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Journal

The Astrophysical Journal, 252(2)

ISSN

0004-637X

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Publication Date

1982

DOI

10.1086/159597

Peer reviewed

STELLAR WEAK INTERACTION RATES¹ FOR INTERMEDIATE-MASS NUCLEI. II. $A=21$ TO $A=60$

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Received 1981 June 12; accepted 1981 August 3

ABSTRACT

Astrophysical electron and positron emission, continuum electron and positron capture rates, as well as the associated neutrino energy loss rates are calculated for free nucleons and 226 nuclei with masses between $A=21$ and 60. Measured nuclear level information and matrix elements are used where available. Unmeasured matrix elements for allowed transitions are assigned as in Paper I. Simple shell model arguments are used to estimate Gamow-Teller sum rules and collective state resonance excitation energies. The discrete state contribution to the rates, dominated by experimental information and the Fermi transitions, determines the nuclear rates in the regime of temperatures and densities characteristic of the hydrostatic phases of presupernova stellar evolution. At the higher temperatures and densities characteristic of the supernova collapse phase, the nuclear rates are dominated by the Fermi and the Gamow-Teller collective resonance contributions. Also included is the important effect of neutron shell closure blocking of electron capture on neutron-rich nuclei. Uncertainties in the rate calculation are discussed. Reference is made to other treatments of the problem. Results of the calculations on a detailed temperature-density grid are available in computer readable form on magnetic tape upon request to M. J. N.

Subject headings: equation of state — neutrinos — nuclear reactions — stars: collapsed — stars: interiors — stars: supernovae

I. INTRODUCTION

The problem of nuclear β -decay in the stellar environment has been discussed in Paper I of this series (Fuller, Fowler, and Newman 1980, hereafter referred to as F²N I). In that work the positron emission, continuum electron capture, and neutrino energy loss rates were calculated for $^{26}\text{Al} \rightarrow ^{26}\text{Mg}$, $^{30}\text{P} \rightarrow ^{30}\text{Si}$, $^{31}\text{S} \rightarrow ^{31}\text{P}$, $^{32}\text{S} \rightarrow ^{32}\text{P}$, $^{33}\text{S} \rightarrow ^{33}\text{P}$, and $^{35}\text{Cl} \rightarrow ^{35}\text{S}$. These rates are important in determining nucleosynthesis, neutronization, and neutrino cooling during the phases of presupernova evolution corresponding to core carbon, oxygen, and silicon burning. The temperatures and densities most relevant to these stellar environments are $0.1 \leq T_9 \leq 5$ and $10^5 \leq \rho/\mu_e \leq 10^9$, encompassing conditions from mild degeneracy up to electron Fermi energies near 5 MeV.

In this work the set of nuclei considered is extended to include free neutrons and protons and 226 nuclei between mass 21 and 60 inclusive. The present calculation

includes positron emission, continuum electron capture, electron emission, and continuum positron capture rates, as well as ν and $\bar{\nu}$ energy loss rates for all nuclei considered. The reaction rates are calculated on an extended grid of temperatures and densities: $0.01 \leq T_9 \leq 100$ and $10 \leq \rho/\mu_e \leq 10^{11}$. This extended thermodynamic regime includes the conditions most appropriate for precollapse hydrostatic stellar evolution as well as the extreme environment of stellar core collapse, wherein temperatures in excess of 1 MeV and densities corresponding to electron Fermi energies near 25 MeV may be encountered before neutrino trapping. In the case of precollapse conditions the nuclear weak interaction rates are dominated by discrete transitions, whereas in the extreme temperature and density environment of the stellar core collapse the rates are dominated by the collective Gamow-Teller resonances.

The discrete state calculations performed in this work proceed as in F²N I. The procedure is very closely tied to experimentally measured nuclear properties. Measured nuclear energy levels, spins, and parities are used to define the discrete states. Experimentally measured weak matrix elements are used where available. Unknown Fermi transitions can be calculated with preci-

¹Supported in part by the National Science Foundation [PHY79-23638] at the California Institute of Technology, and by the U.S. Department of Energy.

²Fannie and John Hertz Foundation Fellow.

sion (cf. F^2N I). Unknown Gamow-Teller transition matrix elements are assigned an average value appropriate to this nuclear mass range ($\log ft = 5.0$, F^2N I; Gleit, Tang, and Coryell 1968; Wilson, Kavanagh, and Mann 1980). Forbidden transitions are neglected, but see Fuller (1981).

The work of Bethe *et al.* (1979, hereafter BBAL) points the way to solving the problem of extreme high density, high temperature electron capture. In the present paper Gamow-Teller collective state strengths and excitation energies are calculated based on a simple shell model picture fashioned after the work of BBAL, Klapdor (1976, 1979), and Gaarde and collaborators (Gaarde *et al.* 1972; Gaarde *et al.* 1980), and guided by the growing volume of experimental data from (p, n) and ($t, {}^3\text{He}$) reactions and conventional delayed proton, neutron, and γ -decay experiments. Also included is the important effect of neutron shell closure blocking of electron capture for neutron-rich nuclei (Fuller 1981).

As the temperature and density rise in the collapsing stellar core, parent nucleus excited states are thermally populated, and many otherwise unattainable electron capture channels are opened among daughter nucleus excited states due to the increasing electron Fermi energy. Eventually the weak interaction rates are dominated by the Gamow-Teller collective resonances, as pointed out by BBAL. In the present paper the Gamow-Teller collective resonances in both parent and daughter are treated as discrete states: one discrete resonance state in the daughter (parent) for each energy level considered in the parent (daughter) nucleus. The energies of these discrete resonance states are estimated using single particle energies from Seeger and Howard (1975), also Seeger and Perisho (1967), and employing an average particle-hole repulsion energy as in BBAL. The collective state transition matrix elements are estimated with a simple shell model sum rule and subtraction of measured strength.

Since the Gamow-Teller resonances are approximated as discrete states, the calculational apparatus employed in F^2N I can be used directly. All of the equations governing the decay rate of nuclei in the stellar environment derived in § II of F^2N I apply here, with the exception that the occupation index P_i in Paper I, equation (9a), is handled differently for resonance state transitions (see § V, this paper).

The importance of obtaining accurate nuclear weak rates both during the hydrostatic phases of stellar evolution (Couch and Arnett 1973; Woosley, Arnett, and Clayton 1972; Weaver and Woosley 1979) and during the collapse phases is a well known problem. Neutronization resulting from electron capture during the phases of stellar evolution up through silicon burning has been treated statistically, utilizing the relatively long time scales associated with the various burning phases (Arnett 1977, 1980). Ultimately detailed neutronization, neutrino

loss rates, and nucleosynthesis yields during presupernova evolution as well as the testing of statistical schemes require a detailed treatment of weak interaction rates. Reliable electron capture rates are required to compute the dynamics of the core infall epoch (Van Riper and Lattimer 1981). The initiation of the collapse in electron-capture supernovae depends on the detailed electron capture rates of intermediate mass nuclei (Couch and Arnett 1967; Nomoto 1980). The very high density electron capture rates determine to some extent the entropy and lepton-to-baryon ratio evolution in the collapsing stellar core, and hence are required for an understanding of the explosion mechanism.

The rates calculated in this work are available on a detailed temperature-density grid in computer readable form on magnetic tape upon request to M.J.N., and will subsequently be published on an abbreviated temperature and density grid in the *Astrophysical Journal Supplement Series* (Fuller, Fowler, and Newman 1982, hereafter F^2N III). The remainder of the present paper (hereafter F^2N II) describes the nuclear transition matrix element calculations, the uncertainties in the rates, and the results for a few interesting nuclei as examples.

II. NUCLEAR MATRIX ELEMENTS

The rate of decay from the i th state of the parent nucleus to the j th state of the daughter is given by

$$\lambda_{ij} = \ln 2 \frac{f_{ij}(T, \rho, U_F)}{(ft)_{ij}}. \quad (1)$$

The $(ft)_{ij}$ are the comparative half-lives and are related to the allowed weak interaction matrix elements by equations (I-2a) and (I-2b), where equations denoted I are found in F^2N I. The phase-space factors f_{ij} are given as integrals over the lepton distribution functions (I-3a) and (I-3b) and hence are sensitive functions of the temperature and density in the stellar interior. Similar expressions hold for the associated neutrino energy loss rates (I-6a) and (I-6b). The total decay or neutrino energy loss rate for a nucleus in the star is given by summing over all transitions between parent and daughter states. The double sum over these discrete state transitions involves weighting nuclear parent levels with a Boltzmann population factor, introducing further temperature sensitivity in the total decay rate. To compute the weak interaction rates for a parent-daughter nucleus pair, discrete state excitation energies, spins, and parities are required for both nuclei, as well as weak interaction transition amplitudes connecting these states.

There has been a vast increase in the amount of measured nuclear level information since the weak rate surveys of Hansen (1966) and Mazurek, Truran, and Cameron (1974) were undertaken. In order to exploit these data and keep the rate calculations close to experi-

ment, measured nuclear level information is used wherever possible. Typically, about 20 discrete states are included in each nucleus; resonance states will be discussed below. The nuclear level excitation energies, spins, and parities are taken from the latest nuclear information tabulations of Endt and van der Leun (1978) and Lederer *et al.* (1978) where available. For some proton- or neutron-rich nuclei the measured level information may be incomplete. In these cases, the nuclear level data can be supplemented by adroit attention to valence neutron and proton numbers and the shell model. For example, where excitation energies are known, but spins and parities are lacking, a zero order shell model for the valence nucleons can be used to generate a "bank" of J^π values which can then be assigned to the experimental states as needed. Where the measurement of spin for a given nuclear level is uncertain and several values are possible, the largest value is chosen in this work.

Finally, it is sometimes necessary to make estimates of excitation energies of levels as well as spins and parities. This is accomplished here by using nuclear level information from a well measured isotope, isotone, or isobar closer to the valley of β -stability which has its valence nucleons filling the same j -subshells as the unmeasured nucleus. Further, the measured nucleus must be even-even, odd-odd, odd-even, or even-odd, just as for the unmeasured nucleus. For instance, in $^{33}_{14}\text{Si}_{19}$, no excitation energies, spins or parities are known. The isotone $^{35}_{16}\text{S}_{19}$ is used to supply the missing information. In very neutron- or proton-rich nuclei, not even the masses are known accurately. In addition to the above techniques to fill out the unknown level information, theoretical nuclear mass values are taken from the calculations of Møller and Nix (1980) where necessary.

In principle, weak interaction matrix elements are required for all transitions between parent and daughter states. Matrix elements for forbidden transitions are not calculated in this work, since we are mainly interested in the weak rates of intermediate mass nuclei in temperature and density environments encountered during the collapse before neutrino trapping sets in. In these conditions allowed transitions and/or experimentally observed transitions, which may be forbidden, dominate the weak interaction rates. In the regime of stellar conditions beyond neutrino trapping ($\rho \gtrsim 5 \times 10^{11} \text{ g cm}^{-3}$; Arnett 1977) most of the transitions are at least partially blocked, the electron captures go through hindered or forbidden transitions, and the bulk of the neutronization is carried by electron capture on free protons (Arnett 1977, 1980; Fuller 1981). In this work then, only allowed transitions between discrete states are considered except where rates for forbidden or hindered transitions are known experimentally.

The treatment of allowed matrix elements proceeds as in § III of F²N I. Briefly, isospin symmetry is exploited

to calculate accurately the Fermi transition matrix element and the energies of the isobaric analog states which carry this Fermi strength. Isospin symmetry is also used to extend the set of measured $\log ft$ values by extracting matrix elements measured in the mirror system, corresponding to the pair of nuclei of interest, after correcting for spin degeneracy factors. After the Fermi and mirror transitions are included in the calculation of a particular reaction, there are typically still many unmeasured Gamow-Teller transitions which can contribute to the weak decay rate. A mean $\log ft = 5.0$ is taken for these transitions when allowed. This mean reflects the general trend of Gamow-Teller matrix elements in intermediate mass nuclei and is the result of an extensive survey of measured and calculated values done in F²N I. As concluded in that work, the experimental and calculated Gamow-Teller and Fermi transitions serve to dominate weak decay calculations and yield a reliable rate for well studied intermediate mass nuclei over most of the range of temperature and density of interest in the hydrostatic phases of stellar evolution. The extension of reliability of these rate calculations to the regime of extreme temperature and density requires a knowledge of the β -strength function over a large range of nuclear excitation energy. In particular, the Gamow-Teller strength function is needed as well as a way to handle large numbers of thermally populated isobaric analog states carrying Fermi strength.

III. FERMİ AND GAMOW-TELLER COLLECTIVE MODES

The Gamow-Teller strength distribution has been a problem of increasing concern to nuclear experimentalists because the details of this distribution are very sensitive to the central isovector spin and tensor components of the nuclear force. However, an increasing amount of experimental and theoretical lore points to the existence of a giant Gamow-Teller resonance which contains much of the collective Gamow-Teller strength in a broad peak (Goodman *et al.* 1980).

The Fermi transitions are illustrative of the resonance collective state phenomena. To each state of a $T^>$ parent nucleus ($T \equiv |Z - N|/2$), there corresponds a $T^>$ isobaric analog state (IAS) in the $T^<$ daughter nucleus. These analog states can be generated by acting on the associated parent states with the isospin raising or lowering operator

$$T_{\pm} = \sum_n \tau_{\pm}(n) \quad (2)$$

given by a sum over nucleons. The selection rules for the Fermi operator are $\Delta\pi=0$; $\Delta T=0$; $\Delta J=0$. This operator commutes with all parts of the nuclear Hamiltonian save the Coulomb force. As a result the analog state is pushed up in excitation by the difference in Coulomb energy between parent and daughter nuclei. Further-

more, the off-diagonal matrix elements of the Coulomb interaction serve to mix the isospin quantum number. In practice the resultant mixing is very small, isospin is a good quantum number, and hence all of the Fermi strength is concentrated in the analog states (Soper 1969; deShalit and Feshbach 1974). There is, of course, no Fermi resonance for a transition from a $T^<$ parent state to a $T^>$ daughter state. The important point here is that isospin is a good quantum number and the Fermi strength is narrowly concentrated even though the isospin operator generates only an approximate symmetry for the nuclear system.

The situation for the Gamow-Teller operator and its associated resonance is somewhat more involved, though qualitatively similar to the Fermi isobaric analog problem. Consider again the decay of a $T^>$ parent nucleus to states in the $T^<$ daughter, but now via the Gamow-Teller (GT) operator,

$$GT = \sum_n \sigma(n) \tau_{\pm}(n), \quad (3)$$

given as a sum over nucleons. The τ_+ and the τ_- are the isospin raising and lowering operators for individual nucleons, and σ is the Pauli spin operator. Note that since GT is a spatial vector and an isovector, the Wigner-Eckart theorem implies the selection rules $\Delta\pi = 0$; $\Delta T = 0, \pm 1$, no $0 \rightarrow 0$; $\Delta J = 0, \pm 1$, no $0 \rightarrow 0$. GT then connects to both $T^<$ and $T^>$ states and can change the spin of a nucleon but cannot change either its orbital angular momentum or its principal quantum number in allowed transitions. Like the Fermi operator, GT does not commute with the total nuclear Hamiltonian.

