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Isomorphic Representations Lead to the Discovery of Different Forms of a Common Strategy with Different Degrees of Generality

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Abstract
This study examines the effects of representational forms on the acquisition and transfer of problem solving strategies. Three isomorphic representations of the Tic-Tac-Toe are used as the experimental tasks. The experiment shows that different representations of a common structure lead to the discovery of different forms of a common strategy with varying degrees of generality. With a better representation, subjects not only learn faster but also acquire more general forms of the strategy. The transfer across different representations can be either positive or negative, and it is based on strategies, not on problem structures.

Introduction
Different isomorphic representations of a common abstract structure can generate dramatically different representational efficiencies, task complexities, and behavioral outcomes. This phenomenon is often called *representational effect* (e.g., Zhang & Norman, 1994). One obvious example is the different difficulty levels of multi-digit multiplication using Arabic and Roman numerals (see Zhang & Norman, 1995, for a detailed study). The study of the representational effect has been focused on performance and transfer, not on acquisition and the relation between acquisition and transfer (e.g., Cheng & Holyoak, 1985; Evans, 1983; Gick & Holyoak, 1980, 1983; Greeno, 1974; Griggs & Hewstead, 1982; Jeffries, Polson, & Razran, 1977; Kotovsky, Hayes, & Simon, 1985; Kotovsky & Simon, 1990; Larkin & Simon, 1987; Novick, 1990; Wason & Johnson-Laird, 1972; Zhang & Norman, 1994).

The purpose of the present study is to examine how representational forms affect the acquisition of problem solving strategies and how the acquired strategies affect transfer. The main hypotheses are (1) different representations of a common structure lead to the discovery of different forms of a common strategy, with a better representation leading to faster learning and to the discover of more general forms of the strategy; and (2) the acquired strategy can cause both positive and negative transfer, depending on the two isomorphs involved.

Tic-Tac-Toe
The Tic-Tac-Toe (henceforth, TTT) and its isomorphs are used for the present study. The TTT is a well-known two-player game. A minor variation of the original game is shown in Figure 1A as the *Line* version. The task for the two players is to select the circles in turn by coloring the circles with different colors, one at a time. The one who first gets three circles on a straight line (horizontal, vertical, or diagonal) wins the game. The TTT is a draw game, i.e., when both players use optimum strategies, neither can win. Figures 1B and 1C are two more isomorphs of the TTT. In the *Number* version (Figure 1B), the task is to select the numbers in turn by coloring the numbers, one at a time. The one who first gets three numbers that exactly add to 15 wins the game. In the *Color* version (Figure 1C), the task is to select the big circles in turn by drawing different background textures. The one who first gets three big circles that contain the same colored small circle wins the game. Figure 2 shows the equivalence of the isomorphs: the center, corners, and sides in *Line* (Figure 2A) correspond to the number five, even numbers, and odd numbers in *Number* (Figure 2B) and the 4-object, 3-object, and 2-object big circles in *Color* (Figure 2C), respectively.

The representational properties of the TTT were analyzed by Zhang (1997) in terms of four formal properties and their representations. The four formal properties are: (1) nine elements; (2) eight winning

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triplets, each of which is a group of three elements that constitute a win; (3) three symmetry categories that group the nine elements, with the elements in a symmetry category being identical to each other; (4) winning invariants of symmetry categories, each of which is the number of winning triplets to which an element belongs. For example, for *Line* in Figure 1A, the nine elements are the nine circles. The eight winning triplets are the 3-circle groups that lie on the 3 horizontal, 3 vertical, and 2 diagonal lines. The three symmetry categories are the center, 4 corners, and 4 sides. The winning invariants of the center, corners, and sides are 4, 3, and 2, respectively. For example, the center is an element of 4 winning triplets: 1 horizontal, 1 vertical, and 2 diagonal lines.

