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Putting the "th" in Tenths: The Role of Labeling Decimals in Revealing Place Value Structure

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Abstract

Language is a powerful cognitive tool. For example, labeling objects or features of problems can support categorization and relational thinking. Less is known about their role in making inferences about the structure of mathematics problems. We test the impact of labeling decimals such as 0.25 using formal place value labels ("two tenths and five hundredths") compared to informal labels ("point two five") or no labels on children's problem-solving performance. Third- and fourth-graders (N = 104) were randomly assigned to one of three conditions (formal labels, informal labels, or no labels) and labeled decimals while playing a magnitude comparison game and number line estimation task. Formal labels facilitated performance on comparison problems that required understanding the role of zero, which highlighted place value structure. However, formal labels hindered performance when explicit understanding of place value magnitudes was required. Findings highlight how the language teachers and students use can impact problem-solving success.

Keywords: mathematics; problem solving; labels; decimals

Introduction

Previous research suggests language may play a critical role in learning and understanding across a variety of domains (Fyfe, McNeil, & Rittle-Johnson, 2015; Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999; Paik & Mix, 2003). Labels in particular have been shown to act as a powerful cognitive tool, recruiting processes that support categorization and relational thinking. For example, providing shared labels encourages children to treat objects similarly and categorize (e.g., Gelman & Markman, 1986; Graham, Kilbreath, & Welder, 2004). Further, children attribute characteristics of ambiguous objects based on their categorical label, rather than relying on perceptual features of the objects (Gelman & Markman, 1986). In addition to supporting categorization, providing shared labels that have a relational meaning enables children to map related sets of objects (Waxman & Gelman, 1986).

Much less is known about the role of shared labels in making inferences about the structure of mathematics problems. Looking for and making use of structure is one of eight mathematical practice standards outlined by the Common Core State Mathematics Standards (2010). For example, mathematically proficient students are able to recognize the relationship between place value location and the value of a digit (e.g., place values decrease from left to right).

Several indirect pieces of evidence suggest that labels may play a role in children's mathematics understanding. First, shared labels facilitated performance on a repeating patterns task (Fyfe et al., 2015). Four- to five-year-olds solved repeating pattern problems and were exposed to either shared, generic labels (e.g., A-B-B-A-B-B) or unshared, specific labels (e.g., blue-red-red-blue-red-red). Children in the formal labels condition solved more pattern problems correctly compared to children in the informal labels condition. The abstract pattern problems required children to make the same kind of pattern as a model pattern by recreating the part that repeats using new materials. The shared labels were generic and arbitrary, which may have helped reveal the structure in the model pattern and generalize it using new materials.

Second, cross-cultural studies suggest differences in number names used in different languages impact mathematics performance. English fraction labels lack information related to the relational magnitudes they represent. In comparison, East Asian languages use verbal names for fractions that explicitly represent part-whole relations. Cross-cultural research compared how Korean, Croatian, and U. S. children performed on a fraction-identification task prior to receiving formal instruction on fractions (Miura et al., 1999). Korean children significantly outperformed Croatian and U.S. children, suggesting differences in fraction labels impacted performance. Additionally, when English-speaking children were provided with fraction names that revealed part-whole relations in a similar way as Korean fraction labels, they outperformed Korean children on a similar fraction-identification task (Paik & Mix, 2003).

Thus, providing children with language that carries meaningful information and can be shared across multiple instances may be one way to support thinking that reveals the mathematical structure of problems. The current study tested how providing different labels for symbolic decimals helps children make inferences about place value structure.

Labels and Decimal Knowledge

How would you say the decimal 0.25? Most adults would name this decimal using informal "point" language (i.e., point two five or point twenty five). In contrast, when children learn to name decimals they are taught to use formal place value labels (i.e., twenty five hundredths). Teachers might also use decomposed place value labels by naming each place value separately (i.e., two tenths and five hundredths). The way we describe or label decimals may impact how children make sense of these numbers. The fractional amounts decimals represent are non-intuitive, and as a result, these symbols are often difficult to interpret. In an effort to understand, children often treat decimals like numbers they have lots of experience with – whole numbers (e.g., Stafylidou & Vosniadou, 2004). For example, when children are asked to compare decimal magnitudes they often think 0.25 is greater than 0.9 because of a whole number bias (25 is greater than 9; Resnick et al.,1998).

