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# An Information-Based Theory
# of Time-Varying Liquidity

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## ABSTRACT

We propose an information-based theory to explain time variation in liquidity and link it to a variety of patterns in asset markets. In "normal times," the market is fully liquid and gains from trade are realized immediately. However, the equilibrium also involves periods during which liquidity "dries up", which leads to endogenous liquidation costs. Traders correctly anticipate such costs, which reduces their willingness to pay. This foresight leads to a novel feedback effect between prices and market liquidity, which are jointly determined in equilibrium. The model also predicts that contagious sell-offs can occur after sufficiently bad news.

JEL classification: G12, G14, C73, D82, D83.

Asset markets are susceptible to periods of illiquidity. Recent examples of this phenomenon include real estate (Clayton, MacKinnon, and Peng (2008)), mortgage-backed securities (Gorton (2009), Acharya and Schnabl (2010), Dwyer and Tkac (2009)), the repo market Gorton and Metrick (2012), structured credit (Brunnermeier (2009)), commercial paper (Anderson and Gascon (2009)), and money market funds (Krishnamurthy, Nagel, and Orlov (2012)). In conjunction with this evidence, a literature has developed that exogenously specifies time variation and risk in the ability to trade securities and then explores the implications for asset prices. Examples include Longstaff (2001, 2009), Acharya and Pedersen (2005), Watanabe and Watanabe (2008), Gârleanu (2009), Ang, Papanikolaou, and Westerfield (2014). Yet the underlying mechanism driving this phenomenon is not well understood. What can explain dramatic changes in the amount of liquidity in markets? What are the implications for asset prices? What is the nature of the interaction between prices and liquidity? For instance, do asset prices affect liquidity in ways that models with exogenously specified illiquidity risk cannot capture? In this paper, we propose answers to these questions by developing a theory in which time variation in market liquidity arises endogenously.

Our theory is developed within a dynamic economy with rational, risk-neutral agents. The model features three key elements. First, traders have private information about the future cash flows generated by their assets. Second, the market receives information about these cash flows, which we refer to as *news* stochastically over time. And third, all agents are subject to idiosyncratic preference shocks (e.g., financial/liquidity constraints); a trader who is hit by a shock has a reason to sell, though she is not forced to do so. This last ingredient implies that a trader who purchases an asset today cares about the expected liquidity of that asset in the future, which we refer to as an (exogenous) *demand for future liquidity*.

We indeed find that the risk of future illiquidity has important implications for asset prices. Consider two traders, $A$, is the current owner of an asset, and $B$, is a potential buyer. Trader $B$, when considering the purchase of the asset, realizes that his ability to sell it in the future may be limited by endogenous frictions stemming from asymmetric information. This foresight reduces his willingness to pay for the asset today. As a result, prices are driven below fundamentals, leading to an *illiquidity discount*, which varies over time with the degree of the information asymmetry. However, because prices and liquidity are jointly determined in our theory, the story does not end here. The reduction in asset prices feeds back into determining liquidity in the market. Trader

*A*, when considering the sale of the asset, is now less inclined to sell at the depressed prices if she has positive (private) information about the asset. This in turn makes Trader *B* even more hesitant to offer a pooling price, which exacerbates the adverse selection problem and leads to further deterioration in market liquidity. In short, the information friction generates illiquidity, and the demand for future liquidity amplifies the consequences. These forces negatively feed back on one another until the price function and degree of liquidity reach a fixed point.

Our formal analysis begins in a single-asset, two-period environment in which we highlight the intuition underlying the mechanism. Then, building on the framework developed in Daley and Green (2012) (hereafter DG12), we extend our analysis to a continuous-time, infinite-horizon model in which news is revealed via a diffusion process, and observable shocks arrive according to a Poisson process. We construct an equilibrium in which the amount of liquidity in the market crucially depends on the market belief about the asset value, which evolves over time. As in DG12, the equilibrium partitions the belief space into three distinct regions: (1) when the belief about the asset's type is favorable, efficient trade occurs immediately at a "fair" price; (2) when the market is pessimistic about the asset, the owner is forced to either sell at a low price or wait (a trader with a low-value asset mixes over these two alternatives, whereas one with a high-value asset waits), and (3) when the market completely breaks down, both sides of the market wait until either sufficient good news restores confidence to (1) or enough bad news forces (2).

Figure 1 illustrates a sample path of the equilibrium dynamics. To elaborate, suppose Trader *A* owns a share of the asset, and she experiences a shock while the market belief happens to be favorable. In this case, which we interpret as "normal times," the market is *fully liquid*; Trader *A* sells immediately to, say, Trader *B*, without affecting the market belief. While Trader *B* is in possession, bad news arrives such that the belief drifts down into the middle region, at which point Trader *B* experiences a shock. In this case, the market is *fully illiquid*; Trader *B* will be unwilling to sell at the highest price buyers are willing to offer, and inefficient delay will ensue. These equilibrium dynamics can explain what is often referred to as "liquidity drying up".[1] If the asset is of low quality and sufficient bad news arrives, Trader *B* may capitulate and sell at a low price. Or she may hold out, waiting for sufficient good news to arrive and market liquidity to be restored. If the asset is of high quality, Trader *B* will wait until enough good news restores market liquidity, meaning that not selling at a low price is a positive signal to the market.

After demonstrating the existence of an equilibrium that features these dramatic changes in market liquidity, we study the implications. First, what are the implications for asset prices? One immediate implication is that during normal times (i.e., when the belief is above $\beta$ in Figure 1), the asset trades at a discount relative to its fundamental value because potential buyers correctly anticipate the future risk of illiquidity. As the market belief increases further above $\beta$, the risk of future illiquidity decreases, and so too does the discount. Another implication is that the price process exhibits excess volatility in response to news, because news provides information not only about fundamentals, but also about future illiquidity risk. That is, bad news decreases the belief about fundamentals as well as the expected future liquidity of the asset. This additional consequence exaggerates the price reaction to news, adding additional volatility.

Second, what is the nature of the interaction between prices and illiquidity? The equilibrium exhibits an important feedback channel between asset prices and the degree of market illiquidity. Illiquidity leads to lower prices, which makes owners of high-value assets less eager to sell. Because high-type owners are less likely to sell, buyers face more severe exposure to the lemons problem. This exacerbates the consequences and further reduces liquidity, which in turn further reduces prices, and so on. This feedback channel leads to a somewhat perverse implication: the more traders demand future liquidity, the more difficulty they will have in consummating a trade today. Intuitively, when traders have greater demand for liquidity, the risk of illiquidity lowers the prices they are willing to pay, setting off the feedback loop just discussed, thereby amplifying the degree of illiquidity.

Part of the rationale for this finding is that greater demand for liquidity increases the severity of the adverse selection problem because an asset owner's private information pertains not only to the underlying cash flows, but also to the future liquidity of the asset. This additional private information arises because future market liquidity depends on the realization of news, which is correlated with the asset's type.

We next extend these results to a setting in which the asset has identical shares, held by $N > 1$ different privately informed owners. Hence, $N$ measures the dispersion of the asset's ownership, which is taken as an exogenous feature of the environment.[2] Ownership dispersion leads to an

*informational externality* among shareholders: the trading behavior of one owner affects the market belief about the asset's type and therefore the payoffs and behavior of other owners. We analyze how this externality affects both trading dynamics and allocative efficiency. One manifestation of this feature is the possibility of a *contagious sell-off*.

A sell-off is often said to occur when a rapid increase in supply coincides with trading activity at low prices. Wall Street traders and analysts also refer to this event as "market capitulation."[3] In our model, sell-offs occur because one seller's willingness to accept a low price reveals information about the asset's type to the market, which reduces the information asymmetry and induces other traders to sell their shares contemporaneously. Because private information is revealed through this trading behavior, the sell-offs in our model lead to higher future liquidity and improve efficiency. These implications are in contrast to the typical view of "fire sales," which can exhibit similar patterns but are often thought to reduce market efficiency and to be driven by accounting standards or large haircuts that "force" owners to sell.[4]

We conclude our analysis by investigating parameterized examples. Doing so allows us to attach magnitudes to our findings and explore implications for trade volume and efficiency. Using parameters targeted at fixed-income securities (e.g., asset-backed securities, corporate debt), the model suggests that even a modest amount of uncertainty can generate severe episodes of illiquidity; the market becomes fully illiquid when the market belief place a 5% to 10% probability on the assets being "bad" (e.g., defaulting), with the possibility of a sell-off occurring at an implied default probability in the range of 20% to 30%. The model generates an illiquidity discount on the order of 2% to 5% in normal times and volatility that can be several times larger than what can be explained by fundamentals.

The allocative inefficiency is most severe when the market belief is just above the lower boundary ($\alpha$ in Figure 1) and accounts for a 6% to 8% loss in total expected surplus. Our results also indicate that a market with dispersed ownership is more efficient; a greater number of informed traders means more trade behavior from which the market can learn, which facilitates efficient trades. Interestingly, this increase in efficiency due to additional information contrasts with the effect of an exogenous increase in the informativeness of the news, which may increase or decrease efficiency.

Interpreted literally, our model is best suited to describe markets with informationally sensitive assets that represent an idiosyncratic component of the economy. For example, the asset could refer

to a mortgage-backed security (MBS), where both the issuer and certain investors have access to private information about the underlying collateral, which was not reliably disclosed. Allegations along these lines were made against numerous banks and asset managers in the aftermath of the financial crisis.[5] Within this environment, it also seems natural to think that shocks are observable and/or verifiable. For example, in the recent financial crisis, it was not difficult to identify financially constrained firms (e.g., Bear Stearns, Lehman Brothers, AIG).

Another natural application is residential or commercial real estate markets. One could interpret owners' private information as pertaining to the quality of the "neighborhood," with news corresponding to information about its relevant characteristics. In the case of residential real estate, news could include school quality rankings or crime reports, with $N$ corresponding to the number of homes with similar observable characteristics within the neighborhood.[6] Indeed, Kurlat and Stroebel (2014) provide evidence that private information about neighborhood characteristics affects home prices. Naturally, prospective real estate buyers are more eager to engage with owners who have a credible reason for selling, such as job relocation or financial distress (i.e., an observable shock). Finally, the "observability" of shocks need not be literal, but can arise in equilibrium due to standard disclosure/unraveling arguments (e.g., Milgrom and Roberts (1986)).[7]

More broadly, the economic forces we identify could apply to a variety of asset markets. For example, in equities or corporate bond markets, a trader in our model can be interpreted as a large stakeholder, with greater access to management and company information, whose decision to sell may influence the market's perception of firm value. Regardless of the application, the illiquidity risk our model generates is *not* diversifiable despite the fact that exogenous shocks are idiosyncratic. The impact on prices is not driven by aversion to risk, but rather by an inefficient allocation that depresses the expected discounted value of future cash flows.[8]

Our continuous-time model builds on DG12, who consider a setting in which there is a single seller and news about the seller's asset is revealed gradually over time. They construct analyze an equilibrium with a similar three-region structure. However, in their model, the asset is traded only once and at a price equal to the fundamental value–buyers do not face resale considerations. Inherently, such a model cannot address the questions we ask in this paper (e.g., How do resale considerations affect prices? How does demand for liquidity in the future affect the market today?).

A necessary theoretical advancement in the present paper is to incorporate demand for future liquidity in order to derive prices endogenously from forward-looking traders. By doing so, we are able to develop a theory in which equilibrium prices and liquidity are jointly determined and vary over time with the market belief. This theory helps us understand not only how liquidity affects prices, but also how prices feed back into determining liquidity. Another theoretical contribution of this paper is to incorporate multiple shares of the asset, which generates a realistic feature of many markets: trading behavior of one seller can reveal information to the market that may facilitate or inhibit the trades of other agents.

Illiquidity can also arise from a variety of other frictions, such as exogenous trading costs (Amihud and Mendelson (1986), Constantinides (1986)), inventory risk (Amihud and Mendelson (1980)), and search (Duffie, Gârleanu, and Pedersen (2005), Duffie, Gârleanu, and Pedersen (2007), Vayanos and Wang (2007), Vayanos and Weill (2008)). A key difference in the exogenous-trading-cost and search literatures is that the "amount" of illiquidity is generally constant over time and is determined by the size of the transaction cost or the arrival rate of a trading partner.[9] Our friction is non-institutional in that nothing in the form of the environment *prevents* efficient trade. Rather, the information asymmetry combined with the strategic considerations of forward-looking traders is what endogenously generates illiquidity, with gradual revelation of information leading to time variation in its degree.

One of our insights is to show that time-varying liquidity naturally leads to excess volatility in prices and returns. Of course, a variety of existing theories can generate excess volatility. To our knowledge, however, all of these channels rely on some additional source of systematic (or non-infinitesimal) risk coupled with risk aversion.[10] By contrast, the shocks in our model are idiosyncratic, and traders are risk neutral. Excess volatility obtains not because of aversion to systematic risk, but because of the information friction that inhibits market liquidity and generates an endogenous source of price volatility.

Our model differs from classic papers studying liquidity and asymmetric information in the microstructure literature (Kyle (1985), Glosten and Milgrom (1985)) in several important respects. First, we do not have noise traders, which facilitates our investigation of allocative efficiency. Second, our model features gradual arrival of information, or news. These two features, along with strategic traders, also distinguish our model from the noisy rational expectations approach

(Grossman and Sitglitz (1980)). Wang (1993) features gradual information arrival but in a setting with risk-averse, competetive traders and noisy supply.

Our work is also related to the growing literature studying adverse selection in dynamic environments Janssen and Roy (2002), Hörner and Vieille (2009), Fuchs and Skrzypacz (2014), Fuchs, Green, and Papanikolaou (2014). We differ from these papers in that our model features both resale considerations and news, which leads to a different equilibrium structure, dynamics, and feedback effects. Guerrieri and Shimer (2014) consider an environment with many different market prices existing simultaneously. In their model, both trading behavior and the degree of liquidity remain constant over time. Another important aspect of our model is that the information that agents possess is "long lived." This feature creates a source of persistence; liquidity tomorrow is likely to be similar to liquidity today. A number of other papers investigate liquidity in the presence of short-lived private information (Gârleanu and Pedersen (2003), Eisfeldt (2004), Wanatabe (2008), Biais, Bossaerts, and Spatt (2010)).

The remainder of the paper is organized as follows. In Section I, we present the two-period model to illustrate the key mechanisms behind our results. Section II presents the continuous-time model with a single share. Section III presents the main theoretical implications. Section IV extends the analysis to an arbitrary number of shares. Section V presents quantitative implications. Section VI discusses extensions and concludes.

# I.    Two-Period Model

In this section, we present a two-period model that illustrates the main ideas of the paper. The model features a single indivisible security that can be traded in a spot market, which is open for two periods. It contains three key ingredients:

- The security's owner has private information about the future value of its cash flows.
- After the first period, the market observes a noisy signal about these cash flows.
- An agent who buys the asset in the first period may want to resell the asset in the second period.

To motivate this setting, consider a credit-constrained bank looking to raise capital by selling an asset-backed security. The bank is privately informed about the quality of the underlying pool

of collateral and therefore has private information about the cash flows the security will generate. Moreover, any trader who purchases the security gains access to this private information, whereas the market observes only a noisy signal of the performance of the underlying assets.

Within this section, we illustrate several points. First, news generates variation in the market's belief about the value of the security, which leads to stochastic variation in its liquidity. Second, because buyers in the first period foresee the liquidity risk in the second period, their willingness to pay for the security decreases. Third, because an owner with a high-value security is optimistic about future liquidity (she expects good news to arrive), she is less inclined to trade in the first period, especially at a depressed price. As a result, buyers are less likely to offer the pooling price, which *amplifies* the degree of illiquidity in the first period. These considerations lead to the finding that the larger is the demand for liquidity in the second period, the more illiquid the market is in the first period.

## A.  Formal Setup

The model involves three dates, $t \in \{0, 1, 2\}$, and a spot market that is open for two periods. At dates 0 and 1, the security can be traded in a competitive market where multiple potential buyers compete in Bertrand fashion by submitting offers. For simplicity, we assume offers are made privately, using pure strategies, and each potential buyer can make at most one offer. We refer to the *bid* in a given period as the highest offer submitted in that period. At $t = 2$, the cash flow from the security, denoted by $C$, is realized. Discounting takes place between dates 0 and 1 and again between dates 1 and 2. Due to financial constraints, the initial owner has a per-period discount factor of $\delta < 1$. Therefore, her payoff from exchanging the security for a price of $w$ at date $t \in \{0, 1\}$ is $\delta^t w$. If the initial owner does not trade the security in either period, she consumes the cash flow realized at $t = 2$, meaning her payoff is $\delta^2 C$.

Ex ante, buyers are more patient than the seller; their discount factor between dates 0 and 1 is normalized to one. However, each buyer faces an idiosyncratic risk of becoming financially constrained. Specifically, after date 0 but prior to date 1, a buyer experiences a shock with probability $\lambda$ that causes her to discount cash flows by the factor $\delta$ between dates 1 and 2. Let $\tilde{\delta}_i$ be the independent random variable representing buyer $i$'s potential of getting shocked. It is realized immediately before date 1, and takes a value of $\delta$ with probability $\lambda$, and a value of one otherwise.

The payoff to a buyer $i$ who purchases the security for a price of $w$ (at either date) and retains it through date 2 is $\tilde{\delta}_i C - w$. A buyer who purchases the security at $t = 0$ for $w_0$ and then sells it at $t = 1$ for $w_1$ receives a payoff of $w_1 - w_0$. Finally, the payoff to a buyer who does not trade is zero.

Notice that if a buyer purchases the security at $t = 0$ and experiences a shock, there are gains from reselling the security in the market at $t = 1$ to another buyer who did not experience a shock.[11] If she is not hit by a shock, the buyer's discount rate remains equal to one and there are no gains from resale. We assume that past trades and shocks are publicly observed, meaning that whether there are gains from trade in each period is common knowledge.[12] Figure 2 summarizes the timing of the two-period model.

$$-\text{Insert Figure 2 here}-$$

The quality of the assets backing the security is $\theta \in \{L, H\}$, and players share a common prior $p_0 = \Pr(\theta = H) \in (0, 1)$. If $\theta = H$, the assets are of high quality and the security pays a cash flow of one at $t = 2$. If they are of low quality ($\theta = L$), the cash flow at $t = 2$ is $\kappa > 0$. The assumption $\kappa < \delta^2$ means that credit constraints are less detrimental than having low-quality assets, and precludes the trivial outcome in which the market is always fully liquid. For analytic convenience, we employ a more restrictive parametric assumption: $\kappa < \frac{\sqrt{\frac{2}{\delta}\delta^2} - 1}{\sqrt{\frac{2}{\delta}} - 1}$.[13] The security's owner knows the quality of the assets, whereas potential buyers do not. Between dates 0 and 1, the market observes news about the quality of the underlying assets. For convenience, suppose the news is a normally distributed random variable, $\tilde{s} \sim N(\mu_\theta, \sigma)$. The information content of the signal is completely summarized by the signal-to-noise ratio $\frac{\mu_H - \mu_L}{\sigma}$. Therefore, we normalize $\mu_H = -\mu_L = 1$.

## B. Equilibrium Analysis

The key determinate of equilibrium trade is the market belief about the value of the security. Let $p_t$ denote the probability that buyers assign to $\theta = H$ at the beginning of period $t$. For any belief $p$, let $\bar{V}(p) \equiv p \cdot 1 + (1 - p) \cdot \kappa$ denote the undiscounted expected cash flow of the security. We refer to $\bar{V}(p)$ as the (expected) *fundamental value* of the security because, given the belief, it is the (expected) value the security generates if efficiently allocated. The strategy of an owner is a mapping from $\theta$ and the history (inclusive of the current offers) to a probability of acceptance.

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We restrict attention to Perfect Bayesian Equilibria (PBE), satisfying a mild refinement on off-equilibrium-path beliefs known as *belief monotonicity*, which requires that an unexpected rejection cannot cause buyers to decrease the weight their belief puts on $\theta = H$.[14]

As we will see, equilibrium behavior at $t = 1$ corresponds to that of the familiar static model. Our primary interest, therefore, is to characterize the equilibrium trading behavior at $t = 0$ and how it depends on $\lambda$. We do so using backward induction.

At $t = 1$: The owner of the security may be constrained or unconstrained.[15] When the owner is unconstrained, we refer to her as a *holder*; because there are no gains from trade, without loss of generality (in terms of payoffs in either period and equilibrium play at $t = 0$), we specify that trade does not occur.[16] When the owner is constrained, we refer to her as a *seller* and the viability of trade depends on the market belief $p_1$. If the market belief is sufficiently low, $p_1 < \bar{p} \equiv \frac{\delta - \kappa}{1 - \kappa}$, a familiar "market for lemons" arises: only the low-value security trades, and the price is $\kappa$. On the other hand, if $p_1 > \bar{p}$, the market is fully liquid and the security trades, regardless of $\theta$, at a price of $\bar{V}(p_1)$. We summarize the key aspects of equilibrium trading behavior at $t = 1$ in the following lemma. The proofs for all formal results in this section are located in Appendix A.

LEMMA 1: *When the owner at time $t = 1$ is a seller, the unique equilibrium outcome for $p_1 \neq \bar{p}$ is as follows:*[17]

- *If $p_1 < \bar{p}$, the bid from buyers is $\kappa$. The low type accepts this offer with probability one. The high type rejects this offer with probability one.*
- *If $p_1 > \bar{p}$, the bid is $\bar{V}(p_1)$ and both seller types accept with probability one.*

*If the $t = 1$ owner is a holder, then in any equilibrium she weakly prefers to reject the bid.*

We note two features of trading at $t = 1$. First, any trade occurs at a price equal to the fundamental value of the traded security. Second, the structure of a seller's payoff at $t = 1$ as it depends on $\theta$ and $p_1$, denoted $F_\theta(p_1)$, is given by

$$F_H(p_1) \equiv \max\{\bar{V}(p_1), \delta\} \quad \text{and} \quad F_L(p_1) \equiv \kappa + \mathbb{I}_{\{p_1 \geq \bar{p}\}}(\bar{V}(p_1) - \kappa). \tag{1}$$

Importantly, $F_H$ is convex in the market belief; it is flat for $p_1 < \bar{p}$ and linearly increasing for $p_1 > \bar{p}$ (see Figure 3(a)).

At $t = 0$: The seller chooses between accepting the bid or retaining the security until the next period. Similar to $t = 1$, the bid will either be $\kappa$ or buyers' unconditional expected value for the security, which is denoted by $B(p_0)$.