Unfortunately, the Gamow-Teller operator does not commute with the strong spin-dependent parts of the nuclear interaction, whereas the force responsible for isospin symmetry breaking is the relatively weaker electromagnetic interaction. As a result, the "isobaric state" corresponding to the collective Gamow-Teller mode is expected to be badly split among the nuclear daughter levels as compared to the Fermi isobaric-analog state resonance.

The strictly nuclear part of the nucleon-nucleon potential can be represented as a sum of central, tensor, and spin-orbit terms. For the most general Hamiltonian invariant under rotations generated by total spin ($J = L + S$) and isospin (T), we have the terms

$$H^{\text{central}} = V^c + V_{\sigma}^c \sigma_1 \cdot \sigma_2 + V_{\tau}^c \tau_1 \cdot \tau_2 + V_{\sigma\tau}^c (\sigma_1 \cdot \tau_1)(\sigma_2 \cdot \tau_2), \quad (4)$$

$$H^{\text{tensor}} = (V' + V'_{\tau} \tau_1 \cdot \tau_2) (3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2), \quad (5)$$

$$H^{\text{spin orbit}} = (V^{\text{so}} + V_{\tau}^{\text{so}} \tau_1 \cdot \tau_2) (L \cdot S), \quad (6)$$

where all of the potentials, V , are functions of r and where $\hat{r} = (r_1 - r_2)/|r_1 - r_2|$ is the normalized nucleon coordinate difference. The coefficients are taken to be the sums of Yukawa potentials in the usual phenomenological effective interaction studies (e.g., Austin 1980). The strengths and ranges of these forces can be found from distorted wave Born approximation (DWBA) analyses of scattering data as well as modeling nuclear spectra, though the parameters of some forces are known better than others. In particular V_{σ}^c , $V_{\sigma\tau}^c$, and V_{τ}^c are comparatively not well known, and these are just the forces most responsible for shaping the Gamow-Teller strength distribution (Halbleib and Sorensen 1967; Fujita and Ikeda 1965).

These forces not only introduce mixing among daughter states, but also can produce particle-hole correlations in the parent nucleus wave functions. The particle-hole forces in the spin-flip isospin-flip mode (i.e., pn^{-1} or $p^{-1}n$) are determined by $V_{\sigma\tau}^c$. The studies by Gaarde *et al.* (1972) and Martinsen and Randrup (1972) show that the effect of this force is to push the Gamow-Teller spin-flip mode strength up in daughter excitation energy, while weakening the no-spin-flip mode strength and spreading it out in excitation energy. This will be developed in more detail below, where a simple shell model is discussed.

As emphasized above, however, even though a nuclear symmetry is only very approximate, it may still be a useful description of the nucleus. In the case of the Fermi operator, the approximation of an isospin-independent interaction insures concentration of all Fermi strength in an isobaric analog state. In the Gamow-Teller case, approximation of a spin and isospin independent nuclear force would ensure concentration of the GT strength in an isobaric resonance state. Specifically, an assumption of a spin and isospin independent nuclear Hamiltonian would result in the Fermi and Gamow-Teller collective strengths being concentrated in a single degenerate daughter nucleus eigenstate: the Wigner supermultiplet state. Gaponov and Lyutostanskii (1974) have shown from a finite Fermi system model that the degenerate supermultiplet state is split by $V_{\sigma\tau}^c$ in such a manner as to concentrate the Gamow-Teller strength in a collective state a few MeV above the analog state in most intermediate mass nuclei. The picture of the Gamow-Teller transition for the ground state of a $T^>$ nucleus to a $T^<$ nucleus, then, is that the collective GT strength is contained in a broad peak centered well above the first analog state, with the no-spin-flip strength relatively more spread out than the concentrated spin-flip strength.

The growing volume of experimental data seems to support this conclusion for the $T^> \rightarrow T^<$ transitions of neutron-rich intermediate mass nuclei, whereas the $T^< \rightarrow T^>$ transition characteristics are much different. The Gamow-Teller strength distribution can be examined

experimentally either in gamma-decay branching or in delayed neutron (e.g., ^{31}Na) and proton (e.g., ^{33}Ar) experiments. The results are available as $\log ft$ -values tabulated in Endt and van der Leun (1978) or Lederer *et al.* (1980), as discussed above. The problem with these techniques is that there are no nuclei whose decays have a large enough Q -value to reach the main part of the β -strength in the daughter nucleus. The $^{33}\text{Ar} \rightarrow ^{33}\text{Cl}$ positron-decay has a large Q -value (11.618 MeV), and even in this case only about 12% of the expected strength is observed. This decay has been analyzed in terms of the simple shell model by Klapdor (1976) in some detail and tends to confirm the trends in the strength function mentioned above.

Another experimental technique which has met with some success involves charged particle reactions to excite the IAS in a nucleus resulting from electron emission. The spin part of the electromagnetic M1 operator corresponds very closely to the Gamow-Teller operator, so the M1 decay from the excited IAS populates the same states as would the electron emission from the parent nucleus ground state. Gaarde *et al.* (1972) have used $^{48}\text{Ca}(p, \gamma)$ to populate the IAS in ^{49}Sc corresponding to the ground state of ^{49}Ca , with results which confirm the theoretical conclusion discussed above that the spin-flip strength is pushed up and strengthened by $V_{\sigma\tau}^c$, while the no-spin-flip strength is weakened and spread out in daughter excitation energy.

By far, however, the most dramatic confirmation of the position of the GT resonance comes from recent charge-exchange experiments, notably (p, n) and $(^3\text{He}, t)$. These reactions mimic electron emission by creating a neutron-hole and proton-particle state in the residual nucleus and hence are especially useful for studying $T^+ \rightarrow T^-$ decays. The advantage of this technique lies with the relative lack of Q -value restrictions on the daughter excitation energy region explored, allowing complete resolution of the high lying Gamow-Teller strength. The incident proton or ^3He energy must be high enough to ensure a predominantly direct component in the scattering amplitude. In this case the scattering amplitude is related in the usual manner to the DWBA T -matrix element, given by

$$T_{FI} = \int \chi_F^\dagger \langle \Psi_f | V_{\sigma\tau}^c | \Psi_i \rangle \chi_I d^3r$$

$$\propto \langle \Psi_f | \sigma \tau_\pm | \Psi_i \rangle, \quad (7)$$

where the χ_F and χ_I are the appropriately advanced and retarded distorted waves and $\langle \Psi_f | V_{\sigma\tau}^c | \Psi_i \rangle$ is the matrix element of the effective nucleon-nucleon potential between initial and final nuclear wave functions Ψ_i and Ψ_f (Austin 1980). Measuring the cross section for a charge exchange reaction in the forward direction assures the dominance of low- l , even partial waves which in turn

determine, through the selection rules, which part of the nuclear interaction contributes.

For example, in the (p, n) reaction, for relative s -wave scattering, isospin must change, and spin can change by 0 or 1 unit of angular momentum. The interactions $V_\tau^c, V_{\sigma\tau}^c, V_\tau^t$ can then contribute, where V_τ^c is responsible for the "Fermi" transition to the IAS and where $V_{\sigma\tau}^c$ is predominantly responsible for the Gamow-Teller spin-flip and no-spin-flip transitions. V_τ^t is not as important, since this force is known to be weak in the lower partial waves, dominating only for high momentum transfer (Austin 1980). The IAS and the associated Fermi strength were first discovered with the (p, n) reaction by Anderson and Wong (1961), while the Gamow-Teller peak was unambiguously identified in the $^{90}\text{Zr}(p, n)^{90}\text{Nb}$ experiment by Doering *et al.* (1975), Bainum *et al.* (1980), Goodman *et al.* (1980), and Sterrenburg *et al.* (1980).

The results for the $^{90}\text{Zr}(p, n)^{90}\text{Nb}$ experiment tend to confirm the overall features discussed above for the $T^+ \rightarrow T^-$ transitions: a broad Gamow-Teller resonance lies roughly 5 MeV above the first IAS in ^{90}Nb (Bainum *et al.* 1980). In general the results for $^{90}\text{Zr}(p, n)^{90}\text{Nb}$ agree well with a simple shell model prediction, though not all of the predicted strength is seen. Such charge exchange experiments have been performed for other Zr isotopes with results in general similar to the $^{90}\text{Zr}(p, n)^{90}\text{Nb}$ case but with details which change in a complicated manner with increasing Zr mass number (Sterrenburg *et al.* 1980; Galonsky 1980). The description of Gamow-Teller resonance behavior for lighter nuclei is better understood as higher resolution (p, n) experiments have been done, for example, in $^{26}\text{Mg}(p, n)^{26}\text{Al}$ by Goodman *et al.* (1980) and in $^{48}\text{Ca}(p, n)^{48}\text{Sc}$ by Anderson *et al.* (1980). The simple-shell model systematics of Klapdor (1976) and others provides the basis for a very simple, compelling explanation for the Gamow-Teller resonance characteristics in this mass range for the $T^+ \rightarrow T^-$ transitions and will be discussed below.

The experimental observations are much less complete for the Gamow-Teller resonance systematics in the $T^- \rightarrow T^+$ transitions, e.g., $^{56}\text{Fe} + e^- \rightarrow ^{56}\text{Mn} + \nu$. The conventional measurements of $\log ft$ -values for discrete state transitions are hindered in the $T^- \rightarrow T^+$ direction by the nuclear Q -values. The T^+ daughter ground state usually lies higher in energy than the T^- parent ground state by at least the Coulomb energy difference, which is considerable in intermediate and heavy mass nuclei. There are some lighter nuclei where weak transition matrix elements have been measured for $T^- \rightarrow T^+$, but these measured matrix elements do not exhaust the Gamow-Teller sum rule. The work of Flynn, Sherman, and Stein (1974) using the $(t, ^3\text{He})$ reaction has identified $0^+ \rightarrow 1^+$ transitions in $^{28}\text{Si}(t, ^3\text{He})^{28}\text{Al}$ and $^{58}\text{Ni}(t, ^3\text{He})^{58}\text{Co}$, among others, which seem to be re-

lated to pieces of the giant M1–GT collective modes. A promising experimental technique which has so far not been employed in searches for Gamow-Teller strength is the (n, p) reaction. Though this reaction suffers in resolution (Brady and Needham 1980), it has the advantage over $(t, {}^3\text{He})$ in not being inhibited strongly by Coulomb-barrier penetration.

Some indication of the Gamow-Teller resonance characteristics for ${}^{56}\text{Fe}$ are given by the detailed shell model calculations of Bloom and Fuller (1981). In that work, the Gamow-Teller strength for the ${}^{56}\text{Fe}$ (ground state) $\rightarrow {}^{56}\text{Mn}$ (continuum) is seen to lie in a broad peak centered at 2.7 MeV in ${}^{56}\text{Mn}$, while the ${}^{56}\text{Fe}$ (first excited state) $\rightarrow {}^{56}\text{Mn}$ (continuum) is located in a broad feature at 4.6 MeV. This calculation is a good confirmation of the simple estimate of the ${}^{56}\text{Fe}$ Gamow-Teller peak energy done by BBAL, and it appears that shell model systematics suffice to predict the trend of strength and excitation energy for the Gamow-Teller resonance in the $T^< \rightarrow T^>$ direction, which is just the isospin change direction most important in neutronization and neutrino loss processes in the late stages of stellar evolution.

IV. SHELL MODEL INTERPRETATION OF GAMOW-TELLER RESONANCE CHARACTERISTICS

A shell-model description of nuclear β -decay processes based on the ideas of Gaarde *et al.* (1972), Klapdor (1976), and BBAL is used in the rate calculations presented in this work to give (1) an approximate collective Gamow-Teller ground state sum rule, and (2) an approximate daughter nucleus excitation energy for this collective state. The Gamow-Teller resonance and its strength, together with the allowed discrete state transitions, serve to outline the β -strength distribution and provide a reliable weak interaction rate at high temperature and density. The differing characteristics of the $T^< \rightarrow T^>$ and the $T^> \rightarrow T^<$ Gamow-Teller strength functions emerge naturally from this procedure, as do important predictions about the variation of collective state strengths with increasing neutron richness.

Consider first the $T^< \rightarrow T^>$ Gamow-Teller strength distribution for transitions from the ground state of an intermediate mass nucleus, e.g., ${}^{33}\text{S}(e^-, \nu_e){}^{33}\text{P}$. The contribution to the stellar rates from the collective resonances for each of the parent excited states will be discussed below. A possible zero-order shell model configuration for the ground state of ${}^{33}\text{S}$, for example, is represented in Figure 1. The valence nucleons are filling the sd -shell, the $1d_{5/2}$ and $2s_{1/2}$ orbits are closed for both protons and neutrons, and there is one unpaired neutron in the $1d_{3/2}$ orbit. The Gamow-Teller selection rules derived above allow both spin-flip and no-spin-flip transition modes, but it is clear from Figure 1 for this $T^< \rightarrow T^>$ case (i.e., continuum electron capture or positron emission) that the no-spin-flip transitions are

not possible, while only one type of spin-flip mode contributes, one of six $1d_{5/2}$ protons transforming into one of three $1d_{3/2}$ neutron holes. In reality, the ground state nuclear wave function for ${}^{33}\text{S}$ will be an admixture of many such simple shell model configurations since the residual nuclear forces polarize the $2s_{1/2}$ – $1d_{5/2}$ closed shells. The resulting holes in the $1d_{5/2}$ neutron shell could allow a no-spin-flip transition of a $1d_{5/2}$ proton into a $1d_{5/2}$ neutron hole. However, in this work it is argued that the ground state wave function is predominantly given by the zero-order model, the more complicated configurations tending to be concentrated in highly excited states due to the substantial single particle energy difference penalty required to excite particles out of closed shells.

For the purposes of these stellar rate calculations it is sufficient to estimate the Gamow-Teller spin-flip mode excitation energy in ${}^{33}\text{P}$, since, as discussed above, experiment shows that the spin-flip mode is relatively concentrated, the no-spin-flip mode being spread out in daughter nucleus excitation energy when it can occur. The Gamow-Teller collective spin-flip mode is given by

$$|\psi\rangle_{\text{CGT}} = \sum_{ij} \langle i | \sigma \tau_- | j \rangle a_i^\dagger a_j |\psi({}^{33}\text{S}^{\text{gs}})\rangle, \quad (8)$$

where $a_i^\dagger$ creates a neutron in single particle orbit i , a_j destroys a proton in single particle orbit j , and gs indicates ground state. With a simple zero-order shell model like the one shown for ${}^{33}\text{S}$ one can solve for the collective state excitation energy relative to the ${}^{33}\text{P}$ ground state. This collective spin-flip configuration (shown in Fig. 2) differs from the ${}^{33}\text{P}$ ground state by the excitation of a $1d_{5/2}$ proton to the $2s_{1/2}$ proton orbit. The equation for the collective resonance energies E_n ($n = 1, 2, \dots$) will be

$$\{\mathcal{E}_a - \mathcal{E}_b - E_n\} \psi^{(n)}(ab) + \sum_{lm} \langle ab | V_{\text{ph}} | lm \rangle \psi^{(n)}(lm) = 0, \quad (9)$$

where $\mathcal{E}_a$, in the case of ${}^{33}\text{P}$, is the single particle energy of the $2s_{1/2}$ proton orbit, $\mathcal{E}_b$ is the energy of the $1d_{5/2}$ proton orbit, V_{ph} is the component of the nucleon-nucleon force which acts in the particle-hole channel, and the $\psi^{(n)}(ab)$ are the coefficients of the particle-hole configurations in $|\psi\rangle_{\text{CGT}}$, i.e., for some n ,

$$|\psi\rangle_{\text{CGT}} \approx \left(\sum_{ij} \psi^{(n)}(ij) a_i^\dagger a_j \right) |\psi({}^{33}\text{P}^{\text{gs}})\rangle. \quad (10)$$

If there are n_p particles in the upper orbit of the final nucleus (e.g., $n_p^f = 2$ protons in $2s_{1/2}$ in ${}^{33}\text{P}^{\text{sf}}$ indicates spin-flip) and if there are n_h^f holes in the lower orbit (e.g., $n_h^f = 1$ proton hole in $1d_{5/2}$ in ${}^{33}\text{P}^{\text{sf}}$), then the

FIG. 1

FIG. 2

FIG. 1.—The zero-order shell model orbit-occupation diagram for the ground state configuration of $^{33}_{16}\text{S}_{17}$ is shown. The closed proton and neutron sp -shells (closed at 8 nucleons each) are shown at the bottom of the diagram. Other closed orbits are denoted with circles.

