UC Irvine

UC Irvine Previously Published Works

Title

Network oriented dyadic time domain green's function for a sequentially excited infinite planar array of dipoles in free space

Permalink

<https://escholarship.org/uc/item/2mb511f1>

Journal

IEEE Transactions on Antennas and Propagation, 55(8)

ISSN

0018-926X

Authors

Capolino, F
Felsen, LB
Della Villa, A

Publication Date

2007-08-01

DOI

10.1109/TAP.2007.901841

Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, available at <https://creativecommons.org/licenses/by/4.0/>

Peer reviewed

Network Oriented Dyadic Time Domain Green's Function for a Sequentially Excited Infinite Planar Array of Dipoles in Free Space

Filippo Capolino, *Senior Member, IEEE*, Leopold B. Felsen, *Fellow, IEEE*, and Alessandro Della Villa

This paper is dedicated to Prof. Leopold B. Felsen, mentor and friend. The paper contains several of his ideas and he checked and wrote parts of it. Unfortunately he did not see the final version.

Abstract—During the past few years we have performed an effective wideband analysis for characterizing the electrodynamic behavior of phased array antennas, infinite periodic structures, frequency selective surfaces and related applications, with emphasis on gaining physical insight into the phenomenology of short-pulse radiation. The present contribution shows the current status of our network-oriented *dyadic* time domain Green's function (TD-GF) for a planar array of sequentially excited dipoles that constitutes a prototype study of sequentially short-pulsed radiation by infinite periodic arrays. The dispersive effects of the TD-Floquet waves (FW) and the consequences of a TE and TM field decomposition are discussed in details. In this network formulation, the equivalent TD transmission line (TL) voltages and currents excited by equivalent TD current generators are determined. An expression for the TM voltage TL-GF is provided for the first time in terms of incomplete Lipschitz-Hankel integrals and Bessel functions together with some remarks for its evaluation. As shown in previous publications for the scalar potentials, real-TD physical observables are determined by pairing TD Floquet waves with positive and negative indices. It is shown that the vectorial TD radiated field is reconstructed at any time with the superposition of a few TD-FWs.

Index Terms—Arrays, Green function (GF), periodic structures, short pulse radiation, time domain (TD) analysis.

I. INTRODUCTION

ENGINEERING interest in ultrawideband/short pulse phenomena pertaining to phased periodic planar arrays (also relevant in the contest of multilayer metamaterials), suggests that an analytic framework parameterized directly in the time domain (TD) might lead to better problem-matched physics, and

thereby to better numerical convergence. Moreover, a network (transmission line) approach has conventionally been utilized in the analysis and design of such structures. These circumstances have motivated the present investigation of a TD network-oriented representation of the vector electromagnetic field radiated by an infinite periodic array of sequentially-pulsed electric dipoles, that constitutes a prototype study of sequentially short-pulsed radiation by infinite periodic arrays.

We have previously investigated canonical TD dipole-excited Green's functions (GF) for infinite [1] and truncated [2] periodic line arrays, and for infinite [3] and semi-infinite [4] periodic planar arrays. The radiated field there has been expressed and parameterized in terms of a few TD Floquet waves (FWs). A generalized frequency domain (FD)-TD transform for linear arrays of dipoles is in [5]. The resulting *scalar* TD-GF has already been used advantageously to construct a fast TD method of moments algorithm for wideband analysis of infinite periodic structures [6].

A principal feature of the network-oriented approach is that E (TM) and H (TE)-type TD-FW modes can be separated and treated individually. The field is expressed in terms of TD transmission line (TL) Green's functions that obey standard network theory. Therefore, possible infinite planar vertically inhomogeneous media may readily be incorporated into the formalism. Interesting causality issues accompany such E and H mode decompositions: it is found that individually, each E and H mode is *noncausal*. Causality on the total TD-FW vector mode field is recovered by summing the E and H mode contributions. In order to parameterize the TD-FW behavior, we begin with the solution in the FD, with subsequent inversion to the TD. The formal aspects of the analysis follow the traditional lines in [7], to which we refer frequently. Thus, the transverse p, q th vector FW mode fields are expressed in terms of transverse FW-mode scalar eigenfunctions. Longitudinal field propagation is described by voltage and current TL-GFs, Z_{pq} and T_{pq}^I ; in the TD, these TL-GFs can be evaluated in terms of Bessel functions and incomplete Lipschitz-Hankel integrals (see [8] for a summary of the main features for the scalar case combined with a preliminary analysis for the vectorial case). Numerical examples of radiation from infinite planar arrays of dipoles with short-pulse band-limited excitation demonstrate the accuracy of the TD-FW algorithm, and illustrate the rapid convergence of the (TD-FW)-based field representation since only a few FW terms are required for describing the off-surface field radiated by the planar array. Results for the case of an array simultaneously excited (nonphased) have already been used in a combined

Manuscript received June 9, 2006; revised December 31, 2006. This work was supported by the EU-funded project METAMORPHOSE (FP6/NMP3-CT-2004-500252).

F. Capolino and A. Della Villa are with the Department of Information Engineering, University of Siena, Via Roma 56, 53100 Siena, Italy (e-mail: capolino@diu.unisi.it).

L. B. Felsen, (*deceased*), was with the Department of Aerospace and Mechanical Engineering and the Department of Electrical and Computer Engineering, Boston University, Boston, MA 02215 USA, and also with Polytechnic University, Brooklyn, NY 11201 USA.

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TAP.2007.901841

Fig. 1. Geometry of an infinite planar periodic array of dipoles. d_x, d_y : interelement spacing along x and y , respectively. $\eta\omega/c = k\eta$: phase gradient of the excitation (i.e., the wavefront) along the direction $\mathbf{1}_u$ (see [3]). $v_u^{(p)} = c/\eta$: phase speed along $\mathbf{1}_u$.

(TD-FW)-FDTD algorithm, shown in [9], for the analysis of periodic arrays of complex scatterers.

The results of this paper lay the foundation to include layered media in the TD-FW analysis to widen the TD-FW applications already shown in [6] and [9].

The nonphased case (all array dipoles simultaneously excited) of the TD analysis shown in this paper is strictly related to the TD analysis of wave propagation in closed waveguides (see, for instance, [10]).

II. STATEMENT OF THE PROBLEM

The canonical problem for the network formulation under study consists of the generic infinite periodic array of electric dipoles, shown in Fig. 1, oriented along the constant vector $\mathbf{J}_t$, with periodicities d_x and d_y along the x and y directions, respectively. The observation point is denoted by $\mathbf{r} = \boldsymbol{\rho} + z\mathbf{1}_z$, with transverse coordinate $\boldsymbol{\rho} = x\mathbf{1}_x + y\mathbf{1}_y$. The m th dipole source in the periodic array is located at $\mathbf{r}' + \boldsymbol{\rho}_{mn}$, where $\mathbf{r}' = \boldsymbol{\rho}' + z'\mathbf{1}_z$, $\boldsymbol{\rho}' = x'\mathbf{1}_x + y'\mathbf{1}_y$ and $\boldsymbol{\rho}_{mn} = md_x\mathbf{1}_x + nd_y\mathbf{1}_y$. Here, bold face symbols define vector quantities and $\mathbf{1}_x, \mathbf{1}_y, \mathbf{1}_z$ denote unit vectors along x, y, z , respectively. The FW-based modal FD and TD fields due to the array are related by the Fourier transform pair

$$\begin{aligned} f(\mathbf{r}, \omega) &= \int_{-\infty}^{\infty} \hat{f}(\mathbf{r}, t) e^{-j\omega t} dt \\ \hat{f}(\mathbf{r}, t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\mathbf{r}, \omega) e^{j\omega t} d\omega \end{aligned} \quad (1)$$

in which f can be either a scalar or a vector quantity; a caret $\hat{}$ denotes time-dependent quantities.

The phased FD and sequentially pulsed TD dipole array currents $J(\mathbf{r}, \omega)$ and $\hat{J}(\mathbf{r}, t)$, respectively, are given by

$$\left. \begin{aligned} J(\mathbf{r}, \omega) \\ \hat{J}(\mathbf{r}, t) \end{aligned} \right\} = \sum_{m,n=-\infty}^{\infty} \delta[\mathbf{r} - (\mathbf{r}' + \boldsymbol{\rho}_{mn})] \left\{ \begin{aligned} &e^{-j\eta k \mathbf{1}_u \cdot \boldsymbol{\rho}_{mn}} \\ &\delta(t - \eta \mathbf{1}_u \cdot \frac{\boldsymbol{\rho}_{mn}}{c}) \end{aligned} \right\} \quad (2)$$

where $\delta[\mathbf{r} - (\mathbf{r}' + \boldsymbol{\rho}_{mn})] = \delta[x - (x' + md_x)]\delta[y - (y' + nd_y)]\delta[z - z']$, and $\mathbf{1}_u \eta = \eta_x \mathbf{1}_x + \eta_y \mathbf{1}_y$. Moreover, $k = \omega/c$ denotes the

ambient wavenumber, $c = 1/\sqrt{\epsilon\mu}$ (with ϵ and μ representing the ambient permittivity and permeability) denotes the ambient wave speed, and $\omega\eta_x/c$ and $\omega\eta_y/c$, with

$$\eta_x = \eta \cos \phi_u, \quad \eta_y = \eta \sin \phi_u \quad (3)$$

are the interelement phase gradients along x and y , respectively [3]. In the FD, the composite linear phasing on the array is along the direction $\mathbf{1}_u$, perpendicular to $\mathbf{1}_v = \mathbf{1}_z \times \mathbf{1}_u$ (see Fig. 1), and rotated through the angle ϕ_u with respect to the x axis. In the TD, this translates into interelement excitation delayed by $\eta_x d_x/c$ and $\eta_y d_y/c$ along x and y , respectively; i.e., the sequentially pulsed dipole elements located at $\mathbf{r} = \mathbf{r}' + \boldsymbol{\rho}_{mn}$ are turned on at time $t_{mn} = \eta \mathbf{1}_u \cdot \boldsymbol{\rho}_{mn}/c$. The important nondimensional *single* parameter

$$\eta = \sqrt{\eta_x^2 + \eta_y^2} = \frac{c}{v_u^{(p)}} \quad (4)$$

combines both phasings η_x and η_y [3]. In (4), $v_u^{(p)} = c/\eta$ is the impressed phase speed along u . Here, we treat the case $\eta < 1$ which implies excitation phase speeds $v_u^{(p)} = c/\eta$ (and corresponding *projected* phase speeds c/η_x and c/η_y) *larger* than the ambient wavespeed c [1], [3]. The nonphased case $\eta = 0$ corresponds to simultaneous excitation of all the dipoles.

