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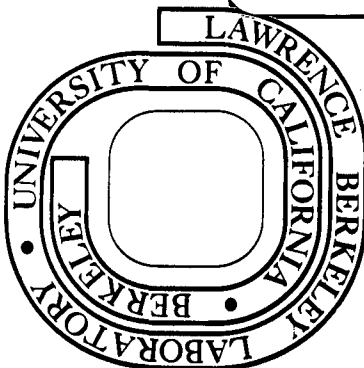
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AN ANALYSIS OF SINGLE PION PHOTOPRODUCTION BETWEEN  
THRESHOLD AND 16 GeV<sup>†</sup>

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## INTRODUCTION

This paper presents the results of the continuation of a previous analysis of single pion photoproduction<sup>(1)</sup>. As in that analysis our method consists of a multipole analysis at low energies, an amplitude analysis at high energies and a combination of both at intermediate energies. We use fixed- $t$  dispersion relations as a constraint on our amplitudes and attempt to fit simultaneously to the data in the resonance region (C. of M. energy  $W \leq 2$  GeV), the intermediate energy region ( $2 \leq W \leq 2.5$  GeV) and the high energy region ( $W \geq 2.5$  GeV). This division of the data may appear to be somewhat arbitrary but in practice is a suitable way to combine parametrizations of the imaginary parts of the photoproduction amplitudes. This procedure, involving a simultaneous multipole and amplitude analysis, allows us to incorporate the general features of earlier fits to data in the resonance and the high energy region and to investigate their relative consistency in the intermediate energy region.

In sections 2, 3 and 4 we describe and discuss our results of two energy dependent analyses of the photoproduction data. We comment on the difficulties in extracting multipole amplitudes from the hybrid parametrization in the intermediate energy region. In section 5 we describe an energy independent method for treating this problem.

## 2. PARAMETRIZATION OF THE PHOTOPRODUCTION AMPLITUDES

The low energy parametrization for the imaginary parts of the photoproduction multipole amplitudes is identical to that used by Crawford<sup>(2)</sup> in an earlier low energy analysis.

In that fit the imaginary parts of resonance contributions were taken to have a Breit-Wigner structure aside from the  $P_{33}$  for which an Omnes-Muskhelishvili form was used to impose the proper behaviour of the complex phase. Background contributions to the imaginary parts of the S-wave multipoles were included as broad Breit-Wigners.

The major difficulty inherent in such a parametrization arises from the divergence of the multipole expansion when it is used in the fixed- $t$  dispersion relation and is consequently required for large negative unphysical values of  $\cos\theta$ , the C. of M. scattering angle<sup>(3)</sup>. In the previous fit we had to introduce a cut-off in the resonance region data set at  $t = -1.5$  (GeV/c)<sup>2</sup> and in the high energy data set at  $t = -.8$  (GeV/c)<sup>2</sup>. In the fits presented below we have extended the high energy fit to  $|t| \leq 1$  (GeV/c)<sup>2</sup> and suppress the high energy amplitudes in the fixed- $t$  dispersion relation for  $|t| > 1.2$  (GeV/c)<sup>2</sup> by a factor  $e^{\lambda(1.2+t)}$  where the parameter  $\lambda$  is determined by the fit.

The difficulties due to the possible divergence of the multipole expansion would not have arisen had we used only an energy independent method to analyse the data. We would then have been able to cover the full physical momentum transfer region (but of course without the constraint of the fixed- $t$  dispersion relations), and projection of the multipole amplitudes from our invariant amplitudes would then have followed a well defined procedure.

The high energy parametrization is essentially the same as that described by Barbour and Moorhouse<sup>(4)</sup>. Previous analyses<sup>(5)</sup> found that the fit to the  $\pi^0$  data was improved by the addition of an exotic exchange contribution to  $\text{Im } A_3^{\pi^0}(s,t)$ . Our earlier analysis found this exchange to be in the + isospin amplitude and we now constrain our amplitudes to satisfy (for  $W \geq 2.5$  GeV)

$$\text{Im } A_3^{0,-}(s,t) = 0$$

together with

$$\text{Im } A_1^{+,-,0}(s,t) = -t \text{Im } A_2^{+,-,0}(s,t)$$

for  $\rho$ ,  $\omega$  and  $A_2$  exchanges.

In the intermediate energy region we take the parametrization to be a combination of the above resonance and high energy parametrizations such that

$$\text{Im } A_i^{\text{INT}}(s,t) = \frac{1}{s_1 - s_2} \text{Im} \left\{ (s - s_2) A_i^{\text{RES}}(s,t) + (s_1 - s) A_i^{\text{REG}}(s,t) \right\} \quad 2.1$$

for  $\sqrt{s_1} = 2 \text{ GeV} < \sqrt{s} < 2.5 \text{ GeV} = \sqrt{s_2}$ .

In this region, there is no longer a good understanding of the  $\pi N$  resonance structure and only the peripheral resonances have been definitely identified from  $\pi N$  partial wave analyses. Thus we are obliged to switch from the low energy to the high energy parametrizations and the hybrid form given above represents one of the simplest ways of achieving this.  $\text{Im } A_i^{\text{RES}}(s,t)$  is parametrized using only the established  $G_{17}(2190)$ ,  $H_{19}(2220)$ ,  $H_{3,11}(2420)$  resonances and the high energy tails of the lower energy resonances. If duality holds for the photoproduction amplitudes,  $\text{Im } A_i^{\text{REG}}(s,t)$  should be in an average sense equivalent to

4.

$\text{Im } A_i^{\text{RES}}(s,t)$ . As a result, the parametrization of the  $G_{17}(2190)$ ,  $H_{19}(2220)$  and especially the  $H_{3,11}(2420)$  does not truly describe the photoproduction of these resonances but is partly describing the local deviation of the Regge parametrization from the actual physical amplitudes. This implies that we cannot rely on the resonance parameters from the fit for  $W > 2$  GeV but rather that the masses and widths of these resonances must come from an analysis of  $A_i^{\text{INT}}(s,t)$ . This we discuss below.

### 3. THE ENERGY DEPENDENT FITS

Tables I and II and Appendix 1 give the results of the two fits to the photoproduction data. The two fits were made independently to essentially the same data set (which is summarized in Appendix II). We attribute the differences in the results from the two fits mainly to random statistical effects and also to minor differences in the data sets used although we consider the latter to have a negligible effect.

The fitting procedures entailed calculating a  $\chi^2$  weighted in favour of the data in the energy region associated with the parameters being varied. It was not practical to minimize the  $\chi^2$  by varying all the parameters simultaneously but, as in our previous analysis, we frequently found that the fits to different regions were complementary since fitting in one region often improved the fit in other regions.

Information about the quality of the fits obtained is given in Table I and the corresponding resonance parameters are presented in Table IIa. Average values of our couplings are presented in Table IIb. Due to the complexity of the fitting program it was not practical to make

the usual error analyses for the couplings. However such errors are purely statistical in nature and are probably significantly less important than the systematic differences between various analyses such as the method of parametrization and the choice of data. To take account of this we have made subjective estimates of our statistical errors based on the variation of the parameters over a number of our fits and have combined them with estimates of possible systematic errors obtained by comparing the values obtained in a number of recent analyses<sup>(9,16)</sup>. The errors obtained cannot be considered as unique or even very objective but hopefully they give a useful and realistic assessment of the uncertainties in the couplings.

Although the  $\chi^2$  for the two fits differ, the resonance couplings below 2 GeV are essentially the same. However, the differences in the resonance couplings from the fits increase with the resonance energy, and this gives an indication of the uncertainty to be associated with values of the couplings. The neutron couplings seem to be particularly poorly determined, especially for the helicity 3/2 amplitudes and at  $W = 2$  GeV the  $D_{35}$  and  $F_{17}$  have couplings which differ by a factor of two in the fits. As would be expected the peripheral resonances are, in general, the best determined with the prominent  $F_{37}$  obtaining close agreement between the fits.

In the intermediate region, both fits find the  $G_{17}$  with a mass of about 2.1 GeV and with similar couplings. At this energy the Breit-Wigner term is dominant (80% of total) and thus we consider that it is reasonable to use the Breit-Wigner couplings as the actual resonance couplings. On the other hand, the  $H_{1,9}$  and  $H_{3,11}$  do not show the same agreement between the fits. In the resonance parametrizations, both tend to lie at higher energies (about 2.3 GeV) than the  $G_{1,7}$  and clearly interference between the Regge and resonance parametrizations is an important factor. Thus the analyses cannot at this stage give



their couplings and separate multipole analyses of the Regge plus resonance amplitudes must be made.

