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A Metric for Situated Difficulty

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A Metric for Situated Difficulty

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Abstract

Analogy, conceptual change and problem reformulation have been central components in the exploration of human problem solving. A Situation Theoretic approach is developed to model analogy and conceptual change. This model is then used to relate a problem's representation to the associated cognitive difficulty. In this Unified framework the cognitive difficulty of isomorphic problem situations is defined in terms of the task, objects and relations of the problem situation. These components are then decomposed based on an Ecological Information Processing user model. The decomposition turns a problem situation the structure and dynamics of the problem; the rules or constraints which are applicable; and the necessary instructions for user interaction. From this, the cognitive difficulty associated with a problem representation is shown to be largely determined by the "instructional" component.

Introduction

The use of analogy as a central mechanism in problem solving has been explored in numerous domains. Computational models of analogy commonly describe analogy as a type–preserving mapping between relation or predicates and objects, or terms, in dissimilar domains. However, it is the dynamic similarity between domains which is central to what humans consider to be good analogies [1]. A differing perspective on analogy is typified by Lakoff and Johnson [13] and Hofstadter and French [10] in which analogy is seen as a pervasive and integrative component of human thought and problem solving.

By these accounts analogy is not a special instance, but rather a general form, of cognition which reveals an underlying associationist architecture. This perspective considers analogy to reflect an actual encoding of information rather than a mapping imposed between separately encoded entities. We assume the "common–place dynamics" of specific types of situation to be automatically processed and therefore imagined without effort. The role of reformulation and conceptual change that results from an analogical encoding is both difficult to design for an difficult to predict.

Despite a long history as an explanatory construct and a central role in Gestalt theories of problem solving problem reformulation, or insight, remains explanatory rather than predictive of performance. Theories of human information processing which focus on problem structure to the exclusion of problem context, or situation–ness are to blame. The defects of this approach are most evident in studies of isomorphic problems ([9]; [12]; [21]) in which performance differences of up to an order of magnitude have been found between problems having identical structure.

To capture the effects of representation on problem solving we hypothesize a model of cognition in which mental models simulating common–place physical events are additionally constrained by mental rules similar to production rules found in traditional models. We assume some portions of the dynamics of the mental models can be automatically processed. The mental rules, by contrast, are assumed to be situation independent, effortful, and consciously used to guide, restrict and interpret the behavior of the model. We operationalize the concept of a *mental model*, within situation theory ([2]; [6]) as a constrained parameteric situation type.

The ecological realist epistemology of Situation Theory holds that through evolution, experience and learning our mental states come to *reflect* those of the world. The mental reflection of environmental constraints is regarded as as attunement of the mental model to the environmental consistency. Because nomic (natural law) constraints exert such a pervasive influence over our experience and development, they are guaranteed to meet the conditions of extended practice and the consistent mapping needed to develop automaticity in cognitive processing. This distinction is psychologically significant because mental processes characterized as automatic:

- are not subject to processing limitation or introspection
- elicit attention automatically
- and are extremely resistant to extinction.

This means that a mental model constrained by nomic attunements should, without mental effort, exclude states or events which would violate those constraints. Thus, situated cognition, occurs within the context of expectations built from a history of experience and perception of the current environment.

Cognition using this model involves simulating events which are jointly constrained by mental constraints associated with the situation–type, referred to as attunements, and mental constraints not integral to the situation–type, referred to as *instructions*. The interplay between the expectations (attunements) and explicit constraints (instructions) is based on the context that the user is interacting with. This context contains a set of objects, relations, features and behaviors which impact cognition via the expectations they license and the constraints being applied. Defining the situation of an interaction as a relational composition of the objects and dynamics provides an epistemology for hierarchically, and recursively typing the situation, objects, and associated constraints.

The assumptions of this cognitive modeling result in predictable differences in the processing of constraints in a situation. Constraints to which the user has developed an attunement will be processed automatically. Whereas the constraints which require the application of explicit rules are processed by controlled cognitive and perceptual operations. The constraint to which an agent is attuned are determined by the type of situation which in turn is determined by the types of objects and relations involved. So for example, any mental model involving solid physical objects would be constrained by nomic attunements to exclude events in which two objects came to occupy the same location at the same time. Additional conscious constraints can be imposed, as instructions, on the nomically constrained mental model to further restrict its behavior.

In the game of chess for example, a naive player's nomic attunements would preclude pieces changing locations or shape without being moved. However, the constraint preventing a Knight from moving three squares in a line would require application of a rule not available through nomic attunement, i.e. explicit instructions.