FIG. 2.—The zero-order shell model orbit-occupation diagram for the spin-flip configuration in $^{33}_{15}\text{P}_{18}$ is shown. This configuration can be generated from the Fig. 1 diagram by transforming a $1d_{5/2}$ proton into a hole in the $1d_{3/2}$ neutron orbit.

problem reduces to finding the eigenvectors and eigenvalues of an $n_p^f n_h^f \times n_p^f n_h^f$ matrix.

Although the residual force acting between nucleons is generally attractive, in the particle-hole channel it is usually repulsive, as can be seen by applying the Pandya relations (Schiffer and True 1976; Koonin 1978). Further, as is implicit in the discussion of β -decay in BBAL, this particle-hole repulsion energy is taken as constant in this work, and in particular

$$\langle ab | V_{ph} | lm \rangle \approx v_{ph}. \quad (11)$$

If the sum on particle-hole pairs (ab) is taken over just the orbits relevant for the spin-flip excitation (e.g., two $2s_{1/2}$ proton particles and one $1d_{5/2}$ proton hole in the $^{33}\text{P}^{sf}$ case) and the equation (11) approximation is employed, then equation (9) becomes

$$\begin{aligned} & (\mathcal{E}(2s_{1/2}) - \mathcal{E}(1d_{5/2}) - E_n) \cdot \begin{pmatrix} \psi_1^n \\ \psi_2^n \\ \vdots \end{pmatrix} \\ &= \begin{pmatrix} 1 & 1 & \dots \\ 1 & 1 & \\ \vdots & & \ddots \end{pmatrix} \begin{pmatrix} \psi_1^n \\ \psi_2^n \\ \vdots \end{pmatrix}. \end{aligned} \quad (12)$$

The eigenvalues of an $n_p^f n_h^f \times n_p^f n_h^f$ matrix of 1's are zero and $n_p^f n_h^f$, so that the collective mode eigenstate is the coherent sum of the equally weighted $\psi^n(ab)$'s:

$$|\psi\rangle_{\text{CGT}} = \frac{1}{(n_p n_h)^{1/2}} \sum_{ij} a_i^\dagger a_j |\psi(^{33}\text{P}^{gs})\rangle \quad (13)$$

corresponding to the excitation energy eigenvalue

$$\begin{aligned} E(^{33}\text{P}^{sf}) &= \mathcal{E}(2s_{1/2}) - \mathcal{E}(1d_{5/2}) + n_p^f n_h^f \cdot v_{ph} \\ &= 7.80 \text{ MeV}. \end{aligned} \quad (14)$$

This is similar to the calculation made in BBAL to arrive at the prediction of $\Delta_n = 3$ MeV in the $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ transition, which agrees very well with the detailed shell model value of 2.7 MeV from Bloom and Fuller (1981). In this work the value of $n_p^f n_h^f \cdot v_{ph}$ for each nucleus is approximated as 2.0 MeV (Bertsch 1980), and this agrees with the shell model calculations and experimental data fairly well.

The total Gamow-Teller strength in the collective mode can be estimated from equation (13). In principle, for a collective state which corresponds to an eigenstate of the daughter nucleus, the total Gamow-Teller strength

from the ground state of the parent is defined to be

$$|M_{GT}|^2 = \frac{1}{2J_i + 1} \sum_{m_{CGT}} \sum_{m_i} \sum_k |\langle \psi_{CGT} m_{CGT} | \sigma_k \tau_{\pm} | \psi_i m_i \rangle|^2. \quad (15)$$

In practice, and as experiment shows, the collective Gamow-Teller state is split among many daughter nucleus eigenstates. In this case, the collective strength is given as an incoherent sum of the strengths from each daughter state. From the expression for the collective Gamow-Teller wave function and from equation (15) it can be shown that the incoherent strength is

$$|M_{GT}|^2 = \sum_{if} \frac{n_p^i n_h^f}{2j_f + 1} |M_{GT}^{sp}|_{if}^2, \quad (16)$$

where the sum is over initial orbits i and final orbits f . The number of particles in orbit i is n_p^i , n_h^f is the number of holes in orbit f , and $2j_f + 1$ is the degeneracy of the final orbit. $|M_{GT}^{sp}|_{if}^2$ is the single particle Gamow-Teller matrix element connecting particle state i and hole state f . $|M_{GT}^{sp}|^2$ for $j_i = l \pm \frac{1}{2}$ and $j_f = l \pm \frac{1}{2}$ is given by

$$|M_{GT}^{sp}|_{if}^2 = 6(2j_f + 1) \cdot \begin{Bmatrix} 1/2 & 1/2 & 1 \\ j_i & j_f & l \end{Bmatrix}^2. \quad (17)$$

As shown in deShalit and Feshbach (1974), the six- j symbol is easily evaluated to yield the table of $|M_{GT}^{sp}|^2$ shown in Table 1. For instance, the single particle matrix element table gives $|M_{GT}^{sp}|^2 = 12/7$ for $f_{7/2} \rightarrow f_{5/2}$. In general for each particle orbit j_i , there are two possible final hole orbits, corresponding to no-spin-flip (nsf), $j_f = j_i$, and spin-flip (sf), $j_f = j_i \pm \frac{1}{2}$. The single valence particle orbit case then gives, from equation (16),

$$|M_{GT}|^2 = \frac{n_p n_h^{nsf}}{2j_f^{nsf} + 1} |M_{GT}^{sp}|_{nsf}^2 + \frac{n_p n_h^{sf}}{2j_f^{sf} + 1} |M_{GT}^{sp}|_{sf}^2. \quad (18)$$

Note that the ratio $n_h/(2j_f + 1)$ is the fractional number of holes available in the parent nucleus. For positron emission and electron capture decay, each term in equation (18) is the number of protons in the valence orbit times the fractional number of holes in the neutron orbit, times the characteristic single particle matrix element. In the case of electron emission or positron capture the roles of neutrons and protons are reversed.

An example of the application of the expression in equation (16) is given by the zero-order shell model picture for ^{33}S presented in Figure 1. Consider first $^{33}\text{S}(e^-, \nu_e)^{33}\text{P}$. There are six valence protons in the $1d_{5/2}$ orbit, and two in the $2s_{1/2}$ orbit. Deeper lying proton orbits need not be considered, as these give zero

TABLE 1
SINGLE-PARTICLE GAMOW-TELLER
MATRIX ELEMENTS

j_i	j_f	
	$l+1/2$	$l-1/2$
$l+1/2 \dots$	$\frac{j_i+1}{j_i}$	$\frac{2j_i-1}{j_i}$
$l-1/2 \dots$	$\frac{2j_i+3}{j_i+1}$	$\frac{j_i}{j_i+1}$

β -strength; that is, all corresponding neutron orbits are filled (blocked), the Gamow-Teller operator cannot change principal quantum number, and hence cannot allow particle transitions between major shells. Likewise, the no-spin-flip transition is not allowed for the six $1d_{5/2}$ protons in ^{33}S as there are no neutron holes in $1d_{5/2}$. Since the Gamow-Teller operator cannot change orbital angular momentum, the two protons in $2s_{1/2}$ can only transform into $2s_{1/2}$ neutron holes, of which there are none in ^{33}S . These two protons give zero β -strength. This is an example of neutron blocking, which occurs whenever all final allowed orbits for valence protons are completely filled by neutrons. A similar blocking can occur in electron emission or positron capture when there are no allowed proton-hole slots for valence neutron transitions. In the example of ^{33}S , the only unblocked transition is for $1d_{5/2}$ protons transforming into neutron holes in the $1d_{3/2}$ orbital, a spin-flip transition. The single particle matrix element for $1d_{5/2} \rightarrow 1d_{3/2}$ is $|M_{GT}^{sp}|^2 = 8/5$, so that with $n_p = 6$ and $n_h/2j_f + 1 = \frac{3}{4}$ one has

$$|M_{GT}|^2 = 6 \cdot 3/4 \cdot 8/5 = 36/5, \quad (19)$$

which, from equation (I-2a) gives $\log ft = 2.739$. For the reverse transition $^{33}\text{P}(e^-, \bar{\nu}_e)^{33}\text{S}$, the spin-flip decay of six $1d_{5/2}$ neutrons to four $1d_{3/2}$ proton holes contributes, as do two no-spin-flip transitions: two $1d_{3/2}$ neutrons into four $1d_{3/2}$ proton holes, and two $2s_{1/2}$ neutrons in one $2s_{1/2}$ proton hole. Applying equation (16) and equation (I-2a) gives a total $|M_{GT}|^2 = 69/5$ so that $\log ft = 2.456$. This is faster than the $\log ft = 2.739$ for the positron emission, electron capture transitions because ^{33}P is a neutron-rich nucleus and therefore has more unblocked neutron particles than ^{33}S has unblocked proton particles. In the case of large neutron excess, where both spin-flip and no-spin-flip proton orbits corresponding to the filled or partially filled neutron orbits are empty, then $n_h/(2j_f + 1) = 1$, and equation (16) reduces to

$$|M_{GT}|^2 = \sum_i n_p^i \{ |M_{GT}^{sp}|_{isf}^2 + |M_{GT}^{sp}|_{insf}^2 \} = 3 \cdot |N - Z|, \quad (20)$$

where $N - Z$ is the difference in neutron and proton numbers for the parent nucleus. With the use of the absolute value of $N - Z$, equation (20) is generalized to the case of unblocked proton orbits for $Z > N$. This is the expression for $|M_{GT}|^2$ derived by Gapanov and Lyutostanskii (1974). Equation (20) makes good physical sense, since it states that there are $|N - Z|$ completely unblocked nucleons, and, since each free nucleon has an unblocked Gamow-Teller matrix element $|M_{GT}|^2 = 3$, the total Gamow-Teller strength is $3 \cdot |N - Z|$. Equation (I-11) for the Fermi transition matrix element can be recast, using $T_z^i = (Z - N)/2$ as

$$|M_F|^2 = |N - Z| \quad (21)$$

and again, since for a free nucleon $|M_F|^2 = 1$, equation (21) is easily interpreted. The total β -strength possessed by a free nucleon remains the same when placed in a nucleus, and only blocking can make this strength unavailable.

In comparing to the moment-method shell model calculations done by Bloom and Fuller (1981) the zero-order shell model result of equation (16) does quite well. The moment-method calculations for electron capture in $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ yields $|M_{GT}|^2 = 9.125$ ($\log ft = 2.64$) for the total beta strength from the 0^+ ground state of ^{56}Fe , and $|M_{GT}|^2 = 9.656$ ($\log ft = 2.61$) from the 2^+ first excited state of ^{56}Fe . The application of equation (16) to the zero-order shell model ground configuration of ^{56}Fe yields $|M_{GT}|^2 = 72/7 = 10.29$ ($\log ft = 2.58$), which agrees well with the moment-method shell model results. In the present paper the strengths for collective state resonances corresponding to the excited states of the parent nucleus are taken to be the same as for the ground state i.e., the zero-order shell model result, equation (16). This conjecture will be discussed below, but note that it seems to be borne out in the ^{56}Fe results. For $^{60}\text{Fe} \rightarrow ^{60}\text{Mn}$ the zero-order shell model and equation (16) predict $|M_{GT}|^2 = 48/7 = 6.86$ ($\log ft = 2.76$), whereas Bloom and Fuller calculate $|M_{GT}|^2 = 7.33$ ($\log ft = 2.73$) from the ground state and $|M_{GT}|^2 = 7.04$ ($\log ft = 2.75$) from the first excited state of ^{60}Fe , and again good agreement obtains. In the case of electron capture in $^{64}\text{Fe} \rightarrow ^{64}\text{Mn}$ the Bloom and Fuller result gives $|M_{GT}|^2 = 1.06$ ($\log ft = 3.57$) from the ^{64}Fe ground state and $|M_{GT}|^2 = 1.05$ ($\log ft = 3.57$) from the ^{64}Fe first excited state. The present paper uses the energy and ordering of the shell model orbits for protons and neutrons given by Seeger and Howard (1975). The $1f_{5/2}$ neutron orbit is then taken to lie lower than the $2p_{1/2}$ neutron orbit. Since there are 38 neutrons in ^{64}Fe , in the zero-order shell model employed here, the $1f_{5/2}$ neutron orbit is completely filled, the $^{64}\text{Fe} \rightarrow ^{64}\text{Mn}$ transition is blocked, and the application of equation (16) yields $|M_{GT}|^2 = 0$. The Bloom and Fuller strength result is small but not zero, corresponding to the value of a

typical Gamow-Teller transition which reflects the mixing of configurations due to the residual nucleon-nucleon interaction. The Bloom and Fuller $^{64}\text{Fe} \rightarrow ^{64}\text{Mn}$ strength is about a factor of 10 less than equation (16) would predict were the $2p_{1/2}$ neutron orbit to lie lower in energy than the $1f_{5/2}$ neutron orbit in the zero-order model employed here, opening neutron holes in the $1f_{5/2}$ shell.

The $^{54}\text{Fe}(^3\text{He}, t)^{54}\text{Co}$ experiment performed and analyzed by Gaarde *et al.* (1980) also tends to confirm the validity of equation (16). A two-particle shell model calculation of the total weak interaction strength in the $^{54}\text{Fe} \rightarrow ^{54}\text{Co}$ transition made by these authors yields $|M_{GT}|^2 = 13.8$, which agrees well with the total weak interaction strength inferred from the $^{54}\text{Fe}(^3\text{He}, t)^{54}\text{Co}$ experiment. In a zero-order shell model picture of ^{54}Fe , in which the valence fp -shell has six protons in the $1f_{7/2}$ proton orbit and eight neutrons in the $1f_{7/2}$ neutron orbit, equation (16) yields $|M_{GT}|^2 = 114/7 = 16.3$ (note that Gaarde *et al.* also quote $|M_{GT}|^2 = 16.3$ derived from an expression similar to equation [20]). The sum rule expression employed in the present paper and given by equation (16) then seems to give good agreement with theoretical and experimental results.

