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LIE-TRANSFORM DERIVATION OF
THE GYROKINETIC HAMILTONIAN SYSTEM

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THE GYROKINETIC HAMILTONIAN SYSTEM

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ABSTRACT. The Hamiltonian structures of the self-consistent gyrokinetic equations, and of the cold guiding-center plasma, are derived from first principles. The Vlasov system is first formulated as an action principle, and is then subjected to a single-particle Lie transform. The resulting expression then yields the Poisson structure, on functionals of the guiding-center distribution, and the system Hamiltonian. The self-consistent field is eliminated by a subsidiary condition.

The Lie transform [1,3,4,11] has proved to be a very successful tool for systematically reducing the Hamiltonian description of single-particle motion. In the area of plasma physics, the chief applications have been to the concepts of oscillation-center motion [2,6,7] and guiding-center motion [10,12]. This method transforms to higher order the effects of oscillatory motion and gyration, respectively. These higher order terms then contribute, for example, to ponderomotive effects [2,6] and to wave coupling [7].

In order to obtain a (self-consistent) system Hamiltonian corresponding to this reduced description, it is necessary to imbed the single-particle Lie transform in a many-particle or continuum framework which includes the self-consistent fields. In the companion paper [9], a Lagrangian principle is presented for the Vlasov equation with Coulomb interaction, which leads to the accepted Hamiltonian structure of the Vlasov equation. However, that principle utilizes particle orbits in configuration space, and thus is of no help for the Lie transform, which operates in particle phase space.

In this paper, the action principle for the Vlasov system is first formulated in particle phase space. The invariance of the particle action integral under canonical transformations then enables us to effect the Lie

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transform directly. The treatment of the self-consistent field requires special care, of course. In the new variables, a system Hamiltonian is then obtained in terms of the reduced description. The standard Poisson structure [8] then yields the desired nonlinear Vlasov equation.

To illustrate this methodology, we treat the gyrokinetic model studied by Dubin, Krommes, Oberman, and Lee [5]. These authors consider a Vlasov plasma in a uniform magnetic field, with only Coulomb interaction. The self-consistent Coulomb potential is assumed to vary slowly on the gyrofrequency time-scale, and in addition to be weak relative to the particle kinetic energy. Further, its spatial variation along the magnetic field is weak.

These authors present a careful treatment of the Lie transform to the guiding-center description, wherein the gyrophase is systematically eliminated from the particle Hamiltonian. The Poisson equation for the self-consistent potential is then expressed in terms of the Vlasov guiding-center distribution. Finally, an energy invariant is found for the coupled Vlasov and Poisson equations. As we shall see below, the energy is actually the system Hamiltonian which generates the nonlinear evolution.

Our starting point is an action principle for the self-consistent Vlasov system:

$$S(\underline{p}, \underline{q}, \phi) = \int dt \int d^3x |\nabla \phi(\underline{x}, t)|^2 / 8\pi + \int dt \int d^6z_0 f_0(z_0) (\underline{p}(z_0, t) \cdot \underline{\dot{q}}(z_0, t) - h(\underline{p}, \underline{q}, \phi)) \quad (1)$$

Here $\underline{p}$ and $\underline{q}$ are two conjugate vector-valued fields on (z_0, t) space, where z_0 represents the six components of particle-initial-condition. The initial Vlasov distribution is $f_0(z_0)$. The particle Hamiltonian function is h , expressed in terms of $\underline{p}, \underline{q}$ and the scalar potential field $\phi(\underline{x}, t)$. (Appropriate summation over species is implicit).