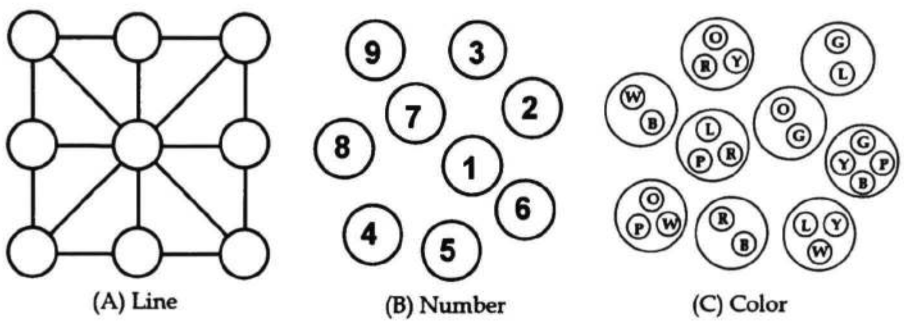

Figure 1. Three TTT isomorphs. (A) *Line*. Getting three circles on a straight line is a win. (B) *Number*. Getting three numbers that exactly add to 15 is a win. (C) *Color*. Getting three big circles that contain the same colored small circle is a win. The letters inside the circles indicate the colors used in the experiment: B = Blue, G = Green, L = Light Blue, O = Orange, P = Pink, R = Red, Y = Yellow, W = Brown.

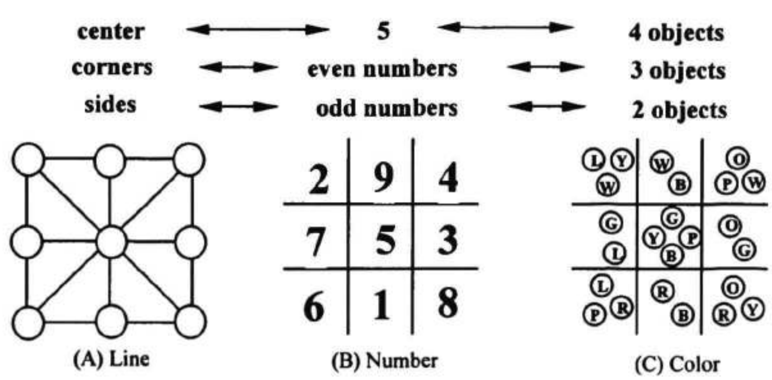

Figure 2. The mappings of the TTT isomorphs. The center, corners, and sides in *Line* correspond to five, even numbers, and odd numbers in *Number*, and 4-object, 3-object, and 2-object big circles in *Color*, respectively.

The formal properties of the TTT are represented differently in the three TTT isomorphs. The nine elements are represented by nine circles in *Line*, by nine numbers in *Number*, and by nine big circles with 2, 3, and 4 small circles in *Color*. The eight winning triplets are represented by the 3 horizontal, 3 vertical, and 2 diagonal straight lines in *Line*, by the eight number triplets with the sum of the three numbers in each triplet equal to 15 in *Number*, and by the eight different colors of the small circles in *Color*. The three symmetry categories are represented by spatial symmetry in *Line* (the center, 4 corners, and 4 sides), by parity in *Number* (the number five, 4 even numbers, and 4 odd numbers), and by the quantity of objects in a big circle in *Color* (4-object, 3-object, and 2-object big circles). The winning invariant of a symmetry category is represented by the number of straight lines connecting a circle in *Line* (4, 3, and 2 for the center, the corners, and the sides, respectively) and by the quantity of objects in a big circle in *Color* (4, 3, and 2). In *Number*, however, the winning invariants (4 for the number five, 3 for even numbers, and 2 for odd numbers) are not directly represented. To get the winning invariant of a number, it must be grouped with all possible pairs of other numbers to form number triplets and the sums of the three numbers in all number triplets have to be mentally computed to see whether each sum is 15. Even with extensive mental computations, this task is very difficult if not impossible.

Because all TTT isomorphs have the same formal structures, for convenience, we use the three symmetry categories of *Number*, i.e., five, even numbers, and odd numbers, to refer to the three symmetry categories of all TTT isomorphs for the rest of this article. For example, when we talk about even numbers in *Color*, we actually refer to the 3-object big circles (see Figure 2).