Labeling decimals using formal, decomposed place value labels might help children understand decimal magnitudes for at least two reasons. First, these labels could help reveal place value structure. Decomposed place value labels assign each digit with an associated value or magnitude, which may encourage children to make place value comparisons by providing a shared place value label. For example, when comparing 0.25 and 0.9, distinct place value labels may encourage children to compare 2 and 9 instead of comparing 25 and 9. Further, providing shared labels promotes relational thinking potentially by revealing the mathematical structure of problems (Fyfe et al., 2015). Second, these labels may help children distinguish decimals from whole numbers by reducing a whole number bias.

There are also compelling reasons to predict that informal point labels will aid or harm thinking. In comparison to formal place value labels, informal "point" labels that reflect familiar language adults use may activate partial understanding of decimal magnitudes children acquire during everyday experiences. Children are exposed to these labels for decimals in everyday environments in which we often label decimal amounts, such as reading thermometers and discussing weight. Mix et al. (2014) found that children as young as 3 years showed surprising understandings of multidigit place values on simple tasks focusing on mappings between spoken number names to written numerals, dots, or block representations. The authors argued that these partial understandings were likely acquired through statistical learning processes that occur in everyday environments rich with multidigit numerals and verbal number names. If children develop these partial understandings in a similar way with decimal magnitudes, labeling decimals using informal, familiar labels could activate this knowledge.

However, using informal labels may harm thinking by activating whole number misconceptions. Labeling digits using only their number names may encourage children to treat decimals like whole numbers. Activating misconceptions has been shown to hinder problem-solving performance (McNeil & Alibali, 2005), in part because children perseverate on using incorrect strategies (Fyfe, Rittle-Johnson & DeCaro, 2012). Thus, informal labels may encourage a whole number bias that interferes with children's problem-solving success.

Current Study

We examined the influence of naming decimals using formal, decomposed place value labels compared to informal, everyday labels or no labels on children's decimal magnitude problem-solving performance. Decimal magnitude knowledge was examined using two main performance measures (i.e., magnitude comparison and number line estimation) and several follow-up transfer tasks. When comparing symbolic decimals, children's success rates and the types of errors they make vary depending on features of the symbolic decimals they are comparing (e.g., Desmet, Grégoire, & Mussolin, 2010; Durkin & Rittle-Johnson, 2015; Resnick et al., 1998). For example, children are influenced by the length or number of digits (using whole number logic, assume decimals with more digits are greater than decimals with fewer digits) and the value of the digits (decimals that include zeros as placeholders are misunderstood).

We hypothesized that formal labels would facilitate performance on magnitude comparison problems that highlight place value structure by including zeros as placeholders and problems that require ignoring a whole number response. Additionally, we predicted that the advantages of informal labels would be counteracted by the activation of whole number misconceptions that interfere with problem solving. Therefore, we predicted children in the informal labels condition would perform similarly to the no labels condition.

Number line estimation was included as a more general measure of symbolic mapping knowledge for decimals. We hypothesized the same pattern of results for this task as the magnitude comparison task but expected effects to be weaker given that symbolic mapping knowledge requires additional knowledge of the specific quantity represented by a given written numeral that may draw more on children's prior knowledge.

Method

Participants

Participants were 121 third- and fourth-grade children. A pretest was given to identify children who did not already demonstrate a high level of decimal magnitude knowledge and thus would perform near ceiling regardless of label condition. Thirteen were excluded from participation because they scored above 75% on the pretest measure. Four additional children were excluded from analysis because they had diagnosed learning disabilities. The final sample included 104 children (*M* age = 9 yrs, 7 mos; 56% female; 26% ethnic minorities).