The two fits find the same leading Regge parameters for the imaginary parts of the photoproduction amplitudes above 2.5 GeV and marginally different values for the momentum dependence of the exotic exchange and amounts of natural parity daughter exchange.

In general, at all energies above the third resonance region, our fit to the  $\pi^0$  photoproduction data is not as good as that to the  $\pi^\pm$  data. This could well be a reflection of the different dynamical mechanisms<sup>(6)</sup> governing single pion photoproduction at these energies. The real parts of the  $\pi^0$  amplitudes arise from delicate energy dependent interferences between each resonance and with the Regge contribution whereas for the  $\pi^\pm$  amplitudes, the Born terms dominate in the forward direction and vary smoothly with energy. These amplitudes are hence less sensitive to a correct description of the energy dependence of the resonance and Regge contributions. This is an obvious limitation on any energy dependent analysis.

The  $\chi^2$  for the fits in the intermediate energy region is almost comparable to that for the fits below 2 GeV. However the data set for this region contains relatively few points.

Due to its preliminary nature we chose not to include the recent Daresbury measurements<sup>(7)</sup> of P,  $\Sigma$  and T for  $\gamma P \rightarrow \pi^+ n$ . However we obtain qualitative agreement with this data and we do not believe that the final results of this experiment will present us any difficulty. Similarly we were unable to use the preliminary measurements<sup>(8)</sup> of H at Daresbury. Figs. 1 and 2 show our predictions for G and H between  $E_\gamma = 0.6$  and 1.6 GeV and gives a comparison with the Daresbury measurements of H. The rapid variation of G at  $\theta = 120^\circ$  is a notable feature of our predictions.

## 4. DISCUSSION OF RESULTS

A comparison of the resonance couplings with those predicted by the quark model<sup>(9,10)</sup> reveals good qualitative agreement as shown in Table IIb.

In the  $\{\overline{70}, L = 1^-\}$  multiplet, however, disagreement is noted with the couplings predicted for the  $S_{11}(1535)$ ,  $S_{11}(1705)$  and the  $A_{\frac{1}{2}}^n$  and  $A_{3/2}^n$  of the  $D_{13}(1700)$ .

However, by introducing some configuration mixing:-

$$S_{11}(1535) = \cos\theta \{(\overline{8}, 2), J=\frac{1}{2}\} - \sin\theta \{(\overline{8}, 4), J=\frac{1}{2}\}$$

$$S_{11}(1705) = \cos\theta \{(\overline{8}, 4), J=\frac{1}{2}\} + \sin\theta \{(\overline{8}, 2), J=\frac{1}{2}\}$$

with  $\theta \approx 30^\circ$ , where  $\{(\overline{8}, 2S+1), J = \frac{1}{2}\}$  refers to the nucleon member of spin  $\frac{1}{2}$  in the SU(3) octet of total quark spin S, we obtain reasonable agreement between our analysis and the quark model prediction for the  $S_{11}$  resonances. The disagreement of the quark model predictions for the  $D_{13}(1700)$  can be explained when one takes into account of the large uncertainties in the numbers obtained by the analysis and the fact that, as we have already mentioned, these uncertainties are by no means unique statistical errors.

In the  $\{\overline{56}, L = 2^+\}$  multiplet good agreement is again revealed.

Disagreements are evident, however, in the  $A_{\frac{1}{2}}^{V3}$  amplitude of the  $F_{35}(1890)$  the  $A_{3/2}^{V3}$  amplitude of the  $P_{33}(1690)$  and the  $A_{\frac{1}{2}}^{V3}$  amplitude of the  $P_{31}(1910)$ . These quark model predictions may well be greatly improved by including a substantial quark spin-orbit term in the  $\gamma N^*$  interaction as suggested by the Melosh transformation<sup>(11,12)</sup>.

The quark model gives values of 30 and -50 for the  $A_{\frac{1}{2}}^{V3}$  and  $A_{3/2}^{V3}$  amplitudes, respectively, of the  $P_{33}(1690)$ , as compared with the values of  $0 \pm 30$  obtained from our analysis.

Kubota and Ohta<sup>(13)</sup> apply relativistic "spin-orbit" corrections, which they obtain by applying the Foldy-Wouthuysen-Tani transformation on the relativistic Hamiltonian, to the non-relativistic quark model. They obtain much better agreement with the amplitudes of the  $F_{35}$ ,  $P_{33}$  and  $P_{31}$ , as well as improved agreements with the  $D_{13}(1520)$ ,  $S_{11}(1535)$  and  $F_{15}(1685)$ . Substantial mixing of the  $S_{11}$ 's is still required, however.

An interesting observation in reference (12) is that when our amplitudes are analysed within the context of the Melosh transformation the relative sign of the P-wave and F-wave pionic decays of the  $\{56, L = 2^+\}$  multiplet disagrees with that found in an analysis of  $\pi N \rightarrow \pi \Delta$ <sup>(14)</sup>.

We would like to interpret this as evidence for the existence of a  $\{\overline{70}, L = 2^+\}$  or  $\{\overline{70}, L = 0^+\}$  multiplet, which mixes fairly strongly with the  $\{\overline{56}, L = 2^+\}$ . This ties in very well with our need for a resonance in the  $F_{17}$  wave at 1990 MeV. Such a resonance could only be assigned to a  $\{\overline{70}, 2^+\}$  multiplet.

As usual the quark model predictions for the radial states are in disagreement with those of this analysis. These predictions, however, are extremely sensitive to the precise form used for the quark model wave-functions.

## 5. THE ENERGY INDEPENDENT FITS

As described above, the resonance parameters for the  $H_{19}(2220)$  and  $H_{311}(2420)$  are not determined by the above energy dependent fits. We therefore performed energy independent fits for  $W < 2.5$  GeV to the

amplitudes obtained in the above fits ( $|t| < 1.5$  (GeV/c)) together with differential cross-section data in the backward direction for  $\pi^+ n$  and  $\pi^0 p$  photoproduction. This analysis must be considered exploratory in that the backward differential cross-section data places additional constraints only on  $F_1 - F_2$ . Our motivation at present is to study the stability of the multipole solutions obtained and hence to estimate how clearly they are determined.

The energy independent analysis is performed by treating the Born contributions exactly and fitting the remainder to the multipole expansion cut off at  $\ell \sim 9$ . We find that the multipole amplitudes thus obtained are not sensitive to changes  $|\Delta\ell| \leq 2$  for  $W < 2.5$  GeV and that a good fit to the amplitudes obtained in the earlier fits and the backward data is maintained.

The results for the multipole amplitudes (without Born contributions) to which the  $G_{17}$ ,  $H_{19}$  and  $H_{311}$  couple are shown in Figs. 3, 4 and 5. In general the two fits have qualitatively the same structures in each multipole but differ quantitatively in the energy dependence of the real background.

The resonance parameters for the  $H_{19}$  differ markedly between the two fits but Fig. 4 shows that the multipole amplitudes projected from the resonance and Regge contributions are remarkably similar. This illustrates our comments above concerning the need to consider the combined effects of the two parametrizations before extracting resonance couplings. One would conclude that the  $H_{19}$  has a mass between 2.1 and 2.15 GeV with a half width of about 0.15 to 0.2 GeV coupling predominantly to the electric multipole.

The  $H_{311}$  couples significantly to the  $M_{5+}$  multipole in both fits. In fit 2 the shape is distorted by a steadily increasing real background but the energy dependence of the imaginary parts obtained from both fits are very similar. The Becchi-Morpurgo<sup>(15)</sup> selection rule required that the  $H_{311}$  should only couple weakly to the electric multipole. As can be seen from Fig. 5 this selection rule is satisfied by Fit 1 but may or may not be satisfied by Fit 2. In both fits the imaginary part seems to be increasing steadily with energy and the difference between the two solutions appears to lie in the behaviour of their real parts.

We would consider the above analysis of the intermediate energy region encouraging, indicating that a more complete energy independent analysis linked to the amplitudes from the fixed- $t$  dispersion relations could extract useful information about this energy region.