The simple shell model utilized in this work serves to provide a weak interaction sum rule for $T^< \rightleftharpoons T^>$ as outlined above and, in addition, gives a simple interpretation for the location of the $T^< \rightarrow T^>$ Gamow-Teller resonance. In particular the shell model procedure predicts the daughter nucleus excitation energy of the Gamow-Teller collective state resonance in $T^< \rightarrow T^>$ in terms of single particle energy differences for spin-flip and an average particle-hole repulsion energy. The accuracy of these predicted excitation energies will be examined below, but first the application of the shell model to the characteristics of the $T^> \rightarrow T^<$ Gamow-Teller strength distribution must be addressed.

The $T^> \rightarrow T^<$ and $T^< \rightarrow T^>$ Gamow-Teller strength distributions differ markedly, as the experimental data and theoretical results outlined previously indicate. The principal difference between these distributions reflects the fact that in the $T^> \rightarrow T^<$ transition both $T^<$ and $T^>$ states in the $T^<$ daughter nucleus may be populated by the Gamow-Teller operator. To understand how these differences arise in the context of the simple shell model requires a discussion of the so-called M1 method for determining the resonance excitation energies for both $T^< \rightarrow T^>$ and $T^> \rightarrow T^<$ transitions.

Following Klapdor, the M1 method for $T^> \rightarrow T^<$ nuclear transitions is best explained in terms of a doubly magic, neutron-rich parent nucleus. A schematic shell model diagram for such a nucleus is shown in Figure 3a. In Figure 3b an excited configuration of this nucleus is shown, wherein a neutron is promoted from a completely filled $j = l + \frac{1}{2}$ single particle orbit to the higher-lying $j = l - \frac{1}{2}$ orbit. This is the M1 excitation, and the

FIG. 3.—Doubly magic nucleus with $N > Z$. (a) The zero-order ground state configuration diagram for a neutron-rich nucleus. (b) An excited configuration for this nucleus wherein a neutron is promoted from a $j = l + \frac{1}{2}$ orbit to a higher-lying, empty, $j = l - \frac{1}{2}$ orbit.

configuration in Figure 3b represents the giant M1 configuration. Another such giant M1 configuration could be constructed by promoting a proton in a single particle orbit up to its unfilled spin-orbit partner. Such proton M1 configurations will be discussed below in connection with $T^< \rightarrow T^>$ transitions of neutron-rich nuclei.

Returning to the neutron M1 configuration in the doubly magic neutron-rich nucleus, note that the excitation energy of this configuration relative to the Figure 3a ground state could be calculated via an expression involving single particle energy differences and a particle-hole repulsion as in equation (12). In this case the excitation would amount to the difference in energy between the filled $j = l + \frac{1}{2}$ neutron orbital and its empty $j = l - \frac{1}{2}$ spin-orbit partner, plus the particle-hole repulsion energy and a pairing energy if a neutron pair must be broken in the excitation. Thus, the excitation energy of this neutron M1 configuration can be found in the $T^>$ parent nucleus and, furthermore, its isospin must be equal to $T^>$.

The neutron M1 configuration in Figure 3b can now be operated on with the isospin raising operator

$$T_+ = \sum_i \tau_+^i$$

(in this work, $\tau_3^n = -\frac{1}{2}$, $\tau_3^p = +\frac{1}{2}$), where the sum on i is over all neutrons. The operator τ_+ acting on a neutron creates a proton in the same orbit with a factor 1; hence acting on the Figure 3b configuration with T_+ creates the $T = T^>$ configuration mixture shown in Figure 4a, the coefficients multiplying each component configuration following from the number of neutrons in each orbit with appropriate normalization. (T_0 is defined such that $2T_0 + 1 = |N - Z|$ for the original $T^>$ nucleus shown in Fig. 3.) The result shown in Figure 4a is the isobaric

analog state (IAS) in the $T^<$ daughter corresponding to the neutron M1 excitation in the $T^>$ parent. For very neutron-rich nuclei, where T_0 is large but with the $j = l - \frac{1}{2}$ orbit empty, the analog of the M1 configuration will clearly consist predominantly of the second of the two configurations shown in Figure 4a.

The anti-isobaric-analog state (AIAS) corresponding to the M1 analog in Figure 4a can be constructed from the same basis configurations by using orthogonality. Interchanging the coefficients and reversing the sign between the configurations in Figure 4a results in the orthogonal AIAS configuration shown in Figure 4b, which must have isospin $T = T^<$. Because of the reversal of coefficients, the M1-AIAS will be predominantly the first configuration in Figure 4b for very neutron-rich nuclei.

Note that the first, predominant configuration in the Figure 4b M1-AIAS is just the spin-flip configuration generated from the $T^>$ parent ground state configuration, Figure 3a, by transforming a neutron with $j = l + \frac{1}{2}$ into an empty proton orbit with $j = l - \frac{1}{2}$. As discussed in § IIIa, the experimental and theoretical results point to this spin-flip configuration as the carrier of most of the relatively concentrated Gamow-Teller strength. Therefore, for this doubly magic neutron-rich nucleus, the Gamow-Teller resonance could be expected to lie near the excitation energy of the AIAS of the M1 excitation, since for large T_0 the M1-AIAS contains most of the spin-flip configuration.

The excitation energy of the M1-AIAS can be estimated in the following way. The excitation energy of the M1 configuration in the $T^>$ parent is calculated as previously discussed. The IAS of this M1 excitation in the $T^<$ daughter has an excitation energy which is the energy of the first analog state plus the M1 excitation energy in the $T^>$ parent. The M1-AIAS differs from the M1-IAS only in total isospin; other than that, both states contain the same configurations. In the absence of an isospin-dependent residual interaction, the M1-IAS and M1-AIAS would be degenerate in energy. In fact, the nucleon-nucleon force in equation (4) contains a $\tau_1 \cdot \tau_2$ term, and Lane (1962) has shown that this results in an isospin potential, the Lane potential, which splits otherwise identical states differing in isospin. The form of the Lane potential used here is

$$V = \frac{V_\tau (2T_0 + 1)}{2A} \approx \frac{50}{A} |N - Z|, \quad (22)$$

and the M1-AIAS lies lower than the M1-IAS by the value of this potential.

To summarize the proposed method for finding the excitation energy of the Gamow-Teller resonance in the $T^<$ daughter, the $T^>$ parent M1 excitation energy is added to the first IAS energy in the $T^<$ daughter, yielding the M1-IAS excitation energy, from which is

FIG. 4a

FIG. 4b

FIG. 4.—(a) The neutron M1 analog is shown for a neutron-rich nucleus. This configuration can be generated from the excited configuration in Fig. 3b by operating with T_+ . (b) The neutron M1 anti-analog corresponding to the M1 analog in Fig. 4a is shown. This configuration can be generated from that in Fig. 4a by noting that the anti-analog state must be identical to the analog state in all quantum numbers save total isospin T . It is therefore orthogonal to the Fig. 4a configuration.

subtracted the Lane potential, to finally yield the excitation energy of the M1-AIAS containing the greater part of the Gamow-Teller strength. As will be made clear in the example below, this is not quite the procedure adopted for $T^> \rightarrow T^<$ transitions in this paper.

The method for computing the parameters of the $T^> \rightarrow T^<$ Gamow-Teller resonance adopted later will, however, require the first analog state excitation energy in the $T^<$ nucleus. The first analog state energy is taken from experiment where available, or else calculated using the parametrized formulae of Fowler and Woosley (1982). For a nucleus with Z_A protons, N_A neutrons, and mass excess ΔM_A , which has an analog state in a nucleus with Z_C protons, N_C neutrons, and mass excess ΔM_C , the expression for the analog state excitation energy E_{IAS} for $N_C \geq Z_C$ and $Z_A = Z_C - 1$, $N_A = N_C + 1$, is

$$E_{IAS} = \Delta M_A - \Delta M_C - 0.7824 + \frac{1.728(Z_C - 1)}{R} \text{ MeV} \quad (23)$$

where $(6/5)e^2 = 1.728$ MeV-fermis and the nuclear radius R (for both nuclei) is in fermis. Fowler and Woosley (1982) give $R = 1.12A^{1/3} + 0.78$ fermis. This is the customary way to calculate E_{IAS} . However, in some cases ΔM_A is known for a proton-rich nucleus across the $Z = N$ line which can yield the energy of the $T^>$ analog in the $T^<$ nucleus. In this case $N_C \geq Z_C$ and $Z_A = N_C + 1$

so that

$$E_{IAS} = \Delta M_A - \Delta M_C + 0.7824(Z_A - Z_C) - 0.864 \frac{N_C(N_C + 1) - Z_C(Z_C - 1)}{R} \text{ MeV.} \quad (24)$$

For $N_C \leq Z_C$ and $Z_A = Z_C + 1$ the result is

$$E_{IAS} = \Delta M_A - \Delta M_C + 0.7824 - \frac{1.728 Z_C}{R} \text{ MeV;} \quad (25)$$

and, finally, for $N_C \leq Z_C$ and $Z_A = N_C - 1$ the result is

$$E_{IAS} = \Delta M_A - \Delta M_C - 0.7824(Z_C - Z_A) + 0.864 \frac{[Z_C(Z_C - 1) - (N_C - 1)(N_C - 2)]}{R} \text{ MeV.} \quad (26)$$

In the application of the M1 method to the neutron-rich nucleus discussed above, it was clear that most of the Gamow-Teller spin-flip configuration was concentrated in the M1-AIAS. For less neutron-rich nuclei, including many of those encountered in this survey, this

FIG. 5.—The zero-order orbit-occupation diagram for the ground state configuration of $^{33}\text{Ar}_{15}$ is shown. Notation is as in Fig. 1. Note that ^{33}Ar is proton rich.

is not always the case and a significant portion of the spin-flip configuration may be contained in the M1-IAS. The M1-IAS lies at a higher excitation energy than the M1-AIAS, and so the question of the splitting of the Gamow-Teller strength between them is an important point to resolve in calculating the stellar rates.

The question of M1-IAS/M1-AIAS Gamow-Teller strength splitting in $T^+ \rightarrow T^-$ transitions is best illuminated with an example, $^{33}\text{Ar}_{15} \rightarrow ^{33}\text{Cl}_{16}$. This is a $T^+ \rightarrow T^-$ transition corresponding to positron emission or electron capture, and has a ground state to ground state nuclear mass difference of 11.107 MeV, so that a considerable number of discrete transitions from the ^{33}Ar ground state have been measured.

The zero-order shell model ground state for ^{33}Ar is shown in Figure 5. The proton M1 excitation in ^{33}Ar can be made by promoting a $1d_{5/2}$ proton up to the partially filled $1d_{3/2}$ proton orbit. If now the usual M1 method is employed, the M1-IAS in ^{33}Cl can be constructed by operating on the ^{33}Ar proton M1 excitation with T^- . This schematic diagram for the M1-IAS is shown in Figure 6. Note that the first configuration in the M1-IAS state in ^{33}Cl corresponds to the spin-flip configuration which can be made from the ^{33}Ar ground state configuration by transforming a $1d_{5/2}$ proton into a $1d_{3/2}$ neutron. Of course the lower-lying M1-AIAS would consist of a mixture of the same configurations, but with the coefficients interchanged and the relative signs reversed. The $^{33}\text{Ar} \rightarrow ^{33}\text{Cl}$ reaction is an example of one in which most of the Gamow-Teller spin-flip strength is concentrated closer to the M1-IAS, rather than to the M1-AIAS.

The equation (14) result can be used to estimate the excitation energy of the M1-IAS and M1-AIAS in the following manner. First, the proton M1 excitation in ^{33}Ar would have an excitation energy of

$$E^{M1}(^{33}\text{Ar}) = \mathcal{E}(1d_{3/2}) - \mathcal{E}(1d_{5/2}) + \Delta E_{\text{pair}} + \Delta E_{\text{ph}}$$

where the difference in the proton single particle energies is taken from the tabulation of Seeger and Howard (1975), the particle hole repulsion energy, $\Delta E_{\text{ph}} \equiv n_p n_h \nu_{\text{ph}}$, is taken as 2.0 MeV, and, since a proton pair is broken in the proton M1 excitation energy, a pairing energy of $\Delta E_{\text{pair}} = 12/A^{1/2}$ is added (Klapdor 1976). Since the first analog state in ^{33}Cl is known to lie at an excitation energy of 5.544 MeV, the M1-IAS will have an excitation energy in ^{33}Cl of $E_{\text{IAS}}^{M1}(^{33}\text{Cl}) = 9.1717 + 5.544 = 14.715$; whereas the M1-AIAS will lie at this excitation energy less the value of the Lane potential in equation (22), or $E_{\text{AIAS}}^{M1} = 14.715 - (50/A)|N - Z| =$

FIG. 6.—The orbit-occupation diagram for the ^{33}Cl proton M1 analog state, which can be generated from the Fig. 5, ^{33}Ar ground state, configuration by the promotion of a proton from $1d_{5/2}$ to $1d_{3/2}$ and the application of T^- .

10.169 MeV. In the simple zero-order shell model applied here, 75% of the Gamow-Teller spin-flip strength should be carried by the M1-IAS, while the other 25% would be concentrated on the M1-AIAS.

If the usual M1 method were employed, all of the strength would be lumped in a resonance near the M1-AIAS at 10.169 MeV excitation. The application of the equation (16) sum rule to the ground state configuration of ^{33}Ar (Fig. 5) yields $|M_{\text{GT}}^{\text{sf}}|^2 = 48/5 = 9.6$, $|M_{\text{GT}}^{\text{nsf}}|^2 = 21/5 = 4.2$ and $|M_{\text{GT}}|^2 = 69/5 = 13.8$. Since the Gamow-Teller resonance is believed to be very broad, as discussed in the last section, it would be reasonable to expect a considerable amount of strength below the resonance at 10.169 MeV. In contrast, only $\log(\Sigma 1/ft)^{-1} = 4.09$ ($|M_{\text{GT}}|^2 = 0.321$) is measured from 0 to 5.446 MeV excitation in ^{33}Cl and only $\log(\Sigma 1/ft)^{-1} = 3.49$ ($|M_{\text{GT}}|^2 = 1.276$) from 5.675 to 8.969 MeV (Endt and van der Leun 1978, Table 33.21). In other words, approximately 12% of the total sum rule strength is located at excitation energies less than 8.969 MeV. This is inconsistent with the broad Gamow-Teller resonance being located near the M1-AIAS and indicates that the zero-order shell model is correct in predicting the Gamow-Teller strength peak nearer the M1-IAS at 14.715 MeV.

The shell structure in the $^{33}\text{Ar} \rightarrow ^{33}\text{Cl}$ transition which results in the Gamow-Teller strength being concentrated more on the M1-IAS rather than the M1-AIAS is common to many of the $T^+ \rightarrow T^-$ transitions considered in this survey of weak rates. In light of this fact, the following procedure is adopted here for computing the excitation energy of the Gamow-Teller resonance in all $T^+ \rightarrow T^-$ transitions considered in this work. First, a zero-order shell model diagram is constructed for the ground state of the T^+ parent. The appropriate Gamow-Teller spin-flip configuration in the T^- daughter is constructed as outlined above. The excitation energy of this spin-flip configuration relative to the T^- daughter ground state is reckoned in the usual manner using equation (12), with the addition that in the case of $T^+ \rightarrow T^-$ transitions, the first analog state energy is added. The equation (16) sum rule strength is computed for the T^+ parent ground state, all discrete state transition strength included in the stellar rate computation is subtracted from the sum rule strength, and the remaining strength is lumped into the Gamow-Teller resonance at the calculated energy.

