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Publication Date
1962-10-26

UCRL-10440

University of California

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Lawrence Radiation Laboratory
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Contract No. W-7405-eng-48

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October 26, 1962

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Abstract

The object of this work is to calculate the dose rate and integrated dose, inside a shield, due to an arbitrary proton flux. To this end, it is necessary to ascertain the effect of the shield on the incident flux. The work is concerned with this aspect of the problem.

The analysis takes into account both proton ionization energy loss and secondary-particle production, using the transport method of successive generations. The secondaries considered are protons, neutrons, and π mesons.

Introduction

The system chosen for analysis is a one-dimensional slab of width a, upon which is incident a uniform and isotropic proton flux with an arbitrary spectrum. An expression is developed for the uncollided proton flux, termed the primary flux, as a function of energy, angle, and position in the slab.

The first-generation secondary flux--i.e., the flux due to direct nuclear collisions of the primary flux--is then calculated. Similarly, a second-generation secondary flux, due to nuclear collisions of the first-generation flux is calculated. Succeeding generations of the secondary flux are calculated until the magnitude of the flux in a particular generation is sufficiently small to be neglected.

The secondary flux is subdivided into protons, neutrons, and π mesons, each of which is considered individually in the calculations.

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[*] Work done under the auspices of the U. S. Atomic Energy Commission, and the Joint Atomic Energy Commission—NASA Space Radiation Program.

Primary Proton Flux

The primary or uncollided proton flux is given as

$$\phi_p^{(0)}(x, E, \mu) = \frac{1}{2\pi}\, \phi_0(E_0)\, \exp\left\{-\int_E^{E_0} \frac{\Sigma_p(E')}{f(E')}\, dE'\right\}, \tag{1}$$

where $\phi_0(E_0)$ = incident proton flux,

$\Sigma_p(E)$ = total macroscopic proton cross section of the slab medium,

$f(E)$ = ionization energy loss of the slab medium.

The relation between E and E_0 is given as

$$E_0 = E + \int_0^{x/\mu} f(E')\, dx' , \tag{2}$$

which states that the initial proton energy E_0 is equal to the energy E at x plus the energy lost by ionization.

The exponential term in Eq. 1 represents the proton attenuation due to nuclear interactions.

Data on ionization energy loss are well known for a large number of elements. Similarly, low-energy proton cross sections are known. Unfortunately, only a small amount of high-energy cross-section data is available. Some estimates[1] and some experimental measurements[2] have been made. A useful approximation is the equivalence of neutron and proton cross sections at high energies, allowing the use of high-energy neutron cross-section data.[2]

Because of the complex form of f(E), the integration in Eqs. 1 and 2 must, in general, be performed numerically. The calculations are minimized by noting that the primary flux may be expressed as a function of two independent variables: E and x/μ.

Secondary Particle Fluxes

For the sake of brevity, meson fluxes are not discussed in the following sections. It is pertinent to note that the equations describing π^0 mesons are analogous to those describing neutrons. A similar correspondence exists for $\pi^{\pm}$ mesons and protons.

First-Generation Fluxes

The flux of particle i at some point $\vec{r}$, where i may represent protons or neutrons, is calculated by considering the production of i particles by proton interactions within a volume element at $\vec{r}'$, which contribute to the flux at $\vec{r}$, and then integrating over all $\vec{r}'$. The results for slab geometry are

$$\phi_i^{(1)}(x,E,\mu) = \frac{2\pi}{|\mu|} \int_{-1}^{1} d\mu' \int dE' \int_0^a dx' \, G(x';\mu)\,\Sigma_p(E')\,\phi_p^{(0)}(x',E',\mu') \tag{3}$$

$$\cdot \, N_{ip}(E' \rightarrow E_0, |\mu''|) \cdot \exp[-h_i(E,x',x)]$$

with the following definitions:

The term μ'' is the cosine of the difference in angle between the direction of the initial proton and the direction of the final particle i. It is related to μ and μ' by

$$\mu'' = \mu\mu' + \sqrt{[1-(\mu)^2][1-(\mu')^2]} \, . \tag{4}$$

The quantity E_0 is that energy which the particle i must have at x' so as to arrive at x with energy E. For neutrons, E_0 is identically E; but for protons, E_0 is greater than E by an amount equal to the ionization energy loss experienced by the proton in migrating from x' to x. The relationship between E and E_0 in this case is

$$E_0 = E + \int_0^{|x'-x| \, / \, |\mu|} f(E'')\, dx'' \, . \tag{5}$$

The function $G(x,\mu')$ serves to set the limits on the x' integration, and is defined by

$$G(x', \mu) = \begin{array}{llll} 1 \ldots & 0 < x' < x, & \text{for } \mu > 0, \\ 0 \ldots & x < x' \leqslant a, & \text{for } \mu > 0, \\ 0 \ldots & 0 \leqslant x' < x, & \text{for } \mu < 0, \\ 1 \ldots & x < x' \leqslant a, & \text{for } \mu < 0. \end{array} \tag{6}$$

The quantity $\exp[-h_i(E,x',x)]$ is the fraction of particles i that travel from x' to x without undergoing any nuclear interactions. The term $h_i(E,x',x)$ is given by

$$\begin{aligned} h_i(E,x',x) &= \Sigma_N(E)\,\frac{x'-x}{\mu} && \text{-- for neutrons,} \\ h_i(E,x',x) &= \int_E^{E_0} \frac{\Sigma p(E'')}{f(E'')}\, dE'' && \text{-- for protons,} \end{aligned} \tag{7}$$

where Σ_n = total macroscopic cross section for neutrons.

The superscripts attached to the fluxes denote generation number, with the primary proton flux being defined as the zero-order generation.

The function $N_{ij}(E' \rightarrow E_0, |\mu''|)$ represents the number of particles i with energy E_0 and direction μ produced per interaction of a proton of energy E' and direction μ'.

Higher-Generation Fluxes

The calculation of the higher-order fluxes is analogous to that for the first-generation fluxes, with one difference: particles i may be produced not only by nuclear interactions of protons, but also from secondary neutrons and mesons. The expressions for the nth-generation fluxes are

$$\phi_i^{(N)}(x, E, \mu) = 2\pi \int_{-1}^{1} \frac{d\mu'}{|\mu|} \int dE' \int_0^a dx' \exp[-h_i(E, x', x)] \; G(x', \mu) \cdot \sum_j \phi_j^{(N-1)}(x', E', \mu') \Sigma_j(E') N_{ij}(E' \rightarrow E_0, |\mu''|. \tag{8}$$

The summations in Eq. 8 extend over proton, neutron, and meson interactions.

Integral Evaluation

The integrals appearing in Eqs. 3 and 8 must be evaluated numerically for several reasons, the strongest of which is the quite complex form of the functions N_{ij}. Furthermore, the primary flux is in general specified as numerical data, as previously discussed.

At present, direct triple numerical integration of these equations appears to be the most likely method of attack.

Secondary-Particle Production

One of the major difficulties is the great lack of information about the function $N_{ij}(E' \rightarrow E_0, |\mu''|)$. The sparse data available are due primarily to Metropolis[3] and Moyer.[4]

The function N_{ij} represents the angular, energy, and number distributions of secondary particles j, and is therefore quite complicated. It is expected that more complete data on this function will be made available in the future.

Conclusions

The primary and secondary fluxes can, in principle, be determined to a high degree of accuracy by means of the foregoing analysis. In practice, however, there are two limitations on the obtainable accuracy, the more important by far being the lack of accurate data on high-energy cross sections and secondary-particle production.

The second limitation arises from truncation errors produced in the numerical integrations. The magnitude of these errors is determined by the sizes of the coordinate differences, which are unfortunately limited by computer capacities.

The desired end result is the fluxes of the various particles at the inner surface of the slab. A detailed knowledge of these fluxes is of major importance in determining the dose away from the inner slab surface.

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