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Leptogenesis via axion oscillations after inflation

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The evolution of an axionic field after inflation offers an explanation for the matter-antimatter asymmetry of the universe. During inflation, light scalar fields, including axions expected to arise from string theory and in various field-theoretic models, develop large expectation values. These fields relax to the minima of their effective potentials during or after reheating. An oscillating axion coupled to the electroweak gauge fields generates an effective chemical potential for the fermion number, which, in the presence of lepton number-violating processes, generates a lepton asymmetry that is partly converted to a baryon asymmetry by sphalerons. The observed matter-antimatter asymmetry can be explained in a broad range of parameter values with the reheating temperatures being at least of order $10^{12}$ GeV, and for all right-handed neutrino masses close to the scale of grand unification. Our mechanism is hence complementary to thermal leptogenesis with respect to the range of allowed parameter values.

The Peccei–Quinn (PQ) solution to the strong CP problem [1] has led to the prediction of a light scalar field called the axion [2], which appears generically in a number of models [3, 4] and which has a characteristic coupling to the gauge fields of the form $f_a a \Phi \tilde{F}$, where $f_a$ is the scale of PQ symmetry breaking. However, the motivation for considering axionic fields extends well beyond the context of the strong CP problem. Axions are ubiquitous in string theory, where at least one such field is generally associated with the Green–Schwarz mechanism of anomaly cancellation [5] and the scale $f_a$ of which lies a few orders of magnitude below the Planck scale [6], but multiple other axions can also appear. While the model-independent axion is coupled to all gauge groups with a universal coupling strength, the additional axionic fields can couple to different groups with couplings that depend on both the gauge group and the particle content of the model [7]. The masses of these model-dependent axions can be different as they arise from their couplings to different anomalous groups. We will focus, in particular, on the axion (or linear combination of axions) that has a coupling to the electroweak $SU(2)$ gauge fields.

During inflation, light scalar fields develop large expectation values [8]. The relaxation of the axion field to the minimum of its effective potential begins once the Hubble rate drops below the axion mass. While the field $a(t)$ oscillates, its coupling to the $SU(2)$ gauge fields and, via the anomaly, to the fermionic current $j^\mu = \bar{\psi} \gamma^\mu \psi$ induces an effective CPT-violating term $a(t) \Phi \tilde{F} \propto (\partial_t a(t)) j^0$, which serves as a chemical potential for fields carrying nonzero baryon or lepton number. Then, in presence of lepton number-violating processes in the plasma—for example, due to the exchange of virtual heavy right-handed neutrinos—the conditions for successful leptogenesis are satisfied.

A similar scenario was discussed in connection with a flat direction that carries no baryon or lepton number [9]. Our treatment of the asymmetry is different, and we obtain very different results. Our scenario is also similar to leptogenesis via Higgs relaxation [10], where the Higgs coupling to $F \tilde{F}$ is assumed to arise from a higher-dimensional operator, unlike in the present scenario, where the required coupling appears generically for any axion coupled to the electroweak gauge fields. One can also draw an analogy to models of spontaneous electroweak baryogenesis [11], in which the Higgs field in the expanding bubble wall generates an effective chemical potential for the fermions. Our scenario is different in that the “wall” is represented by an axionic field moving in the timelike direction uniformly in space, unlike the bubble wall moving in a spacelike direction.

Provided that the axion $a$ is to be identified with the pseudo-Nambu-Goldstone boson of a spontaneously broken $U(1)$ symmetry with a compact global topology, its initial value at the end of inflation is $a_0 = f_a \theta_0$, where the angle $\theta_0$ takes a random value in the range $\theta_0 \in [0, 2\pi]$. Assuming that the PQ symmetry is broken sufficiently early before the end of inflation (and is not restored during reheating), this initial value ends up being constant on superhorizon scales. For definiteness, we set $a_0 = f_a$ and treat $f_a$ as a free parameter in the following. Anticipating that the final baryon asymmetry will depend on $a_0$, we require that the baryonic isocurvature perturbations induced by the quantum fluctuations of the axion field during inflation be smaller than the observational upper limit. This implies a constraint on the Hubble rate during inflation: $H_{inf}/(2\pi)/a_0 \lesssim 10^{-5}$ [12], or

$$H_{inf} \lesssim 6 \times 10^{11} \text{GeV} \left( \frac{f_a}{10^{15} \text{GeV}} \right). \tag{1}$$

