw, 1989). Although my focus on gender-identity in the present work allowed for an in-depth analysis of how masculinity may operate in graduate education, by no means was the work intended to generalize Kayla's and Morgan's experiences to all women in graduate mathematics. This work raised new questions about how other salient social and cultural identities may influence how historically underrepresented graduate students navigate graduate mathematics education and engage with discursive practices. Future work can adopt intersectional lenses to examine how other identities (e.g., racial identities, sexual orientation) intersect with dominant discourses in mathematics (e.g., Whiteness and anti-blackness, heteronormativity).

Conclusion

Ultimately, examining the lived experiences and narratives of women graduate students in mathematics is essential for dismantling the inequities that persist within the field. By highlighting the challenges and most importantly, the successes and resilience Kayla and Morgan exhibited, this research not only illuminates the path forward for achieving gender equity but also enriches the mathematical community with diverse voices and perspectives. As institutions and departments strive to create more inclusive learning environments, the insights gained from this research, mainly identifying and disrupting masculine discourses in doctoral programs, will be invaluable in guiding their efforts and ensuring that mathematics is a field where all students can thrive.

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CHAPTER FOUR

Dimensions of Mathematics Doctoral Students' Mental Models of Mathematics Teaching

Graduate education is a formative period when students are introduced to the multifaceted roles of university teaching, yet many enter these roles with limited formal pedagogical training (Ellis et al., 2016; Oleson & Hora, 2014). In mathematics, many graduate students begin teaching as graduate teaching assistants (TAs), with this experience often being their first exposure to formal teaching roles (Friedberg, 2005). Beisiegel and Simmt (2012) noted, “After all, it is through the formative experiences of graduate school that mathematics graduate students’ identities are shaped as mathematicians, researchers, teachers, or more generally as professors of mathematics” (p. 35). For aspiring faculty, these critical experiences, such as serving as TAs, play a crucial role in shaping their instructional practices and professional identities (Deshler et al., 2015; Musgrave & Carlson, 2017; Olarte, Swartz et al., 2024). Given that students’ experiences can establish lasting patterns that influence instructional practices beyond graduate school, understanding how graduate students make sense of undergraduate mathematics teaching can position departments to better support their development as future educators.

To examine this sensemaking, I conceptualize graduate students’ understanding of mathematics teaching through the lens of multidimensional mental models (Genter & Stevens, 1983; Jones et al., 2011). Mental models reflect how “individuals’ cognition shapes their understanding of a particular issue, concept, or event” (Holcombe & Kezar, 2018, p. 93). In the context of undergraduate mathematics teaching, these internal representations of teaching and learning can shape classroom decisions, teaching priorities, and engagement with students (Manrique & Abchi, 2015; Mesa et al., 2014; Speer, 2008). I extend mental

models as a construct to mathematics graduate students by framing their mental models of undergraduate mathematics teaching as multidimensional configurations of beliefs, values, prior experiences, and disciplinary norms that together shape how they interpret and enact instructional practices (Beisiegel & Simmt, 2012; Musgrave & Carlson, 2017).

Importantly, mental models of teaching are not static constructs (Greca & Moreira, 2000). Graduate students come to graduate school with preexisting conceptions of teaching and learning that inform how they approach their early graduate teaching experiences *and* how they continue to learn about pedagogy (e.g., Beisiegel et al., 2019; Speer et al., 2005). As they engage with and navigate a variety of teaching roles and positions, these models both guide their pedagogical decisions and are simultaneously reshaped through experience. As such, mental models function as dynamic and evolving structures that bridge cognition and practice (Greca & Moreira, 2000; Jones et al., 2011). Examining graduate students' reflections on their teaching therefore serves as a window into the underlying conceptions that inform their teaching practices while also shedding light on how these models may have been shaped over time, particularly during graduate school.

Undergraduate mathematics teaching is a central component of the professional lives of mathematics professors (Clarke et al., 2013; Mesa et al., 2014). By focusing on how graduate students reflect on their teaching practices and underlying pedagogical conceptions, this work contributes to research on graduate teaching preparation and faculty development, as it illuminates beliefs and experiences that structure mathematics graduate students' understanding of their teaching. Investigating these multidimensional mental models of mathematics teaching offers insights into the knowledge and resources students bring to their instructional roles and highlights how departments can scaffold their development as

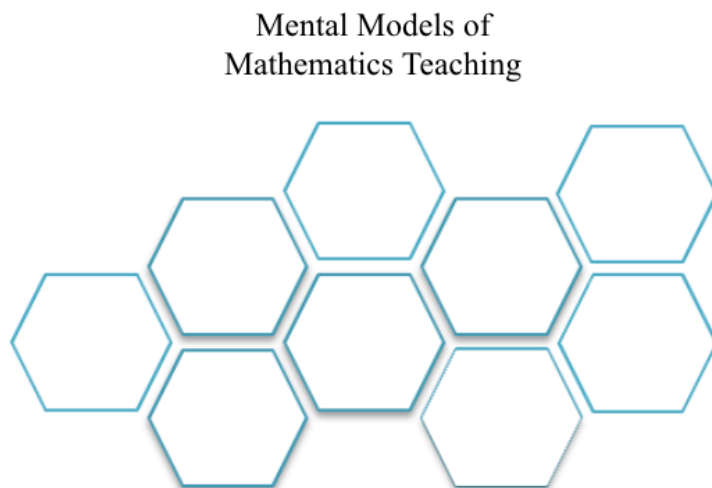
teachers, both during graduate school and as they transition into faculty positions. The research questions that guided this study were: (1) What experiences and beliefs most structured mathematics graduate students' mental models of mathematics teaching?; and (2) In what ways did these dimensions inform their current instructional approaches?

Conceptual Framing and Background Literature

This study is guided by perspectives that conceptualize graduate students' mental models of undergraduate mathematics teaching as dynamic and multidimensional (see Figure 8), encompassing beliefs, values, and prior experiences that together undergird their instructional approaches and teaching commitments. Aligning with prior characterizations of mental models (e.g., Jones et al., 2011), I view mental models of teaching as flexible, evolving structures that both guide graduate students' pedagogical decision-making and are, in turn, reshaped through their varied teaching experiences.

Figure 8

A Multidimensional Representation of Mental Models of Mathematics Teaching



Note. The figure is my representation of the multidimensional nature of mental models.

By inviting participants to reflect on their teaching practices and commitments, I used students' reflections as a lens into the underlying mental models that inform, and are continually informed by, their experiences in the classroom. Below, I discuss the pertinent perspectives of mental models – mainly *what* they are and *how* they are formed – that guided my exploration of the dimensions that configure graduate students' mental models of undergraduate mathematics teaching.

What Are Mental Models?

From the perspective of cognitive psychology and development, mental models are complex, socially constructed structures that form the basis of one's reasoning and decision-making (Jones et al., 2011). Johnson-Laird (1983) and Genter and Stevens (1983) helped conceptualize mental models by offering perspectives regarding how individuals form mental models, the nature of these cognitive representations, and how mental models guide real-world actions. Johnson-Laird described mental models as dynamic, analogical representations of reality that individuals use to interpret the world around them. These working models do not have to be complete; rather, they are continually constructed and revised as individuals encounter new information and experiences. Building on this, Genter and Stevens focused on the functionality of mental models and conceptualized them as tools for explaining and predicting phenomena in the world. Taken together, these perspectives illustrate both the structure of mental models (i.e., multidimensional, evolving, internal representations) and their function (i.e., sensemaking and reasoning about the world).

Throughout the literature, mental models have been used synonymously with terms such as belief systems, cognitive maps, and most often, schema (e.g., Ernest, 1989) and result from one's "own perceptive and manipulative experience with this world" (Greca and

Moreira, 2000, p. 3). In this sense, everyday experiences, such as graduate students engaging in teaching, shape their mental models. Accordingly, mental models are also often understood to be qualitative, incomplete, and continually evolving as individuals interact with and make meaning of their experiences. I considered Jones et al.'s (2011) description of mental models particularly fitting for the phenomenon of undergraduate mathematics teaching. They wrote,

Mental models are personal, internal representations of external reality that people use to interact with the world around them. They are constructed by individuals based on their unique life experiences, perceptions, and understandings of the world. Mental models are used to reason and make decisions and can be the basis of individual behaviors. (Jones et al., 2011, p. 1)

Jones et al.'s perspective clarifies the connection between cognition and practice and can be situated within undergraduate mathematics teaching as a perspective for examining graduate students' reflections on their teaching experiences and their understandings of mathematics teaching and student learning. For mathematics graduate students, mental models of teaching are socially constructed representations shaped by their experiences, perceptions, and conceptions of what it means to teach mathematics. These models function as cognitive frameworks that guide instructional decision-making, shape how pedagogical knowledge is interpreted, and mediate the adoption or resistance of new teaching approaches.