#### ACKNOWLEDGMENTS

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APPENDIX I

We list here the imaginary parts of the photoproduction isoscalar (0) and isovector (+,-) invariant amplitudes valid for  $W > 2.5$  GeV as found in fit I. The parameters obtained in fit II are given in brackets when they differ significantly from those of fit I. These amplitudes also contribute to the imaginary parts of the photoproduction amplitudes for  $2.0 \text{ GeV} < W < 2.5 \text{ GeV}$  when combined as in equation 2.1 with the Breit-Wigner contributions summarized in Table IIa

The amplitudes are normalized as in reference (17) and 1 GeV is taken as the unit of energy.

$$\text{Im } A_1^0 = \left[ 0.39e^{-2.8t} \begin{matrix} (0.24) \\ -0.26 \end{matrix} \left( \frac{s}{s_0} \right) e^{-3.0t} \right] J_2 \left( \frac{s}{s_0} \right)^{\alpha_\rho - 1}$$

$$\text{Im } A_1^- = \left[ 0.25e^{-2.9t} \begin{matrix} (7.42) \\ + 6.79 \end{matrix} \left( \frac{s}{s_0} \right) e^{5.5t} \right] J_2 \left( \frac{s}{s_0} \right)^{\alpha_{A_2} - 1}$$

$$\text{Im } A_1^+ = \left\{ 0.48e^{-0.41t} \begin{matrix} (0.68) \\ J_4 \end{matrix} + \left[ -0.93e^{+2.1t} \begin{matrix} (+1.8) \\ + 3.73 \end{matrix} \left( \frac{s}{s_0} \right) e^{1.4t} \right] J_2 \right\} \left( \frac{s}{s_0} \right)^{\alpha_\omega - 1}$$

$$\text{Im } t A_2^0 = -\text{Im } A_1^0 + 0.29e^{-0.87t} \begin{matrix} (-0.54) \\ J_2 \end{matrix} \left( \frac{s}{s_0} \right)^{\alpha_B - 1}$$

$$\text{Im } t A_2^- = -\text{Im } A_1^-$$

$$\text{Im } t A_2^+ = -\text{Im } A_1^+$$

$$\text{Im } A_3^0 = 0$$

$$\text{Im } A_3^- = 0$$

$$\text{Im } \sqrt{-t} A_3^+ = -32.6e^{+0.96t} \begin{matrix} (+2.0) \\ J_1 \end{matrix} \left( \frac{s}{s_0} \right)^{\alpha_V - 1}$$

$$\text{Im } \sqrt{-t} A_4^0 = 0.33e^{12.0t} \begin{matrix} (15.0) \\ J_1 \end{matrix} \left( \frac{s}{s_0} \right)^{\alpha_\rho - 1}$$

$$\text{Im } \sqrt{-t} A_4^- = 0.19e^{-1.4t} \begin{matrix} (-1.9) \\ J_1 \end{matrix} \left( \frac{s}{s_0} \right)^{\alpha_{A_2} - 1}$$

$$\text{Im } \sqrt{-t} A_4^+ = \left\{ 2.51e^{-1.3t} J_1 - 0.11e^{+0.51t} J_3 \right\} \left( \frac{s}{s_0} \right)^{\alpha_\omega - 1}$$

$$\text{with } s_0 = 2 \text{ GeV}^2.$$

The Bessel functions  $J_n$  are functions of  $R\sqrt{-t}$  with  $R = 1.05F$ .

In both fits the cut-off parameter  $\lambda$  was found to be small

$\lambda = 0.2 \text{ (GeV/c)}^{-1}$  for fit I and  $-0.4 \text{ (GeV/c)}^2$  for fit II. The

trajectories for the Regge exchanges included were found to be

$$\alpha_\rho = 0.48 + 1.00t \qquad \alpha_B = 0.025 + 1.05t$$

$$\alpha_{A_2} = 0.43 + 0.99t \qquad \alpha_V = -3.0 + 1.5t$$

$$\alpha_\omega = 0.34 + 1.05t$$

APPENDIX II

## SOURCES OF EXPERIMENTAL DATA USED IN THE ANALYSIS

 $\gamma p \rightarrow \pi^+ n$  differential cross sections

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$\gamma p \rightarrow \pi^+ n$  Polarized photon asymmetry ( $\Sigma$ )

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 $\gamma p \rightarrow \pi^+ n$  Recoil nucleon polarization (P)

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$\gamma p \rightarrow \pi^0 p$  differential cross section

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$\gamma p \rightarrow \pi^0 p$  Polarized Photon Asymmetry ( $\Sigma$ )

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TABLE CAPTIONS

- Table I            The  $\chi^2$  for the two fits to the data in the low, intermediate and high energy regions.
- Table IIa         The couplings, masses and widths for the two fits. All the masses and widths except for the  $P_{33}(1690)$  were varied during the fitting.
- Table IIb         The average couplings from the two fits compared with the predictions of the quark model. The method of obtaining the errors and their significance are discussed in the text. The asterisks indicate quark model predictions which are free from strong cancellations and hence those whose signs can be considered to be well determined.

TABLE I

Type of data	Fit	Threshold to $W=2.0\text{GeV}$		$2.0 < W < 2.5 \text{ GeV}$		$W=2.5\text{GeV to } 5.6\text{GeV}$	
		No. of data	$\chi^2/N$	No. of data	$\chi^2/N$	No. of data	$\chi^2/N$
$\gamma p \rightarrow \pi^+ n$ differential cross section	1	2140	1.59	46	1.87	83	1.76
	2	2182	1.53	46	1.60	83	1.87
$\gamma p \rightarrow \pi^+ n$ recoil nucleon polarisation	1	26	2.88	-	-	-	-
	2	26	3.22	-	-	-	-
$\gamma p \rightarrow \pi^+ n$ polarised photon asymmetry	1	207	1.83	14	1.18	22	1.23
	2	261	1.51	9	0.67	22	1.18
$\gamma p \rightarrow \pi^+ n$ polarised target asymmetry	1	61	2.48	7	4.90	10	0.70
	2	61	2.06	7	4.30	10	0.78
$\gamma p \rightarrow \pi^0 p$ differential cross section	1	2194	3.06	61	8.35	97	3.31
	2	2033	2.71	61	4.80	97	3.14
$\gamma p \rightarrow \pi^0 p$ recoil nucleon polarisation	1	197	3.03	1	0.60	12	1.27
	2	207	2.90	1	0.11	12	1.38
$\gamma p \rightarrow \pi^0 p$ polarised photon asymmetry	1	252	5.18	101	2.93	25	1.07
	2	263	4.24	123	1.57	25	1.10
$\gamma p \rightarrow \pi^0 p$ polarised target asymmetry	1	149	1.69	18	1.36	35	2.60
	2	141	1.68	18	0.95	35	2.85
$\gamma n \rightarrow \pi^- p$ differential cross section	1	491	1.77	8	2.89	3	1.19
	2	491	1.57	9	2.27	3	1.56
$\gamma n \rightarrow \pi^- p$ polarised photon asymmetry	1	113	2.49	9	3.67	14	5.31
	2	113	2.21	8	1.42	14	5.08
$\gamma n \rightarrow \pi^- p / \gamma p \rightarrow \pi^+ n$ ratio of differential cross sections	1	101	2.24	1	6.74	56	1.15
	2	101	2.21	1	10.66	56	1.15
$\gamma n \rightarrow \pi^0 n / \gamma p \rightarrow \pi^0 p$ ratio of differential cross sections	1	0	-	0	-	20	0.76
	2	0	-	0	-	20	0.75
Totals	1	5931	2.40	266	3.87	377	2.10
	2	5879	2.15	283	2.32	377	2.10



TABLE IIa

Resonance	Fit	Mass (MeV)	Width (MeV)	Couplings (GeV) <sup>-1/2</sup> × 10 <sup>3</sup>			
				A <sub>1/2</sub> <sup>P</sup>	A <sub>3/2</sub> <sup>P</sup>	A <sub>1/2</sub> <sup>n</sup>	A <sub>3/2</sub> <sup>n</sup>
P <sub>11</sub> (1470)	1	1415	331	-74	-	58	-
	2	1419	331	-75	-	60	-
D <sub>13</sub> (1520)	1	1504	137	-14	157	-56	-138
	2	1502	133	-18	157	-54	-144
S <sub>11</sub> (1535)	1	1510	133	75	-	-111	-
	2	1512	130	88	-	-112	-
D <sub>15</sub> (1670)	1	1682	195	18	12	-66	-73
	2	1677	189	25	7	-66	-72
F <sub>15</sub> (1685)	1	1679	119	-4	138	37	-36
	2	1681	118	-5	138	36	-39
S <sub>11</sub> (1700)	1	1689	197	50	-	-38	-
	2	1698	188	45	-	-52	-
D <sub>13</sub> (1705)	1	1726	100	-27	-8	49	15
	2	1712	151	-38	-19	50	54
P <sub>11</sub> (1780)	1	1735	128	-2	-	-34	-
	2	1706	205	3	-	-21	-
P <sub>13</sub> (1810)	1	1807	271	110	-57	13	42
	2	1810	299	111	-68	0	60
F <sub>17</sub> (1990)	1	1990	210	46	13	-93	-43
	2	2008	221	34	-6	-45	-100
G <sub>17</sub> (2190)	1	2135	201	-41	180	-83	-5
	2	2098	238	-18	179	-87	19
P <sub>33</sub> (1232)	1	1231.1	111.3	-141	-269	-	-
	2	1231.2	110.7	-143	-272	-	-
S <sub>31</sub> (1650)	1	1666	201	40	-	-	-
	2	1657	159	28	-	-	-
D <sub>33</sub> (1670)	1	1629	217	136	98	-	-
	2	1628	214	123	98	-	-
P <sub>33</sub> (1690)	1	-	-	-	-	-	-
	2	1690	225	0	0	-	-
F <sub>35</sub> (1890)	1	1899	132	35	-59	-	-
	2	1844	186	31	-51	-	-
P <sub>31</sub> (1910)	1	1936	208	-37	-	-	-
	2	1861	252	-32	-	-	-
F <sub>37</sub> (1950)	1	1911	175	-58	-79	-	-
	2	1913	220	-58	-71	-	-
D <sub>35</sub> (2000)	1	2024	489	-81	10	-	-
	2	2024	434	-43	27	-	-
H <sub>19</sub> (2220)	1	2184	303	1	29	-13	-27
	2	2345	284	-43	110	67	-19
H <sub>3,11</sub> (2420)	1	2394	327	-21	-33	-	-
	2	2303	324	0	-70	-	-