In the current study, the task for the subjects is to discover the *Even-Even* strategy, which requires them to select any even numbers for the first and second moves to get draws (for the detailed algorithm, see the appendix of Zhang, 1997). The Even-Even strategy is necessary and sufficient for subjects to get draws. The first and the second moves are crucial: if either or both are made incorrectly, then subjects always lose, regardless of how other moves are made. Under this strategy, subjects only have to make decisions for the first and second moves because for all other situations, the subjects only have one choice—blocking the piece that can lead to an immediate win for the computer. In addition, subjects' first and second moves only depend on the symmetry categories (i.e., five, even numbers, and odd numbers), not on specific elements.

Zhang (1997) found that for the Even-Even strategy, the difficulty order was, from hardest to easiest: *Number* > *Color* ≥ *Line*. Two factors contributed to this difficulty order. The first factor is the perception of the symmetry categories. Subjects had better perception of the symmetry categories for *Line* than for *Color*, and had no perception of the symmetry categories for *Number*. The second factor is biases. For *Line* and *Color*, subjects had a perceptual bias called *more-is-better* bias, which is the tendency to pick an element that has higher winning invariant (e.g., the center for *Line* and the 4-object big circle for *Color*). This perceptual bias was consistent with the Even-Even strategy, thus it helped make *Line* and *Color* easier. For *Number*, instead of the more-is-better bias, subjects had a cognitive bias called *larger-is-better* bias, which is the tendency to pick up a larger number (e.g., 9 or 8). This cognitive bias is irrelevant to the Even-Even strategy, thus it did not help make *Number* easier. Based on this finding of the difficulty order of the three isomorphs, *Line* can be called the most effective representation and *Number* the least effective representation for the Even-Even strategy.

The experiment described below will examine whether the different representations of the three TTT isomorphs lead to the discovery of different forms of the Even-Even strategy and whether the acquired strategies affect transfer across isomorphs. The main hypotheses are (1) the three isomorphs lead to the discovery of different forms of the Even-Even strategy, with the most effective representation (*Line*) leading to the fastest learning and to the discover of the most general form of the Even-Even strategy; and (2) the different forms of the acquired strategy can cause both positive and negative transfer, depending on the two isomorphs involved.

Experiment

Method

Subjects. 90 undergraduate students enrolled in introductory psychology courses at The Ohio State University participated in the experiment to earn course credit.

Stimuli. The three TTT isomorphs in Figure 1 were the experimental tasks. They were programmed in SuperCard on Macintosh computers. The three TTT isomorphs were controlled by the same program because they have the same formal structure. The computer always made the first move in all games. Its strategy was designed such that the subjects had to discover the Even-Even strategy to get draws. Subjects made moves by clicking the pieces with a mouse. The pieces selected by the computer and subjects were in different colors or background patterns such that they could be distinguished.

Design & Procedure. The design is shown in Figure 3. Each subject played two of the three TTT isomorphs. Subjects were told that the best they could get was a draw and were instructed to play the games against the computer until they got 10 draws in a row. They were not told about any relations between the first and second games. Complete move sequences and time stamps for all games were recorded by the computer. After each game, subjects were asked to write down the strategies they discovered.

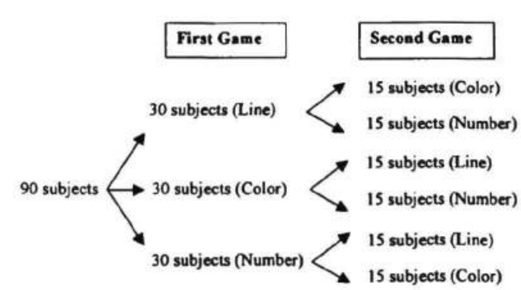

Figure 3. The design of the experiment.

Results

Pre-Transfer Performance. The results for first games are shown in Figure 4, with 30 subjects for each isomorph. In terms of the number of games needed to get 10 draws in a row (Figure 4A), *Line* was marginally easier than *Color* (Tukey HSD, $p = 0.08$), which in turn was significantly easier than *Number* (Tukey HSD, $p < 0.01$). In terms of the number of games needed to get the first draw (Figure 4B), *Line* and *Color*, which did not differ from each other significantly (Tukey HSD, $p = 0.38$), were both significantly easier than *Number* (Tukey HSD, $p < 0.001$, for both cases).