Design and Procedure

Children completed a brief pretest in their classrooms and participated in a single individual session lasting approximately 40 minutes. Children were randomly assigned to one of three conditions: formal labels (*n* = 56), informal labels (*n* = 55), or control (*n* = 56). The only difference between conditions was the labels the experimenter and children used to name decimals during the decimal comparison game and number line task. The remaining transfer tasks were administered without using any labels. All tasks were presented on a laptop computer with the exception of the decimal comparison card game. The experimenter read aloud each question and recorded the child's verbal response.

Materials

Pretest An abbreviated version of a validated assessment measured children's decimal magnitude knowledge (Durkin & Rittle-Johnson, 2015). Sample items included comparing decimal magnitudes (e.g., circle the decimal that is greater), identifying decimals worth the same amount (e.g., 0.5 and 0.50), writing a decimal that comes between two decimals (e.g., 0.4 and 0.5), and locating decimals on number lines.

Decimal Comparison Game The decimal labels manipulation occurred while children played a decimal magnitude comparison game (e.g., which decimal is greater?). The game had the same rules as the card game War. Children played the game with the experimenter using a deck of decimal cards, and the player with the greater decimal won each round. Children read aloud the decimal labels printed on the cards before choosing the greater decimal. The printed labels were removed halfway through game play to give children an opportunity to practice generating the decimal labels on their own with feedback from the experimenter. During game play, children compared the magnitudes of 40 pairs of decimals. Pairs were designed to reveal different levels of understanding based on previous research that has identified common errors children make when comparing decimal magnitudes (Desmet, Grégoire, & Mussolin, 2010; Durkin & Rittle-Johnson, 2015; Resnick et al., 1998). Comparisons fell into three different categories. On *benchmark comparisons* (n = 5), children compared a decimal to a familiar 0 or 1 benchmark. The second comparison type included *congruent and incongruent* pairs (n = 17). Congruent comparisons can be solved correctly by comparing decimals as whole numbers (e.g., 0.68 and 0.2; n = 7), whereas incongruent comparisons cannot be solved correctly using whole number rules (e.g., 0.51 and 0.8; n = 10). Finally, *role of zero* comparisons included decimals with a zero in either the tenths or hundredths place. Children often apply rules for the role of zero in whole numbers to decimals. For example, children ignore a leading zero (e.g., 0.04 is the same amount as 0.4) and think a trailing zero increases a decimal's magnitude (e.g., 0.40 is greater than 0.4). Eleven of these pairs had identical non-zero digits (e.g., 0.40 and 0.4 or 0.09 and 0.9). The remaining 7 pairs had different non-zero digits and a zero in the tenths place only (e.g., 0.07 and 0.1 or 0.8 and 0.02), so competing strategies of either comparing the digit values or comparing the length of the decimals could be used.

Decimal Number Line Estimation To measure magnitude knowledge, a 0-1 decimal number line task was created (18 trials; adapted from Siegler, Thompson, & Schneider, 2011). Children were instructed to name each decimal according to their assigned label condition before placing the decimal on the number line. The decimals were taken from previous work and included decimals with one or two digits (Rittle-Johnson, Siegler, & Alibali, 2001). Percent absolute errors (PAE) were calculated reflecting the absolute difference between the student's estimate and the correct location. Lower PAEs indicate more accurate estimates. Each child received an average PAE score across all 18 trials. Because children are often influenced by the number of digits a decimal has, we calculated an average PAE score for hundredths trials (e.g., 0.46; n = 5) and for tenths trials (e.g., 0.2; n = 5). An average PAE score was also calculated for the remaining 8 trials that included decimals with a zero in the tenths or hundredths place (e.g., 0.40 and 0.09) because children often experience confusion about the role of zero.

Decimal Comparison Transfer Decimal comparison transfer items (n = 10; adapted from Durkin & Rittle-Johnson, 2015; Rittle-Johnson, Siegler, & Alibali, 2001) included problems with decimals that included digits in either the thousandths or ones places, which children were not exposed to during the comparison game. Half of the problems were comparisons involving the role of zero (e.g., 3.3 and 3.300).