We consider independent variations of the three fields $\phi, \underline{p}, \underline{q}$, and demand that S be stationary. Varying $\phi(\underline{x}, t)$, we obtain

$$0 = \delta S / \delta \phi(\underline{x}, t) = - \nabla^2 \phi(\underline{x}, t) / 4\pi - \int d^6z_0 f_0(z_0) \delta(\int h dt) / \delta \phi(\underline{x}, t). \quad (2)$$

We write the particle Hamiltonian as

$$h(\underline{p}, \underline{q}, \phi) = h_{\text{kin}}(\underline{p}, \underline{q}) + \int d^3x \phi(\underline{x}, t) \rho(\underline{x}; \underline{p}, \underline{q}), \quad (3)$$

where

$$\rho(\underline{x}; \underline{p}, \underline{q}) = e \delta^3(\underline{x} - \underline{r}(\underline{p}, \underline{q})) \quad (4)$$

is the single-particle charge density, while $\underline{r}(\underline{p}, \underline{q})$ is particle position in terms of the chosen representation $\underline{p}, \underline{q}$. Inserting (3) in (2), we obtain

$$- \nabla^2 \phi(\underline{x}, t) / 4\pi = \int d^6z_0 f_0(z_0) \rho(\underline{x}; \underline{p}(z_0, t), \underline{q}(z_0, t)). \quad (5)$$

We define the Vlasov distribution in $\underline{p}, \underline{q}$ space by

$$f(\underline{p}_1, \underline{q}_1; t) = \int d^6 z_0 f_0(z_0) \delta^3(\underline{p}_1 - \underline{p}(z_0, t)) \delta^3(\underline{q}_1 - \underline{q}(z_0, t)) \quad (6)$$

Thus (5) reads

$$-\nabla^2 \phi(\underline{x}, t) / 4\pi = \rho(\underline{x}; f) \quad (7)$$

where

$$\rho(\underline{x}; f) = \int d^6 z f(\underline{p}, \underline{q}; t) \rho(\underline{x}; \underline{p}, \underline{q}) \quad (8)$$

is the system charge density. The Poisson equation (7) enables ϕ to be expressed as a functional of f .

Returning to (1), we vary S with respect to the fields $\underline{p}$ and $\underline{q}$, obtaining

$$\dot{\underline{q}}(z_0, t) = \partial h(\underline{p}, \underline{q}, \phi) / \partial \underline{p}, \quad \dot{\underline{p}}(z_0, t) = -\partial h(\underline{p}, \underline{q}, \phi) / \partial \underline{q}, \quad (9)$$

the particle Hamiltonian equations. As a result, the Vlasov distribution (6) satisfies the Vlasov equation

$$\partial f(\underline{p}, \underline{q}; t) / \partial t = -[f, h], \quad (10)$$

where

$$[a, b] = \frac{\partial a}{\partial \underline{q}} \cdot \frac{\partial b}{\partial \underline{p}} - \frac{\partial a}{\partial \underline{p}} \cdot \frac{\partial b}{\partial \underline{q}} \quad (11)$$

is the canonical bracket in particle phase space.

Considering the action S as the time integral of a Lagrangian, we see from (1) that the field canonically conjugate to $\underline{q}(z_0, t)$ is $f_0(z_0) \underline{p}(z_0, t)$, in agreement with Eq. (4) of the companion paper. The methods of that paper then lead again to the Poisson bracket on system functionals:

$$\{A_1, A_2\} = \int d^6 z f(z) [\partial A_1 / \partial f(z), \partial A_2 / \partial f(z)]. \quad (12)$$

The system Hamiltonian H is read off from the action (1), interpreted as

$$S = \int dt (\int d^6 z_0 \dot{\underline{q}} \cdot \underline{p} f_0) - H dt. \quad (13)$$

We thus obtain

$$H(f) = \int d^6 z f(z) h(z, \phi) - \int d^3 x |\nabla \phi|^2 / 8\pi, \quad (14)$$

where we have used (6), and ϕ is a functional of f from (7). Considering the right side of (14) as $H(f, \phi(f))$, we note that the particle Hamiltonian

$$h(z) = \delta H / \delta f(z) \quad (15)$$

can be separated into explicit and implicit parts:

$$h(z) = \delta H(f, \phi) / \delta f(z) + \int d^3 x \delta H(f, \phi) / \delta \phi(\underline{x}) \delta \phi(\underline{x}) / \delta f(z) \quad (16)$$

But the factor $\delta H/\delta$ vanishes, as a result of Eq. (2). Hence, the first term of (16) yields the identity $h = h$. Although the form (14) is more convenient for calculation, we note that, from (3) and (7), it can be expressed as

$$H(f) = \int d^6 z f(z) h_{\text{kin}}(z) + \int d^3 x |\nabla \phi(f)|^2 / 8\pi, \quad (17)$$

in agreement with the final equation of the companion paper.