Figure 4C shows the percentage of the subjects who got draws for each game position for the three isomorphs. Consistent with the results on 10 draws and first draw, it is clear that subjects acquired the Even-Even strategy fastest for *Line* and slowest for *Number*.

One critical finding of this experiment is shown in Figure 4D. We say a subject discovered or acquired the Even-Even strategy if the subject could get 10 draws in a row. However, the Even-Even strategy acquired by subjects can be in different forms. Remember that the Even-Even strategy requires that the first and second moves be any even numbers. If a subject could not perceive the symmetry categories, they might use a fixed move sequence for all ten draws in a row. This is the *Fixed* form of the Even-Even strategy. For example, a subject might always select 2 as the first move and always select 4 as the second move for all draw games, even if any even numbers would be equally correct. If the ten continuous draw games of a subject all had the same move sequence and the subject did not indicate any knowledge of the symmetry in the written report, then the Even-Even strategy acquired by this subject is classified as *Fixed.* All other forms are considered as *Non-Fixed.* Figure 4D shows that more subjects acquired the Fixed form of the Even-Even strategy in *Number* than in *Color* and in *Line* ($\chi^2 = 25.5$, $p < 0.001$; $\chi^2 = 15.6$, $p < 0.001$), which did not differ from each other significantly ($\chi^2 = 1.8$, $p = 0.12$).

Transfer Effect. The transfer data were analyzed for the six 15-subject groups corresponding to the six combinations of first and second games. In terms of the number of games needed to get 10 draws in a row (Figure 5A), there were a significant positive transfer from *Number* to *Color* ($t(28) = -2.34$, $p < 0.05$) and a significant negative transfer from *Color* to *Line* ($t(28) = -3.02$, $p < 0.005$). Other transfers were not significant (largest $|t(28)| = 1.77$ with smallest $p = 0.09$). In terms of the number of games to first draw (Figure 5B), none of the transfers were significant (largest $|t(28)| = 1.17$ with smallest $p = 0.25$).

It appeared that the positive transfer from *Number* to *Color* and the negative transfer from *Color* to *Line* were due to the transfer of the different forms of the Even-Even strategy. Figure 5C shows that the percentage of subjects who acquired the Fixed form of the Even-Even strategy for the three isomorphs when they were played as the first game and when they were played as the second game. Although not significant ($\chi^2 = 1.20$, $p = 0.20$), slightly more subjects used the Fixed form of the Even-Even strategy when *Color* was played after *Number* than when *Color* was played before *Number* (60% vs. 40%). After acquiring the Fixed form of the Even-Even strategy, it seems that subjects tended to use a fixed move sequence to get 10 draws in a row when they played Color because they simply wanted to get 10 draws in a row as soon as possible to satisfy the goal of the task. Therefore, subjects needed fewer games to get 10 draws in a row when *Color* was played after *Number* than when Color was played before *Number*.

The negative transfer from *Color* to *Line* can be explained in a similar manner. After solving *Color*, the Even-Even strategy was acquired as a Non-Fixed form by 53% of the subjects. When *Line* was played after *Color*, the Non-Fixed form acquired from *Color* tended to make subjects use Non-Fixed form in *Line*, causing them to play more games before getting 10 draws in a row. Therefore, slightly more subjects used the Fixed form of the Even-Even strategy when *Line* was played before *Color* than when *Line* was played after *Color* (40% vs. 20%, $\chi^2 = 1.42$, $p = 0.16$).

The Pattern of Subjects' Moves. Figures 6A-6C show the averaged frequencies of even and odd numbers selected by the subjects as the first moves in the initial 10 games for the six transfer pairs. The rightmost column in each graph shows the expected frequencies for random moves with replacement. No transfer effect was observed for the selection of the first moves (largest $t(28) = 1.43$ with smallest $p = 0.16$).

Figures 6D-6F show the distributions of subjects selecting even and odd numbers as their second moves for the first game in which an even number was selected as the first move. Chi-Square tests showed that there was no significant change of the selection of even numbers for any of the transfer pairs (largest $\chi^2 = 2.40$ with smallest $p = 0.12$).

