Title

Field Harmonics in the 18 cm Wide SUPERBEND Dipole Magnet

Author

Caspi, S.

Publication Date

1995-03-24

SC-MAG-494

LBL-36988

Field Harmonics in the 18 cm Wide *SUPERBEND* Dipole Magnet.*

Shlomo Caspi and Clyde Taylor

Lawrence Berkeley Laboratory
University Of California
Berkeley, CA 94720

March, 21, 1995

* This was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, High Energy Physics Division, U. S. Department of Energy, under Contract No. DE-AC03-76SF00098.

Introduction

Superbend, a superconducting dipole magnet for the Advance Light Source (ALS), will replace existing conventional dipole magnets with a high field superconducting magnet that will fit directly over the existing beam structure. The width of this short dipole (12 cm pole length along the beam) is in the process of being optimized with values currently considered between 18 and 24 cm. Whereas a wide (24 cm pole or more) structure has an inherent low field harmonic content, a reduced pole width with an 18 cm pole that operates at the same 5 Tesla field point will suffer from a field with reduced quality. Such a reduction in field quality may however be well justified in terms of cost saving on a smaller structure as long as the increase in harmonic content is manageable by the existing correction scheme of the ALS.

The Superconducting High Field Magnet Group has developed a format for harmonic representation which is especially useful in short highly 3D magnets[bc]. The above procedure was used in conjunction with the Superband magnet design where field points calculated by the program TOSCA (18 cm transverse case) are used in calculating various A's values (summary and results included in the Appendix). In this note we shall develop the field expansion as a function of x in terms of the local A's functions and compare such results with a similar expansion that follows an integration along the magnet z axis.

Local lens approximation

In beam dynamics simulations the Superbend magnet is treated as a thin lens approximation where the field and coefficients correspond to a Taylor expansion series along the magnet midplane x, at z=0.

$$B_y(x,0,0) = \sum_{n=1} b_n x^{n-1} = \sum_{n=1} \frac{1}{(n-1)!} \frac{\partial^{n-1} B_y(x,0,0)}{\partial x^{n-1}} \Big|_{x=0} x^{n-1} \tag{1}$$

The above expression does not correspond to the integrated field (in z) and therefore care should be taken in the interpretation of such coefficients. In a short magnet such coefficients are NOT what are commonly called the normal multipole —e.g. quadrupole , sextupole etc, but rather a combination of both normal and pseudo multipole harmonics. That can easily be seen by examining the expression for B_y at y=0 derived from the 3D analysis :

$$B_y(x,0,z) = \sum_{n=1} \sum_{k=0} (-1)^{k+1} \frac{n!n}{2^{2k} k!(n+k)!} A_n^{(2k)}(z) x^{2k+n-1} \tag{2}$$

or explicitly :

$$\begin{aligned} -B_y(x,0,z) = {} & A_1 + 2A_2 x + \left(3A_3 - \frac{1}{8}A_1''\right)x^2 + \left(4A_4 - \frac{1}{6}A_2''\right)x^3 \\ & + \left(5A_5 - \frac{3}{16}A_3'' + \frac{1}{96}A_1''''\right)x^4 + \left(6A_6 - \frac{1}{5}A_4'' + \frac{1}{192}A_2''''\right)x^5 \\ & + \left(7A_7 - \frac{5}{24}A_5'' + \frac{3}{640}A_3'''' - \frac{1}{9216}A_1^{(6)}\right)x^6 \\ & + \left(8A_8 - \frac{3}{14}A_6'' + \frac{1}{240}A_4'''' - \frac{1}{11520}A_2^{(6)}\right)x^7 + \cdots \end{aligned}$$

[b] 3D Field Harmonics — S.Caspi , M.Helm , and L.J. Laslett , SC-MAG-328 , LBL-30313 , March 1991.

[c] An Approach To 3D Magnetic Field Calculation Using Numerical and Differential Algebra Methods — S.Caspi , M.Helm , and L.J. Laslett , SC-MAG-395 , LBL-32624 , July 1992.

where A_n are functions of z and A_n primes denote differentiation with respect to z. The coefficient associated with such a polynomial expansion (Eq.1 and Eq.2) are therefore equivalent :

$$
\begin{aligned}
B_y(0,0,z) &= -A_1(z) \\
\frac{\partial B_y(0,0,z)}{\partial x} &= -2A_2 \\
\frac{1}{2}\frac{\partial^2 B_y(0,0,z)}{\partial x^2} &= -\left(3A_3 - \frac{1}{8}A_1''\right) \\
\frac{1}{6}\frac{\partial^3 B_y(0,0,z)}{\partial x^3} &= -\left(4A_4 - \frac{1}{6}A_2''\right) \\
\frac{1}{24}\frac{\partial^4 B_y(0,0,z)}{\partial x^4} &= -\left(5A_5 - \frac{3}{16}A_3'' + \frac{1}{96}A_1''''\right) \\
\frac{1}{120}\frac{\partial^5 B_y(0,0,z)}{\partial x^5} &= -\left(6A_6 - \frac{1}{5}A_4'' + \frac{1}{192}A_2''''\right) \\
\frac{1}{720}\frac{\partial^6 B_y(0,0,z)}{\partial x^6} &= -\left(7A_7 - \frac{5}{24}A_5'' + \frac{3}{640}A_3'''' - \frac{1}{9216}A_1^{(6)}\right) \\
\frac{1}{5040}\frac{\partial^7 B_y(0,0,z)}{\partial x^7} &= -\left(8A_8 - \frac{3}{14}A_6'' + \frac{1}{240}A_4'''' - \frac{1}{11520}A_2^{(6)}\right) \\
&\ldots
\end{aligned}
\tag{3}
$$

Equation 2 may also be normalized (specifically at z=0) :

$$
\begin{aligned}
\frac{-B_y(x,0,0)}{B_y(0,0,0)} &= 1 + 2\frac{A_2(0)}{A_1(0)}x + \left(3\frac{A_3(0)}{A_1(0)} - \frac{1}{8}\frac{A_1''(0)}{A_1(0)}\right)x^2 \\
&+ \left(4\frac{A_4(0)}{A_1(0)} - \frac{1}{6}\frac{A_2''(0)}{A_1(0)}\right)x^3 \ldots
\end{aligned}
\tag{4}
$$

Thin Lens Approximation

In a true thin lens approximation the above expressions will have to be integrated in z and averaged over a "magnetic length" corresponding to a "hard edge" magnet approximation. In applying the integration to Eq. 3 we may note that all pseudo harmonics integrate to zero, that is:

$$
\int_{-\infty}^{+\infty} A_n^{(k)}(z)dz = 0
$$

and Eq. 3 assumes the form :

$$
\begin{aligned}
\int_{-\infty}^{+\infty} B_y(0,0,z)dz &= -\int_{-\infty}^{+\infty} A_1 dz \\
\int_{-\infty}^{+\infty} \frac{\partial B_y(0,0,z)}{\partial x} dz &= -2\int_{-\infty}^{+\infty} A_2 dz \\
\frac{1}{2}\int_{-\infty}^{+\infty} \frac{\partial^2 B_y(0,z)}{\partial x^2} dz &= -3\int_{-\infty}^{+\infty} A_3 dz \\
\frac{1}{6}\int_{-\infty}^{+\infty} \frac{\partial^3 B_y(0,0,z)}{\partial x^3} dz &= -4\int_{-\infty}^{+\infty} A_4 dz \\
\frac{1}{24}\int_{-\infty}^{+\infty} \frac{\partial^4 B_y(0,0,z)}{\partial x^4} dz &= -5\int_{-\infty}^{+\infty} A_5 dz \\
\frac{1}{120}\int_{-\infty}^{+\infty} \frac{\partial^5 B_y(0,0,z)}{\partial x^5} dz &= -6\int_{-\infty}^{+\infty} A_6 dz \\
&\cdots
\end{aligned}
\tag{5}
$$

