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LONG RANGE ABSORPTION AND OTHER OPTICAL
MODEL EFFECTS FROM STRONG INELASTIC COUPLING

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ABSTRACT

An optical potential component is constructed to represent the effect of a strongly coupled inelastic excitation upon elastic scattering. In the particular case of quadrupole Coulomb excitation a long range imaginary potential component is derived in closed form. The effects of long range absorption upon the elastic scattering are considered in a general way by inserting this potential into a weak absorption model and deriving an elastic scattering cross section in closed form. Below the Coulomb-barrier the formula takes a simple form which may be related to the semi-classical theory of Coulomb excitation.

The potential component arising from nuclear excitation of an inelastic state may be evaluated numerically on a computer. Two examples computed (50 MeV α scattering on ^{154}Sm and 60 MeV ^{16}O scattering on ^{40}Ca) exhibit strong ℓ -dependence in the potential component.

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1. Introduction

Strongly coupled inelastic transitions can have important effects on elastic scattering cross sections in nuclear physics. Experimental data which clearly bear this out include transitions that are primarily nuclear, such as $^{154}\text{Sm}(\alpha, \alpha')$ at 50 MeV,¹ and also transitions which exhibit strong Coulomb excitation effects such as $^{184}\text{W}(^{18}\text{O}, ^{18}\text{O}')$ at 90 MeV.² In both these cases the original analysis in terms of coupled channels calculations provided a satisfactory description of the data.

An alternative theoretical description is the construction of an optical model component arising from the particular inelastic contribution to elastic scattering. The point of this paper is to investigate some aspects of this approach. We will first construct this non-local potential in a general way and then define its local equivalent. For the quadrupole Coulomb interaction we then obtain an approximate analytical form for the potential which turns out to be negative imaginary. This potential is then incorporated into a weak absorption model to obtain analytical forms for the elastic scattering cross section in the presence of long range absorption.³ Below the Coulomb barrier our analytical formula takes a particularly simple form. This form can be related to the semi-classical theory of Coulomb excitation by summing the Born series on-energy-shell. Finally, we consider the short range potential generated by nuclear inelastic scattering. In two particular examples studied we find a potential component that is highly ℓ -dependent in both real and imaginary parts.

2. The Potential Component and Its Local Equivalent

We begin by writing down the set of coupled equations whose effect is to be represented by an optical model component:

$$(E_1 - H) \chi_1 = V_{12} \chi_2 \quad (2.1)$$

$$(E_2 - H) \chi_2 = V_{21} \chi_1 \quad (2.2)$$

V_{12} is the inelastic transition form factor and H is an optical model Hamiltonian. For tractability we ignore reorientation couplings. Eq. (2.2) may be rewritten as an integral equation

$$\chi_2 = G_2^{(+)} V_{21} \chi_1, \quad (2.3)$$

where $G_2^{(+)}$ is the outgoing boundary condition distorted-wave Green's function operator, $(E_2 - H)^{-1}$, and this result can be substituted into Eq. (2.1) to obtain

$$(E_1 - H) \chi_1 = V_{12} G_2^{(+)} V_{21} \chi_1. \quad (2.4)$$

The elastic channel is thus formally uncoupled, with the non-local potential operator, $V_{12} G_2^{(+)} V_{21}$, bringing in the effects of coupling to all orders upon the elastic channel. This representation is equivalent to the Feshbach projection operator formalism.⁴

The potential component to be evaluated may be written in coordinate space

$$V(r, r') = V_{12}(r) G_2^{(+)}(r, r') V_{21}(r'). \quad (2.5)$$

V_{12} and V_{21} are the multipole operators connecting ground and excited state, i.e.,

$$V_{12} = V(r) \sum_M Y_{LM}^* (\hat{r}) \quad (2.6)$$

$$V_{21} = V(r') \sum_{M'} Y_{LM'} (\hat{r}'). \quad (2.7)$$

A partial wave expansion of $G_2^{(+)}$ may be made in coordinate space

$$G_2 = - \frac{2\mu}{rr'k\hbar^2} \sum_{\ell'm'} f_{\ell'}(r_<) h_{\ell'}^{(+)}(r_>) Y_{\ell'm'}(\hat{r}) Y_{\ell'm'}^*(\hat{r}'), \quad (2.8)$$

where $f_{\ell}(r_<)$ and $h_{\ell}^{(+)}(r_>)$ are optical model wave functions with regular and outgoing boundary conditions respectively normalized so that asymptotically they approach the respective Coulomb wave functions.

The standard method of treating optical model calculations is via a partial wave series. With the Z axis along the incoming beam axis we obtain the distorted waves

$$\chi_i^{(+)}(\hat{r}') = \frac{1}{kr} \sum_{\ell} i^{\ell} e^{i\sigma_{\ell}} \sqrt{4\pi(2\ell+1)} f_{\ell}(r') Y_{\ell 0}(\hat{r}') \quad (2.9)$$

$$\chi_f^{(-)*}(\hat{r}) = \frac{4\pi}{kr} \sum_{\ell''m} i^{-\ell''} e^{-i\sigma_{\ell''}} f_{\ell''}(r) Y_{\ell''m}(\hat{k}_f) Y_{\ell''m}^*(\hat{r}). \quad (2.10)$$

Since for even-even nuclei any optical model component is necessarily scalar, $m = 0$ and $\ell'' = \ell$. We may then project out the ℓ -dependent, non-local radial potential component

$$U_{\ell}(r, r') = \int d\hat{r} \int d\hat{r}' Y_{\ell 0}^*(\hat{r}) V(r, r') Y_{\ell 0}(\hat{r}') \quad (2.11)$$

which will be used in the solution of the radial Schroedinger equation.

Combining (2.5), (2.6), (2.7), and (2.8) with (2.11) we obtain

$$U_{\ell}(r, r') = - \frac{2\mu}{k\hbar^2} V(r)V(r') \sum_{\ell' m'} f_{\ell'}(r_{<}) h_{\ell'}^{(+)}(r_{>}) \int d\hat{r} \int d\hat{r}'$$

$$\times Y_{\ell' m'}(\hat{r}) Y_{\ell' m'}^*(\hat{r}') Y_{\ell 0}^*(\hat{r}) Y_{\ell 0}(\hat{r}') \sum_M Y_{LM}^*(\hat{r}) \sum_M Y_{LM}(\hat{r}'). \quad (2.12)$$

The integrals over the spherical harmonics may be carried out along with the sum on m' yielding

$$U_{\ell}(r, r') = - \frac{2\mu}{k\hbar^2} V(r)V(r') \sum_{\ell'} f_{\ell'}(r_{<}) h_{\ell'}^{(+)}(r_{>})$$

$$\times \frac{2L+1}{4\pi} \langle \ell 0 \ell 0 | \ell' 0 \rangle^2. \quad (2.13)$$

This is the ℓ -dependent, non-local optical potential component corresponding to the effects of the inelastic excitation upon the elastic channel.

By incorporating this potential component into our solution of the optical model as in Eq. (2.4) we obtain a result still completely equivalent to the coupled-channels solution with no reorientation.

At this point it is useful to consider a "trivially equivalent local potential".

$$U_{\ell}(r) = \frac{1}{\chi_{\ell}(r)} \int dr' U_{\ell}(r, r') \chi_{\ell}(r'). \quad (2.14)$$

Of course to evaluate this potential exactly one must know the solution of the Schroedinger equation which includes its effect. We have chosen to solve

the problem by iteration, i.e.,

$$(E_1 - H)f_\ell^0 = 0 \quad (2.15)$$

$$U_\ell^0(r) = \frac{1}{f_\ell^0(r)} \int dr' U_\ell(r, r') f_\ell^0(r') \quad (2.16)$$

$$(E_1 - H - U_\ell^0(r))f_\ell^1(r) = 0 \quad (2.17)$$

$$U_\ell^1(r) = \frac{1}{f_\ell^1(r)} \int dr' U_\ell(r, r') f_\ell^1(r') \quad (2.18)$$

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$$(E_1 - H - U_\ell^{n-1}(r))f_\ell^n(r) = 0 \quad (2.19)$$

$$U_\ell^n(r) = \frac{1}{f_\ell^n(r)} \int dr' U_\ell(r, r') f_\ell^n(r') \quad (2.20)$$

Assuming that the iteration scheme converges, we have a local Schroedinger equation (2.19) to solve for the coupled channels equivalent, and we can construct and inspect the local potential equivalent Eq. (2.20). Specific cases of computer evaluation of these equations will be discussed later. For comparison we have also calculated the wave functions utilizing an iterated Born series whose n-th iteration is

$$f_\ell^n(r) = f_\ell^0(r) + \int dr' G_{1\ell}^{(+)}(r, r') \int dr'' U_\ell(r', r'') f_\ell^{n-1}(r'') \quad (2.21)$$

with $G_{1\ell}^{(+)}(r, r')$ the unperturbed Green's function for the ground state.

It now is necessary only to specify the radial parts of the multipole operators to complete our general treatment of the optical potential component.