Table IIb

Resonance	Mass (MeV)	Width (MeV)	Couplings (GeV) <sup>-1/2</sup> × 10 <sup>3</sup>							
			A <sub>1/2</sub> <sup>P</sup>	A <sub>1/2</sub> <sup>P(QM)</sup>	A <sub>3/2</sub> <sup>P</sup>	A <sub>3/2</sub> <sup>P(QM)</sup>	A <sub>1/2</sub> <sup>n</sup>	A <sub>1/2</sub> <sup>n(QM)</sup>	A <sub>3/2</sub> <sup>n</sup>	A <sub>3/2</sub> <sup>n(QM)</sup>
P <sub>11</sub> (1470)	1417	331	-75±15	27	-	-	59±16	-18	-	-
D <sub>13</sub> (1520)	1503	135	-16± 8	-34	157± 7	109*	-55±14	-31	-141 15	-109*
S <sub>11</sub> (1535)	1511	132	82±19	156	-	-	-112±34	-108	-	-
D <sub>15</sub> (1670)	1680	192	22±10	0*	15± 6	0*	-66±20	-38*	-73±14	-53*
F <sub>15</sub> (1685)	1680	119	-5±15	-10	138±21	60*	37±10	30*	-38±18	0*
S <sub>11</sub> (1700)	1694	193	48±17	0	-	-	-45±24	30	-	-
D <sub>13</sub> (1705)	1719	126	-33±21	0*	-14±25	0*	50±42	-10*	35±30	-40*
P <sub>11</sub> (1780)	1721	167	1±39	-40	-	-	-28±45	10	-	-
P <sub>13</sub> (1810)	1809	285	111±47	100	-63±32	-30	7±20	-30	51±51	0*
F <sub>17</sub> (1990)	1999	216	40		4		-69		-72	
P <sub>33</sub> (1232)	1231.2	111	-142± 7	-108*	-271±10	-187*	-	-	-	-
S <sub>31</sub> (1650)	1662	180	34±28	47	-	-	-	-	-	-
D <sub>33</sub> (1670)	1629	216	130±37	88	98±36	84*	-	-	-	-
P <sub>33</sub> (1690)	1690	225	0±30	23	0±45	39	-	-	-	-
F <sub>35</sub> (1890)	1892	159	33±18	-20*	-55±19	-90*	-	-	-	-
P <sub>31</sub> (1910)	1899	230	-35±21	30	-	-	-	-	-	-
F <sub>37</sub> (1950)	1912	198	-58±13	-50*	-75±20	-70*	-	-	-	-
D <sub>35</sub> (2000)	2024	462	-62±64		19±54		-	-	-	-
G <sub>17</sub> (2190)	2117	220	-30		180		-85		7	

FIGURE CAPTIONS

- Fig. 1        Predictions for the asymmetry parameter G from fit 2. The predictions of fit 1 are essentially identical.
- Fig. 2        Comparison of the predictions from fit 2 for the asymmetry parameter H with the Daresbury measurements. The predictions of fit 1 are essentially the same.
- Fig. 3        Argand diagrams for the  $G_{17}$  multipoles in the intermediate region. The Born term contributions to the real parts are not included. Fit 1 is given by the dashed line and fit 2 by the solid line. The units correspond to  $\hbar$ , c and the pion mass set equal to one.
- Fig. 4        Argand diagrams for the  $H_{19}$  multipoles. The same comments as in Fig. 3 apply.
- Fig. 5        Argand diagrams for the  $H_{3,11}$  multipoles. The same comments as in Fig. 3 apply.

