LBL-14327
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Physics, Computer Science & Mathematics Division

Submitted to Physical Review Letters

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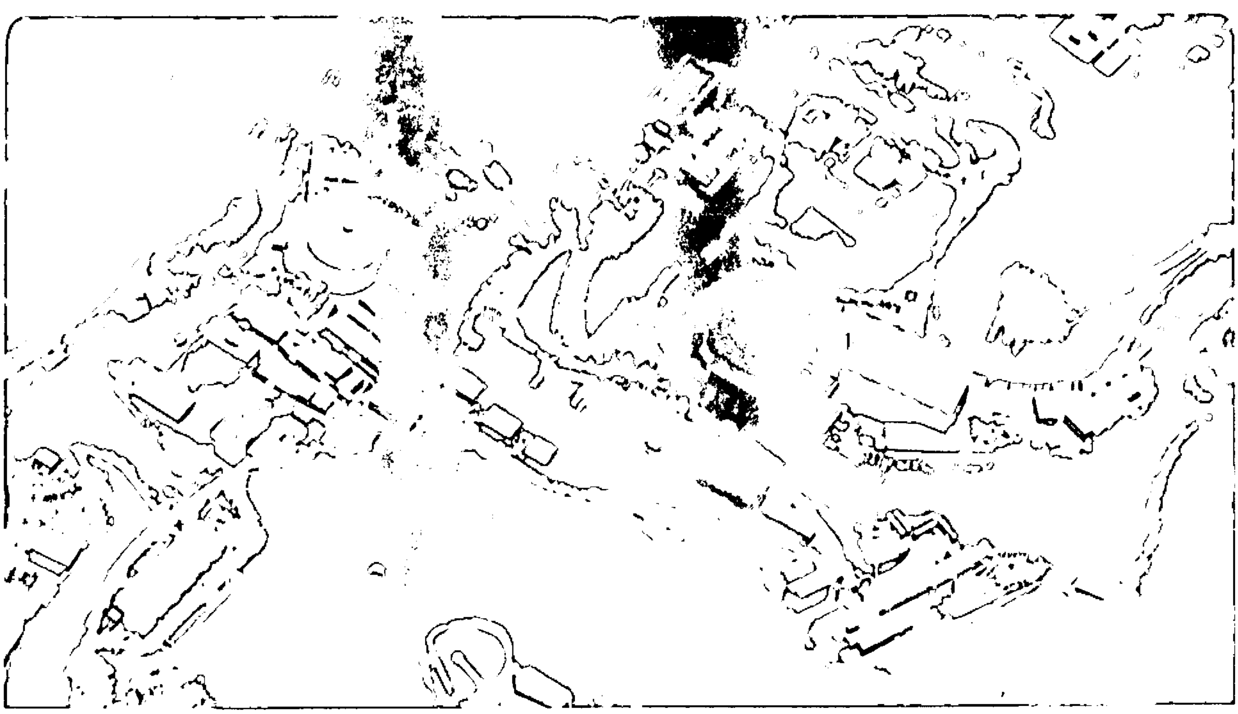
March 1982



Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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COMMENTS ON A PREVIOUS PAPER ABOUT BELL'S THEOREM*

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March 25, 1982

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ABSTRACT

In answer to a paper published recently arguments are given against considering Bell's inequalities to be equivalent with determinism. Possible misinterpretations of the conflict between quantum mechanics and these inequalities are pointed out. Using results obtained in previous papers on this subject, it is shown that locality rather than determinism is the issue.

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

In a recent issue of Physical Review Letters,[1] the relationship between hidden variables, joint probability, and the Bell inequalities[2] was discussed. It was claimed that an equivalence exists between the requirement that the inequalities hold and the existence of a deterministic hidden variables model. It was concluded that the demonstration of the inequalities imposes requirements to make well defined quantities whose rejection is the very essence of quantum mechanics. It is the intent of this paper to show that such claims are likely to be misleading. For this purpose, three key statements made in Ref. 1 will be critically analyzed.