Having assured ourselves that the action principle (1) yields correct results, we now perform a canonical transformation on particle phase space:

$$\underline{p}, \underline{q}, h(\underline{p}, \underline{q}, \phi) \rightarrow \underline{P}, \underline{Q}, K(\underline{P}, \underline{Q}, \phi). \quad (18)$$

We shall use the Lie transform to effect this transformation, but for now we may treat the transformation as arbitrary. We use the covariance property of the particle action integral: for given z_0 ,

$$\int dt (\underline{p}(z_0, t) \cdot \dot{\underline{q}}(z_0, t) - h(\underline{p}, \underline{q}, \phi)) = \int dt (\underline{P}(z_0, t) \cdot \dot{\underline{Q}}(z_0, t) - K(\underline{P}, \underline{Q}, \phi)). \quad (19)$$

Then, defining the Vlasov distribution in $\underline{P}, \underline{Q}$ space analogously to (6):

$$F(\underline{P}_1, \underline{Q}_1; t) = \int d^6 z_0 f_0(z_0) \delta^3(\underline{P}_1 - \underline{P}(z_0, t)) \delta^3(\underline{Q}_1 - \underline{Q}(z_0, t)), \quad (20)$$

we obtain the corresponding Vlasov equation,

$$\partial F(\underline{P}, \underline{Q}; t) / \partial t = - [F, K] \quad (21)$$

on varying S with respect to the fields $\underline{P}(z_0, t)$, $\underline{Q}(z_0, t)$. In (21), $[,]$ is the canonical bracket in $\underline{Q}, \underline{P}$ space.

Before varying S with respect to ϕ , we expand the new particle Hamiltonian K in powers of ϕ . If the Lie generating function is linear in ϕ , the Hamiltonian K contains terms bilinear in ϕ and higher order. We express this expansion in ϕ formally as

$$\begin{aligned} K(\underline{P}, \underline{Q}, \phi) &= h_{\text{kin}}(\underline{P}, \underline{Q}) + \int d^3 x \phi(\underline{x}) \bar{\rho}(\underline{x}; \underline{P}, \underline{Q}) \\ &+ \frac{1}{2} \int d^3 x \int d^3 x' \phi(\underline{x}) \phi(\underline{x}') \sigma(\underline{x}, \underline{x}'; \underline{P}, \underline{Q}) + \text{h.o.t.} \end{aligned} \quad (22)$$

The action principle thus reads

$$\begin{aligned} S(\underline{P}, \underline{Q}, \phi) &= \int dt \int d^6 z_0 f_0(z_0) (\underline{P}(z_0, t) \cdot \dot{\underline{Q}}(z_0, t) - h_{\text{kin}}(\underline{P}, \underline{Q})) \\ &+ \frac{1}{2} \int dt \int d^3 x \int d^3 x' \phi(\underline{x}, t) \phi(\underline{x}', t) \epsilon(\underline{x}, \underline{x}'; F) \\ &- \int dt \int d^3 x \phi(\underline{x}, t) \bar{\rho}(\underline{x}; F) + \text{h.o.t.}, \end{aligned} \quad (23)$$

where

$$\epsilon(\underline{x}, \underline{x}'; F) = (\partial^2 \delta^3(\underline{x} - \underline{x}') / \partial \underline{x} \cdot \partial \underline{x}') / 4\pi - \int d^6 Z F(Z) \sigma(\underline{x}, \underline{x}'; \underline{P}, \underline{Q}) \quad (24)$$

Variation of (23) with respect to ϕ , at constant $\underline{p}, \underline{q}$, now yields

$$\int d^3x' \epsilon(\underline{x}, \underline{x}'; F) \phi(\underline{x}') = \bar{\rho}(\underline{x}; F) \quad (25)$$

This expresses the potential in terms of the guiding-center charge density ρ , and the dielectric kernel ϵ . By (24), we see that ϵ includes the polarization kernel σ of the guiding-center Hamiltonian (22). As before, we suppose that (25) is solved formally for ϕ as a functional of F .