Fig. 1

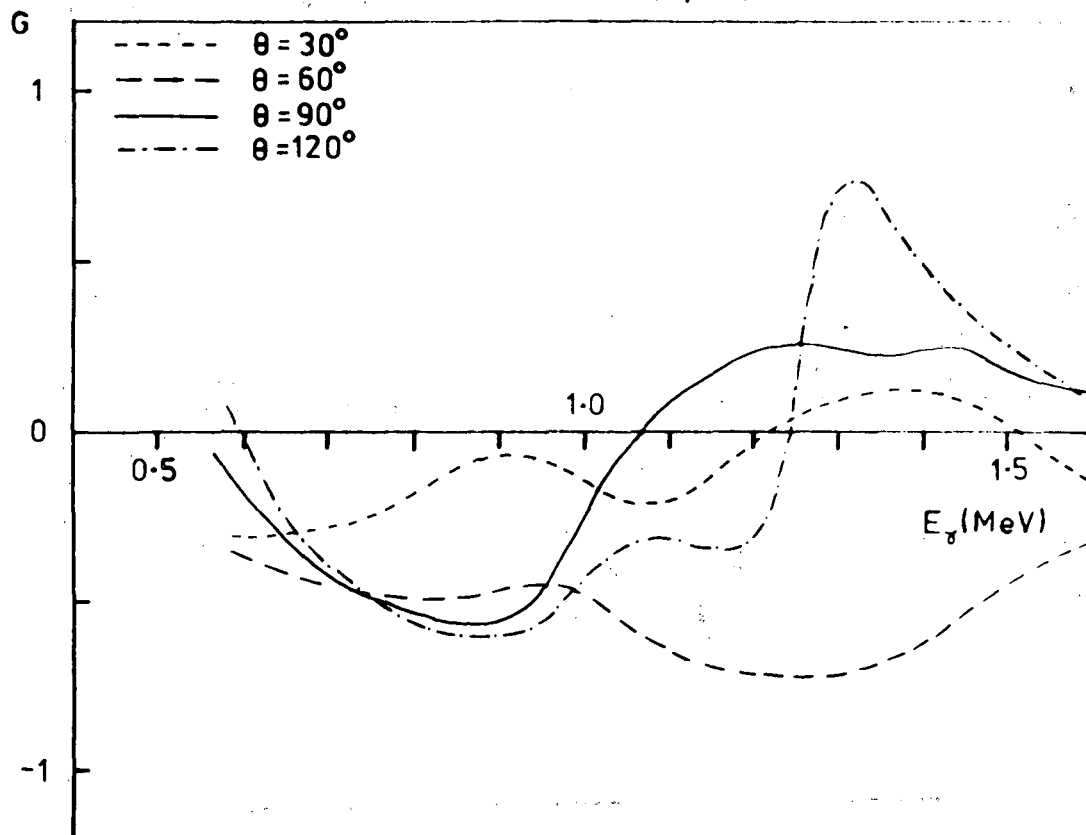


Fig. 2a

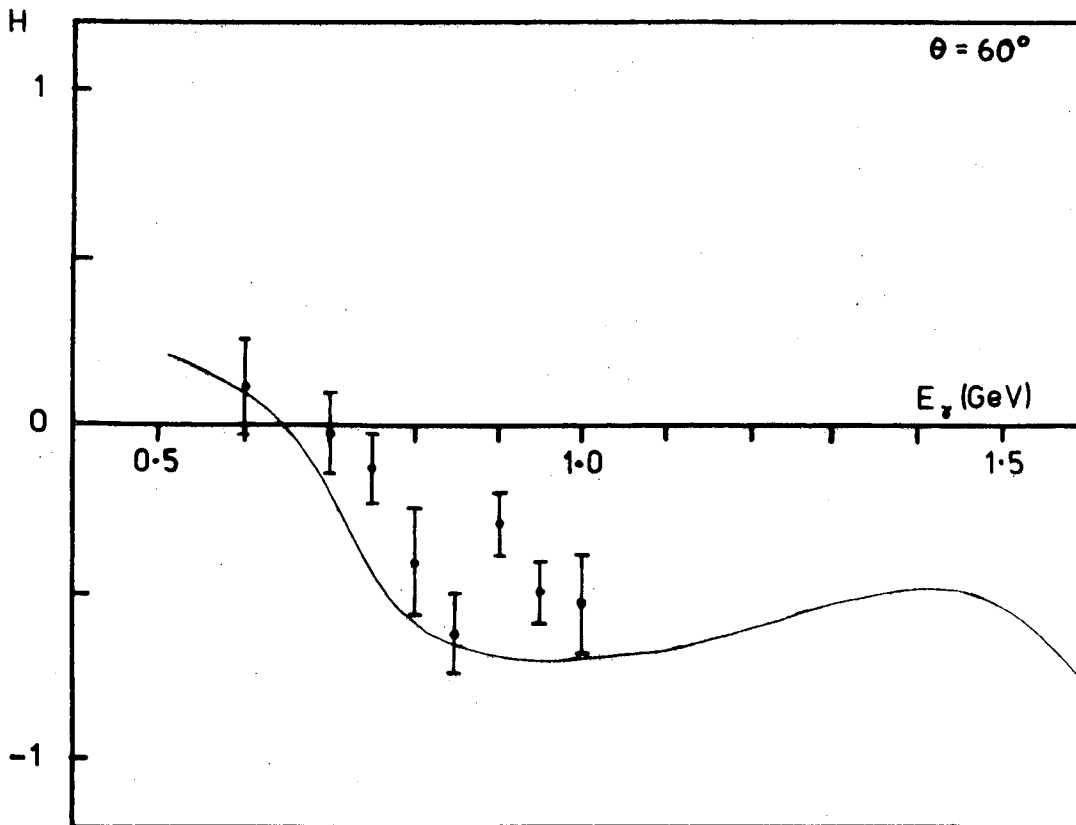


Fig. 2b

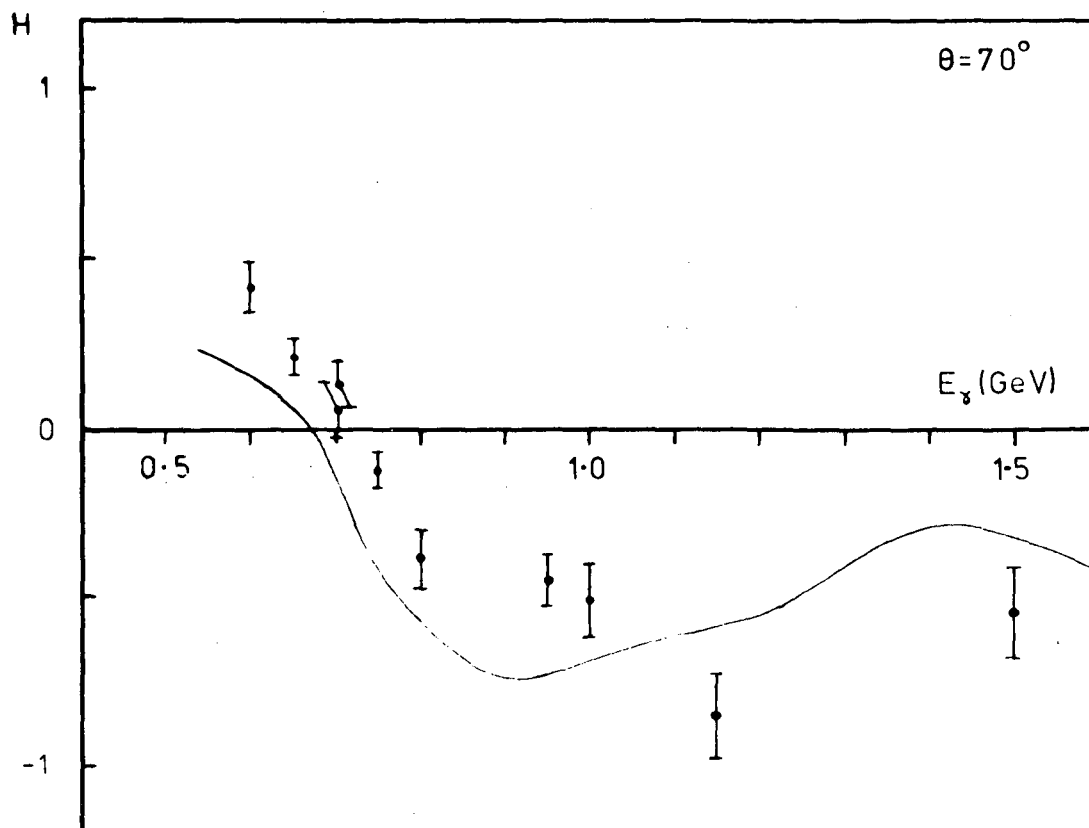


Fig. 2c