The evolution of the homogeneous axion field in its effective potential $V_{eff}$ around the origin is described by

$$\ddot{a} + 3 H \dot{a} = -\partial_a V_{eff}, \quad V_{eff} = \frac{1}{2} m_a^2 a^2. \tag{2}$$

Here, $m_a$ denotes the axion mass, which we assume to
arise via dimensional transmutation, i.e., from an additional coupling of the axion to the gauge fields of some strongly coupled hidden sector. Given a dynamical scale $\Lambda_f$ in this hidden sector, the axion mass is then of $\mathcal{O}(\Lambda_f^2/f_a)$. For consistency, we require $m_a$ to be smaller than $H_{\text{inf}}$, the Hubble rate at the end of inflation:

$$m_a \lesssim H_{\text{inf}}.$$  \hspace{1cm} (3)

When inflation is over, the axion field remains practically at rest until the Hubble parameter drops to $H_{\text{osc}} = m_a$. Once the axion field is in motion, the effective Lagrangian contains the term

$$\mathcal{L}_{\text{eff}} \supset \frac{g_2^2}{32\pi^2} \frac{a(t)}{f_a} F^2 - \frac{a(t)}{N_f f_a} \partial_\mu (\bar{\psi} \gamma^\mu \psi)$$  \hspace{1cm} (4)

$$= \frac{\partial a(t)}{N_f f_a} (\bar{\psi} \gamma^0 \psi) + \cdots + \mu_{\text{eff}} j^0 + \cdots,$$  \hspace{1cm} (5)

with $g_2$ being the $SU(2)$ gauge coupling and $N_f = 3$ the number of fermion generations in the standard model, where we have used the anomaly equation in Eq. [1], and integration by parts in Eq. [3]. In the following, we will absorb $N_f$ in our definition of $f_a$ and simply determine the effective chemical potential as $\mu_{\text{eff}} = \dot{a}/f_a$.

Now the necessary conditions for generating a lepton asymmetry are satisfied. A nonzero effective chemical potential shifts the energy levels of particles as compared to antiparticles. If lepton number is not conserved, the minimum of the free energy in the plasma is reached for a different number density of leptons than for antileptons, i.e., for $n_L \equiv n_e - n_\bar{e} \neq 0$. Instead, if the lepton number violation is very rapid, the minimum of the free energy is obtained for an equilibrium number density of

$$n_{L}^{\text{eq}} = \frac{4}{\pi^2} \mu_{\text{eff}} T^2.$$  \hspace{1cm} (6)

Lepton number violation is mediated by the exchange of right-handed neutrinos. In contrast to thermal leptogenesis [13], we will assume all heavy right-handed neutrino masses to be close to the scale of grand unification (GUT), $M_i \sim \mathcal{O}(10^{14}\cdots1)\Lambda_{\text{GUT}} \sim 10^{15}\cdots10^{16}$ GeV, so that the heavy neutrinos are never thermally produced on the mass shell, i.e., $T \ll M_i$ at all times. In the expanding universe, the evolution of the lepton number density $n_L$ is described by the Boltzmann equation

$$\dot{n}_L + 3Hn_L \simeq -4n_L^{\text{eq}} \sigma_{\text{eff}} (n_L - n_L^{\text{eq}}),$$  \hspace{1cm} (7)

where $n_L^{\text{eq}} = 2/\pi^2 T^3$ and with $\sigma_{\text{eff}} \equiv \langle \sigma_{\Delta L = 2} v \rangle$ denoting the thermally averaged cross section of two-to-two scattering processes with heavy neutrinos in the intermediate state that violate the lepton number by two units,

$$\Delta L = 2 : \; \ell_i \ell_j \leftrightarrow HH, \; \ell_i H \leftrightarrow \ell_j H,$$  \hspace{1cm} (8)

$$\ell_i^T = (\nu_i, \bar{e}_i), \; H^T = (h_+ \; h_0), \; i, j = 1, 2, 3.$$  

We note that the term proportional to $n_L^{\text{eq}}$ now acts as a novel production term for the lepton asymmetry, as long as the axion field is in motion. For center-of-mass energies much smaller than the heavy neutrino mass scale, $\sqrt{s} \ll M_i$, the effective cross section $\sigma_{\text{eff}}$ is practically fixed by the experimental data on the light neutrino sector [14], assuming the seesaw mass matrix [15].