The context is a well-known quantum correlation experiment.[3] There are two well separated regions of space, R_1 and R_2. In R_1 (R_2) two non-commuting observables A and A' (B and B') are defined.[4] It is possible to measure simultaneously any of the four combinations of two commuting observables, A and B, A and B', A' and B, or A' and B'. Each measurable combination above corresponds to a probability distribution $P_{00}(AB)$, $P_{01}(AB')$, $P_{10}(A'B)$, or $P_{11}(A'B')$, respectively.[4] These distributions are functions of the values A, A', B, and B' which A, A', B, and B' can take. All four distributions can be computed using the recipes of quantum mechanics. Central to the discussion is whether or not at least one positive definite function p(AA'BB') exists with the following properties.

$$
\begin{aligned}
P_{00}(AB) &= \sum_{A'B'} p(AA'BB') \\
P_{01}(AB') &= \sum_{A'B} p(AA'BB') \\
P_{10}(A'B) &= \sum_{AB'} p(AA'BB') \\
P_{11}(A'B') &= \sum_{AB} p(AA'BB') .
\end{aligned}
\tag{1}
$$

In Ref.1, whenever such a function p(AA'BB') exists, it is interpreted as a joint probability for the four observables A, A', B, and B'. Then it is correctly demonstrated that the existence of one or several such functions p(AA'BB') implies that the functions P_{00}(AB), P_{01}(AB'), P_{10}(A'B), and P_{11}(A'B') satisfy Bell's inequalities and vice versa, if each of the observables A, A', B, and B' is two valued. However, other statements were also made.

(a) "the existence of a deterministic hidden variables model is strictly equivalent to the existence of a joint probability distribution p(AA'BB')".

Statement (a) is correct only if an unusually restrictive meaning is given to the word "deterministic".[5] In general, determinism means that the evolution of a system is determined by its initial state and by its environment. Then, the outcome of any experiment depends only on some variables which specify the state of the system, on the interactions with other systems, and on all the apparatus that are connected to make measurements. In this general sense, any probability distribution can be reproduced by a deterministic hidden variables model,[6] whether or not Bell's inequalities hold, therefore whether or not a joint probability distribution p(AA'BB') exists. Any computerized Monte Carlo simulation is actually a deterministic hidden variables model for the theory it simulates and there is no limit to the kind of quantum mechanical probability distribution the Monte Carlo technique can reproduce.[7] The Monte Carlo generation of the results for all the measurements which can be made simultaneously depends on the preparation of the system, on its evolution, and on the entire measuring apparatus. The generation of A in R_1 when B is measured in R_2 may have to be different from the generation of A when B' is measured in R_2. However, if the algorithm that generates the data is allowed to have enough mathematical dependence on variables describing the entire measuring set up in R_1 and in R_2, it can be done.

The argument through which statement (a) is made in Ref.1 contains an assumption which restricts the meaning of the word "deterministic". The hidden variables of the deterministic model are labeled λ and their statistical distribution is $\rho(\lambda)$. The observables A, A', B and B' are assumed to be able to take values +1 and -1 only. Four functions $a(\lambda)$, $a'(\lambda)$, $b(\lambda)$, and $b'(\lambda)$ represent the outcome of the measurement of A, A', B, and B' for a given λ.[4] They are used to express the probability distributions $P_{00}(AB)$, $P_{01}(AB')$, $P_{10}(A'B)$, and $P_{11}(A'B')$.

$$
\begin{aligned}
P_{00}(AB) &= \int \delta^{A}_{a(\lambda)}\ \delta^{B}_{b(\lambda)}\ \rho(\lambda)\ d\lambda \\
P_{01}(AB') &= \int \delta^{A}_{a(\lambda)}\ \delta^{B'}_{b'(\lambda)}\ \rho(\lambda)\ d\lambda \\
P_{10}(A'B) &= \int \delta^{A'}_{a'(\lambda)}\ \delta^{B}_{b(\lambda)}\ \rho(\lambda)\ d\lambda \\
P_{11}(A'B') &= \int \delta^{A'}_{a'(\lambda)}\ \delta^{B'}_{b'(\lambda)}\ \rho(\lambda)\ d\lambda
\end{aligned}
\tag{2}
$$

where δ^{x}_{y} is the Kronecker symbol.