To derive $B(p_0)$, define $q(s, p)$ to be the posterior belief updated from an arbitrary prior of $p$ given a realization of the signal, $s$.[18] Next, notice that a buyer faces the following potential outcomes. If he acquires the security at $t = 0$, then with probability $1 - \lambda$ he will not be shocked at $t = 1$ and will therefore consume the (unconditional) expected cash flow $\mathbb{E}[\bar{V}(q(s, p_0))] = \bar{V}(p_0)$ (by linearity of $\bar{V}$ and the martingale property of beliefs). On the other hand, with probability $\lambda$, he will be shocked. In this case, the value he can expect to enjoy at $t = 1$ is $\mathbb{E}[F_\theta(q(s, p_0))]$. Therefore, a buyers value is given by

$$B(p_0) \equiv (1 - \lambda)\bar{V}(p_0) + \lambda\mathbb{E}\left[F_\theta(q(s, p_0))\right]. \tag{2}$$

Notice that when $\lambda = 0$, $B$ is simply equal to the fundamental value because buyers do not have demand for future liquidity. On the other hand, when $\lambda > 0$, a buyer is not willing to pay the fundamental value $(B < \bar{V})$ even if both seller types are willing to sell, because he anticipates the potential for a reason to resell at $t = 1$, at which point the market may not be fully liquid (Lemma 1)–that is, he faces liquidity risk. The magnitude of the risk can be measured by

$$\bar{V}(p_0) - B(p_0) = (1 - \delta) \cdot \left(\lambda p_0 \Pr\left(q(s, p_0) < \bar{p}|\theta = H\right)\right). \tag{3}$$

The difference between $\bar{V}$ and $B$ is easily interpreted: it is the probability that $(i)$ the buyer will be hit by a shock, $(ii)$ the security is of high value, and $(iii)$ the realized signal is sufficiently low that he will be unable to resell it, multiplied by the lost value from this occurrence.

Turning now to the seller's decision, let $C_\theta(p)$ denote the continuation payoff to the seller given the market belief is $p$ after observing a rejection but prior to observing the signal:

$$C_\theta(p) \equiv \delta\mathbb{E}\left[F_\theta(q(s, p))|\theta\right].$$

Equilibrium behavior at $t = 0$ will be characterized by two belief thresholds. One threshold, denoted by $b$, is the belief level such that the high type is indifferent between selling at a price of

$B(b)$ or waiting until the next period. That is, $b$ is defined implicitly by $B(b) = C_H(b)$.

LEMMA 2: *There exists a unique $b \in (0,1)$ that solves $B(b) = C_H(b)$, where $C_H(p)$ is strictly greater than $B(p)$ for all $p < b$ and strictly less than $B(p)$ for all $p > b$.*

This lemma essentially implies that for $p_0 < b$, the market cannot be fully liquid; the high type's option to wait (recall the convexity of $F_H$) is more valuable than accepting $B(p_0)$. On the other hand, for $p_0 \geq b$, the high type is willing to trade at the buyers' value and a fully liquid market remains feasible.

For a given $p_0$, if the high-type seller is not trading, the low type can either trade at $\kappa$ or retain the security and hope for good news. Thus, the second relevant threshold, denoted by $a$, is the belief at which a low type is indifferent between these two options. That is, $a$ is defined implicitly by $\kappa = C_L(a)$.

LEMMA 3: *There exists a unique $a \in (0,1)$ that solves $\kappa = C_L(a)$, where $C_L(p)$ is strictly less than $\kappa$ for all $p < a$ and strictly greater than $\kappa$ for all $p > a$.*

Together, Lemmas 2 and 3 imply that if $a < b$ and $p_0 \in (a, b)$, any offer the seller finds acceptable yields negative expected profit to the buyer. Hence, in this region, trade occurs with probability zero. Further, this *fully illiquid* region corresponds to intermediate belief levels, meaning that market liquidity is nonmonotonic in the market belief. On the other hand, if $a > b$, then similar to $t = 1$, the equilibrium structure involves two regions, with higher $p_0$ corresponding to higher liquidity.[19] These results are summarized by the following proposition.[20]

PROPOSITION 1: *Equilibrium behavior at $t = 0$ is characterized as follows:*

(i) *If $p_0 > \max\{a, b\}$, the market is fully liquid, meaning that the bid is $B(p_0)$ and both types accept with probability one.*

(ii) *If $p_0 < \min\{a, b\}$, the market is partially liquid, meaning that the bid is $\kappa$, the high type rejects with probability one, and the low type accepts with probability $\frac{a - p_0}{a(1 - p_0)}$.*

(iii) *If $a < b$ and $p_0 \in (a, b)$, the market is fully illiquid, meaning that the bid is weakly below $C_L(p_0)$ and both types reject with probability one.*

(iv) *If $b < a$ and $p_0 \in (b, a)$, there exists $c \in (b, a)$ such that the market is fully liquid for all $p_0 \in (c, a)$. For $p_0 \in (b, c)$, there are two equilibrium outcomes (fully liquid or partially*

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*liquid).*

We note three features of the equilibrium. First, the amount of liquidity in the market can look quite different from one period to the next, as it depends on the initial belief and the realization of the signal. Second, because of illiquidity risk, traders shade their offers downward at $t = 0$. This drives the equilibrium price below the fundamental value when the market is fully liquid, which we interpret as an *illiquidity discount* and is characterized by (3). Third, a fully illiquid region is possible at $t = 0$. In what follows, we illustrate another key insight: greater demand for liquidity at $t = 1$ has a feedback effect that further reduces the amount of liquidity in the market at $t = 0$.

— Insert Figure 3 here —

*C. Role of Resale Considerations*

Consider the comparative static effect of $\lambda$. When $\lambda = 0$, buyers do not have demand for future liquidity and hence illiquidity in the market at $t = 1$ is irrelevant for their value at $t = 0$. As a result, there is no discount (i.e., $B = \bar{V}$). As $\lambda$ increases, so too does traders' demand for future liquidity. Because the market is not fully liquid in all states at $t = 1$, higher $\lambda$ depresses traders' value at $t = 0$. At this new lower value, a high type is no longer willing to accept $B(b)$ (recall that $C_H$ is independent of $\lambda$); hence buyers are no longer willing to offer it and $b$ increases. Thus, an increase in $\lambda$ affects not only the price level, but also the viability of trades as summarized by the following proposition.

PROPOSITION 2: *As traders' demand for future liquidity ($\lambda$) increases, the following hold:*

(i) *$B(p)$ strictly decreases for all $p \in (0, 1)$.*

(ii) *The high type's indifference threshold, b, strictly increases, whereas the low type's threshold, a, is unchanged.*

(iii) *The probability of trade in equilibrium at $t = 0$ weakly decreases for all $p_0$ and strictly decreases for some $p_0$.*

For perhaps the starkest illustration of (*iii*), consider the case in which $\lambda_1 < \lambda_2$ leads to $b_1 < a < b_2$ (recall that $a$ is unchanged by $\lambda$). Then for $\lambda_1$, the market is always either partially

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or fully liquid. However, for $\lambda_2$, the increased demand for future liquidity *creates* a fully illiquid region when one did not exist for $\lambda_1$.

Intuitively, adverse selection becomes more severe as $\lambda$ increases because the seller's private information is not only about the underlying cash flows, but also about the future liquidity of the security. This additional private information arises because market liquidity at $t = 1$ depends on the realization of the signal, which is correlated with the seller's type. That is, conditional on $\theta = H$, the market is more likely to be fully liquid at $t = 1$ than if $\theta = L$. Hence, a high type expects more favorable market conditions than does a low type. When $\lambda = 0$, this information is irrelevant since buyers have no value for future liquidity, but as $\lambda$ increases, so too does its relevance and the adverse consequences.

## D. Connection to the Continuous-Time Model

With intuition for the key mechanism established, in Section sec:model we develop an infinite-horizon, continuous-time model. Among other things, this model delivers a richer and more tractable characterization of the illiquidity discount and its implications for price dynamics. It also allows us to clearly illustrate several results that obtain only in a model with a longer horizon and that are difficult to conceptualize within a non-stationary setting.[21] An overview of how the continuous-time model enriches the present two-period version is as follows:

- There is an infinite horizon and the security delivers cash flows continuously, rather than a finite horizon with a cash flow in the final date. Constrained traders have positive holding costs relative to unconstrained ones, which generates gains from trade.
- The market is open continuously rather than at discrete periods of time; news is revealed according to a diffusion process with type-dependent drift; and shocks, which increase an owner's holding costs, arrive according to a Poisson process.

In Section IV, we illustrate how informational externalities among owners affect trading behavior. To do so, we generalize the model and extend our results to a setting where

- There are $N > 1$ shares of the security. Each agent can own at most one share.

# II.   Continuous-Time Model

There is a single asset in the economy, which has a fixed type $\theta \in \{L, H\}$. At every moment in time $t \in [0, \infty)$, an agent who is privately informed of $\theta$ owns the asset. We refer to this agent as the *owner* at time $t$, formally denoted by $A_t$.[22] The asset generates a cash flow to its current owner that depends on $\theta$ and the owner's (financial) *status*, which is either *constrained* or *unconstrained*. An unconstrained owner of a type-$\theta$ asset obtains an instantaneous cash flow of $v_\theta$, whereas a constrained owner has positive holding costs and obtains only $k_\theta < v_\theta$.[23] All agents are risk neutral and discount future payoffs at rate $r$. Let $V_\theta = \frac{v_\theta}{r}$ and $K_\theta = \frac{k_\theta}{r}$ denote the value of a share being held *ad infinitum* by an unconstrained and constrained agent, respectively. We assume $K_H > V_L$, meaning that the potential for a lemons problem exists.

Any owner is subject to an observable shock that arrives according to a Poisson process, with intensity $\lambda > 0$; if $A_t$ is unconstrained at time $t$, then the arrival of the first shock after time $t$ induces a positive holding cost, thereby transforming the owner into a constrained agent. As in the previous section, we refer to an owner as a *seller* if she is constrained, and as a *holder* otherwise. For simplicity, the seller is unaffected by subsequent arrivals and maintains a positive holding cost indefinitely. Let $I_t$ be an indicator that is equal to one if and only if $A_t$ is a seller at time $t$.

The market opens at $t = 0$, with the asset owned by $A_0$, whose status is commonly known. At every $t \geq 0$, buyers in the market generate an outstanding *bid*, and the seller can accept the bid in exchange for ownership rights to the asset. If the asset trades, its new owner learns the asset's type, and the previous owner exits the economy. If a seller rejects the current bid, she retains the asset, receives the flow payoff, and can entertain future bids.

The bid process is a convenient modeling device for aggregated buyer behavior, and is motivated by multiple potential buyers always being in the marketplace. In this setting, the precise mechanism for trading is largely unimportant. In each of the following trading mechanisms, it can be shown that there exists an equilibrium with trading behavior identical to the one on which we focus.[24]

EXAMPLE 1 (Decentralized or Over-the-Counter Markets): *At every $t \geq 0$, the owner is approached by multiple buyers who can each make a purchase offer. The owner observes offers and decides if, when, and to whom to sell.*[25]

EXAMPLE 2 (Posted Prices): *At every $t \geq 0$, the owner posts a price at which she is willing to*

*sell. Buyers observe posted prices and choose if and when to buy.*

Though it is perhaps more appropriate for the model with multiple shares (see Section IV), we offer a third example here in which a market maker facilitates trade.

EXAMPLE 3 (Facilitation by a Market Maker): *There is a market maker privy to the public history. At every $t \geq 0$, the market maker generates a bid price at which he will buy (shares of) the asset, and an ask price at which he will sell (shares of) the asset to buyers. He obtains weakly lower flow value from the asset than a seller and aims to maximize his own expected profit, but competitive pressures drive this profit to zero.*

## A.   Public Information

A key feature of the model is that *news* about the asset's type is continuously and publicly revealed via a diffusion process, $X$, where for all $t \geq 0$,

$$X_t = \mu_\theta t + \sigma B_t,$$

and $B$ is a standard Brownian motion. Define the signal-to-noise ratio $\phi \equiv \frac{\mu_H - \mu_L}{\sigma}$, which we assume to be strictly greater than zero. Larger values of $\phi$ imply higher quality news; $\phi = 0$ would correspond to a model without informative news.

To formalize the information structure, we introduce the probability space $(\Omega, \mathcal{F}, \mathcal{Q})$ in which $\theta$, $B$, and the shock process, denoted $L = \{L_t : 0 \leq t \leq \infty\}$, are mutually independent. The *public history* at time $t$, which also corresponds to the information set of buyers at time $t$, contains:

- The initial status of the original owner: $\{I_0\}$.
- The arrival times of shocks: $\{L_s : 0 \leq s \leq t\}$.
- The history of news: $\{X_s : 0 \leq s \leq t\}$.
- The history of all past trades.

Let $\{\mathcal{F}_t\}_{t \geq 0}$ denote the filtration generated by the public history. Finally, it will be convenient to keep track of the set of times at which the asset trades, which we denote by $\mathcal{T}$.[26]

## B. Bid Process and Owner Strategies

The bid process $W$ is a real-valued stochastic process, progressively measurable with respect to $\mathcal{F}_t$. To prevent trades based solely on the expectation of ever-increasing prices, we impose the standard transversality condition.[27]

An owner's information set contains the public history, the asset type, and a randomization device to allow for mixing. Let $\{\mathcal{G}_t\}_{t \geq 0}$ denote the filtration generated by the information sets of owners. A strategy for an owner of a type-$\theta$ asset who becomes a seller at time $t$, hereafter a "$(\theta, t)$-seller," is a stopping time adapted to $\mathcal{G}_h$ and denoted by $\tau^{\theta,t} \geq t$. It will be convenient to represent a seller's strategy by the distribution it induces over $\mathcal{F}_h$-adapted stopping times. Thus, let $S^{\theta,t} = \left\{ S_h^{\theta,t}, t \leq h \leq \infty \right\}$ denote the progressively measurable process with respect to $\mathcal{F}_h$, where

$$S_h^{\theta,t}(\omega) \equiv \Pr\left( \tau^{\theta,t}(\omega) \leq h | \mathcal{F}_h \right).$$

From the buyer's perspective, $S_h^{\theta,t}$ keeps track of how much probability mass the $(\theta, t)$-seller has "used up" as of time $h$ by assigning positive probability to accepting bids at times $s \in [t, h]$. An upward jump in $S^{\theta,t}$ corresponds to the $(\theta, t)$-seller accepting with an atom of mass. For any given sample path, $S^{\theta,t}$ is a CDF over a $(\theta, t)$-seller's acceptance time.

## C. Market Beliefs

Naturally, we will require that the market belief about $\theta$ be consistent with the public history and the sellers' strategies in equilibrium (see Definition 1 below). It will be convenient to derive the belief process consistent with arbitrary strategies as a prerequisite to discussing the equilibrium concept in the next subsection. We begin with the belief process that updates *based only on news* and then incorporate the information content from the public history due to strategic effects into a second component. Using a convenient change of variables, the market belief can be represented by the sum of these two processes.

The market begins with a common prior $P_0 = \Pr(\theta = H) \in (0, 1)$. Let $f_t^\theta$ denote the density function of type $\theta$'s news at time $t$, which is normally distributed with mean $\mu_\theta t$ and variance $\sigma^2 t$. Define $\hat{P}$ to be the belief process for a Bayesian who updates based only on news starting from the

prior (i.e., $\hat{P}_0 = P_0$):

$$\hat{P}_t \equiv \frac{\hat{P}_0 f_t^H(X_t)}{\hat{P}_0 f_t^H(X_t) + (1 - \hat{P}_0) f_t^L(X_t)}. \tag{4}$$

It is useful to define a new process $\hat{Z} \equiv \ln(\hat{P}/(1-\hat{P}))$, which represents the belief in terms of its log-likelihood ratio. Because the mapping from $\hat{P}$ to $\hat{Z}$ is injective, there is no loss in making this transformation. By definition,

$$\hat{Z}_t = \ln\left(\frac{\hat{P}_t}{1-\hat{P}_t}\right) = \underbrace{\ln\left(\frac{\hat{P}_0}{1-\hat{P}_0}\right)}_{\hat{Z}_0} + \underbrace{\ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right)}_{\frac{\phi}{\sigma}\left(X_t - \frac{t(\mu_H + \mu_L)}{2}\right)},$$

and thus

$$d\hat{Z}_t = -\frac{\phi}{2\sigma}(\mu_H + \mu_L)dt + \frac{\phi}{\sigma}dX_t. \tag{5}$$

Now define $P = \{P_t, 0 \le t < \infty\}$ to be the equilibrium market belief process. Note that $P_t$ differs from $\hat{P}_t$ because it accounts for information contained in trades, or the lack thereof, before time $t$. Define $Z \equiv \ln(P/(1-P))$. As before, there is no information lost in making this transformation. Because Bayes rule is linear in log-likelihoods, we can decompose $Z$ as $Z = \hat{Z} + Q$, where $Q$ is the stochastic process that keeps track of the information conveyed by the history of past acceptances and rejections. For example, because the market belief is consistent with seller strategies, along the equilibrium path and for all $h \in (t_i, t_{i+1})$, where $t_i \in \mathcal{T}$ denotes the time of the $i$th trade,[28]

$$Z_h = Z_{t_i} + \underbrace{\ln\left(\frac{f_{h-t_i}^H(X_h - X_{t_i})}{f_{h-t_i}^L(X_h - X_{t_i})}\right)}_{\hat{Z}_h - \hat{Z}_{t_i}} + \underbrace{\ln\left(\frac{1 - S_{h^-}^{H,t_i}}{1 - S_{h^-}^{L,t_i}}\right)}_{Q_h - Q_{t_i}}. \tag{6}$$

The third term on the right-hand side of (6) shows how beliefs can update over time due to strategic effects, even if trade does not occur. For example, suppose that in equilibrium, the likelihood of trade at time $t$ is larger if $\theta = L$ than if $\theta = H$. Then if trade occurs at $t$, $Q$ decreases while if trade does not occur at $t$, $Q$ increases.

*D. Equilibrium Definition*

Given $W$, the problem a $(\theta, t)$-seller faces is to select a stopping rule that for all $h \geq t$ solves

$$\sup_{\tau \geq h} \mathbb{E}^{\theta} \left[ \int_h^{\tau} e^{-r(s-h)} k_{\theta} ds + e^{-r(\tau-h)} W_{\tau} | \mathcal{G}_h \right]. \tag{$SP_{\theta,t}$}$$

Let $\mathcal{S}^{\theta,t} = \text{supp}(S^{\theta,t})$. We say that $S^{\theta,t}$ solves $(SP_{\theta,t})$ if each $\tau \in \mathcal{S}^{\theta,t}$ solves $(SP_{\theta,t})$. Now define $F_{\theta,t}(h, \omega)$ to be the expected payoff to the $(\theta, t)$-seller, who chooses a $\tau$ that solves $(SP_{\theta,t})$, starting from time $h \geq t$. In addition, because a holder waits to become a seller at some future time $t$ when the shock arrives, let

$$G_{\theta}(s, \omega) \equiv \mathbb{E}^{\theta} \left[ \int_s^t e^{-r(x-s)} v_{\theta} dx + e^{-r(t-s)} F_{\theta,t}(t, \omega) | \mathcal{G}_s \right] \tag{7}$$

denote the expected payoff to the holder of a type-$\theta$ asset starting from time $s$.

DEFINITION 1: *An equilibrium consists of $\{S^{L,t}, S^{H,t}\}_{t \in \mathbb{R}_+}, W$, and $Z$ such that*

(i) *Owner Optimality: Given $W$, for all $(\theta, t)$, $S^{\theta,t}$ solves $(SP_{\theta,t})$.*

(ii) *Belief Consistency: For any $t$ and history such that $\mathcal{F}_t \neq \emptyset$, $Z_t$ satisfies Bayes rule.*

(iii) *Zero Profit: If $\mathcal{F}_t \cap \{t \in \mathcal{T}\} \neq \emptyset$, then*

$$W_t = \mathbb{E}[G_{\theta}(t^+, \omega) | \mathcal{F}_t, t \in \mathcal{T}].$$

(iv) *No Deals: If $I_t(\omega) = 1$, there does not exist $y \in \mathbb{R}$ such that*

$$\mathbb{E}\left[ G_{\theta}(t^+, \omega) | \mathcal{F}_t, F_{\theta,h}(t, \omega) \leq y, A_t \neq A_{t^+} \right] - y > 0.$$

The first two conditions, *Owner Optimality* and *Belief Consistency*, represent standard criteria: a seller in possession of the asset at time $t$ must choose a strategy that maximizes her payoff, and beliefs must follow from Bayes rule along the equilibrium path (i.e., $\mathcal{F}_t \neq \emptyset$). The interpretation of *Zero Profit* is that any executed trade must deliver zero expected surplus to the purchasing buyer due to the presence of competing buyers. If *No Deals* does not hold, then there exists an offer that will earn a buyer a positive expected payoff; hence, this condition reflects the equilibrium requirement that no buyer can profitably deviate by making an offer $y$ that the seller would be

willing to accept with positive probability.

# III. Equilibrium: Liquidity, Prices, and Volatility

In this section, we present formal results demonstrating the existence of an equilibrium featuring stochastic, time-varying liquidity. We further show that these features generate an illiquidity discount, which in turn leads to excess volatility in prices and returns as well as feedback effects that further deteriorate market liquidity. To focus on the main results and their implications, we relegate the details of the equilibrium construction to Appendix B.

The equilibrium, formally characterized by Definition 2 and Theorem 1, is a natural extension of the $t = 0$ behavior in the two-period model and the equilibrium of focus in DG12 (see Benchmark 2 in Section III.A). It is stationary with respect to the current market belief, $z$, and owners' status, $i$.[29] Equilibrium play can be characterized by a pair of belief thresholds $(\alpha, \beta) \in \mathbb{R}^2$, $\alpha < \beta$, and an increasing function $B : \mathbb{R} \to [V_L, V_H]$, where $B(z)$ represents a buyer's expected value for the asset given the belief $z$. For states in which the owner is a seller $(i = 1)$, the equilibrium is characterized as follows:

- When $z > \beta$, the market is *fully liquid*. The bid is $B(z)$ and both seller types accept with probability one.

- When $z < \alpha$, the market is *partially liquid*. The bid is $V_L$, the low-type seller accepts with positive probability, and the high type rejects with probability one. If trade occurs, the market belief jumps immediately to $z = -\infty$. If trade does not occur, it jumps to $\alpha$.

- When beliefs are intermediate, the market is *fully illiquid*. For all $z \in (\alpha, \beta)$, the asset is not traded. The bid is unacceptable to either type of seller, and both sides of the market wait for more information to be revealed.

For states in which the owner is a holder $(i = 0)$, trade cannot occur because it is common knowledge that there are no gains from trade and the seller is privately informed.

Intuition for the trading dynamics is similar to that at $t = 0$ in the two-period model with $(\alpha, \beta)$ corresponding to $(a, b)$. When the owner is a seller, the market belief evolves according to both news and trading behavior with $\alpha$ forming a lower boundary of the belief process unless a sale at $W_t = V_L$ reveals $\theta = L$. Conversely, when the owner is a holder, beliefs evolve only according to

the news and may drift below $\alpha$.