$$\sigma_{\text{eff}} \approx \frac{3}{32\pi} \frac{m_i}{v_{\text{ew}}} \simeq 3 \times 10^{-31} \text{GeV}^{-2}, \; \bar{m}^2 = \sum_{i=1}^3 m_i^2,$$  \hspace{1cm} (9)

where $v_{\text{ew}} \simeq 174$ GeV and where we have assumed that the sum of the light neutrino masses squared is of the same order of magnitude as the atmospheric neutrino mass difference, $\Delta m^2_{\text{atm}} \simeq 2.4 \times 10^{-3}$ eV$^2$ [16].

For $a_0 \ll M_{\text{Pl}}$, and as long as $H \gg m_a$, i.e., prior to the onset of the axion oscillations, the axion energy density $\rho_a$ is much smaller than the total energy density $\rho_{\text{tot}} = \rho_\varphi + \rho_R + \rho_a \approx \rho_\varphi + \rho_R$, where $\rho_\varphi$ and $\rho_R$ are the energy densities of the inflaton and of radiation. Reheating is described by a system of equations:

$$\dot{\rho}_\varphi + 3H\rho_\varphi = -\Gamma_\varphi \rho_\varphi, \quad \dot{\rho}_R + 4H\rho_R = +\Gamma_\varphi \rho_\varphi,$$  \hspace{1cm} (10)

$$H^2 \equiv (\dot{R}/R)^2 = \frac{\rho_{\text{tot}}}{3M_{\text{Pl}}^2}, \quad \rho_{\text{tot}} \approx (\rho_\varphi + \rho_R),$$  \hspace{1cm} (11)

where $\Gamma_\varphi$ is the inflaton decay rate. The inflaton must not decay before the end of inflation, which implies

$$\Gamma_\varphi \lesssim H_{\text{inf}}.$$  \hspace{1cm} (12)

The solution for the temperature, $T^4 \equiv \pi^2/3\rho_\varphi, \rho_R$, according to Eqs. (10) and (11) shows the following characteristic behavior: within roughly one Hubble time after the end of inflation, $T$ quickly rises to its maximal value,

$$T_{\text{max}} \simeq 5 \times 10^{13} \text{GeV} \left(\frac{\Gamma_\varphi}{10^9 \text{GeV}}\right)^{1/4} \left(\frac{H_{\text{inf}}}{10^{11} \text{GeV}}\right)^{1/2},$$  \hspace{1cm} (13)

after which the temperature decreases because the energy density is dominated by the inflaton oscillations (which scale as matter). During reheating, the temperature drops as $T \propto R^{-3/8}$ until radiation comes to dominate at time $t = t_{\text{rh}} \simeq \Gamma_\varphi^{-1}$, when $\rho_\varphi = \rho_R$, and the reheating temperature is

$$T_{\text{rh}} \simeq 2 \times 10^{13} \text{GeV} \left(\frac{\Gamma_\varphi}{10^9 \text{GeV}}\right)^{1/2}.$$  \hspace{1cm} (14)

After the end of reheating, i.e., for $t > t_{\text{rh}}$, the expansion is then driven by relativistic radiation and the temperature simply decreases adiabatically, $T \propto R^{-1}$. In the case of a large axion decay constant, this phase of radiation domination, however, does not last all the way to the time of primordial nucleosynthesis. Instead, the axion comes to dominate the total energy density at some time prior to its decay, which marks the beginning of yet another
stage of matter domination. The decay of the axion into relativistic gauge bosons and the corresponding renewed transition to radiation domination then represent a second installment of reheating, which can be described by the same set of equations as the primary reheating process, cf. Eqs. (10) and (11). With the axion decay rate
\[ \Gamma_a \approx \frac{\alpha^2 m^3_a}{64\pi^3 f_a^2}, \quad \alpha = \frac{g^2}{4\pi}, \] (15)
and using Eq. (14), we find for the secondary reheating temperature or axion decay temperature
\[ T_{\text{dec}} \approx 1 \times 10^4 \text{ GeV} \left( \frac{m_a}{10^9 \text{ GeV}} \right)^{3/2} \left( \frac{10^{15} \text{ GeV}}{f_a} \right). \] (16)
This temperature should be at least of $O(10)\text{ MeV}$, which imposes a lower bound on $m_a$:
\[ m_a \gtrsim 8 \times 10^4 \text{ GeV} \left( \frac{f_a}{10^{15} \text{ GeV}} \right)^{2/3}. \] (17)