If we now define a "magnetic length" :

$$
L = \frac{\int_{-\infty}^{\infty} B_y(0,0,z)dz}{B_y(0,0,0)} = \frac{\int_{-\infty}^{\infty} A_1(z)dz}{A_1(0)}
$$

we can express the average field expansion as :

$$
\overline{B_y}(x,0) = \frac{1}{L}\int_{-\infty}^{\infty} B_y(x,0,z)dz = -\sum_{n=1} nA_1(0)\frac{\int_{-\infty}^{\infty} A_n(z)dz}{\int_{-\infty}^{\infty} A_1(z)dz}x^{n-1}
$$

and therefore

$$
\frac{1}{(n-1)!}\frac{\partial^{n-1}\overline{B_y}(x,0)}{\partial x^{n-1}}\Big|_{x=0} = -nA_1(0)\frac{\int_{-\infty}^{\infty} A_n(z)dz}{\int_{-\infty}^{\infty} A_1(z)dz}
\tag{6}
$$

or in analogy to Eq. 4

$$
\begin{aligned}
\frac{-\overline{B_y}(x,0,0)}{B_y(0,0,0)} &= 1 + 2\frac{\int A_2(z)dz}{\int A_1(z)dz}x + 3\frac{\int A_3(z)dz}{\int A_1(z)dz}x^2 \\
&+ 4\frac{\int A_4(z)dz}{\int A_1(z)dz}x^3 \cdots
\end{aligned}
$$

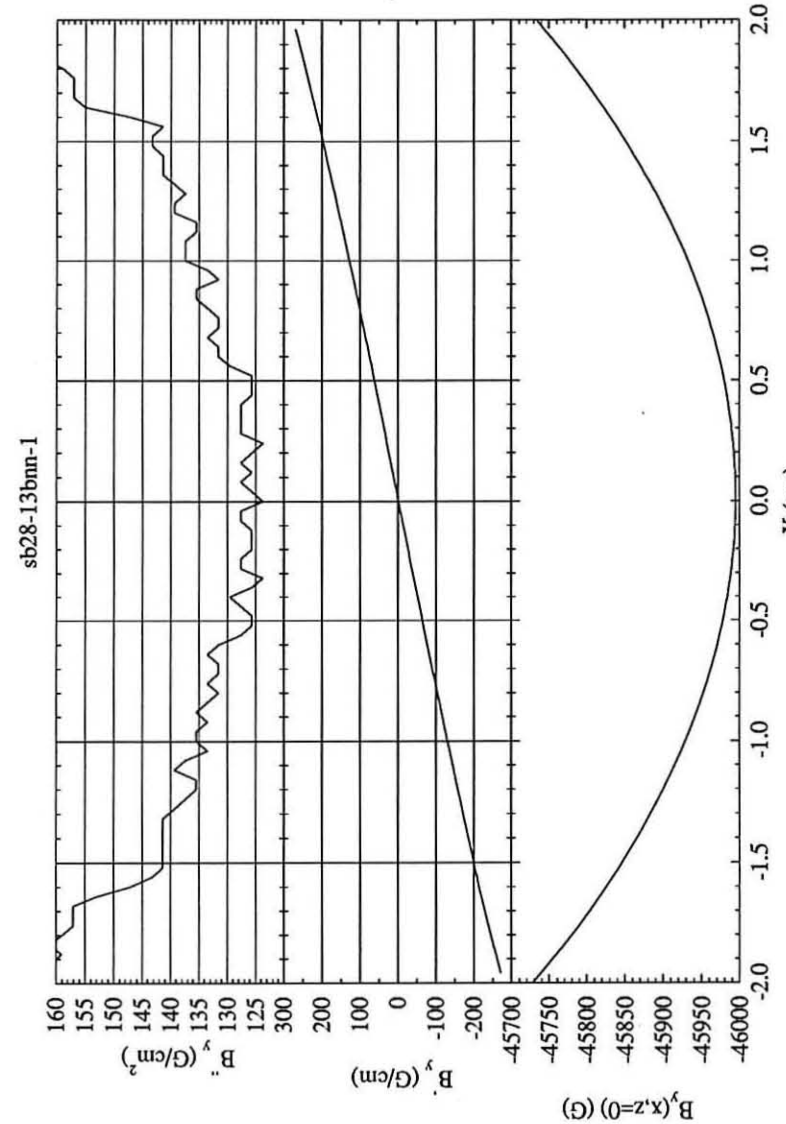


Figure 1 B_y computed with TOSCA as a function of x at z=0 with first and second derivatives.

We have shown that any order differentiation on the axis of B_y with respect to x can be calculated directly or can be evaluated as a combination of normal and pseudo multipole coefficients. Such values may or may not differ from a similar differentiation performed after all harmonics have been integrated in z. We demonstrate such results using the 18 cm wide Superbend magnet and computations performed by the program TOSCA. We have calculated B_y at several x locations (y=0, z=0) and performed a numerical differentiation with respect to x. We have also calculated B_x, B_y and B_z on a grid and have used the program FIGEND to reduce such values into 3D harmonics. Results are compared in the following tables. From Fig.1 the following local values have been computed (18 cm pole) :

$$B_y(0,0,0) = 45995.0 \quad G$$

$$\frac{\partial B_y(0,0,0)}{\partial x} = -1.414 \quad G/cm$$

$$\frac{1}{2}\frac{\partial^2 B_y(0,0,0)}{\partial x^2} = 63.1 \quad G/cm^2$$

$$\frac{1}{24}\frac{\partial^4 B_y(0,0,0)}{\partial x^4} = 0.83 \quad G/cm^4$$

	$A_n(0)$	$A_n^{'}(0)$	$A_n^{''}(0)$	$\int_{-\infty}^{\infty} A_n(z)dz$	$\int_{-\infty}^{\infty} A_n^{'}(z)dz$	$\int_{-\infty}^{\infty} A_n^{''}(z)dz$
n=1	46006.0 G	0.2	-378.2	897763.5	0.02	-8.886
n=2	0.74236 G/cm	-2.3e-5	0.0061	48.37	1e-7	-0.035
n=3	-37.45 G/cm^2	-0.0236	-1.53	-410.53	1e-7	-0.155
n=5	-0.17 G/cm^4	0.00062	-0.042	-3.026	1e-7	0.0013

Table 1 Local and integrated values of A's

$\frac{\partial B_y(0,0,0)}{\partial x}$ Eq. 1	$\frac{\partial B_y(0,0,0)}{\partial x} = -2A_2$ Eq. 3	$\frac{\partial \overline{B_y}(0,0,0)}{\partial x} = -2A_1 \frac{\int_{-\infty}^{\infty} A_2 dz}{\int_{-\infty}^{\infty} A_1 dz}$, Eq. 6
-1.414	-1.48	-4.96

$\frac{1}{2}\frac{\partial^2 B_y(0,0,0)}{\partial x^2}$	$\frac{1}{2}\frac{\partial^2 B_y(0,0,0)}{\partial x^2} = -\left(3A_3 - \frac{1}{8}A_1^{''}\right)$	$\frac{1}{2}\frac{\partial^2 \overline{B_y}(0,0,0)}{\partial x^2} = -3A_1 \frac{\int_{-\infty}^{\infty} A_3 dz}{\int_{-\infty}^{\infty} A_1 dz}$
63.10	65.1	63.10

$\frac{1}{24}\frac{\partial^4 B_y(0,0,0)}{\partial x^4}$	$\frac{1}{24}\frac{\partial^4 B_y(0,0,0)}{\partial x^4} = -\left(5A_5 - \frac{3}{16}A_3^{''} + \frac{1}{96}A_1^{''''}\right)$	$\frac{1}{24}\frac{\partial^4 \overline{B_y}(0,0,0)}{\partial x^4} = -5A_1 \frac{\int_{-\infty}^{\infty} A_5 dz}{\int_{-\infty}^{\infty} A_1 dz}$
0.83	$0.56-\frac{1}{96}A_1^{''''}$	0.775

Table 2 Values of coefficients computed several different ways.

It is interesting to note that the values in column I calculated directly from derivatives of $B_y(x,0,0)$ with respect to x, are very close to the one in column III, computed after integrating B_y in z. There is no apparent reason why that should be so (Jackson Laslett where are you?).

