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Journal
Physical Review Letters, 91(5)

ISSN
0031-9007

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Publication Date
2003-08-01

Peer reviewed

Kolmogorov-Kraichnan Scaling in the Inverse Energy Cascade of Two-Dimensional Plasma Turbulence

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(Received 14 February 2003; published 1 August 2003)

Turbulence in plasmas that are magnetically confined, such as tokamaks or linear devices, is two dimensional or at least quasi two dimensional due to the strong magnetic field, which leads to extreme elongation of the fluctuations, if any, in the direction parallel to the magnetic field. These plasmas are also compressible fluid flows obeying the compressible Navier-Stokes equations. This Letter presents the first comprehensive scaling of the structure functions of the density and velocity fields up to 10th order in the PISCES linear plasma device and up to 6th order in the Mega-Ampère Spherical Tokamak (MAST). In the two devices, it is found that the scaling of the turbulent fields is in good agreement with the prediction of the Kolmogorov-Kraichnan theory for two-dimensional turbulence in the energy cascade subrange.

DOI: 10.1103/PhysRevLett.91.055002 PACS numbers: 52.35.Ra, 47.27.–i, 52.25.Fi

Magnetically confined plasmas are promising candidates for future energy production by fusion reaction. The study of turbulence in these plasmas is emphasized by the fact that turbulent fluctuations enhance the radial transport perpendicular to the magnetic field lines and thus reduce considerably the confinement times of particles and energy [1]. Turbulence in magnetically confined plasmas like tokamaks or linear devices is two dimensional due to the strong magnetic field $\vec{B}$ freezing the particle motion along the magnetic field lines due to the Lorenz force. The scale about which parallel motion to the magnetic field can occur is several meters, but no proof that turbulence exists in this direction has been reported in fusion devices. In neutral fluids, the research on two-dimensional turbulence was first developed and motivated by meteorologists [2]. Until recently, two-dimensional turbulence was investigated using computer simulations where high resolution and relatively large Reynolds numbers can be achieved [3–5]. Experimental investigations in the laboratory were conducted using mainly soap films [6,7] and conducting liquids subject to electromagnetic forces [8]. Both of these methods use incompressible fluid flows.

This Letter presents the first comprehensive results on the scaling of density and velocity fields up to 10th order in the PISCES linear device and up to 6th order in the Mega-Ampère Spherical Tokamak (MAST). We found that the Kolmogorov-Kraichnan theory rather accurately describes turbulence inside the plasma. Furthermore, we believe that the experimental results presented here are also important for the neutral fluid community as they provide evidence that the Kolmogorov-Kraichnan scaling describes accurately the energy cascade of turbulent fluctuations in compressible flows.

We recall that in two-dimensional turbulence, there exist two scaling regions on either side of the production scale where turbulence is forced, the scaling being reflective of self-similarity. The enstrophy transfer rate (β) feeds turbulence to scales r smaller than the production scale leading to an energy with the form $E = C\beta^{2/3}r^2$ using dimensional analysis *à la* Kolmogorov [9]. The second domain extends over scales larger than the production scale and is called the energy or the inverse cascade subrange. It is maintained by an energy transfer rate (ε) leading to $E = C'\varepsilon^{2/3}r^{2/3}$ [10]. The above two relations can be generalized to the pth order leading to $E_p \sim r^p$ and $E_p \sim r^{p/3}$ in the enstrophy and energy cascade, respectively $[E_p(\vec{r}) = \langle[\vec{v}(\vec{x}) - \vec{x}(\vec{x} + \vec{r})]^p\rangle]$. The linear increase of the scaling exponent with p is called the Kolmogorov-Kraichnan scaling [10]. In 2D turbulence of incompressible flows, the Kolmogorov-Kraichnan scaling was found to describe rather well within the experimental precision the energy cascade [11,12]. In the enstrophy cascade, data from experiments [13] and numerical simulations [14,15] showed exponents that vary between -3 and -4. The surface friction forces present in the different experiments are believed to play a role in setting the scaling exponent [16].