For a rotational nucleus we follow the usual prescription for the deformed potential component to obtain the nuclear and Coulomb interaction potential.⁶

However, for use in the next section we write the exterior Coulomb interaction potential in terms of the reduced electromagnetic transition rate

$B(EL)\uparrow$. For a vibrational nucleus we use the first derivative of the spherical potential for the form factor with the strength given by a β in the usual way.⁶

3. The Long Range Absorptive Potential Due to Coulomb Excitation

Recent elastic scattering data of 90 MeV ^{18}O on ^{184}W exhibited a Fresnel pattern damped below the Rutherford cross section that is well reproduced by a coupled channels calculation which includes Coulomb excitation of the 111 keV 2^+ rotational state in ^{184}W .² An alternative theoretical description was effected by Love, Terasawa, and Satchler with an optical model component arising from the contribution of the 2^+ state to the elastic scattering.⁵ Their approximation was to use plane waves for the intermediate state and ground state in an equation analogous to Eq. (2.14). A classical correction was then made for the Coulomb braking. The potential obtained was dominantly negative-imaginary, and apart from finite size corrections, has a radial dependence of $r^{-5} [1 - (Z_1 Z_2 e^2 / r E_{\text{cm}})]^{-1/2}$. (We will refer to this potential as the LTS potential.)

In this section we show that it is possible to derive a more exact expression for this long range potential by making use of Coulomb-distorted scattering states and a Coulomb-distorted Green's function. The result shows some interesting differences from the LTS potential.

For quadrupole Coulomb excitation the external radial part of the transition operators Eq. (2.6) and (2.7) takes the form

$$V(r) = \frac{4\pi Z_P e(B(E2) \uparrow)^{1/2}}{5\sqrt{5}} \frac{1}{r^3} \quad (3.1)$$

The ℓ -dependent, non-local optical potential component Eq. (2.13) then becomes

$$U_{\ell}(r, r') = - \frac{2\mu}{k\hbar^2} \frac{4\pi}{25} Z_p^2 e^2 B(E2) \sum_{\ell'} \langle \ell 0 2 0 | \ell' 0 \rangle^2 \times \frac{1}{r^3} \frac{1}{r'^3} F_{\ell'}(r_{<}) H_{\ell'}(r_{>}). \quad (3.2)$$

where $F_{\ell}(r_{<})$ and $H_{\ell}(r_{>})$ will be taken to be the regular and outgoing boundary Coulomb wave functions respectively. Below the Coulomb barrier and for high partial waves (corresponding to distances of closest approach outside the nuclear field) our formula is still exact, except for the restriction of coupling to one excited 2^+ state without reorientation. However, from this non-local potential we will calculate perturbatively its local equivalent by employing our first iteration Eq. (2.18) and Coulomb wave functions

$$U_{\ell}(r) = \frac{1}{F_{\ell}(r)} \int dr' U_{\ell}(r, r') F_{\ell}(r'). \quad (3.3)$$

Recalling that

$$H_{\ell}(r_{>}) = G_{\ell}(r_{>}) + i F_{\ell}(r_{>}), \quad (3.4)$$

where $G_{\ell}(r_{>})$ is the irregular Coulomb wave function, the local potential takes the form

TABLE 1. Semiclassical Energy Loss Correction Factor for E2 Coulomb Excitation⁵

ξ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$g_2(\xi)$	1.0	0.960	0.815	0.627	0.452	0.311	0.206	0.133	0.084	0.052	0.032

$$\begin{aligned}
 U_{\ell}(r) = & -\frac{2\mu}{\hbar^2} \frac{4\pi}{25} Z_p^2 e^2 B(E2) \uparrow \sum_{\ell'} \langle 2020 | \ell' 0 \rangle^2 \frac{1}{r^3} \\
 & \times \left[1 \frac{F_{\ell'}(r)}{F_{\ell}(r)} \int_0^{\infty} dr' F_{\ell'}(r') \frac{1}{r'^3} F_{\ell}(r') \right. \\
 & + \frac{F_{\ell'}(r)}{F_{\ell}(r)} \int_r^{\infty} dr' G_{\ell'}(r') \frac{1}{r'^3} F_{\ell}(r') \\
 & \left. + \frac{G_{\ell'}(r)}{F_{\ell}(r)} \int_0^r dr' F_{\ell'}(r') \frac{1}{r'^3} F_{\ell}(r') \right]. \quad (3.5)
 \end{aligned}$$

Notice here the clean separation of the real and imaginary parts of this potential. Since G_{ℓ} , the irregular Coulomb wave function, is asymptotically 90° out of phase with F_{ℓ} , both real components will oscillate in sign as a function of r ; this behavior is confirmed in computer evaluation of Eq. (3.5). These real components merely serve to put insignificant "hair" on top of the real Coulomb potential and will not be considered further. On the other hand, we can evaluate the imaginary component in closed form.

For the sake of simplicity in derivation we assume no energy loss in the quadrupole transition. However, a semiclassical energy loss factor $g_2(\xi)$ is applied to our results at the end.⁵ $g_2(\xi)$ is merely the ratio $f_2(\eta, \xi)/f_2(\eta, 0)$, where ξ is the adiabaticity parameter $\xi = \frac{1}{2} \eta \Delta E / E_{cm}$, and $f_2(\eta, \xi)$ is the standard factor of Alder et al.⁷ which we assume for $\eta = \infty$. (We reproduce Table 1 of $g_2(\xi)$ from Ref. (5) for the reader's convenience.) We will also ignore terms of order $1/\ell$. In Appendix A we show that our formula, derived for the case of large ℓ , also holds for large η independent of ℓ , the usual semiclassical situation.

For a given ℓ our imaginary potential will consist of three radial integral terms:

$$\begin{aligned}
 U_{\ell}(r) = & -i \frac{2\mu}{\hbar^2} \frac{4\pi}{25} Z_p^2 e^2 B(E2) \frac{1}{r^3} \\
 & \times [\langle \ell 0 2 0 | \ell + 2 \rangle^2 \frac{F_{\ell+2}(r)}{F_{\ell}(r)} \int_0^{\infty} dr' F_{\ell+2}(r') \frac{1}{r'^3} F_{\ell}(r') \\
 & + \langle \ell 0 2 0 | \ell 0 \rangle^2 \int_0^{\infty} dr' F_{\ell}(r') \frac{1}{r'^3} F_{\ell}(r') \\
 & + \langle \ell 0 2 0 | \ell - 2 \rangle^2 \frac{F_{\ell-2}(r)}{F_{\ell}(r)} \int_0^{\infty} dr' F_{\ell-2}(r') \frac{1}{r'^3} F_{\ell}(r')] . \quad (3.6)
 \end{aligned}$$

In evaluating this formula we substitute the semiclassical $\hat{\ell} = \ell + \frac{1}{2}$ for any sums of ℓ and an integer or half-integer, i.e., $\ell + 2$, $\ell - 1$, ℓ , $\ell + 1$; and of course $2\ell + 1 = 2\hat{\ell}$. Corrections to an $\ell + 2$ in the first term tend to be cancelled by corrections to an $\ell - 1$ in the third term. In such an approximation we have for the Clebsch-Gordan coefficients

$$\langle \ell 0 2 0 | \ell + 2 \rangle^2 = \langle \ell 0 2 0 | \ell - 2 \rangle^2 = \frac{3}{8} , \quad (3.7)$$

$$\langle \ell 0 2 0 | \ell 0 \rangle^2 = \frac{1}{4} . \quad (3.8)$$

There are closed forms for the integrals^{8,7} which become in our approximation

$$\int_0^{\infty} dr' F_{\ell+2}(r') \frac{1}{r'^3} F_{\ell}(r') = \frac{k^2}{6} \frac{1}{(\hat{\ell}^2 + \eta^2)} \quad (3.9)$$

$$\int_0^\infty dr' F_{\ell-2}(r') \frac{1}{r'^3} F_\ell(r') = \frac{k^2}{6} \frac{1}{(\hat{\ell}^2 + \eta^2)} \quad (3.10)$$

$$\int_0^\infty dr' F_\ell(r') \frac{1}{r'^3} F_\ell(r') = \frac{k^2}{2\hat{\ell}^2} \left(1 - \frac{\eta}{\hat{\ell}} \arctan \frac{\hat{\ell}}{\eta}\right) \quad (3.11)$$

In Eq. (3.11) we have maintained a consistent semi-classical ansatz by replacing a sum over $(\hat{\ell}^2 + \eta^2)^{-1}$ with an integral to obtain the arctan term. Equation (3.6) now becomes

$$\begin{aligned} U_\ell(r) = & -1 \frac{2\mu}{\hbar^2} \frac{4\pi}{25} Z_P^2 e^{2B(E2)} k^2 \frac{1}{r^3} \\ & \times \left[\frac{1}{8\hat{\ell}^2} \left(1 - \frac{\eta}{\hat{\ell}} \arctan \frac{\hat{\ell}}{\eta}\right) \right. \\ & \left. + \frac{1}{16(\hat{\ell}^2 + \eta^2)} \left(\frac{F_{\ell+2}(r)}{F_\ell(r)} + \frac{F_{\ell-2}(r)}{F_\ell(r)} \right) \right] \quad (3.12) \end{aligned}$$

This is a neat closed form except for the wave function ratios. To deal with these ratios we will make use of the Coulomb wave function recurrence relation¹⁰ expressed in the appropriate approximate form

$$F_{\ell+1} + F_{\ell-1} = \frac{2}{(\hat{\ell}^2 + \eta^2)^{1/2}} \left[\eta + \frac{\hat{\ell}^2}{kr} \right] F_\ell \quad (3.13)$$