The five differential equations in Eqs. (2), (7), (10), and (11) allow one to compute the present value of the baryon asymmetry (i.e. the baryon-to-photon ratio),
\[ \eta_B^0 \approx \frac{n_B^0}{n_\gamma} = c_{\text{sph}} \frac{g_{\ast_s}^{0}}{g_*} \eta_L^0 \approx 0.013 \eta_L^0, \] (18)
where the sphaleron factor $c_{\text{sph}}$ accounts for the conversion of the lepton asymmetry into baryon asymmetry by sphalerons. Here, $g_{\ast_s}^{0}$ and $g_{\ast}$ denote the effective numbers of degrees of freedom contributing to the entropy density in the present epoch and during reheating, respectively. In the standard model $c_{\text{sph}} = 28/79$, $g_{\ast_s}^{0} = 43/11$, and $g_{\ast} = 427/4$. Last but not least, $\eta_L^0$ in Eq. (18) stands for the final lepton asymmetry after the decay of the axion around $t \approx \Gamma_a^{-1}$.

We determine $\eta_L^a$ by solving the five differential equations in Eqs. (2), (7), (10), and (11) numerically. We also present approximate analytical solutions, which will be discussed in detail in an upcoming publication. It is convenient to parametrize $\eta_L^a$ as follows:
\[ \eta_L^a = C \Delta_a^{-1} \Delta_f^{-1} \eta_L^{\text{max}} e^{-\kappa}. \] (19)

The approximate analytical results agree with the numerical results, as shown in Fig. 1. In the following, we shall present analytical expressions for the individual factors on the right-hand side of Eq. (19). $\eta_L^{\text{max}}$ denotes the all-time maximum value of the lepton asymmetry, which is reached around the time when the axion oscillations set it, i.e. at $t \sim t_{\text{osc}} \approx m_a^{-1}$. Integrating the Boltzmann equation for the lepton asymmetry up to $t \sim t_{\text{osc}}$, one approximately finds
\[ \eta_L^{\text{max}} \approx \frac{\sigma_{\text{eff}} a_{0}}{g_s^{1/2} f_a} m_a M_{\text{Pl}} \left\{ \begin{array}{ll} \left( \frac{\Gamma_f}{m_a} \right)^{1/2} & ; \quad m_a \gtrsim \Gamma_f, \\ 1 & ; \quad m_a \lesssim \Gamma_f \end{array} \right. \] (20)

Remarkably enough, this expression is rather insensitive to the axion decay constant; it only depends on the ratio $a_0/f_a$, which is expected to be $O(1)$. Furthermore, $\eta_L^{\text{max}}$ turns out to be directly proportional to the effective cross section $\sigma_{\text{eff}}$. For $a_0 = f_a$ and given the value of $\sigma_{\text{eff}}$ in Eq. (9), the maximal asymmetry is hence typically much larger than the observed value, $\eta_B^{\text{obs}} \approx 6 \times 10^{-10}$ [13],
\[ \eta_L^{\text{max}} \approx 2 \times 10^{-5} \left( \frac{m_a}{10^9 \text{ GeV}} \right)^p \left( \frac{\Gamma_f}{10^9 \text{ GeV}} \right)^q, \] (21)

where the powers $p$ and $q$ are given as Eq. (20).