The procedure for the $^{33}\text{Ar} \rightarrow ^{33}\text{Cl}$ transition is illustrative of this process. The Gamow-Teller spin-flip configuration generated from the ^{33}Ar ground state is shown in the first configuration in Figure 6. It differs from the zero-order ^{33}Cl ground state by the excitation of a $1d_{5/2}$ proton to the $1d_{3/2}$ proton orbit and a $2s_{1/2}$ neutron to the $1d_{3/2}$ neutron orbit, by the breaking of a $2s_{1/2}$ neutron pair, and by a particle-hole excitation

energy, so that the modified equation (12) result will be

$$\begin{aligned} E(^{33}\text{Cl}^{\text{GT}}) &= E(^{33}\text{Cl}^{\text{sf}}) = [\mathcal{E}(1d_{3/2}) - \mathcal{E}(1d_{5/2})]_p \\ &\quad + [\mathcal{E}(1d_{3/2}) - \mathcal{E}(2s_{1/2})]_n \\ &\quad + \Delta E_{\text{ph}} + \Delta E_{\text{pair}} + E_{\text{IAS}} \\ &= \frac{16.3}{(33)^{1/3}} + \frac{4.8}{(33)^{1/3}} + 2.0 + 2.089 + 5.544 \\ &= 16.211 \text{ MeV}, \end{aligned} \quad (28)$$

where the Seeger and Howard (1975) single particle energies are used along with the usual formula for the pairing energy, and the first analog state energy, 5.544 MeV, is included.

This is the procedure followed in this paper for all $T^+ \rightarrow T^-$ transitions. Adding in the energy of the first IAS yields an estimated Gamow-Teller excitation energy which may be too high relative to the values given by the M1 method. The experimental evidence in the $^{33}\text{Ar} \rightarrow ^{33}\text{Cl}$ transition is that most of the Gamow-Teller strength must lie well beyond 9 MeV, and, as shown, the shell model estimate predicts most of the strength near the M1-IAS at 14.7 MeV, and so the $F^2\text{N II}$ method is still somewhat high compared to the M1 method but is not in disagreement with the limited experimental data which do not reveal the full sum-rule strength. Additional experiments are needed.

Of course, for $T^+ \rightarrow T^-$ transitions in very proton or neutron-rich nuclei, most of the Gamow-Teller spin-flip strength will lie closer to the M1-AIAS state than the M1-IAS state, and the $F^2\text{N II}$ procedure will be in greater disagreement with the M1 method. In these cases the $F^2\text{N II}$ result will lie higher by the first IAS energy or, more accurately, by the Lane potential. From the discussion in the last sections it was clear that the Gamow-Teller strength is expected to lie at excitation energies above the first IAS in the T^- daughter.

The $^{90}\text{Zr}(p, n)^{90}\text{Nb}$ data obtained by Bainum *et al.* (1980) was discussed above in connection with the behavior of the strength distribution in $T^+ \rightarrow T^-$ transitions. These authors observe a broad $T^- = 4$ Gamow-Teller spin-flip peak centered at 8.7 MeV in ^{90}Nb , a few MeV above the first IAS at 5.1 MeV and the $T^+ = 5$ peak at 13.4 MeV. Several smaller Gamow-Teller non-spin-flip features were reported below the main peaks. The application of the equation (16) sum rule to the ^{90}Zr ground state yields a total Gamow-Teller strength of $|M_{\text{GT}}|^2 = 30$, yet Bainum *et al.* observe only 38% of this strength below 20 MeV excitation. The remainder of the strength lies at very high excitation energy as discussed in Bertsch (1981) and Bohr and Mottelson (1981).

The F^2N II $T^> \rightarrow T^<$ resonance procedure would put this strength at 11.85 MeV. The M1 method places 53.3% of this strength at 8.938 MeV and 5.3% at 14.493 MeV. The remainder, the non-spin-flip strength, is spread over a large range in energy. It is clear in this case that both methods of calculation are in error. However, the placing of the full strength at a single resonance energy equal to 11.85 MeV compensates approximately for the observed spread from 8.7 MeV to greater than 20 MeV.

In the very neutron- or proton-rich $T^> \rightarrow T^<$ transitions in which the F^2N II procedure places the strength too high, the first IAS in the $T^<$ daughter will lie higher in excitation above the transitions available to the $T^>$ parent states, i.e., E_{IAS} (in $T^<$) $> Q_n(T^> - T^<)$. In such circumstances the placement of the Gamow-Teller resonance in the $T^<$ daughter is not crucial; what is crucial is the strength within the range of the nuclear Q -value, and the F^2N II stellar rate computations attempt to take this strength into account with discrete state transitions with observed or calculated $\log ft$ values or with $\log ft = 5$ assumed on average for allowed transitions.

An example of a $T^> \rightarrow T^<$ transition treated in this survey in which discrete state transition strength plays an important role is the positron capture transition $^{26}\text{Mg}(T=1) \rightarrow ^{26}\text{Al}(T=0)$. Of course, positron capture is of little interest in the applications of our results. But this example has experimental strength functions from the $^{26}\text{Mg}(p, n)^{26}\text{Al}$ reaction studies. The zero-order shell model state for ^{26}Mg consists of a filled $1d_{5/2}$ neutron orbit and two holes in the $1d_{5/2}$ proton orbit, and is shown in Figure 7a. Figure 7b shows the excited spin-flip configuration in daughter ^{26}Al , wherein one of six $1d_{5/2}$ neutrons in ^{26}Mg has transformed into any of four $1d_{3/2}$ proton holes. The ^{26}Al zero-order ground state

configuration is shown in Figure 8, and it is clear that the spin-flip configuration differs from it by the excitation of an unpaired $1d_{5/2}$ proton to the $1d_{3/2}$ orbit. In accordance with the F^2N II resonance procedure for $T^> \rightarrow T^<$ transitions, the excitation energy of the Gamow-Teller resonance in ^{26}Al is calculated as

$$E(^{26}\text{Al}^{GT}) = [\mathcal{E}(1d_{3/2}) - \mathcal{E}(1d_{5/2})]_p + \Delta E_{ph} + E_{IAS} \\ = \frac{73.6 - 57.3}{(26)^{1/3}} + 2.0 + 0.228 = 7.730 \text{ MeV}, \quad (29)$$

where $E_{IAS} = 0.228$ MeV is the energy of the first $0^+; 1$ state in ^{26}Al . Note that ^{26}Al is relatively atypical in having such a low-lying analog state.

The result obtained above in the F^2N II resonance calculation for $^{26}\text{Mg} \rightarrow ^{26}\text{Al}$ can be compared with the recent results of the $^{26}\text{Mg}(p, n)^{26}\text{Al}$ charge exchange experiment done by Goodman *et al.* (1980). Their Gamow-Teller strength distribution consists of three broad peaks. The first of these peaks is resolved into three narrow peaks which correspond to the three discrete $J=1^+$ states in ^{26}Al at 1.058, 1.851, and 2.072 MeV excitation energy, respectively. The $\log ft$ and $|M_{GT}|^2$ for the transitions from the ground state of ^{26}Mg to these three ^{26}Al states can be obtained from isospin symmetry from measurements made in the $^{26}\text{Si} \rightarrow ^{26}\text{Al}$ system (see discussion on p. 453 of F^2N I). The other broad peaks in the Goodman *et al.* spectrum occur at 5.0 and 10.6 MeV.

In the stellar rate computations for $^{26}\text{Mg} \rightarrow ^{26}\text{Al}$ in this work the mirror transitions to the three discrete states are, of course, explicitly included. In accordance with the above procedure, the discrete strength is

FIG. 7.—(a) The zero-order orbit-occupation diagram for the ground state configuration of $^{26}_{12}\text{Mg}_{14}$. (b) The zero-order orbit-occupation diagram of the $^{26}_{13}\text{Al}_{13}$ spin-flip configuration generated from (a) by the transformation of a $1d_{5/2}$ neutron into a $1d_{3/2}$ proton hole.

FIG. 8.—The zero-order orbit-occupation diagram for the ground state configuration of $^{26}\text{Al}_{13}$.

summed, subtracted from the equation (16) sum rule, and the remaining strength placed at 7.73 MeV. This resonance then should mock-up the two higher-lying strength peaks in the Goodman *et al.* spectrum. The rough energy centroid of the strength in these peaks is at about 8 MeV, so the agreement with the F^2N II $T^> \rightarrow T^<$ resonance procedure is good. For $T^> \rightarrow T^<$ transitions in which the energy of the first IAS is larger than in this case, the F^2N II procedure will not give such good agreement, but it is argued here that the discrete state transitions will then determine the stellar rates.

It should be noted that the $T^< \rightarrow T^>$ transitions can also be examined with the M1 method. The procedure is to operate on the spin-flip configuration in the $T^>$ daughter with $T^\pm$ to generate its analog in the $T^<$ parent. T^+ operates on nuclei with $N > Z$, and T^- operates on nuclei with $Z > N$. Orthogonality can be used to generate one or more spin-flip-anti-analog states with $T = T^<$. These anti-analog states will contain most of the M1 excitation configuration. The excitation energy of the M1 excitation and hence the anti-analog states can be estimated by the usual techniques, and then the Lane potential can be added to yield the energy of the analog state. Subtraction of the first analog state energy from the estimated spin-flip-analog energy yields an estimate of the excitation of the spin-flip configuration in the $T^>$ daughter. The M1 method for studying $T^< \rightarrow T^>$ transitions is discussed in connection with ($t, {}^3\text{He}$) reactions by Flynn and Garrett (1972). Though complicated, this method confirms and illuminates the $T^< \rightarrow T^>$ resonance procedure adopted in this rate survey.

To illustrate both the M1 method and the F^2N II technique for treating $T^< \rightarrow T^>$ transitions, consider the

electron capture reaction ${}^{56}\text{Fe} \rightarrow {}^{56}\text{Mn}$ at high electron Fermi energies. The zero-order shell model ground states for ${}^{56}\text{Fe}(T=2)$ and ${}^{56}\text{Mn}(T=3)$ are shown in Figures 9a and 9b. The ${}^{56}\text{Mn}$ spin-flip configuration can be constructed from the ${}^{56}\text{Fe}$ ground state configuration by transforming one of six $1f_{7/2}$ protons into a neutron in the empty $1f_{5/2}$ neutron orbital. This spin-flip configuration is shown in Figure 10.

Turning first to the M1 method, the ${}^{56}\text{Mn}$ spin-flip configuration can be operated on with T^+ in the usual manner to yield the $T^>$ spin-flip analog state in ${}^{56}\text{Fe}$, shown in Figure 11, with the subscript 1 coefficients. Two $T^<$ anti-analog states can now be constructed using orthogonality. (In our example $T^>=3$, $T^<=2$). These states, designated $T^<_+$ and $T^<_-$, are shown in Figure 11 with the appropriate coefficients multiplying the $|1\rangle$, $|2\rangle$, and $|3\rangle$ basis configurations. The $T^<$ spin-flip anti-analog state contains two-thirds of the proton M1 excitation in ${}^{56}\text{Fe}$, the $|1\rangle$ configuration. The excitation energies of the $T^<_+$ and $T^<_-$ states can be estimated in the usual manner by comparing the basis configurations to the ${}^{56}\text{Fe}$ ground state configuration, applying the modified equation (12) result, and weighting the energy for each basis configuration by the appropriate coefficients in $T^<_-$ or $T^<_+$.

Carrying out this procedure with the Seeger and Howard (1975) single particle energies yields the following excitation energies for the spin-flip anti-analog states:

$$E(T^<_+) = 10.756 \text{ MeV}, \quad E(T^<_-) = 10.037 \text{ MeV}.$$

(30)

The $T^>$ spin-flip analog and the $T^<_+$ anti-analog differ

FIG. 9.—(a) The ${}^{56}\text{Fe}_{30}$ ground state configuration zero-order orbit-occupation diagram. Note the closed proton and neutron sd and sp-shells (closed at 20 nucleons each) indicated at the bottom of the diagram. (b) The ${}^{56}\text{Mn}_{31}$ ground state configuration zero-order orbit-occupation diagram.

FIG. 10.—The zero-order orbit-occupation diagram for the spin-flip configuration in ^{56}Mn is shown. This configuration is generated from that in Fig. 9a by the transformation of a $1f_{7/2}$ proton into a $1f_{5/2}$ neutron hole.

only in total isospin, since both states consist of the same three basis configurations with coefficients of the same magnitude. As a result, these states are separated by the value of the Lane potential, and thus the excitation of the spin-flip analog is

$$E(T^+) = 10.756 + \frac{50 \times 6}{56} = 16.113 \text{ MeV.} \quad (31)$$

Subtracting the excitation energy of the first analog state in ^{56}Fe , 11.440 MeV, gives the energy of the ^{56}Mn

spin-flip configuration as

$$E(^{56}\text{Mn}^{\text{sf}}) = 4.673 \text{ MeV.} \quad (32)$$

The technique adopted in this paper for computing the parameters of the Gamow-Teller resonance for $T^+ \rightarrow T^+$ transitions can now be applied to the $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ reaction. In the F^2N II technique, as previously outlined, the spin-flip configuration in the T^+ daughter is constructed, and its excitation energy relative to the T^+ daughter ground state is estimated as in equation (12). The ^{56}Mn spin-flip configuration in Figure 10 differs from the ^{56}Mn ground state, Figure 9b, by the excitation of a $2p_{3/2}$ neutron to the $1f_{5/2}$ neutron orbit, with no nucleon pair broken, so that

$$\begin{aligned} E(^{56}\text{Mn}^{\text{sf}}) &= [\epsilon(1f_{5/2}) - \epsilon(2p_{3/2})]_n + \Delta E_{\text{ph}} \\ &= 3.777 \text{ MeV,} \end{aligned} \quad (33)$$

where the Seeger and Howard (1980) single particle energies have been used. The F^2N II spin-flip mode excitation energy differs from the M1 method result by about 0.9 MeV.

Part of this discrepancy results from the necessity of adding the Lane potential to the excitation energy of T_+^+ state in the M1 method. The T^+ spin-flip analog configuration cannot be directly compared with the ^{56}Fe ground state in estimating an excitation energy since these states differ in total isospin. The Lane potential is parametrized by the strength of the V_r^c component of the residual interaction, which varies from nucleus to nucleus. On the other hand, some of the discrepancy between the M1 method and the F^2N II technique is a result of using the Seeger and Howard (1975) single particle energies which differ for protons and neutrons.

FIG. 11.—This figure shows the zero-order orbit-occupation diagrams for several states generated from the ^{56}Mn spin-flip configuration by (a) acting with T^+ to give the T^+ spin-flip-analog in ^{56}Fe with indicated coefficients α , β , γ for the $|1\rangle$, $|2\rangle$, and $|3\rangle$ coefficients, respectively; (b) and (c) are the T_+^+ and T_-^+ spin-flip anti-analog states generated from the configuration in (a) by orthogonality, with coefficients of the $|1\rangle$, $|2\rangle$, and $|3\rangle$ configurations as indicated.