Notice that $\alpha < \beta$, meaning that a fully illiquid region always exists, in contrast to the equilibrium of the two-period model. This difference is due to the increased frequency of trading opportunities. In either model, there are beliefs at which the market is partially liquid, meaning the low type is indifferent between trading or waiting. To maintain this indifference, the low type must expect a nontrivial delay before a better price will be offered. In the two-period model, this feature is built directly into the discrete-time formulation. However, in continuous time, there is no delay until the next trading opportunity, meaning it must be part of the equilibrium structure. That is, the possibility of $a > b$ in Proposition 1 is an artifact of the discrete-time environment.

The following definition formalizes the description provided above.

DEFINITION 2: *For any pair $(\alpha, \beta) \in \mathbb{R}^2, \alpha < \beta$, and measurable $B : \mathbb{R} \to [V_L, V_H]$, define*
$m_t = \sup\{s \leq t : I_s = 1\}$, $Q_t^\alpha = \max\{\alpha - \inf_{s \leq m_t} \hat{Z}_s, 0\}$, $Q_{0-}^\alpha = 0$, *and* $\Xi(\alpha, \beta, B)$ *to be the belief process and strategy profile such that for all $t, h \geq 0$,*

$$
Z_t = \begin{cases} -\infty & \text{if there exists } s < t \text{ when the asset sold and } Z_s \leq \alpha \\ \hat{Z}_t + Q_{t-}^\alpha & \text{otherwise} \end{cases}
$$

$$
S_h^{H,t} = \begin{cases} 1 & \text{if there exists } s \in [t, h] \text{ such that } Z_s \geq \beta \\ 0 & \text{otherwise} \end{cases}
$$

$$
S_h^{L,t} = \begin{cases} 1 & \text{if there exists } s \in [t, h] \text{ such that } Z_s \geq \beta \\ 1 - e^{-\left(Q_h^\alpha - Q_{t-}^\alpha\right)} & \text{otherwise} \end{cases}
$$

$$
W_t = \begin{cases} V_L & \text{if } Z_t \leq \alpha \\ K_L + \mathbb{E}_t^L[e^{-rT(\beta,t)}](B(\beta) - K_L) & \text{if } Z_t \in (\alpha, \beta) \\ B(Z_t) & \text{if } Z_t \geq \beta, \end{cases}
$$

*where $\mathbb{E}_t^L$ is the expectation with respect to the probability law of the process $Z$ conditional on $\mathcal{F}_t$ and $\theta = L$, and $T(\beta, t) \equiv \inf\{s \geq t : Z_s \geq \beta\}$.*

Our informal description of the equilibrium did not specify the bid in states where trade does not occur, because it is not uniquely pinned down. In such states, the definition above specifies $W$ to be the *highest* bid consistent with the candidate being an equilibrium.[30]

THEOREM 1 (Existence): *There exists an $(\alpha^*, \beta^*, B^*)$ such that $\Xi(\alpha^*, \beta^*, B^*)$ is an equilibrium.*

The theorem is established by construction. The main challenge is that the price a buyer is willing to pay in the fully liquid region $z \geq \beta$, $B(z)$, depends on the degree of future liquidity risk he faces, as captured by $(\alpha, \beta)$. At the same time, the boundaries that jointly solve the seller's stopping problem depend on the prices buyers are willing to pay, $B$. The key steps in the proof of the theorem involve formally deriving this interdependence and demonstrating that a fixed point to the system exists. Appendix B derives necessary conditions that any candidate $\Xi$ must satisfy. The Internet Appendix demonstrates that these conditions are also sufficient and proves existence.[31]

Having established its existence, we turn to characterizing properties of the fixed point relative to several benchmarks. Equilibrium value functions play a key role in the subsequent analysis. For each $\theta$, let $F_\theta$ and $G_\theta$ denote the value function of a type-$\theta$ seller and holder, respectively (see Figure 4 below). Because the equilibrium is stationary, each function depends only on $z$.

REMARK 1 (Multiplicity): *We have verified numerically that for a broad range of parameters, there is a unique $(\alpha, \beta, B)$ satisfying the necessary and sufficient conditions for $\Xi(\alpha, \beta, B)$ to be an equilibrium. However, without additional equilibrium restrictions, there exist other equilibria that are not of the $\Xi$ form. One reason for the multiplicity is that the seller's decision can signal her private information, and signaling games canonically have multiple equilibria without restrictions on off-path beliefs.[32] Motivated by their relative simplicity and, more importantly, the economic predictions they generate, we focus on equilibria of the $\Xi$ form.*

## A. Benchmark cases

In this section, we establish properties of two natural benchmark cases that will be useful for comparison to the equilibrium described above. We first adapt the definition of the asset's *fundamental value* to the continuous-time model.

DEFINITION 3 (Fundamental Value): *For any belief $Z_t$, let $p(Z_t) \equiv \frac{e^{Z_t}}{1+e^{Z_t}}$ denote the probability assigned to $\theta = H$ given $Z_t$, and $\bar{V}(Z_t) \equiv \mathbb{E}[V_\theta | Z_t] = V_L + p(Z_t)(V_H - V_L)$ denote the* fundamental value *of the asset.*

—Insert Figure 4 here—

*Benchmark 1: The symmetric information model.* Consider an economy in which owners have no private information.[33] In this symmetric information setting, equilibrium behavior is straight-forward: (i) $Z_t = \hat{Z}_t$ , because nothing can be learned from trading behavior, (ii) buyers are always willing to pay the fundamental value of the asset, $W_t = \bar{V}(Z_t)$, and (iii) if $A_t$ is hit with a shock at time $t$, she sells immediately. Therefore, the asset value to both sellers and holders is equal to $\bar{V}$. Because the asset spends zero time in the possession of constrained agents, the equilibrium is fully efficient and the price aligns perfectly with the fundamental value. These statements hold re-gardless of the value of $\lambda$; because no institutional frictions exist (e.g., search or transaction costs), shocks in isolation do not generate inefficiency or cause prices to deviate from fundamentals.

*Benchmark 2: The model without resale.* Restoring the information asymmetry, consider the model without demand for future liquidity (i.e., $\lambda = 0$). A holder is never shocked, never becomes a seller, and therefore retains the asset in perpetuity. Hence, a type-$\theta$ holder's value, $G_\theta$, is simply $V_\theta$. Further, buyers face no desire to resell in the future. Correspondingly, given any belief $Z_t$, their expected value of possessing the asset, which we have denoted by $B(Z_t)$, is simply the fundamental value, $\bar{V}(Z_t)$. This is the situation considered in DG12, in which the main result is as follows.

THEOREM 2 (DG12): *If $\lambda = 0$, there exists a unique $\Xi$-equilibrium. It has the following properties:*

- $G_\theta(z) = V_\theta$ *for all $z$.*
- $B(z) = \bar{V}(z)$ *for all $z$.*

*Further, this is the unique stationary equilibrium satisfying belief monotonicity (i.e., $Q$ is non-decreasing).*

The key takeaway is that without shocks, when a share trades, it does so at its fundamental value. Thus, future liquidity considerations are necessary, though not sufficient (recall Benchmark 1), for the divergence of prices from fundamentals.

## B. Effect of Illiquidity on Prices and Volatility

Notice that in both benchmarks, buyers do not face future liquidity risk. In Benchmark 1, the market is always fully liquid. In Benchmark 2, a buyer never faces a need to resell the asset. For this reason, in both benchmarks, (i) $B(z) = \bar{V}(z)$ for all $z$, and (ii) whenever the asset trades, the price equals the fundamental value (conditional on trade). Both (i) and (ii) fail when a demand

for future liquidity is combined with asymmetric information and news.

We now demonstrate how this liquidity risk creates an illiquidity discount and excess price volatility even during "normal times" (i.e., a fully liquid market).

PROPOSITION 3 (fully liquid prices): *In any $\Xi$-equilibrium, $B(z) < \bar{V}(z)$ for all $z$. In particular, in the fully liquid region, $B$ is of the form*

$$B(z) = \bar{V}(z) - \gamma(1 + e^z)^{-1} e^{-\frac{1}{2}\left(\sqrt{1+r/\phi^2}-1\right)z},$$ (8)

*where $\gamma > 0$ is a constant that is determined by parameters.*

Hence, even while the market is fully liquid, buyers anticipate the potential for illiquidity in the future, causing their value to drop below fundamentals. Recalling that in the fully liquid region, the asset trades at a price of $B(z)$, the following corollary is immediate.

COROLLARY 1 (Illiquidity discount): *In the fully liquid region, the asset trades at a price strictly less than its fundamental value.*

The illiquidity discount is characterized by the second term in equation (8)—just as was done by equation (3) in the two-period model. Having identified its functional form, a few properties are immediate. First, the discount is decreasing in $z$; as the market belief increases from $\beta$, the probability of getting enough bad news to reach the fully illiquid region decreases. Second, the illiquidity discount goes to zero as informational asymmetry disappears (i.e., as $z \to \infty$); just as in Benchmark 1, without an informational asymmetry, the market is always fully liquid.

To establish the effect on the volatility of equilibrium prices, a comparison to Benchmark 1 is again useful. Recall that in this case, the price is equal to the fundamental value $\bar{V}$, and therefore the *fundamental volatility* of prices is given by $\phi\bar{V}'(z)$, which follows directly from Ito's Lemma and the law of motion of $\hat{Z}$. Similarly, in the fully liquid region of $\Xi$, the price is $B(z)$, and because trades reveal no information, the law of motion of $Z$ is the same as that of $\hat{Z}$. Therefore, the volatility of equilibrium prices is given by $\phi B'(z)$. Using (8), we arrive at the following additional corollary.

COROLLARY 2 (Excess volatility): *In the fully liquid region, the volatility of both equilibrium prices and returns is strictly greater than the fundamental volatility of prices and returns.*

The intuition for this result is that bad news about fundamentals means two things: lower expected cash flows and higher expected liquidation costs. To see this, consider any $z > \beta$. Over the next short interval of time, bad news would move the belief downward toward $\beta$, which increases the probability of future illiquidity. Similarly, good news would increase the belief further away from $\beta$, which decreases the illiquidity risk. Hence, the effect of news gets compounded, which leads to higher price volatility in equilibrium than can be justified based solely on fundamentals. Because $B < \bar{V}$, the volatility of equilibrium *returns* (i.e., the volatility of $dB(Z_t)/B(Z_t)$) is also strictly greater than the fundamental volatility of returns $(d\bar{V}(Z_t)/\bar{V}(Z_t))$.

## C.   *How Does Demand for Future Liquidity Affect Market Liquidity Today?*

Having explored the impact on prices, it is natural to ask how the demand for future liquidity interacts with the information asymmetry and affects the resulting market liquidity today. In the two-period model, we found that an increase in the probability of wanting to resell at date 1 leads to an increase in $b$ and hence negatively affects the market illiquidity at date 0. The intuition is that an owner with a high-value asset is optimistic about future liquidity relative to buyers because she expects good news to arrive. She is therefore less inclined to trade in the initial period, especially at a depressed price. As a result, buyers are less likely to offer the pooling price, which amplifies illiquidity in the initial period. The same force is at play within the continuous-time model; the possibility of becoming constrained in the future increases $\beta$ and hence shortens the duration of normal times. Unlike the two-period model, these considerations extend beyond just liquidity in the "next period."

There is also a secondary effect through which market liquidity is affected that was not present in the two-period model. In particular, because of the increase in $\beta$, a low-type seller now has to wait longer (in expectation) to get the pooling price. If $\alpha$ were to remain fixed, a low type's payoff would drop below $V_L$, violating the *No Deals* condition. Therefore, $\alpha$ must also increase to compensate the low type, which therefore *increases* the probability of trade in "abnormal" times. However, such trades entail substantial price impact since they occur at a price of $V_L$. As we show in the following proposition, the first effect tends to dominate the second in that the overall size of the fully illiquid region increases.

PROPOSITION 4 (Amplification): *Fix all parameters except $\lambda$. Let $\Xi(\alpha_0, \beta_0, \bar{V})$ denote the $\Xi$-equilibrium with $\lambda = 0$, and $\Xi(\alpha_1, \beta_1, B)$ be any $\Xi$-equilibrium with $\lambda > 0$. Then $\beta_1 > \beta_0$ and $\beta_1 - \alpha_1 \geq \beta_0 - \alpha_0$.*

Note that without an information asymmetry, $\lambda$ has no effect on prices or market liquidity (Benchmark 1). Hence, the interaction between the demand for future liquidity and the strategic considerations of asymmetrically informed traders drives the result. Our finding here is also quite different from what obtains if shocks are of the Diamond and Dyvbig (1983) flavor, which effectively force a trader to sell (regardless of the bid) upon their arrival. Shocks of this nature eliminate sellers' strategic considerations and thereby lead to immediate trade. Hence, equilibrium trading behavior and prices would be identical to those in Benchmark 1 (i.e., full efficiency and prices equal to fundamentals).

# IV. Markets with Multiple Shares

In this section, we extend our analysis to a setting in which the asset has multiple identical shares and multiple informed owners. The key additional consideration in this environment is that the trading behavior of one owner may affect the market belief about $\theta$, and therefore the value derived by, and consequently the behavior of, other owners. In other words, there is an informational externality among owners. We first show that the externality gives rise to the possibility of "contagious sell-offs" in any equilibrium (Proposition 5), and then investigate the extension of $\Xi$ with these new considerations.

## A.  *Extending the Model*

Extending the model is a straightforward exercise in which the key components are as follows. Let there be $N \in \{1, 2, ..., \infty\}$ shares, and let $n$ refer to a generic share. Each share $n$ endows a cash flow of $k_\theta$ or $v_\theta$ depending on the owner's status, with $k_\theta$ and $v_\theta$ satisfying the same conditions as in Section II;[34] $A_t^n$ denotes the owner of share $n$ at time $t$, and we assume each agent can own at most one share; $L^n = \{L_t^n : 0 \leq t \leq \infty\}$ is the publicly observable shock process for the owner of share $n$, where the processes are assumed to be mutually independent; and $I_t^n$ denotes the status of the owner of share $n$ at time $t$. Because the shares are derived from a common underlying asset,

there remains just one type, $\theta$, common to all shares, one news process, one market belief about $\theta$, and one bid process available to all sellers. We let $\{\mathcal{F}_t\}_{t\geq 0}$ continue to denote the filtration generated by the public history, and let $\{\mathcal{G}_t\}_{t\geq 0}$ denote the filtration generated by the information sets of owners, which for technical reasons (see the discussion following Theorem 3) contains a randomization device that allows for correlated mixed strategies. Finally, our equilibrium notion (Definition 1) generalizes in a straightforward way by requiring the conditions to hold for all shares.

As discussed in the introduction, larger $N$ can be interpreted as more dispersed ownership, provided that the marketplace is transparent. Small $N$ may correspond either to markets for obscure, heterogeneous products (e.g., private-label MBS), in which only a few securities are backed by the underlying collateral, or to assets with a large number of identical physical shares but a decentralized marketplace (e.g., corporate bonds prior to the introduction of TRACE).[35] In the latter case, $N$ should be interpreted as the number of shareholders for which agents have information. Conversely, large $N$ corresponds to markets for assets with both dispersed ownership *and* transparent trading (e.g., corporate bonds after the introduction of TRACE).

## B. Contagious Sell-Offs

Our first result is perhaps the starkest illustration of the informational externality among owners. For simplicity, consider $N = 2$ and fix *any* equilibrium (i.e., not necessarily one resembling $\Xi$). Suppose that at time $t$, both owners are sellers ($I_t^1 = I_t^2 = 1$) and, given the history, the equilibrium calls for at least one of them to trade with positive probability if and only if $\theta = L$. By the *Zero Profit* condition, $W_t$ must be $V_L$. The key element is, of course, that if one seller does accept the bid at time $t$, the asset's type is perfectly revealed to be $L$. The other seller then has no further incentive to delay trade, and immediately follows suit. This logic extends to arbitrary $N > 1$. Hence, the trade of one share at a low price is *contagious*, inducing a sell-off of other shares at low prices over a short period of time.

Formally, because the model is posed in continuous time, "immediately follows suit" translates to "instantaneously," and both shares trade at time $t$. In addition, contagious sell-offs are not just a possibility, they *must* occur with positive probability in any equilibrium if beliefs are sufficiently unfavorable.

PROPOSITION 5 (Contagious sell-offs): *Fix arbitrary $N > 1$. If, in any equilibrium, any share $n$ trades at time $t$ at a price $W_t < K_H$, then:*

(i) *That price is $W_t = V_L$.*

(ii) *Any other share $m$ in the possession of a seller, $I_t^m = 1$, also trades at time $t$.*

*Further, there exists a nondegenerate $z^*$ such that for any equilibrium and history in which $Z_t < z^*$, the probability that a contagious sell-off occurs at some $t' \geq t$ is strictly positive.*

## C.   $\Xi^N$-Equilibrium

We now describe the generalization of $\Xi$ to the case of multiple shares. For arbitrary $N$, let $\Xi^N(\alpha, \beta, B)$ be the strategy profile and belief process in which each seller follows the strategy described in $\Xi(\alpha, \beta, B)$, with the acceptance/rejection behavior of all contemporaneous low-type sellers being perfectly correlated, where $Z$ is the belief process that is Bayesian consistent with this strategy profile, and $W$ is as in $\Xi(\alpha, \beta, B)$ (note that $\Xi$ and $\Xi^1$ are synonymous). A $\Xi^N$-equilibrium is described by a system of equations, $\mathbb{S}^N$, that is derived in Appendix B.

THEOREM 3: *For arbitrary $N \in \{1, 2, \ldots, \infty\}$, an equilibrium of the form $\Xi^N(\alpha, \beta, B)$ is characterized by the system of equations $\mathbb{S}^N$ as defined by B32 in Appendix B. That is, a solution to the equations is both necessary and sufficient for an equilibrium of this form.*

Notice that $\Xi^N$ specifies that the mixing behavior of contemporaneous low-type sellers is perfectly correlated: if $\theta = L$ and $z \leq \alpha$, either all current sellers trade or none of them do. In fact, the probability that they trade is independent of the number of sellers, because regardless of the number of sellers, a unique aggregate probability of trading is needed for the Bayesian-consistent posterior belief to jump to $\alpha$ conditional on no trades occurring.

The necessity of this correlation is merely an artifact of continuous time. As discussed in Section IV.B, if a share of the low-type asset trades at price $V_L$, it reveals $\theta = L$, meaning any remaining sellers will wish to trade at their next opportunity. In continuous time, there is no "next" opportunity. This results in an existence problem, which is resolved by the correlation. If time periods were discrete but short, mixing behavior could be independent across sellers: to keep the probability that at least one seller trades constant, the mixing of each individual seller would put

less weight on selling as the number of sellers increases. If the realization of this mixing induces a trade, all remaining sellers trade in the next period. As time periods become arbitrarily short, the observed trading behavior converges in distribution to that specified by $\Xi^N$.

The analytic characterization of the equilibrium candidate becomes somewhat more involved for arbitrary finite $N > 1$. Doing so requires an additional state variable because, as discussed below, the value to a holder depends on whether other sellers are in the market (see Appendix B). This contingency is irrelevant for fully liquid states (i.e., $z \geq \beta$) and matters most when $z \leq \alpha$ as this is when the behavior of sellers affects the evolution of the belief.[36] Unlike the $N = 1$ case, we do not analytically prove the existence of a solution to $\mathbb{S}^N$ for $N > 1$. We have verified numerically that a solution, and hence (by Theorem 3) a $\Xi^N$-equilibrium, exists for a wide range of parameters. It is also straightforward to verify that in any such equilibrium, the essential properties of each of the benchmark cases investigated in Section III.A obtain, and that Proposition 3 and its subsequent corollaries (i.e., illiquidity discount and excess volatility) hold for any $N$.

## D.   Informational Externalities in $\Xi^N$

Clearly, if $\theta = L$, contagious sell-offs (Proposition 5) are a feature of $\Xi^N$. However, these sell-offs are not the sole manifestation of multi-share informational externalities. For example, consider the owner of share 1 who is a holder at time $t$ ($I_t^1 = 0$). When $N = 1$, there are no other traders whose behavior the market might learn from, and hence beliefs evolve only based on news. When $N > 1$, the evolution of the market belief, and therefore the holder's continuation value, depends on the status of other owners. If no sellers are present, the belief continues to evolve based only on news. But if a seller is present in the market ($I_t^n = 1$, for some $n \neq 1$), her trading behavior has information content. Specifically, if $z \leq \alpha$, a trade reveals that $\theta = L$ and, inversely, no-trade increases the belief beyond what is revealed by news alone (all while $I_t^1 = 0$). By contrast, if the owner of share 1 is a seller ($I_t^1 = 1$), the presence or absence of other sellers in the market has no effect on her value because, as just discussed, the evolution of the market belief is the same in either case.

To explicitly capture the dependence of holder values on the presence of sellers in the stationary equilibrium $\Xi^N$, let $i^n$ denote the status of the owner of share $n$ and $\vec{i} = (i^1, i^2, ...)$. A type-$\theta$ holder's value function depends on both $z$ and $\max(\vec{i}) \in \{0, 1\}$, denoted $G_\theta(z, \max(\vec{i}))$. Our next result

illustrates that the total effect of the additional information gleaned from the trading behavior of sellers is beneficial to holders on average, which offers insight regarding the efficiency consequences of the informational externalities.

PROPOSITION 6: *For any $\Xi^N$-equilibrium, $N \notin \{1, \infty\}$, and $z$, $\mathbb{E}[G_\theta(z,1)|z] \geq \mathbb{E}[G_\theta(z,0)|z]$.*

We can build on this result to generate intuition about the effects of increased $N$. Proposition 6 says that holders gain, on average, from the presence of sellers. Of course, the larger $N$ is, the more likely at least one other owner will become a seller by any point in time. Hence, we would expect average holder value to increase with $N$. Because buyers become holders upon purchase, their value for a share of the asset depends on average holder value and would then likewise increase with $N$, also raising prices (and therefore, seller values), liquidity, and efficiency.

In Section V, we substantiate this intuition by comparing the two polar cases: $N = 1$ and $N = \infty$. Note that these two cases are excluded from Proposition 6 because, when $N = 1$, there is *never* simultaneously a holder and a seller, and when $N = \infty$, a seller is (effectively) *always* present. So, in the former case holders never experience the presence of sellers, and in the latter case, holders always experience it. These two cases therefore represent the extrema for investigating this force.

In sum, the key difference with multiple shares is in the information available in the market and hence the evolution of market beliefs. When $N = 1$, there are periods of time in which information is revealed only by news, whereas when $N$ is arbitrarily large, the economy is always learning from the trading behavior of sellers in addition to news. In this way, beliefs evolve based on more information for larger $N$. In Section V.D, we contrast these findings with the effect of increasing news quality, $\phi$, which has surprisingly different implications for allocative efficiency.

# V. Quantitative results

In this section, we parameterize the model to explore the quantitative significance of our main results. In addition, we explore comparative statics with respect to news quality, number of shares, and the frequency of shocks, as well as the model's predictions for trade volume and market efficiency. Before doing so, we briefly discuss our choice of parameters, which is summarized in

Table I.