PISCES is a linear plasma device [17] where stationary discharges allow us to record long data strings (10^6 points at an acquisition frequency of 5 MHz) under stationary plasma conditions. Hot electrons ionize hydrogen gas that continuously feeds the chamber at a constant pressure between 0.2 and 5 mTorr. An axial magnetic field is continuously applied to the plasma while amplitude, being between 800 and 2300 Gauss, remains strong enough to maintain the bidimensionality of turbulence. The plasma density and temperature are, respectively, $n \simeq 2 \times 10^{17}$ m^{-3} and $T \simeq 15$ eV. The level of fluctuations inside the main plasma column is about 10%. We use a multitip Langmuir probe to record the signals. The tip biased to the ion saturation current yields the density fluctuations. Two poloidally (radially) separated floating tips yield the poloidal (radial) electric field and thus v_r (v_θ) assuming

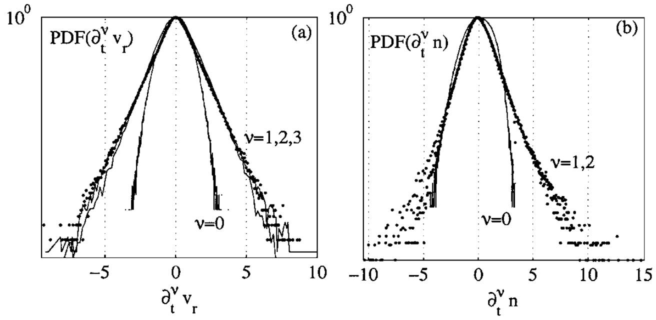

FIG. 1. (a) and (b) simultaneously illustrate the pdf of the radial velocity and density fluctuations normalized to the standard deviation. For comparison, the pdf's are normalized to their maximum values.

$\vec{v} \sim \vec{\mathcal{E}} \times \vec{B}$, where $\vec{\mathcal{E}}$ is the electric field. The assumptions made here by neglecting the temperature fluctuations are commonly used in plasma turbulence and were verified by different groups.

One of the main hypotheses in the Kolmogorov-Kraichnan theory is the randomness of the fluctuations. This is checked using the probability distribution function (pdf). In Fig. 1 is plotted the pdf's of the radial velocity and the density fluctuations as well as their derivatives normalized to the standard deviation. Both signals are Gaussian but the derivatives show exponential tails. However, unlike 3D turbulence the deviation from Gaussian does not increase with the order of derivative. A similar result was obtained in 2D turbulence by Baroud *et al.* [18] using rapidly rotating incompressible flow. The pdf's were also found self-similar with respect to the order of derivative.

Before presenting the scaling, we aim first at determining the highest order moment that converges given the number of points and the acquisition frequency. In order to do that, the structure function of the pth is put in the form

$$\langle \Delta n(\tau)^p \rangle = \int d\Delta n(\tau) P(\Delta n(\tau)) \Delta n(\tau)^p,$$

where $\Delta n(\tau) = n(t) - n(t + \tau)$ and $P(\Delta n)$ is the probability distribution function of the density increments. Figure 2 shows the integrand as a function of $\Delta n(\tau = 4\ \mu s)$, allowing us to conclude that the highest order moment that can be achieved is about 10.

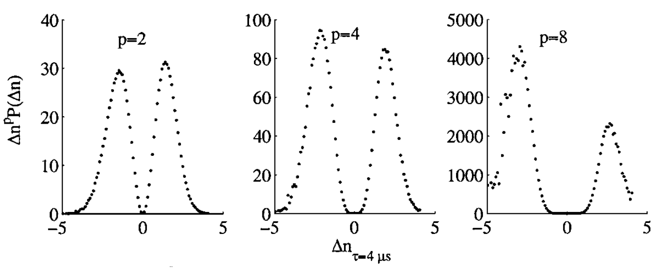

FIG. 2. The plots show $P(\Delta n(\tau))\Delta n(\tau)^p$ taken for $\tau = 4\ \mu s$ and $p = 2$, 4, and 8. The data are recorded inside the main plasma column in PISCES where the pdf's are Gaussian; outside the main plasma column, $p = 4$ is the maximum order moment that converges.

The generalized structure function $G_{n,p}(\tau) = \langle |n(t) - n(t + \tau)|^p \rangle$ is used. Hereafter, the subscripts, n, v_r, and v_θ, denote density, radial, and poloidal velocity fluctuations. The absolute value helps to make the odd moments better defined while keeping the even moments unchanged. In Fig. 3 is plotted $G_{n,p}(\tau)$ as a function of τ for $p = 2$, 3, 4, and 5. Clearly one cannot use the self-similarity (SS) assumption (τ^{ξ_p}) to determine the scaling exponents ξ_p because no scaling region is detected. This is probably due to the production-dissipation energy spectrum which extends over a large region affecting the shape of G_p. In neutral fluids, the lack of self-similarity was also reported and the difficulty is overcome by using the extended self-similarity (ESS) method [19]. This technique scales moments among each other rather than scaling moments against spatial or temporal separation. Therefore, the constraint of having an inertial range far from the production and dissipation scales of the system is relaxed. In the case where an inertial range exists, it was shown that either the ESS or SS methods yield the same scaling law [19]. The ESS method does not exclude deviations from the Kolmogorov law nor other possible scalings. Because the scaling of the third-order moment can be deduced from the Navier-Stokes equation $[\langle |\Delta v(r)|^3 \rangle \sim r]$, the scaling $\langle |\Delta v|^p \rangle = A \langle |\Delta v|^3 \rangle^{\xi_p}$ is used here. In principle, however, one can use any moment to conduct the scaling.