We have

$$\frac{F_{\ell+2}}{F_\ell} = -1 + \frac{2}{(\hat{\ell}^2 + \eta^2)^{1/2}} \left[\eta + \frac{\hat{\ell}^2}{kr} \right] \frac{F_{\ell+1}}{F_\ell} \quad (3.14)$$

$$\frac{F_{\ell-2}}{F_{\ell}} = -1 + \frac{2}{(\hat{\ell}^2 + \eta^2)^{1/2}} \left[\eta + \frac{\hat{\ell}^2}{kr} \right] \frac{F_{\ell-1}}{F_{\ell}}, \quad (3.15)$$

and

$$\begin{aligned} \frac{F_{\ell+2}}{F_{\ell}} + \frac{F_{\ell-2}}{F_{\ell}} &= -2 + \frac{2}{(\hat{\ell}^2 + \eta^2)^{1/2}} \left[\eta + \frac{\hat{\ell}^2}{kr} \right] \left[\frac{F_{\ell+1}}{F_{\ell}} + \frac{F_{\ell-1}}{F_{\ell}} \right] \\ &= -2 + \frac{4}{(\hat{\ell}^2 + \eta^2)} \left[\eta + \frac{\hat{\ell}^2}{kr} \right]^2. \end{aligned} \quad (3.16)$$

Thus the wave function ratios will provide additional r -dependence on top of the $1/r^3$ dependence already present. Putting in the $g_2(\xi)$ factor one obtains finally the long range imaginary potential for a given partial wave ℓ :

$$\begin{aligned} U_{\ell}(r) &= -\frac{i2\mu}{\hbar^2} \frac{\pi}{50} Z_p^2 e^2 B(E2)^\dagger g_2(\xi) \\ &\times \left[\left(\frac{\eta^2 k^2 (3\hat{\ell}^2 + \eta^2)}{\hat{\ell}^2 (\hat{\ell}^2 + \eta^2)^2} - \frac{\eta k^2}{\hat{\ell}^3} \arctan \frac{\hat{\ell}}{\eta} \right) \frac{1}{r^3} \right. \\ &\left. + \frac{4\eta k \hat{\ell}^2}{(\hat{\ell}^2 + \eta^2)^2} \frac{1}{r^4} + \frac{2\hat{\ell}^4}{(\hat{\ell}^2 + \eta^2)^2} \frac{1}{r^5} \right] \end{aligned} \quad (3.17)$$

where μ is the reduced mass of the system, Z_p is the projectile charge, k is the wave number, and η is the usual Sommerfeld parameter. Of course if more than one low-lying collective quadrupole state contributes to the absorption then the factor $Z_p^2 B(E2)^\dagger g_2(\xi)$ should be replaced by an appropriate sum over such factors. For example a

quadrupole state in the projectile would add a term $Z_T^2 B_P(E2) g_2(\xi_P)$ to the target term.

This potential at first sight seems quite different from the LTS potential. It is specifically ℓ -dependent with $1/r^5$, $1/r^4$, and $1/r^3$ radially dependent terms in contrast to the ℓ -independent, dominantly $1/r^5$, LTS behavior. As $\ell \rightarrow \infty$, the $1/r^5$ term dominates in our potential and the ratio of LTS to ours approaches 4/3. The physical correspondence between the approximate LTS potential and our more exact form may be seen in Fig. 1. The LTS potential crosses our ℓ -dependent potential several Fermis outside of the classical turning point for the small and intermediate ℓ values of interest. Paradoxically, due to the ℓ -dependence, our $1/r^5$ term has the longest range, while the $1/r^3$ term has the shortest range.

Our formula has been compared with the results of a computer evaluation of the imaginary part of Eq. (3.6) for the case in Fig. 1, and for all partial waves agreement is quite good (to within several percent, except for computationally unstable points where $1/F_\ell(r)$ becomes large). For the lower partial waves in above-barrier scattering, one should properly consider nuclear effects both in the wave functions and in the quadrupole operator, but for the present we do not consider these questions, arguing that our potential is sufficient as it stands to describe the unambiguously long-range part of the imaginary optical potential.

This ℓ -dependent long range absorptive potential has been incorporated into an optical model code and the resulting cross section (Fig. 2) is

practically indistinguishable from the corresponding calculations using the LTS potential for $^{18}\text{O} + ^{184}\text{W}$ at 90 MeV in the angular region of experimental interest. We see in the following section why there is such a close correspondence between the results of the two potentials for this case and also where the close correspondence breaks down.

4. Analytical Expressions for Elastic Scattering Incorporating Long Range Absorption

We have briefly mentioned the incorporation of the long range imaginary potential into an optical model code which would include also the usual potential geometry. An alternative approach which has the possibility of yielding a broader physical insight, is to attempt to derive approximately, in analytical closed form, the effect of the long range absorption on the elastic scattering amplitude. We have succeeded in doing this via a two-step procedure. First, the contribution which the long range absorptive potential makes to phase shifts beyond the critical angular momentum is evaluated in a semiclassical approximation. Then these phase shift contributions are inserted into a modified form of Frahn's strong absorption formula⁹ for the elastic scattering amplitude. The resulting closed form for the elastic scattering amplitude greatly simplifies below the Coulomb barrier, which allows a very general comparison of the predictions of the LTS potential with those of ours in this regime. A similar closed form has recently been independently proposed by Frahn and Hill.¹¹

To calculate the contributions to phase shifts beyond the critical angular momentum which arise from the long range absorption, we insert our potential, Eq. (3.17), into a perturbative JWKB integral which is evaluated along the Coulomb trajectory¹²

$$\delta_{U_\ell}(\hat{\ell}) \approx -\frac{1}{2E_{\text{cm}}} \int_0^\infty U_\ell \left(\frac{1}{k} (\eta + (\hat{\ell}^2 + \eta^2 + \rho^2)^{1/2}) \right) \left(1 + \frac{\eta}{(\hat{\ell}^2 + \eta^2 + \rho^2)^{1/2}} \right) d\rho. \quad (4.1)$$

where $\hat{\ell} \geq \Lambda$, the critical angular momentum for strong absorption.⁹ Note that Eq.(4.1) reduces to the eikonal approximation¹³ when $\eta \rightarrow 0$.

Of course, the same procedure may be applied to the LTS potential. The principal mathematical steps in the approximate evaluation in closed form of Eq. (4.1) both for our and the LTS potential are explained in Appendix B.

Due to the relative weakness and very smooth behavior of $U_{\ell}(r)$, both as a function of r and of $\hat{\ell}$ (e.g., see Fig. 1), the scattering coefficient component due to the long range absorption

$$M(\hat{\ell}) \equiv \exp[2i\delta_{U_{\ell}}(\hat{\ell})] \quad (4.2)$$

is a very smoothly varying function of $\hat{\ell}$. It decreases monotonically with decreasing $\hat{\ell}$, going from unity at large values of $\hat{\ell}$ to a non-negative value at $\hat{\ell} = \Lambda$, with no regions of large derivative. Thus $M(\hat{\ell})$, being smooth, may simply be factored out of integral approximations to the partial wave sum⁹ at critical points such as those of stationary phase or critical angular momentum. The upshot is that the long range absorption superimposes a quasi-classical weak absorption effect⁹ on the usual strong absorption phenomena. The modified form of Frahn's strong absorption formula which results for the total scattering amplitude, $f_T(\theta)$, may be written schematically as follows

$$f_T(\theta) = H(\theta_{\Lambda} - \theta) M(\lambda_R(\theta)) f_R(\theta) + M(\Lambda) \mathcal{F}(\theta, \Lambda, \Delta, B) \quad (4.3)$$

where $\theta_{\Lambda} = 2 \arctan (\eta/\Lambda)$ is the critical angle, H is the Heaviside unit step function, $\lambda_R(\theta) = \eta \cot (\theta/2)$ is the Coulomb stationary phase point, $f_R(\theta)$ is the Rutherford scattering amplitude, and $\mathcal{F}(\theta, \Lambda, \Delta, B)$ is the

damped diffractive portion of the scattering amplitude developed in detail by Frahn.⁹ $\mathcal{F}(\theta, \Lambda, \Delta, B)$ depends on Δ , which parameterizes the width of the nuclear-surface strong absorption profile in angular momentum space, as well as on the parameter Λ , which locates the center of that profile, and also on B , which parameterizes the short range nuclear refraction. A detailed description of $\mathcal{F}(\theta, \Lambda, \Delta, B)$ is given in Appendix C.

The result of fitting Eq. (4.3) to Brookhaven data² for $^{18}\text{O} + ^{184}\text{W}$ at 90 MeV lab energy is shown in Fig. 3. The $B(E2, 0^+ \rightarrow 2^+)$ of ^{184}W was taken to be $3.76 \times 10^4 \text{ e}^2 \text{ fm}^4$ while that of ^{18}O was taken to be $48 \text{ e}^2 \text{ fm}^4$. The parameters Λ, Δ , and B were varied "by hand". A reasonably satisfactory fit was obtained to the data of the September, 1976, run, but a number of points of the previous August, 1976, run - whose data had generally smaller error bars - proved simply not possible to fit so well.

Below the Coulomb barrier, Eq. (4.3) undergoes great simplification, for all strong absorption and nuclear refraction effects cease to operate. In this case we simply have the weak absorption formula

$$f_T(\theta) = M(\lambda_R(\theta)) f_R(\theta). \quad (4.4)$$

Comparing Eq. (4.4) with Eq. (4.3), we see that below the Coulomb barrier we may follow the effect of the long range absorption all the way to $\theta = \pi$ (corresponding to $\hat{\ell} \approx 0$) instead of being stopped by strong

absorption effects at $\theta = \theta_\Lambda$ (corresponding to $\hat{\ell} \approx \Lambda$). One clearly obtains the most information about long range absorption by studying sub-Coulomb elastic scattering. Such experiments represent a nuclear analogy to eclipsing the solar disc to better observe the corona.