The two $\Delta$ factors in Eq. (19) account for the entropy production in inflaton and axion decays during reheating and at late times, respectively. In case the axion oscillations already set in before the end of reheating, i.e. for $m_a \gtrsim \Gamma_f$, we find for $\Delta_f$,
\[ \Delta_f \approx \left( \frac{m_a}{\Gamma_f} \right)^{5/4}, \quad m_a \gtrsim \Gamma_f. \] (22)

On the other hand, if $m_a \lesssim \Gamma_f$, the maximal lepton asymmetry $\eta_L^{\text{max}}$ is only reached after the end of reheating and there is no dilution from the inflaton decay. In this case, one can simply set $\Delta_a = 1$. At the same time, we obtain for the axion dilution factor $\Delta_a$,
\[ \Delta_a \approx \frac{2\pi^2}{\alpha} \frac{a_0}{a_{0}} \frac{m_a}{M_{\text{Pl}}} \left\{ \begin{array}{ll} \left( \frac{\Gamma_f}{m_a} \right)^{1/2} & ; \quad m_a \gtrsim \Gamma_f, \\ 1 & ; \quad m_a \lesssim \Gamma_f \end{array} \right. \] (23)
This expression is valid only as long as the axion dominates the total energy density shortly prior to its decay, i.e. as long as $\Delta_a \gtrsim 1$. Otherwise, the entropy production during the decay of the axion is negligible and one must set $\Delta_a = 1$. In the region of parameter space in which we are able to successfully reproduce $\eta_{B}^{\text{obs}}$, entropy production during the decay of the axion begins to play a role for $f_a$ values around $3 \times 10^{13} \text{GeV}$, cf. Fig. 2. For smaller values of $f_a$, we always have $\Delta_a = 1$ in the entire region of parameters of interest.

The factor $e^{-\kappa}$ in Eq. (19) accounts for the washout of $\eta_{B}^{\text{max}}$ during reheating due to the $\Delta L = 2$ washout processes, cf. the term proportional to $-\sigma_{\text{eff}} \eta L$ in Eq. (7). In case the axion begins to oscillate before the end of reheating, one can estimate

$$\kappa \sim 5 \frac{\sigma_{\text{eff}}}{g_{*}^{1/2}} M_{Pl} T_{\text{rh}} \simeq 1 \left( \frac{T_{\text{rh}}}{10^{15} \text{GeV}} \right).$$

(24)

For $m_a \lesssim \Gamma_{\varphi}$ on the other hand, washout is always negligible, so that $\kappa$ can be safely set to $\kappa = 0$. A more careful treatment of the effect of washout on the final baryon asymmetry in our scenario is left for future work.

Finally, the factor $C$ in Eq. (19) is a numerical fudge factor, which can, in principle, be estimated analytically, but which, in practice, is best determined by fitting the analytical expression for $\eta_{B}^{0}$ in Eq. (19) to the outcome of our numerical analysis. Specifically, we find $C \simeq 1.5$ for $m_a \lesssim \Gamma_{\varphi}$ and $C \simeq 2.2$ for $m_a \gtrsim \Gamma_{\varphi}$. The fact that these values are both of $O(1)$ confirms the accuracy of our analytical estimate.

Altogether, the parameter dependence of the final lepton asymmetry in Eq. (19) can be summarized as follows (here, we neglect the effect of washout and set $\kappa \rightarrow 0$),

$$\eta_{B}^{0} \propto \frac{\sigma_{\text{eff}}}{g_{*}^{1/2}} M_{Pl} a_{0} f_{a}^{-1} \begin{cases} m_{a}^{-3/4} \Gamma_{\varphi}^{7/4} M_{Pl} a_{0} f_{a}^{-1} & m_{a} \lesssim \Gamma_{\varphi}, \Delta_a = 1 \\ m_{a} M_{Pl} a_{0} f_{a}^{-1} & m_{a} \gtrsim \Gamma_{\varphi}, \Delta_a = 1 \\ m_{a}^{-3/4} \Gamma_{\varphi}^{7/4} M_{Pl} \sigma_{\text{eff}} f_{a}^{-2} & m_{a} \lesssim \Gamma_{\varphi}, \Delta_a > 1 \\ m_{a}^{2} M_{Pl} a_{0} f_{a}^{-1} & m_{a} \gtrsim \Gamma_{\varphi}, \Delta_a > 1 \end{cases}$$