III. FD GREEN'S FUNCTION: DECOMPOSITION INTO TE AND TM MODES AND PREPARATION FOR FOURIER INVERSION

The FD electric and magnetic vector fields $\mathbf{E}(\mathbf{r}, \mathbf{r}', \omega)$ and $\mathbf{H}(\mathbf{r}, \mathbf{r}', \omega)$ at location $\mathbf{r}$, excited by the array of time-harmonic transverse dipole sources given in (2), oriented along $\mathbf{J}_t$, are represented as a sum of FW modes

$$\mathbf{E}(\mathbf{r}, \mathbf{r}', \omega) = \sum_{pq} \mathbf{E}_{pq}(\mathbf{r}, \mathbf{r}', \omega) \quad (5)$$

$$\mathbf{H}(\mathbf{r}, \mathbf{r}', \omega) = \sum_{pq} \mathbf{H}_{pq}(\mathbf{r}, \mathbf{r}', \omega). \quad (6)$$

Each pq th indexed FW is explicitly separated into its E and H parts

$$\mathbf{E}_{pq}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{E}_{pq}^H(\mathbf{r}, \mathbf{r}', \omega) + \mathbf{E}_{pq}^E(\mathbf{r}, \mathbf{r}', \omega) \quad (7)$$

$$\mathbf{H}_{pq}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{H}_{pq}^H(\mathbf{r}, \mathbf{r}', \omega) + \mathbf{H}_{pq}^E(\mathbf{r}, \mathbf{r}', \omega). \quad (8)$$

The steps detailed in Appendix A lead to the following convenient form for the *transverse* components $\mathbf{E}_{t,pq}$ and $\mathbf{H}_{t,pq}$ of the modes $\mathbf{E}_{pq}$ and $\mathbf{H}_{pq}$

$$\mathbf{E}_{t,pq}^E = \mathbf{k}_{t,pq}(\mathbf{k}_{t,pq} \cdot \mathbf{J}_t) F_{pq} Z_{pq}^E(z, z', \omega) \quad (9)$$

$$\mathbf{E}_{t,pq}^H = (\mathbf{k}_{t,pq} \times \mathbf{1}_z)[(\mathbf{k}_{t,pq} \times \mathbf{1}_z) \cdot \mathbf{J}_t] F_{pq} Z_{pq}^H(z, z', \omega) \quad (10)$$

and

$$\mathbf{H}_{t,pq}^E = (\mathbf{k}_{t,pq} \times \mathbf{1}_z)[\mathbf{k}_{t,pq} \cdot \mathbf{J}_t] F_{pq} T_{pq}^I(z, z', \omega) \quad (11)$$

$$\mathbf{H}_{t,pq}^H = \mathbf{k}_{t,pq}[(\mathbf{k}_{t,pq} \times \mathbf{1}_z) \cdot \mathbf{J}_t] F_{pq} T_{pq}^I(z, z', \omega). \quad (12)$$

Fig. 2. Frequency and time domain FW-based transmission line (TL) voltage and current Green's functions excited by a unit current generator at z' .

where the $F_{pq}(\omega)$ factor is given by

$$F_{pq}(\omega) = \frac{e^{-j\mathbf{k}_{t,pq} \cdot (\boldsymbol{\rho} - \boldsymbol{\rho}')}}{d_x d_y k_{t,pq}^2}. \quad (13)$$

The present analysis is limited to transverse-to- z dipoles, the simpler case with dipoles oriented along the z axis can be treated analogously. Here, $Z_{pq}^E(z, z', \omega)$ and $Z_{pq}^H(z, z', \omega)$ are the E and H voltages at z excited by a delta current generator at z' (see Fig. 2), while $T_{pq}^I(z, z', \omega)$ is the current at z excited by the same generator [7, Sec. 2.3c]. The vector wavenumber

$$\mathbf{k}_{t,pq}(\omega) = k_{xp}\mathbf{1}_x + k_{yq}\mathbf{1}_y = \eta k \mathbf{1}_u + \boldsymbol{\alpha}_{pq} \quad (14)$$

$$\boldsymbol{\alpha}_{pq} = \alpha_{xp}\mathbf{1}_x + \alpha_{yq}\mathbf{1}_y \quad (15)$$

combines the two Floquet-type dispersion relations

$$k_{xp}(\omega) = \eta k \cos \phi_u + \alpha_{xp}, \quad \alpha_{xp} = \frac{2\pi p}{d_x} \quad (16)$$

$$k_{yq}(\omega) = \eta k \sin \phi_u + \alpha_{yq}, \quad \alpha_{yq} = \frac{2\pi q}{d_y} \quad (17)$$

with $p, q = 0, \pm 1, \pm 2, \dots$. The subscript “ t ” on $\mathbf{k}_{t,pq}$ denotes the vector component transverse to z , and $\boldsymbol{\alpha}_{pq}$ represents the ω -independent part of the vector dispersion relation in (14). Floquet waves with transverse propagation constants $k_{t,pq} < k$ or $k_{t,pq} > k$, with $k_{t,pq} = (k_{xp}^2 + k_{yq}^2)^{1/2}$, are propagating or evanescent, respectively, along the z -direction (we have assumed a lossless environment).

In (9)–(12) [as well as in (64)–(68)] we have introduced the FD admittance and voltage TL Green's functions

$$\left. \begin{array}{l} Z_{pq}(z, z', \omega) \\ Y_{pq}(z, z', \omega) \\ T_{pq}^I(z, z', \omega) \end{array} \right\} = \frac{1}{2} \left\{ \begin{array}{l} Z_{0,pq}(\omega) \\ \frac{1}{Z_{0,pq}(\omega)} \\ 1 \end{array} \right\} e^{-jk_{z,pq}(\omega)|z-z'|} \quad (18)$$

shown in Fig. 2, where the longitudinal Floquet wavenumber is

$$\begin{aligned} k_{z,pq}(\omega) &= \sqrt{\left(\frac{\omega}{c}\right)^2 - k_{t,pq}^2(\omega)} = \\ &= \frac{\sqrt{1-\eta^2}}{c} \sqrt{(\omega - \bar{\omega}_{pq})^2 - \tilde{\omega}_{pq}^2} \end{aligned} \quad (19)$$

Fig. 3. Topology of the ω plane for the evaluation of the inverse Fourier transform.

with definitions

$$\bar{\omega}_{pq} = \frac{\eta c}{1 - \eta^2} \mathbf{1}_u \cdot \boldsymbol{\alpha}_{pq} \quad (20)$$

$$\tilde{\omega}_{pq} = \frac{c}{1 - \eta^2} \sqrt{\alpha_{pq}^2 - \eta^2(\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v)^2}. \quad (21)$$

The quantity $Y_{pq}(z, z', \omega)$ represents the current at z produced by a voltage generator at z' , and it is here used in the derivation in Appendix A. The top Riemann ω -sheet in Fig. 3 is defined by $\text{Im}k_{z,pq}(\omega) < 0$, and $\text{Re}k_{z,pq}(\omega) \geq 0$ or ≤ 0 for $\omega > 0$ or $\omega < 0$, respectively [7, p. 207], [1], [3]. The characteristic impedances for E and H modes are

$$Z_0^E(\omega) = \frac{k_{z,pq}(\omega)}{\omega \epsilon}, \quad Z_0^H(\omega) = \frac{\omega \mu}{k_{z,pq}(\omega)}. \quad (22)$$

The z -components of the fields can be determined from the transverse components.

IV. TD GREEN'S FUNCTION: FOURIER INVERSION OF THE FD MODAL EXPANSION

The total TD field is obtained as a superposition of the TD-FWs

$$\hat{\mathbf{E}}(\mathbf{r}, \mathbf{r}', t) = \sum_{pq} \hat{\mathbf{E}}_{pq}(\mathbf{r}, \mathbf{r}', t) \quad (23)$$

$$\hat{\mathbf{H}}(\mathbf{r}, \mathbf{r}', t) = \sum_{pq} \hat{\mathbf{H}}_{pq}(\mathbf{r}, \mathbf{r}', t) \quad (24)$$

obtained by Fourier-inverting the FD modes in (5) and (6). Accordingly, each TD mode is decomposed into its E and H components

$$\hat{\mathbf{E}}_{pq}(\mathbf{r}, \mathbf{r}', t) = \hat{\mathbf{E}}_{pq}^H(\mathbf{r}, \mathbf{r}', t) + \hat{\mathbf{E}}_{pq}^E(\mathbf{r}, \mathbf{r}', t) \quad (25)$$

$$\hat{\mathbf{H}}_{pq}(\mathbf{r}, \mathbf{r}', t) = \hat{\mathbf{H}}_{pq}^H(\mathbf{r}, \mathbf{r}', t) + \hat{\mathbf{H}}_{pq}^E(\mathbf{r}, \mathbf{r}', t). \quad (26)$$

Here, each pq th TD-FW is determined by Fourier inversion of (7) and (8) and is carried out by first analyzing the critical points in the complex ω -plane and then applying standard analytic Fourier transforms.

A. Analysis of Singularities in the Complex ω -Plane

1) *Physical Branch Points*, $\omega_{b,pq}$: The expressions for the modal admittance and voltage TL Green's functions in (18) contain the longitudinal wavenumber $k_{z,pq}(\omega)$ given in (19). Therefore they contain pq -dependent branch points at $\omega = \omega_{b,pq}^{\pm} \equiv \bar{\omega}_{pq} \pm \tilde{\omega}_{pq}$ shown in Fig. 3 that cause $k_{z,pq}(\omega)$ vanish. They are physical because they are related to the usual radiation condition at $|z| = \infty$ [3]. Branch cuts are defined by imposing $\text{Im}k_{z,pq}(\omega) = 0$, and they separate the two Riemann sheets where the spectral wave components exponentially decay (the top one, where $\text{Im}k_{z,pq} < 0$) or grow (the bottom one) at $|z| = \infty$.