In Fig. 4 is shown the scaling of $G_{n,p}$, $G_{v_r,p}$, and $G_{v_\theta,p}$ with respect to the third-order moments. It is clear that the scaling is considerably enhanced when compared to Fig. 3 and extends over almost the entire domain. A linear fit in the logarithmic plot yields the scaling exponents for the density ($\xi_{n,p}$) as well as for the velocity fluctuations ($\xi_{v_r,p}$ and $\xi_{v_\theta,p}$). To our knowledge, this is the first time such scaling is obtained in magnetically confined plasmas. The corresponding scaling exponents are plotted in Fig. 5(a) with respect to p. Note that the values of $\xi_{v_r,p}$ should be disregarded for $p \geq 8$ because no scaling is detected. Moreover, in Fig. 5(b) $\xi_{n,p}$ is shown for

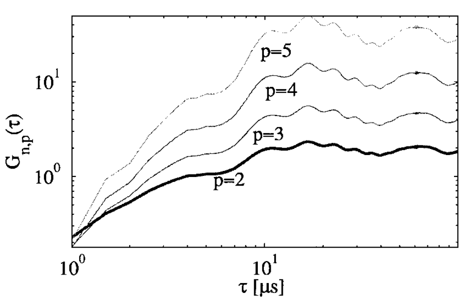

FIG. 3. $G_{n,p}(\tau)$ vs τ for $p = 2$, 3, 4, and 5. Note that no scaling region can be identified.

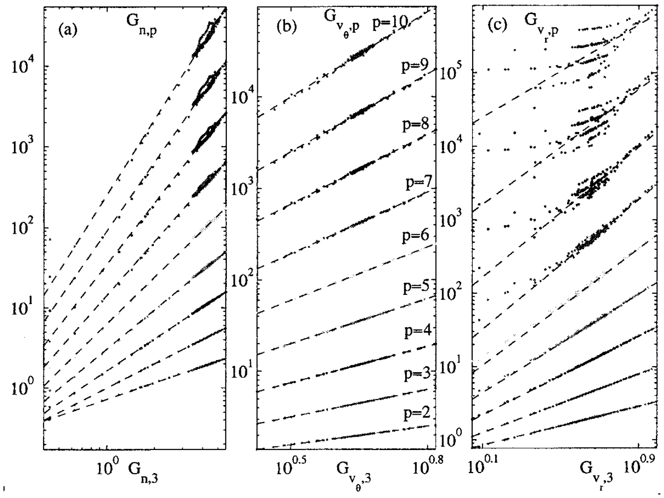

FIG. 4. The structure functions of pth order as a function of the third-order moment for the density (a), v_θ (b), and v_r (c). Note the good scaling that is reached except for v_r and for $p \geq 8$.

different operational parameters in PISCES. Within the experimental precision, the dependence of $\xi_{n,p}$ on p is linear with a slope equal to $1/3$ in agreement with the Kolmogorov-Kraichnan theory for two-dimensional turbulence in the energy cascade subrange. The magnetic Reynolds number (R_m), being about 3, explains why ξ_p does not follow the scaling for magnetohydrodynamic turbulence of Iroshnikov-Kraichnan which is valid for $R_m \gg 1$ [20,21].

The same procedure, using the same type of diagnostic, is applied to data obtained on the MAST [22]. In this toroidal device probes cannot stay long inside the last closed flux surface because of the high ion and electron fluxes which seriously perturb the measurement. For the present experiment, stationary plasma conditions were achieved for about 15 ms, the acquisition frequency being 1 MHz. Consequently, the maximum order moment that converges is about 6. In Fig. 6(a) is shown the structure functions $G_{n,p}$ plotted as a function of $G_{n,3}$ indicating a satisfactory scaling. In Fig. 6(b) is shown the scaling exponents of the radial velocity and density. Here again, the scaling is in agreement with the linear $p/3$ prediction of the Kolmogorov-Kraichnan theory.