Place Value Two items assessed children's place value knowledge as used in Rittle-Johnson et al. (2001). These items were administered after the labels manipulation had occurred to determine if using formal labels helped children understand place value concepts. One item presented the number 413.728 and asked how much the 2 was worth from a list of 5 choices: 0.2, 2 tenths, 2 hundredths, 2 tens, or 2 hundreds. The second item asked how many tenths were in 30 hundredths.

Additional Measures Several additional items (n = 13) were included for exploratory purposes. Due to poor reliability and few condition differences, results are not reported for these measures.

Analysis and Results

To examine children's performance on the primary outcome measures (performance on decimal comparison game, decimal number line estimation accuracy, and decimal comparison transfer), a series of ANCOVAs with condition as a between-subject variable were performed. Specifically, condition was dummy coded with formal labels and informal labels entered into the models, and no labels as the reference group. In all models, children's age, grade, and their score on the pretest were included as covariates. Preliminary analyses revealed no interactions with age, grade, or pretest scores so these interaction terms were not retained in the final models.

Pretest

On the pretest, children answered a minority of problems correctly (M = 32% correct, SD = 14%). Importantly, there were no differences by condition at pretest, $F < 1$.

Decimal Comparison Game Performance

Across conditions children solved about half of the decimal magnitude comparison problems correctly (see Table 1). There were no significant effects between any of the label type conditions for overall performance, *F's* < 2.5. Children performed above chance (33%), $t(103) = 7.05$, $p < .001$, but condition differences were not reliable. Comparison types were designed with common errors and misunderstandings in mind, and some comparisons could be solved correctly using whole number rules. Thus, we compared performance on the three comparison types.

Table 1: Summary of Performance by Condition

Task	Formal *M (SD)*	Informal *M (SD)*	Control *M (SD)*
Comparison Game Accuracy	.53 (.25)	.44 (.18)	.47 (.20)
Benchmark	.74 (.28)	.71 (.27)	.71 (.28)
Congruent	.91 (.26)	.96 (.18)	.94 (.21)
Incongruent	.31 (.43)	.14 (.31)	.16 (.35)
Role of zero			
Same digits	.43 (.38)*	.20 (.34)	.24 (.39)
Different digits	.45 (.38)*	.53 (.28)	.62 (.21)
Number Line Est. PAE	20 (12)*	18 (6)	16 (4)
Tenths	30 (16)	36 (13)	35 (13)
Hundredths	13 (10)	10 (8)	9 (5)
Role of zero	18 (17)*	11 (11)	8 (5)
Comparison Transfer Accuracy	.48 (.16)*	.39 (.13)	.43 (.12)
Congruent & Incong.	.42 (.19)	.39 (.13)	.38 (.12)
Role of zero	.54 (.18)*	.39 (.18)	.47 (.21)

As expected, children's percent correct on *benchmark comparisons* was high and similar across conditions (*F's* < 1; see Table 1). Children across all three conditions were also successful on *congruent comparison*s that could be solved correctly using a whole number rule (e.g., 0.62 and 0.2; *F's* < 1; see Table 1). However, for *incongruent comparisons* in which a whole number rule produced an incorrect answer (e.g., 0.51 and 0.8), children's percent correct with formal labels was highest and lower with informal and no labels (see Table 1). These condition differences were not reliable, though. Children in the formal labels condition performed at chance (33%), $t(34) = -.26$, $p = .80$, but children in the informal labels and no labels conditions performed significantly below chance, $t(33) = -3.56$, $p = .001$ and $t(34) = -2.91$, $p = .006$, respectively.

Children's percent correct on *role of zero comparisons* with identical non-zero digit values (e.g., 0.40 and 0.4) was highest with formal labels and lower with informal and no labels (see Table 1). There was a significant effect of formal labels relative to no labels, $F(1, 98) = 4.94$, $p = .03$, $\eta_p^2 = .05$, and no effect of informal labels relative to no labels, $p = .72$. A follow-up analysis revealed a significant effect of formal labels relative to informal labels, $F(1, 98) = 6.55$, $p = .01$, $\eta_p^2 = .06$. Children's accuracy on role of zero comparisons with different non-zero digits (e.g., 0.07 and 0.1) was highest with no labels, lower with informal labels, and lowest with formal labels (see Table 1). There was a significant, negative effect of formal labels relative to no labels, $F(1, 98) = 7.50$, $p = .01$, $\eta_p^2 = .07$. There was no significant effect of informal labels relative to no labels, $p = .21$, or between the two label types, $p = .15$.