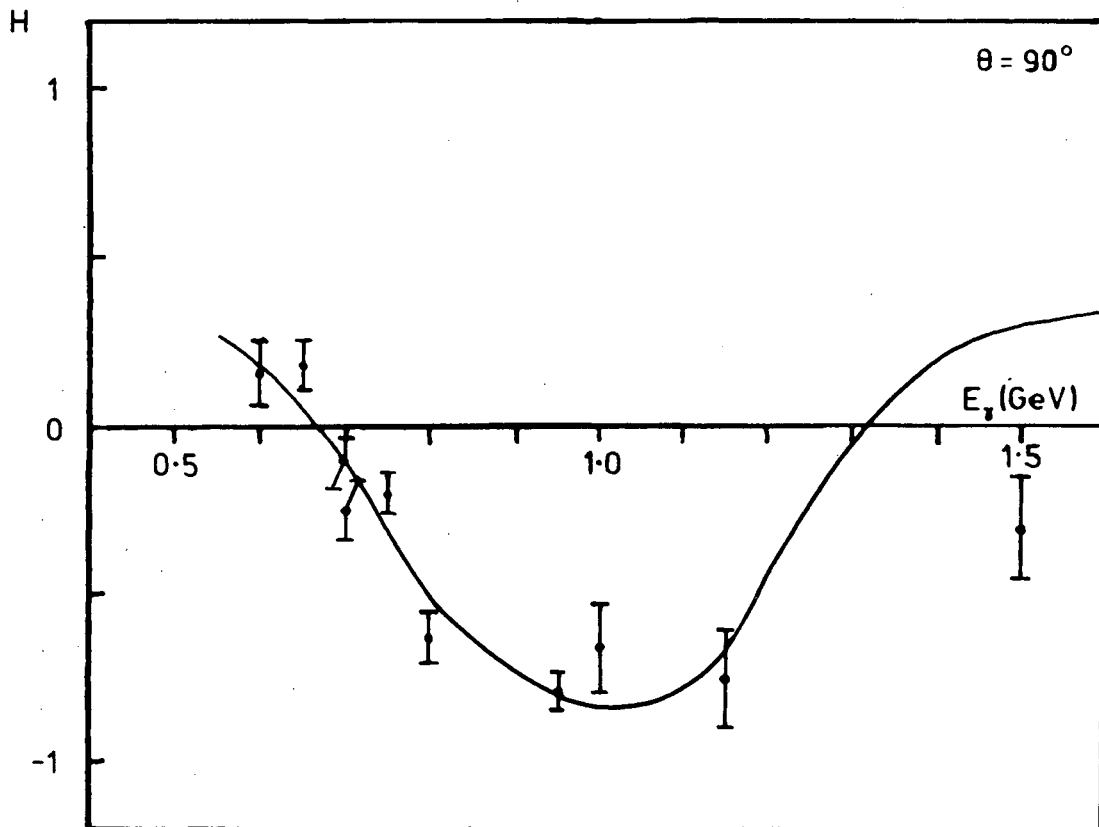


Fig. 2d

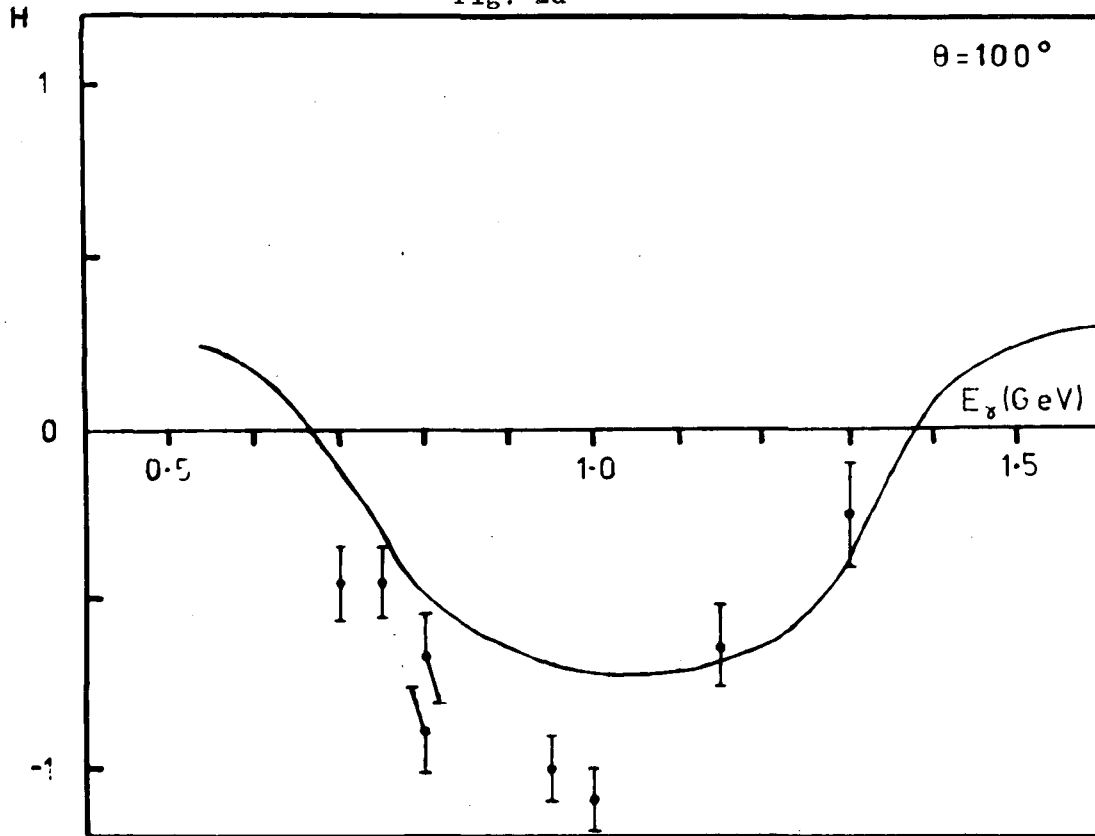




Fig. 3a

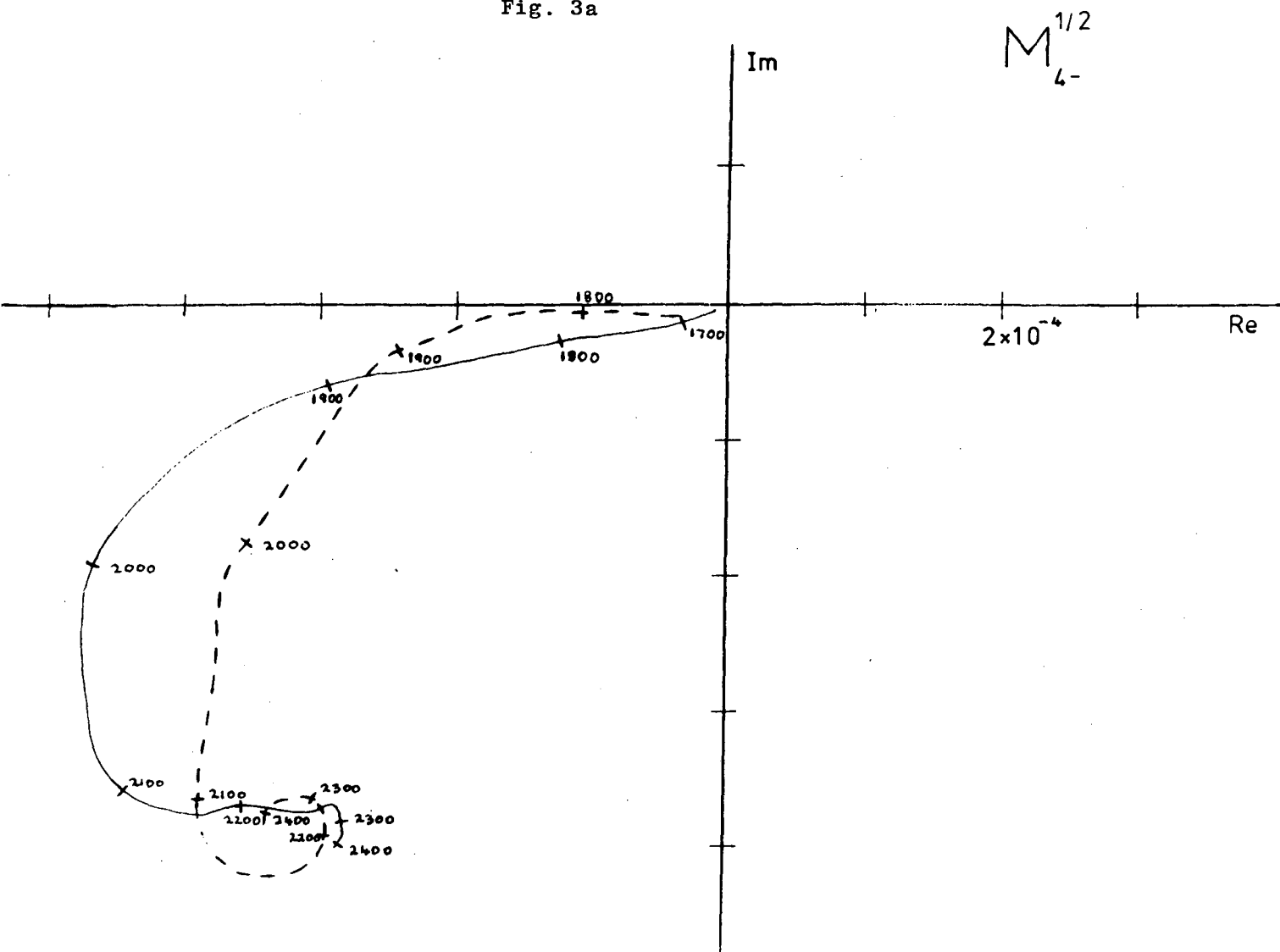


Fig. 3b