(25)

In the various regions of parameter space, typical values for $m_a$, $\Gamma_{\varphi}$ and $f_a$ then yield the following final baryon asymmetries (again for $a_{0} = f_{a}$ and $\sigma_{\text{eff}}$ as in Eq. (9)),

<table>
<thead>
<tr>
<th>$f_a$ [GeV]</th>
<th>$m_a$ [GeV]</th>
<th>$\Gamma_{\varphi}$ [GeV]</th>
<th>$\eta_{B}^{0}$</th>
<th>$\Delta_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12}$</td>
<td>$3 \times 10^{7}$</td>
<td>$3 \times 10^{6}$</td>
<td>$3 \times 10^{-10}$</td>
<td>1</td>
</tr>
<tr>
<td>$10^{13}$</td>
<td>$3 \times 10^{6}$</td>
<td>$3 \times 10^{7}$</td>
<td>$1 \times 10^{-9}$</td>
<td>1</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>$1 \times 10^{10}$</td>
<td>$1 \times 10^{9}$</td>
<td>$7 \times 10^{-9}$</td>
<td>3</td>
</tr>
<tr>
<td>$10^{17}$</td>
<td>$1 \times 10^{9}$</td>
<td>$1 \times 10^{10}$</td>
<td>$6 \times 10^{-9}$</td>
<td>80</td>
</tr>
</tbody>
</table>

Let us now determine the range of parameters that admit the correct value of the baryon asymmetry in view of the constraints in Eqs. (1), (6), (12), and (17). The results are presented in Fig. 2 which shows the contour lines of successful leptogenesis for different values of the axion decay constant $f_a$. The allowed range of $f_a$ values spans five orders of magnitude,

$$4 \times 10^{10} \text{GeV} \lesssim f_a \lesssim 4 \times 10^{15} \text{GeV}.$$  

(26)

For smaller values of $f_a$, it is not possible to generate a sufficiently large baryon asymmetry, while keeping the baryonic isocurvature perturbations small enough. For larger values of $f_a$, the dilution of the asymmetry during the late-time decay of the axion is too strong. Varying $f_a$ within the interval in Eq. (26), we then find that $m_a$, $\Gamma_{\varphi}$, and $T_{rh}$ can take values within the following ranges:

$$1 \times 10^{6} \text{GeV} \lesssim m_a \lesssim 2 \times 10^{11} \text{GeV},$$

(27)

$$3 \times 10^{6} \text{GeV} \lesssim \Gamma_{\varphi} \lesssim 3 \times 10^{11} \text{GeV},$$

(28)

$$9 \times 10^{11} \text{GeV} \lesssim T_{rh} \lesssim 3 \times 10^{14} \text{GeV}.$$  

(29)

These ranges of parameters are consistent with models of dynamical axions, as well as string axion models.

In summary, the axion-driven leptogenesis mechanism described above appears to be an attractive alternative to the conventional scenario of thermal leptogenesis. In contrast to thermal leptogenesis, the baryon asymmetry does not depend on the CP violation in the neutrino mass matrix, and the right-handed neutrino masses lie much closer to the GUT scale. Hence, while thermal leptogenesis assumes some of the heavy neutrino Yukawa couplings to be much smaller than 1, our mechanism rather applies in the case of Yukawa couplings of $O(10^{-1} \cdots 1)$. Furthermore, as one can see from Eqs. (6) and (19), the larger the neutrino masses $m_i$, the larger is the baryon

FIG. 2: Contour lines for successful leptogenesis ($\eta_{B}^{0} = \eta_{B}^{\text{obs}}$) in the $m_a$–$\Gamma_{\varphi}$ plane for different values of the axion decay constant $f_a$. The dashed segments along the individual contours mark the regions where either $m_a$ or $f_a$ become comparable to the maximally allowed Hubble rate $H_{\text{max}}^{\text{obs}}$, cf. Eq. (1).
asymmetry. While thermal leptogenesis imposes an upper bound on the neutrino mass scale, $\bar{m} \lesssim 0.2 \text{ eV}$, to avoid strong washout \cite{19}, our scenario works for all experimentally allowed values of the light neutrino masses.

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