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ABSTRACT

The Green's function method is adapted to the description of strongly interacting particles which become more loosely bound as their number increases.

A NOTE ON THE GREEN'S FUNCTION METHOD

Maurice Neuman

Formally closed expressions for propagation functions, vertex, mass and polarization operators have recently been constructed by Schwinger[1]. An expansion of these in terms of free particle propagators reproduces the covariant Feynman-Dyson perturbation theory. The attempted improvements on the latter have centered around efforts to obtain solutions to equations in which the propagation functions are left in tact and only the lowest powers in the expansion of the remaining operators retained. This procedure is partially justified on the grounds of the presumed weakness of certain interactions.

1 J. Schwinger, Proc. Natl. Acad. Sci. U. S. 37, 452 (1951). We employ the notation of this reference.

It is the purpose of this note to put the theory in a form appropriate to the description of a somewhat different physical situation. No interactions between particles are assumed to be weak. A complex of particles will, however, be taken to be the more tightly bound the fewer the particles in it. As their number increases a looser structure results, which tends to break up into smaller, more tightly bound units. Thus, the propagation function associated with a large number of particles may be expanded in terms of those of smaller numbers, whereas the latter must be treated rigorously.

The result of this procedure is a finite set of equations involving propagation functions up to a certain order. Only in the equation for the highest order propagation function is a generalized vertex operator introduced explicitly; in the others it is effectively replaced by Green's functions of a higher order than the one in which the vertex operator would ordinarily appear. Since this quantity is associated with transition amplitudes involving creation and annihilation of particles and its expansion is equivalent to conventional perturbation theory, the modified procedure is suitable for a non-perturbation treatment of nonstationary processes like the production of mesons in nucleon-nucleon collisions.

The equations for the propagation function of one nucleon, S , and one meson, Δ , are

$$\left[\gamma p + m - g\, \tau_\xi \Gamma_0 \phi + i g\, \tau_\xi \Gamma_0 \frac{\delta}{\delta J}\right] S = 1 \qquad (1a)$$

$$(p^2 + \mu^2)\Delta - i g\, \tau_r \Gamma_0 \frac{\delta S}{\delta g J} = 1 \qquad (1b)$$

where Γ_0 is the zero'th order vertex operator explicitly defined for a given meson nucleon coupling. If the system $1N + 1M$ is assumed to be loosely bound, the vertex operator

$$\Gamma(\xi) = -\frac{\delta S^{-1}}{\delta g \phi(\xi)} \qquad (2)$$

is introduced at this point. The equations then assume the form

$$(\gamma p + m - g\,\mathrm{Tr}_\xi\,\Gamma^0\phi + \Sigma)\,S = 1 \qquad (1'a)$$

$$(p^2 + \mu^2 + \Pi)\,\Delta = 1 \qquad (1'b)$$

with

$$\Sigma = i g^2\,\mathrm{Tr}_\xi\,\Gamma^0 S \Delta \Gamma \qquad (3a)$$

$$\Pi = -i g^2\,\mathrm{Tr}_2\,\Gamma^0 S \Gamma S \qquad (3b)$$

The operator $\Gamma(\xi)$ may be expanded in terms of Δ and S :

$$\Gamma(\xi) = \Gamma_0(\xi) - \frac{\delta\Sigma}{\delta g\phi(\xi)} \qquad (4)$$

This procedure is inappropriate if not only 1N and 1M but also 1N + 1M are regarded as being tightly bound. The Green's function corresponding to 1N + 1M will be denoted by ${}^{1M}S$. Taking the variational derivative of (1a) with respect to $J(\xi)$ and observing that

$$\frac{\delta^2 S}{\delta J(\xi)\,\delta J(\xi')} = i\left[{}^{1M}S(\xi\xi') - \Delta(\xi\xi')\,S\right] \qquad (5)$$

we obtain

$$\frac{\delta S}{\delta J(\xi)} = g\,S_0\,\Gamma_0(\xi')\;{}^{1M}S(\xi'\xi) \qquad (6)$$

where the notation

$$S_0 = (\gamma p + m - g\,\mathrm{T}_{\xi}\,\Gamma_0 \phi)^{-1} \tag{7a}$$

$$\Delta_0 = (p^2 + \mu^2)^{-1} \tag{7b}$$

was introduced. Equations (1) now become

$$S = S_0 - i g^2\,\mathrm{T}_{\xi}\, S_0 \Gamma_0 S_0 \Gamma_0 \,{}^{1M}S \tag{1''a}$$

$$\Delta = \Delta_0 + i g^2\,\mathrm{Tr}\, \Delta_0 \Gamma_0 S_0 \Gamma_0 \,{}^{1M}S \quad , \tag{1''b}$$

explicit expression of S and Δ in terms of ${}^{1M}S$. Differentiating (1a) twice with respect to J and making use of (5) and (6) we obtain an equation for ${}^{1M}S$ [2]:

2 This equation, in a somewhat different form, was previously given by S. Deser and P. C. Martin, Phys. Rev. 90, 1075 (1953).

$$
\begin{aligned}
&{}^{1M}S(\xi\bar{\xi}) + i g\, S_0 \Gamma_0(\xi') \frac{\delta\, {}^{1M}S(\xi\bar{\xi})}{\delta J(\xi')} + \\
&+ i g^2 S_0 \Gamma_0(\xi') \Delta(\xi'\xi) S_0 \Gamma_0(\xi'')\, {}^{1M}S(\xi''\bar{\xi}) + i g^2 S_0 \Gamma^0(\xi') \Delta(\xi'\bar{\xi}) S_0 \Gamma_0(\xi'')\, {}^{1M}S(\xi''\xi) = \\
&= S_0 \Delta(\xi\bar{\xi})
\end{aligned}
\tag{8}
$$

With the aid of (1") we then have

$$
{}^{1M}S(\xi\bar{\xi}) + ig^2 S_0 \Gamma_0(\xi') \frac{\delta\, {}^{1M}S(\xi\bar{\xi})}{\delta g J(\xi')} =
$$

$$
\begin{aligned}
&= \Delta_0(\xi\bar{\xi}) S_0 + ig^2 S_0 \, Tr\, \Delta_0(\xi\xi')\Gamma_0(\xi')S_0\Gamma_0(\xi'')\, {}^{1M}S(\xi''\bar{\xi}) \\
&- ig^2 S_0 \Delta_0(\xi\xi')\Gamma_0(\xi')S_0\Gamma_0(\xi'')\, {}^{1M}S(\xi''\bar{\xi}) \\
&+ g^4 S_0 \Gamma_0(\xi')\Delta_0(\xi'\xi''')\, Tr\left[\Gamma_0(\xi''')S_0\Gamma_0(\xi^{IV})\, {}^{1M}S(\xi^{IV}\xi)\right] S_0\Gamma_0(\xi'')\, {}^{1M}S(\xi''\bar{\xi}) \\
&+ [\text{last two terms with } \xi \leftrightarrow \bar{\xi}]
\end{aligned}
\tag{9}
$$

At this point we may decide that a system more complex than $1N + 1M$ is loosely bound and express the variational derivative occurring in (9) by means of a generalized vertex operator

$$
\begin{aligned}
\frac{\delta\, {}^{1M}S(\xi\bar{\xi})}{\delta g J(\xi')} &= {}^{1M}S(\xi\xi'')\, {}^{1M}\Gamma(\xi''|\xi^{IV}|\xi''')\Delta(\xi^{IV}\xi')\, {}^{1M}S(\xi'''\bar{\xi})\,, \\
{}^{1M}\Gamma(\xi|\xi'|\bar{\xi}) &= -\frac{\delta\, {}^{1M}S^{-1}(\xi\bar{\xi})}{\delta g \phi(\xi')} \cong \Gamma^0(\xi')\Delta^{-1}(\xi\bar{\xi})
\end{aligned}
\tag{10}
$$

more explicitly defined by an expansion in terms of Δ, S and ${}^{1M}S$. The variational derivative term then gives rise to an interaction with the character of a self mass for the nucleon meson system:

$$
\begin{aligned}
&F^{1M} S(\xi\bar{\xi}) - \Delta(\xi\bar{\xi}) \\
&+ ig^2 \Gamma_0(\xi')\Delta(\xi'\xi^{IV})\left[{}^{1M}S(\xi\xi'')\,{}^{1M}\Gamma(\xi''|\xi^{IV}|\xi''') - \delta(\xi\xi''')S\Gamma(\xi^{IV})\right]{}^{1M}S(\xi'''\bar{\xi}) \qquad (8') \\
&+ ig^2 \Gamma_0(\xi')\Delta(\xi'\xi)S_0\Gamma_0(\xi'')\,{}^{1M}S(\xi''\bar{\xi}) + ig^2 \Gamma_0(\xi')\Delta(\xi'\bar{\xi})S_0\Gamma_0(\xi'')\,{}^{1M}S(\xi''\xi) = 0
\end{aligned}
$$

Equation (1") and (8') with (10) treat complexes of particles 1N, 1M, 1N + 1M rigorously and those with 1N + nM ($n \geqslant 2$) only approximately. An extension of this procedure would be to derive an expression for $\dfrac{\delta\, {}^{1M}S(\xi\bar{\xi})}{\delta J(\xi')}$ in terms of ${}^{2M}S$ and to augment (1") and (9) with an equation for the latter quantity. The introduction of a vertex operator ${}^{2M}\Gamma$ in it would imply that systems 1N + nM ($n \geqslant 3$) are regarded as loosely bound.