When the results of the phase shift integral, Eq. (4.1), are inserted into the formula for $M(\hat{\ell})$, Eq. (4.2), one is able to derive from Eq. (4.4) a simple form for the sub-Coulomb elastic scattering ratio-to-Rutherford cross section

$$\frac{\sigma(\theta)}{\sigma_R(\theta)} = \left| \frac{f_T(\theta)}{f_R(\theta)} \right|^2 = M^2(\lambda_R(\theta)) = \exp(-Kf(\theta)) \quad (4.5)$$

where all specific parameters of the reaction are contained in the constant

$$K = \frac{16\pi}{225} \frac{k^4}{\eta^2} \left[\frac{B_T(E_2) \dagger g_2(\xi_T)}{Z_T^2 e^2} + \frac{B_P(E_2) \dagger g_2(\xi_P)}{Z_P^2 e^2} \right] \quad (4.6)$$

and $f(\theta)$ is a universal function of angle

$$\begin{aligned} f(\theta) = & \frac{9}{4} \left[\left(\cos \frac{\theta}{2} \right)^4 \left(\frac{4}{3} D^4 + \frac{104}{105} D^5 \right) \right. \\ & + (\sin \theta)^2 \left(\frac{\pi}{4} D^3 + \left(\frac{64-15\pi}{30} \right) D^4 \right) \\ & \left. + \left(\left(3 + \left(\tan \frac{\theta}{2} \right)^2 \right) \left(\sin \frac{\theta}{2} \right)^4 - \left(\left(\tan \frac{\theta}{2} \right)^3 \left(\frac{\pi-\theta}{2} \right) \right) \right) \left(D^2 + \frac{2}{3} D^3 \right) \right] \quad (4.7) \end{aligned}$$

with

$$D = \left(1 + \csc \frac{\theta}{2}\right)^{-1}. \quad (4.8)$$

This analytical form for $f(\theta)$ has the smooth behavior exhibited in Figure 4a.

By the same procedure, we obtain an expression almost identical to Eq. (4.5) for the sub-Coulomb cross section produced by the LTS potential (omitting unimportant finite size corrections). The only difference is that a different universal function of angle, $\bar{f}(\theta)$, is involved

$$\bar{f}(\theta) = D^4 \left[4 + 6 \log \left(1 + \left(\frac{8e^{-5/2}}{\csc \frac{\theta}{2} - 1} \right) \right) \right]. \quad (4.9)$$

We have plotted the universal below barrier ratio $\bar{f}(\theta)/f(\theta)$ in Fig. 4b. This ratio deviates from unity by up to 33-1/3% at forward angles, but this will not show up in most reactions due to the small magnitude of $f(\theta)$. At intermediate angles of about 40° to 110° the ratio deviates little from unity, implying excellent agreement for the predictions of the two potentials. However, beyond 110° (corresponding to LTS cut-off at $R_d/0.9$ of the Coulomb correction factor) there is no theory from the LTS potential but only a possible prescription. For the sake of analytical tractability we have merely ignored the cut-off in the ratio calculation. Clearly without the arbitrary cut-off, LTS predictions deviate substantially from those of our potential at very large angles as is illustrated in Fig. 4c. Here are plotted

cross sections in a realistic case for which data exists at two angles:¹⁴
 $^{16}_0 + ^{162}\text{Dy}$ at 48 MeV. There is also similar data for $^{16}_0 + ^{152}\text{Sm}$
 for which σ/σ_R is 0.56(1) at 120° (lab) and 0.51(1) at 140° as compared
 with our calculated values of 0.57 and 0.49, respectively.

Recently more complete angular distributions have been obtained for
 sub-Coulomb 70 MeV ^{20}Ne scattering on Sm isotopes.¹⁵ In Figure 5
 we see comparison of data with our formula which includes here also a term
 for excitation of the 2^+ state in ^{20}Ne (dashed curve). While the quali-
 tative agreement is good, at backward angles discrepancies occur especially
 for ^{148}Sm and ^{150}Sm . These discrepancies may be at least partially attributed
 to the larger energy loss factors ξ , which are only described approximately
 by the angle independent factor $g_2(\xi)$. At angles farther forward and
 especially for cases with a very low lying 2^+ state (small ξ) we expect
 both our potential and cross section formula to have greater validity.
 Furthermore, at more forward angles (corresponding to a greater distance of
 closest approach) there is less multiple Coulomb excitation to higher states.
 However, multiple Coulomb excitation will have an effect less direct upon
 the elastic scattering than on the inelastic 2^+ scattering in general.

5. Summing the Born Series for Coulomb Excitation

Recently Cotanch and Vincent¹⁶ have used a Greens' function separable in its two coordinates to sum the distorted wave series for elastic and transfer channel scattering amplitudes. Recalling that the approximate form of Greens function utilized in Section 3 to obtain the long range potential was on-energy-shell and thereby separable, we are led to the parallel approach of summing the Born series on-energy-shell to obtain elastic and inelastic scattering amplitudes for Coulomb excitation.

We begin by writing down the Born series for below barrier elastic scattering in the presence of quadrupole Coulomb excitation

$$\begin{aligned}
 T_{0+}^{\ell} = & \iint F_{\ell}(r) U_{\ell}(r, r') F_{\ell}(r') dr' dr \\
 & + \iiint F_{\ell}(r) U_{\ell}(r, r') G_{\ell}^{(+)}(r', r'') U_{\ell}(r'', r''') F_{\ell}(r''') dr dr' dr'' dr''' \\
 & + \iiint F_{\ell}(r) U_{\ell}(r, r') G_{\ell}^{(+)}(r', r'') U_{\ell}(r'', r''') G_{\ell}^{(+)}(r''', r''''') \\
 & \times U_{\ell}(r''''', r''''') F_{\ell}(r''''') dr dr' dr'' dr''' dr''''' \\
 & + \dots
 \end{aligned} \tag{5.1}$$

$U_{\ell}(r, r')$ is defined by Eq. (3.2) and

$$G_{\ell}^{(+)}(r, r') = - \frac{2\mu}{k\hbar^2} F_{\ell}(r_{<}) H_{\ell}(r_{>}) \tag{5.2}$$

is the elastic channel Greens function. We make the on-energy-shell approximation

$$G_{\ell}^{(+)}(r, r') = -i \frac{2\mu}{\hbar^2} F_{\ell}(r) F_{\ell}(r') , \quad (5.3)$$

and likewise for the non-local potential

$$U_{\ell}(r, r') = -i \frac{2\mu}{\hbar^2} \frac{4\pi}{25} Z_p^2 \ell^2 B(E2) \sum_{\ell'} \langle \ell 0 2 0 | \ell' 0 \rangle \frac{1}{r^3} \frac{1}{r'^3} F_{\ell'}(r) F_{\ell'}(r') . \quad (5.4)$$

The Born series may now be rewritten

$$\begin{aligned} T_{0+} &= i D_{\ell}^{020} \sum_{n=0}^{\infty} (D_{\ell}^{020})^n \\ &= i D_{\ell}^{020} / (1 - D_{\ell}^{020}) \end{aligned} \quad (5.5)$$

where

$$\begin{aligned} D_{\ell}^{020} &= - \left(\frac{2\mu}{\hbar^2} \right)^2 \frac{4\pi}{25} Z_p^2 \ell^2 B(E2) \sum_{\ell'} \langle \ell 0 2 0 | \ell' 0 \rangle^2 \\ &\times \left[\int F_{\ell}(r) \frac{1}{r^3} F_{\ell'}(r) dr \right]^2 . \end{aligned}$$

Making use of the semi-classical ansatz of Eqs. (3.7)-(3.11) we write down

$\hat{D}_{\ell}^{020}$ in closed form

$$\begin{aligned} \hat{D}_{\ell}^{020} &= - \left(\frac{2\mu}{\hbar^2} \right)^2 \frac{4}{25} Z_p^2 e^2 B(E2) + k^4 \\ &\times \left[\frac{1}{48} \frac{1}{(\hat{\ell}^2 + n^2)^2} + \frac{1}{16 \hat{\ell}^4} \left(1 - \frac{n}{\hat{\ell}} \arctan \frac{\hat{\ell}}{n} \right)^2 \right] . \end{aligned} \quad (5.6)$$

Continuing with the semi-classical

$$\hat{\ell} = \eta \cot \frac{\theta}{2}$$

we obtain the angle dependent

$$D^{020}(\theta) = - \frac{2\mu}{k\hbar^2}^2 \frac{4\pi}{25} Z_p^2 e^2 B(E2) \left(\frac{k}{\eta}\right)^4 \left[\frac{1}{48} \left(\sin \frac{\theta}{2}\right)^4 + \frac{1}{16} \left(\tan \frac{\theta}{2}\right)^4 \left(1 - \left(\tan \frac{\theta}{2}\right) \left(\frac{\pi-\theta}{2}\right)\right)^2 \right]. \quad (5.7)$$

For convenience this may be rewritten

$$D^{020}(\theta) = - \frac{1}{4} K g(\theta) \quad (5.8)$$

where K is identical to the K in Eq. (4.6) (with energy loss factor and projectile excitation term added) and

$$g(\theta) = \frac{9}{4} \left[\frac{1}{3} \left(\sin \frac{\theta}{2}\right)^4 + \left(\tan \frac{\theta}{2}\right)^4 \left(1 - \left(\tan \frac{\theta}{2}\right) \left(\frac{\pi-\theta}{2}\right)\right)^2 \right]. \quad (5.9)$$

We notice that $g(\theta)$ is identical to $f(\theta)$ of Eq. (4.7) at 180° and the maximum deviation of the two functions at any angle is less than 4 percent.