To see the effect of this, the calculations can be repeated with the Nilsson single particle energies (cf. Hillman and Grover 1969, Table 1) which are the same for protons and neutrons. Repeating first the F^2N II technique in equation (33) yields

$$E(^{56}\text{Mn}^{\text{sf}}) = 2.810 \text{ MeV} \quad (34)$$

for the spin-flip mode excitation in ^{56}Mn .

The arguments of the M1 method leading to equation (30) can be repeated with the Nilsson single particle energies. In this case the $T_+^<$ and $T_-^<$ spin-flip anti-analog states are found to be degenerate in energy, with

$$E(T_+^<) = E(T_-^<) = 7.876 \text{ MeV}. \quad (35)$$

Adding the Lane potential and then subtracting the first analog state energy yields an estimate of the ^{56}Mn spin-flip excitation energy of

$$E(^{56}\text{Mn}^{\text{sf}}) = 7.876 + \frac{50 \times 6}{56} - 11.440 = 1.793 \text{ MeV}. \quad (36)$$

The results in equations (34) and (36) should, in principle, agree. That they do not reflect the use of a particular value of the Lane potential. If the Lane potential parameter were chosen as $V_7^c/2 = 60$, which is common in many works (Klapdor 1976), then the M1 method employed in equation (36) would give an estimated ^{56}Mn spin-flip mode excitation energy of 2.865 MeV, in good agreement with the F^2N II result (2.810 MeV) employing the Nilsson energies.

There is an alternative approach to the M1 method for the $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ transition. The proton $1f_{7/2} \rightarrow 1f_{5/2}$ and neutron $1f_{7/2} \rightarrow 1f_{5/2}$ M1 excitations can be constructed and the resulting configurations can be combined to yield a $T^<(T=2)$ M1 state in ^{56}Fe , whose excitation energy can be estimated in the standard manner. For reasons of simplicity we neglect the M1 excitation of the $2p_{3/2}$ neutrons in ^{56}Fe to the $2p_{1/2}$ state. A $T^>(T=3)$ state can be constructed from the M1 basis states using orthogonality. The excitation energy of this state will be the $T^<$ M1 state energy plus the Lane potential. Operating on the $T^>$ M1 state with T^+ yields the $T=3$ spin-flip configuration in ^{56}Mn , with excitation energy equal to the difference in energies of the $T^>$ M1 state and the first analog state in ^{56}Fe . For the proper choice of the Lane potential parameter this method yields an estimate of the ^{56}Mn spin-flip excitation energy in good agreement with the F^2N II result using the Nilsson energies (eq. [34]).

As discussed previously, the Bloom and Fuller (1981) shell model calculation of the electron capture β -strength distribution for $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ yields a fairly broad distribution with a centroid at 2.7 MeV, which again is in

reasonable agreement with the F^2N II calculation. The resonance energy for the electron capture transitions from the ground state of ^{60}Fe is calculated as 2.0 MeV in ^{60}Mn with the F^2N II technique. The Bloom and Fuller result for Gamow-Teller strength centroid in ^{60}Mn for this transition is 1.5 MeV and again agrees well with the F^2N II results.

In $T^< \rightarrow T^>$ transitions in which several spin-flip transitions are possible the above procedure is carried out for each spin-flip, and several resonances may be placed in the $T^>$ daughter. In contrast, for $T^> \rightarrow T^<$ transitions in which several spin-flip transitions are possible, the resulting resonance excitation energies are averaged and the total sum rule strength lumped in a single resonance at this average excitation energy. This procedure is justified for $T^> \rightarrow T^<$ transitions because a $T^>$ nucleus, which is neutron-rich enough to have several possible spin-flip transitions, will have Gamow-Teller resonances at high excitation energy in the $T^<$ daughter.

Finally, a few nuclear transitions in this work have possible back-spin-flip transitions, for example, $1d_{3/2} \rightarrow 1d_{5/2}$ in $^{30}\text{Al}(e^- \bar{\nu}_e)^{30}\text{Si}$. This Gamow-Teller transition strength is expected to lie at low excitation energy (Klapdor 1976) and thus, as for the no-spin-flip transitions, the discrete state transitions should include most of this strength.

At this point a summary of the Gamow-Teller resonance procedure employed in this work is in order. Typically, some 20 discrete nuclear levels are employed in the calculation for both $T^<$ and $T^>$ nuclei. Gamow-Teller matrix elements for these $T^> \rightleftharpoons T^<$ discrete state transitions are assigned as in F^2N I. A zero-order shell model diagram is constructed for both $T^<$ and $T^>$ ground states, and equation (16) is employed to give the total Gamow-Teller strength for the $T^< \rightarrow T^>$ and $T^> \rightarrow T^<$ transitions. The discrete state transition strength from the ground state of the $T^<(T^>)$ parent to all states of the $T^>(T^<)$ daughter is summed and subtracted from the sum rule result. This remaining Gamow-Teller strength is placed in a collective resonance in the $T^>(T^<)$ daughter which connects to the $T^<(T^>)$ parent ground state. The excitation energy of this resonance is chosen to be the excitation energy of the spin-flip mode in the $T^>(T^<)$ daughter. The spin-flip mode excitation energy is calculated by the techniques discussed above, differing for $T^< \rightarrow T^>$ and $T^> \rightarrow T^<$ transitions.

V. RESONANCE TRANSITIONS FOR EXCITED STATES

The previous discussion reviews the calculations used in this work to estimate Fermi and Gamow-Teller sum rules and resonance excitation energies for transitions from the ground states of the $T^<$ and $T^>$ nuclei. The rate calculations described in this paper use only these ground-state resonance transitions; resonances corre-

sponding to all of the excited discrete states included in both $T^<$ and $T^>$ nuclei are taken into account here through the use of a special occupation index for ground-state $\rightleftharpoons$ resonance-state transitions. The mathematical manipulations involved in the use of this special occupation index are based on reasonable assumptions about nuclear shell structure.

The basic $T^<\rightleftharpoons T^>$ rate calculation problem is depicted in Figure 12. $E^<$ and $E^>$ denote the discrete states in the $T^<$ and $T^>$ nuclei, respectively. These symbols will also be used to designate the energy of these states. These discrete states represent those states included in the rate calculation where excitation energies, spins, and parities are taken from experimental tabulations. $R^>$ designates Gamow-Teller resonances for each of the $E^<$ states, while the $R^<$ resonances correspond to the $E^>$ discrete states. In the $T^<\rightarrow T^>$ transition, the $R^<\rightarrow E^>$ transitions contribute to the stellar rates through the thermal population of the $R^<$

FIG. 12.—A schematic representation of a typical excited state problem is shown. $E_1^<, E_2^<, \dots, E_i^<$ are the discrete states of the $T^<$ nucleus. $E_1^>, E_2^>, \dots, E_j^>$ are the discrete states in the $T^>$ nucleus. $A_1^<, A_2^<, \dots, A_j^<$ are the analogs of $E_1^>, E_2^>, \dots, E_j^>$. Transitions from the discrete states of the $T^<$ nucleus to Gamow-Teller collective resonances ($R_1^>, R_2^>, \dots, R_i^>$) in the $T^>$ nucleus, and the reverse transitions, are shown with rates $\lambda_{ii} = \lambda_{er}$. Similarly, transitions from the discrete states of the $T^>$ nucleus to collective Gamow-Teller resonances in the $T^<$ nucleus ($R_1^<, R_2^<, \dots, R_j^<$), and the reverse transitions are shown with rates $\lambda_{jj} = \lambda_{re}$. For clarity, the Fermi transitions λ_{jj}^F between $E_j^> \rightleftharpoons A_j^<$ are not illustrated, nor are the discrete state to discrete state transitions $E_i^< \rightleftharpoons E_j^>$ illustrated.

collective states. Likewise, in the reverse $T^>\rightarrow T^<$ transition, the $R^>\rightarrow E^<$ transitions contribute through the thermal population of the $R^>$ collective states. It will be clear that the $T^<\rightarrow T^>$ and the $T^>\rightarrow T^<$ transitions are symmetrical. In what follows, the $T^<\rightarrow T^>$ transitions are discussed in detail. Generalization to $T^>\rightarrow T^<$ will be obvious.

The resonance problem is simplified by making two assumptions. First, it is assumed here that *on the average* each $E^<$ and $E^>$ discrete state has a similar number of valence particles and holes in the same single particle orbits as does the assigned shell model ground state configuration. In this case, from equation (16) for the Gamow-Teller sum rule, all $E^<\rightarrow R^>$ transitions have equal strengths and all $R^<\rightarrow E^>$ transitions have equal strengths. The second assumption made here is that the excitation energies of the $R^<$ ($R^>$) resonances scale just as do their corresponding $E^>$ ($E^<$) discrete states; for example, if the ground state and first excited state in the $T^<$ nucleus differ by 0.847 MeV as in ^{56}Fe , then the first two $R^>$ resonances in the $T^>$ nucleus, ^{56}Mn in this example, differ in excitation energy by 0.847 MeV. In other words, each $E^<\rightarrow R^>$ transition has the same Q -value as does the ground-state $\rightarrow$ resonance transition.

Similarly, each $R^<\rightarrow E^>$ transition has the same Q -value as does the resonance $\rightarrow$ ground state transition. Of course, the residual interaction will produce considerable configuration mixing, and these assumptions of constant sum rule and transition Q -value will be only approximately valid. It is argued here, however, that the approximate validity of these assumptions will serve to simulate the effect of excited-discrete-state $\rightleftharpoons$ collective-resonance transitions on the stellar rates.

How valid are these assumptions? Weak rates are available only for a few excited states of nuclei (isomers), and therefore comparison to theoretical shell model calculations must be made for most excited states. The Bloom and Fuller (1981) moment calculations give a strength function centroid of 2.8 MeV and sum rule of $|M_{GT}|^2 = 9.04$ for the $^{56}\text{Fe}(\text{ground state}) \rightarrow ^{56}\text{Mn}$ transition. For $^{56}\text{Fe}(\text{first excited state}) \rightarrow ^{56}\text{Mn}$ transition, those authors calculate a strength centroid of 4.3 MeV and a sum rule of $|M_{GT}|^2 = 9.25$. The agreement of the ground state data with the F^2N II resonance procedure is discussed above, but note that since the first excited state of $^{56}\text{Fe}(J^\pi = 2^+)$ lies at an excitation energy of 0.847 MeV, the F^2N II scaling assumption would put the corresponding Gamow-Teller resonance at $3.777 + 0.847 = 4.624$ MeV in ^{56}Mn with $|M_{GT}|^2 = 72/7 = 10.29$. This is in good agreement with the shell model calculation results. In general, the differences in the shell model structure for each wave function will tend to average out for at least the important low-lying discrete states of the $T^<$ and $T^>$ nuclei, with the result that the Q -value and sum rule strength scaling procedure used here will allow excited-discrete-state $\rightleftharpoons$ resonance-state transitions to

be taken into account with a special treatment of the occupation index for ground-state $\rightleftharpoons$ resonance transitions.

Consider first the $E^< \rightarrow R^>$ transitions; the transitions, $R^< \rightarrow E^>$, which proceed through the thermal population of the states collectively treated as a single resonance state are slightly more complicated and will be discussed later. As illustrated in Figure 12, the discrete states in the $T^<$ nucleus are designated by $E_i^<$, where $i=1,2,\dots$, while the discrete states in the $T^>$ nucleus are designated by $E_j^>$, where $j=1,2,\dots$. Since there is a one-to-one correspondence between the resonance in the $T^<$ nucleus and the $E^>$ states, these resonances are designated by $R_j^<$ and similarly the resonances in the $T^>$ nucleus are designated by $R_i^>$.

From equations (I-7) and (I-8) the overall transition rate from the $T^<$ nucleus is

$$\lambda^< = \sum_i P_i^< \sum_j \lambda_{ij}^< \quad (37)$$

where the occupation index, $P_i^<$, is defined by

$$P_i^< = G_i^< / G^< = (2J_i^< + 1) \exp(-E_i^< / kT) / G^< \quad (38)$$

and the partition function for the $T^<$ nucleus is given by

$$G^< = \sum_i G_i^< = \sum_i (2J_i^< + 1) \exp(-E_i^< / kT). \quad (39)$$

For the $E^< \rightarrow R^>$ contribution to $\lambda^<$ it is clear that

$$\sum_j \lambda_{ij}^< = \lambda_{ii}^< \quad (40)$$

If this contribution is designated by λ_{ER} , it follows that

$$\lambda_{ER}^< = \sum_i P_i^< \lambda_{ii}^<, \quad (41)$$

which is completely general. If it is assumed, in addition, that all of the $\lambda_{ii}^<$ are identical as discussed previously and are equal, say, to $\lambda_{er}^<$ as calculated from the Gamow-Teller matrix elements, then

$$\lambda_{ER}^< = \lambda_{er}^< \sum_i P_i^<. \quad (42)$$

But

$$\sum_i P_i^< = 1, \quad (43)$$

so that

$$\lambda_{ER}^< = \lambda_{er}^<. \quad (44)$$

In other words, the contribution of the $E^< \rightarrow R^>$ transitions to the total $T^< \rightarrow T^>$ transition rate can be calculated by including only the $E_1^< \rightarrow R_1^>$ transition and setting the occupation index for the $E_1^<$ state in this transition equal to unity. The two basic assumptions made in reaching this conclusion were discussed at the beginning of this section. It will be immediately obvious that the errors arising from the use of a finite number of discrete states are magnified by this procedure especially at high temperature. Another source of error in the calculations discussed here is the inclusion of the $R^<$ collective states in the calculation of $G^<$. At low temperatures the small Boltzmann factors for high-lying states make these errors negligible, whereas at high temperatures (especially $T_9=100$), where the errors in the calculation of G may be considerable, the stellar rates are dominated by the resonance transitions and do not depend sensitively on G .