Therefore, the positive transfer from *Number* to *Color* and the negative transfer from *Color* to *Line* were not due to the transfer of problem structures that are reflected by the selection of moves.

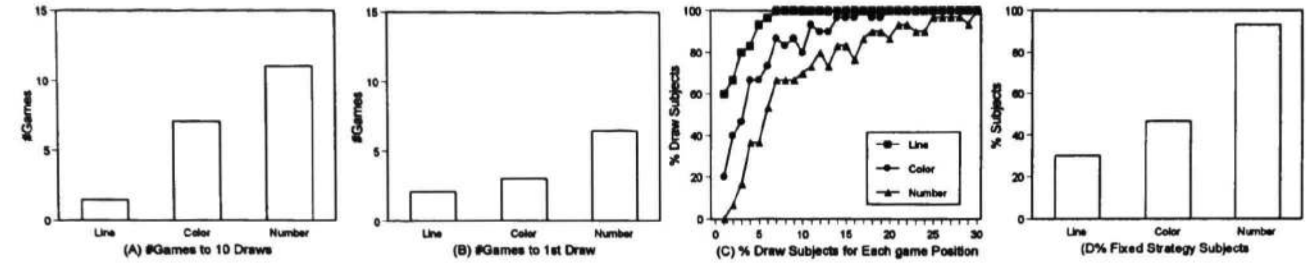

Figure 4. Pre-transfer results. (A) The number of games needed to get 10 draws in a row (excluding the 10 draws). (B) The number of games needed to get the first draw (including the first draw). (C) Percentage of subjects who got draws for each game position. (D) Percentage of subjects who used the Fixed form of the Even-Even strategy.

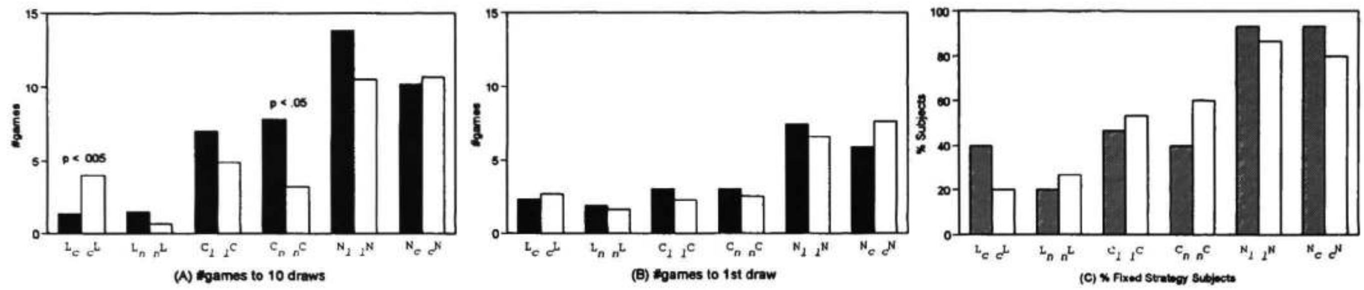

Figure 5. Transfer results. (A) The number of games needed to get 10 draws in a row (excluding the 10 draws). (B) The number of games needed to get the first draw (including the first draw). (C) Percentage of subjects who used the Fixed form of the Even-Even strategy. C = *Color*, L = *Line*, N = *Number*. The subscripts indicate the preceding or succeeding games. For example, L_C is for *Line* that was played before *Color* and cL is for *Line* what was played after *Color*.