The elastic scattering T matrix element then becomes

$$T_{0+} = - i \frac{1}{4} K g(\theta) / \left(1 + \frac{1}{4} g(\theta)\right). \quad (5.10)$$

Now the S matrix element $e^{2i\delta_l(\theta)}$ may be written

$$\begin{aligned} e^{2i\delta_l(\theta)} &= 1 - 2i T_{0+} \\ &= \frac{1 - \frac{1}{4}K g(\theta)}{1 + \frac{1}{4}K g(\theta)}. \end{aligned} \quad (5.11)$$

Following the approach of Section 4 we write down the sub-Coulomb ratio to Rutherford elastic scattering cross section

$$\frac{\sigma(\theta)}{\sigma_R(\theta)} = \left(\frac{1 - \frac{1}{4}K g(\theta)}{1 + \frac{1}{4}K g(\theta)} \right)^2. \quad (5.12)$$

This is the on-shell Born series formula for sub-Coulomb elastic scattering. It is instructive to compare this formula with the JWKB formulation based on the long range absorptive potential

$$\frac{\sigma(\theta)}{\sigma_R(\theta)} = e^{-K f(\theta)}. \quad (4.5)$$

At 180° these formulas agree exactly to second order in K (which is equivalent to fourth order in the interaction). At other angles the same correspondence is broken only by the small deviation between $g(\theta)$ and $f(\theta)$. Thus to a very good approximation the JWKB optical model approach of Section 4 is equivalent to summing the scattering series on-energy-shell for the case of sub-Coulomb elastic scattering.

In a parallel manner the Coulomb Born series may be summed for the amplitude of inelastic Coulomb excitation to the 2^+ state. The exact inelastic

amplitude for the model problem may be written schematically

$$T_{2+}^{\ell'} = \sum_{\ell} c_{\ell} \int F_{\ell}(r) V(r) X_{\ell}(r) dr \quad (5.13)$$

where $F_{\ell}(r)$ is the Coulomb scattering wave function for the $2+$ state, $V(r)$ is the Coulomb quadrupole operator, and $X_{\ell}(r)$ is the exact wave function for the ground state. The Coulomb Born series for $X_{\ell}(r)$ may be written

$$\begin{aligned} X_{\ell}(r) = & F_{\ell}(r) + \iint G_{\ell}^{(+)}(r, r') U_{\ell}(r', r'') F_{\ell}(r'') dr' dr'' \\ & + \iiint G_{\ell}^{(+)}(r, r') U_{\ell}(r', r'') G_{\ell}^{(+)}(r'', r''') \\ & \times U_{\ell}(r''', r'''') F_{\ell}(r'''') dr' dr'' dr''' dr'''' \\ & + \dots \end{aligned} \quad (5.14)$$

If, as before, we make the on-energy-shell approximation we obtain

$$\begin{aligned} X_{\ell}(r) = & F_{\ell}(r) \sum_{M=0}^{\infty} (D_{\ell}^{020})^M \\ = & F_{\ell}(r) / (1 - D_{\ell}^{020}). \end{aligned} \quad (5.15)$$

The inelastic scattering amplitude then becomes

$$T_{2+}^{\ell'} = \sum_{\ell} c_{\ell} \int F_{\ell}(r) V(r) F_{\ell}(r) / (1 - D_{\ell}^{020}). \quad (5.16)$$

Thus, the first order inelastic scattering amplitude is renormalized by the same denominator as the elastic scattering denominator. Noting that the first order semi-classical result for Coulomb excitation is

$$\frac{\sigma_{(1)}^{2+}(\theta)}{\sigma_R(\theta)} = K g(\theta) \quad (5.17)$$

we have the corrected expression for the inelastic scattering

$$\frac{\sigma^{2+}(\theta)}{\sigma_R} = \frac{-K g(\theta)}{\left[1 + \frac{1}{4}K g(\theta)\right]^2} \quad (5.18)$$

For convenience we may rewrite the equations for elastic and inelastic scattering in terms of the first order inelastic cross section

$$\frac{\sigma^{0+}(\theta)}{\sigma_R(\theta)} = \left(\frac{1 - \frac{1}{4} \frac{\sigma^{2+}_{(1)}(\theta)}{\sigma_R(\theta)}}{1 + \frac{1}{4} \frac{\sigma^{2+}_{(1)}(\theta)}{\sigma_R(\theta)}} \right)^2 \quad (5.19)$$

$$\sigma^{2+}(\theta) = \frac{\sigma^{2+}_{(1)}(\theta)}{\left(1 + \frac{1}{4} \frac{\sigma^{2+}_{(1)}(\theta)}{\sigma_R(\theta)}\right)^2} \quad (5.20)$$

Note that these formulas preserve the quasi-classical unitary relationship at every angle

$$\sigma^{0+}(\theta) + \sigma^{2+}(\theta) = \sigma_R(\theta) . \quad (5.21)$$

	V	W	r	a	r _c
148,150,152,154 _{Sm} coupled channels	65.9	27.3	1.440	.637	1.440
154 _{Sm} pure elastic	34.6	29.4	1.404	.819	1.4

TABLE 2. Optical Model Parameters for 50 MeV α Particles Scattered on Sm Isotopes.

6.1 Nuclear Effects: α Scattering

It was previously suggested by Glendenning, Hendrie, and Jarvis that the effect of inelastic scattering could be represented by an optical potential component and these authors considered the case of 50 MeV α scattering on $^{148,150,152,154}\text{Sm}^1$. The coupling is dominantly nuclear and it reflects the change from a spherical vibrational nucleus ^{148}Sm to a rotational nucleus ^{154}Sm . It was found that a single optical potential could describe both spherical and deformed Sm isotope elastic scattering when the strongly coupled excited states were treated explicitly. However, in the absence of explicit coupling to excited states, the optical potential that reproduces the elastic scattering is quite different in the two cases. Moreover, while the optical model parameters for ^{148}Sm differ little from the coupled channels parameters, the optical model parameters for ^{154}Sm are quite different from the coupled channels parameters (see Table 2). In this section we will calculate the component of the optical potential arising from the strong rotational coupling in ^{154}Sm and compare it with the results of the previous empirical analysis.

The computer calculations of the potential component were done using the iteration scheme Eqs. (2.15)-(2.20). A deformed optical model was utilized, as in the work of Glendenning, Hendrie, and Jarvis, with the usual monopole component of deformation included in the optical model Hamiltonian H and the quadrupole component of deformation as the interaction, $V(r)$, causing the transition. Reorientation effects were of necessity ignored

in the calculation of the potential, but their effects are not dominant. The calculated cross section from the wave functions of our iteration scheme deviates some from the coupled channels with reorientation calculations, but both show a greater deviation from the spherical optical model calculation. As a check of the iteration scheme we compared with a coupled channels code calculation with reorientation turned off and obtained good agreement with the iterated calculations.

The real part of the calculated optical potential component is exhibited in Figure 6 as a function of orbital angular momentum ℓ . Clearly it is highly ℓ -dependent, repulsive in the low partial waves, increasing in magnitude to the surface, changing sign and becoming attractive, and then decreasing in magnitude for high partial waves. The empirical optical model component to be compared with the calculation is ℓ -independent and repulsive as is also seen in Figure 6.

The imaginary part of the optical model component is shown in Figure 7. It is ℓ -dependent, but absorptive for all partial waves. The empirical imaginary potential component is of small magnitude, but relatively diffuse in its small absorption outside the surface.

6.2 Nuclear Effects: $^{16}_0$ Scattering

We have investigated a second case of recent interest, 60 MeV $^{16}_0$ scattering on ^{40}Ca .¹⁷ In this case a coupled channels calculation was able simultaneously to reproduce the 0^+ , 3^- , 5^- and 2^+ states in ^{40}Ca , while DWBA calculations using parameters fitted to elastic scattering failed to reproduce the angular distributions for the 3^- and 5^- inelastic scattering. For this coupled channel calculation in which only the 3^- state was coupled to the ground state and no reorientation was assumed, our optical model formulation Eq. (2.14) is exactly equivalent to the coupled channels formulation. In Figures 8 and 9 we show the real and imaginary parts of the ℓ -dependent local equivalent potential component which exactly represent the effect of the coupling of the 3^- state upon the elastic scattering. The general pattern is similar to the α -Sm case of Figures 6 and 7: the real potential component is repulsive for low partial waves and attractive for high partial waves; the imaginary potential component is dominantly absorptive with an ℓ -dependence of strength peaking in the surface partial waves. In both cases the ℓ -dependence of the imaginary potential seems to reflect the ℓ -window of a direct reaction in the presence of a strongly absorptive background potential; flux is lost from the elastic channel into the inelastic channel primarily in the surface partial waves.