2) *Nonphysical Poles*, $\omega_{t,pq}$: Introducing the frequency shift $\omega' = \omega + (c/\eta)(\mathbf{1}_u \cdot \boldsymbol{\alpha}_{pq})$, the vector $\mathbf{k}_{t,pq}$ in (14) is transformed into

$$\mathbf{k}_{t,pq}(\omega) = \eta \frac{\omega'}{c} \mathbf{1}_u + (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v) \mathbf{1}_v \quad (27)$$

and

$$k_{t,pq}^2(\omega) \equiv \mathbf{k}_{t,pq} \cdot \mathbf{k}_{t,pq} = \left(\eta \frac{\omega'}{c} \right)^2 + (\mathbf{1}_v \cdot \boldsymbol{\alpha}_{pq})^2. \quad (28)$$

The pq th E and H mode inversion integrals arising from (9) and (10) have poles at frequencies $\omega = \omega_{t,pq}^{\pm}$, where $k_{t,pq}^2(\omega) = 0$, i.e., for

$$\omega_{t,pq}^{\pm} = \frac{c}{\eta} (-\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_u \pm j|\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v|) \quad (29)$$

with the sign $+/-$ denoting the pole above/below the real ω axis (see Fig. 3). Note that the poles move to ∞ for the non-phased case $\eta \rightarrow 0$. These poles are nonphysical since their contribution violates causality for the *individual* E and H modes. However, when the pq th E and H mode contributions are superposed to obtain the *total* pq th TD-FW field, the pole singularities cancel out, and the sum in (7) is causal, having only branch point singularities *above* the ω -integration path in Fig. 3. The cancellation is demonstrated by first observing that (see Appendix B)

$$\mathbf{k}_{t,pq}(\omega_{t,pq}^{\pm}) \times \mathbf{1}_z = \mp j \text{sgn}(\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v) \mathbf{k}_{t,pq}(\omega_{t,pq}^{\pm}). \quad (30)$$

Next, the two characteristic impedances in (22), evaluated at $\omega_{t,pq}^{\pm}$, become

$$Z_0^E(\omega_{t,pq}^{\pm}) = Z_0^H(\omega_{t,pq}^{\pm}) = \mp \zeta, \quad \zeta = \sqrt{\frac{\mu}{\epsilon}} \quad (31)$$

where we have used the fact that, by definition, the z directed wavenumber in (19) for $k_{t,pq}^2(\omega_{t,pq}^{\pm}) = 0$ becomes $k_{z,pq}(\omega_{t,pq}^{\pm}) = \mp \omega_{t,pq}^{\pm}/c$. This implies that for $\omega \approx \omega_{t,pq}^{\pm}$ (near the nonphysical poles),

$$Z_{pq}^E(z, z', \omega) \approx Z_{pq}^H(z, z', \omega). \quad (32)$$

Therefore, (9) and (10) for $\omega \approx \omega_{t,pq}^{\pm}$, are such that $\mathbf{E}_{t,pq}^H(\mathbf{r}, \mathbf{r}', \omega) \approx -\mathbf{E}_{t,pq}^E(\mathbf{r}, \mathbf{r}', \omega)$, which proves the pole cancellation, and restores causality for the sum of the E and H modes in (7).

B. TD Transmission Line Green's Functions

A TD version of the TL Green's functions, schematized in Fig. 2, is obtained by Fourier-inverting the FD-TL Green's functions in (18)

$$\left. \begin{aligned} \hat{Z}_{pq}(z, z', t) \\ \hat{Y}_{pq}(z, z', t) \\ \hat{T}_{pq}^I(z, z', t) \end{aligned} \right\} = \frac{1}{4\pi} \int_{-\infty}^{\infty} \left\{ \frac{Z_{0,pq}(\omega)}{Z_{0,pq}(\omega)} \right\} e^{j(\omega t - k_{z,pq}(\omega)|z - z'|)} d\omega \quad (33)$$

in which $k_{z,pq}(\omega)$ is given in (19). The integrand in (33) has branch points at $\omega_{b,pq}^{\pm}$, as shown in Fig. 3, and the indentation of the integration path is chosen in accord with the radiation condition at ∞ (causality) for any ω ; therefore the integration path from $-\infty$ to $+\infty$ is shifted below the branch cuts (see Fig. 3), where $\text{Re}(k_{z,pq}) \geq 0$ or ≤ 0 for $\omega > 0$ or $\omega < 0$ in accord with the radiation condition specified in the text after (21) (see also [7, p. 35] where, to ensure the existence of the Fourier transform pair in (1), the ω variable and therefore the contour of integration in (33) is shifted slightly below the real ω -axis into $\text{Im}\omega < 0$). The dashed area in Fig. 3 represents the side of the branch cut where $\text{Re}(k_{z,pq}) \geq 0$ (see also [5]). Note that, because of the ω -dependency in (22), the evaluation of $\hat{Y}_{pq}^{E,H}(z, z', t)$ is analogous to that of $\hat{Z}_{pq}^{H,E}(z, z', t)$.

1) *The Nondispersive Case* $p = q = 0$: For the nondispersive case $p = q = 0$, $k_{z,00} = (1 - \eta^2)^{1/2} \omega/c$, and the TD voltage and current are evaluated exactly as

$$\left. \begin{aligned} \hat{Z}_{00}^H(z, z', t) \\ \hat{Z}_{00}^E(z, z', t) \\ \hat{T}_{00}^I(z, z', t) \end{aligned} \right\} = \frac{1}{2} \left\{ \frac{\zeta}{\sqrt{1-\eta^2}} \right\} \delta(t - \tau_0) \quad (34)$$

where

$$\tau_0 = \sqrt{1 - \eta^2} \frac{|z - z'|}{c} \quad (35)$$

defines the turn-on time at which the impulsive wavefront reaches the observer at $\mathbf{r} = (x, y, z)$, and $\zeta = \sqrt{\mu/\epsilon}$ is the free space impedance.

2) *The Dispersive Case*, $p \neq 0$ or $q \neq 0$: Dispersive, higher order (p, q) -mode voltages and currents are evaluated as in Appendix C (see also [9] for the nonphased case, and [11]–[13] for similar dispersive problems), yielding the closed form expressions

$$\hat{Z}_{pq}^H(z, z', t) = \frac{\zeta e^{j\bar{\omega}_{pq}t}}{2\sqrt{1-\eta^2}} \left\{ \delta(t - \tau_0) + \left[j\bar{\omega}_{pq}J_0(b) - \bar{\omega}_{pq}^2 \frac{J_1(b)}{b} \right] U(t - \tau_0) \right\} \quad (36)$$

$$\hat{Z}_{pq}^E(z, z', t) = \frac{\zeta \sqrt{1-\eta^2}}{2} e^{j\bar{\omega}_{pq}t} \left\{ \delta(t - \tau_0) + \left[\hat{Q}_{pq}(t) + j\bar{\omega}_{pq}J_0(b) + \bar{\omega}_{pq}^2 \frac{J_1(b)}{b} \right] U(t - \tau_0) \right\} \quad (37)$$

$$\hat{T}_{pq}^I(z, z', t) = \frac{e^{j\bar{\omega}_{pq}t}}{2} \left\{ \delta(t - \tau_0) - \bar{\omega}_{pq}^2 \tau_0 \frac{J_1(b)}{b} U(t - \tau_0) \right\} \quad (38)$$

with

$$\hat{Q}_{pq}(t) = \frac{\sqrt{\tilde{\omega}_{pq}^2 - \tilde{\omega}_{pq}^2}}{2} \left[e^{a+b} \left(\sqrt{1 + a_+^2} J_{e0}(a_+, b) - 1 \right) + e^{a-b} \left(\sqrt{1 + a_-^2} J_{e0}(a_-, b) - 1 \right) \right]. \quad (39)$$

Here, J_0 and J_1 are the zero-th and first order Bessel functions of the first kind, and $J_{e0}(a, b) = \int_0^b e^{-a\xi} J_0(\xi) d\xi$ is the zeroth-order incomplete Lipschitz-Hankel integral of the first kind (see [14] and references therein for more details). In (36)–(39)

$$a_{\pm} = \frac{(-j\tilde{\omega}_{pq}t \pm \tau_0 \sqrt{\tilde{\omega}_{pq}^2 - \tilde{\omega}_{pq}^2})}{b} \quad (40)$$

$$b = \tilde{\omega}_{pq} \sqrt{t^2 - \tau_0^2}. \quad (41)$$

The evaluation of $\hat{Z}_{pq}^E(z, z', t)$ is rather complex and it is carried out in this paper for the first time. The main steps of its derivation are shown in Appendix C. Note that *all* the pq th TD-TL GF have the same turn-on time τ_0 , defined by the unit step function $U(t - \tau_0)$.