The power of this hybrid approach is in its ability to characterize the composition of imagined processes, visual inference, intuition, and controlled information processing within a single model. For example, an aeronautical engineer's mental model of wing behavior might compose a commonsense understanding of fluids and fluid flows with specialized engineering knowledge of airfoil design., The commonsense understanding which is unlikely to be verbalized, is described by attunements (constraints) associated with situations involving fluids. The other, more specialized, engineering knowledge, expressed as learned heuristics and equations, are the instruction which further constrain the commonsense model.

With this epistemology, it then becomes possible to model differing situation types, and their associated constraints. The model is grounded in the perceptual and cognitive experience of the user and the task which they are attempting to perform within a context. Situational contexts which are consistent with the user's experience and which closely model the problem task will be less complex that those are not so consistent. Determination of this "closeness", or "intuitiveness", is accomplished through the unification of problem task, context and user model. The comparison of alternative problem contexts, for the same abstract problem task, becomes realisable due to the normalization of representation. The attunements and constraints licensed by each context determine the amount of cognitive and perceptual effort necessary for a user to interact. Maintaining constraints from the abstract problem as constraints in the alternative context results in greater difficulty than for contexts in which attunements are able to represent some of the same relations. The presence of this dichotomy, provides the framework to establish a mechanism for determining and evaluating the inherent complexity of a situation with respect to the interactions of a user and their particular problem–solving task.

Difficulty in Situated Cognition

Accepting the conventional characterization of cognitive tasks as search, the difficulty that a particular situation presents is determined by the degree to which controlled processing is needed to constrain search, and by the size and complexity of the problem space. Based on the Situation Theoretic approach outline above, it is convenient to think to the user's task as involving two problem spaces:

1. A cognitive space which may be traversed with minimal effort but may also be very large. This is defined as a problem context, S, which is constraind by attunements, A, and written as $A \bullet S$.
2. A non–cognitive, or formal, problem space which may require substantial effort to traverse but contains only feasible transitions. This is defined as a problem situation, S, constrained by the task constraints, C, and written as $C \bullet S$.

The relation between the problem situation, task constraints, and attuned constraints can be expressed as:

$$C \bullet S \equiv f \bullet A \bullet S$$

where f is the extrinsic constraints necessary to provide a match between the two spaces. The complexity and character of these explicit constraints depend on the degree of correspondence between the problem situation, $C \bullet S$, and the user's cognitive situation, $A \bullet S$. Thus the difficulty of a task can be thought of as *reducible* based on the difference between the two problem spaces. When presented with the circumstance that several different, isomorphic, cognitive situations ($S', S'', S''', \ldots$) may be used to instantiate the same abstract problem space, this decomposition allows each one, and its associated attunements ($A', A'', A''', \ldots$) to be contrasted to determine which is minimally complex. i.e.

$$C \bullet S \equiv f \bullet A \bullet S \equiv f' \bullet A' \bullet S' \equiv f'' \bullet A'' \bullet S''$$

It may be useful to approach this model, and its implications for the evaluation of alternative representations ($f \bullet A \bullet S$), from a slightly different angle. We can define the extrinsic difficulty of a task, in a representation, as being determined by the constraints, f, which must be supplied by using controlled processing. The constraints needed in f in turn depend on the task constraints, from C, which are not supplied to the analog representation through automatic processing by the attunements, A. The availability of automatically processed attunements are themselves determined by the type of the situation, S. From this chain of dependencies, the only way to reduce the difficulty, of the $C \bullet S$ problem is to introduce a new, but equivalent, situation S' whose attunements, A, correspond more closely to the problem constraints C. As a result, controlled processing constraints are shifted to A and thus processed automatically. Any constraints which do not map to attunements are maintained as instructions, f, and thus also maintain their status as requiring controlled processing.

This model predicts that analogy will be the only route to reducing problem difficulty, and even then only by shifting the controlled processing constraints of the problem situation onto the automatically processed attunements of the analogous situation. Introducing the analogous situation S', however, requires defining an analogy as a translation M between S and S' written as:

$$M : f \bullet A \bullet S \rightarrow f' \bullet A' \bullet S'$$

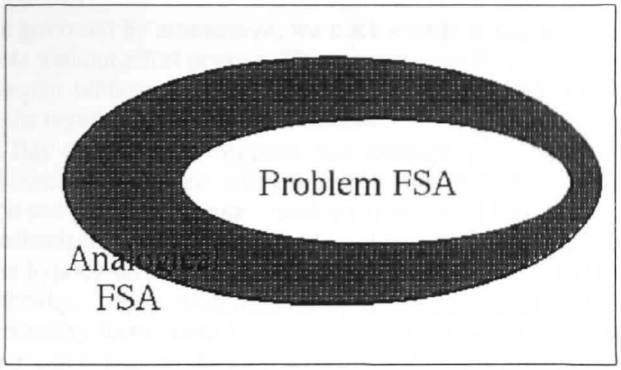

Figure 1: Necessary Instructions

As with the traditional problem reformulation systems [20], the complexity of M may also need to be considered. If the task is incompletely characterized (there may be additional goals or constraints associated with the object or relations of S) then resource consumption associated with M will increase the complexity of the representation. Assuming M to require controlled processing, a measure of cognitive difficulty, D, for a task can be defined as:

$$D(f \bullet A' \bullet S') + D(M)$$

If we presume constraint through attunement to be only nominally difficult, and certainly less difficult than enforcing constraints through instructions, we can rank representations for task difficulty by ordering them according to difficulty of their instructions. i.e.