The Construction of Mental Models of Teaching

While mental models are inherently personal and unique to the individual, existing research has identified recurring themes in how individuals learn to teach across various educational contexts (e.g., Friedberg, 2005; Hill et al., 2014; Jensen, 2011), offering insights into the complex processes through which mental models develop. Bransford et al. (2000)

theorized that teachers learn not only from their own practice but also through a combination of formal and informal experiences. In higher education, informal learning can occur through faculty mentorship, peer discussions, and self-reflection on teaching experiences, while formal learning may take place in pedagogical workshops, teaching seminars, or structured professional development programs (Manrique & Abchi, 2015; Oleson & Hora, 2014). Graduate students navigate these diverse learning contexts, filtering new insights through their existing mental models and disciplinary beliefs. However, learning how to teach does not necessarily equate to changing one's mental model; individuals may incorporate new pedagogical knowledge, modify their existing models, or reject new instructional approaches that conflict with their prior beliefs (Greca & Moreira, 2000).

Institutional contexts also play a critical role in shaping graduate students' mental models of teaching. Research on higher education faculty development suggests that graduate students' interactions with faculty (e.g., through advising and observation), their early teaching experiences (e.g., teaching assistantships), and their interactions with the broader institutional climate significantly influence the formation of these models (Manrique & Abchi, 2015). Faculty's modeling of teaching practices, along with departmental norms and institutional expectations of graduate students, influence how they make sense of their teaching roles and responsibilities. Nasser-Abu Alhija and Fresko (2020, 2021) explored how departmental and institutional norms often reinforce power hierarchies that position graduate students with limited agency in the classroom, and Slack and Pownall (2023) characterized this experience as graduate students being treated as "spare parts" (p. 1272) within undergraduate mathematics education contexts. This constrained sense of ownership over their teaching can diminish the extent to which graduate students see themselves as

legitimate instructors. Taken together, these institutional conditions play a role in how graduate students develop and refine their mental models of teaching.

Beliefs about the discipline and subject matter can further influence graduate students' mental models of teaching (Beisiegel & Simmt, 2012; Friedberg, 2005; Leinhardt & Smith, 1985). Oleson and Hora (2014) referred to the disciplinary context as the “primary cultural unit” (p. 42) in which STEM faculty are socialized into higher education professions and develop their pedagogical practices. Specifically in mathematics, Ernest (1989) identified three perspectives on the nature of mathematics that influence instructional approaches: (a) the instrumentalist view, which emphasizes facts, rules, and procedures; (b) the Platonist view, which sees mathematics as a static body of knowledge to be discovered rather than created; and (c) the problem-solving view, which conceptualizes mathematics as a dynamic and evolving cultural product. These epistemological perspectives are not just abstract beliefs but actively informed how instructors approached teaching. Instructors who view mathematics as an authoritative, rigid discipline may adopt pedagogical practices that prioritize procedural rigor over conceptual exploration. In contrast, those who embrace a problem-solving perspective may be more inclined to implement inquiry-based or student-centered approaches (Beisiegel & Simmt, 2012; Mesa et al., 2014).

In previous work, I also explored how cultures and norms associated with mathematics influenced how graduate students made sense of their approaches to teaching, suggesting a salient dimension of their mental models. Olarte, Swartz et al. (2024) found that graduate students' perceptions of mathematics and specific courses influenced their instructional choices, like adapting their mentoring practices to meet the unique needs of students in introductory proof courses who are continuing on to pursue more abstract

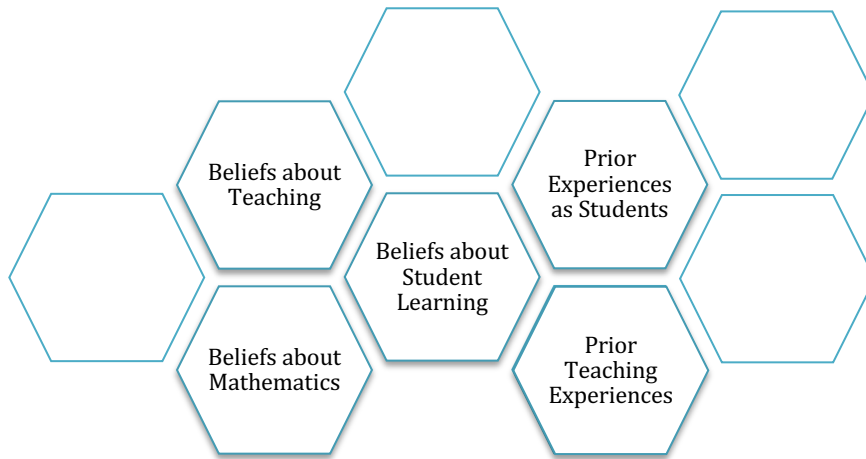
mathematics courses. Olarte and Street (2025) continued to explore discipline-specific influences by illuminating how women-identifying graduate students' values about undergraduate mathematics teaching and learning were often in tension with dominant narratives (e.g., math is individual, math is objective). Across these bodies of work, I found that graduate students were deeply aware of discipline-specific norms and narratives and structured their teaching approaches in response to these norms and narratives.

Connecting Prior Perspectives on Mental Models

In all, I consider the varied experiences and features of graduate education—encompassing prior student and teaching experiences, institutional influences, and disciplinary beliefs—to contribute to the evolution of graduate students' mental models of teaching. Over time, these models are refined as students engage in teaching, interact with others, and confront new pedagogical knowledge. However, these refinements are not always additive; graduate students may interpret new experiences in ways that reinforce their existing models or resist perspectives that challenge their held beliefs about mathematics teaching and learning (Greca & Moreira, 2000). I examine graduate students' mental models of mathematics teaching through five key dimensions (see Figure 9): (1) beliefs about student learning, (2) beliefs about teaching, (3) beliefs about mathematics, (4) prior experiences as students, and (5) prior teaching experiences.

Figure 9

Dimensions of Graduate Students' Mental Models of Mathematics Teaching



Note. The figure depicts the dimensions of mental models that will be discussed in the paper.

These dimensions reflect the alignment between established scholarship on mental models (e.g. Jones et al., 2011) and mathematics graduate students' teaching approaches (e.g., Beisiegel et al., 2019; Speer et al., 2005), and the emergent themes from participants' reflections. Although the present study focuses on five dimensions, I recognize the complexity of mental models and acknowledge that other dimensions of experiences (e.g., early mathematics experiences, experiences outside academia) also constitute their mental models of teaching (i.e., visually represented by the blank dimensions in Figure 9).

Method

Guided by conceptualizations of mental models as tools for sensemaking and decision-making, I employed a thematic analysis (Miles et al., 2020; Creswell & Poth, 2016)

of participants' reflections on their reported teaching approaches to identify and describe various dimensions of their mental models of undergraduate mathematics teaching.

Participants

Purposeful sampling (Miles et al., 2020) was used to recruit doctoral students from the mathematics department at Spruce University who reported they were actively pursuing mathematics faculty positions. Six doctoral students agreed to participate. See Table 4 for a more detailed profile of each participant.

Table 4

Participant Profiles and Teaching Experience

Participant	Year in Program	Race/Ethnicity	Gender	Prior Postsecondary Teaching Experience
Connor	3 rd	White/Caucasian	Male/Masculine	Teaching Assistant
Kayla	1 st	Latinx/Chicanx	Female/Feminine	Teaching Assistant, Graduate Instructor of Record
Kendall	5 th	White/Caucasian	Female/Feminine	Teaching Assistant, Graduate Instructor of Record
Landon	2 nd	White/Caucasian	Male/Masculine	Teaching Assistant
Martin	1 st	Latinx/Chicanx	Genderqueer/ Genderfluid	Teaching Assistant
Morgan	5 th	White/Caucasian	Female/Feminine	Teaching Assistant, Graduate Instructor of Record, Adjunct Instructor

Note. Demographics are self-reported.

The Instructional Context of Spruce University

Understanding the institutional and departmental norms is essential for contextualizing participants' reported teaching practices and developing mental models. This study was conducted at a large research-intensive university on the West Coast of the United States called Spruce University (a pseudonym). Spruce University approximately enrolls

2,900-3,000 graduate students and during the data collection year of 2023-2024, the mathematics department had 74 graduate students.

At Spruce University, all participants served as TAs. Notably, Kendall and Morgan also had prior experience teaching as instructors of record and adjunct faculty, respectively. In the mathematics department at Spruce University, graduate TAs typically lead recitation sections under the supervision of a faculty instructor. Rather than introducing content, their role is to reinforce concepts covered in instructor-led lectures through supplemental activities. On the other hand, instructors of record oversee course content, instructional activities, and pacing. While some faculty instructors allow TAs to design activities if they align with the course, others provide strict guidelines, prescribing the materials and plans for facilitating section. As a result, TAs may navigate inconsistent expectations and varying levels of agency (Brownell & Tanner, 2012), shaping how they enact instructional approaches. Situated within these norms, this study explores how graduate students reflect on their teaching practices, primarily as TAs, and posits how these practices reflect their underlying mental models of undergraduate mathematics teaching.

Data Collection

Participants took part in 90-minute, semi-structured interviews (Rubin & Rubin, 2011) over Zoom about their graduate education experiences and understanding of mathematics teaching. Sections of the interview protocol were specifically designed to prompt participants to reflect on their instructional approaches, how they learned to teach, and their broader understanding of the roles and characteristics of successful mathematics professors. This reflected the theoretical connection between pedagogical decision-making and mental models of teaching (e.g., Holcombe & Kezar, 2018; Manrique & Abchi, 2015).