## A. *Parameterization*

The selection of cash flow parameters depends on the type of asset markets under consideration. For example, one might expect the severity of the adverse selection problem (i.e., $v_H - v_L$) to be larger for more obscure securities. Thus, we consider two different sets of parameters to represent two different asset markets.

First, we set $N = 1$ and choose cash flow parameters consistent with over-the-counter markets for assets such as MBS or corporate debt by setting $v_H = 10\%$ and $v_L = 5\%$ with a holding cost of two percentage points ($k_\theta = v_\theta - 0.02$). To motivate these parameter values, suppose "type" corresponds to whether the security will default. Low-type securities eventually default, whereas high-type securities do not.[37] This parameterization is consistent with a 50% recovery rate on defaulted securities as estimated by Moody's for Ba to A rated tranches of mortgage-backed CDOs (Gluck and Remeza (2000)). It is also in line with average recovery rates for defaulted corporate bonds reported by Acharya, Bharath, and Srinivasan (2003) and Altman et al. (2005).

Next, we let $N = \infty$ and select cash flow parameters consistent with shares in a firm that makes a new investment of uncertain type. The current operations of the firm generate dividends of 3%. A "good" investment ($\theta = H$) will increase dividends in perpetuity by 0.5% ($v_H = 3.5\%$), whereas if $\theta = L$, dividends remain constant ($v_L = 3\%$). The holding cost is set to 0.5% ($k_\theta = v_\theta - 0.005$).

—Insert Table I here —

Of particular interest is how changes in the demand for future liquidity (measured by $\lambda$) and news quality (measured by $\phi$) affect prices, liquidity, and efficiency. Therefore, for each of the parameterizations above, we consider two values for both $\lambda$ and $\phi$. In our base specification, we set $\lambda = \frac{1}{4}$, which is consistent with a trader experiencing a shock once every four years. We also consider the case in which $\lambda = 1$, which means that traders have greater demand for future liquidity. For news quality, we consider $\phi = 0.2$ and $\phi = 0.5$. If we interpret the news process as being the cash flows (see footnote 23), then $\mu_\theta = v_\theta$ and the two $\phi$ values are consistent with cash-flow volatility of $\sigma = 25\%$ and $\sigma = 10\%$ (respectively) for the $N = 1$ parameterization. For the $N = \infty$ parameterization, in which the news process is the firm's realized dividends, these $\phi$-values would

imply unrealistically low values for dividend volatility (2.5% and 1%, respectively). In this case, the parameter values for $\phi$ can be justified by the fact that firms with a large number of shares will typically be associated with additional sources of information beyond realized dividends (e.g., analyst coverage).

More generally, one may expect the appropriate choice of $\lambda$ and $\phi$ to depend on the type of asset and characteristics of its participants. For example, hedge funds may be more susceptible to shocks than large institutional investors; blue-chip stocks should be associated with higher quality news than an obscure pool of collateral. Nevertheless, for reasons articulated above, we believe the chosen values for these two parameters are of an appropriate order of magnitude across a variety of markets and asset classes.

## B.   Regions of Illiquidity

In Section III, we provide a descriptive characterization of how liquidity varies over time with the market belief. One way to quantify the amount of liquidity in the market is through the region boundaries, $\alpha$ and $\beta$, which we compute for each parameter configuration. For ease of interpretation, we convert the boundaries from log-likelihoods to probabilities, denoted $a$ and $b$, respectively, and report them in Table II.

—Insert Table II here —

Focusing on the first row of the $N = 1$ parameterization, the implied default probability at which the market becomes fully illiquid is 11% (i.e., $b = 0.89$).[38] This order of magnitude is broadly consistent with (corporate) default rates during financial crises as reported by Jurek and Stafford (2013). An interesting empirical exercise for future work would be to compare the model-implied estimates to the ex-post performance of MBS, the market for which collapsed during the recent financial crisis.

For an increase to $\lambda = 1$, the fully illiquid region occurs at a lower implied default probability. This is consistent with feedback effects demonstrated in Propositions 2 and 4; when resale considerations are more imminent, less uncertainty about fundamentals is needed for liquidity to "dry up." For the $N = \infty$ parametrization, a larger degree of uncertainty can be sustained before the market becomes illiquid, primarily because the spread between high- and low-quality investments

is smaller. As a result, an owner of a high-value asset has less to gain by waiting, and buyers worry less about being stuck with a low-value asset.

## C.   Illiquidity Discount and Excess Volatility

The results in Section III.B illustrate that even when the market is fully liquid, time variation in future liquidity leads to an illiquidity discount and excess volatility. To quantify these effects, we measure the illiquidity discount, denoted by $D(z)$, as the amount, in percentage terms, that the price deviates from fundamentals in state $z \geq \beta$:

$$D(z) = \frac{\bar{V}(z) - B(z)}{\bar{V}(z)}. \tag{9}$$

Similarly, we measure equilibrium volatility, $\sigma_e$, in percentage terms (i.e., $\sigma_e = \phi B'/B$) and compare it to fundamental volatility, $\sigma_f = \phi \bar{V}'/\bar{V}$. For expositional convenience, we present the illiquidity discount and excess volatility at $z = \beta$, where their values are greatest. The results are given in Table III.

For our base set of parameters, the illiquidity discount is 3.7% for $N = 1$ and 1.9% for $N = \infty$. Illiquidity risk also leads to volatility that is more than double that based solely on fundamentals. Notice that the comparative statics on $D$ and $\sigma_e/\sigma_f$ are similar. Intuitively, if the discount is higher, the cost of getting bad news, and hence the risk due to illiquidity, increases. As $\lambda$ increases, so does $D$; because traders face costly liquidation more frequently, they require a larger discount, which feeds back into greater illiquidity, and so on (see Section III.C). Higher-quality news "speeds things up," which reduces the liquidity premium at $\beta$ as equilibrium beliefs spend less time in the fully illiquid region.

—Insert Table III here —

## D.   Implications for Volume and Efficiency

An integral part of our theory is that the level of trading activity is determined endogenously, and, further, that trade volume is intimately related to market efficiency. Because it is never efficient for a share to remain in the possession of a constrained agent, the uniquely efficient trade pattern is for every share to trade the moment its owner experiences a shock. Of course, we have

seen that informational frictions cause illiquidity, which creates an inefficiency. We now turn to investigating how volume and efficiency vary with key parameters of the model.

## D.1. Scaled Trade Volume

With regard to trade volume, both $N$ and $\lambda$ have two effects. First, fixing the equilibrium structure, a greater number of shares or more frequent trading needs would mechanically produce higher trade volume. Second, and more interestingly, both $N$ and $\lambda$ affect the equilibrium structure. To focus on the latter effects, we calculate the expected number of times that an arbitrary share of the asset is traded over any length of time, normalized by the (expected) frequency with which shocks arrive, $\lambda^{-1}$, which we refer to as *scaled* volume. Let $\nu_t^n$ denote the counting process, which keeps track of the number of trades that occur in $[0, t]$ for an arbitrary share $n$. That is,

$$d\nu_t^n = 1_{\{A_t^n \neq A_{t+}^n\}}, \text{ where } \nu_{0-}^n = 0.$$

Clearly, the turnover of a share depends on the status of its owner, denoted by $i$. We let $\bar{f}$ and $\bar{g}$ denote the functions mapping $(t, z)$ to *expected* scaled trade volume conditional on the share being owned by a seller ($i^n = 1$) and holder ($i^n = 0$), respectively. That is, $\bar{f}(t, z) \equiv \lambda^{-1}\mathbb{E}[\nu_t^n | (Z_0, I_0^n) = (z, 1)]$ and $\bar{g}(t, z) \equiv \lambda^{-1}\mathbb{E}[\nu_t^n | (Z_0, I_0^n) = (z, 0)]$. In Appendix C (Proposition C.4), we derive the system of interdependent PDEs that fully characterize these functions. Using this result, we then solve the system of PDEs for our parameterizations. In Figure 5, we plot scaled volume over a unit interval of time beginning with a holder. In this case, the first-best/symmetric information benchmark level is equal to one for all $z$. As expected, the model predicts that the information asymmetry reduces volume and the structure of the equilibrium implies that volume is lowest for intermediate beliefs and highest in the extremes.

More interestingly, Figure 5(a) illustrates the amplifying feedback effect of resale considerations on scaled volume. The increase in $\lambda$ leads to both a decrease in the overall level and a "shift" rightward. That is, the amplification effect causes scaled volume to decrease dramatically for higher beliefs (since $\beta$ increases, see Table II), but perhaps surprisingly, leads to slightly larger volume for lower beliefs since $\alpha$ also increases. This pattern also obtains for the $N = 1$ parameterization (not pictured).

On the other hand, the effect of news quality differs across the two parameterizations. Higher $\phi$ increases the rate at which the uncertainty about $\theta$ is resolved, which tends to increase the overall level of volume. This in turn has general equilibrium effects: the incentives of the sellers and hence the boundaries of the fully illiquid region also shift. In both cases, higher $\phi$ decreases volume for higher beliefs (again because $\beta$ increases), but volume increases for lower beliefs in Figure 5(b) whereas it actually decreases slightly in Figure 5(c). This discrepancy arises because news quality has an ambiguous effect on the lower boundary, $\alpha$. In the $N = \infty$ parametrization, the increase in $\beta$ makes low types more willing to trade, and therefore $\alpha$ and volume increase with $\phi$. In the $N = 1$ parameterization, however, the faster resolution of uncertainty makes low types more willing to wait, and therefore $\alpha$ and volume decrease with $\phi$.

—Insert Figure 5 here —

## D.2.   Efficiency

If the market is always fully liquid, then the asset is always efficiently allocated and the expected discounted value derived from a share of the asset is $\bar{V}$. Our measure of efficiency compares the value derived from a share of the asset in equilibrium to this benchmark. Because all buyers earn zero profit, the discounted expected value of a share is $\mathbb{E}[F_\theta(z)|z]$ or $\mathbb{E}[G_\theta(z)|z]$, depending on whether the current owner is a seller or a holder. We look at the percentage efficiency loss per share by defining

$$\mathcal{L}^F \equiv \frac{\bar{V}(z) - \mathbb{E}[F_\theta(z)|z]}{\bar{V}(z)} \quad \text{and} \quad \mathcal{L}^G \equiv \frac{\bar{V}(z) - \mathbb{E}[G_\theta(z)|z]}{\bar{V}(z)}.$$

Figure 6 shows that both measures of efficiency loss are positive for all $z$ (which follows from Proposition 3), single peaked with maximal inefficiency occurring in the fully illiquid region, and tend to zero as $z \to \pm\infty$.[39] Intuitively, when $z$ is in the fully illiquid region, not only are any efficient trades postponed today, but the likelihood of future periods of illiquidity is also higher. Conversely, as $z \to \pm\infty$, the information asymmetry that impedes efficient trading, both now and in the future, is removed. Except when otherwise noted, efficiency figures use the first parameterization from Section V.A, with $\lambda = 0.25$ and $\phi = 0.5$. Below we discuss how inefficiency varies with the key parameters.

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–Insert Figure 6 here —

*News quality (φ):* Because the market is always fully liquid, and hence fully efficient, under symmetric information, one might think that increasing news quality will bring the market closer to the efficient benchmark and improve efficiency. As shown for $\mathcal{L}^F$ in DG12 (i.e., if $\lambda = 0$) and replicated in Figure 7(a) for $\mathcal{L}^G$ and $\lambda > 0$, this is not necessarily the case.[40] As $\phi$ increases, the inefficiency decreases for lower states but *increases* in higher states. Increasing $\phi$ generates two offsetting effects on efficiency. First, beliefs move more quickly through the fully illiquid region, reducing the amount of time shares are inefficiently allocated. Second, high-type sellers expect good news to be revealed more quickly and therefore have more incentive to wait; both $\beta$ and the size of the fully illiquid region increase. Whether increasing news quality leads to more or less efficient markets depends on both the initial state and the initial quality.

—Insert Figure 7 here —

*Number of shares (N):* As we discuss in Section IV, the key difference as $N$ varies pertains to the information available to uninformed market participants. When $N = \infty$, sellers are always present, providing an additional channel through which learning can take place. By contrast, when $N = 1$, periods exist during which no sellers are present and the market relies solely on news. In line with the discussion following Proposition 6, Figure 7(b) suggests that the informational externality from higher $N$ aids efficiency for lower $z$ (for which the market is not fully liquid) but has little impact for higher beliefs (for which the market is fully liquid).

Note that the efficiency consequences of a greater number of shares and of higher news quality are different, despite the fact that both generate information for the market. The different circumstances under which the information is revealed explains the difference in their effects. In the case of greater $N$, additional information is revealed only for $z \leq \alpha$ (i.e., when the market is functioning poorly), which has a nontrivial *level* effect for efficiency but an insignificant effect on a seller's *marginal* consideration of whether to sell at $z = \beta$, thereby not disturbing the market when it is already well functioning.[41] In the case of higher $\phi$, additional information is revealed in all states and therefore has first-order consequences on the expected benefit to the seller of waiting for a higher price. Although improved news quality does increase efficiency when the market is functioning poorly, because it gives the high-type seller increased incentive to delay trade, increasing

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$\beta$, it disturbs what would otherwise have been a well-functioning market in these states.

*Demand for future liquidity ($\lambda$):* As Figure 7(c) depicts, inefficiency increases with the demand for future liquidity. Recall that $\lambda$ affects neither the efficient value of a share, $\bar{V}$, nor the ability of a symmetrically informed economy to achieve this value. Hence, the result follows from the equilibrium behavior of the agents in the economy, introducing periods of illiquidity in which shares remain in the hands of constrained owners. Such periods occur more frequently as $\lambda$ increases, both directly decreasing allocative efficiency and depressing asset prices, which feeds back to amplify the amount of illiquidity, generating even greater inefficiency.

*Borrowing costs (c):* In each of the figures thus far, $k_\theta = v_\theta - c$, where $c = 0.02$. That is, the shock induces an additive holding cost of two percentage points. Let us maintain the additivity assumption and consider the effect of varying $c$.[42] Two countering forces arise. An increased holding cost means that sellers receive lower net cash flows while in possession of the asset, hurting their welfare and reducing efficiency. However, the increased holding cost also implies that the high-type seller is more willing to sell, decreasing $\beta$, which in turn makes the equilibrium more efficient for high levels of $z$ (Figure 7(d)). These results imply that government policies aimed at "easing" credit constraints of distressed financial institutions can have detrimental side effects. Indeed, part of the motivation for Federal Reserve credit easing during the financial crisis was aimed at "preventing a liquidation of assets at distressed prices to avoid destabilizing affects" (Carlson et al. (2009)). Our results suggest that mitigating destabilization may come at the cost of slower reallocation.

## VI. Discussion of Assumptions

The model entails a number of simplifying assumptions, which are made primarily to facilitate a tractable analysis and to keep the intuition for the main forces accessible. Below we interpret these assumptions and discuss the robustness of the predictions to various generalizations and extensions.

*Shocks.* That shocks are observable (or verifiable; see footnote 7) is an important feature of both the model and its application. Although this assumption is restrictive, we believe it is

natural in certain applications. One possible interpretation is that observable shocks correspond to a marketplace dominated by large institutional traders or banks whose financial constraints are transparent. Investors must be able to discern between traders with a credible reason for trading and speculators. For example, in the recent financial crisis, it was not difficult to identify constrained firms (e.g., Bear Stearns, Lehman Brothers, AIG). A model with unobservable shocks could correspond to either a marketplace dominated by private firms (e.g., hedge funds) or one in which the identity of trading partners remains anonymous (e.g., dark pools), where the motivation for trading is often unclear. If shocks were unobservable, then in favorable market conditions, the holder of a share of a low-value asset would prefer to sell before being hit by a shock, exacerbating buyers' exposure to the lemons problem.

To formally demonstrate the consequences of unobservable shocks, modify the two-period model (Section I) so that shocks are no longer publicly observed. Consequently, if the asset traded at $t = 0$, then at $t = 1$ the owner privately knows her type-status pair (e.g., high-type seller, low-type holder, etc.).[43] Obviously, because in this case buyers no longer know the owner's status, the bid price cannot be conditioned on it, as in Lemma 1.

Next, notice two things. First, a high-type holder has no gains from trade and buyers face an adverse selection problem, therefore a high-type holder will not trade in equilibrium. Second, because $\kappa < \delta$, a low-type holder strictly prefers to accept any bid the high-type seller is willing to accept. Hence, equilibrium play is determined by the buyers' belief that the security is high value, conditional on the owner being either a high-type seller or a low type (holder or seller). Denote this belief as $\tilde{p}_1$. Using $\tilde{p}_1$, the equilibrium trading behavior is analogous to that in Lemma 1. If $\tilde{p}_1 > \bar{p}$, then a high-type seller and both low types (holder and seller) trade, meaning that the market is fully liquid (i.e., all surplus-enhancing trades are realized); if $\tilde{p}_1 < \bar{p}$, then only low types trade, meaning that the market is only partially liquid.

Of course, our interest lies in how the unobservability of shocks affects market liquidity at $t = 0$. Because the original owner's status is common knowledge, if trade does not occur at $t = 0$, buyers know the owner is a seller at $t = 1$, and the market equilibrium at $t = 1$ is identical to that in Lemma 1. Hence, the continuation values for the original owner $(C_L, C_H)$ also remain the same.

What remains is to determine how the unobservability of shocks affects a buyer's (unconditional) value for the asset at $t = 0$, which we denote by $B^{un}$. To account for the changes at $t = 1$ discussed

above, we replace the $q(s, p_0)$ term in equation (3) with

$$\tilde{q}(s, p_0) \equiv \frac{\lambda p_0 f_H(s)}{\lambda p_0 f_H(s) + (1 - p_0) f_L(s)} = \frac{\lambda q(s, p_0)}{\lambda q(s, p_0) + (1 - q(s, p_0))}. \tag{10}$$

Noting that $\tilde{q} < q$ when $p_0$ and $\lambda$ are interior, it is clear that the risk of illiquidity at $t = 1$ is greater when the shock is unobservable, which leads to the following result.

PROPOSITION 7: *In the two-period model, when the shock is unobservable, a buyer's (unconditional) value for the asset at $t = 0$ is given by*

$$B^{un}(p_0) = \bar{V}(p_0) - (1 - \delta) \cdot \left( \lambda p_0 \Pr\left( \tilde{q}(s, p_0) < \bar{p} | H \right) \right),$$

*which is strictly lower than $B(p_0)$ for all $\lambda$ and $p_0$ both in $(0, 1)$.*[44]

Proposition 7 says that the effect of unobservable shocks is much like the effect of an increase in $\lambda$ in Proposition 2. Since buyers' value is lower, the high-type seller's threshold at $t = 0$ will increase while the low type's threshold remains unchanged, which endogenously generates less liquidity (i.e., a lower probability of trade). The moral is that an observable shock provides the owner with a credible reason to liquidate. Without this credibility, buyers face even more severe exposure to the lemons problem, which exacerbates the forces studied in this paper.

Finally, we limit attention to shocks that increase the holding cost of the owner. Including "reverse" shocks—shocks that turn sellers back into holders—would change little qualitatively. Doing so would simply decrease the gains from trade, because the seller may return to being a holder if she rejects bids, which would, of course, sometimes happen on the equilibrium path. In addition, our modeling of the buyer side of the market makes no mention of the possibility that some buyers may be constrained. Provided sufficiently many unconstrained buyers are present, the presence of constrained buyers is irrelevant.

*Learning.* In our model, the asset type is fixed, and a new owner learns the type perfectly upon purchasing a share. We make these assumptions primarily for tractability. Allowing the asset type to switch over time would reduce high types' incentive to delay, increase low types' incentive to pool, and change the evolution of market beliefs, but the key forces would persist. In addition, an analysis similar to ours would apply to a model in which the purchasing buyer obtains some

noisy, binary signal of asset quality that is subsequently, gradually revealed to the market through news. The model would become more complicated, though, if asset owners obtain a noisy signal and subsequently learn additional information from news, because one would need to keep track of multiple sets of beliefs. In such a model, enough bad news could induce even a "high-initial-signal" owner to sell at low prices.

*Risk Neutrality.* One can interpret the assumption that agents are risk neutral either literally or as an "as if" stand-in for risk-averse traders that have hedged their idiosyncratic risk in this asset with a portfolio of other holdings. If, instead, agents were averse to the risk imbued by this asset, there would be two off-setting effects. On the one hand, risk aversion incentivizes sellers to trade more quickly, shrinking the no-trade region and reducing the effect of information asymmetry. On the other hand, sufficient good news becomes more valuable as it not only increases the mean of traders' expectations, but also reduces the variance, providing more incentive for sellers to delay trade. Which of these forces dominates, and the implications for the interaction of risk premia and liquidity, seems an interesting question for subsequent work.

*Binary Types.* Our results are derived from a setting with binary asset types; yet the key forces would persist in a more general environment. Regardless of the number of types, trade remains inefficient, provided holding costs are not overly punitive—higher-type sellers have incentive to wait for news when beliefs are not favorable—and thus prices remain below fundamentals, and selling at a low price reveals negative information about $\theta$, which can facilitate future trades.

## VII.   Conclusion

In this paper, we present a model that features news arrival and idiosyncratic shocks in a market with asymmetrically informed traders. The combination of these three features generates time variation in liquidity in an environment without institutional frictions. We elucidate an important feedback effect between illiquidity and asset prices that reduced-form models do not capture: endogenous liquidation costs cause the asset to trade at a discount relative to its fundamental value, which feeds back to amplify illiquidity, which further depresses prices, and so on. These forces also generate excess volatility, reduced trade volumes (even when normalized to account for trading needs), and more allocative inefficiency. The presence of other informed traders has informational

externalities, which can lead to contagious sell-offs. The framework provides a unified setting from which one can derive implications for asset prices, volatility, illiquidity, trade volume, and efficiency.

# Appendix A.   Proofs for Section II

Throughout this appendix we employ the following notation: following an arbitrary public history,[45] denoted $h_t$, $w_t^j$ is the offer of an individual buyer $j$, $w_t$ is the bid (i.e., highest offer), $w_t'$ is the second-highest offer (ties inclusive), and $y_\theta^t$ is the probability with which the type-$\theta$ seller trades. Finally, $p^r$ is the market belief following rejection at $t = 0$, but prior to the realization of the signal. The following four observations will be useful for characterizing behavior in the two-period model.