$$D(f \bullet A \bullet S) > D(f' \bullet A' \bullet S')$$

if $D(f) > D(f')$ and generally that $D(A) \gg D(f)$.

This translates the measure of difficulty between isomorphic problem spaces into the determination of complexity of the necessary instructional component. Figure 1 shows the Necessary Instructions as the area of mis–match between the space of the analogical situation and the space of the problem situation. If we were to treat these spaces as Finite State Automata, the Necessary Instructions are simply a prescriptive list of states and state-transitions which are illegal in the analogical context. This list is added to the analogical situation as explicit constraints (instructions) and becomes part of the specification of the mapping, M. Thus the difficulty of an analogical situation is increased by a mis–matched mapping from the problem situation. The larger the mis–match, the more instructions are necessary and the greater the complexity which is as one would expect.

It is convenient to think of nomic attunements as being those problem constraints which go without mentioning, such as object constancy. All other constraints are then those which require explicit instructions in order to be conveyed. The ability of a representation to convey constraints through instructions makes the automatic identification of communicative analogies practical by providing what is essentially an "error term." This allows a relatively narrow range of nominally constrained situations to fit a much broader range of situations.

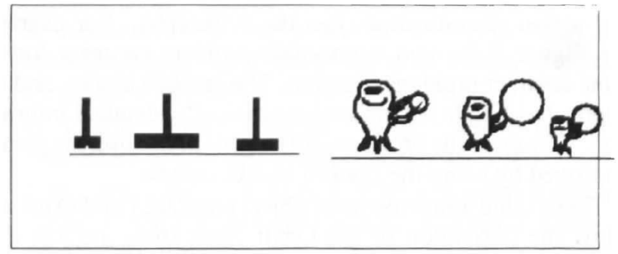

Figure 2: Tower of Hanoi Isomorphs

An operational approach is taken for establishing the difficulty associated with a set of instructions. Adopting the assumptions of Larkin and Simon [14], and Casner [4], that the difficulty of using a rule is proportional to the perceptual/memorial operations needed to determine its applicability we are provided with a relatively simple measure of cognitive difficulty. This measure is experimentally verified in Cleveland and McGill [5], and operationalized in Mackinlay [17], for the automated design of graphs and by Casner [4] for the automated design of task-oriented graphics.

The cognitive/perceptual simplicity of instructional conditions is evaluated based on the results of task-like decomposition. The actual reduction of instructions usually involves the presentation of conjunctive information at a single spatial location. this is the primary mechanism Larkin and Simon [14], use to explain cognitive efficiencies resulting form the use of diagrams and is central to many of the advantages demonstrated by Casner [4], in presenting airline reservation information. In other contexts it appears as a primary source of the effectiveness of task analytic approaches to display formatting such as Mitchell and Sasi [18], and a key mechanism involved in the presentation of spatially correlated data in scientific visualization.

Experimental Support

For problem tasks in differing analogs of an isomorphic domain, marked differences in human performance have been seen for the situational–analogues which possess ecological advantages ([12]; [16]; [3]; [21]). A large body of research into display design has also shown a similar trend. for example, Edlund and Lewis [7] showed that displays which present information and dynamics in a situational–analogue that utilizes the attunements which the users possess toward physical objects and interactions, gain a performance advantage over displays which rely more on instructions and controlled processing. This performance advantage existed both for control tasks and for error detection and diagnostic tasks. Moray [19], showed that a graphic rendering of the Rankin cycle, which makes physical constraints visible, allows steam plant operators to more accurately control and monitor for failures than did the more traditional collection of Piping and Instrumentation displays.

Lewis [16] showed that a simple attunement involving object constancy and extension allows a block stacking version of the Towers of Hanoi problem to be identified as simpler and involving fewer necessary instructions than alternatives with isomorphic problem spaces. The block–stacking representation was shown to provide more efficient state discrimination and action identification than the isomorphs. For example see Figure 2 for two isomorphic problem contexts for the same abstract problem situation. The stacked blocks, and the Globe–holding Monsters present logically identical information. However, the problem–solving ability of users is greatly improved by using the stacked blocks context.