Some of the interview questions included: (1) Reflecting on your teaching experiences, how would you describe your approaches to teaching?; (2) How do you think students best learn?; (3) What most shaped your approaches to teaching?; and (4) Describe the characteristics of a successful undergraduate mathematics instructor. I then transcribed the interviews, and these transcripts served as the primary data source for this study (Miles et al., 2020).

Data Analysis

Each turn at talk of the interview transcripts served as a unit of analysis (Miles et al., 2020) because responses during the semi-structured interview represented sequential understanding of ideas emerging from conversation. The graduate student participants and I, as the interviewer, constructed meanings moment-by-moment, so each turn at talk serves as a meaningful contribution and block of dialogue. I analyzed the data utilizing a combination of a priori and open coding (Saldaña, 2013) to explore how participants reflected on their instructional practices and the dimensions of their mental models of mathematics teaching. I was sensitive to existing literature, like Oleson and Hora's (2014) study, which examined the formative experiences that shaped faculty members' teaching, as well as my prior work on TAs' beliefs about mathematics teaching and learning (Olarte, Swartz et al., 2024; Olarte & Street, 2025). These studies provided a starting point for interpreting participants' mental models of teaching while allowing space for emergent ideas to surface during the process of coding.

I engaged in Saldaña's (2013) approach of inductive coding to examine how participants described how they learned to teach and to illuminate the dimensions of their mental models most salient to their current instructional approaches. My goal was to identify the beliefs, values, and prior experiences most influential to participants' understanding and

conceptions of mathematics teaching. I employed Saldaña's method of pre-coding, first-cycle, and second-cycle coding the interview transcripts. I engaged in pre-coding by reading through each transcript and identifying any instances that participants articulated beliefs, values, and experiences related to mathematics teaching and student learning. I was especially cognizant of participants suggesting relationships between these beliefs, values, and experiences to reported instructional practices. For example, when prompted to share what informed their student-centered approaches to mathematics teaching, many cited their experiences as undergraduate students and noticing their own professors' pedagogical approaches that they wanted to mirror or disrupt.

The first-cycle coding process (Saldaña, 2013) involved reading through the transcripts again and reconciling the codes and initial dimensions of mental models that emerged during pre-coding. For example, in the pre-coding, I had identified experiences such as serving as TAs, reviewing end-of-quarter course evaluations from students, and engaging with students during office hours. Then, in the first cycle, I consolidated these into the code *prior teaching experiences*. I developed five codes to represent the multidimensionality of graduate students' mental models: *beliefs about student learning*, *beliefs about teaching*, *beliefs about mathematics*, *prior experiences as students*, and *prior teaching experiences*.

My second-cycle coding process (Saldaña, 2013) involved using the first-cycle codes to build themes and identify patterns across participants' dimensions of mental models related to teaching and learning mathematics. For example, within the dimension of prior teaching experiences, I developed themes surrounding perceived success and positive feedback from students, to learning from prior instructional challenges. Additionally, it was during the second cycle that I identified another critical dimension: *awareness of discourses*

in mathematics, which I unpack in the Discussion section. Throughout this analysis process, I wrote notes and memos (Miles et al., 2020) capturing the progression of the themes and ideas. My analytic memos focused on describing the multidimensional nature of mental models and the ways that participants linked these understandings to their current instructional approaches.

Findings

I found that the dimensions and experiences that emerged as most central to graduate students' conceptions of mathematics teaching were: (a) beliefs about student learning, (b) beliefs about teaching, (c) beliefs about mathematics, (d) prior experiences as students, and (e) prior teaching experiences. These beliefs and critical experiences informed how they approached mathematics instruction, engaged with students, and made pedagogical decisions – reflective of how prior scholars have conceptualized mental models to bridge cognition and practice (e.g., Bransford et al., 2000; Jones et al., 2011). The following sections explore the dimensions of the graduate students' mental models in more depth, unpacking how the participants' beliefs, disciplinary orientations, and experiences informed their evolving instructional practices and understanding of undergraduate mathematics teaching.

Beliefs About Student Learning

The first key dimension is graduate students' beliefs about student learning in mathematics. Participants varied in their views of learning, with some viewing learning as a process of acquiring and mastering procedures (Ernest, 1989), while others emphasized conceptual understanding and sense-making (Mesa et al., 2014). Their perspectives on student learning often reflected their own experiences as learners and the instructional norms

they encountered in their own mathematics learning. Below, I highlight themes in participants' beliefs about how students best learn and barriers to student learning.

How Students Best Learn

Five of the six participants explicitly stated that collaborations and working together was how students best learned mathematics, leading them to incorporate group work and to facilitate opportunities for students to share and explain their thinking to one another. For example, Kendall said, "I do think math classes need more group work, because I think it's super important for people to explain how they think about something and compare that to how other people think about it." Similarly, Morgan shared, "I think it's when you work together, and kind of talk about it with your peers, that's probably when you learn the most." Kendall's and Morgan's beliefs that collaborations were an important feature of learning motivated them to encourage students to work together and to make sense of the mathematics with their peers.

Connor, Martin, and Landon all described assigning students many problems during section, working through examples together, and breaking down the procedures, because they believed that students learn best through frequent problem-solving efforts. Martin shared,

I think doing the problems, I think they'd best learn like that, by being hands-on with the material, because you can sit there and you can read a book, or you can sit there and take notes, but you're not engaging with it, then you're not making that connection in your brain, like, how to do it.

Martin believed that actively engaging and making sense of the mathematics supported student learning more than taking notes or passively sitting through lectures. Alternatively, Connor explained, "I imagine that for some of them, they would learn better if I just walked

through the theory and walked through the entire class doing – this is the theory, this is step 1, 2, 3, and then we're done.” This belief in a more procedural focus was exemplified in Connor’s adoption of more traditional instructional approaches (McMinn et al., 2022; Mesa et al., 2014), as he attempted to work through as many problems with his students during section, because he believed that this approach would bridge students’ understanding of theories and associated procedures. These examples illustrate nuances in beliefs about student learning that influenced how participants facilitated their sections.

Barriers to Student Learning

Participants also shared beliefs about features of instruction that may hinder students’ learning – leading them to align with practices that combatted these learning barriers. For example, a few participants believed that students should not be passive participants in the classroom and that sitting through long lectures does not support their learning. Kayla commented, “I definitely don't think students best learn when it's just 50-minutes of a professor just writing on the board going off.” Moreover, in opposition to undergraduate students often passively sitting through sections or lectures, some participants believed that students needed to engage in “hands-on” work with the material. Thus, encouraging group work or assigning many problems during section was a common feature in how participants described their sections.

Beliefs About Teaching

Closely linked to their views on student learning, participants’ beliefs about teaching were also reflected in their instructional choices, classroom presence, and pedagogical priorities. Some participants conceptualized teaching primarily as delivering content, ensuring clarity and efficiency in explanations, while others saw it as supporting students in

ways that extended beyond the mathematics content. At times, participants' beliefs reflected broader disciplinary norms in mathematics, such as their perceived importance of rigor and logical precision, but often, participants described beliefs about teaching that pushed back against hegemonic discourses in mathematics (Ataide Pinheiro, 2021; Gutiérrez, 2017; Herzig, 2004; Olarte & Street, 2025), such as the historic characterization of mathematics professors as apathetic, detached, and distant.

Classroom Presence and Engagement with Students

First, participants expressed beliefs about instructors' classroom presence, as it related to supporting students' mathematical learning. For example, a few participants believed that instructors need to be engaging and enthusiastic, as opposed to being perceived as monotone or disinterested, because this would help students learn the mathematics content. They believed that students' perception of their instructors significantly affected students' connection to the class and their overall learning process (Olarte, Swartz et al., 2023).

Landon explained,

[Having an] energetic [presence] is a big one. Maybe the most important thing ... the students sometimes, subconsciously, can tell if it seems like you [the TA] want to be there or whether you hate being in section just as much as them and you're just waiting for it to be over. I feel like when the students get the feeling that you want to be there and you're excited to talk to them. I think that's the most important trait.

For Landon, his commitment to presenting as energetic and invested in the course was guided by his belief about the importance of a positive classroom presence.

Participants also associated effective undergraduate mathematics instruction with traits like approachability and personability. Many expressed a desire to be instructors who

students felt comfortable approaching for both mathematical and non-mathematical support. Morgan emphasized the importance of being available to students and fostering an environment where they felt empowered to seek help. Similarly, Kayla described teaching as not only delivering content but also mentoring students and building connections. She explained that “having an actual personality is nice to have in a professor versus just like coming in every day and all I know about you is you do math.” She believed that sharing some details about her life made her classrooms friendlier and more inviting. Kayla expanded on her beliefs and priorities saying,

Definitely trying to make mathematics a more welcoming community to a lot of people and just kind of breaking the stereotype of like, “Oh, you have to be like this math person or you have to be super scared of your professors or messing up.” I’ve had professors get upset because you don’t know something. And I’m like, yeah, no, I’m not going to be that kind of person.

Kayla, like other participants, saw their teaching as disruptive to patterns of negative instruction they experienced and observed. This was further reflected in their commitment to humanizing mathematics instruction (Gutiérrez, 2016, 2017) and being kind, supportive, compassionate, and nurturing of all students – often described as a higher priority than being skilled at explaining or breaking down the mathematics content.