The derivation of the system Poisson bracket proceeds as before, and we obtain, for functionals $A(F)$:

$$\{A_1(F), A_2(F)\} = \int d^6Z F(Z) [\delta A_1 / \delta F(Z), \delta A_2 / \delta F(Z)]. \quad (26)$$

From (23), we read off the system Hamiltonian

$$\begin{aligned} H(F) = & \int d^6Z F(Z) h_{kin}(Z) + \int d^3x \phi(\underline{x}) \bar{\rho}(\underline{x}; F) \\ & - \frac{1}{2} \int d^3x \int d^3x' \phi(\underline{x}) \phi(\underline{x}') \epsilon(\underline{x}, \underline{x}'; F) + \text{h.o.t.} \end{aligned} \quad (27)$$

The guiding-center Hamiltonian is thus

$$\begin{aligned} K(Z; F) = \delta H(F) / \delta F(Z) = & h_{kin}(Z) + \int d^3x \phi(\underline{x}) \delta \bar{\rho}(\underline{x}) / \delta F(Z) \\ & - \frac{1}{2} \int d^3x \int d^3x' \phi(\underline{x}) \phi(\underline{x}') \delta \epsilon(\underline{x}, \underline{x}') / \delta F(Z) + \text{h.o.t.} \end{aligned} \quad (28)$$

The last term of (28) expresses the previously discovered relation between linear susceptibility and ponderomotive Hamiltonian. Again, we may use (25) to re-express the system Hamiltonian as

$$H(F) = \int d^6Z F(Z) h_{kin}(Z) + \frac{1}{2} \int d^3x \int d^3x' \phi(\underline{x}; F) \phi(\underline{x}; F) \epsilon(\underline{x}, \underline{x}'; F) + \text{h.o.t.} \quad (29)$$

Since the Hamiltonian (27) or (29) contains no explicit time dependence, it is invariant in time.

It is particularly instructive to treat the cold plasma limit, wherein the gyroradius r_g vanishes, and the motion along the magnetic field is ignored. We choose $(\underline{X}, \theta, u)$ as the four original phase space coordinates of a particle; $\underline{X}$ represents the guiding center position:

$$\underline{X} = \underline{r} - \underline{r}_g, \quad \underline{r}_g = (\hat{x} v_y - \hat{y} v_x) / \Omega, \quad (30)$$

θ is the gyrophase:

$$v_x = -v \sin \theta, \quad v_y = -v \cos \theta, \quad (31)$$

μ is the gyromomentum:

$$\mu = \frac{1}{2} m v^2 / \Omega, \quad (32)$$

and the gyrofrequency is $\Omega = eB/mc$. The kinetic energy is thus

$$h_{kin} = \mu \Omega, \quad (33)$$

while the potential energy is

$$e \phi(\underline{r}, t) = e \phi(\underline{X} + \underline{r}_g, t) = e \phi(\underline{X}, t) - \underline{\pi} \cdot \underline{E}(\underline{X}, t), \quad (34)$$

where

$$\underline{\pi} = e \underline{r}_g \quad (35)$$

is the electric dipole moment of the gyration relative to $\underline{X}$.

The dipole term in (34) is transformed to higher order by the Lie generating function w , satisfying

$$\frac{\partial w}{\partial \theta} \frac{\partial}{\partial \mu} (\mu \Omega) = \underline{\pi}(\mu, \theta) \cdot \underline{E}(\underline{X}, t) \quad (36)$$

One then obtains, by standard Lie transform techniques, the new Hamiltonian

$$K(\mu, \underline{X}, t) = \mu \Omega - e \phi(\underline{X}, t) - \frac{1}{2} \frac{mc^2}{B^2} E^2(\underline{X}, t) + \text{h.o.t.} \quad (37)$$