C. Fourier Inversion of the E-H-Decomposed pq th FD-FWs

The TD electric and magnetic fields in (25) and (26), decomposed into their E and H components, are now rewritten as

$$\hat{\mathbf{E}}_{t,pq}^{E,H}(\mathbf{r}, \mathbf{r}', t) = f_{pq} \hat{\mathbf{D}}_E^{E,H}(t) \cdot \mathbf{J}_t \otimes \hat{Z}_{pq}^{E,H}(z, z', \tau) \quad (42)$$

$$\hat{\mathbf{H}}_{t,pq}^{E,H}(\mathbf{r}, \mathbf{r}', t) = f_{pq} \hat{\mathbf{D}}_H^{E,H}(t) \cdot \mathbf{J}_t \otimes \hat{T}_{pq}^I(z, z', \tau) \quad (43)$$

in which $\otimes$ denotes time convolution, and

$$f_{pq} = \frac{e^{-j\alpha_{pq} \cdot (\boldsymbol{\rho} - \boldsymbol{\rho}')}}{d_x d_y}. \quad (44)$$

The retarded time

$$\tau = t - \eta \mathbf{1}_u \cdot \frac{(\boldsymbol{\rho} - \boldsymbol{\rho}')}{c} \quad (45)$$

in the TL GFs in (42) and (43) [cfr. (36)–(38)] appears because of the $\exp[-j\omega\eta \mathbf{1}_u \cdot (\boldsymbol{\rho} - \boldsymbol{\rho}')/c]$ factor in F_{pq} in (9)–(12). The TD dyads in (42) and (43) are obtained by Fourier-inverting a corresponding group of terms in the FD-FW expressions in (9)–(12). We evaluate here explicitly only

$$\hat{\mathbf{D}}_E^E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\mathbf{k}_{t,pq}(\omega) \mathbf{k}_{t,pq}(\omega)}{k_{t,pq}^2(\omega)} e^{j\omega t} d\omega. \quad (46)$$

From (27)

$$\begin{aligned} \mathbf{k}_{t,pq}(\omega) \mathbf{k}_{t,pq}(\omega) &= \omega'^2 \frac{\eta^2}{c^2} \mathbf{1}_u \mathbf{1}_u \\ &+ \omega' \frac{\eta}{c} (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v) (\mathbf{1}_u \mathbf{1}_v + \mathbf{1}_v \mathbf{1}_u) + (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v)^2 \mathbf{1}_v \mathbf{1}_v \end{aligned} \quad (47)$$

where we have used the shifted frequency ω' defined just before (27). After decomposition of (46) into three integrals with distinct ω' dependency, we obtain the expression

$$\hat{\mathbf{D}}_E^E(t) = \hat{a}_{pq}(t) \mathbf{1}_u \mathbf{1}_u + \hat{b}_{pq}(t) (\mathbf{1}_u \mathbf{1}_v + \mathbf{1}_v \mathbf{1}_u) + \hat{c}_{pq}(t) \mathbf{1}_v \mathbf{1}_v \quad (48)$$

with TD coefficients

$$\begin{aligned} \hat{c}_{pq}(t) &= \frac{1}{\pi} (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v)^2 \hat{e}(t) \int_{-\infty}^{\infty} \frac{e^{j\omega' t}}{k_{t,pq}^2(\omega)} d\omega' = \\ &= \frac{c}{\eta} |\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v| \hat{e}(t) e^{-c/\eta |\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v| |t|} \end{aligned} \quad (49)$$

$$\begin{aligned} \hat{b}_{pq}(t) &= \frac{1}{\pi} \frac{\eta}{c} (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v) \hat{e}(t) \int_{-\infty}^{\infty} \frac{\omega' e^{j\omega' t}}{k_{t,pq}^2(\omega)} d\omega' \\ &= j \operatorname{sgn}[(\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v)t] \hat{c}_{pq}(t) \end{aligned} \quad (50)$$

$$\hat{a}_{pq}(t) = \frac{1}{\pi} \frac{\eta^2}{c^2} \hat{e}(t) \int_{-\infty}^{\infty} \frac{\omega'^2 e^{j\omega' t}}{k_{t,pq}^2(\omega)} d\omega' = \delta(t) - \hat{c}_{pq}(t) \quad (51)$$

where $k_{t,pq}^2(\omega)$ is a function of ω' as shown in (28), and the common factor $\hat{e}(t) = \exp[-jc/\eta (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_u)t]/2$ arises from the mentioned shift of frequency. Note that $\hat{e}(t)$ is a noncausal function since it has been evaluated by applying Cauchy's theorem with residue contributions from the nonphysical poles at $\omega_{t,pq}^{\pm}$ in (29), both above and below the ω' integration path which is located slightly below the real axis (Fig. 3). The remaining dyad for the electric field in (42) is then evaluated by using the property $\hat{\mathbf{D}}_E^H = -\mathbf{1}_z \times \hat{\mathbf{D}}_E^E \times \mathbf{1}_z$ that leads to

$$\begin{aligned} \hat{\mathbf{D}}_E^H(t) &= \hat{c}_{pq}(t) \mathbf{1}_u \mathbf{1}_u - \hat{b}_{pq}(t) (\mathbf{1}_u \mathbf{1}_v + \mathbf{1}_v \mathbf{1}_u) \\ &+ \hat{a}_{pq}(t) \mathbf{1}_v \mathbf{1}_v. \end{aligned} \quad (52)$$

Similarly, those for the magnetic field in (43) are evaluated using $\hat{\mathbf{D}}_H^E = -\mathbf{1}_z \times \hat{\mathbf{D}}_H^H$ and $\hat{\mathbf{D}}_H^H = \hat{\mathbf{D}}_H^E \times \mathbf{1}_z$ that yield

$$\begin{aligned} \hat{\mathbf{D}}_H^E(t) &= -\hat{c}_{pq}(t) \mathbf{1}_u \mathbf{1}_v - \hat{b}_{pq}(t) (\mathbf{1}_u \mathbf{1}_u - \mathbf{1}_v \mathbf{1}_v) \\ &+ \hat{a}_{pq}(t) \mathbf{1}_v \mathbf{1}_u \end{aligned} \quad (53)$$

$$\begin{aligned} \hat{\mathbf{D}}_H^H(t) &= -\hat{a}_{pq}(t) \mathbf{1}_u \mathbf{1}_v + \hat{b}_{pq}(t) (\mathbf{1}_u \mathbf{1}_u - \mathbf{1}_v \mathbf{1}_v) \\ &+ \hat{c}_{pq}(t) \mathbf{1}_v \mathbf{1}_u. \end{aligned} \quad (54)$$

Note that in (42) and (43) the TD-TL Green's functions defined in (36)–(38) are causal since the signal arrives at $\mathbf{r}$ at the turn-on time $t = t_0$ ($\tau = \tau_0$) (see [3] for a physical interpretation of t and τ reference systems in terms of moving coordinates.) The E, H -decomposed TD field expressions in (42) and (43) are noncausal since they are determined by a time convolution between casual TD-TL Green's functions and noncausal dyads that are time-spread around $t = 0$. However, as demonstrated in Section IV-A, causality is recovered by summing the E and H constituents in (25) and (26). This can easily be seen directly in the TD for the magnetic field in free space where the TL Green's functions $\hat{T}^I$ for the E and H cases coincide: the sum of the E and H components is equivalent to the sum of the TD dyads $\hat{\mathbf{D}}_H^E(t) + \hat{\mathbf{D}}_H^H(t) = -\delta(t) \mathbf{1}_u \mathbf{1}_v + \delta(t) \mathbf{1}_v \mathbf{1}_u$, which is not spread around $t = 0$.

1) *The Case of the Array Elements Simultaneously Excited (Nonphased):* When the array dipoles are simultaneously excited (the nonphased case) $\eta = 0$ and the TD coefficients in the dyads reduce to

$$\begin{aligned} \hat{a}_{pq}(t) &= \delta(t) \frac{(\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_u)^2}{\alpha_{pq}^2}, \quad \hat{b}_{pq}(t) = 0 \\ \hat{c}_{pq}(t) &= \delta(t) \frac{(\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v)^2}{\alpha_{pq}^2}. \end{aligned} \quad (55)$$

These equations can be obtained from the frequency domain or as a limit case from (49)–(51). The expressions in (42) and (43) become again strictly causal, and no time domain convolution between the TLGF ($\hat{Z}_{pq}(z, z', \tau)$ and $T_{pq}^I(z, z', \tau)$) and the TD dyads $\hat{\mathbf{D}}(t)$ is required in (42) and (43).

V. SUMMARY OF RESULTS AND THE PHYSICALLY OBSERVABLE RADIATED FIELD

The representation for the electric field excited by an array of impulsive dipole currents described in the previous sections is given by (23) with (25) and (42), where TD-TL Green's function and dyadic expressions are given in (36)–(38) and (46)–(54), respectively.

Although obtained by conventional Fourier inversion from the frequency domain, the results in (42), (43) are complex for $p \neq 0$ or $q \neq 0$. Indeed, in this case $\alpha_{pq} \neq 0$ and, from (19), $\bar{\omega}_{pq} \neq 0$ and thus the expressions (36)–(38) are complex. The phenomenology is analogous to that observed previously for the TD potential radiated by a linear [1] and planar [3] array of dipoles, and is thus addressed as in [1], [3] by $(+p, +q)$, $(-p, -q)$ pairing to obtain the “physically observable” *real* TD-FW field. Noting from (19) that $\alpha_{-p, -q} = -\alpha_{p, q}$ and $\bar{\omega}_{-p, -q} = -\bar{\omega}_{p, q}$, while $\tilde{\omega}_{-p, -q} = \tilde{\omega}_{p, q}$, it follows that in (42) the following properties hold: $\hat{\mathbf{D}}_{E, -p, -q}^{E, H} = (\hat{\mathbf{D}}_{E, pq}^{E, H})^*$ and $\hat{Z}_{-p, -q}^{E, H} = (\hat{Z}_{pq}^{E, H})^*$, where the asterisk $*$ denotes complex conjugation. The property $\hat{Z}_{-p, -q}^E = (\hat{Z}_{pq}^E)^*$ follows from noticing that $\hat{Q}_{pq}(t)$ in (37) is real and from $\hat{Q}_{-p, -q}(t) = \hat{Q}_{p, q}(t)$. Therefore, the total electric field can be represented as a sum of “physically observable” *real* TD-FWs as

$$\hat{\mathbf{E}}_t(\mathbf{r}, \mathbf{r}', t) = \sum_{pq} \text{Re} \left(\hat{\mathbf{E}}_{t, pq}^E + \hat{\mathbf{E}}_{t, pq}^H \right) \quad (56)$$

since

$$\hat{\mathbf{E}}_{t, pq}^{E, H} + \hat{\mathbf{E}}_{t, -p, -q}^{E, H} = 2\text{Re} \left(\hat{\mathbf{E}}_{t, pq}^{E, H} \right). \quad (57)$$

The terms in the series on the right-hand side of (56) can also be rearranged so as to include only positive (and zero) p, q indexes. Analogous conclusions can be drawn for the magnetic field.