Responsive to Students’ Varied Experiences

Participants’ beliefs about teaching reflected their sensemaking efforts of the varied ways students experience undergraduate mathematics. They sought to demonstrate support and care for students through exercising compassion, being understanding and empathetic of students’ lives, and serving as emotional guides. Connor noted, “Having a lot of compassion

and understanding of what students are actually going through is pretty necessary,” while Kayla similarly believed that “having compassion for students just as people is a pretty big one.”

The graduate students also personally understood their students’ experiences with learning more abstract mathematical content. Several connected the need for compassion to these specific experiences and challenges, suggesting that instructors must be especially supportive and encouraging when students encounter abstract mathematics for the first time. For example, Martin emphasized the importance of patience, explaining that students are “probably seeing this material for the first time. So, you need to be patient with them ... That’s really important to be able to take the time and guide them through the stuff.” Similarly, Kendall acknowledged the difficulties students face in their mathematics courses and viewed encouragement as a necessary form of support. She noted,

A lot of students get disheartened. You see so many success stories. You see so many kids who fail both midterms and then pass the final and with flying colors and you’re like, “I’ve seen it happen. You can do this.” I think a lot of students don’t necessarily have all these or their friends don’t tell them if they’re going through the same thing. The kids feel so alone, and I think it’s part of our job as an instructor or a TA, especially as a TA, because they feel more comfortable with us, to be there when they need a mentor and then to remind them that they can still get this stuff and they can still learn all they need to learn in whatever amount of time they have left.

Participants deeply empathized with students’ struggles and adopted practices that attended to a multitude of experiences, like learning abstract mathematics for the first time or feeling alone during their undergraduate journeys.

Disrupting Negative Narratives and Discourses

Lastly, participants believed that mathematics teaching was an avenue for combatting long-held narratives about mathematicians and “math persons.” This was linked to teaching practices that aimed to rehumanize mathematics and push back against narratives of mathematics being abstract, objective, nonpersonal, and detached from reality (Herzig, 2004b, Mendick, 2006). Kayla described her teaching commitment as, “trying to reconfigure the narrative of mathematicians, being these godlike people that you can’t talk to and just trying to let them know I am as normal as I could get as a person.” She sought to demonstrate a different, more relatable, mathematical experience. She expanded on the sentiment above saying,

And then I would just hate for students to come in feeling like there aren’t other people that are like them. So that’s what dissuades them from it. And it’s the math person thing, like thinking of math people as these gods. So, it’s like, “Oh, you’re smart because you know math.” It’s like, well, you could be a humanities person and be just as smart, if not smarter. So, I think that’s been kind of hard.

Similarly critiquing the elitism and competitive values associated with mathematics, Morgan felt that many mathematics professors only “teach to the top one percent of the class.” She added, “I think that a lot of math professors do that, and I think that it makes students feel dumb.” Like Kayla, Morgan considered how her teaching practices can reframe perceptions about mathematics ability and can prevent students from feeling “dumb” in class. These conceptions influenced their adoption of student-centered approaches (Mesa et al., 2014) to support students' learning processes.

Beliefs About Mathematics

The next dimension of graduate students' mental models of teaching are their beliefs about mathematics and what counts as mathematical knowledge. Some participants viewed mathematics as an abstract body of knowledge consisting of definitions, theorems, and proofs (Ernest, 1989) – often reflected in emphasizing procedures and prioritizing showing students examples of how definitions and theorems are applied (Mesa et al., 2014). Others saw the discipline as a more dynamic and creative discipline centered on collaborative problem-solving and sense-making (Olarie & Street, 2025). These beliefs influenced how they presented mathematical content, the types of explanations they provided during sections, and the ways they engaged students in mathematical activities.

Participants criticized how mathematics is often presented in abstract ways and connections to real world contexts and applications were not emphasized enough. Kayla explained, “With the math, it's hard. You can't really give a real-world application. Sometimes it's like, ‘Nope, this is just math, we don't use it for anything.’” She described challenges with communicating practical significance to her students, which may have negatively impacted their learning. Similarly, Morgan critiqued the traditional conceptions of the mathematics as constituted by “definition, theorem, definition, theorem.” For both participants, this belief seemed to underscore their efforts to engage students in mathematical problems situated in the real world to better support their connection-making.

Another notable belief about mathematics was related to what the field has historically considered knowledge and how it has traditionally assessed this knowledge. Kendall and Connor believed that mathematics has emphasized “objective” assessments and grades – highlighting their beliefs about mathematical knowledge. Kendall believed that the

overemphasis of grades in undergraduate mathematics education negatively impacts student learning. She said, “Grades. This whole obsession with, ‘Oh, I have a 93, but I really needed 94 to get the A.’ ... You're thinking about how many points, and that's just terrible. We need a better system.” She described instances when she reminded students about the value of making sense of mathematics without the goal of simply getting points. Expanding on assessment practices in undergraduate mathematics, Connor believed that objective assessments often fail to recognize the students’ breadth of knowledge and experiences. He noted,

Especially in math, it's easy to lean into the trap of objective assessments, just because we have this very objective metric. It's tempting to use [such assessments], but I think it's important to be able to understand that not everyone has the same background or the same experiences. And it's [students’ varied backgrounds] something that should be accounted for and not just harshly penalized on assessments.

Connor recognized that traditional forms of objective assessments have been prioritized in mathematics, and this has often penalized students in unfair ways. He valued multiple ways of knowing and doing mathematics, which is contrast to how the field has historically defined and assessed mathematical knowledge (Mendick, 2006).

Kayla also reflected on how mathematical knowledge and ability are defined, critiquing how the field has historically celebrated so-called “math gods.” By valuing a specific way of knowing and doing mathematics, the discipline elevates certain individuals, reinforcing an exclusionary culture. This, in turn, shapes narratives about who is considered successful in mathematics. For students who do not fit the image of naturally gifted in

mathematics, these perceptions can be discouraging and detrimental to their learning. She shared,

I would just hate for students to come in feeling like there aren't other people that are like them. So that's what dissuades them from it. And it's the math person thing, like thinking of math people as these gods. So, it's like, "Oh, you're smart because you know math." It's like, well, you could be a humanities person and be just as smart, if not smarter. So, I think that's been kind of hard.

To combat such detrimental student views, she implemented practices that encouraged and fostered positive relationships with students – such as getting to know students as people and being open about her own identities and background to encourage students, especially those who resonated with her lived experiences. Participants' views about mathematics affected their expectations of students' abilities, their interpretations of what constitutes mathematical understanding, and their desire to combat narratives and values historically associated with mathematics.

Prior Experiences as Students

While participants' mental models were strongly shaped by their beliefs about teaching, learning, and mathematics, their accounts also revealed the central role of lived experiences as a distinct, yet interconnected, dimension. As participants reflected on their instructional approaches, they consistently drew on their own histories as students, suggesting that these experiences not only informed but actively structured how they made sense of teaching. Kendall articulated this dimension clearly, explaining,

I try to take an approach that I am the student, like trying to be in their shoes. I try to remember what I was like as a student, what I wanted from a class. I know exactly what that feels like.

Reflecting on Their Own Mathematical Learning

Across interviews, participants reflected on both positive and negative experiences in courses, instructors, and learning environments when describing the underpinnings of their current teaching practices. For example, Kayla noted, “I have learned best from lecture but other times, I feel like I learned better when I’m just reading on my own from my book.” She further explained that lectures were most impactful when professors provided “expert exposition” at the beginning of class. These experiences informed her current instructional approach, including adopting practices aligned with meaning-making approaches (Mesa et al., 2014), such as connecting mathematical content to real-world contexts to enhance its relatability. Martin similarly reflected on their most effective learning experiences, emphasizing the importance of visuals, worked procedures, and multiple examples. Their current approach of working through numerous examples during sections was shaped in contrast to prior experiences with instructors who focused primarily on presenting definitions or theorems without application. As Martin explained, “I’ve had that experience where it was like, ‘Well, here’s the theorem or whatever, do the problem.’ And I’m like, ‘Well, that’s not helpful.’ So, I try to do a lot of examples.”

Modeling Their Teaching After Prior Professors

A shared storyline across participants’ experiences as students centered on professors who intentionally reframed the goals of learning and cultivated a genuine interest in mathematics. Participants recalled multiple instances in which their professors supported

their own mathematical sensemaking, often by shifting the focus away from performance and toward understanding and curiosity. These experiences were not only memorable but also influential in shaping how participants came to think about teaching. For example, Connor described how some of his former professors explicitly communicated that grades were not the primary priority of the course, a message that has since fundamentally shaped his own instructional approach. In addition to de-emphasizing grades, this professor created space for Connor to explore mathematics beyond the prescribed course content, taking time to engage with his questions and nurture his curiosity. Such experiences positioned mathematics as something to be explored rather than simply mastered, and they continue to inform how Connor, and other participants, envision fostering similar environments in their own classrooms. Connor reflected,

This professor was very open and made very clear to us that grades were not the object, it was learning. He just made sure to take the time to teach not only what we needed to learn, but way more than [that] ... [the professor] would always be open to have conversations after class. I talked with him for more than an hour after class sometimes, about additional math topics. If I didn't have that, I don't know if I would like math as much as I do now. Having that as my initial base on which I would try to emulate has pushed me towards the style that I try to adopt now.