LEMMA A.1: *In any equilibrium and for any public history $h_t$, the seller plays a reservation strategy: the seller rejects if the bid is below, and accepts if the bid is above, a reservation value $r_\theta^t$, which is equal to her continuation value. Further, $r_L^t < r_H^t$, except if $p^r = 1$, in which case $r_L^0 = r_H^0 = \delta$.*

*Proof.* If $t = 1$, the lemma is immediate with $r_L^1 = \delta\kappa$ and $r_H^1 = \delta$. If $t = 0$, any equilibrium specifies a post-rejection, pre-signal-realization belief $p^r$. Since offers are private, $p^r$ does not vary if a buyer deviates to an off-path offer. Hence, the type-$\theta$ seller strictly prefers to reject if $w_0 < C_\theta(p^r)$, strictly prefers to accept if $w_0 > C_\theta(p^r)$, and is indifferent if $w_0 = C_\theta(p^r)$. For the final claim, note that for all $p^r < 1$, $\delta\mathbb{E}[F_H(q(s,p^r))|H] = C_H(p^r) > C_L(p^r) = \delta\mathbb{E}[F_L(q(s,p^r))|L]$, since $F_H \geq F_L$ are both increasing and $q(s,p^r|H)$ first-order stochastically dominates $q(s,p^r|L)$, and that $C_H(1) = C_L(1) = \delta$. □

LEMMA A.2: *In any equilibrium and for any public history $h_t$, if the owner is a seller, $w_t > r_L^t$, and $w_t \neq r_H^t$, then $w_t' = w_t$.*

*Proof.* Fix an equilibrium candidate and history $h_t$, and without loss of generality, let $w_t^j = w_t \geq w_t'$. If $w_t \in (r_L^t, r_H^t)$, then $y_L^t = 1$ and $y_H^t = 0$ (Lemma A.1). If, however, $w_t' < w_t$, then buyer $j$ can profitably deviate by lowering his offer to $w \in (\max\{r_L^t, w_t'\}, w_t)$ and pay a lower price without changing the probability with which he trades with either type. An analogous argument applies if $w_t > r_H^t$. □

LEMMA A.3: *In any equilibrium, all buyers make zero (expected) profit.*

*Proof.* For $t = 0, 1$, a buyer earns zero by not trading, which he can guarantee himself by offering $w_t^j = 0$. If at time $t = 1$ the owner is a holder, the result is immediate since a seller's willingness to accept the bid implies that the common value for the security is no greater than the bid.

Now consider an arbitrary history $h_t$ in which the owner is a seller. Suppose that buyer $j$ is making positive profits in equilibrium by offering $w_t^j = w_t < r_H^t$. Letting $\rho$ be the value a buyer gets from acquiring the low-type security at time $t$ in this equilibrium, this implies that $\rho - w_t > 0$, and the total payoff to buyers is no greater than $(1 - p_t)(\rho - w_t)$. Hence, there must be at least one buyer $k$ whose payoff is less than $\frac{1}{2}(1 - p_t)(\rho - w_t)$. For $\varepsilon > 0$ small enough, by deviating to $w_t^k = w_t + \varepsilon$, buyer $k$ attracts the low type w.p.1 so improves his payoff to $(1 - p_t)(\rho - w_t - \varepsilon)$, contradicting the equilibrium. An analogous argument demonstrates that no buyer can earn positive profits in equilibrium with a bid of $w_t^j \geq r_H^t$. □

LEMMA A.4: *Let $x_0 \equiv B(p_0)$ and $x_1 \equiv \bar{V}(p_1)$. Fix any equilibrium and public history $h_t$. If the owner at time $t$ is a seller, then at least one of the following holds: (i) $w_t \leq r_L^t$ and $y_L^t = y_H^t = 0$; (ii) $w_t = \kappa$; or (iii) $w_t = x_t$ and $y_L^t = y_H^t = 1$.*

*Proof.* First, suppose $r_L^t < r_H^t$. Lemma A.1 rules out that both $y_H^t > 0$ and $y_L^t < 1$. Hence, three possibilities remain.

(1) $y_L^t = y_H^t = 0$, which implies $w_t \leq r_L^t$ by Lemma A.1, and therefore (i).
(2) $y_L^t > 0$, $y_H^t = 0$, which, noting that in either period a buyer grosses $\kappa$ by trading only with the low type in equilibrium, implies (ii) by zero-profit (Lemma A.3).
(3) $y_L^t = 1$, $y_H^t > 0$. Note that total buyer profits are increasing in $y_H^t$. Suppose $w_t > x_t$; total buyer profits are no greater than $x_t - w_t < 0$, violating zero-profit. Suppose instead that $w_t < x_t$; by zero-profit, $y_H^t \in (0, 1)$ and $x_t - w_t > 0$. But then for $\varepsilon > 0$ small enough, a buyer $k$ can deviate to $w_t^k = w_t + \varepsilon$, attracting both types w.p.1 and hence a payoff of $x_t - w_t - \varepsilon > 0$. Therefore, $w_t = x_t$, and zero-profit implies $y_L^t = y_H^t = 1$, and thus (iii).

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Next, suppose that $r_L^t = r_H^t$, meaning that $t = 0$, $p^r = 1$, and $r_L^t = r_H^t = \delta$ (Lemma A.1). For $p^r = 1$, it must be the case that $y_L^0 = 1$ and either $y_H^0 = 1$ (rejection is off-path) or $y_H^0 < 1$ (rejection is on-path). In the former, zero-profit implies $w_0 = x_0$ and hence (iii). In the latter, the same argument given for (3) above shows that this is inconsistent with zero-profit. $\qquad\square$

*Proof of Lemma 1.* Consider first the case in which the owner at $t = 1$ is a seller. Recall that the seller reservation values are $r_L^1 = \delta\kappa < r_H^1 = \delta$ (Lemma A.1).

- Suppose $p_1 < \bar{p}$. The optimality of the proposed seller play follows from Lemma A.1. Further, no buyer $j$ can profitably deviate. Deviating to $w_1^j < \kappa$ will be ignored because $w_1' = \kappa$ (Lemma A.2). Deviating to $w_1^j \in (\kappa, \delta)$ attracts only the low type (i.e., $y_L^1 = 1$, $y_H^1 = 0$) and therefore earns negative profits. Deviating to $w_1^j \geq \delta$ leads to $y_L^1 = 1$, $y_H^1 \leq 1$, and hence a payoff no greater than $\bar{V}(p_1) - \delta < 0$ since $p_1 < \bar{p}$. For the uniqueness claim, note that since $\kappa \in (r_L^1, r_H^1)$, Lemma A.1 implies that the equilibrium outcome just verified is the unique one in which $w_1 = \kappa$. We need to rule out $(i)$ and $(iii)$ of Lemma A.4. If $(i)$, buyer $j$ can deviate to $w_1^j \in (\delta\kappa, \kappa)$, which will attract the low type w.p.1 and, therefore, positive profits. Finally, $(iii)$ is inconsistent with Lemma A.1, since $p_1 < \bar{p}$ implies $\bar{V}(p_1) < \delta = r_H^1$.
- Suppose $p_1 > \bar{p}$. The optimality of the proposed seller play follows from Lemma A.1. Further, no buyer $j$ can profitably deviate. Deviating to $w_1^j < \bar{V}(p_1)$ will be ignored because $w_1' = \bar{V}(p_1)$ (Lemma A.2). Deviating to $w_1^j > \bar{V}(p_1)$ leads to $y_L^1 = y_H^1 = 1$, and hence a payoff of $\bar{V}(p_1) - w_1^j < 0$. For the uniqueness claim, note that since $\bar{V}(p_1) > r_H^1 > r_L^1$, Lemma A.1 implies that the equilibrium outcome just verified is the unique one in which $w_1 = \bar{V}(p_1)$. We need to rule out (i) and (ii) of Lemma A.4. In either case, a buyer $j$ can deviate to $w_1^j \in (\delta, \bar{V}(p_1))$, which will attract both types w.p.1 and therefore earns positive profits.

Finally, suppose the owner at $t = 1$ is a holder, which can only happen if the security traded $t = 0$. Hence, by Lemma A.1, the post-*acceptance*, pre-signal belief is no greater than $p_0$ and therefore less than one. Because the signal is never perfectly informative, it also holds that $p_1 < 1$. Further, the low- and high-type values for retaining the security are $\kappa$ and one, respectively. Hence, any $w_1 \geq 1$ earns no more than $\bar{V}(p_1) - 1 < 0$, so cannot be the equilibrium bid. Any $w_1 \in (\kappa, 1)$ attracts only the low type and earns $\kappa - w_1 < 0$, so cannot be the equilibrium bid. Hence, $w_1 \leq \kappa$, which both types weakly prefer to reject. $\qquad\square$

*Proof of Lemma 2.* Note first that both $C_H(p)$ and $B(p)$ are continuous with $B(0) = \kappa < C_H(0) = \delta^2$ and $C_H(1) = \delta < B(1) = 1$. Existence of $b$ such that $C_H(b) = B(b)$ follows from the intermediate value theorem.

For uniqueness of $b$, define $\bar{F}(p) = \mathbb{E}[F_\theta(q(s, p))]$ and note that (i) $B$ is a convex combination of $\bar{V}$ and $\bar{F}$, (ii) $\bar{F} < \bar{V}$ on $(0, 1)$, and (iii) letting $\underline{p}$ be implicitly defined as $\bar{V}(\underline{p}) = \delta^2$, we have $C_H(p) \geq \delta^2 > \bar{V}(p)$ for all $p < \underline{p}$. To prove uniqueness of $b$ and the claimed rankings of $B$ and $C_H$, it is therefore sufficient to show $C_H'(p) < \bar{V}'(p)$ and $C_H'(p) < \bar{F}(p)$ for all $p > \underline{p}$, since this implies that $B$ must be strictly steeper at any point of intersection and thus can intersect $C_H$ at most once.

To demonstrate $C_H'(p) < \bar{V}'(p)$ for $p > \underline{p}$, let $R(s) = \frac{f_L(s)}{f_H(s)}$ denote the likelihood ratio of the signal $s$, and let $\bar{r}(p)$ be the likelihood ratio such that a Bayesian updates a prior of $p$ to a posterior of $\bar{p}$ based on a signal realization with likelihood ratio $\bar{r}(p)$ (i.e., $\frac{p}{p + (1-p)\bar{r}(p)} = \bar{p}$). Note that both $\bar{r}(.)$ and $R(.)$ are strictly monotone and thus we can define $\bar{s}(p) = R^{-1}(\bar{r}(p)) = -\frac{1}{2}\sigma^2 \log(\bar{r}(p))$—the level of the signal above which the posterior will be above $\bar{p}$. With this notation, we can write

$$C_H(p) = \delta\left(\kappa + \int_{-\infty}^{\bar{s}(p)} (\delta - \kappa)f_H(s)ds + (1 - \kappa)\int_{\bar{s}(p)}^{\infty} \frac{p}{p + (1-p)R(s)}f_H(s)ds\right),$$

and therefore

$$\frac{1}{\delta(1-\kappa)}C_H'(p) = \int_{\bar{s}(p)}^{\infty} \frac{1}{(p + (1-p)R(s))^2}f_L(s)ds = \int_0^{\bar{r}(p)} \frac{1}{(p + (1-p)r)^2}\underbrace{\left(\frac{\sigma^2}{2r}f_L(S(r))\right)}_{q_L(r)}dr, \qquad \text{(A1)}$$

where the second equality uses a change of variables: $r = R(s)$, $dr = \frac{-2r}{\sigma^2}ds$. Integrating by parts with $u = \frac{1}{(p+(1-p)r)^2}$ and $dv = q_L(r)dr$, we get

$$\frac{1}{\delta(1-\kappa)}C_H'(p) = Q_L(r)\frac{1}{(p+(1-p)r)^2}\Big|_0^{\bar{r}(p)} - \frac{2}{\sigma^2}\int_0^{\bar{r}(p)} Q_L(r)\frac{r(1-p)}{(p+(1-p)r)^3}dr$$

$$\leq \left(\frac{1-\bar{p}^2Q_L(\bar{r}(p))}{p^2}\right) - Q_L(\bar{r}(p))\frac{2}{\sigma^2}\int_0^{\bar{r}(p)}\frac{r(1-p)}{(p+(1-p)r)^3}dr$$

$$= \frac{1-Q_L(\bar{r}(p))}{p^2}.$$

For $p > \bar{p}$, note that $1 - Q_L(\bar{s}(p)) = \Pr(s > \bar{s}(p)|\theta = L) < p$, which implies that $C_H'(p) \leq \delta(1-\kappa)\frac{1}{p} < 1 - \kappa$ since $\bar{p} > \delta$. For $p < \bar{p}$, note that $Q_L(\bar{s}(p)) \geq \frac{1}{2}$, so we get that $C_H'(p) \leq \frac{\delta(1-\kappa)}{2p^2} < 1 - \kappa$, where the second inequality employs the parametric assumption $\frac{\delta}{2} < \underline{p}^2$ (or equivalently, $\kappa < \frac{\sqrt{\frac{2}{\delta}}\delta^2-1}{\sqrt{\frac{2}{\delta}}-1}$). Noting that $\bar{V}'(p) = 1 - \kappa$ for all $p$, we have $\bar{V}' > C_H'$ for $p > \underline{p}$.

To demonstrate $\bar{F}' > C_H'$ for $p > \underline{p}$, note that $\bar{F}' - C_H' = F_H - F_L + (p-\delta)F_H' + (1-p)F_L'$, which by inspection is strictly positive for all $p \geq \delta$. For $p \in [\underline{p}, \delta]$, we have that

$$\bar{F}'(p) - C_H'(p) > (1-\kappa)\int_0^{\bar{r}(p)}\left(\frac{p}{p+(1-p)r}(q_H(r) - q_L(r)) + \frac{(1-p)q_L(r) - (\delta-p)q_H(r)}{(p+(1-p)r)^2}\right)dr$$

$$> (1-\kappa)\int_0^{\bar{r}(p)}\frac{p^2 - r(1-p)^2}{(p+(1-p)r)^2}\left(q_H(r) - q_L(r)\right)dr.$$

We claim that $p^2 - r(1-p)^2$ is strictly positive for $p \in [\underline{p}, \bar{p}]$ and $r \in [0, \bar{r}(p)]$. To see this, note first that $p^2 - r(1-p)^2$ is increasing in $p$ and decreasing in $r$ and therefore

$$p^2 - r(1-p)^2 > \underline{p}^2 - \bar{r}(\underline{p})(1-\underline{p})^2 = \underline{p}^2 - \frac{\underline{p}}{\bar{p}}(1-\bar{p})(1-\underline{p}) > 0,$$

where the last inequality uses the parametric restriction $\kappa < \frac{\sqrt{\frac{2}{\delta}}\delta^2-1}{\sqrt{\frac{2}{\delta}}-1}$. □

*Proof of Lemma 3.* Note first that $C_L(p) = \delta\mathbb{E}\left[F_L(q(s,p))|\theta = L\right]$ is continuous and strictly increasing in $p$, with $C_L(0) = \delta\kappa < \kappa < C_L(1) = \delta$. Existence of $a$ such that $C_L(a) = \kappa$ follows from the intermediate value theorem. The remaining claims follow from the monotonicity and boundary values of $C_L$. □

*Proof of Proposition 1.* For each of the four cases, we rule out outcomes that do not satisfy the claims, and then demonstrate the existence of an equilibrium that does. For any $p_0$, note that $p^r$ must be weakly greater than $p_0$ by Lemma A.1 (if rejection at $t = 0$ is on-path) and belief monotonicity (if rejection at $t = 0$ is off-path).

Let $p_0 > \max\{a, b\}$. From Lemma A.4, we need to rule out $(i)$ and $(ii)$. If $(i)$ then, by equilibrium belief consistency, $p^r = p_0$ and a buyer $j$ can deviate to $w_0^j \in (C_H(p_0), B(p_0))$ and attract both types w.p.1, and can therefore earn positive profits. If $(ii)$, then $p^r \geq p_0 > a$ implies that $C_L(p^r) \geq C_H(p_0) > C_L(a) = \kappa$, so $w_0 = \kappa$ is rejected by both types w.p.1 and $p^r = p^0$. Hence, the deviation to $w_0^j \in (C_H(p_0), B(p_0))$ is again profitable. The only remaining possibility is $(iii)$, as claimed in the proposition. For an equilibrium consistent with the claims, let $p^r = p^0$, and note that $B(p_0) > C_H(p_0) > C_L(p_0)$. Hence, both types prefer to accept the bid of $B(p_0)$, which also earns any buyer zero profit. A buyer's unilateral deviation to a lower offer will be ignored since $w_0' = B(p_0)$ (Lemma A.2), and deviating to a higher offer $w > B(p_0)$ will attract both types w.p.1 and therefore earn $B(p_0) - w < 0$.

Let $p_0 < \min\{a, b\}$. From Lemma A.4, we need to rule out $(i)$ and $(iii)$. If $(i)$, then $p^r = p_0$ and a buyer $j$ can deviate to $w_0^j \in (C_L(p_0), \kappa)$, which attracts the low type w.p.1 and therefore earns positive profits. Next, $p_0 \leq p^r$ and $p_0 < b$ imply $C_H(p^r) \geq C_H(p_0) > B(p_0)$, meaning that $(iii)$ is inconsistent with Lemma A.1. Hence, the only possibility is $(ii)$. Given $w_0 = \kappa$ and the specified $y_L^0$, equilibrium belief consistency

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requires that $p^r = a$. By definition of $a$, this implies the low type is indifferent ($C_L(a) = \kappa = w_0$) and hence willing to mix. The high type strictly prefers to reject since $\kappa < C_H(a)$. A buyer's unilateral deviation to a lower offer will be rejected by both types w.p.1, leaving his payoff unchanged. Deviating to any offer in $(\kappa, C_H(a))$ will attract only the low type, and so earn negative profits, whereas deviating to an even higher offer will earn no more than $B(p_0) - C_H(a) < 0$.

Let $a < b$ and $p_0 \in (a, b)$. Ruling out $(iii)$ of Lemma A.4 is done by the same argument given for $p_0 < \min\{a, b\}$. Next, by $p^r \geq p_0 > a$, we have $C_L(p^r) \geq C_H(p_0) > C_L(a) = \kappa$. Hence, in this case, $(ii)$ of Lemma A.4 implies $(i)$. Finally, we verify the proposed equilibrium. Given the proposed play, equilibrium belief consistency requires that $p^r = p_0$. That both seller types are willing to reject the bid $w_0 \leq C_L(p_0)$ is immediate. A buyer's unilateral deviation to a lower offer will continue to be rejected. Deviating to any offer in $(C_L(p_0), C_H(p_0))$ will attract only the low type, so earn $\kappa - C_L(p_0) < 0$, whereas deviating to an even higher offer will earn no more than $B(p_0) - C_H(p_0) < 0$.

Let $b < a$. First, let $c$ solve $C_H(a) = B(c)$, and $p_0 \in (c, a)$. From Lemma A.4, we need to rule out $(i)$ and $(ii)$. If $(i)$, then $p^r = p_0$ and a buyer $j$ can deviate to $w_0^j \in (C_H(p_0), B(p_0))$ and attract both types w.p.1, therefore earning positive profits. If $(ii)$ and $p^r > a$, then $y_L^0 = y_H^0 = 0$. Equilibrium belief consistency requires $p^r = p_0 < a$, contradicting the supposition that $p^r > a$. If $(ii)$ and $p^r \leq a$, then, because $p_0 \in (c, a)$, we have $C_H(p^r) \leq C_H(a) < B(p_0)$. But then a buyer $j$ can deviate to $w_0^j \in (C_H(p^r), B(p_0))$ and attract both types w.p.1 and therefore earn positive profit. Hence, the only possibility is $(iii)$, as claimed in the proposition. Verifying this equilibrium outcome is done by the same argument given for $p_0 > \max\{a, b\}$. Now let $p_0 \in (b, c)$. We need to rule out $(i)$ of Lemma A.4. If $(i)$, then $p^r = p_0$ and a buyer $j$ can deviate to $w_0^j \in (C_L(p_0), \kappa)$, which attracts the low type w.p.1 and therefore earns positive profits. Verifying the fully liquid and partially liquid equilibrium outcomes follows the same arguments given for the $p_0 > \max\{a, b\}$ and $p_0 < \min\{a, b\}$ cases, respectively. $\square$

*Proof of Proposition 2.* Consider any $\lambda_2 > \lambda_1 \geq 0$. Let $B_i$ denote the buyers' value given the probability of the shock is $\lambda_i$. Similarly let $a_i, b_i$ denote the respective thresholds (likewise $c_i$ if $b_i < a_i$).

For $(i)$, that $B_1 > B_2$ follows from equation (3) and by noting that the terms on the RHS (save $\lambda$) are independent of $\lambda$ and strictly positive for any $p_0 \in (0, 1)$. For $(ii)$, suppose that $b_2 \leq b_1$, then $B_2(b_2) = C_H(b_2) \geq B_1(b_2)$ (by Lemma 2), contradicting $(i)$. That $a$ remains constant is immediate. For $(iii)$, note that from $(ii)$ we have $a_1 = a_2$, and thus we drop the subscript. Generically, there are three cases to consider:

*Case 1:* $a < b_1 < b_2$. The probability of trade is unchanged for $p_0 < b_1$ and $p_0 > b_2$. For $p_0 \in (b_1, b_2)$, the probability of trade drops from one to zero.

*Case 2:* $b_1 < a < b_2$. The probability of trade is unchanged for $p_0 < b_1$ and $p_0 > b_2$. For $p_0 \in (a, b_2)$, the probability of trade drops from one to zero. For $p_0 \in (b_1, a)$, the unique equilibrium under $\lambda_2$ is the partially liquid one, which involves weakly lower probability of trade than any equilibrium for these priors under $\lambda_1$.

*Case 3:* $b_1 < b_2 < a$. The probability of trade is unchanged for $p_0 < b_1$ and $p_0 > c_2$. For the remaining states, equilibrium multiplicity complicates the comparison. For any $p_0 \in (b_1, c_1)$, under $\lambda_1$ there are both partially liquid and fully liquid equilibria, whereas under $\lambda_2$ either the equilibrium is uniquely the partially liquid one or the equilibrium set is identical to that under $\lambda_1$. For any $p_0 \in (c_1, c_2)$, under $\lambda_1$ the unique equilibrium is the fully liquid one, whereas under $\lambda_2$ either the equilibria is uniquely the partially-liquid one or there is both the partially liquid and fully liquid equilibria. If one compares the respective equilibria with the maximum or minimum probability of trade for each $p_0$ across the two $\lambda$ values, the proposition holds. Further, if $c_1 < b_2$, then for $p_0 \in (c_1, b_2)$ the equilibrium is unique for both $\lambda$ values, being fully liquid under $\lambda_1$ but only partially liquid under $\lambda_2$. $\square$

*Proof of Proposition 7.* It is straightforward to adapt Lemma A.3 to show that buyers at $t = 1$ make zero expected profit. Hence, any buyer's unconditional value for the security at $t = 0$ equals the expected discounted realized stream of cash flows. To establish the claimed form of $B^{un}$, it is sufficient to demonstrate the following: in equilibrium, for generic $\tilde{p}_1$, the security is efficiently allocated at $t = 2$, unless $\theta = H$, the $t = 1$ owner is constrained, and $\tilde{p}_1 < \bar{p}$. The logic for the claim is analogous to that of Lemma 1. Note that the first part of Lemma A.1 now applies to all owners, with the continuation/reservation values remaining $\delta$ and $\delta\kappa$ for the constrained high and low types, respectively, and being one and $\kappa$ for the unconstrained high and low types, respectively.