$E_{4-}^{1/2}$

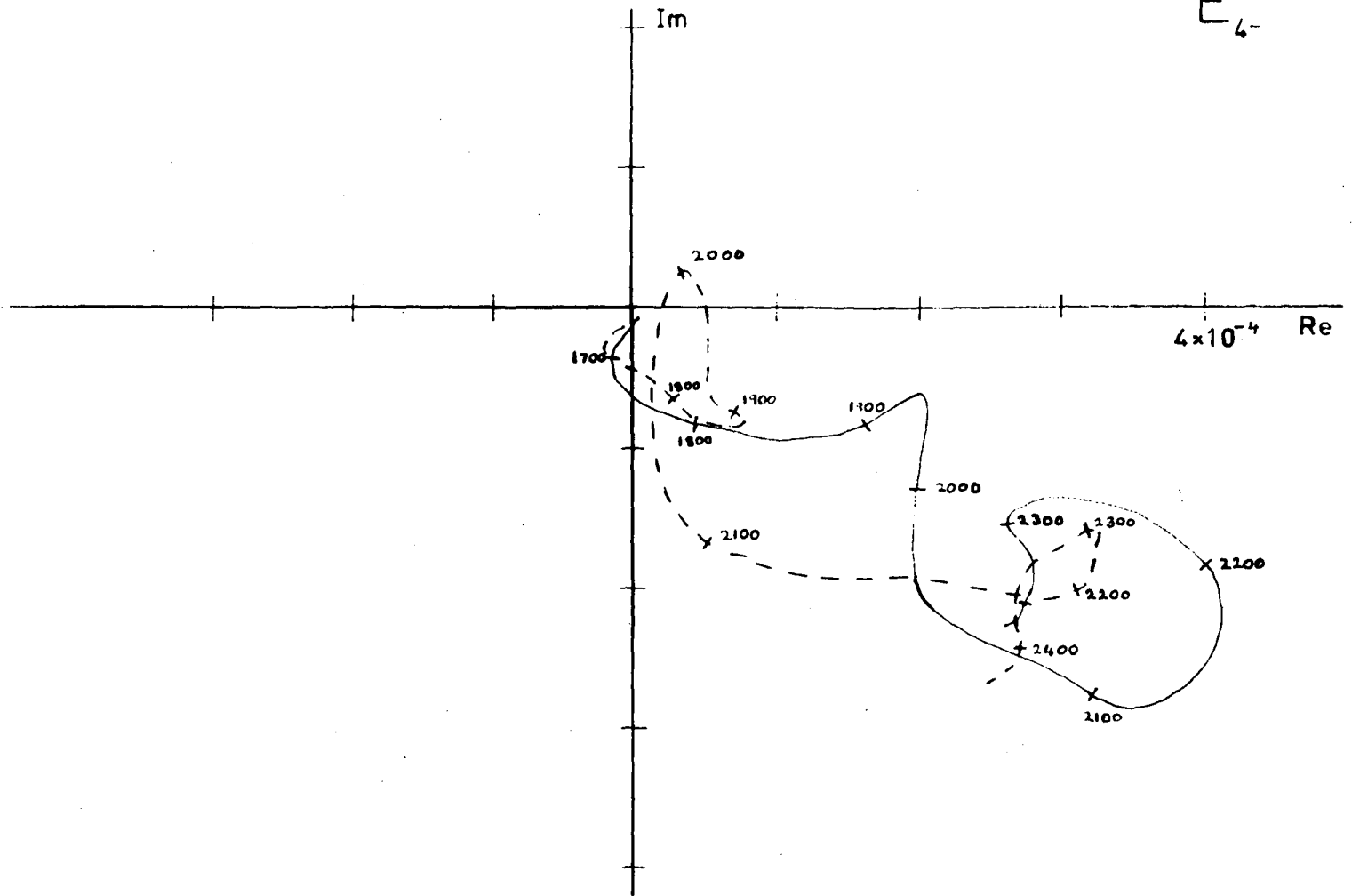


Fig. 4a

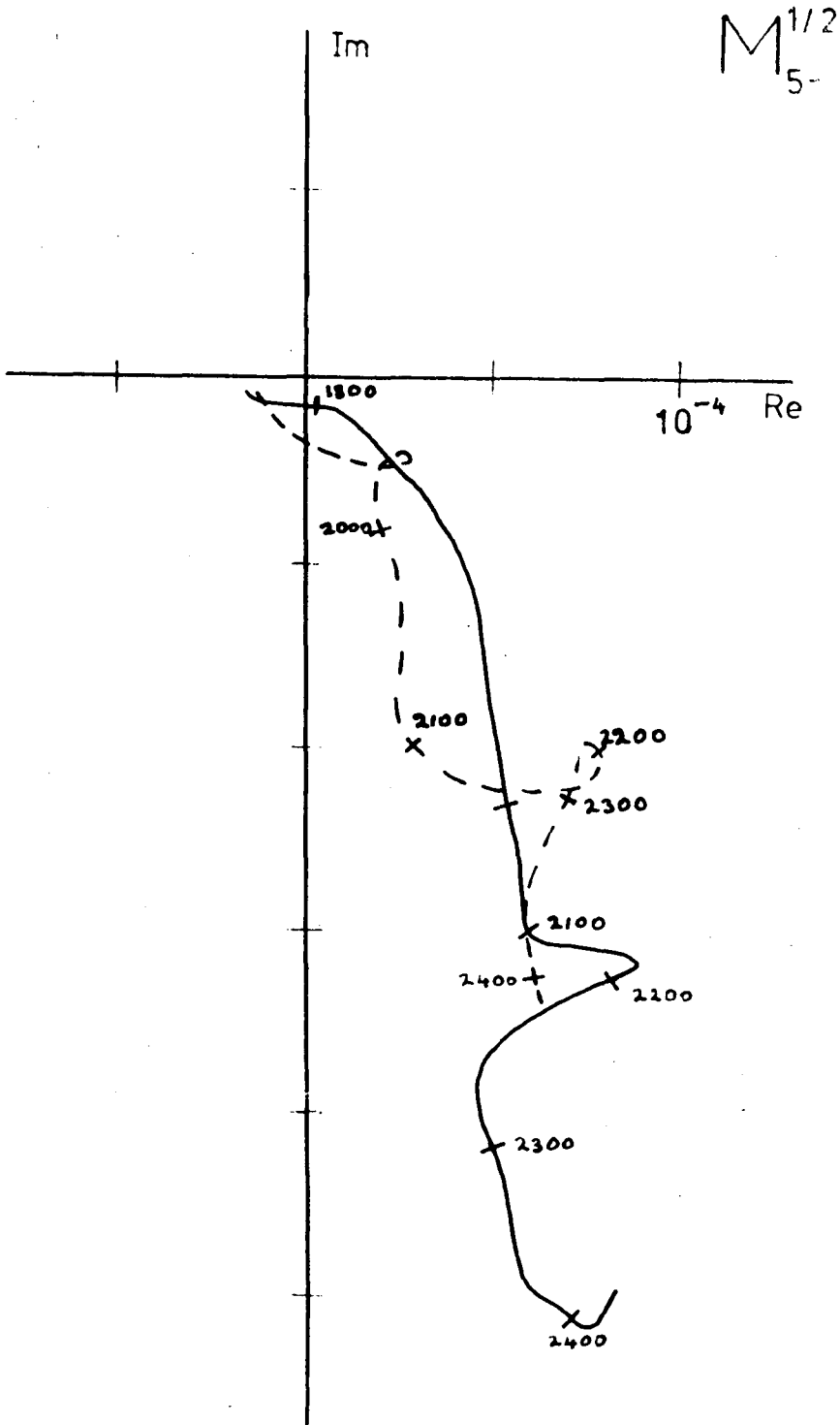


Fig. 4b

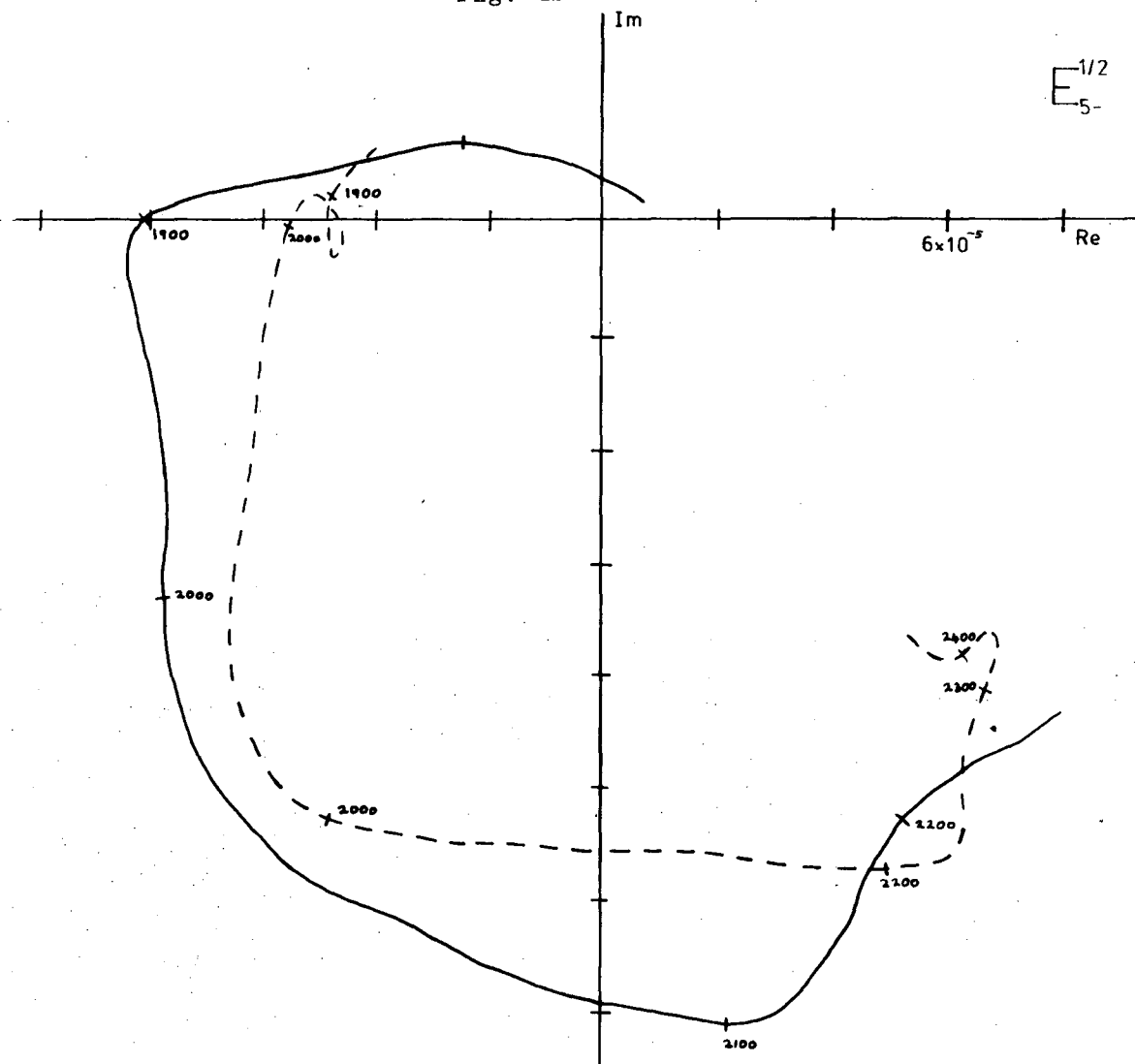