Connor identified a memorable experience as a student that informed his mental model of mathematics teaching. He mirrored his instructional approaches after the practices of this professor, who cultivated spaces where Connor felt comfortable asking questions and exploring new mathematics topics. Landon similarly described instances where highly relational dynamics between professors and students made the learning environment more

engaging. He said, “I’ve definitely had some classes like that, where people are asking lots of questions, a lot of good back and forth with the professor and it feels like everyone’s learning and having a good time.” For many participants, their current instructional practices reflected professors who supported their mathematical sensemaking through practices like encouraging questions, being personable, and creating learning environments where students feel comfortable engaging with each other.

Experiencing Humanizing Mathematics Instruction

Lastly, participants reflected on how their professors fostered a sense of safety, comfort, and care that extended beyond the mathematics content, practices aligned with student-support approaches (McMinn et al., 2022; Mesa et al., 2014). They consistently valued instructors who were personable, approachable, and attentive to students’ lives both within and outside the classroom. For many, these relational dimensions of teaching were central to their learning experiences and shaped how they now conceptualized effective instruction. Kayla, in particular, resonated with professors with whom she felt a personal connection, noting that their willingness to share aspects of their lives and to get to know students helped challenge dominant narratives about mathematicians as distant or impersonal. She explained:

I think that when I reflect on just who my teachers have been and who my professors have been, it’s the ones that I’m most comfortable with or that I know more about as a human being. [They] are the ones that I’ve learned the best from versus being like closed off with my personal life and stuff.

These prior experiences as undergraduate students enabled participants to more deeply understand their own students’ needs and informed how they enacted supportive teaching

practices. Examining how graduate students draw on and reinterpret these experiences highlights how their mental models of teaching are shaped not only by beliefs, but also by the pedagogical and relational practices they encountered as undergraduate mathematics students themselves.

Prior Teaching Experiences

Lastly, while prior experiences as students informed graduate students' mental models of teaching, their teaching experiences further refined, reinforced, or challenged these conceptions. As participants navigated the complexities of serving as TAs or instructors of record, they encountered real-time feedback, student successes and struggles, and external expectations that shaped their evolving perspectives. Additionally, with all participants serving as TAs during the time of this study, they shared experiences related to this role that shaped their understanding of their teaching – such as pedagogical learning through TA training and reviewing the course evaluations that their students completed at the end of each quarter. I considered these experiences as aspects of their collective prior teaching experiences and share how these informed participants' conceptions of undergraduate mathematics teaching.

Affirming Teaching Moments

First, participants' mental models of teaching were reinforced when their students positively responded to their instructional decisions. Some of their most memorable teaching moments were noticing students expressing motivation to learn and feeling connection to the content. Kendall recalled, "The most enjoyable thing was probably the students. I had a really great group of students. They wanted help, they wanted to learn, they wanted to do well. They were motivated." She connected this experience to her current instructional

practices that aimed to support students to ask questions, and continue to foster genuine interest. Similarly, Landon enjoyed the moments when he felt that he helped students understand difficult concepts. He shared,

I think when you can see that something that they clearly didn't understand before clicks for them, and they're able to really understand it and connect it to something else or they make a comment that makes it clear that now they do understand what's going on here. That's really rewarding.

Moments when graduate students felt like they meaningfully advanced students' learning reinforced their instructional approaches and likely stabilized aspects of their mental models.

Beyond mathematics instruction, some participants found that the most rewarding aspect of teaching was building relationships and connecting with students on a personal level, often tied to adopting more student-support approaches. For example, Kayla described how she regularly shared details about her background and took time to get to know her students. In previous courses she had taught, she noticed that students deeply appreciated these efforts and felt cared for, which solidified this practice as part of her mental model of teaching. She recalled, "I remember one time in my section, I said that [information about her background] and I saw someone's face light up. Like, they're, 'Oh my gosh, she's probably from where I'm from, and I can't believe she's teaching section.'" Recognizing the impact of representation, Kayla believed that by sharing aspects of her identity, background, and lived experiences, she could foster a greater sense of belonging for her students in mathematics.

Making Sense of Past Teaching Challenges

Participants' prior teaching experiences also included making sense of challenges and difficulties, particularly when they implemented instructional approaches that they later

realized did not effectively support student learning or when students expressed dissatisfaction with their methods. These moments became valuable learning experiences, prompting graduate students to refine their teaching strategies and adopt practices that better aligned with their evolving mental models. Morgan recalled instances in class when small mistakes on the board became a distraction for students, which she believed negatively impacted their learning. She reflected, “I think that I sometimes make really small mistakes at the board and that can be really distracting to students. I get that feedback a lot, where it's like, ‘Oh, she made a lot of little mistakes.’” As a result, Morgan associated successful mathematics instruction with being well-prepared and minimizing errors during lectures. Similarly, Connor reflected on a previous challenge related to covering too much content too quickly, which left students feeling lost and discouraged. He explained, “Too many steps at once has sometimes hindered people ... People were getting lost along the way and needed additional encouragement because it was so easy to get discouraged.” Drawing from this experience, Connor has since prioritized giving students time to make sense of the mathematics, encouraging students, reassuring them during difficult moments in the course, and remaining mindful of the stress they may be experiencing.

Navigating the Instructional Context

In addition to their individual teaching experiences, some participants noted the influence of departmental and institutional structures on their instructional decisions, such as course evaluations and TA training programs. Martin’s emphasis on facilitating small group work to foster a sense of community among students was reinforced by positive feedback in his course evaluations at Spruce University. He shared, “When I was looking through my reviews a couple days ago, a lot of [students] were like, ‘Because we had a small group, we

really felt connected, and it was really friendly. We could just talk.’” This feedback further solidified his commitment to making his sections accessible and engaging, emphasizing a dynamic in which he and his students are “on the same level.” Kayla highlighted the impact of the department TA training, where graduate students were encouraged to be transparent and accountable during moments when they do not know answers to students’ questions. She internalized this pedagogical move and actively incorporated this practice, seeing it as a way to humanize mathematics and challenge traditional perceptions of instructors. In her view, this transparency helps build positive relationships with students, ultimately supporting their learning. Kayla said,

I know another big one that they teach us in TA training is just owning up to your mistakes. If you're wrong, it's fine. It's actually pretty encouraging for students to see when you do make a mistake, to being able to take accountability for it, and just correct yourself versus just getting defensive about it. It's usually better to kind of stop yourself and correct, or if a student asks you a question, and you don't know it, telling them like, “Oh, I'll come back to it and look into it more.” It's something that I like that professors do.

This experience exemplified a moment that shaped Kayla’s mental model of teaching and was directly linked to her instructional goals of reconfiguring narratives and discourses about mathematicians and instructors. She resonated with the advice from the TA training to embrace one’s own mistakes, to be accountable, and to be transparent with students. Overall, participants found that their initial assumptions about teaching aligned well with their prior teaching experiences in the classroom, while also confronting unexpected challenges that prompted them to rethink their approaches. These prior teaching experiences served as

critical moments for reflection, allowing graduate students to refine their mental models and to develop greater pedagogical awareness. In particular, I found that direct instructional experiences, student interactions, and exposures to different teaching contexts contributed to shifts in graduate students' teaching conceptions.

Discussion

Consistent with prior scholarship on mental models and pedagogical development, a mental models perspective provided a useful lens for understanding how mathematics graduate students made sense of their teaching worlds and translated those understandings into practice. In this study, I examined participants' reflections of their teaching practices to identify the dimensions most salient to their mental models of mathematics teaching. The focal research questions were: (1) What experiences and beliefs most structured mathematics graduate students' mental models of mathematics teaching?; and (2) In what ways did these dimensions inform their current instructional approaches?

Addressing Research Question One: Multidimensional Mental Models

Participants' reflections revealed connections between cognition and practice, aligning with prior conceptualizations of the functionality of mental models (e.g., Bransford et al., 2000; Greca & Moreira, 2000; Jones et al., 2011; Speer, 2008). Graduate students' beliefs and prior experiences informed how they approached instructional decisions, while their ongoing teaching experiences simultaneously reshaped those understandings. Across narratives, their conceptions of teaching were not static but evolved over time, as evidenced by how participants connected formative experiences as undergraduate students to their current instructional approaches. Importantly, the dimensions of their mental models, particularly beliefs about teaching, students, and mathematics, alongside critical prior

experiences, emerged as both multidimensional and deeply interconnected, rather than discrete or isolated components (Genter & Stevens, 1983; Johnson-Laird, 1983). Attending to participants' reflections on their teaching practices proved especially generative in surfacing these underlying structures, offering insight into how graduate students interpret, enact, and refine their approaches to undergraduate mathematics teaching.

My findings resonated with Oleson and Hora's (2014) framework of how STEM faculty reported learning how to teach, likely due to the similarities in the institutional contexts and subject matter. Oleson and Hora found that the following factors informed how higher education STEM faculty learned how to teach (i.e., how they constructed their mental models of teaching): prior experiences as an instructor, prior experiences as a student, prior experiences in non-academic roles, and prior experiences as a researcher. The STEM faculty in their study and the graduate students in my study cited how these dimensions directly shaped their current instructional approaches. Additionally, scholars who have studied higher education faculty's teaching approaches have posited that experiences with faculty (e.g., through advising, through observation), the early exposures to teaching (e.g., teaching assistantships), and the institutional climates (i.e., the unique school cultures) are influential to the formation of these models (Manrique & Abchi, 2015). My analysis yielded that participants' mental models of teaching were most informed by beliefs about teaching, beliefs about student learning, beliefs about mathematics, prior teaching experiences, and prior experiences as students.