These same attunements of object constancy and extension allow the derivation of the Gantt chart from the job shop scheduling problem ([15]; [8]), and the derivation of stacking keyed blocks (See Figure 3)as a display for the more constrained flow shop problem. Hume [11] again experimentally show, that displays relying on attunement to reduce difficulty substantially improves performance. The ability of this model to order the difficulty of the problem isomorphs in agreement with experimental results; to derive proven display formats from their problem representation; and to generate novel formats in accordance with the same principles, lend empirical support to this approach.

In an example from the airline reservation domain, the choice of a nonstop flight to a particular destination, with an interest in cost and departure time is shown by Casner [4], to be efficiently presented on a graph with time on the horizontal axis and cost on the vertical. A customer with the goal of choosing a flight for an approximate time and cost need only examine a single spatial region to identify flights matching her requirements. Experimental results show that the presentations which take advantage of spatial conjunction of information and which in turn reduce the complexity of the cognitive/perceptual operations necessary to administer an instruction, result in faster and more accurate task performance. Where a user's goal involves extracting conjunctive information from static data this form of conjunctive display is almost always superior to tabular presentations.

Conclusion

The determination of instructional complexity in isomorphic representation of a problem, is central to establishing an overall difficulty measure for the representation. The use of Situation Theory and an Ecological user model allow us to represent a person's problem solving goals, task environment and expectations in a unified framework. Within this framework, difficulty is modeled as a measure of resource allocation. Cognitive and perceptual tasks divide the problem constraints between the effortless, automatic operations (attunements) and the controlled operations (instructional constraints).

The contents and the functional role of the problem instructions can now be determined for the various alternative representations for a problem situation. Comparison of the relative perceptual/cognitive difficulties of the instruction sets is used to rank the difficulty associated with each representation. Research into the methodology and accuracy of instructional task decomposition is currently being pursued.

The ecological cognitive model we have presented, supplements information processing models of cognition by proposing a framework in which intuition, imagined events, mental models and production rule–like constraints interact and are described within a unified model. Classic computational problems, as solved by human, such as the frame/ramification problems are explained by the proposed distinctions between attuned and instructed constraints. Where our expectations

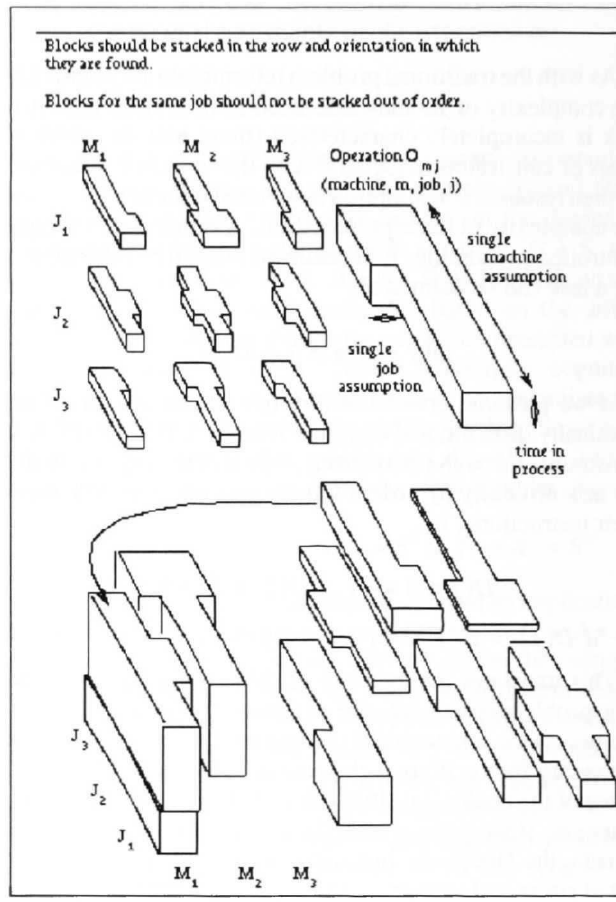

Figure 3: Flow–shop Block Stacking Display

are governed by attunement, we track events in our environment without effort or error. This happens, not by performing complex sentential updates but by our unconscious adaptation to the regularities of our environment.

This development suggests that analogy, as an instance of conceptual change, plays a central role in human cognition and problem solving. Analogy is modeled here not as a mechanism of valid inference or as a heuristic guide to inductive hypotheses, but as a mechanism for reducing cognitive difficulty. This is accomplished by shifting the allocation of processing from controlled to automatic processes over the course of designing the representation for the cognitive space equivalent of the problem situation. In this manner we can evaluated the implications of representational, and thus conceptual, change on the cognitive complexity for a problem solving task.

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