Consider next the $R^< \rightarrow E^>$ contribution to the transition rate which will be designated by $\lambda_{RE}^<$. In these transitions only the $R_j^<$ resonances in the $T^<$ nucleus take part, and

$$\sum_i \lambda_{ij}^< = \lambda_{jj}^<, \quad (45)$$

so that

$$\lambda_{RE}^< = \sum_j P_j^< \lambda_{jj}^<. \quad (46)$$

If it is now assumed that all the $\lambda_{jj}^<$ are identical and equal, say, to $\lambda_{re}^<$, then

$$\lambda_{RE}^< = \lambda_{re}^< \sum_j P_j^<. \quad (47)$$

The calculation of the sum over j is now somewhat more subtle than in equations (42) and (43). The $P_j^<$ are given by

$$P_j^< = (2J_j^< + 1) \exp(-R_j^< / kT) / G^<. \quad (48)$$

The complications involved in the multiplicity in spins (≤ 3) and the corresponding statistical weight factors for $R_j^<$ resonances associated with each $E_j^>$ state can be treated simply by replacing $J_j^<$ in equation (48) with $J_j^>$. The $\lambda_{re}^<$ can be most simply calculated by using the fact that the product of $|M_{GT}|^2$ by the level multiplicity is the same for the two directions of the transition. This is guaranteed by detailed balance arguments. In addition

$$R_j^< = R_1^< + E_j^> - E_1^>. \quad (49)$$

One is free to set $E_1^> = 0$ and thus

$$P_j^< = (2J_j^> + 1) \exp(-E_j^> / kT) \exp(-R_1^< / kT) / G^<, \quad (50)$$

which in turn yields

$$\sum_j P_j^< = \frac{\exp(-R_1^</kT)}{G^<} \sum_j (2J_j^> + 1) \exp(-E_j^>/kT). \quad (51)$$

The sum over j will be recognized as $G^>$ and thus finally

$$\sum_j P_j^< = (G^>/G^<) \exp(-R_1^</kT) \quad (52)$$

and

$$\lambda_{RE}^< = \lambda_{re}^< (G^>/G^<) \exp(-R_1^</kT). \quad (53)$$

In other words, the contribution of the $R^< \rightarrow E^>$ transitions to the total $T^< \rightarrow T^>$ transition rate can be calculated by including only the rate for the $R_1^< \rightarrow E_1^>$ transition and setting the occupation index for the $R_1^<$ resonance equal to the ratio of the partition function for the $T^>$ nucleus to that for the $T^<$ nucleus multiplied by the Boltzmann factor for the first resonance in the $T^<$ nucleus.

Throughout this paper we have referred to ground states whereas the simple shell model gives a lower configuration which yields in general a number of degenerate ground states. This point is particularly relevant to the choice of $G^>$ and $G^<$ in equation (53). In principle these G 's should be calculated by summing over the states arising from the appropriate configurations. This was not done. In the spirit of using experimental information as much as possible, the G 's in equation (53) were calculated using spins and energies of known states. Since the ratio of $G^>/G^<$ occurs in equation (53), the error arising from this procedure is well within the other uncertainties involved.

These occupation index manipulations allow the effect of all the resonances to be taken into account by calculating only the ground state to resonance transition rate as a representative average value. At very high temperatures where the F^2N II partition function calculation breaks down, the stellar rates will be almost completely dominated by these resonance transitions. It is argued here that the F^2N II occupation index manipulations allow reasonable estimates of the stellar rates even at very high temperature, since the expressions for the occupation indices for the resonances involve only ratios of partition functions and, hence, errors in the partition functions will tend to cancel. There remains the question whether the matrix elements calculated for transitions involving the lowest shell model configurations are representative for high excitations. In electron capture, for example, neutron shell blocking will not be effective and matrix elements will increase, but not by unreasonable factors except perhaps at $T_9 =$

100 where uncertainties of the order of a factor of two from this and other causes must be admitted.

In generalizing to $T^> \rightarrow T^<$ transitions the counterpart of equation (44) will be an identical statement equating the total rate to that for the ground state $\rightarrow$ resonance rate. The counterpart of equation (53) will contain $(G^</G^>) \exp(-R_1^>/kT)$.

To each discrete state in a $T^>$ nucleus there corresponds a $T^>$ analog state in the $T^<$ nucleus. The discrete state $\rightleftharpoons$ analog state Fermi transitions precisely obey the Q -value scaling and constancy of the sum rules which were approximations for the Gamow-Teller transitions. Again, the deep similarity between the behavior of the Fermi and Gamow-Teller analog resonances becomes apparent. It would be desirable not to have to include in the stellar rate computations an analog state for every $T^>$ discrete state, and yet still include their effect. The occupation index, partition function manipulations proceed exactly as before, and the analog states are handled just like the Gamow-Teller resonances. The ground-state $\rightarrow$ first-isobaric-analog-state transition is given an occupation index equal to unity, whereas the thermally populated isobaric analog state in the $T^<$ nucleus which transforms to the $T^>$ nucleus ground state is given an occupation index equal to $(G^>/G^<) \exp(-A_1^</kT)$, where the G 's are the respective nuclear partition functions and $A_1^< \equiv E_{IAS}$ is the excitation energy of the first isobaric analog state in the $T^<$ nucleus. We show $A_1^<, A_2^<, A_3^<, \dots, A_j^<$ in Figure 12. The individual transition rates corresponding to the $\lambda_{er}^<$ and the $\lambda_{er}^>$ will be equal and can be calculated from the appropriate Fermi matrix element.

VI. DISCUSSION OF THE STELLAR RATES

The salient point of the F^2N I discussion of stellar weak rates was the close reliance of the calculations on experimental nuclear data. In this paper the weak rate calculations have been extended to include 226 nuclei up to mass 60 and free protons and neutrons; further, these calculations have been performed on an extended temperature and density grid. The larger variety of nuclei and the considerably more extreme stellar conditions involved in the rate calculations of this paper require a far better coverage of the weak interaction strength distribution than in the F^2N I calculations. In this work the validity of the rate calculations relies on the experimental discrete states to reproduce the detailed, relatively low excitation strength, and on at most a few Fermi and Gamow-Teller resonances to yield the high-excitation-energy strength function characteristics so important to the regime of extreme temperatures and density. The last section discussed the nuclear physics underlying the Fermi and Gamow-Teller strength distribution in intermediate mass nuclei and on calculations of the Fermi and Gamow-Teller resonance excitation

energies and strengths used in the rate calculations presented here. The sensitivity of the stellar weak rates to the parameters of the resonance procedure presented in the last section must be tested, much as the discrete-state transition matrix element assignment procedure introduced in F²N I was tested in § III of that paper. It will be seen that the resonance procedure used in these rate calculations is adequate in most cases to ensure accurate rates in extreme conditions. The central point of this paper is that the calculated stellar rates should be valid at the lower temperatures and densities characteristic of hydrostatic stellar evolution because of the discrete state transitions, while at the more extreme conditions the Gamow-Teller resonances and Fermi analog transitions should adequately determine the rates.

a) The Temperature and Density Grid

The temperature and density grid on which the rate calculations were performed consists of 143 points: for each of the temperatures ($T_9 = 0.01, 0.1, 0.2, 0.4, 0.7, 1.0, 1.5, 2.0, 3.0, 5.0, 10.0, 30.0, 100.0$) the rates are computed at the densities $\log(\rho/\mu_e) = 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0, 11.0$, where the notation follows F²N I. Note that this grid covers conditions from electron nondegeneracy up to electron Fermi energies near 25 MeV. Neutrino trapping and hence neutrino blocking of electron capture is expected to set in for densities in excess of $10^{11} \text{ g cm}^{-3}$ (Arnett 1977), while many nuclei in nuclear statistical equilibrium at these conditions will have allowed electron capture blocked. This means electron capture on heavy nuclei at densities above $\rho = 10^{11} \text{ g cm}^{-3}$ may not be very important. Future calculations to extend the present work to nuclear masses in the $A = 61$ to $A = 70$ range will be done on a grid of temperatures and densities which includes $\rho = 10^{12} \text{ g cm}^{-3}$.

The temperature-density grid used in this work should be adequate to cover most of the conditions of interest in stellar evolution. The rate tables available on magnetic tape and printed in abbreviated form in F²N III are suitable for interpolation between temperature-density points. The interpolation error in the detailed grid is probably comparable to the systematic errors due to the approximate nuclear physics in the rate calculations. Note, however, that in this calculation screening effects are neglected (cf. F²N I, p. 451, for a discussion of the effects of screening). Continuum electron capture is included in the calculation, but bound state electron capture is not. A discussion of this point and a table of terrestrial bound state electron capture rates for the nuclei considered here are presented in F²N III. That table also includes all known terrestrial weak decay rates for the nuclei included in this survey.

At the lower temperatures and densities characteristic of the hydrostatic phases of stellar evolution, the discrete state transitions dominate the stellar electron-

capture rates, but as the electron Fermi energy rises above 5 MeV ($\rho/\mu_e \gtrsim 10^9 \text{ g cm}^{-3}$) or when the temperatures approach $T_9 = 10$ ($kT \sim 1 \text{ MeV}$), the dominant transitions involve the Gamow-Teller and Fermi resonances. To illustrate this point, consider the electron capture rate of $^{26}\text{Al}(e^-, \nu_e)^{26}\text{Mg}$. The F²N I and F²N II calculations contain the same discrete state information and the same Fermi transitions, but the F²N II calculations also have Gamow-Teller resonances whose excitation energies and strengths are calculated with the procedure set down in § IV of this paper. The $^{26}\text{Al}(e^-, \nu_e)^{26}\text{Mg}$ rate is $\log \epsilon^- = -3.123$ at $T_9 = 1.0$, $\log(\rho/\mu_e) = 7$ in the F²N I calculation, while at the same temperature and density the F²N II result is $\log \epsilon^- = -3.119$. Clearly, in these typical carbon/oxygen burning conditions the Fermi and discrete state transitions are dominating the rate. For $T_9 = 0.01$ and $\log(\rho/\mu_e) = 9.0$, the electron Fermi energy is, using equation (I-4c), $U_F \approx 5.0 \text{ MeV}$, and so electrons do not have quite enough energy to reach the Gamow-Teller resonance in ^{26}Mg at 11.408 MeV excitation which requires an electron energy equal to $11.408 - 4.004 = 7.404 \text{ MeV}$; consequently, the rates are again similar, with $\log \epsilon^- = 0.271$ for F²N I and $\log \epsilon^- = 0.282$ for F²N II.

The added effect of the Gamow-Teller resonance can be seen in the electron capture rate $^{56}\text{Co}(e^-, \nu_e)^{56}\text{Fe}$. The Gamow-Teller resonance lies at 5.380 MeV excitation in ^{56}Fe , while the Q_n for the reaction is -4.056 , so that an electron inducing the $^{56}\text{Co}(\text{ground state}) \rightarrow ^{56}\text{Fe}(\text{resonance state})$ transition must have total energy equal to at least $5.380 - 4.056 = 1.324 \text{ MeV}$. At $T_9 = 0.01$ and $\log(\rho/\mu_e) = 7.0$ the total electron Fermi energy is 1.222 MeV, clearly not high enough to reach the resonance, and thus the F²N II electron capture rate ($\log \epsilon^- = -4.545$) should be dominated by the discrete state transitions. At the same temperature and $\log(\rho/\mu_e) = 8.0$ the total electron Fermi energy is 2.447 MeV and the Gamow-Teller resonance in ^{56}Fe can be reached. The $^{56}\text{Co}(\text{ground state}) \rightarrow ^{56}\text{Fe}(\text{resonance state})$ contribution to the overall electron capture rate can be computed setting the occupation index of the ground state equal to unity and employing the equation (I-3b) phase-space factor result for a degenerate electron gas to yield $\log \epsilon^- (\text{ground state} \rightarrow \text{resonance}) = -0.585$. The F²N II calculation yields $\log \epsilon^- = -0.540$ for the overall ^{56}Co electron capture rate, and thus the resonance is beginning to dominate the overall transition rate at this density. At $T_9 = 0.01$ and $\log(\rho/\mu_e) = 11$ the total electron Fermi energy is 23.930 MeV and the resonance transition can be expected to completely dominate the rate. Applying the equation (I-3b) result yields $\log \epsilon^- (\text{ground state} \rightarrow \text{resonance}) = 5.168$, while the full F²N II computation gives $\log \epsilon^- = 5.194$ for the overall electron capture rate and thus, indeed, the resonance transition dominates.

Where the resonances dominate, they make substantive differences in the stellar weak rates. The above example is an indication of how much faster the neutronization rate during stellar collapse/silicon burning can be, due to low-lying Gamow-Teller resonances, as first pointed out by BBAL. On the other hand, blocked nuclei at these extreme conditions have stellar weak rates characterized only by discrete state Gamow-Teller and Fermi transitions, the lack of resonance transitions implying a relatively less drastic increase in the rates as the temperature and density increase. Unblocking by configuration mixing, thermal excitation and forbidden transitions is discussed by Fuller (1981).

In conclusion, it is clear that the nuclear physics which dominates the rate calculations meshes well with the two regimes of stellar conditions represented in the temperature-density grid used here. At the lower temperatures and densities characteristic of the hydrostatic phases of stellar evolution, very accurate stellar weak rates may be required to determine the nucleosynthesis of nuclear species, the overall neutrino energy loss rates which may affect the temperature, and the detailed lepton/baryon ratio which becomes very important going into stellar collapse. In these conditions the F^2N II rates are dominated by discrete state Gamow-Teller and Fermi transitions which, as pointed out above, are heavily reliant on the best experimental nuclear information available and so serve accurately to determine the stellar rates. In the extreme conditions encountered in the later phases of silicon burning, and on into the collapse phase, overall neutronization rates and neutrino production rates become the most interesting quantities. What is needed there are reliable estimates of the stellar rates based on the total amount of Gamow-Teller strength and how much of it can be reached in a given nucleus at a given temperature and density. The Gamow-Teller resonance procedure is designed to do this by approximating the high-lying Gamow-Teller strength as one or two narrow resonances.

b) Tests of the Gamow-Teller Resonance Procedure

In principle, of course, the most accurate stellar weak rate calculations require a complete and detailed Gamow-Teller strength distribution in the daughter nucleus for each parent nucleus discrete state. Section III of this paper discussed the underlying nuclear physics of the Gamow-Teller strength distribution as well as how the problem is handled in this calculation. In brief, as many discrete states are included in each nucleus as possible in order to outline accurately the Gamow-Teller distribution out to the excitation energy of the last discrete state. The discrete state strength is summed and subtracted from the zero-order shell model sum rule result, the remaining strength is typically lumped in a discrete Gamow-Teller resonance state at an excitation

energy calculated in accordance with the procedure discussed in § III. At higher nuclear excitation energy the state density is very large and as a result the Gamow-Teller strength distribution is nearly continuous. How much error is being made in these rate computations by approximating what is known experimentally to be a broad strength distribution by one or two narrow resonances?

Specifically, there are two major worries regarding this resonance procedure. The first is just how sensitive the rates are to the placement of the strength-distribution centroid and to the total strength; the second involves possible threshold effects. A single narrow resonance would not contribute to, say, an electron capture reaction until the electron Fermi energy was sufficient to exceed the Q -value. The danger is that the stellar rates might show an unphysical jump just as the electron Fermi energy reaches the resonance Q -value. This latter problem is expected to be ameliorated by three effects. First, discrete state transitions serve to broaden the Gamow-Teller strength distribution, as outlined above. Second, the high temperatures encountered in the typical stellar conditions where Gamow-Teller resonances are important serve to thermally populate many parent states, coupling into the calculation many more decay channels than just the resonance transitions and, thus, helping to effectively smear out any resonance threshold effects. Finally, and most important, these high temperatures smear out the electron Fermi-Dirac distribution function, increasing the length of the high energy exponential tail. Thus some electrons will be energetic enough to reach the Gamow-Teller resonance even though the electron Fermi energy has not reached the resonance Q -value, serving to wash out a threshold effect.