Participants' beliefs about teaching ranged from viewing instruction as the transmission of knowledge to facilitating student-centered inquiry. Similarly, beliefs about student learning varied, with some participants emphasizing mastery through rote practice,

while others prioritized conceptual understanding and building tenacity through struggling and preserving through difficult problems. Their views on the nature of mathematics, whether as a rigid, procedural system or a dynamic, meaning-making discipline, shaped instructional decisions, resonating with literature on beliefs in mathematics education (Ernest, 1989; Pekhonen, 1999; Speer, 2008). Interestingly, many participants critiqued long-held beliefs about mathematics and mathematicians (e.g., math ability is innate) and actively attempted to redefine these narratives through their teaching (Herzig, 2008; Gutiérrez, 2016, 2017). Participants' teaching experiences as TAs or instructors of record refined these beliefs over time as they were able to implement practices and to consider the success of their implementations through reflection or direct student feedback. Additionally, their experiences as students were especially influential. For example, those who personally struggled with mathematics also reported adopting more empathetic and student-centered approaches to ensure a better learning experience for their students. Moreover, for those who experienced impactful professors, they described wanting to copy or mirror the practices that motivated them to learn.

Addressing Research Question Two: Reported Instructional Practices

Across participants' reflections on their teaching, I observed that dimensions of their mental models motivated many of their reported practices, illustrating the connections between cognition and practice. For example, their prior experiences as students informed their efforts to empathize with students learning abstract mathematics for the first time, or their beliefs about mathematical knowledge reflected their departure from an overemphasis on grades and points. However, their mental models did not rigidly determine their teaching practices; rather, they provided a flexible framework through which they interpreted and

adapted to classroom dynamics. This was evidenced by participants drawing from a range of practices (McMinn et al., 2022; Mesa et al., 2014) blending aspects based on context, experience, and evolving pedagogical understanding. This fluidity highlights the complexity of how graduate students develop their pedagogical knowledge and teaching identities through making sense of departmental norms, teaching roles and expectations, students' emergent needs, and personal values, rather than adhering strictly to a single pedagogical approach (e.g., a traditional approach like only lecturing during section). Thus, even if graduate students have unique underlying mental models, their enacted instructional practices may converge, reflecting the multifaceted and adaptive nature of learning to teach in higher education.

These findings both align with and extend existing literature on mental models of teaching, and broadly instructors' understanding and conceptions of undergraduate teaching. While research acknowledges that instructors' beliefs influence their teaching (Fennema & Franke, 1992; Manrique & Abchi, 2015; Oleson & Hora, 2014), this study highlights the interplay between past experiences and evolving pedagogical perspectives, particularly the emotional and identity-related dimensions of teaching. Participants' reflections suggested a profound awareness of students' affect and emotional wellbeing. Notably, their own prior experiences as students enabled them to empathize with students' experiences, impacting their sense of authority and responsibility in the classroom. They considered dismantling authority and power structures between instructors and students would support students' overall classroom experience, and they felt compelled to be emotional guides and supporters of students. These findings emphasize the deeply personal and situated nature of university mathematics teaching (Hill et al., 2008; Jensen, 2011; Speer, 2008). Overall, examining these

dimensions of mental models of undergraduate mathematics teaching provided insight into how graduate students interpreted students' struggles, the strategies they employed to support learning, and the extent to which they prioritized engagement, exploration, and independent reasoning in their teaching.

Broadening the Mental Models: Awareness of Discourses in Mathematics

An important dimension that emerged from my analysis of participants' mental models of teaching extended beyond their beliefs and prior experiences. Some participants explicitly referenced dominant discourses and narratives within mathematics (McBride, 1989; Mendick, 2006) and in broader societal perceptions. They saw their roles as instructors and educators as opportunities to challenge and disrupt these narratives, particularly those shaping how mathematicians are perceived and how these perceptions influence students' learning experiences in undergraduate mathematics spaces. One of the primary discourses they commonly identified was the concept of “math gods” and the elitism, detachment, and individualism often associated with those who do mathematics (Ataide Pinheiro, 2021; Mendick, 2006)—including TAs, instructors, and faculty.

This discourse reinforced hierarchies of mathematical ability, positioning some individuals as inherently gifted while alienating others. Kayla reflected on this, stating, “And it's the math person thing, like thinking of math people as these gods. So, it's like, ‘Oh, you're smart because you know math.’” She critiqued the way intelligence is often equated with mathematical ability and highlighted how this perception can create distance between students and their instructors. To counter this, she intentionally shared details about her personal life to “reconfigure the narrative of mathematicians, being these godlike people that you can't talk to and just trying to let them know I am as normal as I could get as a person.”

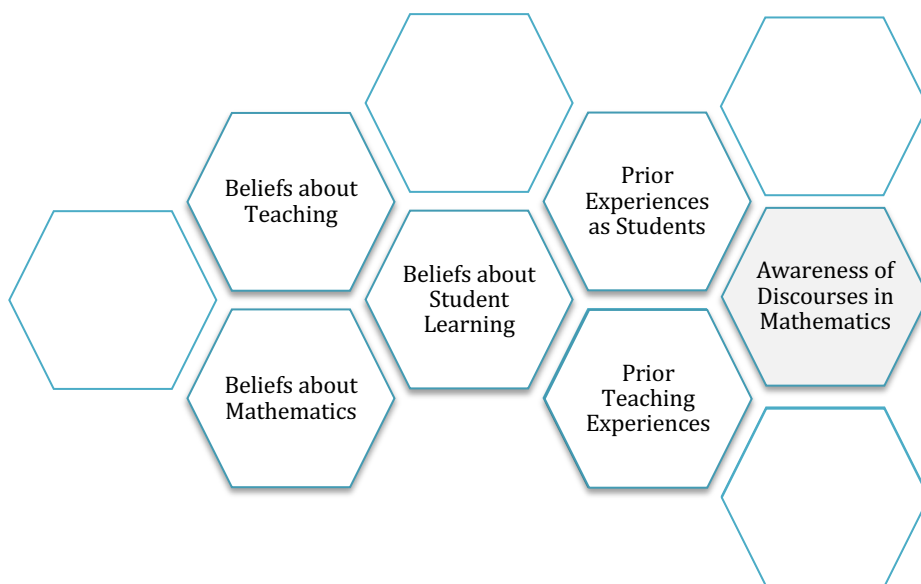
Similarly, Morgan challenged the notion of innate mathematical ability and the ways university instructors perpetuate this idea through their teaching. She explained, “I think that a lot of math professors do that, and I think that it makes students feel dumb. A lot of people have that issue with math where they're like, ‘Oh, I'm too dumb to be in math.’” By recognizing how these narratives shape students’ self-perceptions, Morgan reinforced her commitment to teaching and supporting all students, not just those in “the top one percent of their class.”

Lastly, I observed instances where participants’ understanding of undergraduate mathematics teaching were connected to their gender identities (Olarie & Street, 2025), and consequently gendered discourses within mathematics (Hottinger, 2016; Mendick, 2006). For example, Kayla described her teaching to be deeply informed by her identities as a Christian, Hispanic, first-generation woman. Kayla was aware of the underrepresentation of her identities in higher education mathematics and how this underrepresentation can continue to position students who are not part of the dominant cultures and identities (i.e., White, male, cisgender) as outsiders of mathematics. Her awareness of this discourse was intertwined with her beliefs about mathematics, beliefs about teaching, and prior experiences. She recalled a moment with a student saying: “I remember one time in my section, I said that and I saw someone's face light up. Like, they're, ‘Oh my gosh. She's probably from where I'm from.’ And, ‘I can't believe she's teaching section.’” Kayla felt that sharing her background and educational journeys can inspire students by increasing the representation of Christian, Hispanic, first-generation women in higher education mathematics. This was done to disrupt discourses that have historically claimed who *can* or *cannot* do mathematics. These examples illustrated that awareness of these hegemonic discourses within mathematics, including how

society views mathematicians, guided their reasoning and decision-making (Jones et al., 2011), thus emerging as central components of participants' mental models of mathematics teaching (see Figure 10).

Figure 10

Added Dimension of Mental Models: Awareness of Discourses in Mathematics



Note. The figure depicts the added dimension to graduate students' mental models of mathematics teaching that emerged from my analysis.

Implications & Future Work

Understanding how graduate students perceive their current instructional practices and the underlying mental models that guide their teaching can equip higher education institutions to better support them in their current teaching roles and prepare them to teach as faculty. At the individual student level, being metacognitive and self-reflecting on their teaching is recognized as an important dimension of how to improve their instructional practices (Manrique & Abchi, 2015). Higher education institutions and mathematics departments can cultivate more scaffolded opportunities for reflective practices (Bell, 2001)

beyond a single annual training or provide graduate students with tools that develop their agency to self-reflect (e.g., heuristics for intentional reflection on their teaching practices). My findings highlight how participants enacted a number of instructional practices (McMinn et al., 2022; Mesa et al., 2014) and that teaching roles, such as serving as a TA, afforded opportunities to implement such practices and deepen their understanding and conceptions of mathematics teaching. Deshler et al. (2019) noted that graduate students have limited exposure to evidence-based instructional practices, yet the participants in the study reported practices that are aligned with both content-centered and student-centered practices. Professional learning experiences that allow graduate students to reflect on their current practices and connect those with contemporary undergraduate mathematics education research on evidence-based, equity-centered practices can support their broader pedagogical development. Moreover, departments can provide more structured, and frequent, pedagogical support for graduate students beyond an annual TA training.