Appendix A 3D harmonic coefficients

The field components inside the curl free region of a magnet can be expressed as :

$$\begin{aligned} B_r &= -\frac{\partial V}{\partial r} = \sum_n \left[g_{rn} r^{n-1} \sin n\theta - \tilde{g}_{rn} r^{n-1} \cos n\theta \right] \\ B_\theta &= -\frac{n}{r} V = \sum_n \left[g_{\theta n} r^{n-1} \cos n\theta + \tilde{g}_{\theta n} r^{n-1} \sin n\theta \right] \\ B_z &= -\frac{\partial V}{\partial z} = \sum_n \left[g_{zn} r^{n} \sin n\theta - \tilde{g}_{zn} r^{n} \cos n\theta \right] \end{aligned} \tag{1}$$

where

$$\begin{aligned} g_{rn} &\equiv \tilde{g}_{rn} \\ g_{\theta n} &\equiv \tilde{g}_{\theta n} \\ g_{zn} &\equiv \tilde{g}_{zn} \end{aligned}$$

are general functions of r and z that include the appropriate "normal" and "skew" terms $A_n(z)$ *and* $\tilde{A}_n(z)$.

In order that the series for the potential V_n satisfy the differential equation we introduce the functions $A_n(z)$ and express the coefficients g_{rn} , $g_{\theta n}$, g_{zn} as general functions of r and z as shown below :

$$\begin{aligned} g_{rn}(r,z) &= \sum_{k=0} (-1)^{k+1} \frac{n!(n+2k)}{2^{2k} k!(n+k)!} A_n^{(2k)}(z) r^{2k} \\ g_{\theta n}(r,z) &= \sum_{k=0} (-1)^{k+1} \frac{n!n}{2^{2k} k!(n+k)!} A_n^{(2k)}(z) r^{2k} \\ g_{zn}(r,z) &= \sum_{k=0} (-1)^{k+1} \frac{n!}{2^{2k} k!(n+k)!} A_n^{(2k+1)} r^{2k} \end{aligned} \tag{2}$$

Explicitly we can write the above as :

$$\begin{aligned} g_{rn}(r,z) &= -nA_n(z) + \frac{n+2}{4(n+1)} A_n''(z) r^2 - \frac{n+4}{32(n+1)(n+2)} A_n''''(z) r^4 \\ &\quad + \frac{n+6}{384(n+1)(n+2)(n+3)} A_n''''''(z) r^6 - \ldots \\ g_{\theta n}(r,z) &= -nA_n(z) + \frac{n}{4(n+1)} A_n''(z) r^2 - \frac{n}{32(n+1)(n+2)} A_n''''(z) r^4 \\ &\quad + \frac{n}{384(n+1)(n+2)(n+3)} A_n''''''(z) r^6 - \ldots \\ g_{zn}(r,z) &= -A_n'(z) + \frac{1}{4(n+1)} A_n'''(z) r^2 - \frac{1}{32(n+1)(n+2)} A_n'''''(z) r^4 \ldots\ldots \end{aligned}$$

For the expressions of the skew terms just replace g_{rn} , $g_{\theta n}$, g_{zn} with $\tilde{g}_{rn}$, $\tilde{g}_{\theta n}$, $\tilde{g}_{zn}$ and $A_n(z)$ with $\tilde{A}_n(z)$