Fig. 5a

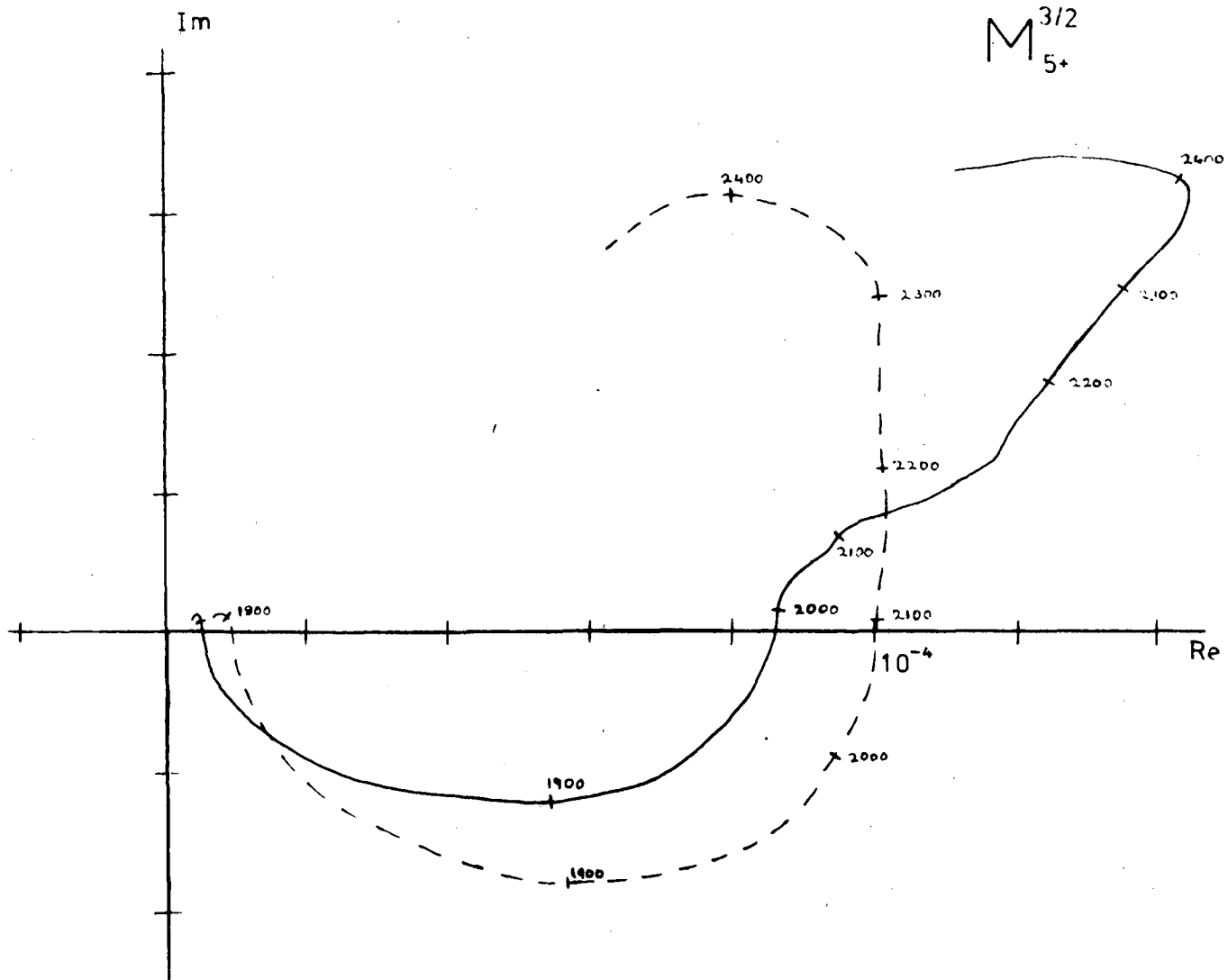
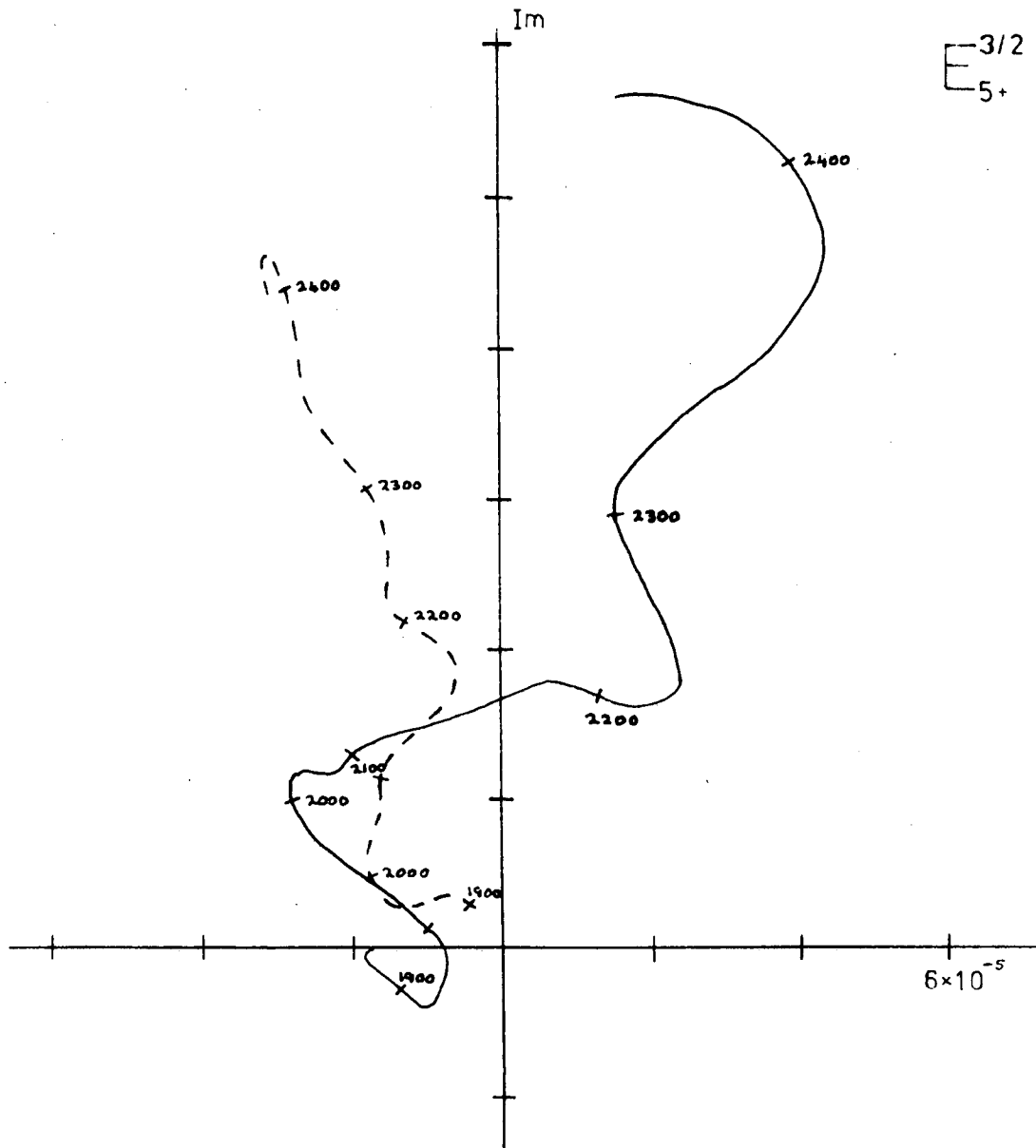


Fig. 5b



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