If we wish to look at the amplitude for inelastic scattering to the 3^- excitation in this particular case we can obtain it directly from an equation of the form of Eq. (2.3),

$$x_{3^-} = G_{3^- 3^-}^{(+)} V_{3^- 0^+} x_{0^+} \quad (6.1)$$

Taking a partial wave we find the asymptotic form

$$\chi_{\ell}^{3-}(r \rightarrow \infty) = h_{\ell}^{3-}(r \rightarrow \infty) \sum_{\ell'} c_{\ell'} \int_0^{\infty} f_{\ell'}^{3-}(r') V(r') \chi_{\ell'}^{0+}(r') dr'. \quad (6.2)$$

Since the coefficient of the outgoing wave function is the scattering amplitude we have the coupled channels equivalent for the inelastic scattering transition amplitude in the form of DWBA. All 0^+-3^- coupling effects enter through the ground state wave function $\chi_{0+}(r')$, and the excited state wave function $f_3(r')$ is just an optical model wave function without the effect of the strong coupling to the 0^+ ground state. We have in fact incorporated the ground state wave functions $\chi_{0+}(r')$ into a DWBA code, and very good numerical agreement is obtained with calculations using the coupled channels code CHUCK¹⁸ for the 3^- cross section in the $^{16}_0 + ^{40}\text{Ca}$ case. This optical potential method for calculating coupled elastic and inelastic cross sections may be straightforwardly generalized to a sum of excited states coupled only to the ground state.

As Ascutto et al.¹⁹ have pointed out for the $^{16}_0 + ^{40}\text{Ca}$ case, it is most crucial in fitting the angular distribution of the 3^- and 5^- cross sections with a DWBA type cross section that the effects of the specific 3^- coupling not be included in the final channel wave functions. (This is clear in our Eq. (6.2)). In this particular case the strong ℓ -dependence of the calculated potentials is of less importance in obtaining a DWBA type cross section similar to that generated by coupled channels calculations. However, this case is not typical of heavy-ion reactions because it involves

one excited state coupled strongly only to the ground state as the prime mechanism for explaining a DWBA anomaly. In the usual DWBA angular distribution anomalies, particle transfer (a weak process) is involved and thus the proper DWBA wave functions should be consistent with elastic scattering in both channels. On the other hand, the usual optical model prescription for fitting elastic scattering makes use of an ℓ -independent potential. Recalling the strong ℓ -dependence of the calculated potentials arising from direct inelastic channels, one must question the usefulness of wave functions generated by this procedure when large direct reaction strength is present. The heavy ion DWBA angular distribution anomalies may not be unrelated to the use of an ℓ -independent optical potential even when a large percentage of flux is going into direct channels.

7. Conclusions

The optical model approach to strong coupling effects in elastic scattering has proved to be instructive in several ways. In the case of quadrupole Coulomb excitation, we have shown that the main effect on the elastic channel may be represented by an ℓ -dependent imaginary potential of analytical form. By use of closed forms for the sub-Coulomb elastic cross section in the presence of this long range absorption, we have been able to compare our potential with the approximate ℓ -independent potential of Love, Terasawa, and Satchler in a very general way. The good agreement in the angular range $\sim 40^\circ$ - 110° indicates that specific r -dependence of the LTS potential is to some extent able to mock up both the r - and ℓ -dependence of our more exact form.

Above the Coulomb barrier we have shown how to extend the Frahn model for elastic scattering by incorporating long range absorption.

Summing the Coulomb-Born series on-energy-shell for quadrupole excitation was interesting in itself in that closed forms were obtained for the elastic and inelastic cross sections. It was pointed out that the first order approximation to our inelastic scattering expression is just the usual semi-classical formulation. The elastic scattering expression, however was shown to be quite similar to our expression from the JWKB optical model analysis, thus indicating that putting our long range potential into an optical model code has roughly the effect of summing the Born series on-energy-shell for quadrupole Coulomb excitation effects (at least for higher partial waves where nuclear distortion is negligible).

The calculation of the optical potential component due to nuclear quadrupole inelastic scattering in the α on ^{154}Sm case exhibited a severe ℓ -dependence in the real component. Although this component has been previously parameterized in an ℓ -independent way, such an empirical simplification may not prove generally useful. While all elastic scattering coupled channels effects may be represented by optical potential components, the present exercise (including the ^{16}O on ^{48}Ca case) suggests that the form that those components will take can be highly ℓ -dependent.

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APPENDIX A

Our potential has been derived in the large ℓ limit. Here we will show that it also holds for large η independent of ℓ . We begin by writing down the more exact forms of Eqs. (3.7)-(3.11).

$$\langle \ell 0 2 0 | \ell + 2 0 \rangle^2 = \frac{3}{2} \frac{(\ell + 2)(\ell + 1)}{(2\ell + 3)(2\ell + 1)} \quad (A1)$$

$$\langle \ell 0 2 0 | \ell 0 \rangle^2 = \frac{(\ell + 1)\ell}{(2\ell + 3)(2\ell - 1)} \quad (A2)$$

$$\langle \ell 0 2 0 | \ell - 2 0 \rangle^2 = \frac{3}{2} \frac{\ell(\ell - 1)}{(2\ell + 1)(2\ell - 1)} \quad (A3)$$

$$\int_0^\infty dr' F_{\ell+2}(r') \frac{1}{r'^3} F_\ell(r') = \frac{k^2}{6} \frac{1}{[(\ell+1)^2 + \eta^2]^{1/2} [(\ell+2)^2 + \eta^2]^{1/2}} \quad (A4)$$

$$\int_0^\infty dr' F_\ell(r') \frac{1}{r'^3} F_\ell(r') = \frac{k^2}{\ell(\ell+1)(2\ell+1)} \left[\ell + 1 - \eta^2 \sum_{n=0}^{\ell} \frac{1}{n^2 + \eta^2} \right] \quad (A5)$$

$$\int_0^\infty dr' F_{\ell-2}(r') \frac{1}{r'^3} F_\ell(r') = \frac{k^2}{6} \frac{1}{[(\ell-1)^2 + \eta^2]^{1/2} [\ell^2 + \eta^2]^{1/2}} \quad (A6)$$

Substituting into Eq. (3.6) we obtain the more exact form of Eq. (3.12)

$$\begin{aligned}
 U_{\ell}(r) = & -\frac{12\eta}{kh^2} \frac{4\pi}{25} Z_P^2 e^{2B(E2)+k^2} \frac{1}{r^3} \\
 & \times \left[\frac{1}{(2\ell+3)(2\ell+1)(2\ell-1)} \left[\ell+1 - \eta^2 \sum_{n=0}^{\ell} \frac{1}{n^2+\eta^2} \right] \right. \\
 & + \frac{1}{4} \frac{(\ell+2)(\ell+1)}{(2\ell+3)(2\ell+1)} \frac{1}{[(\ell+1)^2+\eta^2]^{1/2} [(\ell+2)^2+\eta^2]^{1/2}} \frac{F_{\ell+2}(r)}{F_{\ell}(r)} \\
 & \left. + \frac{1}{4} \frac{\ell(\ell-1)}{(2\ell+1)(2\ell-1)} \frac{1}{[(\ell-1)^2+\eta^2]^{1/2} [\ell^2+\eta^2]^{1/2}} \frac{F_{\ell-2}(r)}{F_{\ell}(r)} \right]. \quad (A7)
 \end{aligned}$$

Corresponding to Eq. (3.14) we have the exact form for the Coulomb wave function recurrence relation¹⁰

$$\frac{F_{\ell+2}}{F_{\ell}} = -\frac{(\ell+2)}{(\ell+1)} \frac{[(\ell+1)^2+\eta^2]^{1/2}}{[(\ell+2)^2+\eta^2]^{1/2}} + \frac{(2\ell+3)}{(\ell+1)[(\ell+2)^2+\eta^2]^{1/2}} \left(\eta + \frac{(\ell+1)(\ell+2)}{kr} \right) \frac{F_{\ell+1}}{F_{\ell}} \quad (A8)$$

A second recurrence relation involving the derivative of the Coulomb wave function F'_{ℓ} will now be employed to evaluate the remaining wave function ratio

$$\frac{F_{\ell+1}}{F_{\ell}} = \frac{1}{[(\ell+1)^2+\eta^2]^{1/2}} \left(\frac{(\ell+1)^2}{kr} + \eta \right) - \frac{(\ell+1)}{[(\ell+1)^2+\eta^2]^{1/2}} \frac{F'_{\ell}}{F_{\ell}} \quad (A9)$$

Now for large kr $F_{\ell} \rightarrow \sin\theta_{\ell}$, $G_{\ell} \rightarrow \cos\theta_{\ell}$ and $F'_{\ell} \rightarrow \cos\theta_{\ell}$ where $\theta_{\ell} = kr - \eta \log 2kr - \frac{\ell\pi}{2} + \sigma_{\ell}$.¹⁰ Thus

$$\frac{F'_{\ell}}{F_{\ell}} \sim \frac{G_{\ell}}{F_{\ell}} \sim \cot \theta_{\ell} \quad (A10)$$

and F'_{ℓ}/F_{ℓ} oscillates violently. In the spirit of the approximations made, we will set this oscillating term to zero. We now have

$$\begin{aligned} \frac{F_{\ell+2}}{F_{\ell}} = & - \frac{(\ell+2) [(\ell+1)^2 + \eta^2]^{1/2}}{(\ell+1) [(\ell+2)^2 + \eta^2]^{1/2}} \\ & + \frac{(2\ell+3)}{(\ell+1) [(\ell+2)^2 + \eta^2]^{1/2} [(\ell+1)^2 + \eta^2]^{1/2}} \left(\eta + \frac{(\ell+1)(\ell+2)}{kr} \right) \left(\eta + \frac{(\ell+1)^2}{kr} \right). \end{aligned} \quad (A11)$$