VI. TD FIELD PRODUCED BY BAND-LIMITED SHORT-PULSE DIPOLE EXCITATION

In the previous Sections the dipoles were excited by impulsive currents. In the more realistic case of short-pulse current excitation $\hat{P}(t)$ with band-limited (BL) spectrum $P(\omega)$, the Dirac delta function in the TD expressions in (2) is replaced by $\hat{P}(t - \eta \mathbf{1}_u \cdot \boldsymbol{\rho}_{mn}/c)$. Therefore the fields radiated by the array, $\hat{\mathbf{E}}^{BL}(\mathbf{r}, \mathbf{r}', t) = \sum_{pq} \hat{\mathbf{E}}_{pq}^{BL}(\mathbf{r}, \mathbf{r}', t)$ and $\hat{\mathbf{H}}^{BL}(\mathbf{r}, \mathbf{r}', t) = \sum_{pq} \hat{\mathbf{H}}_{pq}^{BL}(\mathbf{r}, \mathbf{r}', t)$, is given by the convolution of the waveform $\hat{P}(t)$ with the impulsive fields in (23) and (24). Accordingly, each BL-TD-FW is decomposed into its E and H components given by the time convolution

$$\hat{\mathbf{E}}_{pq}^{BL}(\mathbf{r}, \mathbf{r}', t) = \hat{P}(t) \otimes \hat{\mathbf{E}}_{pq}(\mathbf{r}, \mathbf{r}', t) \quad (58)$$

$$\hat{\mathbf{H}}_{pq}^{BL}(\mathbf{r}, \mathbf{r}', t) = \hat{P}(t) \otimes \hat{\mathbf{H}}_{pq}(\mathbf{r}, \mathbf{r}', t). \quad (59)$$

Fig. 4. Topology of the ω plane and a different choice of the path for the inverse Fourier transform.

For numerical purposes however it may be convenient to evaluate the TD fields as follows. First, performing and storing the time-convolution between the TD dyads and the BL exciting signal: $\hat{P}(t) \otimes \hat{\mathbf{D}}(t)$. Next, the resulting equivalent “dyadic waveform excitation” is time-convolved with the TD-TL Green's functions $\hat{Z}_{pq}(z, z', t)$ and $\hat{T}_{pq}^I(z, z', t)$. This may be convenient since both $\hat{P}(t)$ and $\hat{\mathbf{D}}(t)$ have a short time duration (the dyad is almost like an impulse in the case $\eta \approx 0$.) Furthermore, once the convolution $\hat{P}(t) \otimes \hat{\mathbf{D}}(t)$ is evaluated and stored, it can be re-used for TD field evaluations at various heights z from the array plane. The number of pq -Floquet waves sufficient to reconstruct the total field is proportional to the bandwidth of the exciting waveform $\hat{P}(t)$. Indeed, one should include all pq -Floquet waves whose pq -cutoff frequencies $\omega_{pq, i}^{\text{cutoff}} = \bar{\omega}_{pq} + (\pm 1)^i \tilde{\omega}_{pq}$ with $i = 1, 2$, are within the bandwidth of $\hat{P}(t)$, as shown in [3, Sec. IV.C and V]

VII. ALTERNATIVE REPRESENTATION

The representation for the pq th FW excited by an array of impulsive dipole currents described in the previous sections is given by (42) where the causal TD-TL Green's functions $\hat{Z}_{pq}^{E, H}(z, z', t)$ are time-convolved with the noncausal dyadic expressions $\hat{\mathbf{D}}_{E, H}^{E, H}(t)$. We have already seen that though the E and H FWs are individually noncausal their sum restores causality. However, it is possible to have causal dyadic expressions for individual E and H components if one chooses the path shown in Fig. 4 for the inverse Fourier transform of the pq th electric field in (7). Indeed, the poles $\omega_{t, pq}^{\pm}$ shown in Figs. 3 and 4 are not present in (7) and therefore one can choose either the path in Fig. 3 (as in this paper) or the one in Fig. 4. If one chooses the Fourier inversion path as in Fig. 4, the decomposition into E and H modes leads to a field representation similar to that in (42) with the only difference that now the dyads have causal coefficients

$$\hat{c}_{pq}(t) = -2 \frac{c}{\eta} |\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v| \hat{e}(t) \sinh \left(\frac{c}{\eta} |\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v| t \right) U(t) \quad (60)$$

$$\hat{b}_{pq}(t) = 2j \frac{c}{\eta} (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v) \hat{e}(t) \cosh \left(\frac{c}{\eta} |\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v| t \right) U(t) \quad (61)$$

$$\hat{a}_{pq}(t) = \delta(t) - \hat{c}_{pq}(t). \quad (62)$$

Though simpler in appearance, one can easily recognize that due to the hyperbolic functions these coefficients grow exponentially in almost all practical cases. Analytically, the correct numbers are restored by the sum of the E and H modes but round-off errors may appear in the calculations.

VIII. A NOTE ON THE EVALUATION OF THE $\hat{Q}_{pq}(t)$ FUNCTION IN (39)

The complex exponent $a_{\pm}b$ in (39) may give rise to extremely large numbers of the order of $\exp(a_{\pm}b) \propto \exp(\tau_0 \sqrt{\hat{\omega}_{pq}^2 - \bar{\omega}_{pq}^2})$ that may cause overflow or round-off errors when either high pq -order FWs are needed (wide band excitation waveforms) or for observation points far from the array plane (large τ_0). The arrangement and grouping shown in (37) with (39) limit this effect since the exponential growth is canceled by the term $(\sqrt{1 + a_{\pm}^2} J_{e0}(a_{\pm}, b) - 1)$. This is shown by using the asymptotic expansion of $J_{e0}(a_{\pm}, b)$ in [15, Eq. (3)] (see also [14]), valid for $|b(1 + a_{\pm}^2)| \gg 1$, that when inserted in (39) leads to

$$e^{a_{\pm}b} \left(\sqrt{1 + a_{\pm}^2} J_{e0}(a_{\pm}, b) - 1 \right) \sim \frac{1}{\sqrt{1 + a_{\pm}^2}} \times \sum_{k=0}^{+\infty} \frac{\Gamma(k + \frac{1}{2})}{\Gamma(\frac{1}{2})} \left[\frac{2}{b(1 + a_{\pm}^2)} \right]^k [J_{k+1}(b) - a_{\pm} J_k(b)] \quad (63)$$

where the exponentially growing terms are canceled analytically. The above formula is also used to avoid the numerical evaluation of the $J_{e0}(a_{\pm}, b)$ function when $|b(1 + a_{\pm}^2)| \gg 1$ resulting in a fast evaluation when only a few terms are retained.

Furthermore, one can show that the $\hat{Q}_{pq}(t)$ function in (39) has pure real values for any t , and thus we suggest to take its real part *a priori*.

IX. ILLUSTRATIVE EXAMPLES

To check the accuracy of the TD-FW-based network formalism for the pulse-excited planar phased array of dipoles, we have implemented the electric field solution (23), with (25), (36) and (37) convolved with a BL excitation as in (58). The solution is compared here with a reference solution obtained via element-by-element summation over the pulsed radiations from all dipoles [16]. It consists of the natural superposition of different “spatial” contributions, and it is considered an independent solution compared to the spectral TD-FW sum. Agreement of both methods implies that they have been both implemented correctly. The various spherical TD contributions reach the observer at the delayed times $t_{mn} = \eta \mathbf{1}_u \cdot \boldsymbol{\rho}_{mn}/c + R_{mn}/c$, with $\mathbf{R}_{mn} = \mathbf{r} - (\mathbf{r}' + \boldsymbol{\rho}_{mn})$, and $m, n = 0, \pm 1, \pm 2, \dots$. The element-by-element mn -sum has been truncated when contributions from far-away elements are negligible, i.e., retaining elements with $|m|, |n| < 100$. Further elements would contribute only to the tail of the TD signal, not shown in the time scale of the following figures. The selected BL excitation is a normalized Rayleigh pulse $\hat{P}(t) = \text{Ree}[j/(j + \omega_M t/4)^5]$ (i.e., $\hat{P}(0) = 1$) [17], with FD spectrum $P(\omega) = \pi(6\omega_M)^{-1}(j4\omega/\omega_M)^4 \exp(-4|\omega|/\omega_M)$ and central radian frequency ω_M , which corresponds to a central wavelength $\lambda_M = 2\pi c/\omega_M$ that determines the time

Fig. 5. $\hat{E}_y(\mathbf{r}, t)$ component of the electric field radiated by an infinite pulsed planar array of y -directed dipoles, observed at $\mathbf{r} = (x, y, z) = (0, 0, 10d_x)$ as a function of time t . Both nonphased $\eta = 0$ and phased $\eta = 0.7$ cases are shown. Parameters: $d_x = d_y = 0.1$ m; $\lambda_M = 8d_x$. The TD-FW expansion consists of only the fundamental FW with $p = q = 0$, and agrees well with the element-by-element reference solution.

duration of the pulse (see [3, Fig. 6] where the same pulse has been denoted by the symbol $\hat{G}(t)$). In [8] only a result for the magnetic field, which involves the TL-GFs $\hat{T}_{pq}^I(z, z', t)$, was shown. Here, we present results for the electric field that have never been shown previously because they are computed by using the more involved TL-GF $\hat{Z}_{pq}^E(z, z', t)$ in (37) whose analytic expression has been shown here for the first time. Consider an array of dipoles oriented along $\mathbf{1}_y$, with periods $d_x = d_y = 0.1$ m.

In Fig. 5 the exciting waveform $\hat{P}(t)$ is chosen such that $\lambda_M = 8d_x$ so that the average length of the pulse is eight times the period of the array (see [3] for more details). Results for both nonphased ($\eta = 0$) and phased ($\eta = 0.7$ along the direction $\mathbf{1}_u \equiv \mathbf{1}_x$) cases are displayed (more appropriately we should say that the dipoles are sequentially excited with a straight wavefront moving along the direction $\mathbf{1}_u$ as in Fig. 1.) The electric field $\hat{E}_y$ is observed at the location $(x, y, z) = (0, 0, 10d_x)$, versus time t . The exciting TD waveform $\hat{P}(t)$ is centered at $t = 0$, and the signal arrival time $t_0 = \eta \mathbf{1}_u \cdot (\boldsymbol{\rho} - \boldsymbol{\rho}')/c + \tau_0$ to the observer is determined by using (45) and (35) in (42) (for the geometry considered $(\boldsymbol{\rho} - \boldsymbol{\rho}') = 0$).