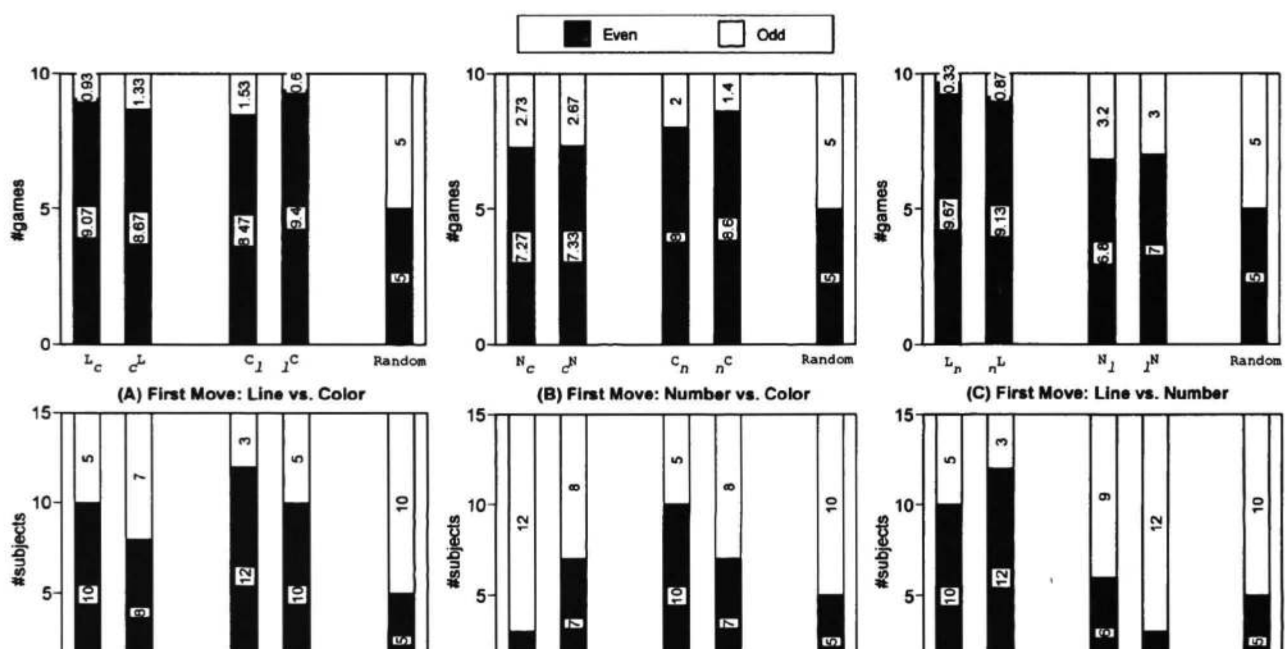

Figure 6. Move patterns. (A)-(C) The average frequencies of even and odd numbers selected by each subject as first moves for the initial 10 games for the *Line-Color*, *Number-Color*, and *Line-Number* transfer pairs. (D)-(F) Frequency distributions of subjects who selected even and odd numbers as their second moves in the first game in which five was selected as the first move for the *Line* and *Color*, *Number* and *Color*, and *Line* and *Number* pairs.

Conclusion

The present study examined how the forms of representations affect what strategies were acquired and how the acquired strategies were transferred. The experiment showed the following results.

First, different representations of a common structure led to the discovery of different forms of a common strategy with varying degrees of generality. With a better representation, subjects not only learned faster but also acquired more general forms of the strategy.

Second, the transfer across different isomorphic representations could be either positive or negative. Although a less effective representation (*Number*) led to the acquisition of a less general form of the Even-Even strategy, this less general form of the strategy led to a positive transfer to a more effective representation (*Color*). In contrast, the more effective representation (*Color*) led to the acquisition of a more general form of the Even-Even strategy. However, this more general form of the strategy led to a negative transfer to the most effective representation (*Line*).

Third, the positive and negative transfers mentioned above were not due to the transfer of the structures of the task. In fact, the structures of the task, reflected by the patterns of subjects' moves, were not transferred from one isomorph to another. This result is consistent with the general finding of minimal spontaneous transfer across problems with different surface representations in the studies of analogical problem solving (e.g., Gick and Holyoak, 1980, 1983; Holyoak & Koh, 1987; Ross, 1984).

In conclusion, the present study is another demonstration of the ubiquitous representational effect. The major contribution is the demonstration that different representations of a common underlying structure can lead to the discovery of different properties of the underlying structure in terms of different forms of strategies, which can not only determine problem difficulties but also affect the pattern of knowledge transfer.

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