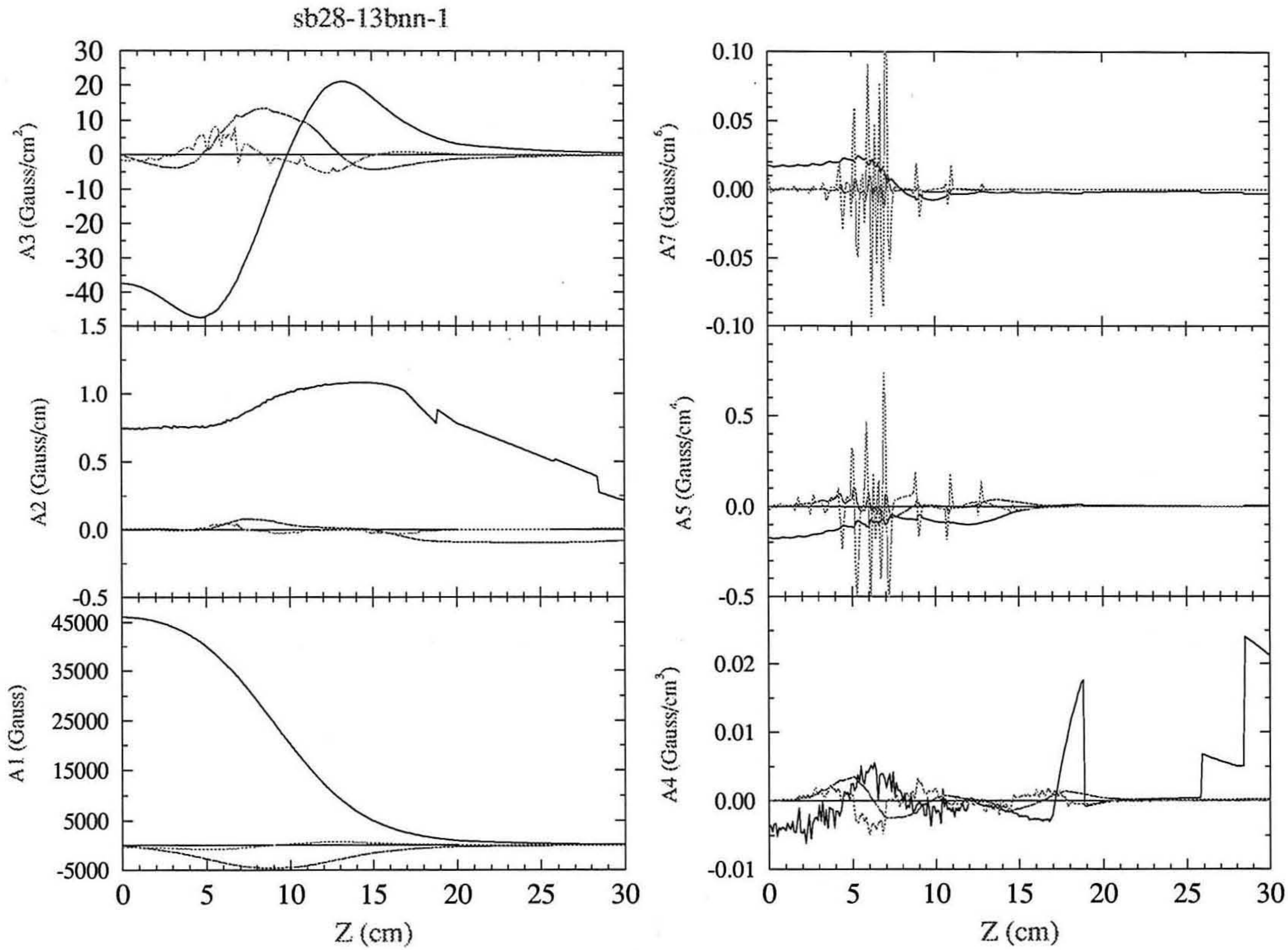


Figure 2 Normal and pseudo multipoles

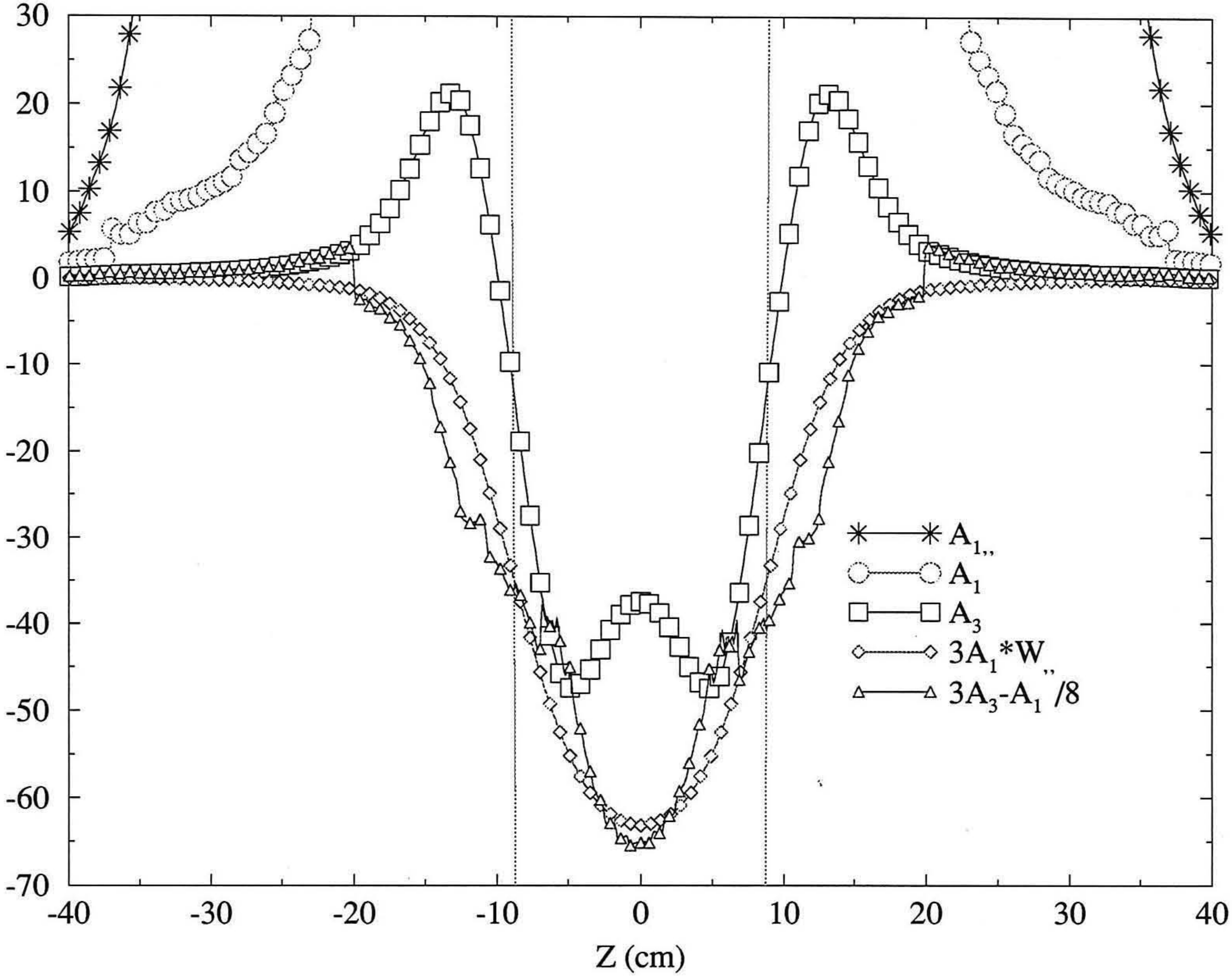
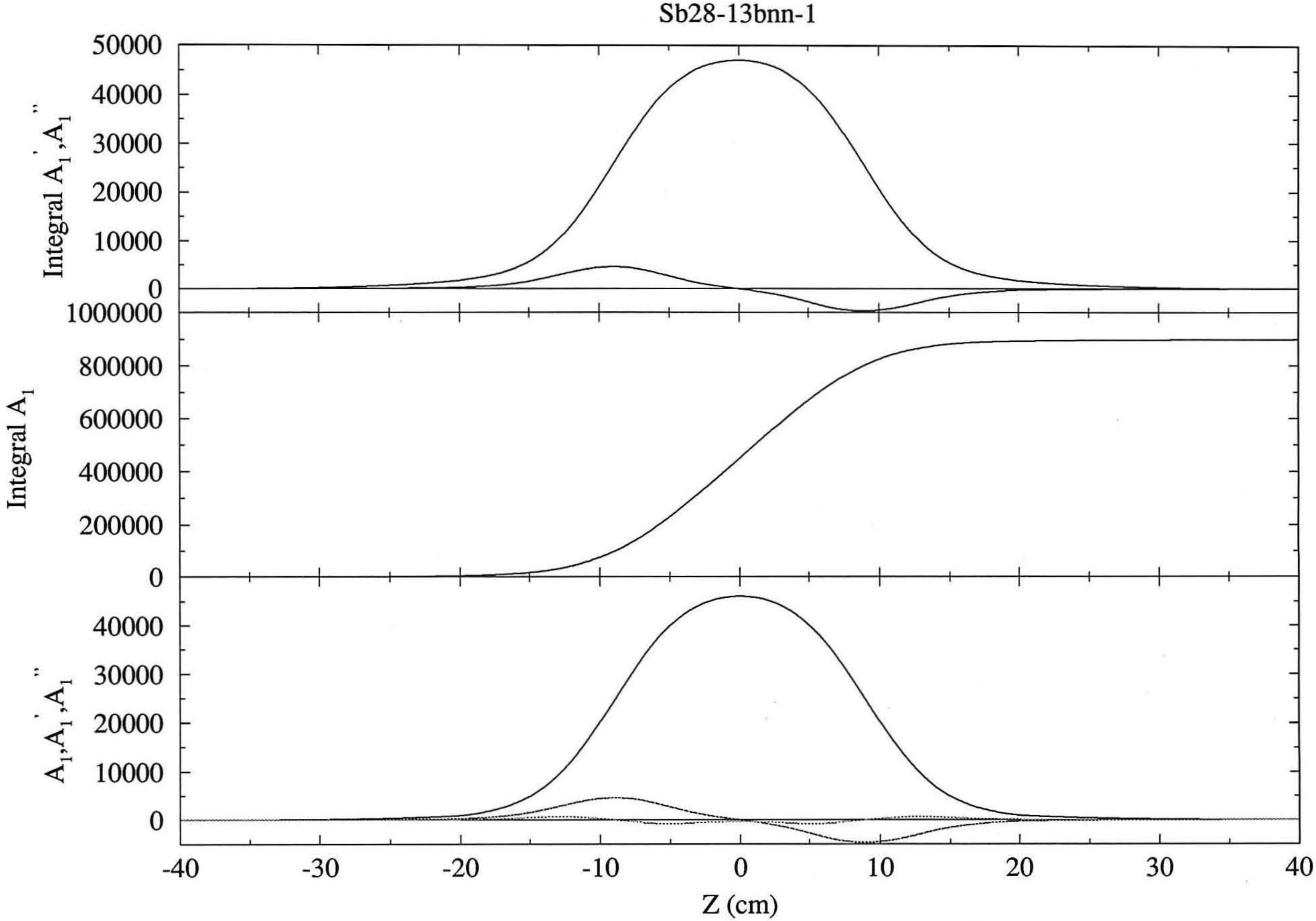
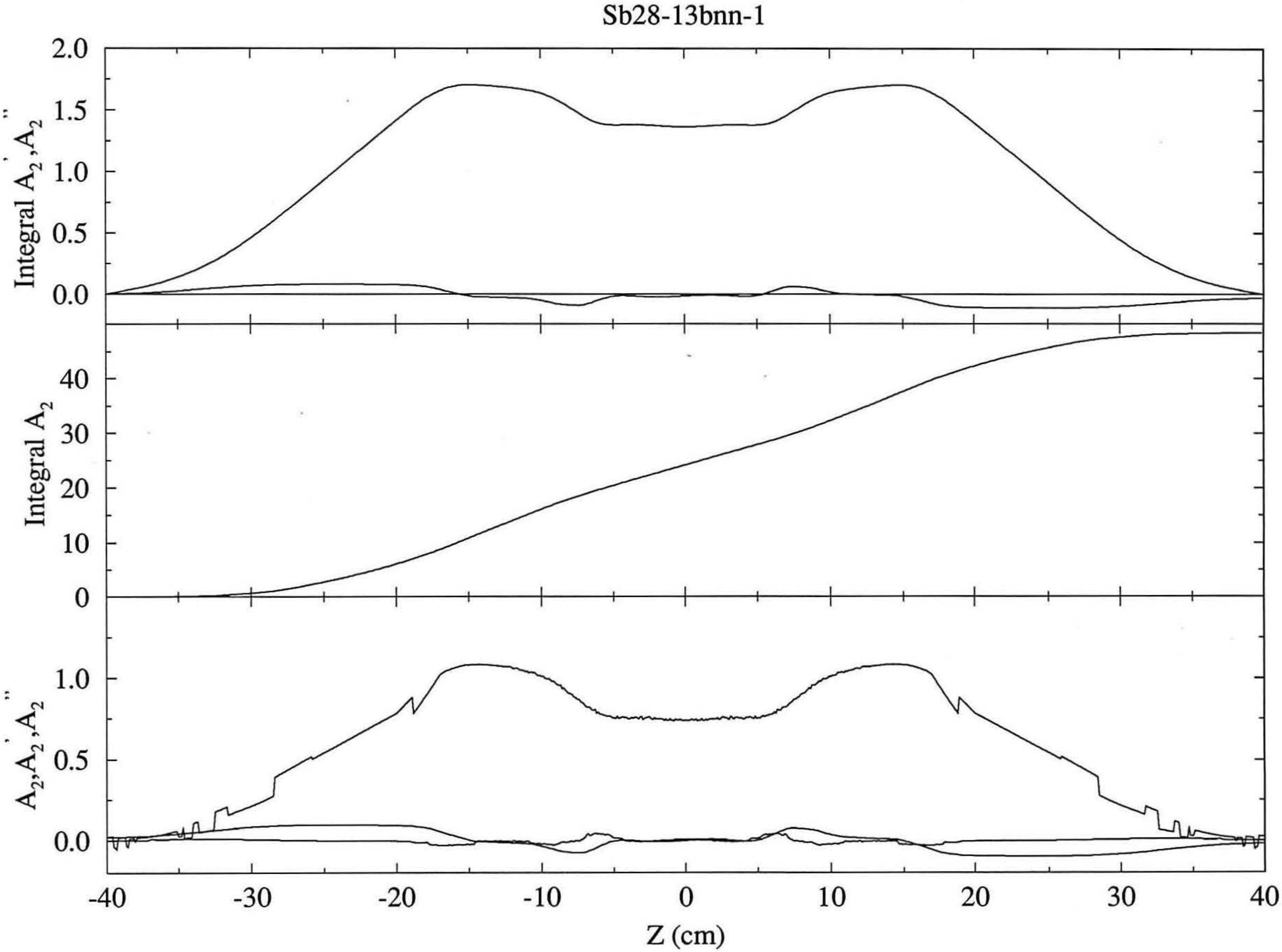