The same function $a(\lambda)$ is used in the expression of $P_{00}(AB)$ and of $P_{01}(AB')$. Therefore the result A in R_1 depends on λ but not on the kind of measurement, B or B', made in the location R_2. Similar properties affect $a'(\lambda)$, $b(\lambda)$, and $b'(\lambda)$ in equation (2). All these properties imply that the measurements made in $R_1(R_2)$ are independent of the experimental setup existing in $R_2(R_1)$. It is an independence condition which fits the ordinary definition of locality used for local deterministic hidden variables models. It is a necessary condition for the demonstration of Ref. 1 as it has been for the previous demonstrations of Bell's theorem dealing with deterministic models.[8] Statement (a) would not be true without this additional assumption.

Statement (a) appears in different forms at different places in Ref.1. Everywhere the words "deterministic hidden variables" appear, it would be less misleading for the reader to have the words "deterministic hidden variables in the restrictive sense imposed by equation (2)."

(b) The propositions demonstrated in Ref.1 show what hidden variables (in the restrictive sense of equation (2)) and the Bell inequalities are all about, namely imposing requirements to make well-defined precisely those probability distributions for non-commuting observables whose rejection is the very essence of quantum mechanics".

Ref.1 does not spell out the meaning of the words "well defined". Neither does it elaborate on how and why quantum mechanics rejects those probability distributions, thus how and why it rejects Bell's inequalities. However, some predictions of quantum mechanics are known to violate these inequalities. In this light, several interpretations that a reader may have of statement (b) should be considered.

1) Statement (b) could be thought to mean that the above mentioned rejection requires a violation of some Bell inequality in every experiment. If it were interpreted this way, it would be incorrect,as can be shown using an example. Consider the case of the pair of dissociation fragments of a metastable molecule.[3] Suppose A and A' (B and B') represent measurements of spin 1/2 components in different directions so that A does not commute with A' and B does not commute with B'. In the case where the initial state is such a mixture that the fragments come out uncorrelated, quantum theory does predict

(3) $$P_{00}(AB) = P_{01}(AB') = P_{10}(A'B) = P_{11}(A'B') = 1/4.$$

These probability distributions satisfy Bell's inequality and, consequently, several positive functions p(AA'BB') satisfying equation (1) can be found. For instance,

(4) $$p(AA'BB') = 1/16.$$

or

(5) $$p(AA'BB') = 1/4\ \delta^{A}_{A'}\ \delta^{B}_{B'}$$

or linear combinations of both. Any one of these linear combinations could generate probability distributions for non-commuting observables in the sense of Ref.1.

2) Another interpretation which could be given to statement (b) is the following: the above mentioned rejection requires a violation of Bell's inequalities at least in some experiments. Then the statement would be true but irrelevant to the demonstrations of Bell's theorem, i.e. of the conflict between locality and quantum mechanics. All demonstrations[8,9,10,11,12,13,14] involve two regions R_1 and R_2 which are separated in space. If, on the contrary, R_1 and R_2 are superposed, the independence condition between the measurement results in $R_1(R_2)$ and the measuring set up in $R_2(R_1)$ cannot be justified by a locality argument, the distributions $P_{00}(AB)$, $P_{01}(AB')$, $P_{10}(A'B)$, and $P_{11}(A'B')$ can violate the inequalities, and no joint probability distribution p(AA'BB') can be defined. Then if the inequalities were holding only when R_1 and R_2 are separated, there would be no conflict between locality and the very essence of quantum mechanics in the sense given to these words in this interpretation of statement (b).

3) What statement (b) was probably intended to mean in Ref.1 was that quantum mechanics requires a violation of the Bell inequalities even in some cases where R_1 and R_2 are separated in space.[15] Then the statement is true, relevant to Bell's theorem, and equivalent to what has been stated before.[8,9,10,11,12,13,14] Computations have been performed to obtain the quantum mechanical predictions in the case of the dissociation fragments of a metastable molecule with strong spin correlations.[3] These computations

make use of the basic principles of quantum mechanics and they show that for certain angles of the spin analyzers, the predictions violate Bell's inequalities. The demonstrations of Ref.1 could also permit one to arrive at the same result if it were demonstrated that, in a relevant case, quantum mechanics mandates that no positive definite function p(AA'BB') or no hidden variables model of the restrictive kind defined by equation (2) can be constructed.[16] However, once this additional demonstration is made, it may still be controversial to claim that the propositions of Ref.1 deeply improve our understanding of the relationship between Bell's theorem and the very essence of quantum mechanics.