To understand the negative effect of formal labels relative to no labels, we examined performance on problems where the correct answer can be achieved using whole number rules. On 4 of these 7 problems, ignoring a zero and choosing the decimal with the greater digit results in the correct answer (e.g. 0.03 and 0.4). Children's percent correct on these 4 problems was highest in the control condition (M = 91%, SD = 38%), lower in the informal labels condition (M = 79%, SD = 37%), and lowest in the formal labels condition (M = 55%, SD = 42%). There was a significant, negative effect of formal labels relative to no labels, $F(1, 98) = 17.64$, $p < .01$, $\eta_p^2 = .15$. There was no significant effect of informal labels relative to no labels, $p = .20$. A follow-up analysis revealed a significant, negative effect of formal labels relative to informal labels, $F(1, 98) = 8.25$, $p = .01$, $\eta_p^2 = .08$. On the remaining 3 problems, the correct answer could not be obtained by ignoring a zero and using whole number rules (e.g., 0.07 and 0.1). Children's percent correct on these problems was highest in the formal labels condition (M = 31%, SD = 41%), lower in the control condition (M = 24%, SD = 38%), and lowest in the informal labels condition (M = 18%, SD = 34%), but condition differences were not reliable, *F's* <1.64.

Comparison Game Performance Summary Performance on difficult incongruent and role of zero comparisons revealed some positive but mixed effects of providing formal labels. Formal labels led to higher performance on role of zero comparisons that isolate place value, but only when there was no competing digit value information (i.e., only for problems that had identical non-zero digits). For role of zero comparisons that had different non-zero digits, formal labels led to lower performance, potentially by reducing a whole number bias that led to the correct answer.

Number Line Estimation Accuracy

Overall, children's estimations of decimal locations on the number line were inaccurate (see Table 1). There was an unexpected significant, negative effect of formal labels relative to no labels, $F(1, 98) = 5.22$, $p = .03$, $\eta_p^2 = .05$. There was no significant effect of informal labels relative to no labels and no significant effect between the two label types, *p's* > .21. To understand the negative effect of formal labels relative to no labels, we examined performance on trials in which children may have been influenced by the number of digits in a decimal and could have experienced confusion about the role of zero.

Children's estimates were least accurate for one-digit decimals that only included *tenths*, suggesting these were the most difficult trials (see Table 1). There were no significant effects between any of the three label type conditions, *p's* > .10. Children's estimates for two-digit decimal *hundredths trials* were more accurate than estimates for tenths trials (see Table 1), but condition differences were not reliable for these trials. Children's PAE for two-digit decimal *role of zero trials* were similar to hundredths trials (see Table 1). There was a significant, negative effect of formal labels relative to no labels, $F(1, 98) = 12.56$, $p < .01$, $\eta_p^2 = .11$. There was no significant effect of informal labels relative to no labels, $p = .22$. A follow-up analysis revealed a significant, negative effect of formal labels relative to informal labels, $F(1, 98) = 5.16$, $p = .03$, $\eta_p^2 = .05$.

In general, formal labels impeded children's ability to accurately estimate the location of decimals on a 0-1 number line. This negative effect of formal labels was strongest for role of zero trials and was not present for the most difficult tenths trials.

Decimal Comparison Transfer

Performance across conditions was low on transfer magnitude comparison problems, although performance was significantly above chance (33%), $t(103) = 7.43$, $p < .001$ (see Table 1). There was no significant effect of either label type relative to no labels, *p's* > .14. A follow-up analysis revealed a significant effect of formal labels relative to informal labels, $F(1, 98) = 5.16$, $p = .03$, $\eta_p^2 = .05$.