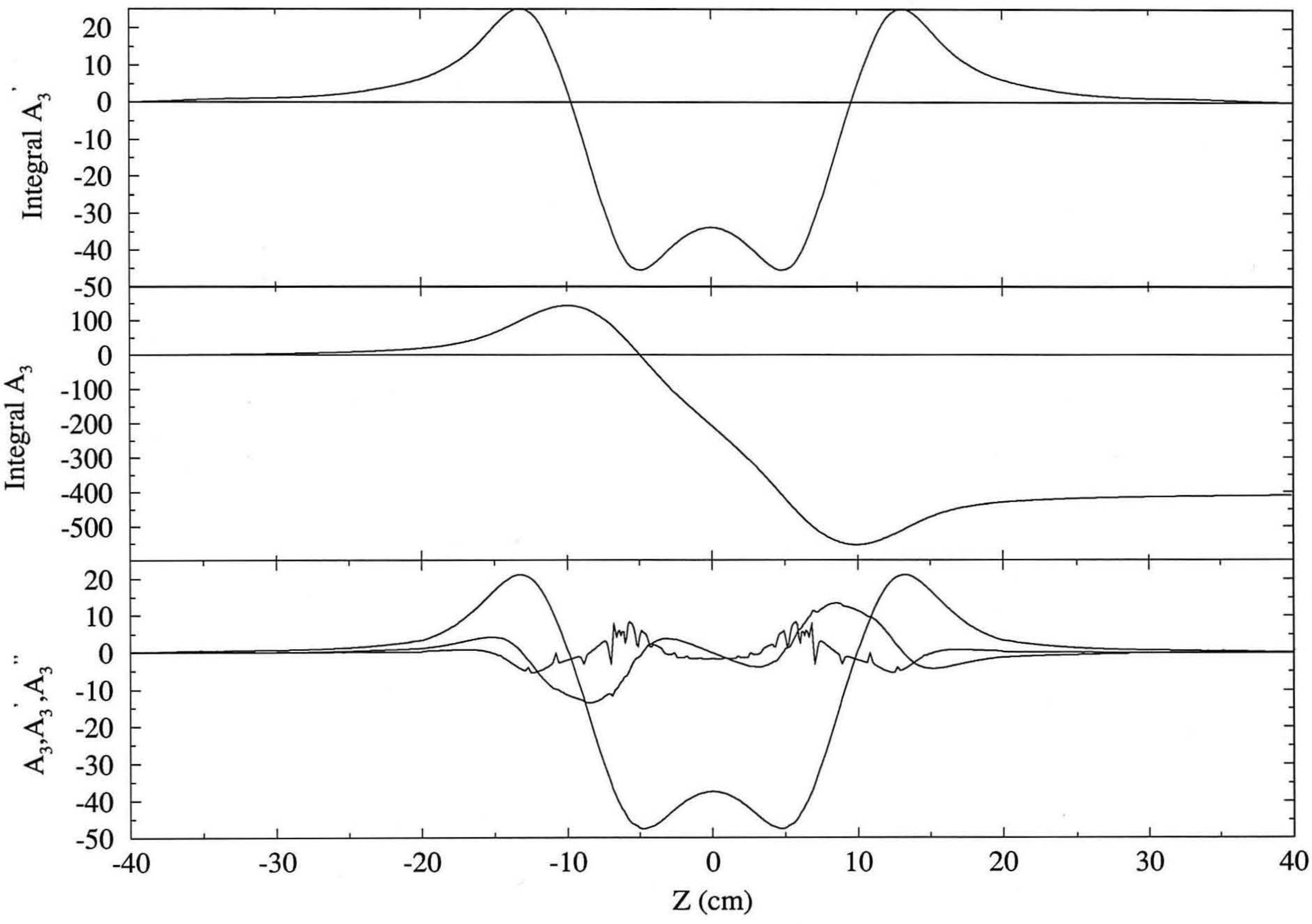
Figure 3 A comparison between local and integrated sextupole ($W = \frac{\int_{-\infty}^{\infty} A_3 dz}{\int_{-\infty}^{\infty} A_1 dz}$)