It is remarkable that only the dominant TD-FW with $p = q = 0$ is necessary to represent the radiated field at any time t . The agreement with the element-by-element reference solution is excellent. As a further check on the numerics, both the TD-FW expansion and the element-by-element reference solution yield a negligible $\hat{E}_x$ component. The total electric field is obtained by numerically summing its E and H -mode constituents; for the phased case $\eta = 0.7$, it has been verified that this sum cancels the higher order small noncausal components, and renders the total signal causal.

In Fig. 6 the exciting waveform $\hat{P}(t)$ is chosen such that $\lambda_M = 2d_x$ so that the average length of the pulse is twice the period of the array. All other parameters are as for the case in Fig. 5. Due to the larger bandwidth of the exciting waveform this time a slightly larger number of TD-FWs (those with $|p|, |q| \leq 2$) is needed, though still showing that a limited

Fig. 6. $\hat{E}_y(\mathbf{r}, t)$ component of the electric field radiated by an infinite pulsed planar array of y -directed dipoles. All parameters are as those in Fig. 5 except that now $\lambda_M = 2d_x$. This time the TD-FW expansion consists of all FWs with $|p|, |q| \leq 2$, and agrees well with the element-by-element reference solution.

number of TD-FW constituents is sufficient to reconstruct the field at any time t .

It has been verified that in all cases encountered in these numerical examples, the parameters needed to compute the incomplete Lipschitz-Hankel integrals in (39) are such that $|b(1 + a_{\pm}^2)| > 10$ and thus the asymptotic form in (63) has been used.

X. CONCLUSIONS

A network-oriented dyadic Green's function has been investigated here for a planar infinite periodic array of sequentially band limited pulse-excited dipoles. Accordingly, the E (TM) and H (TE) mode contributions can be separated and treated individually in a systematic fashion. The thus reduced modal fields are expressed in terms of transmission line (TL) Green's functions that behave according to standard network theory. Therefore, possible infinite-planar transversely homogeneous layers with longitudinal inhomogeneities can be readily incorporated within the formalism (see [18] and [19] for similar problems.) It has been found that individually, each TD-FW E and H mode is non-causal and can be obtained in closed form in terms of a convolution between characteristic noncausal dyadic functions and the causal TL Green's functions. Causality of the total mode field is recovered in the E and H mode sum. The total radiated field can be constructed at any location and at any time within our numerical experiments by retaining only a few TD-FWs. It should also be noted that "physically observable" pq th TD-FWs are synthesized by $(+p, +q)$, $(-p, -q)$ superposition. The attention in this paper has been devoted to a famous network formalism that has been used extensively in the FD [7, Ch. 2 and 3], and we thus felt that an investigation in the TD of the same formalism was necessary to better understand and model short-pulse propagation of the dispersive relation peculiar of radiation and scattering in spatially periodic problems.

The TD field network representation for the case of a truncated semi-infinite planar array of dipoles is not in closed form, and can be treated asymptotically by generalizing the steps in [4] for E and H modes.

APPENDIX A

FD MANIPULATION OF H AND E MODES

The electric and magnetic field expressions in (9)–(12) are obtained as follows. First, the fields are expressed in terms of E -mode (TM_z) and H -mode (TE_z) scalar Hertz potentials Π^E and Π^H , respectively, as in [7, p. 444]. The Hertz potentials are related to scalar functions $\mathcal{S}^E$ and $\mathcal{S}^H$ as shown in [7, p. 445]. From a transmission line analysis along the z axis, with eigenvalues evaluated in the cross section transverse to z , one should represent the scalar potentials $\mathcal{S}^E$ and $\mathcal{S}^H$ in terms of a pq -modal expansion (see [7, pp. 196–198, and p. 446]). Thus, $\mathcal{S}^E$ and $\mathcal{S}^H$ are expressed in terms of scalar transverse eigenfunctions $\Phi_{pq}(\boldsymbol{\rho}) = \Psi_{pq}(\boldsymbol{\rho}) = e^{-j\mathbf{k}_{t,pq} \cdot \boldsymbol{\rho}} / (\sqrt{d_x d_y})$, in which the transverse wavenumber, given in (14), corresponds to the pq th FW, as can be inferred from the plane wave case treated in [7, p. 251]. One should note that for the infinite array in free space, the transverse wavenumber $k_{t,pq}$ in [7, p. 251] is the same for E and H modes.

For each FW (each term in the pq -summation) we use the equivalence $\nabla = -j\mathbf{k}_{pq}$, $\nabla_t = -j\mathbf{k}_{t,pq}$, and $\nabla'_t = +j\mathbf{k}_{t,pq}$, and the scalar Hertz potentials Π^E and Π^H are rewritten as

$$\Pi^E(\mathbf{r}, \mathbf{r}', \omega) = \frac{1}{\omega\epsilon} \sum_{pq} \frac{1}{j\omega\epsilon} (\mathbf{J}_t \cdot \mathbf{k}_{t,pq}) F_{pq} \frac{\partial}{\partial z'} Y_{pq}^E(z, z', \omega) \quad (64)$$

$$\Pi^H(\mathbf{r}, \mathbf{r}', \omega) = \frac{1}{\omega\mu} \sum_{pq} (\mathbf{1}_z \times \mathbf{J}_t) \cdot \mathbf{k}_{t,pq} F_{pq} Z_{pq}^H(z, z', \omega), \quad (65)$$

where the $F_{pq}(\omega)$ factor is given in (13). These expressions are inserted into the field representations [7, p. 444]. After performing the differentiations on the summands, and the use of the equivalence $\mathbf{k}_{pq} \times (\mathbf{k}_{pq} \times \mathbf{1}_z) = \mathbf{k}_{pq} k_{z,pq} - \mathbf{1}_z k^2$ in the E part, the transverse parts of the electric fields in (7) are rewritten as

$$\begin{aligned} \mathbf{E}_{t,pq}^E(\mathbf{r}, \mathbf{r}', \omega) &= -\mathbf{k}_{t,pq} \frac{k_{z,pq}}{j\omega\epsilon} (\mathbf{k}_{t,pq} \cdot \mathbf{J}_t) \\ &\quad \cdot \frac{1}{\omega\epsilon} F_{pq} \frac{\partial}{\partial z'} Y_{pq}^E(z, z', \omega) \\ \mathbf{E}_{t,pq}^H(\mathbf{r}, \mathbf{r}', \omega) &= (\mathbf{k}_{t,pq} \times \mathbf{1}_z) ((\mathbf{k}_{t,pq} \times \mathbf{1}_z) \cdot \mathbf{J}_t) \\ &\quad \cdot F_{pq} Z_{pq}^H(z, z', \omega). \end{aligned} \quad (66)$$

Furthermore, from reciprocity [7, p. 194] $Y_{pq}^E(z, z') = Y_{pq}^E(z', z)$, and from the telegraphist equations $\partial/\partial z' Y_{pq}^E(z', z) = -jk_{z,pq} (Z_0^E)^{-1} T^{V,E}(z', z)$. Also, $(k_{z,pq})(Z_0^E)^{-1} = \omega\epsilon$, and from reciprocity [7, p. 194] $T^{V,E}(z', z) = -T^{I,E}(z, z')$, which leads to $\partial/\partial z' Y_{pq}^E(z, z') = j\omega\epsilon T^{I,E}(z, z')$ and

$$\begin{aligned} \mathbf{E}_{pq}^E(\mathbf{r}, \mathbf{r}', \omega) &= (-1) \mathbf{k}_{t,pq} \frac{k_{z,pq}}{j\omega\epsilon} (\mathbf{k}_{t,pq} \cdot \mathbf{J}_t) \\ &\quad \cdot F_{pq} T_{pq}^{I,E}(z, z', \omega) \\ \mathbf{E}_{pq}^H(\mathbf{r}, \mathbf{r}', \omega) &= (\mathbf{k}_{t,pq} \times \mathbf{1}_z) [(\mathbf{k}_{t,pq} \times \mathbf{1}_z) \cdot \mathbf{J}_t] \\ &\quad \cdot F_{pq} Z_{pq}^H(z, z', \omega). \end{aligned} \quad (67)$$

Again, from the telegraphist equations $\partial/\partial z Z_{pq}^E(z, z') = -jk_{z,pq} Z_{pq}^E T^{I,E}(z, z')$

$$\begin{aligned} \mathbf{E}_{pq}^E(\mathbf{r}, \mathbf{r}', \omega) &= \mathbf{k}_{t,pq}(\mathbf{k}_{t,pq} \cdot \mathbf{J}_t) F_{pq} \\ &\quad \cdot \frac{-1}{k_{z,pq}} \frac{\partial}{\partial z} Z_{pq}^E(z, z', \omega) \\ \mathbf{E}_{pq}^H(\mathbf{r}, \mathbf{r}', \omega) &= (\mathbf{k}_{t,pq} \times \mathbf{1}_z)[(\mathbf{k}_{t,pq} \times \mathbf{1}_z) \cdot \mathbf{J}_t] \\ &\quad \cdot F_{pq} Z_{pq}^H(z, z', \omega). \end{aligned} \quad (68)$$

Note that $\partial/\partial z Z_{pq}^E(z, z') = -\text{sgn}(z - z') k_{z,pq} Z_{pq}^E(z, z')$, which leads to the desired expression for the pq th FD-FW in (9) and (10). Analogous treatment is done to the transverse component of the magnetic field in (8) leading to (11) and (12).