We also examined performance on congruent and incongruent comparisons and role of zero comparisons. For congruent and incongruent comparisons, there were no effects between any conditions, *p's* < .31. For role of zero comparisons, there was no significant effect of either label type relative to no labels. A follow-up analysis revealed a significant effect of formal labels relative to informal labels, $F(1, 98) = 9.60$, $p < .01$, $\eta_p^2 = .09$.

Place Value Knowledge

Despite exposure to formal place value labels, only a quarter of the children in the formal labels condition were able to use the learned labels to correctly identify the hundredths place value (26%). A similar percentage of children in the informal labels condition (18%) and the no labels condition (6%) were successful on this item, χ^2 (2, $N = 104$) = 5.18, $p = .08$.

Children were much more successful at determining how many tenths were in 30 hundredths. More children in the formal labels condition answered this item correctly (69%) compared to children in the informal labels condition (38%) and no labels condition (43%), χ^2 (2, $N = 104$) = 7.43, $p = .02$. Thus, formal labels seemed to reveal place value structure, as evidenced by understanding the relationship between tenths and hundredths.

Discussion

While shared labels have been shown to support categorization and relational thinking (e.g., Gelman & Markman, 1986; Waxman & Gelman, 1986), less is known about their role in making inferences about the structure of mathematics problems. Several indirect pieces of evidence suggest that labels play a role in mathematics understanding (e.g., Fyfe et al, 2015; Miura et al., 1999; Paik & Mix, 2003).

We found that naming decimals using formal, decomposed place value labels had mixed effects on decimal magnitude problem solving performance. Children who learned to name decimals using formal labels (e.g., "two tenths and five hundredths") compared to informal labels (e.g., "point two five") or no labels were better able to solve decimal magnitude problems that required understanding the role of zero. In particular, they solved slightly more incongruent magnitude comparison problems correctly (e.g., Which decimal is greater, 0.51 or 0.8?), solved more role of zero comparison problems correctly with decimals that had identical non-zero digits (e.g., Which decimal is greater, 0.4 or 0.40? 0.09 or 0.9?), and were better able to determine the relationship between tenths and hundredths. In part, they may have been less likely to treat decimals as whole numbers compared to children in the informal and no labels conditions.

However, there were unexpected negative effects of formal labels compared to informal and no labels. Their performance was lower on role of zero magnitude comparison problems and number line estimation problems that required explicit knowledge of how much tenths and hundredths are worth. This decrement in performance compared to the informal and/or no labels conditions on some tasks may reflect a transitional phase when children's performance becomes worse before it becomes better (e.g., "U-shaped development"; McNeil, 2007; Namy et al., 2004; Siegler, 2005). As children learn that their way of thinking only sometimes leads to correct solutions, they begin to reject that way of thinking; however, rejecting an old way of thinking and generating new, correct ways of thinking are separate processes that develop over time (Siegler, 2005), so children's performance can get worse before it gets better. Noticing place value structure seems to reflect a kind of transitional knowledge important for developing decimal magnitude knowledge. Indeed, understanding the role of zero as a placeholder was found to reflect an intermediate level of decimal magnitude knowledge (Resnick et al., 2016). Further, correctly naming decimals using place value language is predictive of symbolic-mapping knowledge for decimals two years later (Mazzacco & Delvin, 2008).

Compared to formal place value labels, using informal labels could have activated contextual knowledge children acquire through everyday experiences (e.g., Mix et al., 2014). Unfortunately, the tasks used in the current study may not have been suitable for revealing this type of knowledge. In general, children in the informal labels condition performed similarly to those in the control condition. There was some concern that informal labels might activate whole number misconceptions. Children in the informal labels condition performed somewhat worse than children in the no labels condition on transfer role of zero magnitude comparisons. However, in general informal language did not seem to activate misconceptions more so than no labels.

In conclusion, findings from the current study extend previous research on the role of language in mathematics learning, and more specifically the use of labels to reveal the mathematical structure of problems (Fyfe et al., 2015). Identifying mathematically meaningful labels may be a powerful first step in the process of impacting students' problem-solving behavior and understanding.

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