First, if $\tilde{p}_1 < \bar{p}$, then $w_1 = \kappa$, which the high-type owner rejects w.p.1, the low-type constrained owner accepts w.p.1, and the low-type unconstrained owner accepts with any probability in $[0, 1]$. The optimality of the owner decisions is immediate given the type-specific continuation values. Further, no buyer $j$ can profitably deviate. Deviating to $w_1^j < \kappa$ will be ignored since at least one other buyer must be offering $\kappa$ in the equilibrium (by an argument analogous to Lemma A.2). Deviating to $w_1^j \in (\kappa, \delta)$ attracts only the low type and therefore negative profits. Deviating to $w_1^j \in [\delta, 1)$, at best, attracts all owners except the high-type unconstrained owner, so leads to a payoff no greater than $\bar{V}(\tilde{p}_1) - \delta < 0$ since $\tilde{p}_1 < \bar{p}$. Deviating to $w_1^j \geq 1$, at best, attracts all owners, so leads to a payoff no greater than $\bar{V}(p_1) - 1 \leq 0$ for any $p_1$. Second, suppose that $\tilde{p}_1 > \bar{p}$. Then $w_1 = \bar{V}(\tilde{p}_1)$, which the high-type unconstrained owner rejects w.p.1, and all other owners accept w.p.1. The optimality of the owner decisions is immediate given the type-specific continuation values. Further, no buyer $j$ can profitably deviate. Deviating to $w_1^j < \bar{V}(\tilde{p}_1)$ will be ignored. Deviating to $w_1^j \in (\bar{V}(\tilde{p}_1), 1)$ attracts all owners except the high-type unconstrained owner, so leads to a payoff of $\bar{V}(\tilde{p}_1) - w_1^j < 0$. Deviating to $w_1^j \geq 1$ leads to a payoff no greater than $\bar{V}(p_1) - 1 \leq 0$ for any $p_1$. Finally, in both cases, uniqueness is established by showing that every other candidate equilibrium bid leads to a profitable deviation for a buyer, using arguments similar to those in the proofs of Lemmas A.4 and 1, which we omit for brevity.

What remains is to demonstrate that $B^{un}(p_0) < B(p_0)$ for all $\lambda$ and $p_0$ both in $(0, 1)$. Using (3),

$$B(p_0) - B^{un}(p_0) = (1 - \delta)\lambda p_0 \big( \Pr(\tilde{q}(s, p_0) < \bar{p}|\theta = H) - \Pr(q(s, p_0) < \bar{p}|\theta = H) \big).$$

Given $(1 - \delta)\lambda p_0 > 0$, we need that $\Pr(\tilde{q}(s, p_0) < \bar{p}|\theta = H) > \Pr(q(s, p_0) < \bar{p}|\theta = H)$. But this follows from $\tilde{q}(s, p_0) < q(s, p_0)$ for all $s$ and $p_0 \in (0, 1)$, given $\lambda \in (0, 1)$—see (10).

$\square$

# Appendix B.   Characterization of Equilibrium System

In this appendix we present the system that characterizes an equilibrium of the form $\Xi^N$, for arbitrary $N \in \{1, 2, \ldots, \infty\}$, on which Theorem 3 relies. The equations govern the necessary optimality and interdependency properties of the equilibrium value functions of sellers, holders, and buyers.

## The Seller Value Function

For any $N$, fix a candidate $\Xi^N(\alpha, \beta, B)$ such that $B$ is differentiable for $z \geq \beta$.[46] Due to the stationary structure of the candidate equilibrium, the state $z$ is sufficient to compute the seller's payoff. Therefore, without loss, fix $t = 0$ and let $T(\beta) \equiv \inf\{s \geq 0 : Z_s \geq \beta\}$. Note that $T(\beta)$ is the strategy prescribed by $\Xi^N$ for the high type and, because she must be indifferent regarding trading when $z = \alpha$, this strategy must yield the low type her $\Xi^N$-equilibrium payoff. Therefore, for each $\theta$, the equilibrium value function must be consistent with this strategy:

$$F_\theta(z) = \mathbb{E}_z^\theta \left[ \int_0^{T(\beta)} e^{-rt} k_\theta dt + e^{-rT(\beta)} B(\beta) \right], \tag{B1}$$

where $\mathbb{E}_z^\theta$ is the expectation over the process $Z$ under the probability law starting at $z$ and conditional on $\theta$ $(\mathcal{Q}_z^\theta)$. For $z \in (\alpha, \beta)$, the seller waits and $Z$ evolves according to news. Therefore,

$$F_\theta(z) = k_\theta dt + e^{-rdt}\mathbb{E}^\theta \left[ F_\theta(z + d\hat{Z}) \right]. \tag{B2}$$

Applying Ito's lemma to $F_\theta$, using the law of motion of $\hat{Z}$, and taking the expectation conditional on $\theta$, (B2) implies a differential equation that $F_\theta$ must satisfy for all $z \in (\alpha, \beta)$. In particular, for a high-type seller

$$rF_H(z) = k_H + \frac{\phi^2}{2}(F_H''(z) + F_H'(z)) \tag{B3}$$

and for a low-type seller

$$rF_L(z) = k_L + \frac{\phi^2}{2}(F_L''(z) - F_L'(z)). \tag{B4}$$

The equilibrium specifies that for all $z \geq \beta$, both types of sellers trade immediately at $B(z)$. Therefore,

$$F_H(z) = F_L(z) = B(z), \qquad \forall z \geq \beta. \tag{B5}$$

For all states $(z, \vec{i})$, $z \leq \alpha$, low-type sellers mix, and the equilibrium belief jumps instantaneously to $\alpha$ conditional on no trade.[47] Therefore,

$$F_H(z) = F_H(\alpha), \qquad F_L(z) = F_L(\alpha), \qquad \forall z \leq \alpha. \tag{B6}$$

Six boundary conditions help pin down the seller's equilibrium value function in the interior of the fully illiquid region. Four of these are *physical* conditions that must be satisfied for the equilibrium value functions to be consistent with (B1). The *value matching* conditions are straightforward:

$$F_L(\beta^-) = B(\beta) \tag{B7}$$
$$F_H(\beta^-) = B(\beta), \tag{B8}$$

where $g(x^-)$ and $g(x^+)$ are used to denote the left and right limits of the function $g$ at $x$. For a high type, the belief process reflects at $z = \alpha$, and therefore the value function must satisfy

$$F_H'(\alpha^+) = 0 \tag{B9}$$

(see Harrison (1985, §5)). According to $\Xi^N$, the low types mix at the lower boundary such that $Z$ is *killed* at the lower boundary at a rate of $\kappa = 1$, implying that $F_L$ must satisfy the *Robin* boundary condition

$$F_L'(\alpha^+) = F_L(\alpha) - V_L \tag{B10}$$

(see Harrison (2013, §9)).[48] The remaining conditions are equilibrium conditions required to ensure that both *Owner Optimality* and *No Deals* hold:

$$F_L'(\alpha^+) = 0 \tag{B11}$$
$$F_H'(\beta^-) = B'(\beta). \tag{B12}$$

The equilibrium argument for (B11) is as follows. According to $\Xi^N$, a low type mixes between accepting $V_L$ at $\alpha$ and rejecting. Therefore, she must be indifferent between these two actions. The first implies a payoff at $\alpha$ of $F_L(\alpha) = V_L$. Using the stopping rule $T(\beta)$ (i.e., always rejecting at $\alpha$) implies that $F_L'(\alpha) = 0$ (since $Z$ reflects conditional on rejection). To be consistent with indifference, both must hold. Note that in conjunction with (B10), any two of these conditions imply the third.

To see why (B12) must hold, suppose that $F_H'(\beta^-) < B'(\beta)$ and consider the following deviation: reject at $z = \beta$ and continue to reject until $z = \beta + \epsilon$ for some arbitrarily small $\epsilon > 0$. Instead of accepting $B(\beta)$, the high type attains a convex combination of $B(\beta + \epsilon)$ and $F_H(\beta - \epsilon)$, which lies strictly above $B(\beta)$, implying the deviation is profitable. On the other hand, if $F_H'(\beta^-) > B'(\beta)$, then $F_H(\beta - \epsilon) < B(\beta - \epsilon)$, the high type would prefer to accept sooner, and buyers will have a profitable deviation, violating *No Deals*.[49]

It remains to determine the buyer value function, $B$, which in turn requires deriving a holder's value for a share of the asset.

### The Holder Value Function for Finite N

We proceed by constructing a holder's value function based on the structure of $\Xi^N$. A holder's value function depends only on $z$ and $\max(\vec{i}) \in \{0,1\}$. Define $G_\theta(z, \max(\vec{i}))$ to be the equilibrium payoff of a type-$\theta$ holder given belief $z$.

Consider first $G_\theta(z, 0)$. The market belief, $Z$, evolves based solely on the realization of news, and the holder waits until either she is shocked and becomes a seller, or another owner is shocked and her value

function becomes $G_\theta(\cdot, 1)$:

$$G_\theta(z,0) = v_\theta dt + \lambda dt F_\theta(z) + (N-1)\lambda dt G_\theta(z,1) + (1 - N\lambda dt)e^{-rdt}\mathbb{E}^\theta\left[G_\theta(z + d\hat{Z}_t, 0)\right]. \tag{B13}$$

As (B2) does for the seller, (B13) implies a differential equation that $G_\theta(\cdot, 0)$ must satisfy for all $z$. In particular, for a high-type and low-type holder,

$$rG_H(z,0) = v_H + \lambda\left(F_H(z) + N[G_H(z,1) - G_H(z,0)] - G_H(z,1)\right) + \frac{\phi^2}{2}\left(G_H'(z,0) + G_H''(z,0)\right) \tag{B14}$$

$$rG_L(z,0) = v_L + \lambda\left(F_L(z) + N[G_L(z,1) - G_L(z,0)] - G_L(z,1)\right) - \frac{\phi^2}{2}\left(G_L'(z,0) - G_L''(z,0)\right). \tag{B15}$$

As $z \to \pm\infty$, the belief becomes degenerate, and the effect of news on equilibrium beliefs goes to zero. A holder waits for the next shock to come. Therefore,

$$\lim_{z \to \infty} G_\theta(z,0) = \frac{rV_\theta + \lambda \lim_{z \to \infty} F_\theta(z) + (N-1)\lambda \lim_{z \to \infty} G_\theta(z,1)}{r + N\lambda} \qquad \theta \in \{L, H\}, \tag{B16}$$

$$\lim_{z \to -\infty} G_\theta(z,0) = \frac{rV_\theta + \lambda \lim_{z \to -\infty} F_\theta(z) + (N-1)\lambda \lim_{z \to -\infty} G_\theta(z,1)}{r + N\lambda} \qquad \theta \in \{L, H\}. \tag{B17}$$

REMARK 2: *When $N = 1$, B14 to B17 simplify, and all $G_\theta(\cdot, 1)$ terms drop out. In this case, the entire system has no dependence on $G_\theta(\cdot, 1)$. With only one share, there is never a history in which both a holder and a seller exist simultaneously. Thus, the analysis of $G_\theta(\cdot, 1)$ is not relevant for the $N = 1$ case, and hence notation simplifies to $G_\theta(\cdot) = G_\theta(\cdot, 0)$ in Section III.*

Now consider $G_\theta(z, 1)$. The market belief, $Z$, evolves based on the realization of both news and the trading behavior of the sellers. We first state the characterization and then explain. Let $q_L(z|\alpha) \equiv \frac{p(\alpha) - p(z)}{p(\alpha)(1 - p(z))}$. Then

$$G_L(z,1) = \begin{cases} q_L(z|\alpha)V_L + (1 - q_L(z|\alpha))G_L(\alpha, 1) & \text{for } z < \alpha \\ \frac{1}{r}\left(v_L + \lambda(F_L(z) - G_L(z,1)) - \frac{\phi^2}{2}(G_L'(z,1) - G_L''(z,1))\right) & \text{for } z \in [\alpha, \beta) \\ G_L(z,0) & \text{for } z \geq \beta \end{cases} \tag{B18}$$

$$G_H(z,1) = \begin{cases} G_H(\alpha, 1) & \text{for } z < \alpha \\ \frac{1}{r}\left(v_H + \lambda(F_H(z) - G_H(z,1)) + \frac{\phi^2}{2}(G_H'(z,1) + G_H''(z,1))\right) & \text{for } z \in [\alpha, \beta) \\ G_H(z,0) & \text{for } z \geq \beta. \end{cases} \tag{B19}$$

If trade occurs when $z < \alpha$, it publicly reveals that $\theta = L$ and leads to a common value of $V_L$ for all owners (see Lemma C.2). Conditional on $\theta = L$, this occurs with probability $q_L(z|\alpha)$. If no sellers sell, then $z$ jumps to $\alpha$ yielding $G_\theta(\alpha, 1)$.

For $z > \alpha$, beliefs evolve based only on news, so the equations are analogous to those derived previously. Note that for $z \geq \beta$, all sellers sell immediately, meaning that sellers are "present" for an arbitrarily short amount of time. Further, there is no information content gleaned from a sale. Thus, $G_\theta(z, 1) = G_\theta(z, 0)$, which implies the following two value matching conditions:

$$G_H(\beta^-, 1) = G_H(\beta, 0) \tag{B20}$$

$$G_L(\beta^-, 1) = G_L(\beta, 0). \tag{B21}$$

The behavior of $Z$ at $\alpha$ requires that

$$G_H'(\alpha^+, 1) = 0 \tag{B22}$$

$$G_L'(\alpha^+, 1) = G_L(\alpha, 1) - V_L. \tag{B23}$$

Similar to (B9), (B22) is due to the reflecting boundary of $Z$ (for high-type owners) when there is at

least one seller present. Similar to (B10), (B23) is the *Robin* condition, which must be satisfied because at $\alpha$, for a low-type holder when sellers are present, the process $Z$ is either reflected (if the sellers reject, yielding the holder $G_L(\alpha, 1)$) or *killed* (if the sellers accept, yielding the holder $G_L(-\infty, 0) = V_L$).

*The Holder Value Function for Countably Infinite N*

As $N \to \infty$, any smooth solution to (B14) and (B15) requires that $|G_\theta(z, 1) - G_\theta(z, 0)| \to 0$ for all $z$. In the limit ($N = \infty$), beliefs always account for the presence of sellers (see Section IV). Thus, we no longer need to distinguish the two cases and simply let $G_\theta^\infty$ be the type-$\theta$ holder's value function, which is given by

$$G_L^\infty(z) = \begin{cases} q_L(z|\alpha)V_L + (1 - q_L(z|\alpha))G_L^\infty(\alpha) & \text{for } z < \alpha, \\ \frac{1}{r}\left(v_L + \lambda\left(F_L(z) - G_L^\infty(z)\right) - \frac{\phi^2}{2}\left(G_L^{\infty\prime}(z) - G_L^{\infty\prime\prime}(z)\right)\right) & \text{for } z \geq \alpha, \end{cases} \tag{B24}$$

$$G_H^\infty(z) = \begin{cases} G_H^\infty(\alpha) & \text{for } z < \alpha, \\ \frac{1}{r}\left(v_H + \lambda\left(F_H(z) - G_H^\infty(z)\right) + \frac{\phi^2}{2}\left(G_H^{\infty\prime}(z) + G_H^{\infty\prime\prime}(z)\right)\right) & \text{for } z \geq \alpha. \end{cases} \tag{B25}$$

This form for $G^\infty$ comes from the fact that above $\alpha$, the belief evolves solely based on news and a holder is simply waiting to get shocked, but once $\alpha$ is reached the behavior of the sellers in the market affects the belief just as described immediately above. Finally, the boundary conditions, which follow from arguments similar to those given previously, are

$$\lim_{z \to \infty} G_\theta^\infty(z) = \frac{rV_\theta + \lambda \lim_{z \to \infty} F_\theta(z)}{r + \lambda} \qquad \text{for } \theta \in \{L, H\}, \tag{B26}$$

$$G_L^{\infty\prime}(\alpha^+) = G_L(\alpha, 1) - V_L, \tag{B27}$$

$$G_H^{\infty\prime}(\alpha^+) = 0. \tag{B28}$$

*The Buyer Value Function*

Finally, we derive the buyer value function. After purchasing a share of the asset, a buyer becomes a holder and therefore a buyer's (unconditional) value for a share is the expected holder value. For finite $N$, this will depend on whether there are sellers present *after* the share is purchased. Because trade occurs at a price of $B$ only when $z \geq \beta$, this dependance has no implications for on-path equilibrium play (as $G_\theta(z, 1) = G_\theta(z, 0)$ for all such $z$). Nevertheless, this dependance is important for checking whether profitable off-path deviations exist. Define

$$B(z) \equiv \begin{cases} \mathbb{E}[G_\theta(z, 1_{\{N>1\}})|z] & \text{for } N < \infty \\ \mathbb{E}[G_\theta^\infty(z)|z] & \text{for } N = \infty. \end{cases} \tag{B29}$$

In Proposition 6 we establish that $\mathbb{E}[G_\theta(z, 1)|z] \geq \mathbb{E}[G_\theta(z, 0)|z]$ when $1 < N < \infty$, which ensures that to check whether a buyer has a profitable deviation it is unnecessary to distinguish whether sellers are present following such a deviation (i.e., demonstrating *No Deals* using $B$ as defined by (B29) is sufficient). From (B29) we see that $B$ is differentiable above $\beta$ (since $G_\theta$ is), as we assume at the outset. In addition, for any finite $N$, we have that

$$\lim_{z \to \infty} B(z) = \frac{rV_H + \lambda \lim_{z \to \infty} F_H(z) + (N-1)\lambda \lim_{z \to \infty} G_H(z, 1)}{r + N\lambda} = V_H, \tag{B30}$$

where the first equality is implied by (B16) and (B19), and the second by (B5) and $B$ bounded. Similarly,

$$\lim_{z \to -\infty} B(z) = \frac{rV_L + \lambda \lim_{z \to -\infty} F_L(z) + (N-1)\lambda \lim_{z \to -\infty} G_L(z, 1)}{r + N\lambda} = V_L. \tag{B31}$$

Analogous arguments establish these same boundary conditions when $N = \infty$.

*Summary and Restatement of Characterization Results*

Collecting the relevant equations for each case, define the system of equations $\mathbb{S}^N$ as follows:

$$\mathbb{S}^N \equiv \begin{cases} (B3) \text{ to } (B12), (B14) \text{ to } (B17), (B29) & \text{for } N = 1 \\ (B3) \text{ to } (B12), (B14) \text{ to } (B23), (B29) & \text{for } 1 < N < \infty \\ (B3) \text{ to } B12), (B24) \text{ to } (B28), (B29) & \text{for } N = \infty. \end{cases} \tag{B32}$$

For convenience, we restate the theorems from Sections III and IV.

*Restatement of Theorem 3: For arbitrary $N \in \{1, 2, \ldots, \infty\}$, an equilibrium of the form $\Xi^N(\alpha, \beta, B)$ is characterized by the system of equations $\mathbb{S}^N$ defined in (B32). That is, a solution to the equations is both necessary and sufficient for an equilibrium of this form.*

*Restatement of Theorem 1: There exists an $(\alpha^*, \beta^*, B^*)$ such that $\Xi^1(\alpha^*, \beta^*, B^*)$ is an equilibrium. Given Theorem 3, this is equivalent to: a solution to $\mathbb{S}^1$ exits.*

The proof of these two theorems can be found in the Internet Appendix. The proofs of all other results from Sections III and IV are found in Appendix C.

# Appendix C.   Remaining Proofs

We first record an intuitive lemma showing that, due to the transversality condition on the bid process, the expected utility of agents in the economy derives only from the expected net cash flows from the asset.