APPENDIX B

WAVENUMBER EVALUATION AT THE NONPHYSICAL POLES

We evaluate here the wavenumbers $\mathbf{k}_{t,pq}(\omega)$ and $\mathbf{k}_{t,pq}(\omega) \times \mathbf{1}_z$ at the frequency poles $\omega_{t,pq}^\pm$. Inserting expression (29) into (27) yields

$$\begin{aligned} \mathbf{k}_{t,pq}(\omega_{t,pq}^\pm) &= \eta \frac{\omega_{t,pq}^\pm}{c} \mathbf{1}_u + \boldsymbol{\alpha}_{pq} = \\ &= [-(\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_u) \pm j[\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_u]] \mathbf{1}_u + \boldsymbol{\alpha}_{pq} \\ &= (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v)(\mathbf{1}_v \pm j\epsilon_{pq} \mathbf{1}_u) \end{aligned} \quad (69)$$

with $\epsilon_{pq} = \text{sgn}(\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v)$. Operating a cross product on (69) yields

$$\mathbf{k}_{t,pq}(\omega_{t,pq}^\pm) \times \mathbf{1}_z = (\boldsymbol{\alpha}_{pq} \cdot \mathbf{1}_v)(\mathbf{1}_u \mp j\epsilon_{pq} \mathbf{1}_v). \quad (70)$$

Comparing (69) with (70) leads to (30).

APPENDIX C

INVERSE FOURIER TRANSFORM OF TL-GFs

The evaluation of $\hat{T}_{pq}^I(z, z', t)$ in (33) is carried out by first using the shift of frequency $\omega' = \omega - \bar{\omega}_{pq}$, then regularizing the integrand at $\omega' = \infty$ by adding and subtracting the asymptotic part $e^{j\omega'(t-\tau_0)}$ as

$$\begin{aligned} \hat{T}_{pq}^I(z, z', t) &= \frac{-e^{j\bar{\omega}_{pq}t}}{4\pi} \int_{-\infty}^{\infty} \left[e^{j\omega'(t-\tau_0)} + \left(e^{jg(\omega')} - e^{j\omega'(t-\tau_0)} \right) \right] d\omega' \quad (71) \end{aligned}$$

where $g(\omega') = (\omega't - \sqrt{\omega'^2 - \tilde{\omega}_{pq}^2} \tau_0)$. The first term produces a delta Dirac while the second term is evaluated as in [20, p. 1027], leading to (38). An analogous argument is used for $\hat{Z}_{pq}^H(z, z', t)$ in (33). First the shift of frequency leads to

$$\begin{aligned} \hat{Z}_{pq}^H(z, z', t) &= \zeta \frac{-e^{j\bar{\omega}_{pq}t}}{4\pi \sqrt{1-\eta^2}} \int_{-\infty}^{\infty} \frac{\omega' + \bar{\omega}_{pq}}{\sqrt{\omega'^2 - \tilde{\omega}_{pq}^2}} e^{jg(\omega')} d\omega' \\ &= \frac{-e^{j\bar{\omega}_{pq}t}}{4\pi \sqrt{1-\eta^2}} \left(\int_{-\infty}^{\infty} \frac{\omega'}{\sqrt{\omega'^2 - \tilde{\omega}_{pq}^2}} e^{jg(\omega')} d\omega' \right. \\ &\quad \left. + \int_{-\infty}^{\infty} \frac{\bar{\omega}_{pq}}{\sqrt{\omega'^2 - \tilde{\omega}_{pq}^2}} e^{jg(\omega')} d\omega' \right). \end{aligned} \quad (72)$$

The first integral is evaluated in a procedure similar to the one in (71) (see the Tables in [21, p. 248]) while the second is exactly the one evaluated for the potential Green's function in [3, Eq. (24)]. This leads to (36). The evaluation of $\hat{Z}_{pq}^E(z, z', t)$ in (33) carried out in the following is rather more involved. After the same shift of frequency used for the two previous cases, the expression of $\hat{Z}_{pq}^E$ is rewritten as

$$\hat{Z}_{pq}^E(z, z', t) = -\frac{e^{j\bar{\omega}_{pq}t}}{2} \zeta \sqrt{1-\eta^2} \frac{d}{d\tau_0} \mathcal{I}(t, \tau_0) \quad (73)$$

with

$$\mathcal{I}(t, \tau_0) = \frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{e^{jg(\omega')}}{\omega' + \bar{\omega}_{pq}} d\omega'. \quad (74)$$

This special integral has already been evaluated in [11, Eq. (11)] and [13, Eq. (7)] under a more general condition, and here readapted as

$$\begin{aligned} \mathcal{I}(t, \tau_0) &= U(t - \tau_0) \left\{ e^{-j\bar{\omega}_{pq}t} \cosh(\sqrt{\tilde{\omega}_{pq}^2 - \bar{\omega}_{pq}^2} \tau_0) \right. \\ &\quad \left. + \frac{1}{2} [A(a_+, b) + A(a_-, b)] \right\} \end{aligned} \quad (75)$$

where

$$A(a_\pm, b) = \mp \sqrt{1 + a_\pm^2} e^{a_\pm b} J_{e0}(a_\pm, b) \quad (76)$$

with arguments $a_\pm$ and b given in (40) and (41). Note that [11, Eq. (31)]

$$\mp \sqrt{1 + a_\pm^2} = \frac{(-j\bar{\omega}_{pq}\tau_0 \mp t\sqrt{\tilde{\omega}_{pq}^2 - \bar{\omega}_{pq}^2})}{b}. \quad (77)$$

The derivative in (73) is rather complex and the main steps of its evaluation are reported in the following. We first write

$$\begin{aligned} \frac{d}{d\tau_0} \mathcal{I}(t, \tau_0) &= -\delta(t - \tau_0) + \\ &\quad + U(t - \tau_0) \left\{ \sqrt{\tilde{\omega}_{pq}^2 - \bar{\omega}_{pq}^2} \sinh(\sqrt{\tilde{\omega}_{pq}^2 - \bar{\omega}_{pq}^2} \tau_0) \right. \\ &\quad \left. + \frac{d}{d\tau_0} [A(a_+, b) + A(a_-, b)] \right\} \end{aligned} \quad (78)$$

then the function A in (76) is decomposed into two parts, $A(a_\pm, b) = \mp 1/2(b\sqrt{1 + a_\pm^2})[b^{-1}e^{a_\pm b} J_{e0}(a_\pm, b)]$, where the derivative of the first part is $d/d\tau_0(b\sqrt{1 + a_\pm^2}) = \mp j\bar{\omega}_{pq}$. The derivative of the second part $d/d\tau_0[b^{-1}e^{a_\pm b} J_{e0}(a_\pm, b)]$ is evaluated by using the equivalence

$$\begin{aligned} [b^{-1}e^{a_\pm b} J_{e0}(a_\pm, b)] &= b^{-1} \int_0^b e^{a_\pm(b-x)} J_0(x) dx \\ &= \int_0^1 e^{a_\pm b(1-y)} J_0(by) dy. \end{aligned} \quad (79)$$

Together with the formula $d/d\tau_0 = (\partial(a_\pm b)/\partial\tau_0)(\partial/\partial(a_\pm b)) + (\partial b/\partial\tau_0)(\partial/\partial b)$, with

$\partial(a_{\pm}b)/\partial\tau_0 = \pm\sqrt{\tilde{\omega}_{pq}^2 - \tilde{\omega}_{pq}^2}$ and $\partial b/\partial\tau_0 = -\tilde{\omega}_{pq}^2\tau_0/b$. Then, after tedious steps, we have

$$\begin{aligned} \frac{\partial}{\partial(a_{\pm}b)} \int_0^1 e^{a_{\pm}b(1-y)} J_0(by) dy = \\ = b^{-1} e^{a_{\pm}b} J_{e0}(a_{\pm}, b) - e^{a_{\pm}b} b^{-2} \\ \left[\frac{e^{-a_{\pm}b}}{-a_{\pm}} b J_0(b) - \frac{1}{-a_{\pm}} [J_{e0}(a_{\pm}, b) - J_{e1}(a_{\pm}, b)] \right] \end{aligned} \quad (80)$$

where we have used the property, $J'_0(x) = J_{-1}(x) = -J_1(x)$, and the definition $J_{e1}(a_{\pm}, b) = \int_0^b e^{-ax} x J_1(x) dx$. The remaining derivative is evaluated as

$$\frac{\partial}{\partial b} \int_0^1 e^{a_{\pm}b(1-y)} J_0(by) dy = -b^{-2} e^{a_{\pm}b} J_{e1}(a_{\pm}, b). \quad (81)$$

Combining these two last derivatives leads to

$$\begin{aligned} \frac{d}{d\tau_0} [b^{-1} e^{a_{\pm}b} J_{e0}(a_{\pm}, b)] = \pm\sqrt{\tilde{\omega}_{pq}^2 - \tilde{\omega}_{pq}^2} \cdot \frac{1}{a_{\pm}b} \{ J_0(b) \\ + b^{-1} e^{a_{\pm}b} J_{e0}(a_{\pm}, b)(a_{\pm}b - 1) + b^{-1} e^{a_{\pm}b} J_{e1}(a_{\pm}, b) \} \\ + \tilde{\omega}_{pq}^2 \tau_0 b^{-3} e^{a_{\pm}b} J_{e1}(a_{\pm}, b). \end{aligned} \quad (82)$$

In summary, the derivatives in (78) are written as

$$\begin{aligned} \frac{d}{d\tau_0} A(a_{\pm}, b) = \\ = \pm \sqrt{1 + a_{\pm}^2} \frac{\sqrt{\tilde{\omega}_{pq}^2 - \tilde{\omega}_{pq}^2}}{a_{\pm}} \{ J_0(b) \\ + b^{-1} e^{a_{\pm}b} J_{e0}(a_{\pm}, b)(a_{\pm}b - 1) + b^{-1} e^{a_{\pm}b} J_{e1}(a_{\pm}, b) \} \\ + \sqrt{1 + a_{\pm}^2} \tilde{\omega}_{pq}^2 \tau_0 b^{-2} e^{a_{\pm}b} J_{e1}(a_{\pm}, b) \\ \mp j\tilde{\omega}_{pq} b^{-1} e^{a_{\pm}b} J_{e0}(a_{\pm}, b). \end{aligned} \quad (83)$$

After using the recurrence relation [15]

$$\begin{aligned} J_{e1}(a_{\pm}, b) = \frac{1}{1 + a_{\pm}^2} \{ J_{e0}(a_{\pm}, b) \\ - e^{-a_{\pm}b} b [J_0(b) + a_{\pm} J_1(b)] \} \end{aligned} \quad (84)$$

and various algebraic manipulations, (83) is transformed into

$$\begin{aligned} \frac{d}{d\tau_0} A(a_{\pm}, b) = - \left[\sqrt{\tilde{\omega}^2 - \tilde{\omega}^2} \sqrt{1 + a_{\pm}^2} J_{e0}^{\pm} e^{a_{\pm}b} \right. \\ \left. + j\tilde{\omega} J_0(b) + \tilde{\omega}^2 t J_1(b) \right]. \end{aligned} \quad (85)$$

Grouping this last result with the cosh in (78), split into exponentials, leads to the result (37) with (39).