The fact that the scaling follows the classical theory is in agreement with other recent investigations. On the Tore Supra tokamak, it was shown that coherent structures are rare inside the plasma [23]. The flatness factor was found to be equal to 3.06 ± 0.02, very close to 3, the Gaussian value. On the other hand, the onset of turbulence in magnetically confined devices is extremely hard to detect because of the various phenomena taking place during the start-up of the discharge. Hence, there exist mostly theoretical studies about the instabilities that can lead to turbulence [24]. The agreement with the $p/3$ scaling suggests that the energy production scale is around or smaller than the range of scales studied here (between 1 mm, the dimension of the probe tip, and a few centimeters, a scale set by the system large-scale dynamics). This is consistent with the idea that the source term may be drift-wave instabilities generated by density or temperature gradients that are about 1 cm in PISCES. Other investigations on the Tore Supra tokamak showed that the autocorrelation time of the three-dimensional energy, which could be linked to the eddy turnover time, decreases with decreasing scale. Because in the enstrophy subrange the eddy turnover time does not depend on the scales, it appears that the studied fluctuations, between 0.4 and 2 cm, lie in the energy cascade subrange [25].

In summary, we performed the first detailed study of high order statistics of the density and the velocity fluctuations in 2D turbulent plasmas. It is found that while the

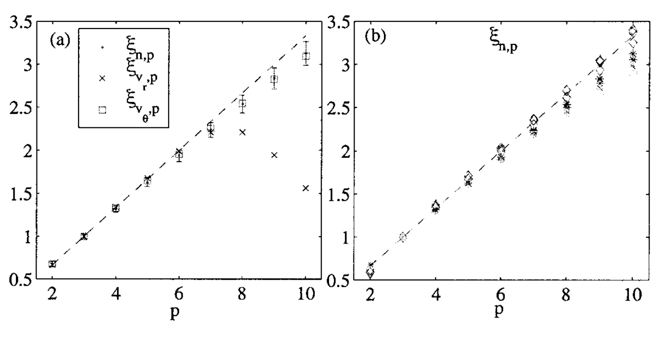

FIG. 5. (a) $\xi_{n,p}$, $\xi_{v_\theta,p}$, and $\xi_{v_r,p}$ as functions of p resulting from the scaling shown in Fig. 4. (b) The different symbols represent the scaling exponents $\xi_{n,p}$ as a function of p in the PISCES linear device under the following conditions: (◇) neutral hydrogen gas pressure of 2 mTorr and magnetic field strength equal to 800, 1200, 1600, and 2000 G, (*) magnetic field of 1200 G and hydrogen neutral pressure equal to 2, 3, and 4 mTorr, (○) helium gas at 2 mTorr and 1200 G. The dashed lines in (a) and (b) represent the Kolmogorov-Kraichnan law in the energy cascade range $p/3$.

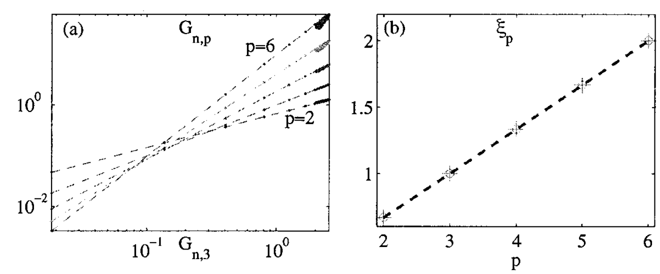

FIG. 6. (a) $G_{n,p}$ vs $G_{n,3}$ in a logarithmic plot showing a large scaling region for p from 2 to 6. (b) The scaling exponents in MAST for the density (+) and the radial velocity fluctuations (○) as a function of p. The discharge in MAST was in L mode with line average plasma density equal to 1.5×10^{20} m^{-3} and a plasma current of 670 kA. The probe is at a fixed position equal to 3 cm inside the last closed flux surface with stationary plasma parameters.

probability distribution functions deviate from Gaussian, they remained self-similar with respect to the order of derivative of the signals. Extended self-similarity was shown to yield good scaling of moments up to ten with respect to the third order in the PISCES linear plasma device. Moreover, we investigated signals recorded on a fusion toroidal device, the MAST spherical tokamak. Even though the two devices are different in many ways, the same behavior is reported inside the main plasma. When good scaling is achieved, ξ_p of either the density or the velocity fields increases linearly according to $p/3$. Moreover, it is found that the $p/3$ law is not affected by modified plasma parameters in PISCES. The results of the experiment show good agreement with the theory of Kolmogorov-Kraichnan for two-dimensional turbulence in the energy cascade subrange. They are also in agreement with the results obtained in laboratory experiments using incompressible neutral fluids.

I am grateful for the support of G. Counsell and Y. Yu at MAST. This work was performed under U.S. Department of Energy Contract No. DE-FG03-95ER-54301.

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