LEMMA C.1: *Fix any equilibrium, and define $\Pi^n(\mathcal{F}_{t_0})$ to be the $\mathcal{F}_{t_0}$-expected utility of the current owner of share $n$, $A_{t_0}^n$, starting from time $t_0$. In any equilibrium,*

$$\Pi^n(\mathcal{F}_{t_0}) = \mathbb{E}\left[\int_{t_0}^\infty e^{-r(t-t_0)}(v_\theta + I_t^n(k_\theta - v_\theta))dt \big| \mathcal{F}_{t_0}\right].$$

*Proof.* Fix any equilibrium. Consider arbitrary share $n$ and public history $\mathcal{F}_{t_0}$ and let $t_1, t_2, t_3, \ldots$ be the (random) times that the share trades for the the first, second, third, etc., times after time $t_0$ (with $t_{j+1} = \infty$ if the asset does not trade more than $j$ times). By *Zero Profit*, for any $j \geq 1$,

$$W_{t_j} = \mathbb{E}\left[\int_{t_j}^{t_{j+1}} e^{-r(t-t_j)}\big(v_\theta + I_t^n(k_\theta - v_\theta)\big)dt + e^{-r(t_{j+1}-t_j)}W_{t_{j+1}} \big| \mathcal{F}_{t_j}\right].$$

Substituting in the analogous expressions for $W_{t_{j+1}}, W_{t_{j+2}}$, etc., and applying the law of iterated expectations, we get that for arbitrary integer $\kappa$,

$$W_{t_j} = \mathbb{E}\left[\int_{t_j}^{t_{j+\kappa}} e^{-r(t-t_j)}\big(v_\theta + I_t^n(k_\theta - v_\theta)\big)dt \big| \mathcal{F}_{t_j}\right] + \mathbb{E}\left[e^{-r(t_{j+\kappa}-t_j)}W_{t_{j+\kappa}} \big| \mathcal{F}_{t_j}\right]. \tag{C1}$$

Because any pair of trades must be separated by a shock arrival, as $\kappa \to \infty$, $(t_{j+\kappa} - t_j) \overset{a.s.}{\to} \infty$. Therefore, by the transversality condition on $W$, the last term in (C1) limits to zero, and, setting $j = 1$, we have

$$W_{t_1} = \mathbb{E}\left[\int_{t_1}^\infty e^{-r(t-t_1)}\big(v_\theta + I_t^n(k_\theta - v_\theta)\big)dt \big| \mathcal{F}_{t_1}\right]. \tag{C2}$$

Next, by definition of $t_1$, the $\mathcal{F}_{t_0}$-expected utility of $A_{t_0}^n$ is

$$\Pi^n(\mathcal{F}_{t_0}) = \mathbb{E}\left[\int_{t_0}^{t_1} e^{-r(t-t_0)}\big(v_\theta + I_t^n(k_\theta - v_\theta)\big)dt + e^{-r(t_1-t_0)}W_{t_1} | \mathcal{F}_{t_0}\right]. \tag{C3}$$

Putting (C2) and (C3) together establishes the result. $\qquad\square$

*Proof of Theorem 2.* Let $t^0 = \inf\{t \geq 0 : I_t = 0\}$. Because $\lambda = 0$, a holder never transitions to a seller, meaning that the asset is held in perpetuity by a holder after $t^0$. Hence, $G_\theta(z) = \int_0^\infty e^{-rt} v_\theta dt = \frac{v_\theta}{r} = V_\theta$ and $B(z) = \mathbb{E}[G_\theta(z)|z] = \bar{V}(z)$, for all $z$ (from (B29)). If $A_0$ is a holder, then $t^0 = 0$ and the model is trivial. Finally, if $A_0$ is a seller, then $B = \bar{V}$ implies that the model is identical to that of DG12, and the result follows from Lemma 3.1 and Theorems 3.1 and 5.1 found therein. $\qquad\square$

*Proof of Proposition 3.* That $B$ takes the functional form given in (8) is shown in Appendix II. For the first claim, using equation (B29), Lemma C.1, and the stationary structure of $\Xi$, we have

$$B(z) = \mathbb{E}[G_\theta(z)|z] = \Pi^1(z, 0) = \mathbb{E}\left[\int_0^\infty e^{-rt}(v_\theta + I_t^n(k_\theta - v_\theta))dt \Big| (Z_0, I_0) = (z, 0)\right].$$

Because $k_\theta < v_\theta$, to prove that $B(z) < \bar{V}(z)$, it is sufficient to argue that

$$E\left[\int_0^\infty e^{-rt}I_t dt | (Z_0, I_0) = (z, 0)\right] > 0.$$

But this is nearly immediate from the structure of the equilibrium. Let $t^1$ be the arrival of the first shock, and $t^2 \geq t^1$ be the time of the first sale thereafter. Hence, if $Z_{t^1} \in (\alpha, \beta)$, then $\text{Prob}(t^2 > t^1) = 1$. Finally, because the shock arrives in finite time w.p.1. and because $Z$ follows a diffusion while $I = 0$, there is positive probability that $Z_{t^1} \in (\alpha, \beta)$, giving

$$0 < \mathbb{E}\left[\int_{t^1}^{t^2} e^{-rt}dt | (Z_0, I_0) = (z, 0)\right] \leq \mathbb{E}\left[\int_0^\infty e^{-rt}I_t dt | (Z_0, I_0) = (z, 0)\right].$$

$\qquad\square$

*Proof of Corollary 1.* Follows immediately from Proposition 3 and the fact that $B(z)$ corresponds to the equilibrium price when $z \geq \beta$ (i.e., in the fully liquid region). $\qquad\square$

*Proof of Corollary 2.* Using Proposition 3, it is clear from inspection of (8) that $B'(z) > \bar{V}'(z)$ for all $z > \beta$, implying the result for price volatility by Ito's Lemma. Combining the two inequalities $B < \bar{V}$ (Corollary 1) and $B' > \bar{V}'$ for all $z > \beta$ implies the statement for return volatility. $\qquad\square$

*Proof of Proposition 4.* For any equilibrium $\Xi(\alpha, \beta, B)$, we have $F_L(\alpha) = K_L + \mathbb{E}_\alpha^L[e^{-rT(\beta)}](B(\beta) - K_L)$ (see IA.1 in the Internet Appendix). The value-matching boundary conditions on the low-type seller's value function then imply $\mathbb{E}_\alpha^L[e^{-rT(\beta)}] = \frac{V_L - K_L}{B(\beta) - K_L}$. Direct calculation yields $\mathbb{E}_\alpha^L[e^{-rT(\beta)}] = \frac{q_1^L - q_2^L}{q_1^L e^{q_2^L(\beta - \alpha)} - q_2^L e^{q_1^L(\beta - \alpha)}}$. Therefore, $B(\beta_1) \geq \bar{V}(\beta_0) \iff (\beta_1 - \alpha_1) \geq (\beta_0 - \alpha_0)$. In addition, Proposition 3 shows that $B < \bar{V}$, meaning $B(\beta_1) \geq \bar{V}(\beta_0) \implies \beta_1 > \beta_0$.

For the purpose of contradiction, suppose that $B(\beta_1) < \bar{V}(\beta_0)$, and therefore $(\beta_1 - \alpha_1) < (\beta_0 - \alpha_0)$. Recalling the functional form of $B_3$ from the Internet Appendix (IA.13), $B(\beta_1) < \bar{V}(\beta_0)$ implies that $\bar{V}(\beta_1) + C_{32}^B \frac{e^{q_4^B \beta}}{1 + e^{\beta_1}} < \bar{V}(\beta_0)$, where $C_{32}^B < 0$. This yields

$$\frac{B'(\beta_1)}{B(\beta_1) - K_H} = \frac{\bar{V}'(\beta_1)}{\bar{V}(\beta_1) + C_{32}^B \frac{e^{q_4^B \beta_1}}{1 + e^z} - K_H} + \frac{\left(\frac{-C_{32}^B e^{q_4^B \beta_1}}{1 + e^{\beta_1}}\right)\left(\frac{(1 - q_4^B)e^{\beta_1} - q_4^B}{(1 + e^{\beta_1})}\right)}{\bar{V}(\beta_1) + C_{32}^B \frac{e^{q_4^B \beta_1}}{1 + e^z} - K_H} > \frac{\bar{V}'(\beta_0)}{\bar{V}(\beta_0) - K_H}. \qquad (C4)$$

However, using Fact IA.1 as above, condition (B12) rearranges to

$$\frac{d}{dz}\mathbb{E}_{z=\beta}^H[e^{-rT(\beta)}] = \frac{B'(\beta)}{B(\beta) - K_H} \qquad (C5)$$

If $(\beta_1 - \alpha_1) < (\beta_0 - \alpha_0)$, then by direct calculation $\frac{d}{dz}\mathbb{E}_{z=\beta_1}^H[e^{-rT(\beta_1)}] < \frac{d}{dz}\mathbb{E}_{z=\beta_0}^H[e^{-rT(\beta_0)}] = \frac{\bar{V}'(\beta_0)}{\bar{V}(\beta_0) - K_H} < \frac{B'(\beta_1)}{B(\beta_1) - K_H}$, where the inequality comes from (C4), but violates (C5), completing the proof. $\qquad\square$

The following lemma will be used in the proof of Proposition 5.

LEMMA C.2: *If, in any equilibrium, $Z_{t_0}$ is degenerate on $\theta$, then $G_\theta(t_0, \omega) = F_{\theta,t}(t_0, \omega) = V_\theta$, for any $t \leq t_0$.*

*Proof.* Lemma C.1 implies that if, in equilibrium, $Z_{t_0}$ is degenerate on $\theta$, then $G_\theta(t_0, \omega), F_{\theta,t}(t_0, \omega) \leq V_\theta$. We now show that this bound is tight for both. Let $\underline{F}_\theta, \underline{G}_\theta$ be the infimums of $F_\theta$ and $G_\theta$ over all possible on-path histories after reaching the degenerate belief (i.e., $\underline{G}_\theta = \inf\{G_\theta(t, \omega) : t \geq t_0, \ \omega \text{ s.t. } Z_{t_0} \text{ is degenerate on } \theta\}$, and analogously for $\underline{F}_\theta$). By (7), the definition of $G$,

$$\underline{G}_\theta \geq \mathbb{E}[(1 - e^{-r(\tau - t)})|\mathcal{G}_t]V_\theta + \mathbb{E}[e^{-r(\tau - t)}|\mathcal{G}_t]\underline{F}_\theta, \tag{C6}$$

where, starting from arbitrary time $t$, $\tau \geq t$ is the time of the next shock. Now suppose there exists an on-path history such that $\underline{F}_\theta < \underline{G}_\theta$. This clearly violates *No Deals*. Therefore, $\underline{F}_\theta \geq \underline{G}_\theta$. This is consistent with (C6) and $\underline{G}_\theta, \underline{F}_\theta \leq V_\theta$ if and only if $\underline{F}_\theta = \underline{G}_\theta = V_\theta$. $\qquad\square$

*Proof of Proposition 5.* By *Owner Optimality* and $K_H > V_L$, it is never on-path for a high-type seller to accept a bid less than $K_H$. Thus, only low types can be trading at $t$, and *Belief Consistency* requires that, for all $t' > t$, $\Pr(Z_{t'} = -\infty) = 1$. Lemma C.2 and *Zero Profit* then require that $W_t = V_L$, which establishes (1). Next, suppose (2) fails. Then there exists $\hat{t} > t$ such that there is positive probability that the seller of share $m$ retains the asset up to $\hat{t}$. Since $\Pr(Z_{\hat{t}} = -\infty) = 1$, Lemma C.2 implies that the seller's payoff falls below $V_L$ by following such a strategy. This violates Lemma C.2, so (2) must hold.

The proof of the final claim requires several steps. First, we establish that if, in any equilibrium, a share trades when the belief is $Z_t$, then $W_t \leq \bar{V}(Z_t)$. Suppose not, then by *Zero Profit* and Lemma C.1, conditional on trade, $Z_{t^+} > Z_t$. By *Belief Consistency* then, not trading is also on-path, and conditional on no trade, $Z_{t^+} < Z_t$. There are two cases to consider: i) not trading is on-path for both types, and ii) not trading is on-path only for the low type. For (i), following no trade, $\min_\theta\{F_\theta(t^+, \omega)\} \leq \mathbb{E}[F_\theta(t^+, \omega)] \leq \bar{V}(Z_{t^+})$, by Lemma C.1. Because, conditional on no trade, $\bar{V}(Z_{t^+}) < \bar{V}(Z_t) < W_t$, at least one type of seller wishes to sell w.p.1 at time $t$, contradicting the equilibrium. For (ii), *Belief Consistency* implies that, conditional on not trading, $Z_{t^+} = -\infty$. By Lemma C.2, $F_L(t^+, \omega) = V_L < W_t$, so *Owner Optimality* is violated for the low type, contradicting the equilibrium.

Second, let $\underline{z}$ be the unique solution to $\bar{V}(\underline{z}) = K_H$. Because $W_t \leq \bar{V}(Z_t)$, by *Owner Optimality*, in any equilibrium the high type never trades if $Z_t < \underline{z}$.

Third, and finally, we establish the proposition's claim of the existence of such a $z^*$. Suppose that for arbitrary starting belief $Z_t < \underline{z}$ there is zero probability of a future contagious sell-off. By the preceding first and second established claims,

$$F_L(t, \omega) < \mathbb{E}^L\left[\int_t^\tau e^{-r(s-t)}k_L ds + e^{-r(\tau - t)}V_H\right],$$

where $\tau = \inf\{s \geq t : Z_t + (\hat{Z}_s - \hat{Z}_t) \geq \underline{z}\}$, since, as there is zero probability of trade by either type before the belief reaches $\underline{z}$, the belief must evolve based only on news. It is straightforward to calculate that as $Z_t \to -\infty$, the term on the right-hand-side limits (continuously) to $K_L$. This means *Owner Optimality* is violated if $Z_t$ is low enough, establishing the proposition's claim. $\qquad\square$

*Proof of Proposition 6.* Without loss of generality we start at $t = 0$, with arbitrary initial state $(z, \vec{i})$ such that some share $n$ satisfies $I_0^n = 0$. Let $\tau = \inf\{t : I_t^n = 1\}$. From the structure of $\Xi^N$, for each $\theta$,

$$G_\theta(z, \max(\vec{i})) = \mathbb{E}^\theta\left[\int_0^\tau e^{-rt}v_\theta dt + e^{-r\tau}F_\theta(Z_\tau)\big|(Z_0, \vec{I}_0) = (z, \vec{i})\right].$$

So,

$$\mathbb{E}[G_\theta(z, \max(\vec{i}))|z] = \mathbb{E}\left[\mathbb{E}^\theta\left[\int_0^\tau e^{-rt}v_\theta dt + e^{-r\tau}F_\theta(Z_\tau)|(Z_0, \vec{I}_0) = (z, \vec{i})\right]\right]$$

$$= \mathbb{E}\left[\int_0^\tau e^{-rt}v_\theta dt\Big|z\right] + \mathbb{E}\left[e^{-r\tau}\mathbb{E}^\theta\left[F_\theta(Z_\tau)|(Z_0, \vec{I}_0) = (z, \vec{i})\right]\right], \qquad \text{(C7)}$$

where the second equality follows from the independence of the shock and news processes.

Let $M$ be the set of shares, other than $n$, initially owned by holders and let $\tau' = \inf\{t : \exists\, m \in M \ s.t. \ I_t^m = 1\}$. Finally, define the process $\widetilde{Z}$ on $[0, \tau]$ as $\widetilde{Z}_0 = z$ and

$$d\widetilde{Z}_t = \begin{cases} d\hat{Z}_t & \text{for } t < \tau' \\ dZ_t & \text{for } t \geq \tau'. \end{cases}$$

Now consider the two cases at $t = 0$: $\max(\vec{i}) = 1$ (i.e., there exists a seller) and $\max(\vec{i}) = 0$. In the latter, by construction, $Z_\tau = \widetilde{Z}_\tau$. In the former, $Z_\tau = \widetilde{Z}_\tau + Q^\alpha_{\min\{\tau,\tau'\}-}$. That is, given any $\tau, \tau'$, and path of $X$ on $[0, \tau]$, the difference between $Z_\tau$ in the two cases is due to updating based on the information content of the low-type sellers' trading behavior prior to $\min\{\tau, \tau'\}$ in the $\max(\vec{i}) = 1$ case versus no such information in the $\max(\vec{i}) = 0$ case. Notice that this additional information is binary in nature (either the sellers traded or they did not), with one realization (trade) perfectly revealing that $\theta = L$ and the other (no trade) increasing the belief that $\theta = H$. Hence, from (C7) it is sufficient to show that any such signal increases the value of $\mathbb{E}[F_\theta(Z_\tau)|\tau, \widetilde{Z}_\tau]$.

To show this result, it will be convenient to transition back to beliefs as probabilities $p = \frac{e^z}{1+e^z}$. Let $f_\theta$ be the type-$\theta$ seller's value function and $\bar{f}(p) = \mathbb{E}[f_\theta(p)|p]$. For any $p \in (0, 1]$, let $R_p$ be the ray from the point $(0, V_L)$ through the point $(p, \bar{f}(p))$, and for any $p' \leq p$, let $R_p(p')$ be the value such that $R_p$ also passes through $(p', R_p(p'))$. Using standard value-of-information arguments, it suffices to show that, for any pair $p' \leq p$, $\bar{f}(p') \leq R_p(p')$.

This last requirement is established via the following properties of $\bar{f}$ (demonstrated below): (i) $\bar{f}(0) = V_L$, $\bar{f}(1) = V_H$, $\bar{f}(p) \leq \bar{V}(p)$ for all $p$, and $\bar{f}$ is continuous and increasing, (ii) $\bar{f}$ is convex below $p(\beta)$ and concave above $p(\beta)$, and (iii) $\bar{f}'(p) > \bar{V}'(p)$, for all $p > p(\beta)$. Given just (i) and (ii), the result is immediate for $p \leq p(\beta)$. For $p > p(\beta)$, properties (i) and (iii) imply that $\bar{f}$ is steeper than $R_p$ at all points $p' \in [p(\beta), p)$, so $\bar{f} \leq R_p$ in this region. Finally, they cannot intersect below $p(\beta)$, since $\bar{f}$ does intersect $R_{p(\beta)}$ in this region and $R_{p(\beta)}$ lies everywhere below $R_p$ due to (ii) and (iii).

(i) By construction in Appendix B.
(ii) That $\bar{f}$ is convex below $p(\beta)$ is immediate for $p < p(\alpha)$ since $\bar{f}$ is linear in this region.

For $p \in (p(\alpha), p(\beta))$, using the functional form in (IA.1), making the change of variables, and taking the expectation over $\theta$, we get

$$\bar{f} = c_1(1-p)\left(\frac{p}{1-p}\right)^{q_1^L} + c_2(1-p)\left(\frac{p}{1-p}\right)^{q_2^L} + pk_H + (1-p)k_L,$$

where $c_i = C_i^H + C_i^L > 0$. Taking the second derivative gives

$$\bar{f}'' = c_1\frac{1}{p^2}q_1^L\left(q_1^L - 1\right)\frac{\left(\frac{p}{1-p}\right)^{q_1^L}}{1-p} + \frac{1}{p^2}c_2q_2^L\left(q_2^L - 1\right)\frac{\left(\frac{p}{1-p}\right)^{q_2^L}}{1-p} > 0,$$

where the inequality follows from $q_1^L > 1, q_2^L < 0$.

For $p \geq p(\beta)$, recall that $F_L(z) = F_H(z) = B_3(z)$. Making the change of variables from $B_3$ using $C_{31}^B = 0$ (which it must be in $\mathbb{S}^N$), we get that

$$\bar{f}(p) = pV_H + (1-p)V_L + C_{32}^B(1-p)\left(\frac{p}{1-p}\right)^{q_2^L}. \qquad \text{(C8)}$$

Recalling that $C_{32}^B < 0$ and taking the second derivative in $p$ gives the result.

(iii) Follows by taking the first derivative of (C8) and noting that $C_{32}^B, q_2^L < 0$. $\qquad\square$

For the remainder of this appendix we drop the superscript $n$ and let $i$ ($I_t$) denote the status (process) of the owner of an arbitrary share. The following lemma will be used in the proof of Proposition C.4.

LEMMA C.3: *Let* $f : \mathbb{R} \times \{0,1\} \to \mathbb{R}$ *denote an arbitrary function that is twice differentiable in its first argument almost everywhere. Let $A$ denote (infinitesimal) generator of $(Z_t, I_t)$ under $\mathcal{Q}$ (i.e., the public measure). For all states such that $z > \alpha$, we have that*

$$Af(z,i) = \frac{\phi^2}{2}\big((2p(z)-1)f_z(z,i) + f_{zz}(z,i)\big) + (1-i)\lambda(f(z,1) - f(z,0)). \tag{C9}$$

*In the case that $N = 1$, the above also holds for all $z$ when $i = 0$.*

*Proof.* For all such states, (i) $dZ_t = d\hat{Z}_t$ and equation (5) gives $\mathbb{E}[dZ_t | \mathcal{F}_t] = \frac{\phi}{2}(2p(Z_t) - 1)dt$, and (ii) $I_t$ follows a jump process with arrival $\lambda$ and fixed jump size $(1-i)$. The result then follows from Applebaum (2004, Theorem 3.3.3). $\qquad\square$

In the next proposition, we characterize the unscaled trade volume per share $f \equiv \lambda \bar{f}$ and $g \equiv \lambda \bar{g}$.

PROPOSITION C.4: *When $N = 1$, for any $t > 0$, the expected (unscaled) trade volume satisfies*

$$f(t,z): \begin{cases} f = \dfrac{p(\alpha) - p(z)}{p(\alpha)}(1 + \lambda t) + \dfrac{p(z)}{p(\alpha)}f(t,\alpha) & \text{for } z \leq \alpha \\[2mm] f_t = \dfrac{\phi^2}{2}\big[(2(p(z)-1))f_z + f_{zz}\big] & \text{for } z \in (\alpha, \beta) \\[2mm] f = 1 + g & \text{for } z \geq \beta \end{cases} \tag{C10}$$

$$g(t,z): \quad g_t = \lambda(f-g) + \frac{\phi^2}{2}\big[(2(p(z)-1))g_z + g_{zz}\big] \quad \text{for all } z \tag{C11}$$

*with boundary conditions*

$$\lim_{z \to \pm\infty} f(t,z) = 1 + \lambda t \tag{C12}$$

$$\lim_{z \to \pm\infty} g(t,z) = \lambda t$$

*and initial conditions*

$$f(0,z) = \begin{cases} \frac{p(\alpha)-p(z)}{p(\alpha)} & z \leq \alpha \\ 0 & z \in (\alpha,\beta) \\ 1 & z \geq \beta \end{cases} \tag{C13}$$

$$g(0,z) = 0.$$

*When $N = \infty$, the above holds except that (C11) becomes*

$$g(t,z): \begin{cases} g(t,z) = \dfrac{p(\alpha) - p(z)}{p(\alpha)}\lambda t + \dfrac{p(z)}{p(\alpha)}g(t,\alpha) & \text{for } z < \alpha \\[2mm] g_t = \lambda(f-g) + \dfrac{\phi^2}{2}\big[(2(p(z)-1))g_z + g_{zz}\big] & \text{for } z \geq \alpha. \end{cases} \tag{C14}$$

*Proof.* The boundary conditions are uniquely pinned down by equilibrium play; when beliefs are degenerate, trade occurs immediately upon arrival of a shock (Lemma C.2). For the remainder of the proof, we break the state space into four different regions enumerated below. With the exception of the region in which $i = 0$, the arguments below are independent of $N$.

1. For $z \leq \alpha, i = 1$: with probability $\frac{p(\alpha)-p(z)}{p(\alpha)}$ trade occurs ($dv_t = 1$) and the state transitions to $(-\infty, 0)$.

With probability $\frac{p(z)}{p(\alpha)}$ trade does not occur and the state transitions to $(\alpha, 1)$. Therefore,

$$f(t,z) = \frac{p(\alpha) - p(z)}{p(\alpha)} \left( 1 + \lim_{z \to -\infty} g(t,z) \right) + \frac{p(z)}{p(\alpha)} f(t,\alpha) = \frac{p(\alpha) - p(z)}{p(\alpha)} (1 + \lambda t) + \frac{p(z)}{p(\alpha)} f(t,\alpha),$$

where the second inequality follows from the boundary condition on $g$.

2. For $z \in (\alpha, \beta), i = 1$: $d\nu_t = 0$ w.p.1. ($f(0,z) = 0$). Applying the Kolmogorov backward equation (e.g., Applebaum (2004, p. 164)) using the generator from (C9) gives $f_t$ in (C10).

3. For $z \geq \beta, i = 1$: $d\nu_t = 1$ w.p.1. (thus $f(0,z) = 1$) and the new owner is a holder. Thus, $f(t,z) = 1 + g(t,z)$.

4. For $i = 0$:

   (i) When $N = 1$, for all $z$: $d\nu_t = 0$ w.p.1. (thus $g(0,z) = 0$). Again, applying the Kolmogorov backward equation using the generator from (C9) gives $g_t$ in (C11).

   (ii) When $N = \infty$, (i) holds for all $z > \alpha$ yielding the second equation in (C14). For $z < \alpha$, with probability $\frac{p(\alpha) - p(z)}{p(\alpha)}$ another trader sells, the asset type is revealed to be low, and all future trade occurs immediately when the shock arrives (i.e., at rate $\lambda$). With complimentary probability, no other traders sell, in which case $z$ transitions to $\alpha$. Taking the expectation yields the first equation in (C14).

$\square$

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# Notes

[1]Reuters, 2008, Prices nosedive, liquidity dries up.

[2]An implicit assumption is that the market is transparent, meaning agents observe information about all trades. If this assumption fails to hold, the appropriate modeling choice for $N$ would be smaller, reflecting not the total number of shares, but the number of shares agents having information regarding. Under this latter interpretation, one can evaluate regulations, such as the introduction and subsequent extensions of TRACE, aimed at improving transparency through the availability of transaction data. See Section IV.