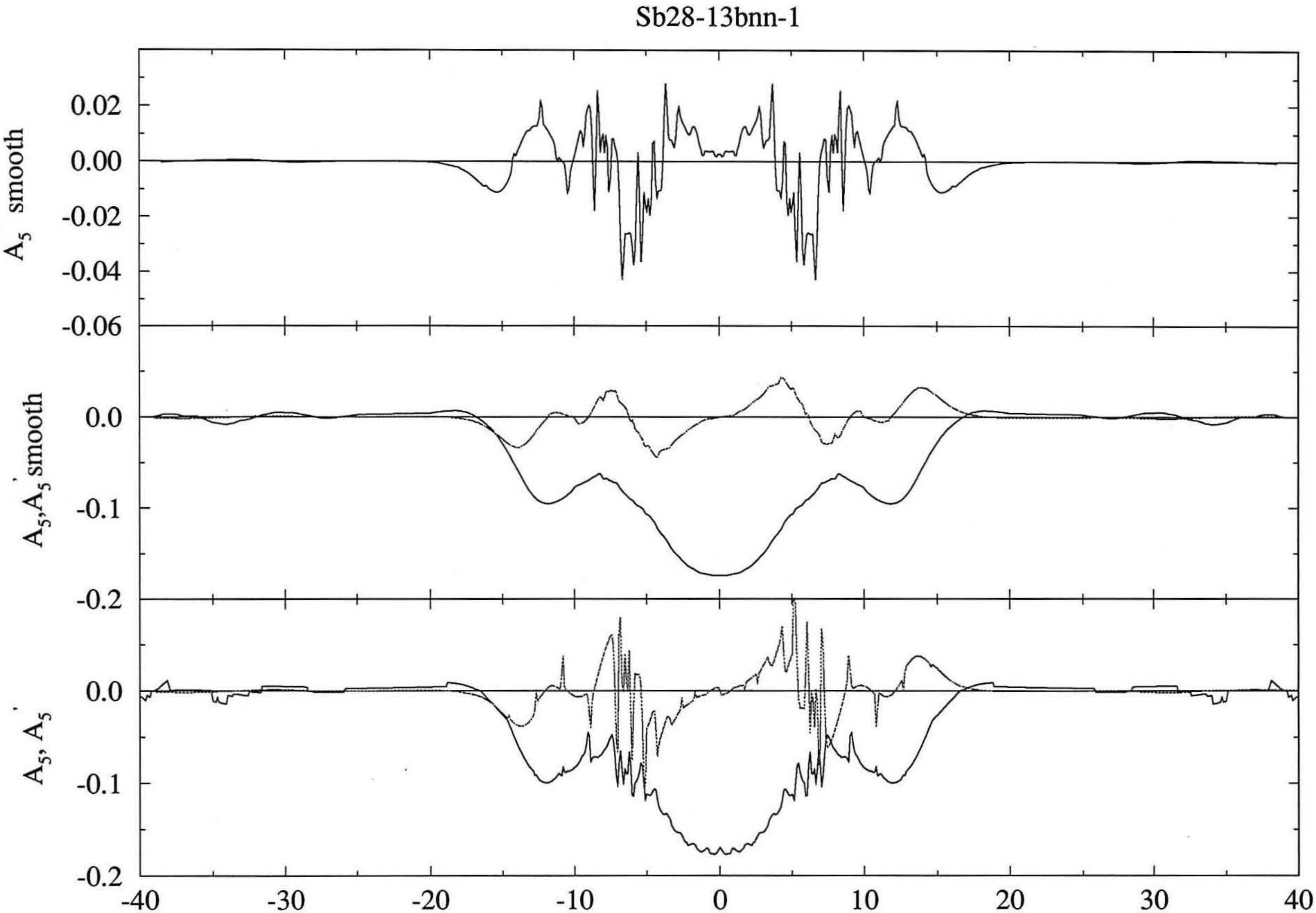

Sb28-13bnn-1
Integral A1',A1"
50000
40000
30000
20000
10000
0
1000000
800000
600000
400000
200000
0
Integral A1
40000
30000
20000
10000
0
A1,A1',A1"
-40
-30
-20
-10
0
10
20
30
40
Z (cm)