This study demonstrated how the dimensions of mental models of teaching can be qualitatively described and condensed into interconnected dimensions that have practical significance to graduate students, and the staff and faculty that are tasked with supporting their pedagogical development and career pursuits. For example, the dimensions of their mental models can anchor discussions for professional development efforts/training, can help identify and attend to any deficit perspectives about students and instruction, and can illuminate why graduate students privilege certain aspects in the classroom (e.g., content, students' wellbeing, general sensemaking). Additionally, these models can be developed through supported discussions with peers, reconciling beliefs about teaching, raising awareness of discourses in mathematics, processing struggles and decision-making with

current roles as teaching assistants, and even challenging their existing mental models (Friedberg, 2005).

A key limitation of this study is that I only examined the practices and mental models of six graduate students within one departmental context. Furthermore, I purposefully focused on how participants reflected on their instructional practices, rather than observing them facilitate sections or teach courses. Future work can research a larger number of graduate students to illustrate more nuance to the dimensions that inform their mental models of mathematics teaching and observe the actual instructional practices of graduate students as they teach. Additionally, given the unique institutional and departmental expectations surrounding graduate students' teaching roles, future work can more closely examine and compare the instructional practices and mental models of graduate students who primarily serve as TAs versus those who are afforded more agency as graduate instructors of record. In this work, I explore five dimensions and offered one possible addition (i.e., *awareness of discourses in mathematics*), however, I acknowledge that the complex and idiosyncratic nature of mental models suggest that there are more experiences and beliefs that contribute to graduate students' understanding and conception of mathematics teaching. For example, Oleson and Hora (2014) found that faculty's non-academic roles or experiences as researchers also informed their development as teachers. However, I did not observe the participants of this study to reflect on such as experiences. Another possible avenue of future research is to examine how other experiences, beliefs, and values may influence graduate students' mental models of mathematics teaching.

Conclusion

Examining graduate students' instructional approaches and mental models of mathematics teaching, particularly the dimensions that shape these models, is a critical area of research with significant implications for higher education institutions and mathematics departments. Given that many mathematics graduate students engage in formal teaching for the first time during graduate school, understanding how they enact instructional practices in their various teaching roles provides valuable insight into their evolving teacher identities and perspectives on university mathematics teaching. Moreover, because mental models of teaching shape how graduate students reason, make pedagogical decisions, and engage in instructional learning, it is essential to describe, analyze, and build upon these models. Strengthening this line of inquiry is crucial for improving postsecondary mathematics instruction and better preparing the next generation of mathematics faculty.

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CHAPTER FIVE

Concluding Thoughts

Becoming a professor is a complex process that involves graduate students negotiating identity development, belonging, and the social worlds of their respective disciplines (Austin & McDaniels, 2006; Gardner, 2008). Inspired by my own journey navigating postsecondary mathematics education and pursuing a career as a professor, I grounded this dissertation in the broad question: What does it mean to become a mathematics professor? I amplified students' identities and narratives to interrogate the systems and structures within mathematics and higher education that both support and hinder their professional development as prospective faculty. Scholars have noted that discipline-specific cultures, values, and norms shape how students experience graduate school (e.g., Clarke et al., 2013). Building on this, I explored the process of becoming a professor with mathematics graduate students as participants, creating multiple avenues for them to share their stories of navigating graduate education and actively pursuing academic careers. Across this work, I remained committed to centering my participants' voices to broaden our understanding of the contours of mathematics graduate education.

I conceptualized the process of becoming a mathematics professor through the metaphor of hiking a trail. This metaphor supported my exploration of participants' narratives as they navigated the doctoral program at Spruce University, attending to both their idiosyncratic identities and experiences and the shared departmental context. This dissertation does not seek to generalize nor homogenize the experiences of Kayla, Martin, Connor, Landon, Kendall, and Morgan; rather, it aims to amplify their lived experiences and to extend the field's understanding of how graduate students are socialized and prepared for the professoriate (Gardner & Barnes, 2007; Tierney & Rhoads, 1993). This process of

socialization was shaped by students' navigation of the social worlds of mathematics as well as their social and cultural identities.

Each chapter examined a different facet of this process (i.e., hiking the trail and developing as a mathematics professor) within an overarching framework grounded in socialization theory and social identity theory. From a socialization perspective, graduate students learn what it means to be a member of the professoriate through participation in disciplinary practices, interactions with others, and engagement with the norms and values of mathematics. At the same time, this process of socialization extends beyond acquiring skills or knowledge; it is deeply influenced by cultural contexts and discourses that shape how individuals are recognized and legitimized within the profession (Gardner, 2008; Haley et al., 2014). Drawing on various identity theories, this work also highlights how students' social and cultural identities shaped how participants navigated, experienced, and made sense of their pathways into the professoriate (Ostrove et al., 2011; Trepte, 2013; Zhang & Wang, 2024). Further, by centering the narratives of women-identifying graduate students, I illustrated how identity and discourse intersect to influence how graduate students are positioned to exercise power (e.g., navigating fluctuating levels of confidence during the qualifying exam process, building community and affirming spaces for other women) within mathematics graduate education.

This dissertation shows that professional identity is not fixed but continually negotiated through participation and belonging. Throughout the hike, graduate students made sense of their everyday responsibilities, personal beliefs and values, and discipline-specific discourses and narratives. Importantly, the influence of mathematics was deeply entangled with all aspects of their experiences and professional identity development (Herzig, 2004).

Although some critical experiences may be shared, longstanding sociohistorical perceptions of mathematics and mathematicians, along with dominant disciplinary discourses, shaped how Kayla, Martin, Connor, Landon, Kendall, and Morgan came to see themselves as future professors. For example, many were driven to pursue the professoriate because of a passion for teaching, and they considered this professional responsibility as an opportunity to reframe dominant narratives about mathematicians and mathematical ability. For women-identifying participants, this was even more pronounced as they saw their professional identities and commitments deeply informed by their gender identity.

I examined three dimensions of their hike: (a) the role identities they enacted, (b) masculine discourses in graduate mathematics education, and (c) mental models of undergraduate mathematics teaching. I now briefly highlight key insights from these chapters and, more importantly, reiterate their implications for theory and practice.

Enacting Different Roles During the Hike

In Chapter 2, I offered a model of professional identity for mathematics graduate students that considered the sustained enactment of role identities to mediate their professional identity development (see Figure 2). By performing and making sense of a multitude of role identities during the hike, graduate students come to integrate these role identities as dimensions of their sense of selves as future mathematics faculty (Colbeck, 2008; Zhang & Wang, 2024). As participants shared their stories of navigating graduate education, I found five distinct, yet interrelated, role identities performed throughout graduate school: *teacher*, *researcher*, *mathematician*, *mentor*, and *peer and colleague*. Beyond just describing actions and responsibilities associated with each role identity, I highlighted different contexts that prompted graduate students to enact these role identities,

different meanings and values they attached to these identities, and the ways participants experienced affirmations and tensions with role identities. Notably, I demonstrated how the mathematics discipline and the departmental context of Spruce University impacted how role identities were understood and enacted, illustrating the socially mediated and context-dependent nature of professional identity development (Zhang & Wang, 2024).

This study contributes both theoretically and practically to understanding how mathematics graduate students develop professional identities in preparation for faculty careers. Professional identity development emerged as deeply shaped by social interactions, departmental contexts, and the broader disciplinary culture of mathematics (Clarke et al., 2013; Solari & Ortega, 2020). Theoretically, this study demonstrates the usefulness of role identity theory for analyzing how graduate students navigate multiple, and at times competing, roles within the liminal space between student and professor, and it advances a conceptualization of identity development as a process of enacting and stabilizing these roles over time (Bale & Anderson, 2022; Jazvac-Martek, 2009; Winstone & Moore, 2017). Practically, the findings point to the importance of structuring graduate education in ways that intentionally support role identity development, particularly by providing early and sustained opportunities for research engagement, teaching development, mentorship, and community building. Along the hike, graduate programs preparing future faculty should attend to how structural elements such as advising relationships, teaching assignments, and peer networks can either expand or constrain students' opportunities to explore, integrate, and sustain these role identities in meaningful ways.

Navigating the Terrain and Encountering Masculine Discourses

In Chapter 3, I amplified the voices of Kayla and Morgan and explored how they encountered masculine discourses throughout the terrain of their hike to the professoriate. Features of the trail, or discursive practices (McBride, 1989), served as sites where Kayla and Morgan encountered and negotiated these masculine discourses. Some of these discursive practices included qualifying exams, relationships with peers, and relationships with faculty. As they navigated these experiences, they confronted three prevalent masculine discourses: *women are positioned as not good at mathematics*, *women have to constantly prove themselves to others*, and *connections are rare yet impactful for women*. While existing scholarship has characterized mathematics as a gendered domain (Leyva, 2017; Mendick, 2006), this work amplified experiences unique to doctoral programs in mathematics to deepen our understanding of how masculinity operates in the practices, norms, and cultures of graduate mathematics education. This chapter demonstrated how social and cultural identities, such as gender identity, influenced how graduate students experience the process of socialization along the hike. For women, who have historically been positioned as outsiders in mathematics (Ataide Pinheiro, 2021), understanding how they negotiate and resist masculine discourses offers insights into how mathematics departments can broaden access and participation for all students.