In a similar manner we also have

$$\frac{F_{\ell-2}}{F_{\ell}} = - \frac{(\ell-1) [\ell^2 + \eta^2]^{1/2}}{\ell [(\ell-1)^2 + \eta^2]^{1/2}} + \frac{(2\ell-1)}{\ell [(\ell-1)^2 + \eta^2]^{1/2}} \left(\eta + \frac{(\ell-1)\ell}{kr} \right) \frac{F_{\ell-1}}{F_{\ell}}, \quad (A12)$$

$$\frac{F_{\ell-1}}{F_{\ell}} = \frac{1}{[\ell^2 + \eta^2]^{1/2}} \left(\frac{\ell^2}{kr} + \eta \right) + \frac{\ell}{[\ell^2 + \eta^2]^{1/2}} \frac{F_{\ell'}}{F_{\ell}} \quad (A13)$$

and

$$\begin{aligned} \frac{F_{\ell-2}}{F_{\ell}} = & - \frac{(\ell-1)}{\ell} \frac{[\ell^2 + \eta^2]^{1/2}}{[(\ell-1)^2 + \eta^2]^{1/2}} \\ & + \frac{(2\ell-1)}{\ell [\ell^2 + \eta^2]^{1/2} [(\ell-1)^2 + \eta^2]^{1/2}} \left(\eta + \frac{(\ell-1)\ell}{kr} \right) \left(\eta + \frac{\ell^2}{kr} \right). \end{aligned} \quad (A14)$$

Substituting Eqs. (A11) and (A14) into Eq. (A7) we now have a potential formula applicable even for small ℓ

$$\begin{aligned}
 U_{\ell}(r) = & -\frac{12\mu}{kh^2} \frac{4\pi}{25} Z_p^2 e^2 B(E2) \uparrow k^2 \frac{1}{r^3} \\
 & \times \left\{ \frac{1}{(2\ell+3)(2\ell+1)(2\ell-1)} \left[\ell + 1 - \eta^2 \sum_{n=0}^{\ell} \frac{1}{n^2 + \eta^2} \right. \right. \\
 & + \frac{1}{4} \frac{(\ell+2)(\ell+1)[\eta^2 - (\ell+2)(\ell+1)]}{(2\ell+3)(2\ell+1)[(\ell+2)^2 + \eta^2][(\ell+1)^2 + \eta^2]} \\
 & \left. + \frac{1}{4} \frac{\ell(\ell-1)[\eta^2 - \ell(\ell-1)]}{(2\ell+1)(2\ell-1)[\ell^2 + \eta^2][(\ell-1)^2 + \eta^2]} \right\} \\
 & + \left(\frac{1}{4} \frac{(\ell+1)(\ell+2)(2\ell+3)}{(2\ell+1)[(\ell+2)^2 + \eta^2][(\ell+1)^2 + \eta^2]} + \frac{1}{4} \frac{\ell(\ell-1)(2\ell-1)}{(2\ell+1)[\ell^2 + \eta^2][(\ell-1)^2 + \eta^2]} \right) \frac{\eta}{kr} \\
 & + \left(\frac{1}{4} \frac{(\ell+2)^2(\ell+1)^3}{(2\ell+1)[(\ell+2)^2 + \eta^2][(\ell+1)^2 + \eta^2]} + \frac{1}{4} \frac{(\ell-1)\ell^3}{(2\ell+1)[\ell^2 + \eta^2][(\ell-1)^2 + \eta^2]} \right) \frac{1}{k^2 r^2} \Big]
 \end{aligned}
 \tag{A15}$$

We now consider Eq. (A15) in the case $\ell \ll \eta$, neglecting terms of order $O(\ell^4/\eta^4)$. Then

$$\eta^2 \sum_{n=0}^{\ell} \frac{1}{n^2 + \eta^2} = \sum_{n=0}^{\ell} \frac{1}{\left(\frac{n}{\eta}\right)^2 + 1} \approx \sum_{n=0}^{\ell} \left(1 - \left(\frac{n}{\eta}\right)^2 \right) \tag{A16}$$

and

$$\sum_{n=0}^{\ell} \left(1 - \left(\frac{n}{\eta}\right)^2 \right) = \ell + 1 - \frac{1}{\eta^2} \sum_{n=0}^{\ell} n^2 = \ell + 1 - \frac{1}{6\eta^2} (2\ell+1)(\ell+1)\ell \tag{A17}$$

Thus in the case $\ell \ll \eta$ the term in curly brackets $\{ \}$ above may be written

$$\begin{aligned}
 \left\{ \right\} &= \frac{1}{(2\ell+3)(2\ell+1)(2\ell-1)} \frac{1}{\eta^2} \frac{1}{6} (2\ell+1)(\ell+1)\ell \\
 &+ \frac{1}{4\eta^2} \frac{(\ell+2)(\ell+1)(2\ell-1)}{(2\ell+3)(2\ell+1)(2\ell-1)} + \frac{1}{4\eta^2} \frac{\ell(\ell-1)(2\ell+3)}{(2\ell+1)(2\ell-1)(2\ell+3)} \\
 &= \frac{1}{24\eta^2} \frac{1}{(2\ell+3)(2\ell+1)(2\ell-1)} [4(2\ell^3 + 3\ell^2 + \ell) \\
 &+ 6(2\ell^3 + 5\ell^2 + \ell - 2) + 6(2\ell^3 + \ell^2 - 3\ell)] \\
 &= \frac{1}{6\eta^2} .
 \end{aligned} \tag{A18}$$

Now let us contrast with the case where $\ell \gg 1$. We extract from Eq. (3.17)

$$\left\{ \right\} = \frac{1}{8} \left(\frac{\eta^2}{\hat{\ell}^2} \frac{(3\hat{\ell}^2 + \eta^2)}{(\hat{\ell}^2 + \eta^2)^2} - \frac{\eta}{\hat{\ell}^3} \arctan \frac{\hat{\ell}}{\eta} \right) \tag{A19}$$

Now as $\frac{\ell}{\eta} \rightarrow 0$

$$\frac{1}{8} \left(\frac{\eta^2}{\hat{\ell}^2} \frac{(3\hat{\ell}^2 + \eta^2)}{(\hat{\ell}^2 + \eta^2)^2} - \frac{\eta}{\hat{\ell}^3} \arctan \frac{\hat{\ell}}{\eta} \right) \rightarrow \frac{1}{6\eta^2} \tag{A20}$$

This essentially completes our demonstration that Eq. (3.17) is good for $\eta \gg 1$ independent of the size of ℓ : Below the Coulomb barrier $kr > 2\eta$ and as $\ell \rightarrow 0$, the $\frac{1}{r^3} \frac{1}{kr}$ term of the potential Eq. (A15) $\rightarrow 0(1/\eta^4)$ and the $\frac{1}{r^3} \frac{1}{k^2 r^2}$ term $\rightarrow 0(1/\eta^6)$. These are to be contrasted

with $\frac{1}{6\eta^2}$ behavior of the $\frac{1}{r^3}$ part of the potential seen above. Since the dominant $\frac{1}{6\eta^2}$ behavior holds for both $\ell \gg 1$ and $\ell \ll \eta$ it must also hold for $1 \ll \eta$ independent of ℓ . This is because if $\ell \gtrsim \eta$ (i.e. $\ell < \eta$), then $\ell \gg 1$ since $\eta \gg 1$.

Appendix B

We will indicate here how the phase shift contribution, Eq. (4.1), was approximated for our potential, Eq. (3.17). We will also give a briefer indication of the same procedure for the LTS potential.

Since our potential consists of terms whose only radial dependence is r^{-p} where $p = 3, 4$, and 5 , we clearly only need to approximate

$$I_p = \int_0^{\infty} \left(\eta + (\hat{\ell}^2 + \eta^2 + \rho^2)^{1/2} \right)^{-p+1} \left(\hat{\ell}^2 + \eta^2 + \rho^2 \right)^{-1/2} d\rho \quad (B1)$$

where $p = 3, 4$, and 5 .

We change variable to

$$x \equiv \frac{(\hat{\ell}^2 + \eta^2 + \rho^2)^{1/2} - (\hat{\ell}^2 + \eta^2)^{1/2}}{\eta + (\hat{\ell}^2 + \eta^2)^{1/2}}. \quad (B2)$$

Defining

$$\alpha \equiv \frac{(\hat{\ell}^2 + \eta^2)^{1/2} - \eta}{(\hat{\ell}^2 + \eta^2)^{1/2} + \eta} \quad (B3)$$

and

$$J_p(\alpha) \equiv \int_0^{\infty} \frac{1}{(x+1)^{p-1}} \frac{dx}{(x(x+1+\alpha))^{1/2}} \quad (B4)$$

we obtain

$$I_p = \left(\eta + (\hat{\ell}^2 + \eta^2)^{1/2} \right)^{-p+1} J_p(\alpha). \quad (B5)$$

Now, α increases monotonically from zero to unity as $\hat{\ell}$ increases from zero to infinity. We see from (B4) that $J_p(\alpha)$ is monotonically decreasing and slowly varying as α increases from zero to unity. We are able to evaluate $J_p(\alpha)$ at $\alpha=0$ and $\alpha=1$ exactly. In view of the slow monotone variation of $J_p(\alpha)$ for $0 \leq \alpha \leq 1$, linear interpolation between its endpoint values ought to be an adequate approximation.