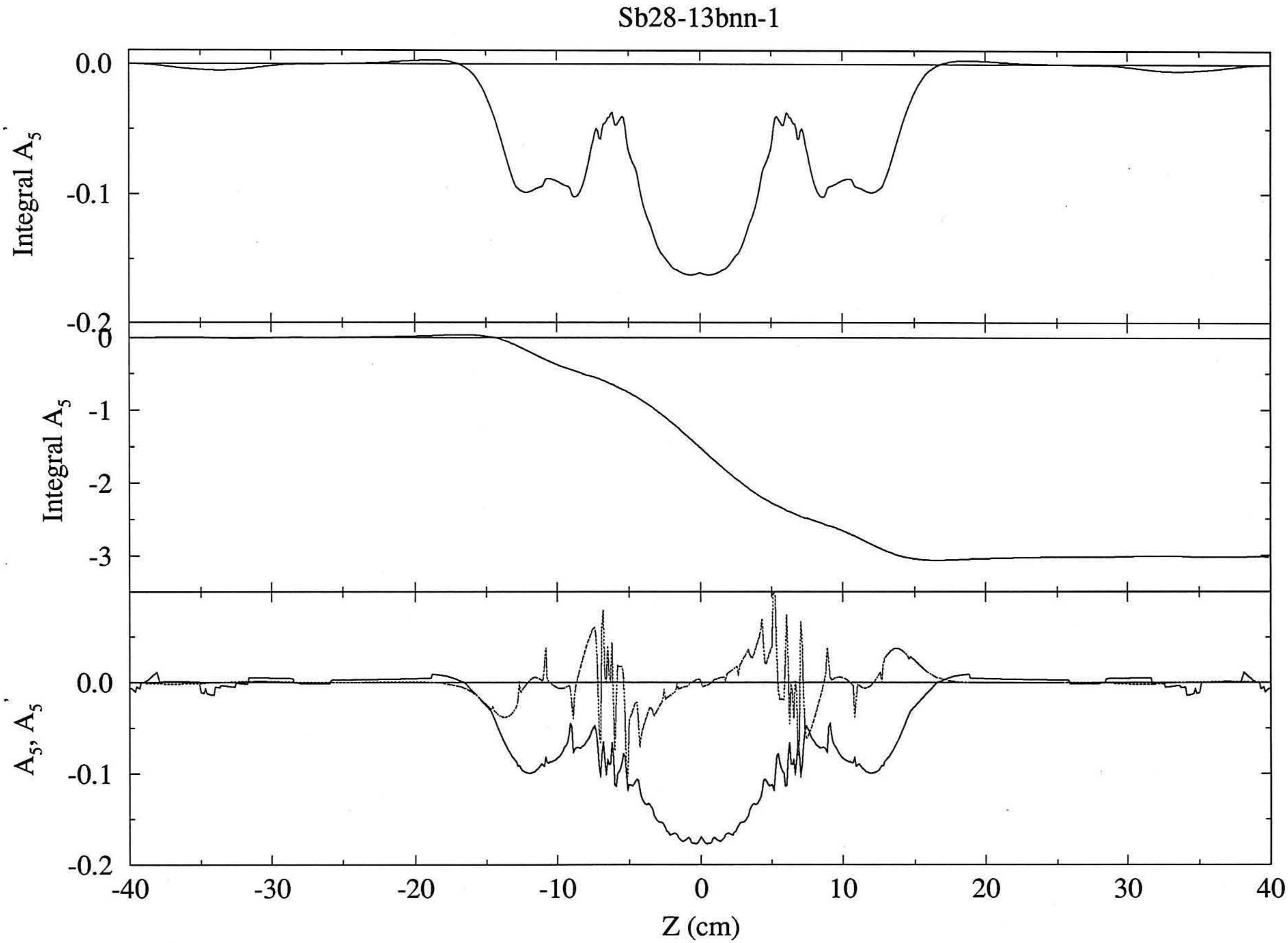


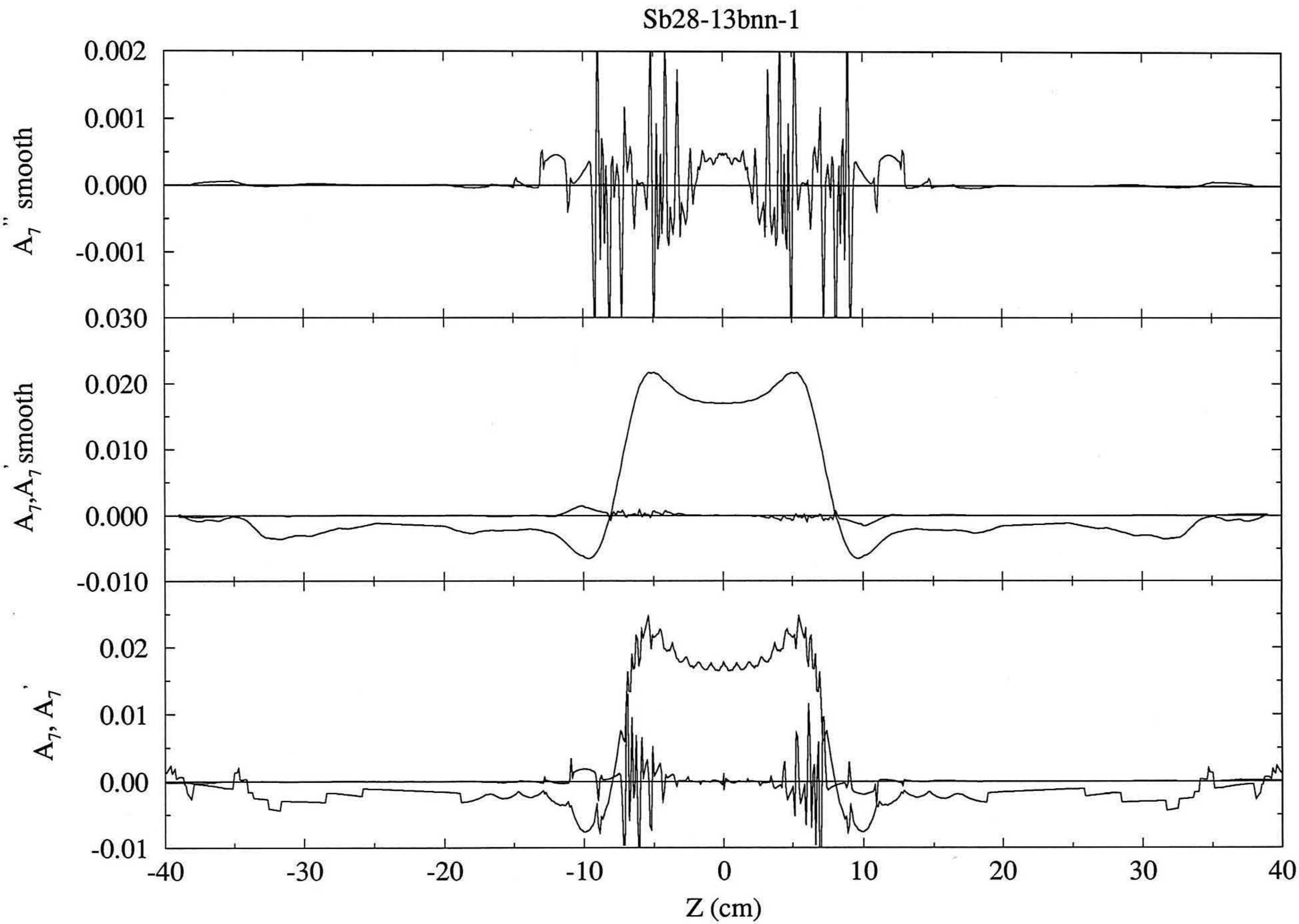
Sb28-13bnn-1
Integral A_2, A_2', A_2''
Integral A_2
A_2, A_2', A_2''
Z (cm)