[3]Zweig, Jason, 2008, Capitulation: When the market throws in the towel  surprisingly, bear markets dont always end with a bang  sometimes its just a whimper, The Wall Street Journal.

[4]See a report from the U.S. Treasury (2009) on the role of fire sales in the recent financial crisis or Shleifer and Vishny (2011) for a survey of the literature on fire sales in financial markets.

[5]See http://www.sec.gov/spotlight/enf-actions-fc.shtml for a list of such allegations.

[6]For commercial real estate, the more relevant characteristics might be population growth or income per capital. Alternatively, the private information could pertain to specific characteristics of the building, with news corresponding to the outcome of inspections, which are required to be publicly disclosed in many states across the U.S.

[7]To flesh this out further, if a cost is associated with listing property for sale and shocks are verifiable, then buyers would be leery of owners who are unable to disclose a credible reason for selling, and in equilibrium, only verifiably constrained owners would bother to list their properties.

[8]If one were interested in applying the model to a systematic component of the market portfolio, it would be natural to incorporate risk-averse traders. See Section VI for comments on this extension.

[9]Recent search-based models in which liquidity is not constant over time include Afonso and Lagos (2012) and He and Milbradt (2014).

[10]For example, the overlapping generations settings of Azariadis (1981), De Long et al. (1990), Spiegel (1998), and others use supply shocks (or noise traders) as a source of risk from which multiple equilibria, including high-volatility ones, emerge as a self-fulfilling prophecy.

[11]For convenience, we assume there are always multiple unconstrained buyers at $t = 1$. This assumption can be motivated by having new (unconstrained) buyers arrive at $t = 1$, or having infinitely many buyers and restricting $\lambda < 1$.

[12]In Section VI, we show that although the predictions are qualitatively similar, the illiquidity problem is further exacerbated if shocks are unobservable.

[13]This restriction has little economic content and is not strictly necessary. For example, alternative proof methods can be used to show that all of the results go through when $\kappa < \delta^2$ under certain conditions on $\sigma$.

[14]See Swinkels (1999) for further discussion of this refinement.

[15]The latter obtains if the security sold at date $t = 0$ and the new owner was not hit by a shock. The former case obtains if either the original owner still owns the security or the security was sold but the new owner experienced a shock.

[16]Payoff-equivalent equilibria exist in which the low-type unconstrained owner engages in non-surplus-enhancing trade at $t = 1$ for a price of $\kappa$. However, the specification that unconstrained owners do not trade mirrors the continuous-time model of subsequent sections, wherein unconstrained owners strictly prefer not to trade whenever the market has any uncertainty regarding $\theta$.

[17]For the knife-edge case of $p_1 = \bar{p}$, both of the above are equilibrium outcomes—but because it is nongeneric, which outcome is specified is irrelevant for behavior or payoffs at $t = 0$. For concreteness, our statements of $F_H, F_L$ in (1) specify the fully liquid outcome at $p_1 = \bar{p}$.

[18]That is, $q(s, p) \equiv \frac{p f_H(s)}{p f_H(s) + (1-p) f_L(s)}$, where $f_\theta$ is the density of $\tilde{s}$ given $\theta$.

[19]Notice that the possibility that $b < a$ contrasts with the structure of equilibria studied in the continuous-time models of DG12 and subsequent sections of this paper, wherein a region with zero probability of trade always exists (i.e., $a < b$). In Section III, we discuss the reasons for this difference. Perhaps surprisingly, $b < a$ arises in the two-period model when the signal-to-noise ratio (i.e., $(\mu_H - \mu_L)/\sigma$) is sufficiently high or sufficiently low, whereas $a < b$ requires intermediate levels of the signal-to-noise ratio.

[20]The proposition focuses on the generic cases in which $p_0 \neq a, b, c$. Analyzing nongeneric cases is not difficult, but their omission simplifies exposition considerably.

[21]Such results include that that prices exhibit excess volatility (Corollary 2), or that resale considerations also influence the low type's threshold (Proposition 4).

[22]We define this ownership process to be left-continuous, meaning that $A_t$ should be interpreted as the owner at the beginning of "period" $t$.

[23]This specification accommodates both additive and proportional holding costs without imposing either. In addition, because agents are risk neutral, nothing substantive changes if the cash flow is random with mean $v_\theta$ or $k_\theta$, depending on the owner's status.

[24]Relying on the rationale that we formalize in Definition 1, because omnipresent buyers compete with one another, there is no uncertainty about the maximum price they are willing to pay following any given history. Therefore, whether buyers bid this price (as in Example 1), the sellers ask for it (as in Example 2, where the bid is not an explicit part of agent strategies), or a market maker facilitates the transaction (as in Example 3) makes no difference for equilibrium outcomes. In all cases, a seller simply decides when to *stop*, at which point she is paid the buyers' value for her share (conditional on her stopping, of course), regardless of whether stopping translates into accepting an offer from a buyer or market maker, or to posting a price equal to the buyers' (conditional) value for a share, which will then be accepted.

[25]Like Duffie, Gârleanu, and Pedersen (2005, 2007), this model of over-the-counter markets features decentralized trading, however, there are no search frictions.

[26]One might argue that it is unreasonable to think agents can keep track of all the information in the public history. This assumption is not crucial. The equilibrium we construct and analyze is stationary; all relevant information prior to time $t$ will be encapsulated in a simple state variable (Section III).

[27]For any $t, \mathcal{F}_t, \lim_{h \to \infty} \mathbb{E}[e^{-rh} W_{t+h} | \mathcal{F}_t] = 0$. See Brunnermeier (2008) for a discussion of how the failure of this

condition can lead to "bubbles."

[28]In general, $Q$ is pinned down along the equilibrium path by Bayes rule; however, writing this process for arbitrary times, given arbitrary strategies, is a cumbersome exercise that provides little insight beyond that found for the case derived in (6). Note also that like the ownership process (footnote 22), the belief process is a left-continuous process corresponding to the interpretation that $Z_t$ is the belief at the beginning of "period" $t$. Hence, the usage of the left-limits (i.e., $S_{h-}^{\theta,t} \equiv \lim_{s \uparrow h} S_s^{\theta,t}$) in (6).

[29]We use $(z, i)$ to refer to any $(t, \omega)$ such that $(Z_t(\omega), I_t(\omega)) = (z, i)$. References to generic $z$ should be understood as $z \in \mathbb{R}$, as opposed to the degenerate belief levels $z = \pm\infty$, unless otherwise stated. In any equilibrium, after reaching a degenerate belief at time $t$, $Z_t \in \{\pm\infty\}$, Lemma C.2 shows that on-path continuation play must be as follows. For all $h \geq t$, $Z_h = Z_t$, $W_h = E[V_\theta | Z_h]$ if $I_h = 1$, and sellers accept with probability one if $W_h \geq K_\theta$, and reject otherwise.

[30]To see this, notice that in $\Xi(\alpha, \beta, B)$, low types mix between accepting and rejecting at $z = \alpha$. Therefore, *Owner Optimality* will require low types to be indifferent between these strategies, including the one that *always* rejects at $z = \alpha$ (i.e., playing according to $T(\beta, t)$).

[31]The Internet Appendix may be found in the online version of this article on the Journal of Finance website.

[32]See DG12 for additional restrictions under which $\Xi$ is the unique equilibrium when $\lambda = 0$.

[33]For example, assume that cash flows and the news process are synonymous (the analysis is unchanged by stochastic cash flows; see footnote 23) and that a seller incurs an additive holding cost that is constant across type: $v_H - k_H = v_L - k_L$. This specification ensures that an owner does not receive any private information from ownership.

[34]If one wishes to avoid an infinite total cash flow when $N = \infty$, $v_\theta, k_\theta$ can be interpreted as "infinitesimal" quantities (see Anderson (2008)).

[35]In July 2002, FINRA introduced regulation to improve the transparency of corporate bond markets by requiring all member broker/dealers to report corporate bonds transactions to TRACE, which then makes transaction data publicly available. More recently, TRACE reporting requirements have extended to a broader class of fixed-income securities.

[36]When $N$ is finite, for $z \in (\alpha, \beta)$, the presence or absence of sellers affects the probability that sellers will be present the next time $z$ reaches $\alpha$, so it also affects the holder value function. However, when $N = \infty$, the probability that there will be sellers present the next time that $\alpha$ is reached is one, regardless of the number of sellers today, so the contingency is again irrelevant.

[37]To complete the mapping, note that our model is isomorphic to one in which the asset type is revealed randomly (i.e., default or prepayment occurs) according to a Poisson arrival with the same arrival rate for either event.

[38]Because all traders are risk neutral in our economy, the physical and risk-neutral measures are identical.

[39]Notice that $\mathcal{L}^F$ and $\mathcal{L}^G$ coincide with the illiquidity discount, $D$, when $z \geq \beta$, because $F_L(z) = F_H(z) = B(z) = E[G_\theta(z)|z]$ in this region.

[40]For ease of exposition, Figure 7 only depicts $\mathcal{L}^G$. The results for $\mathcal{L}^F$ are similar.

[41]Perhaps surprisingly, our numerical results indicate that not only does efficiency increase with $N$, but so too do

seller and holder values, regardless of the asset's type. The additional information generated by greater $N$ causes arbitrary shares to be efficiently allocated more of the time. This, in turn, increases the prices buyers are willing to pay, which benefits even low-type owners.

[42]No qualitative differences arise when using a proportional holding cost structure (i.e., $k_\theta = \delta v_\theta$).

[43]We continue to refer to the owner as a seller if she is constrained, and as a holder if she is unconstrained despite the fact that a low-type holder does sell in equilibrium.

[44]An intuitive feature to note is that as $\lambda \to 0, 1$, whether shocks are observable makes no difference, because then buyers would then have little uncertainty regarding whether the owner is constrained.

[45]The public history at $t = 0$ is empty. The public history at $t = 1$ includes whether trade occurred at $t = 0$, if so then whether a shock arrived, and (in any case) the realization of the signal.

[46]We will later show that $B$ is increasing, continuous, and differentiable almost everywhere. These properties are not required for the present analysis, but may help provide intuition for the arguments.

[47]We use $\vec{i}$ to denote the vector of owners' statuses (i.e., the generalization of $i$ in the single share model).

[48]That the *killing rate* is $\kappa = 1$ follows from the definition of $Q^\alpha$ in $\Xi^N$.

[49]The necessity of high-type-seller indifference at $\beta$, and therefore (B12), hinges on the specification of off-equilibrium-path beliefs imposed by $\Xi^N$. Regardless of this specification, the weaker condition $F_H'(\beta^-) \leq B'(\beta)$ is necessary. Equilibria in which $F_H'(\beta^-) < B'(\beta)$ can be sustained only by imposing "threat beliefs" for off-equilibrium-path rejections (i.e., the probability assigned to a high type *decreases* following an unexpected rejection). The requirement that beliefs cannot decrease following an unexpected rejection makes (B12) necessary.
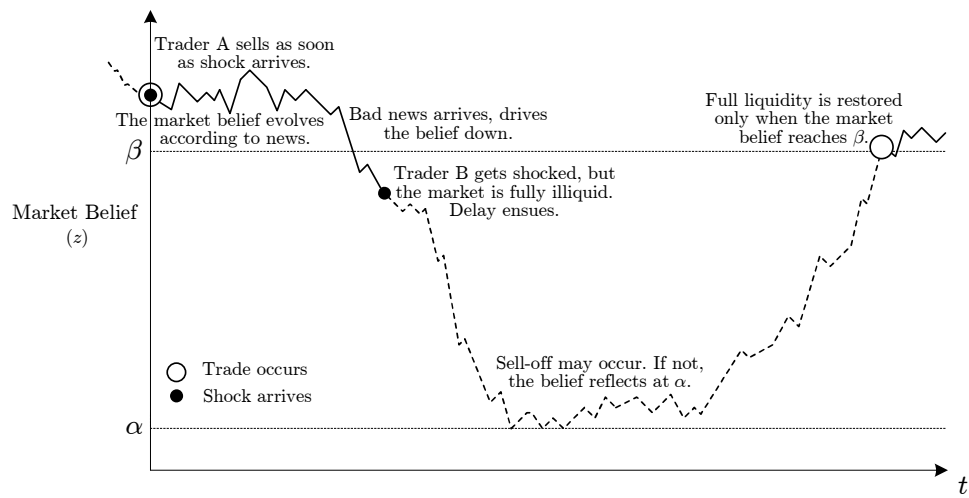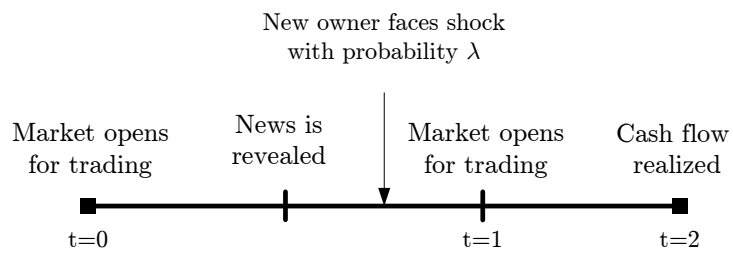
Figure 1. **Sample path of market liquidity dynamics.**

New owner faces shock
with probability $\lambda$

Market opens
for trading

News is
revealed

Market opens
for trading

Cash flow
realized

t=0          t=1          t=2

**Figure 2. Timeline of the two-period model.**
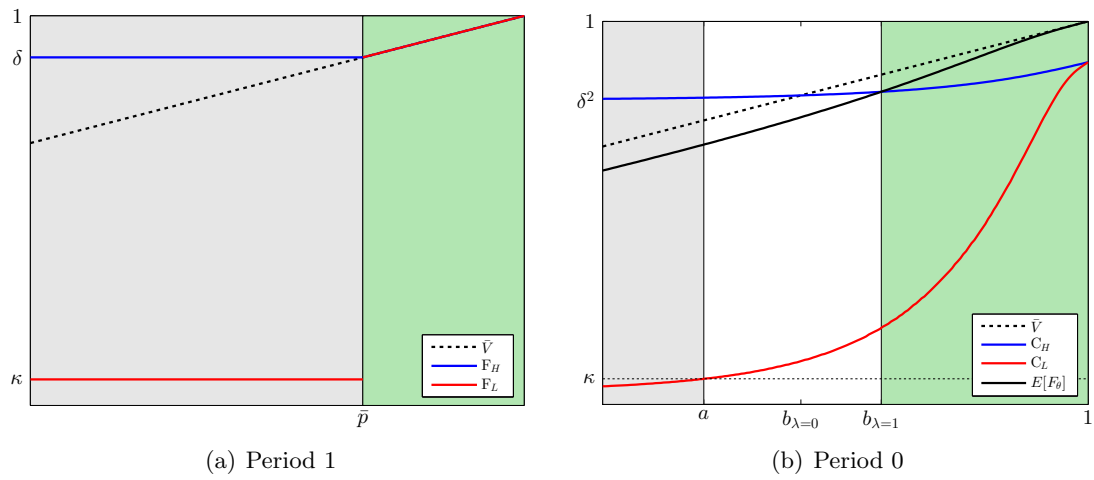
(a) Period 1

(b) Period 0

**Figure 3. Equilibrium trade regions and continuation values in each period.** The leftmost region (shaded in grey) corresponds to the partially liquid region. The rightmost region (shaded in green) corresponds to the fully liquid region. The middle region (shaded in white) in Figure 3(b) corresponds to the fully illiquid region in the case $\lambda = 1$. In the rightmost region of Figure 3(a) all three functions, $\bar{V}$, $F_H$, and $F_L$ coincide.
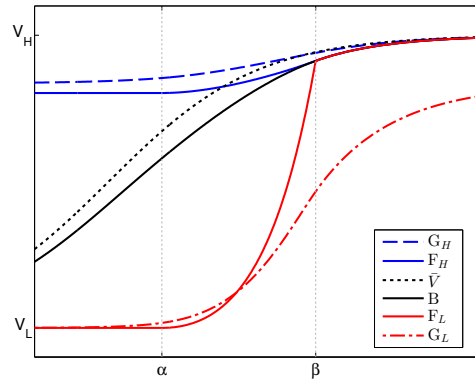
**Figure 4.** $\Xi$-equilibrium value functions as they depend on the underlying market belief ($z$).
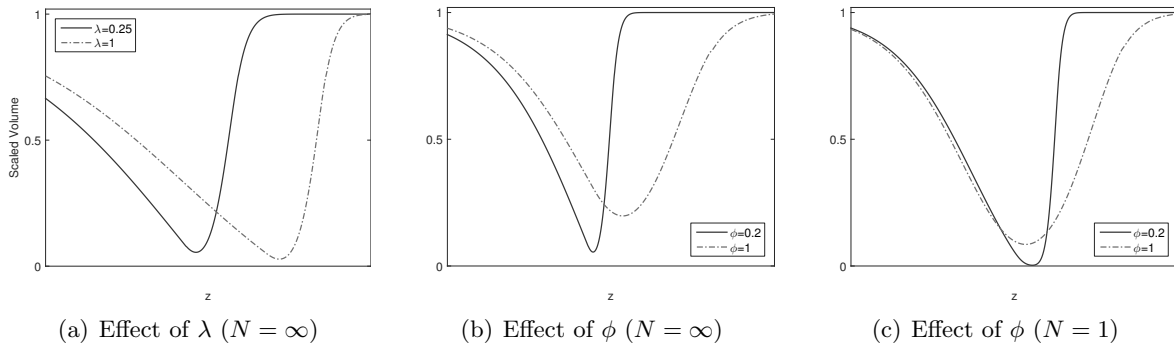
Figure 5. **Scaled volume**. This figure plots the scaled holder volume over a unit interval of time (i.e., $\bar{g}(1,z)$). Note that the first-best efficient trade volume is equal to one for all $z$.
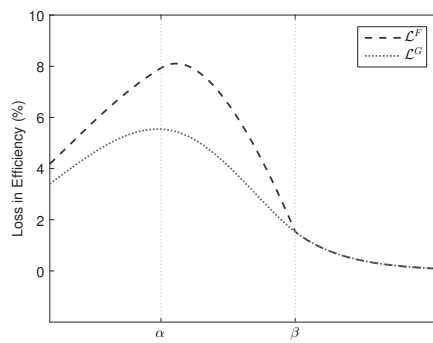
**Figure 6. Efficiency loss.**

(a) $\mathcal{L}^G$ as it varies with $N$

(b) $\mathcal{L}^G$ as it varies with $\phi$

(c) $\mathcal{L}^G$ as it varies with $\lambda$

(d) $\mathcal{L}^G$ as it varies with holding cost
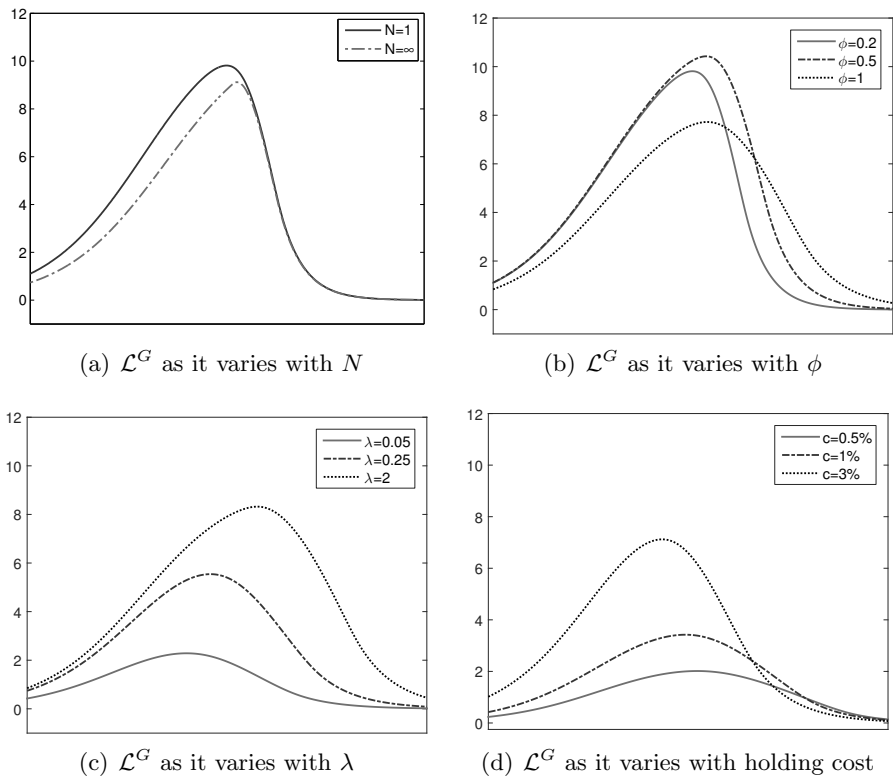
**Figure 7. Efficiency loss as it varies with $N$, $\phi$, $\lambda$, and holding costs.**

**Table I: Parameters**

This table presents the parameters used in all numerical results

|  | $N = 1$ | $N = \infty$ |
| --- | --- | --- |
| $r$ | 5% | 5% |
| $v_H$ | 0.1 | 0.035 |
| $k_H$ | 0.08 | 0.03 |
| $v_L$ | 0.05 | 0.03 |
| $k_L$ | 0.03 | 0.025 |

**Table II: Illiquidity Regions**

This table reports the upper boundary ($b$) and the lower boundary ($a$) of the illiquid region for each set of parameters.

| Arrival Rate | News Quality | $N = 1$ | | $N = \infty$ | |
|---|---|---|---|---|---|
| ($\lambda$) | ($\phi$) | $b$ | $a$ | $b$ | $a$ |
| 0.25 | 0.20 | 0.890 | 0.754 | 0.639 | 0.503 |
| 1.00 | 0.20 | 0.939 | 0.853 | 0.819 | 0.713 |
| 0.25 | 0.50 | 0.942 | 0.672 | 0.837 | 0.549 |
| 1.00 | 0.50 | 0.965 | 0.776 | 0.907 | 0.694 |

**Table III: Liquidity Premium and Excess Volatility**

This table reports the illiquidity discount ($D$) and the excess volatility as given by the ratio of equilibrium volatility to fundamental volatility ($\sigma_e/\sigma_f$) for each of the parameter configurations.

| Arrival Rate | News Quality | $N = 1$ | | $N = \infty$ | |
|---|---|---|---|---|---|
| ($\lambda$) | ($\phi$) | $D$ | $\sigma_e/\sigma_f$ | $D$ | $\sigma_e/\sigma_f$ |
| 0.25 | 0.20 | 0.037 | 2.548 | 0.019 | 2.038 |
| 1.00 | 0.20 | 0.051 | 4.852 | 0.031 | 3.954 |
| 0.25 | 0.50 | 0.015 | 1.707 | 0.009 | 1.550 |
| 1.00 | 0.50 | 0.024 | 2.840 | 0.016 | 2.609 |