First we indicate the calculation of the endpoint values of $J_p(\alpha)$. For the case $\alpha=1$, we change variable in (B4) to $u = (x(x+2))^{1/2}$. We arrive at the result

$$J_p(1) = \int_0^\infty (u^2+1)^{-p/2} du = \begin{cases} 1 & \text{for } p = 3 \\ \frac{1}{4}\pi & \text{for } p = 4 \\ \frac{2}{3} & \text{for } p = 5. \end{cases} \quad (B6)$$

For the case $\alpha = 0$, we change variable in (B4) to $v = x^{1/2}$. We calculate

$$J_p(0) = 2 \int_0^\infty (v^2+1)^{-p+1/2} dv = \begin{cases} 4/3 & \text{if } p = 3 \\ 16/15 & \text{if } p = 4 \\ 32/35 & \text{if } p = 5. \end{cases} \quad (B7)$$

We now make the linear interpolation approximation

$$J_p(\alpha) \approx \begin{cases} 1 + \frac{1}{3}(1-\alpha) & \text{if } p = 3 \\ \frac{1}{4}\pi + (16/15 - \frac{1}{4}\pi)(1-\alpha) & \text{if } p = 4 \\ \frac{2}{3} + (26/105)(1-\alpha) & \text{if } p = 5. \end{cases} \quad (B8)$$

For the LTS potential, neglecting unimportant finite size corrections, we have the radial dependence $r^{-5}(1 - (2\eta/kr))^{-1/2}$. We need to approximate the integral

$$K = \int_0^{\infty} \left(\eta + (\hat{\ell}^2 + \eta^2 + \rho^2)^{1/2} \right)^{-7/2} \left((\hat{\ell}^2 + \eta^2 + \rho^2)^{1/2} - \eta \right)^{-1/2} \left(\hat{\ell}^2 + \eta^2 + \rho^2 \right)^{-1/2} d\rho. \quad (B9)$$

Again changing variable to x and defining

$$L(\alpha) \equiv \int_0^{\infty} \frac{1}{(x+1)^{7/2}} \frac{dx}{(x(x+\alpha)(x+1+\alpha))^{1/2}} \quad (B10)$$

we obtain

$$K = \left(\eta + (\hat{\ell}^2 + \eta^2)^{1/2} \right)^{-4} L(\alpha). \quad (B11)$$

We note that $L(\alpha)$ is monotonic decreasing as α increases from small non-negative values to unity. However $L(\alpha)$ increases without bound as $\alpha \rightarrow 0+$, as a logarithmic singularity is encountered. It proves possible to evaluate $L(\alpha)$ exactly at $\alpha = 1$, and we obtain $L(1) = \frac{2}{3}$. We can also successfully investigate the behavior of $L(\alpha)$ near the singularity at $\alpha = 0$. We obtain

$$L(\alpha) \sim \log(4/\alpha) - (11/6) \quad \text{as } \alpha \rightarrow 0+. \quad (B12)$$

We then approximate $L(\alpha)$ by a formula which interpolates monotonically between these two endpoint behaviors

$$L(\alpha) \approx \frac{2}{3} + \log \left(1 + \frac{(1-\alpha)4 e^{-5/2}}{\alpha} \right). \quad (B13)$$

Appendix C

We write down here in full detail the definition of the damped diffractive portion of the scattering amplitude, $\mathcal{F}(\theta, \Lambda, \Delta, B)$, encountered in Eq. (4.3). It is taken essentially verbatim from the work of Frahn.⁹

Before proceeding, we define $\sigma(\hat{\ell})$ as the semiclassical approximation to the Coulomb phase shift. We note that the Coulomb deflection function is defined in terms of the derivative of $\sigma(\hat{\ell})$

$$\Theta_R(\hat{\ell}) = 2 \frac{d}{d\hat{\ell}} (\sigma(\hat{\ell})) = 2 \arctan(\eta/\hat{\ell}). \quad (C1)$$

We also define $S_N(\hat{\ell})$ as the nuclear part of the scattering coefficient, but excluding the long range absorptive part (which is $M(\hat{\ell})$, given in Eq. (4.2)). Following T. F. Hill²⁰, we introduce Λ, Δ , and B via the following convenient parameterization of $S_N(\hat{\ell})$

$$S_N(\hat{\ell}) = \left[1 + \exp \left(\frac{\Lambda - \hat{\ell}}{\Delta} - 2i \arctan B \right) \right]^{-1}. \quad (C2)$$

We note that for $0 \leq B \leq 1$, $|S_N(\hat{\ell})|$ increases monotonically from zero to unity as $\hat{\ell}$ increases from much smaller than Λ to much larger, and the derivative with respect to $\hat{\ell}$ of the phase of $S_N(\hat{\ell})$ - the nuclear deflection function - is everywhere non-positive with a minimum value of $-(B/2\Delta)$ at $\hat{\ell} = \Lambda$. Taking $B > 1$ causes $|S_N(\hat{\ell})|$ to behave unphysically, so this parameterization cannot describe arbitrarily strong nuclear refraction, but it is adequate for most refraction strengths encountered in practice.

Now we transcribe from Frahn⁹ the definition of the damped diffractive portion of the scattering amplitude

$$\mathcal{F}(\theta, \Lambda, \Delta, B) \equiv \frac{1}{k} \left(\frac{\Lambda}{2\pi \sin \theta} \right)^{1/2} e^{2i\sigma(\Lambda)} \times$$

$$\left[e^{-i(\Lambda\theta - \frac{1}{4}\pi)} \left\langle \{1 + (\theta_{\Lambda} - \theta)a_1\} \{ \text{sgn}(\theta_{\Lambda} - \theta) \gamma(|\theta_{\Lambda} - \theta|) \} - a_0 \right\rangle F(\Delta(\theta_{\Lambda} - \theta)) \right. \quad (C3)$$

$$\left. + e^{+i(\Lambda\theta - \frac{1}{4}\pi)} \frac{F(\Delta(\theta_{\Lambda} + \theta))}{(\theta_{\Lambda} + \theta)} \right]$$

where Frahn's damping function is defined as

$$F(\Delta x) = \int d\hat{\ell} \frac{dS_N(\hat{\ell})}{d\hat{\ell}} e^{i(\hat{\ell} - \Lambda)x} = \left(\frac{\pi \Delta x}{\sinh(\pi \Delta x)} \right) \exp \left(2(\arctan B) \Delta x \right) \quad (C4)$$

and Frahn's Fresnel function is given as

$$\gamma(x) = \left[\frac{\pi}{-2\theta'_R(\Lambda)} \right]^{1/2} e^{i[\frac{1}{4}\pi - \frac{x^2}{2\theta'_R(\Lambda)}]} \text{erfc} \left(e^{i\frac{1}{4}\pi} \frac{x}{[-2\theta'_R(\Lambda)]^{1/2}} \right) \quad (C5)$$

and

$$a_0 = \frac{1}{2\Lambda [-\theta'_R(\Lambda)]} + \left[\frac{1}{3} + i \frac{1}{6} \frac{(\theta_{\Lambda} - \theta)^2}{[-\theta'_R(\Lambda)]} \right] \left[\frac{\theta''_R(\Lambda)}{[\theta'_R(\Lambda)]^2} \right] \quad (C6)$$

$$a_1 = a_0 + \frac{1}{6} \left[\frac{\theta''_R(\Lambda)}{[\theta'_R(\Lambda)]^2} \right]. \quad (C7)$$

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FIGURE CAPTIONS

- Fig. 1. ℓ -dependent imaginary optical potential obtained from Eq. (3.17) compared with the LTS potential for $^{18}\text{O} + ^{184}\text{W}$ at 90 MeV.
- Fig. 2. Cross sections calculated with the LTS potential and our ℓ -dependent potential compared with a calculation without long range absorption. Woods-Saxon optical model parameters are from Ref. 5.
- Fig. 3. Fit of Eq.(4.3) to $^{18}\text{O} + ^{184}\text{W}$ elastic data² at 90 MeV with $A = 41.0$, $\Delta = 1.5$, and $B = 0.4$.
- Fig. 4. (a) Universal function of angle $f(\theta)$.
 (b) Ratio of $\bar{f}(\theta)$ for the LTS potential to $f(\theta)$ for our potential.
 (c) Elastic scattering cross section for $^{16}\text{O} + ^{162}\text{Dy}$ at 48 MeV calculated from Eq. (4.5) incorporating $f(\theta)$ for our potential and $\bar{f}(\theta)$ for the LTS potential. Data are from Lee and Saladin.¹⁴
- Fig. 5. Angular distributions from elastic scattering of ^{20}Ne on samarium nuclei. Dashed curves show calculations using Eq. (4.5) with a term for the $^{20}\text{Ne } 2^+$ excitation added in. Solid curves show coupled channel calculations with both 2^+ states and reorientation included. The lower solid curve for ^{152}Sm shows the calculation without reorientation a significant effect for this isotope.
- Fig. 6. Real potential component for 50 MeV $\alpha + ^{154}\text{Sm}$ scattering.
- Fig. 7. Imaginary potential component for 50 MeV $\alpha + ^{154}\text{Sm}$ scattering.
- Fig. 8. Real potential component for 60 MeV $^{16}\text{O} + ^{40}\text{Ca}$ scattering.
- Fig. 9. Imaginary potential component for 60 MeV $^{16}\text{O} + ^{40}\text{Ca}$ scattering.

XBL 778-1635

FIGURE 1

FIGURE 2

FIGURE 3

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FIGURE 4

FIGURE 5

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FIGURE 6

FIGURE 7

FIGURE 8

FIGURE 9

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