(c) Proposition (2) shows that, despite appearances, no significant generality is achieved by those derivations of the Bell inequalities that dispense with explicit references to hidden variables and/or determinism:[9,13,17] The assumptions of such derivations imply the existence of deterministic hidden variables for any experiment to which they apply.

Proposition (2) of Ref.1 states that "necessary and also sufficient for the existence of a deterministic hidden variables model (in the restrictive sense of equation (2)) is that the Bell/CH inequalities hold for the probabilities of the experiment. Proposition (2) has little relevance to the derivations made in the quoted references. In Ref.13 for instance, the goal was to study the property of all models, stochastic or deterministic, which can reproduce the results predicted by quantum mechanics.[18] For this purpose, a definition of locality was introduced which could be applied to any model regardless of its stochastic character. The definition made reference only to the possible values that the results can take and to the frequency of their occurrences,[19] i.e. to quantities we have to deal with in quantum theory anyway. Then it was shown that this concept of locality implies a form of Bell's inequality when R_1 and R_2 are separated in space. Quantum mechanics violates this form of Bell's inequality too; therefore any deterministic or stochastic model which reproduces the quantum mechanics predictions cannot have the locality property of Ref.13. Of course, proposition (2) of Ref.1 shows that, if the inequalities were satisfied, deterministic hidden variables models in the restrictive sense of equation (2) would also exist and fit the predictions. However, from the violation of the inequalities and from proposition (2), one can only infer that deterministic models referred to in proposition (2) do not exist. One cannot imply any local or non-local property of the stochastic models invoked in Ref.13. Therefore, proposition (2) can neither reproduce nor contradict the conclusions derived in Ref.13.

It has also been shown[14] that a locality condition of the type of Ref.13 would be satisfied if the model were given a more intuitive locality property: Namely that the deterministic or stochastic algorithm that generates the data would be mathematically independent of the measurement set up in R_1 when it generates $\underline{B}$ and $\underline{B'}$ in R_2 and independent of the set up in R_2 when it generates $\underline{A}$ and $\underline{A'}$ in R_1.[20] Therefore, in this sense too, any stochastic simulation of the results predicted by quantum theory has to be non-local.

Then the next question is "How does nature operate?" What is true for models used for simulation of data is also true for any model of reality we may have. It seems an inescapable conclusion that there are processes in nature which involve the same correlation between the results in R_1 and the measurement set up in R_2, or the results in R_2 and the measurement set up in R_1, as the non-local algorithms of our models.[14] This conclusion is not in contradiction with any principle of quantum mechanics since, according to the Copenhagen interpretation, quantum mechanics is supposed only to give recipes to make predictions, not to describe how nature operates.[21] In particular, there is no physical description of the processes occurring during the collapse of the wave function. When measurements are performed in the correlation experiment above,[3] it is not surprising that the wave function collapse may involve non-local processes according to the definitions of Refs. 13 and 14. Quantum field theory abides with a locality condition that is different from all those which imply Bell's inequalities. This condition is based on the commutation of operators outside of the light cone and it prevents observers from acting at a distance.[22] It does not necessarily restrict the natural processes mentioned above the same way as do the other locality conditions. Therefore, quantum mechanics is said to violate or not to violate locality depending on the significance given to the word "locality".[14]

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I am indebted to Drs. J.E. Lys and T.G. Trippe for interesting discussions and for reviewing the manuscript.

Notes and References

1. A. Fine, Phys. Rev. Lett. 48, 291 (1982).

2. By the generic name, Bell's inequalities, this paper refers to all the inequalities based on a locality condition and which were expressed in different forms by different authors. See Refs. 8 and 10 in particular.