The term quadratic in the electric field represents the net polarization energy

$$- \frac{1}{2} \langle \underline{\pi} \rangle \cdot \underline{E} = - \langle \underline{\pi} \rangle \cdot \underline{E} + \frac{1}{2} m (\underline{E} \times \hat{b} c/B)^2, \quad (38)$$

which is the sum of the potential energy of the mean dipole $\langle \underline{\pi} \rangle$ and the kinetic energy of drift. To determine the mean dipole, we apply the Lie transform to (35) and average over gyrophase, obtaining

$$\langle \underline{\pi} \rangle = [w, e \underline{r}_g] = mc^2 \underline{E}(\underline{X}, t) / B^2. \quad (39)$$

This mean dipole represents the displacement of the center of gyration from the guiding center, due to the polarization drift. In terms of the new variables, the mean particle position is

$$\langle \underline{r} \rangle = \underline{X} + \langle \underline{\pi} \rangle / e. \quad (40)$$

The polarization drift is now obtained by differentiating (40):

$$d\langle \underline{r} \rangle / dt = \dot{\underline{X}} + \frac{mc^2}{eB^2} d\underline{E} / dt; \quad (41)$$

thus the guiding-center drift $\dot{\underline{X}}$ excludes the polarization drift. On substituting (39) into (38), we obtain the polarization energy of (37).

We now take the zero temperature limit $\mu \rightarrow 0$ in (37), and obtain the guiding-center Hamiltonian

$$K(\underline{X}, t) = e\phi(\underline{X}, t) - \frac{1}{2} \frac{mc^2}{B^2} E^2(\underline{X}, t). \quad (42)$$

By the arguments which led to (27), we obtain the system Hamiltonian

$$H(N) = \int d^2X \phi(\underline{X}) \bar{\rho}(\underline{X}; N) - \int d^2X \epsilon(\underline{X}; N) E^2(\underline{X})/8\pi, \quad (43)$$

where $N(\underline{X})$ is the guiding-center density, the guiding-center charge density is

$$\bar{\rho}(\underline{X}; N) = \sum_s e_s N_s(\underline{X}) \quad (44)$$

(we now include species label); and the perpendicular dielectric function is

$$\epsilon(\underline{X}; N) = 1 + \sum_s \frac{4\pi N_s(\underline{X}) m_s c^2}{B^2}. \quad (45)$$

The Poisson equation relating ϕ to N is obtained, as before, by

$$0 = \delta H(N, \phi) / \delta \phi(\underline{X}) = \bar{\rho}(\underline{X}; N) - \nabla \cdot (\epsilon(\underline{X}; N) \underline{E}(\underline{X})) / 4\pi. \quad (46)$$

This expresses the field $\underline{E}$ in terms of the guiding-center charge density ρ and the dielectric shielding ϵ .

The Poisson structure is now

$$\{A_1(N), A_2(N)\} = \sum_s \int d^2X N_s(\underline{X}) [\delta A_1 / \delta N_s(\underline{X}), \delta A_2 / \delta N_s(\underline{X})]_s, \quad (47)$$

where

$$[a_1(\underline{X}), a_2(\underline{X})]_s = - (c/e_s B) \hat{b} \cdot \nabla a_1 \times \nabla a_2. \quad (48)$$

The Vlasov equation reads

$$\partial N_s(\underline{X}, t) / \partial t = - [N_s, K_s], \quad (49)$$

with K_s given by (42). The Hamiltonian can be expressed concisely as

$$H(N) = \int d^2X \epsilon(\underline{X}; N) E^2(\underline{X}; N) / 8\pi. \quad (50)$$

In conclusion, we have seen how the single-particle Lie transform can be used in the system action principle, to obtain a Hamiltonian structure for the guiding-center distribution. This structure consists of a Poisson bracket (47) or (26) on functionals, a Hamiltonian (43) or (27), and a subsidiary condition (46) or (25) for the self-consistent field. From these ingredients, one obtains a self-consistent Vlasov equation (49) or (21) for the guiding-center distribution.

The extension of this Lie transform method to wave and oscillation-center distributions [8] is currently under investigation. Further reductions and applications are planned for future publication.

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