REFERENCES

- [1] L. B. Felsen and F. Capolino, "Time domain Green's function for an infinite sequentially excited periodic line array of dipoles," *IEEE Trans. Antennas Propag.*, vol. 48, pp. 921–931, Jun. 2000.
- [2] F. Capolino and L. B. Felsen, "Frequency and time domain Green's functions for a phased semi-infinite periodic line array of dipoles," *IEEE Trans. Antennas Propag.*, vol. 50, pp. 31–41, Feb. 2002.
- [3] F. Capolino and L. B. Felsen, "Time domain Green's functions for an infinite sequentially excited periodic planar array of dipoles," *IEEE Trans. Antennas Propag.*, vol. 51, pp. 160–170, Feb. 2003.

- [4] F. Capolino and L. B. Felsen, "Short-pulse radiation by a sequentially excited semi-infinite periodic planar array of dipoles," *Radio Science-Invited Paper*, vol. 38, no. 2, Mar.-Apr. 2003.
- [5] F. Capolino, "A periodicity-induced generalized Fourier transform pair," *IEEE Trans. Antennas Propag.*, vol. 50, pp. 1167–1169, Aug. 2002.
- [6] N. Chen, M. Lu, F. Capolino, B. Shanker, and E. Michielssen, "Floquet-wave-based analysis of transient scattering from doubly periodic, discretely planar, perfectly conducting structures," *Radio Sci.*, vol. 40, no. 4, Aug. 2005.
- [7] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*. Englewood Cliffs, NJ: Prentice-Hall, 1973.
- [8] F. Capolino and L. B. Felsen, "Toward a network-oriented theory for sequentially pulse-excited periodic arrays: Background and current status," in *Electromagnetics in a Complex World: Challenges and Perspectives*. Berlin: Springer-Verlag, 2004, pp. 259–274.
- [9] G. Marrocco and F. Capolino, "A combined Floquet-wave – FDTD algorithm for the modeling of transient plane waves obliquely incident on periodic structures," in *Electromagnetics in a Complex World: Challenges and Perspectives*. Berlin: Springer-Verlag, 2004, pp. 249–257.
- [10] W. Geyi, "A time-domain theory of waveguide," *PIER (Progress in EM Research)*, vol. 59, pp. 267–297, 2006.
- [11] S. L. Dvorak, "Exact, closed-form expressions for transient fields in homogeneously filled waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 2164–2170, Nov. 1994.
- [12] S. L. Dvorak and D. G. Dudley, "Propagation of ultra-wide band electromagnetic pulses through dispersive media," *IEEE Trans. Electrom. Comput.*, vol. 37, no. 2, pp. 192–200, May 1995.
- [13] H.-Y. Pao, S. L. Dvorak, and D. G. Dudley, "An accurate and efficient analyses for transient plane waves obliquely incident on a conductive half space (TE case)," *IEEE Trans. Antennas Propag.*, vol. 44, pp. 918–924, Jul. 1996.
- [14] S. L. Dvorak and E. F. Kuester, "Numerical computation of the incomplete Lipschitz-Hankel integral $J_{e0}(a, z)$," *J. Comput. Phys.*, vol. 87, no. 2, pp. 301–327, Apr. 1990.
- [15] S. L. Dvorak, "Applications for incomplete Lipschitz-Hankel integrals in electromagnetics," *IEEE Antennas Propag. Mag.*, vol. 36, no. 4, pp. 26–32, Dec. 1994.
- [16] J. Van Bladel, *Electromagnetic Fields*. New York: Taylor and Francis, 1985.
- [17] P. Hubral and M. Tygel, "Analysis of the Rayleigh pulse," *Geophysics*, vol. 54, no. 5, pp. 654–658, May 1989.
- [18] G. W. Hanson and A. B. Yakovlev, "Leaky-wave analysis of transient fields due to sources in planar layered media," *IEEE Trans. Antennas Propag.*, vol. 51, p. 146, Feb. 2003.
- [19] J. C. Oh, E. J. Rothwell, D. P. Nyquist, and M. J. Harvilla, "Natural resonance representation of the conductor-backed lossy layer," *J. Electromagn. Waves Applicat.*, vol. 17, no. 5, pp. 673–694, 2003.
- [20] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1970.
- [21] A. Erdélyi *et al.*, *Tables of Integral Transforms*. New York: McGraw-Hill, 1954, vol. I.

Filippo Capolino (S'94-M'97-SM'04) was born in Florence, Italy, in 1967. He received the Laurea degree (*cum laude*) in electronic engineering and the Ph.D. degree from the University of Florence, Florence, Italy, in 1993 and 1997, respectively.

He is presently employed as an Assistant Professor at the Department of Information Engineering of the University of Siena, Italy. From 2000 to 2001 and in 2006, he was a Research Assistant Visiting Professor with the Department of Electrical and Computer Engineering, University of Houston, Houston, TX, where he is also an Adjunct Assistant Professor. From November to December 2003, he was an Invited Assistant Professor at the Institut Fresnel, Marseille, France. His research interests include theoretical and applied electromagnetics focused on high-frequency, short-pulse radiation, array antennas, periodic structures, numerical modelling, and metamaterials. He is the coordinator of the Siena Unit for the European Network of Excellence "Metamorphose" on Metamaterials.

Dr. Capolino was awarded with an MMET'94 Student Paper Competition Award in 1994, the Raj Mittra Travel Grant for Young Scientists in 1996, and

for Senior Scientists in 2006, the “Barzilai” prize for the best paper at the National Italian Congress of Electromagnetism (XI RiNEm) in 1996, and a Young Scientist Award for participating at the URSI Int. Symposium on Electromagnetics Theory in 1998. He received the R. W. P. King Prize Paper Award from the IEEE Antennas and Propagation Society for the Best Paper of the Year 2000, by an author under 36. He is an Associate Editor for the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION and for the journal *Metamaterials*.

Leopold B. Felsen, *deceased*, (S'47–M'54–SM'55–F'62–LF'90) was born in Munich, Germany, on May 7, 1924. He received the B.E.E., M.E.E., and D.E.E. degrees from the Polytechnic Institute of Brooklyn, Brooklyn, NY, in 1948, 1950, and 1952, respectively.

He emigrated to the United States in 1939 and served in the U.S. Army from 1943 to 1946. After 1952, he remained with the Polytechnic (now Polytechnic University), becoming University Professor in 1978. From 1974 to 1978, he was Dean of Engineering. In 1994, he resigned from the

full-time Polytechnic faculty and was granted the status of University Professor Emeritus. He was Professor of aerospace and mechanical engineering and Professor of electrical and computer engineering at Boston University, Boston, MA (part-time). He was the author or coauthor of more than 350 papers and of several books, including *Radiation and Scattering of Waves* (Piscataway, NJ: IEEE Press, 1994). He was an Associate Editor of several professional journals and was an Editor of the *Wave Phenomena Series* (New York: Springer-Verlag). His research interests encompassed wave propagation and diffraction in complex environments and in various disciplines, high-frequency asymptotic and short-pulse techniques, and phase-space methods with an emphasis on wave-oriented data processing and imaging.

Dr. Felsen was a Member of Sigma Xi and a Fellow of the Optical Society of America and the Acoustical Society of America. He has held named Visiting Professorships and Fellowships at universities in the United States and abroad, including the Guggenheim in 1973 and the Humboldt Foundation Senior Scientist Award in 1981. In 1974 he was an IEEE Antennas and Propagation Society

(APS) Distinguished Lecturer. His “Poet’s Corner” appeared sporadically in the IEEE/APS Magazine. He received the IEEE/APS Best Paper Award for 1969 and was best paper coauthor for 1974 and 1981. He was a contributing author to papers selected for the R. W. P. King Award for 1984, 1986, and 2000. He received the Balthasar van der Pol Gold Medal from the International Union of Radio Science (URSI) in 1975, an Honorary Doctorate from the Technical University of Denmark in 1979, the IEEE Heinrich Hertz Gold Medal for 1991, the APS Distinguished Achievement Award for 1998, the IEEE Third Millennium Medal in 2000, an Honorary Laurea degree from the University of Sannio in Benevento, Italy in 2003, the IEEE Electromagnetics Award for 2003, an honorary doctorate from the Technical University of Munich, Germany in 2004, an Honorary Doctorate from Dogus University in Istanbul, Turkey, three Distinguished Faculty Alumnus Awards from Polytechnic University, and an IEEE Centennial Medal in 1984. In 1977, he was elected to the National Academy of Engineering. He served on the APS Administrative Committee from 1963 to 1966 and was Vice Chairman and Chairman for both the US (1966–1973) and the International (1978–1984) URSI Commission B.

Alessandro Della Villa was born in Rome, Italy, on October 26, 1974. He received the Laurea degree in Telecommunication Engineering from the University of Siena, Siena, Italy, in 2004, where he is working toward the Doctoral Degree.

From April 2004 to September 2004, he was a Visiting Scholar at the Fresnel Institute, University Paul Cézanne Aix-Marseille III, Marseille, France, and from April 2005 to May 2005, he was a Visiting Scholar at the Department of Electrical and Computer Engineering, University of Houston, Houston,

TX. He is also working as a consultant for CommProve, a wireless telecommunications company. His scientific interests include theoretical and numerical studies of propagation and radiation in quasi crystals, periodic structures, leaky wave antennas. He has a strong interest in computer science and its innovative applications on the applied electromagnetic research field.