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Mathematics and Theory in Behavior Analysis: Remarks on Catania (1981), "The Flight From Experimental Analysis"

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Response rate is often a useful measure of behavior, but response rate is far from the only useful measure. Relative measures may produce not only order but deeper insight into behavior. Indeed, response rate, seen in proper perspective, is itself a relative measure.

The chief mark of a mature science is its inclusion of general principles and data-tested theories stated precisely in mathematical terms. Every science that we consider mature has produced such precise mathematical general principles and theories (e.g., relativity theory in physics, laws of thermodynamics in chemistry, the Price Equation in evolutionary biology). The testing of general principles and theories is as much experimental analysis as is the manipulation of guessed-at independent variables that might affect response rate. Indeed, Catania (1981) in his chapter suggests a few experimental analyses that might apply to studies of choice.

Herrnstein's (1961) report of the matching relation began a new era in behavior analysis. Here was a quantitative relation that could be tested for generality and examined in detail for its underlying processes. The relation was first stated in terms of proportions:

$$\frac{B_1}{B_1+B_2} = \frac{r_1}{r_1+r_2} \qquad (1)$$

The variables B1 and B2 denote behavior allocated to Alternatives 1 and 2, the variables r1 and r2 denote the reinforcers provided by Alternatives 1 and 2. The proportion of behavior approximately matches the proportion of reinforcers obtained.

Later, the relation was generalized to accommodate two types of deviation from strict matching:

$$log\frac{B_1}{B_2} = slog\frac{r_1}{r_2} + logb \qquad (2)$$

In Equation 2, the parameter *b* represents any constant bias toward one alternative or the other, and the parameter *s* represents the sensitivity of the choice measure to variation in the reinforcer ratio (Baum, 1974). If *b* and *s* both equal 1.0, strict matching holds, but various factors may cause either of them to deviate from 1.0—factors that can be understood as the result of experimental analysis. Equation 2 has great empirical support (Baum, 1979).

When Equation 2 is generalized to include *n* alternatives, instead of just 2, the result is:

$$logB_i - \frac{1}{n}\sum_{j=1}^{n} logB_j = s_i logr_i + logb_i - \\ - \frac{1}{n}\sum_{j=1}^{n} s_j logr_j - \frac{1}{n}\sum_{j=1}^{n} logb_j \qquad (3)$$

In Equation 3, now each alternative has its own parameters *b* and *s*, and it applies to any alternative out of the *n* activities. Usually sensitivity *s* is considered constant across alternatives (Schneider & Davison, 2005). If *n* equals 2, Equation 3 reduces to Equation 2.

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The number of alternatives n can never equal less than 2, because no situation can be so impoverished as to allow only one possible activity (Herrnstein, 1970). Even if only one activity (e.g., pecking) is recorded, other activities (e.g., grooming, moving about, etc.) also occur.

Finally, when we generalize Equation 3 to variables besides reinforcer rate, we may represent all the variables that affect choice with an aggregate variable v, given by:

$$v_i = \prod_{k=1}^{m} b_{ki} x_{ki}^{s_k} \qquad (4)$$

In Equation 4, the m variables denoted by x are factors like reinforcer rate, amount, and immediacy (a hyperbolic relation to delay). The exponent is considered constant for each variable . Expressing the choice relation in its most general form:

$$logB_i - \frac{1}{n}\sum_{j=1}^{n} logB_j = logv_i -$$
$$-\frac{1}{n}\sum_{j=1}^{n} logv_j \qquad (5)$$

Relative behavior matches relative value v. Equations 2 and 3 are special cases of Equation 5, for reinforcer rate with n alternatives (Equation 3) and with 2 alternatives (Equation 2).

Equation 5 may be considered a tautology, because it takes the form of a law—the Law of Choice—but it is a useful tautology (Rachlin, 1971). *Choice* here refers to the allocation of behavior among the multiple activities that occur in a situation. It is most simply thought of as allocation of time among multiple activities (Baum & Rachlin, 1969). Equation 5 provides a framework within which to think about behavior, both in the laboratory and in everyday life, far superior to the traditional view of behavior as consisting of discrete responses followed by contiguous reinforcers.

The ubiquity of multiple alternative activities gives rise to a general principle: *Behavior is choice*.

That behavior is choice in this sense arises from the matching relation, because no activity—no response rate—exists at a moment; activities and response rates must be assessed over some span of time. From a snapshot, you cannot tell what activity is occurring; only over a sufficient time sample can an activity be identified (Baum, 2002, 2004).

The second insight afforded by the matching relation is that activities have *scale*. The parts of any activity are themselves activities, but on a smaller time scale than the activity of which they are a part (Baum, 2002, 2004). The parts of driving to work, for example, are matters like stopping at intersections, turning, driving down one street or another, slowing in a school zone, and so on (Wallace, 1965). The parts of a concurrent performance are responding on one alternative, responding on the other alternative, and engaging in other, unscheduled, activities.

Response rate is itself an allocation of time. If it is calculated as N/T, where N is number of responses and T is total time, then we can convert it to a time allocation simply by multiplying N by the time required for each response t. Then Nt/T is the proportion of time devoted to the activity of responding (pressing, pecking, or whatever). The equivalence of responses and time has been confirmed empirically (Aparicio & Baum, 1997; Baum, 1976; Dallery & Baum, 1991).

If we were to focus only on response rate and avoid seeking for general principles, we would end up with just a list or catalogue of results, variables that affect response rate and how they do so. That would be a dull science indeed! Even Skinner (1950), while disparaging the learning theories of the time, allowed the possibility of another kind of theory, one that would emerge from data. We have arrived at such a theory.

The matching relation is a useful framework within which to think about behavior, but it is not the final word. In the simple laboratory preparations that we study, we can examine the more basic relations and patterns that may give rise to the experimental result of matching (Baum, 2010; Baum & Davison, 2004; Baum, Schwendiman, & Bell, 1999; Davison & Baum, 2000). That endeavor will take us far beyond the limitations of studying just response rate.

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