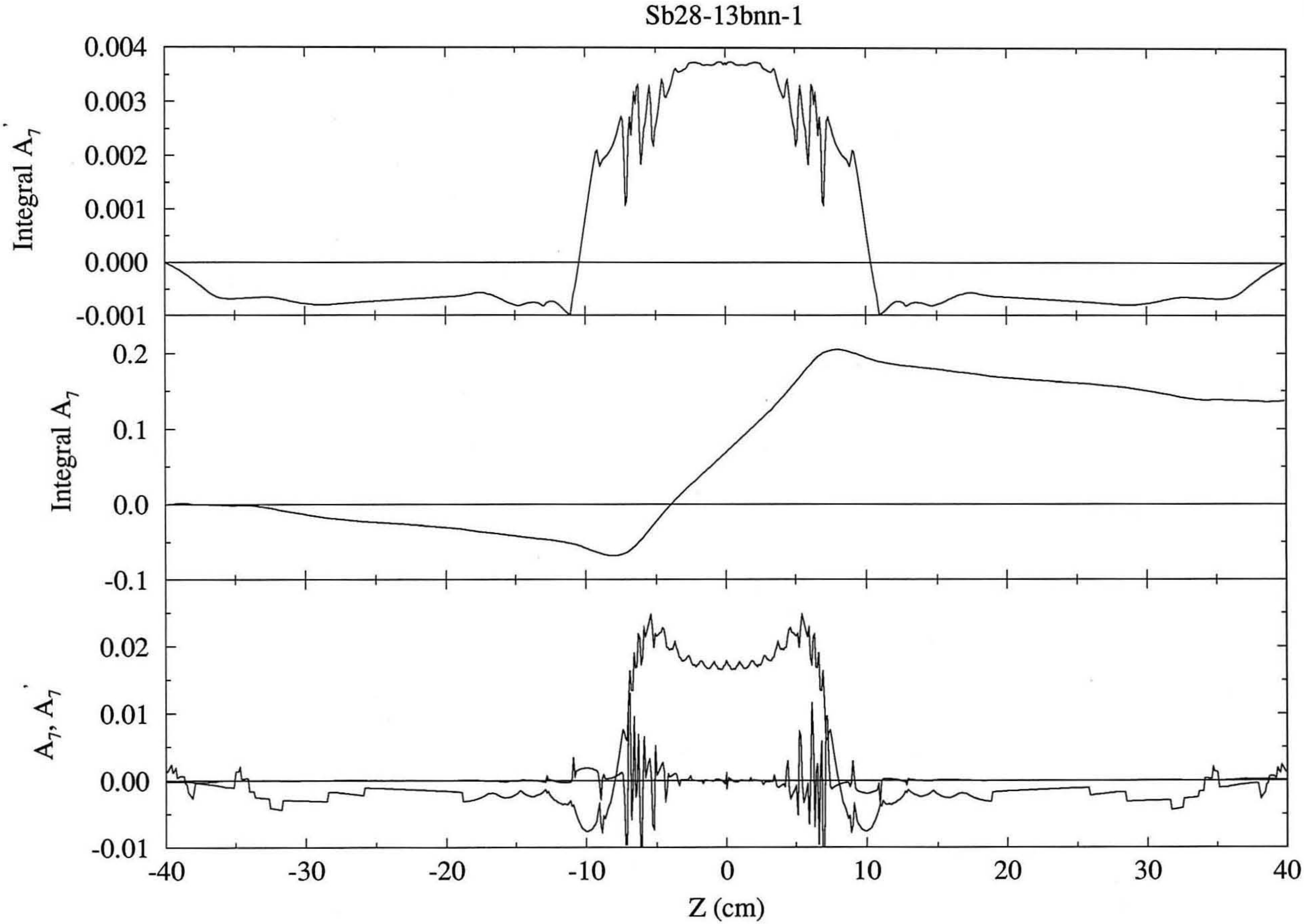
Sb28-13bnn-1

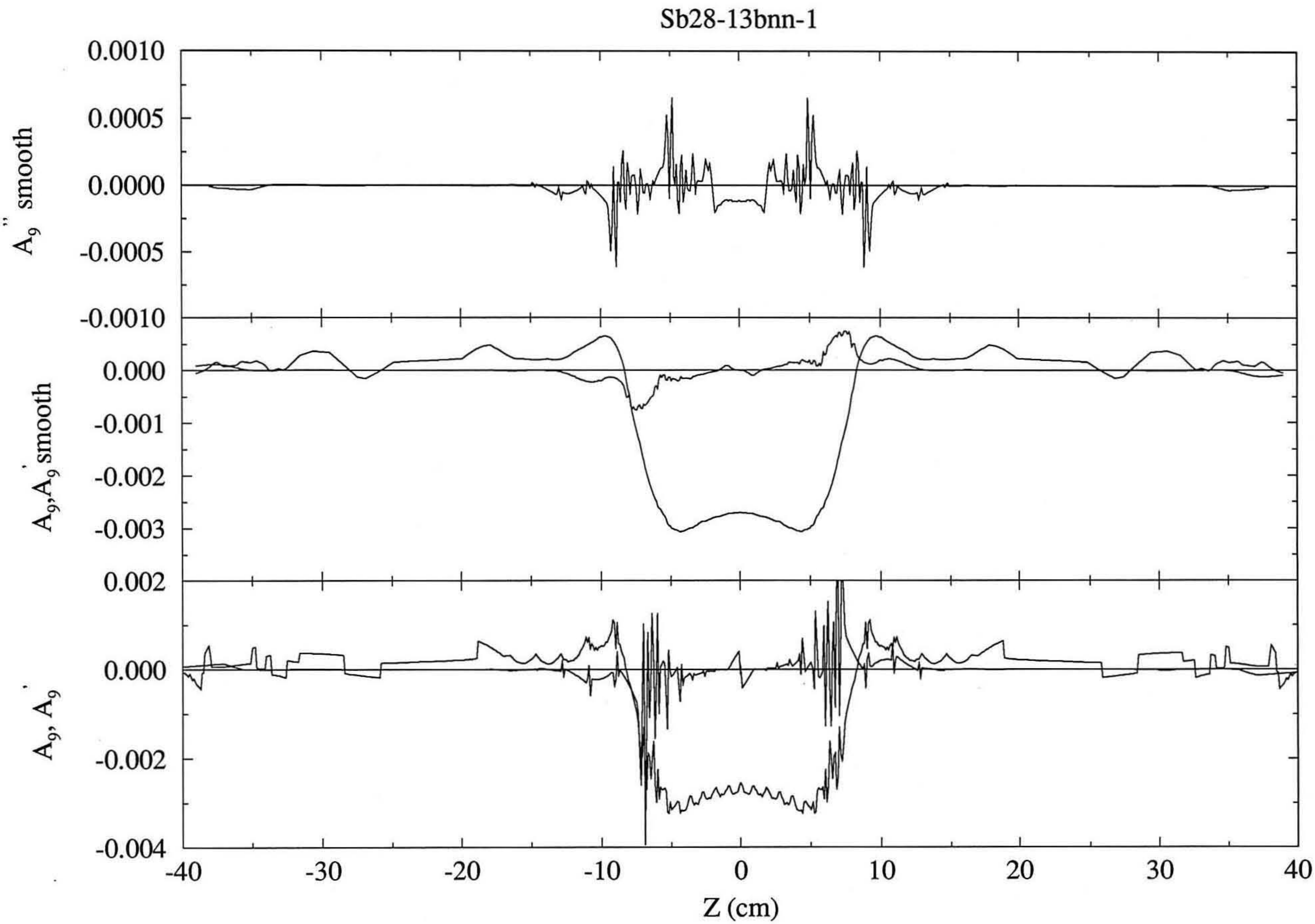


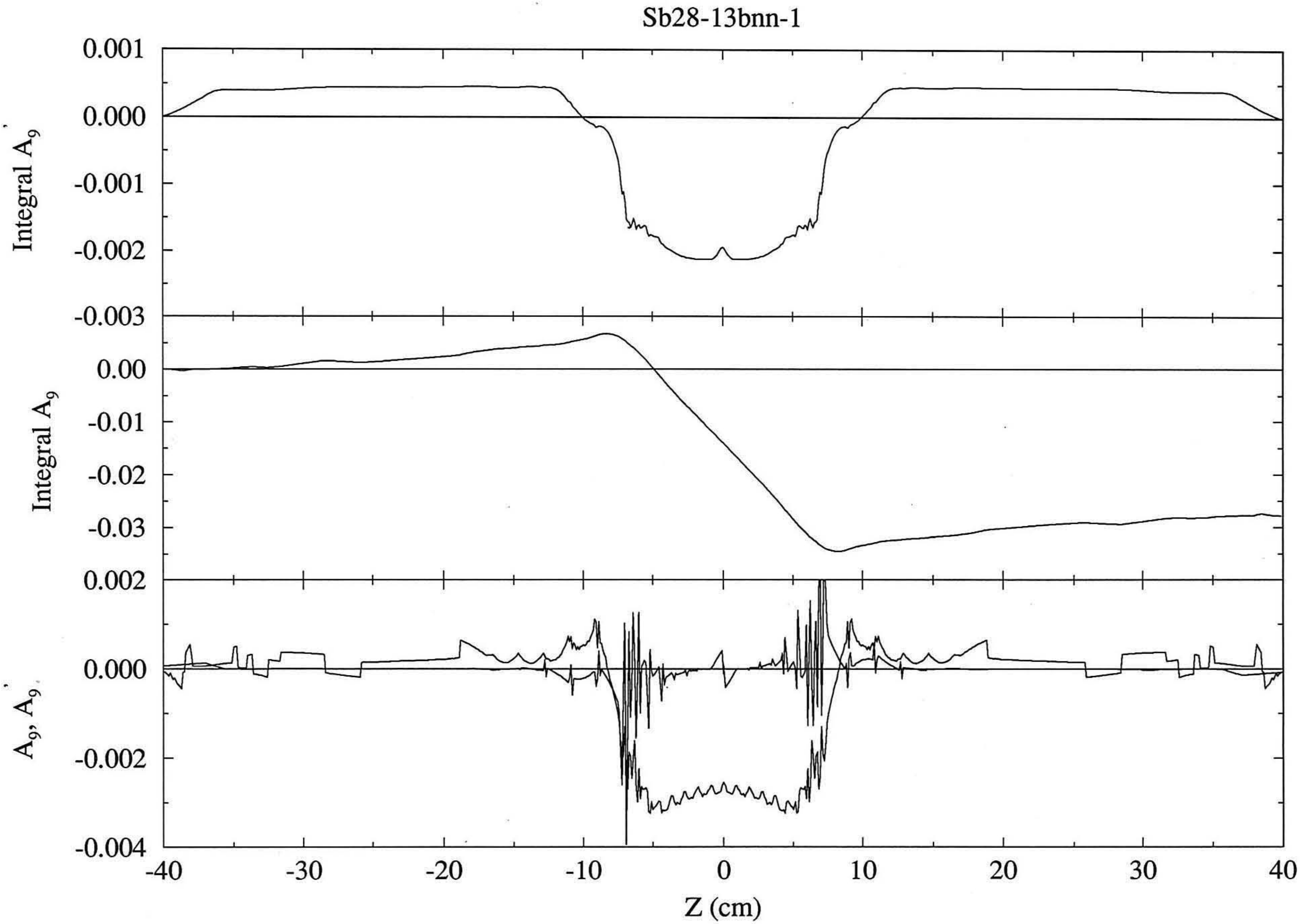

Sb28-13bnn-1
A_5'' smooth
0.02
0.00
-0.02
-0.04
-0.06
A_5, A_5' smooth
0.0
-0.1
-0.2
A_5, A_5'
0.0
-0.1
-0.2
-40
-30
-20
-10
0
10
20
30
40


Sb28-13bnn-1
Integral A_5'
Integral A_5
A_5, A_5'
0.0
-0.1
-0.2
0
-1
-2
-3
-40
-30
-20
-10
0
10
20
30
40
Z (cm)


Sb28-13bnn-1
A7'' smooth
0.002
0.001
0.000
-0.001
0.030
A7,A7' smooth
0.020
0.010
0.000
-0.010
A7, A7'
0.02
0.01
0.00
-0.01
-40
-30
-20
-10
0
10
20
30
40
Z (cm)


Sb28-13bnn-1
Integral A7'
0.004
0.003
0.002
0.001
0.000
-0.001
Integral A7
0.2
0.1
0.0
-0.1
A7, A7'
0.02
0.01
0.00
-0.01
-40
-30
-20
-10
0
10
20
30
40
Z (cm)


Sb28-13bnn-1
A_9'' smooth
A_9, A_9' smooth
A_9, A_9'
0.0010
0.0005
0.0000
-0.0005
-0.0010
0.000
-0.001
-0.002
-0.003
0.002
0.000
-0.002
-0.004
-40
-30
-20
-10
0
10
20
30
40
Z (cm)


Sb28-13bnn-1
Integral A9'
0.001
0.000
-0.001
-0.002
-0.003
Integral A9
0.00
-0.01
-0.02
-0.03
A9, A9'
0.002
0.000
-0.002
-0.004
-40
-30
-20
-10
0
10
20
30
40
Z (cm)