To test the sensitivity of the stellar rates to the resonance procedure, the $^{56}\text{Fe}(e^-, \nu_e)^{56}\text{Mn}$ reaction was examined in some detail. This reaction is particularly appropriate to test, not only because the nuclear physics underlying its resonance calculation was discussed at length above, but also because it is fairly representative of important neutronization reactions in silicon burning, where the Gamow-Teller resonances are becoming important. Briefly, in § III of this paper the resonance calculations for the $^{56}\text{Fe}(e^-, \nu_e)^{56}\text{Mn}$ decay were discussed with the result that the excitation energy of the Gamow-Teller resonance in ^{56}Mn was found to be at 3.777 MeV with a total strength of $|M_{\text{GT}}|^2 = 10.296$ ($\log ft = 2.58$). The Fermi and Gamow-Teller resonances in ^{56}Fe corresponding to the ground state of ^{56}Mn were calculated to be at 11.440 MeV and 24.719 MeV excitation, respectively, and thus are not very important in the overall continuum electron capture reaction at low temperature because they are not populated. The reaction rate calculated by this standard

FIG. 13.—(a) The total $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ weak transition rate is shown for the F^2N II single resonance approximation as a function of temperature T (K) for the densities, $\log(\rho/\mu_e)=7, 8, 9, 10, 11$. (b) The total $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ weak transition rate as shown in Fig. 13a, but now with the four-resonance approximation to the Bloom and Fuller (1981) strength function. There is very little change in the overall transition rate.

technique is shown as a function of temperature for various densities in Figure 13a.

The calculations of Bloom and Fuller (1981) give a detailed strength distribution for the ^{56}Fe (ground state) $\rightarrow ^{56}\text{Mn}$ transition. For purposes of testing, this detailed strength distribution is approximated as four discrete resonances whose excitation energies and strengths are reproduced in Table 2. Note that the total strength in the 4-resonance approximation is $|M_{\text{GT}}|^2=9.036$ ($\log ft=2.64$), or 13% less strength than the F^2N II result. The $^{56}\text{Fe}(e^-, \nu_e)^{56}\text{Mn}$ reaction rate in the 4-resonance approximation was calculated on the standard temperature-density grid and is shown as a function of temperature for various densities in Figure 13b.

A detailed comparison of the $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ rates calculated with the single-resonance F^2N II method and

with the 4-resonance approximation to the Bloom and Fuller strength function serves to illustrate the accuracy of the F^2N II calculations. First, the total electron-emission rate for $^{56}\text{Mn} \rightarrow ^{56}\text{Fe}$ at very high temperature, $T_9=100$, differs in the two calculations by just 13%, reflecting the uniform thermal population of the Gamow-Teller resonances in ^{56}Mn and the 13% difference in total strength in the calculations.

The ground-state to ground-state nuclear mass difference for $^{56}\text{Fe} \rightarrow ^{56}\text{Mn}$ is -4.206 MeV. In the F^2N II calculation, then, the resonance Q -value is $Q_R=3.777+4.206=7.983$ MeV. That is, the total electron Fermi energy must approach the value of Q_R before electron capture proceeds predominantly through the Gamow-Teller resonance. At $\log(\rho/\mu_e)=9.0$ the total electron Fermi energy is 5.182 MeV, while for $\log(\rho/\mu_e)=10$ it

TABLE 2
FOUR RESONANCE APPROXIMATIONS TO BLOOM AND FULLER
 $^{56}\text{Fe}(\text{g.s.}) \rightarrow ^{56}\text{Mn}$ GAMOW-TELLER STRENGTH FUNCTION

EXCITATION ENERGY IN ^{56}Mn (MeV)	PERCENT OF TOTAL STRENGTH	STRENGTH IN EACH RESONANCE	
		$ M_{\text{GT}} ^2$	$\log ft$
1.5	35	3.083	3.107
2.5	25	2.259	3.242
3.8	25	2.259	3.242
4.9	15	1.355	3.464

is 11.119 MeV and so the F^2N II Q_R is reached somewhere between these densities. Similarly, in the 4-resonance calculation the resonance Q_R 's are 5.706 MeV, 6.706 MeV, 8.006 MeV, and 9.106 MeV, respectively, and the total electron Fermi energy exceeds all these energies in the range $9.0 < \log(\rho/\mu_e) < 10$. In comparing Figure 13a with Figure 13b it is seen that the rates are identical at $\log(\rho/\mu_e)=9$ and 11 and differ only 0.2 in the logarithm at $\log(\rho/\mu_e)=10$. No unphysical threshold effects are evident. The F^2N II single resonance procedure reproduces the 4-resonance approximation Bloom-Fuller strength function quite well.

The majority of $T^< \rightarrow T^>$ rates are not very sensitive to small errors in the Gamow-Teller resonance placement, once the electron Fermi energy is larger than Q_R . The $T^> \rightarrow T^<$ rates are even less sensitive to the placement of the $T^<$ resonance, but this is because the resonance usually lies very high in the $T^<$ daughter excitation energy.

c) Trends in Gamow-Teller Resonance Positions and Strengths

In undertaking this survey of 226 nuclei, some important trends in Gamow-Teller peak energies and strengths have become apparent. Here specific examples will be given for some iron group nuclei of importance in neutronization during silicon burning and the onset of core collapse in supernovae models.

In general, as nuclei become more neutron rich, the Gamow-Teller $T^< \rightarrow T^>$ resonance strength decreases due to increasing neutron blocking of allowed transitions. Accompanying this effect is a trend of decreasing daughter resonance excitation energy due in part to the smaller number of single particle excitations required to make the spin-flip transition as neutron blocking is approached. The model in which these effects were predicted was discussed in § III. In Table 3 is listed the total Gamow-Teller strength and the excitation energy for $A=56$ isobars as calculated by the standard F^2N II resonance procedure. The general trend in strength and

excitation energy of the resonances is illustrated, though this A -chain is not carried out to complete blocking.

Where the resonances excitation energies are the same in Table 3, the spin-flip configurations in the $T^>$ daughters involve similar particle excitation and nucleon pair breaking. In the cases where the $T^>$ daughter resonance excitation energy is 2.0 MeV, the spin-flip configuration is coincident with the zero-order shell model configuration, there are no single particle excitations required, the 2.0 MeV simply reflecting the particle-hole repulsion energy. Though one could conclude from Table 3 that the resonance excitation energy decreases while the strength weakens, it is clear that the Q_R for each reaction increases on the average as blocking is approached. This effect was briefly mentioned in § II and can be seen to aid the neutron blocking at low temperature in lowering the electron capture rates as neutronization proceeds. In the two electron capture reactions $^{56}\text{V} \rightarrow ^{56}\text{Ti}$ and $^{56}\text{Ti} \rightarrow ^{56}\text{Sc}$ the rates at $T_9=0.01$ and $\log(\rho/\mu_e)=11$ (total electron Fermi energy = 23.930 MeV) differ by almost a factor of 30. This is due in part to the factor of 1.87 decrease in $T^< \rightarrow T^>$ Gamow-Teller strength between the two cases, but in the main the large decrease in electron capture rate ongoing from $^{56}\text{V} \rightarrow ^{56}\text{Ti}$ to $^{56}\text{Ti} \rightarrow ^{56}\text{Sc}$ results from the increase in Q_R from 6.810 MeV in the former reaction to 16.251 MeV in the latter reaction.

The situation for the electron capture rates in Table 3 is much different for the higher temperature case [$T_9=100$, $\log(\rho/\mu_e)=11$]. The blocking and increasing Q_R effects on the rates are washed out at these temperatures due to the thermal population of the analog and $R^<$ resonances in the parent nuclei. At $\log(\rho/\mu_e)=11$ and $T_9=0.01$ the electron capture rates for $^{56}\text{Ni} \rightarrow ^{56}\text{Co}$ and $^{56}\text{Ti} \rightarrow ^{56}\text{Sc}$ differ by a factor of almost 300, whereas at the same density and $T_9=100$ the rates differ by a little over a factor of 4. In neutron-rich nuclei as blocking is approached the $T^< \rightarrow T^>$ Gamow-Teller strength systematically decreases, but the $T^> \rightarrow T^<$ strength increases due to increasing neutron richness, and at extremely high temperature this strength becomes accessi-

TABLE 3
RATES AT $(\rho/\mu_e)=10^{11}$

REACTION	RESONANCE EXCITATION ENERGY (daughter nucleus) (MeV)	RESONANCE STRENGTH			LOG λ_{ec} RATES (s^{-1}) AT $(\rho/\mu_e)=10^{11}$	
		$ M_{GT} ^2$	$\log ft$	Q_R (MeV)	$T_9=0.01$	$T_9=100$
$p(e^-, \nu_e)n$	0	...	3.035	1.293	4.409	4.982
$^{56}\text{Ni}(e^-, \nu_e)^{56}\text{Co}$ ...	3.777	13.677	2.460	2.153	5.234	6.181
$^{56}\text{Co}(e^-, \nu_e)^{56}\text{Fe}$	5.381	11.995	2.517	1.324	5.194	6.025
$^{56}\text{Fe}(e^-, \nu_e)^{56}\text{Mn}$ ...	3.777	10.280	2.584	7.983	4.727	5.791
$^{56}\text{Mn}(e^-, \nu_e)^{56}\text{Cr}$ ...	5.381	8.570	2.663	7.536	4.685	5.691
$^{56}\text{Cr}(e^-, \nu_e)^{56}\text{V}$	2.0	7.105	2.744	13.106	4.071	5.224
$^{56}\text{V}(e^-, \nu_e)^{56}\text{Ti}$	2.0	4.285	2.964	8.801	4.268	5.798
$^{56}\text{Ti}(e^-, \nu_e)^{56}\text{Sc}$	2.0	2.286	3.237	18.251	2.800	5.557

ble to the $T^- \rightarrow T^+$ electron capture reactions through the thermal populations of the R^- resonances.

Early in supernova core collapse, the overall neutronization should be carried by electron capture on heavy nuclei, but as the mean nucleus becomes more neutron-rich and blocking sets in, an increasing share of the neutronization will be carried by electron capture on free protons. The Fermi matrix element for the free nucleon weak transition is $|M_F|^2 = 1$, while the Gamow-Teller matrix element is $|M_{GT}|^2 = 3$. These matrix elements imply, through equations (I-2a) and (I-2b), an overall $\log ft = 3.035$. The rates for the reactions $p(e^-, \nu_e)n$, $n(e^+, \bar{\nu}_e)p$, $n(e^-, \bar{\nu}_e)p$ and the associated neutrino energy loss rates were calculated with the numerical techniques outlined in F²N I and are reproduced in this work with great accuracy on the standard F²N II temperature-density grid. Note that the $p(e^+ \nu_e)n$ decay is energetically forbidden. At extremely high ambient temperatures and correspondingly high nuclear excitation energies, all nuclear states and configurations become sparsely populated and shell structure blocking cannot occur. As a consequence, nucleons in nuclei act more and more like free nucleons and in some limit it is possible to treat all protons and neutrons, in and out of nuclei, as free particles in the sense that they have maximum weak neutron matrix elements $|M_F|^2 = 1$ and $|M_{GT}|^2 = 3$. Although nuclei are still bound at $T_9 = 100$ ($kT \sim 9$ MeV) according to Tubbs and Koonin (1979) we estimate that at this temperature the maximum sum rule limit is approached. Thus at $T_9 \geq 100$ an upper limit on transition rates can be calculated for electron capture and positron emission by using

$$|M_F|^2 = Z, \quad |M_{GT}|^2 = 3Z; \quad (54)$$

and for electron emission and positron capture by using

$$|M_F|^2 = N; \quad |M_{GT}|^2 = 3N. \quad (55)$$

The electron-capture $^{56}\text{Ni} \rightarrow ^{56}\text{Co}$ in Table 3 illustrates the approach to these limits in the rates calculated in this paper. At $(\rho/\mu_e) = 10^{11}$ and $T_9 = 0.1$ the ratio of the rate for this transition to that for electron capture on the free proton is $\text{dex}(5.234 - 4.409) = 6.68$ from Table 3. The ratio of the products of the appropriate coupling constants times the matrix elements is $(1.567 \times 96/7)/(1 + 1.567 \times 3) = 3.77$. Recall that $[C(\text{GT})C(\text{F})]^2 = 1.567$ from equation (I-2) and $|M_{GT}|^2 = 96/7$ for $E_1^- \rightarrow R_1^+$ in $^{56}\text{Ni} \rightarrow ^{56}\text{Co}$. This factor becomes 6.79 when multiplied by the ratio of the Fermi functions, ~ 1.80 for capture at high electron energy (Rose *et al.* 1955). The agreement is satisfactory and the ratio ~ 7 is to be compared with the expected limit at low temperature, high density, namely $(1.567 \times 3 \times 28)/(1 + 1.567 \times 3) = 23.1$. At $(\rho/\mu_e) = 10^{11}$ and $T_9 = 100$ the ratio of the rate for electron capture on

^{56}Ni to that on the proton is $\text{dex}(6.181 - 4.982) = 15.81$. This is in reasonable agreement with the ratio of the matrix elements we have used assuming that the phase space factors are the same except for the ratio of the Fermi functions given above. In this case the expected limit at high temperature, high density is $Z = 28$. In other words, the calculated values for the last entries in the grid exceed 50% of the maximum sum rule limit. This is a very reasonable result. To within a factor of 2 all neutrons and protons inside and out of nuclei can be treated as free particles in regard to the weak interaction at high temperature, $T_9 \geq 100$, and high density, $\rho/\mu_e \approx 10^{11}$. Similar analysis shows that at $T_9 = 100$ and low density the protons in ^{56}Ni have electron capture rates $\sim 12\%$ of that for free protons.

VII. CONCLUSION

Stellar electron and positron emission, continuum electron and positron capture rates, and all associated neutrino energy losses have been computed for 226 nuclei with masses between $A = 21$ and $A = 60$ as well as for free nucleons. The Fermi and Gamow-Teller strength functions are covered by discrete state transitions at low nuclear excitation energies and by one or two narrow resonances, with the remainder of the sum rule strength, at higher excitation energies. This gives an accurate rate at the lower temperatures and densities characteristic of the hydrostatic phases of stellar evolution where detailed rates are required for nucleosynthesis calculations and gives a good estimate of the rates at the extreme temperatures and densities characteristic of silicon burning and core collapse.

The uncertainties associated with the Gamow-Teller resonance calculations were discussed in the last section, where it was shown that approximating the high excitation strength function as a single narrow resonance does not result in any unphysical threshold effects and reproduces quite well calculations with a broader, more realistic Gamow-Teller strength distribution. The level of uncertainty associated with this resonance procedure can be judged by the results presented in § VI.

The results of the calculations described here are available in computer readable form on magnetic tape upon request to M.J.N. The stellar rates will be presented in tables on a somewhat abbreviated temperature-density grid in a forthcoming publication in the *Astrophysical Journal Supplement Series*.

We would like to thank W. D. Arnett, H. A. Bethe, G. E. Brown, S. E. Koonin, T. A. Weaver, and S. E. Woosley for encouragement and useful discussions and H. S. Wilson, R. W. Kavanagh, and F. M. Mann for making available before publication their extensive determinations of experimental weak interaction matrix elements and spin and parity assignments in inter-

mediate mass nuclei. G. Bertsch and S. D. Bloom provided helpful advice on Gamow-Teller collective phenomenon. The hospitality of A. N. Cox, the Los Alamos

National Scientific Laboratory, and the Lawrence Livermore National Laboratory, is gratefully acknowledged.

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