This chapter highlighted how women's participation in graduate mathematics is shaped by experiences and discursive practices that often reflect and reproduce masculine norms, while also pointing to actionable pathways for disrupting these inequities (Herzig, 2004). In particular, the qualifying exam process emerged as a critical site where norms of competition, lack of clarity, and restricted access to resources can marginalize women,

suggesting the need for greater transparency, earlier access to research and professional development opportunities, and availability of stronger peer and faculty support structures during *and* after the exam process. Faculty interactions likewise play a pivotal role, as they can either reinforce exclusionary dynamics or foster affirming and inclusive environments, underscoring the importance of professional learning that prepares faculty to engage in equitable, responsive mentorship. Methodologically and theoretically, this work contributes an understanding of the use of a narrative-inquiry, intersectionality-informed approach to examine identity and participation in graduate mathematics, offering a foundation for future research that further interrogates how gender intersects with other social identities to shape experiences within the discipline.

Developing Mental Models of Undergraduate Mathematics Teaching During the Hike

In Chapter 4, I focused on one of the most central aspects of the mathematics professoriate – undergraduate mathematics teaching. As graduate students navigate the hike, they develop an understanding of undergraduate mathematics teaching that informs their current instructional practices, often as TAs, while also shaping how they come to develop pedagogical skills as aspiring faculty. I used perspectives that consider mental models to mediate cognition and practice (Bransford et al., 2000; Jones et al., 2011) to illustrate a multidimensional configuration of graduate students' mental models of undergraduate mathematics teaching. I prompted graduate students to reflect on their current teaching practices and explicitly asked about how they learned how to teach, which revealed that their mental models of teaching encompassed five primary dimensions: *beliefs about teaching*, *beliefs about student learning*, *beliefs about mathematics*, *prior experiences as students*, and *prior experiences as teachers*. Participants connected these dimensions to their current

instructional practices, such as adopting student-centered and meaning-making approaches to holistically support students (Mesa et al., 2014). Most importantly, this work revealed that participants actively made sense of exclusionary discourses associated with mathematics (e.g., mathematics ability is innate) and purposefully approached their teaching with the goal of disrupting these long-held narratives (Gutiérrez, 2016, 2017). Although interconnected to the other dimensions of mental models of teaching, *awareness of discourses in mathematics* emerged as another dimension that informed their teaching (see Figure 10).

When we attend to how graduate students make sense of their own teaching across beliefs, experiences, and contextual influences, we gain insight into the underlying logics that shape their pedagogical decisions. This, in turn, positions departments to design and curate experiences that not only support critical reflection but also create structured opportunities for graduate students to enact and refine purposeful instructional practices. By making these mental models visible, faculty and department program coordinators can more effectively guide graduate students in developing robust, equity-oriented approaches to teaching. For instance, professional development spaces, such as TA training, can explicitly engage with discourses and inequities in mathematics and connect these to research-informed teaching practices in undergraduate mathematics education. In doing so, departments move beyond generic pedagogical preparation and instead foster the intentional development of responsive, critically aware individuals who are better equipped for the complexities of the mathematics professoriate.

The Future of the Mathematics Professoriate

As I reflect on the varied identities and narratives shared throughout this body of work, I am both inspired by the future of the mathematics professoriate and more motivated

to disrupt inequities that may continue to diminish the participation, access, and professional identity development of mathematics graduate students. I am excited that the next generation of mathematics professors, including Kayla, Morgan, Kendall, Landon, Connor, and Martin, are not just mathematically competent and exceptional, but are also relational, reflective, and equity-oriented in their practice. What notably stood out was that graduate students are critically aware of inequities within the discipline (e.g., masculine discourses, exclusionary practices, narratives about mathematicians) and within the unique departmental context of Spruce University (e.g., norms and practices around milestones, relationships with certain faculty). Navigating these tensions contributed to participants' commitment to becoming professors who adopt a range of role identities and teaching practices that mark a positive change in the mathematics professoriate.

In addition to preparing for their futures as mathematics professors, they were also actively enacting change within their current contexts and improving the graduate student experiences at Spruce University. They described efforts to create a culture of peer support, advocacy, and collective commitment to improving the experience of hiking the trail – whether that meant supporting each other throughout the hike or changing and improving the trail itself. For example, Morgan's efforts to establish the AWM chapter at Spruce University demonstrated that graduate students could socialize the space that they navigate, rather than simply allowing the discipline and the institution to socialize them into to the academy. Centering their identities and narratives nuances our understanding of the complex process of becoming a mathematics professor and provided actionable insights to improving mathematics departments and higher education institutions, such as increasing structural

support for community, mentorship, representation, and pedagogical approaches that make room for diverse identities and ways of knowing.

Throughout the dissertation process, I gleaned new insights about graduate education mathematics through amplifying participants' identities and narratives. Additionally, I am left with new questions that outline possible avenues for continued research on graduate education. First, questions about how graduate students negotiate tensions among role identities invite more process-oriented and longitudinal work that traces how identities are stabilized, resisted, or reconfigured over time, offering a more dynamic account of professional identity development (Zhang & Wang, 2024). Second, questions attending to social and cultural identities and exclusionary discourses broaden the analytic lens, pointing toward intersectional perspectives that examine how systems, such as whiteness, ableism, and colonialist narratives, shape participation and meaning-making in graduate mathematics education (Gutierrez, 2016, 2017). Third, questions about the development of mental models of teaching connect cognition and practice, suggesting the value of studies that link beliefs, pedagogical decision-making, and classroom enactments across contexts (Jones et al., 2011). Finally, the call for innovative data collection methods encourages methodological expansion, pushing the field to adopt creative, multimodal, and participant-centered approaches that capture the complexity of graduate student experiences. Taken together, these guiding questions chart a research agenda that is more identity-affirming, methodologically diverse, and responsive to persistent inequities in mathematics education. I can summarize these with the following questions:

1. How do graduate students negotiate tensions between their role identities? How do these efforts support how role identities are anchored as aspects of their professional identities?
2. How do other social and cultural identities contribute to graduate students' experiences navigating graduate mathematics education? How do other exclusionary discourses (e.g., those associated with Whiteness, ableism, colonialist ideals) manifest in the discursive practices of graduate education?
3. How do graduate students develop their mental models of teaching and pedagogical approaches over time? How are these practices enacted and implemented in their classrooms?
4. What other innovative data collection methods can help elicit more comprehensive and nuanced stories of navigating graduate education?

Taken together, this dissertation underscores the need to carefully examine and restructure the systems, norms, and supports that shape graduate education to foster more equitable and inclusive pathways into the mathematics professoriate.

Final Reflection: My Own Hike to the Professoriate

Writing this final section feels surreal. In some ways, it resembles the familiar act of finishing a manuscript or submitting a final paper. Yet this moment carries a different weight. Because this work is deeply intertwined with my own experiences and journey, this reflection feels less like a conclusion and more like the final ascent of a long and demanding hike. I began this journey in 2017, after completing my undergraduate degree and preparing to enter graduate study in mathematics. Even then, I knew I wanted to become a professor, initially motivated by a love for teaching mathematics. Over time, that early passion has expanded

and deepened; my commitments now center on mathematics education research, mentoring and advising, and advocating for graduate students.

In many ways, I am who I study. My own professional identity development has been complex, shaped by the same kinds of experiences I explored in this dissertation. Like the participants, I have navigated multiple role identities that continue to evolve and intersect. I have encountered discourses that have marginalized aspects of who I am as a gay, Filipino American immigrant, and I have worked to make sense of and refine my own approaches to teaching undergraduate mathematics. These experiences have not only informed this research, but they have also been inseparable from it.

Despite moments of success and accomplishment, this hike has not been easy. I left a previous institution when I no longer felt able to pursue my passion for mathematics education or to see myself as someone who could succeed in that environment. For years, I questioned whether I could exist as an openly gay scholar in mathematics, whether my work would be valued, and how to navigate challenging relationships with peers and faculty. More recently, broader political pressures and attacks on higher education have made the final stretch especially strenuous. Balancing the demands of completing this dissertation while applying for faculty positions often felt unsustainable. And yet, writing this now just weeks after accepting a tenure-track position in mathematics education, I find myself holding a mix of pride, gratitude, humility, and clarity.

As I reach this final ascent and look toward the next stage of the journey, I carry with me a deeper understanding of what it means to persist, to question, and to belong. I think of those who are still on the hike, those who have stepped away from it, and those who are deciding whether this path is right for them. This dissertation is not only a study of graduate

students' experiences, but also a reflection of the possibilities that emerge when those experiences are seen, valued, and reimagined. As I begin this next chapter, or new trail, I remain committed to creating the kinds of spaces, relationships, and possibilities that I once searched for so that those who come after me might experience a different, more equitable, diverse, and inclusive journey through graduate mathematics education.

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