We now reexamine the 2N equation

$$\frac{1}{2}\left[F^{(1)}\mathcal{Y}^{(2)} + F^{(2)}\mathcal{Y}^{(1)}\right] S^{(12)} = 1^{(12)} \qquad (11)$$

from this point of view. With the variational derivatives written out explicitly (11) becomes

$$\frac{1}{2} F_0^{(12)} S^{(12)} + \frac{ig^2}{2} \gamma^{(12)}(\xi) \frac{\delta' S^{(12)}}{\delta g\, J(\xi)} = 1^{(12)} \qquad (12)$$

where

$$F_0^{(12)} = F^{(1)} F_0^{(2)} + F^{(2)} F_0^{(1)}$$

$$\gamma^{(12)} = F^{(1)} \Gamma_0^{(2)} + F^{(2)} \Gamma_0^{(1)}$$

$$F_0 = S_0^{-1}$$

Assuming at first that 2N is tightly bound and 2N + nM $(n \gg 1)$ is a loose structure we introduce, in analogy to the previous work, a vertex operator

$$\Gamma^{(12)}(\xi) = -\frac{\delta S^{(12)-1}}{\delta g \phi(\xi)} \cong F^{(1)} \Gamma^{(2)}(\xi) + F^{(2)} \Gamma^{(1)}(\xi) = \Gamma_0^{(12)}(\xi) \qquad (13)$$

and rewrite (12) in the form

$$F^{(1)} F^{(2)} S^{(12)} + \frac{1}{2}\left(\Sigma^{(12)} - \Sigma_0^{(12)}\right) S^{(12)} = 1^{(12)} \qquad (14)$$

with

$$\Sigma^{(12)} = i g^2 T_\xi \gamma^{(12)} S^{(12)} \Delta \Gamma^{(12)} \qquad (15)$$

$$\Sigma_0^{(12)} = F^{(1)} \Sigma^{(2)} + F^{(2)} \Sigma^{(1)} \qquad (16)$$

Thus, the interaction appears as the difference of two operators, each having the character of a self-mass. In the approximation $\Gamma_0(\xi)$ of the right member of (13) we obtain after a short calculation

$$\frac{1}{2}\left(\Sigma^{(12)} - \Sigma_0^{(12)}\right) = L^{(12)} F^{(1)} F^{(2)} \tag{17}$$

where

$$L^{(12)} = \frac{ig^2}{2} \operatorname{Tr} \Big\{ \Gamma_0^{(1)} F^{(2)} S^{(12)} F^{(2)} S_0^{(2)} \Gamma_0^{(2)} \, {}^{1M}S^{(2)} + \Gamma_0^{(2)} F^{(1)} S^{(12)} F^{(1)} S_0^{(1)} \Gamma_0^{(1)} \, {}^{1M}S^{(1)}$$
$$+ \Gamma_0^{(1)} \left(F^{(2)} S^{(12)} F^{(1)} - 1 \right) S_0^{(1)} \Gamma_0^{(1)} \, {}^{1M}S^{(1)} + \Gamma_0^{(2)} \left(F^{(1)} S^{(12)} F^{(2)} - 1 \right) S_0^{(2)} \Gamma_0^{(2)} \, {}^{1M}S^{(2)} \Big\} \tag{18}$$

and (12) with all systems $1N + nM$, $2N$ treated rigorously has the form

$$\left(1 + L^{(12)}\right) F^{(1)} F^{(2)} S^{(12)} = 1^{(12)} \; . \tag{19}$$

In the order to include $2N + 1M$ systems in this scheme it is necessary to express the variational derivative in (12) in terms of ${}^{1M}S^{(12)}$. A procedure analogous to the one that yielded (6), and the identity

$$\frac{\delta^2 S^{(12)}}{\delta J(\xi)\, \delta J(\bar{\xi})} = i \left[{}^{1M}S^{(12)}(\xi \bar{\xi}) - \Delta(\xi \bar{\xi})\, S^{(12)} \right] \tag{20}$$

lead in this case to the relation

$$F_0^{(12)} \frac{\tilde{\delta} S^{(12)}}{\delta g J(\xi)} = \gamma^{(12)}(\xi') \; {}^{1M}S^{(12)}(\xi'\xi) + F^{(1)} S_0^{(1)} \Gamma_0^{(1)}(\xi') \; {}^{1M}S^{(1)}(\xi'\xi) + F^{(2)} S_0^{(2)} \Gamma_0^{(2)}(\xi') \; {}^{1M}S^{(2)}(\xi'\xi) \tag{21}$$

The quantity $\frac{\tilde{\delta} S^{(12)}}{\delta g J(\xi)}$ is of considerable physical interest. It is directly related to the transition amplitude for the production of one meson in a nucleon-nucleon collision. If the various propagation functions occurring in (21) could be constructed, the equation would lead to a non-perturbation treatment of this nonstationary process. The extension of this method to the treatment of other nonstationary processes is obvious.

It is probably superfluous to add that the equation here obtained are exceedingly complicated. The general scheme, however, seems to be appropriate to the description of loosely bound large complexes of particles and tightly bound small ones, a notion quite alien to conventional perturbation theory.

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