3. To follow the reasoning of Ref.1 the same experiment is suggested here as an example. It is described by J.F. Clauser and A. Shimony, Rep. Prog. Phys. 41, 1881 (1978).

4. Some changes have been made in the notation of Ref.1 to make sure different mathematical entities are represented by different symbols.

5. This restriction to the definition of determinism has been mentioned in the literature before. It was objected to by J.S. Bell, Rev. Mod. Phys. 38, 447 (1966) in his comments about possible misuse of A.M. Gleason's work, Journ. Math. Mech. 6, 885 (1957).

6. See for instance Appendix A of Ref.14.

7. The Monte Carlo simulation is a technique to generate values of experimental results, statistically distributed like the theoretical probability distribution, using random numbers. I owe to J.S. Bell the idea that Monte Carlo simulation can easily demonstrate that determinism can be restored in quantum theory. J.S. Bell: private communication (July 1977).

8. J.S. Bell, Physics 1, 195 (1964). J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, Phys. Rev. Lett. 23, 80 (1969).

9. H.P. Stapp, Phys. Rev. D3, 1303 (1971).

10. J.F. Clauser and M.A. Horne, Phys. Rev. D10, 5(1974).

11. For example, J.S. Bell, Res. Th., 2053 CERN and communication at the VI GIFT Conference, Jaca, June 2-7, 1975; N. Herbert, Amer. Journ. Phys. 43, 315 (1975). This list is far from being exhaustive.

12. H.P. Stapp, Nuovo Cimento 40B, 191 (1977).

13. P.H. Eberhard, Nuovo Cimento 38B, 75 (1977).

14. P.H. Eberhard, "Bell's Theorem and the Different Concepts of Locality", Nuovo Cimento 46, 392 (1978).

15. Telephone conversation with A.Fine (March 31, 1982).

16. Though this demonstration is not spelled out in Ref.1, it should not be too difficult to make it.

17. B. D'Espagnat, Phys. Rev. D18, 349 (1978).

18. Here, following Ref.1, the word "model" is used to refer to what is called a "theory" in Ref.13.

19. This definition of locality refers to the possible results of a correlation experiment involving a large number N of events. A, A', B, and B' are four series of N numbers A_j, A'_j, B_j, and B'_j equal to +1 or −1, considered to be possible measurement results. For the combination of two series A and B (A and B', A' and B, A' and B'), a correlation parameter C_{00} (C_{01}, C_{10}, C_{11}) is constructed:

$$C_{00}(\text{or } C_{01},\ C_{10},\ C_{11}) = \frac{1}{N}\sum A_jB_j(\text{or } A_jB'_j,\ A'_jB_j,\ A'_jB'_j).$$

A model is said to be local if there are sets of four series A, A', B, and B' for which the correlation parameter for each combination of two series is near the expectation value predicted by the theory for the corresponding experimental set ups in R_1 and in R_2. This mathematical definition of locality is justified in Ref.13 by considering either what would happen if different choices of experimental set up were made or alternatively what results could be found in the records of four different experiments performed at different times.

20. This definition of locality implies the existence of a positive definite function p(AA'BB') satisfying equation (1) while the variables A,A',B, and B' have 2^N possible values. Then, it is trivial to construct another model which would be deterministic in the restrictive sense of equation (2) using p(AA'BB') as a joint probability distribution for the hidden variables. However, there is no requirement that the original model be deterministic at all.

21. For an extensive list of references, see M. Jammer, The Philosophy of Quantum Mechanics (New York, NY, 1974). The advocates of the Copenhagen interpretation also claim that it is not worth investigating the processes by which nature operates, as can be seen from the following quote of R.P. Feynman:

> "I think it is safe to say that no one understands quantum mechanics. Do not keep saying to yourself, if you can possibly avoid it, 'But how can it be like that?' because you will go 'down the drain' into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that."

In the context of the Copenhagen interpretation, the conclusions drawn from Bell's theorem concerning the processes of nature are uninteresting, though not incorrect.

22. This results from a property, called property 4 in Ref.14, which should be sufficient to suppress causal loops in relativity, at least when observers are involved. A demonstration of this property from the quantum field theory commutation rules can be found in Appendix D of Ref.14.

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