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Kent Curtis and Alice McMullen

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ABSTRACT

Integral expressions for the efficiency of a counter arrangement consisting of a cylindrical source and cylindrical counter are derived. An IBM 650 program to evaluate these integrals is described.

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INTRODUCTION

A common problem in the analysis of experiments involving decay reactions of artificially produced elements is finding the efficiency of the counter arrangement used to detect the decay products. In this paper an analysis is presented of an arrangement consisting of a cylindrical source with a known mean path length for the detected particle and a coaxial cylindrical counter that is assumed to have 100% efficiency for counting particles that enter it. As a result of this analysis, two four-fold integrals are derived which may be evaluated to determine the actual counter efficiency. An IBM 650 Computer program has been written by one of the authors (A.M.) to perform these integrations.

The problem is especially simple for this geometry, but the method may be generalized to fit other counter-source arrangements. The principal purpose of this paper is to point out that this problem is easily solvable by use of an automatic computer and, in this specific instance, required about 30 minutes of machine time for the evaluation of the integrals with three figures of accuracy. We also describe a program, which now exists for the IBM 650, that is capable of determining the efficiency of any counter-source arrangement that satisfies the restrictions mentioned above.

An analysis of counter efficiency is shown in Fig. 1. The upper cylinder represents the counter, with thickness t_c and radius R_c , and the lower cylinder represents the source, with thickness t_s and radius R_s . The distance between the two cylinders is b . We shall use subscript c to denote quantities referring to the counter and s for the source throughout this analysis. We assume that any particle that enters the counter is detected. Let us take the origin of coordinates at the center of the base of the source.

Consider a volume element in the source dV_s . The number of particles emitted per second by this volume element is

$$dN_s = S(r_s, \theta_s, Z_s) dV_s$$

where S is the source strength per unit volume. We shall assume

$$\int_V S(r, \theta, Z) dV = 1$$

(Note: if S is constant then we have $S = \frac{1}{V}$). We assume also that particles are emitted with equal probability in any direction. The probability

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Fig. 1. Analysis of efficiency of counter.

that a particle is counted is then the probability that the particle gets out of the source (which is represented by an attenuation factor) times the probability that it strikes the counter (which in turn is simply the solid angle subtended by the counter at the point of emission of the particle).

More explicitly, let dN be the number of particles emitted in volume element dV_s and entering the counter through the element of area dA_c . Let $\vec{R}$ be the vector joining dV_s and dA_c , $\vec{n}$ be the unit vector normal to dA_c , and

$$a(r_s, \theta_s, Z_s, r_c, \theta_c, Z_c)$$

be the attenuation factor. If we have

$$dN = S(r_s, \theta_s, Z_s) \frac{\vec{R} \cdot \vec{n}}{4\pi R^3} a(r_s, \theta_s, Z_s, r_c, \theta_c, Z_c) dA_c dV_s,$$

then the total number of particles counted is $N = \int_{A_c} \int_{V_s} dN$.

It is convenient to break the integral over the counter into two parts, the integral over the base N_B and the integral over the sides N_H . These two parts are then, respectively,

$$N_B = \int_{\text{Base}} \int_{V_s} dN = \int_{\text{Base}} \int_{V_s} S(r_s, \theta_s, Z_s) \frac{\vec{R} \cdot \vec{n}}{4\pi R^3}$$

$$\times a(\vec{r}_s, \vec{r}_c) r_c d\theta_c dr_c dV_s,$$

and

$$N_H = \int_{\text{Sides}} \int_{V_s} S(r_s, \theta_s, Z_s) \frac{\vec{R} \cdot \vec{n}}{4\pi R^3} a(\vec{r}_s, \vec{r}_c) R_c d\theta_c dZ_c dV_s.$$

To evaluate $\frac{\vec{R} \cdot \vec{n}}{R^3}$, let $\vec{Z}_c$ be the vector from the origin of coordinates to

the interaction of a plane parallel to the base of the counter and passing through dA_c and the axis of the counter. We then obtain

$$\vec{R} = \vec{r}_c - \vec{r}_s.$$

Since $\vec{r}_c = \vec{Z}_c + \vec{\rho}$, it follows that $\vec{R} = \vec{Z}_c + \vec{\rho} - \vec{r}_s$, and

$$R^3 = [Z_c^2 - 2\vec{Z}_c \cdot \vec{r}_s + \rho^2 - 2\vec{\rho} \cdot \vec{r}_s + r_s^2]^{3/2}$$

$$= [(Z_c - Z_s)^2 + \rho^2 + r_s^2 - 2\rho r_s \cos(\theta_s - \theta_c)]^{3/2}.$$

We immediately consider the two separate cases:

(I.) Over the base, with $\vec{n} = + \frac{\vec{Z}_c}{|Z_c|}$. Here we have

$$\frac{\vec{R} \cdot \vec{n}}{R^3} = + \frac{(\vec{Z}_c + \vec{\rho} - \vec{r}_s) \cdot \vec{Z}_c}{R^3 Z_c}$$

$$= - \frac{r_s \cdot \vec{Z}_c + Z_c^2}{Z_c R^3} \quad (\text{since } \vec{Z}_c \cdot \vec{\rho} = 0)$$

$$= \frac{-Z_s + Z_c}{[(Z_c - Z_s)^2 + \rho^2 + r_s^2 - 2\rho r_s \cos(\theta_s - \theta_c)]^{3/2}}$$

(II.) Over the sides, with $\vec{n} = \frac{\vec{\rho}}{|\rho|} = \frac{\vec{\rho}}{R_c}$. Here we obtain

$$\frac{\vec{R} \cdot \vec{n}}{R^3} = \frac{(\vec{Z}_c + \vec{\rho} - \vec{r}_s) \cdot \vec{\rho}}{R^3 R_c} = \frac{\rho^2 - \vec{r}_s \cdot \vec{\rho}}{R_c R^3}$$

$$= \frac{R_c^2 - R_c r_s \cos(\theta_s - \theta_c)}{R_c R^3} = \frac{R_c - r_s \cos(\theta_s - \theta_c)}{R^3}.$$

For this case we have a further limitation on θ which is defined as follows. A particle leaving dV_s can enter the counter only in those portions of the side facing it. This means that r_s must be greater than R_c , and also that the maximum and minimum values of θ_c are defined by the points of tangency of $\vec{R}$ and the sides of the counter. At these tangent points $\vec{\rho}$ is perpendicular to $\vec{R}$. Then

$$\vec{\rho} \cdot \vec{R} = \vec{\rho} \cdot (\vec{Z}_c + \vec{\rho} - \vec{r}_s) = \rho^2 - \vec{\rho} \cdot \vec{r}_s = 0.$$

From this we find

$$R_c^2 - R_c r_s \cos(\theta_s - \theta_c) = 0,$$

and

$$\cos(\theta_s - \theta_c) = \frac{R_c}{r_s}.$$

Finally $a(\vec{r}_s, \vec{r}_c)$ is a function only of the distance ℓ the particle must travel through the source. A unit vector in the direction of $\vec{R}$ is $\frac{\vec{R}}{R}$; a unit vector in the direction normal to the surface of the source is $\frac{\vec{Z}_c}{Z_c}$. Then we obtain

$$\begin{aligned} \ell &= \frac{t_s - Z_s}{\frac{\vec{R} \cdot \vec{Z}_c}{R Z_c}} = \frac{t_s - Z_s}{\frac{Z_c^2 - Z_s Z_c}{Z_c R}} = \frac{(t_s - Z_s) R}{Z_c - Z_s} \\ &= \frac{(t_s - Z_s)}{Z_c - Z_s} \left[(Z_c - Z_s)^2 + \rho^2 + r_s^2 - 2\rho r_s \cos(\theta_s - \theta_c) \right]^{\frac{1}{2}} \end{aligned}$$

Then we have

$$\begin{aligned} N_B &= \int_0^{2\pi} d\theta_c \int_0^{R_c} d\rho_c \int_0^{2\pi} d\theta_s \int_0^{R_s} dr_s \int_0^t dZ_s \\ &\times \frac{\rho_c r_s (Z_c - Z_s)}{4 \pi R^3} S(\vec{r}_s) a(\ell) \text{ with } Z_c = t_s + d, \end{aligned}$$

and

$$N_H = \int_{\theta_c \text{ min.}}^{\theta_c \text{ max.}} d\theta_c \int_{t_s + d}^{t_s + d + t_c} dZ_c \int_0^{2\pi} d\theta_s \int_{R_c}^{R_s} dr_s \int_0^{t_s} dZ_s$$

$$\times \frac{R_c r_s [R_c = r_s \cos(\theta_s - \theta_c)]}{4\pi R^3} S(\vec{r}_s) a(l).$$

We note that the angle variables always enter as $\cos(\theta_s - \theta_c)$. For N_B we have $0 \leq \theta_s \leq 2\pi$ and $0 \leq \theta_c \leq 2\pi$. In the $\theta_s - \theta_c$ plane this is an integration over the area defined by the coordinate axes and the lines $\theta_c = 2\pi$, $\theta_s = 2\pi$ (Fig. 2).

Fig. 2. Region of integration over the $\theta_c - \theta_s$ plane.

Because the $\cos(\theta_s - \theta_c)$ is an even function, this is just twice the integral over region 1 or region 2. Let us change variables

$$\left. \begin{aligned} \phi &= \theta_s - \theta_c \\ \theta_c &= \theta_c \end{aligned} \right\} \quad \text{The Jacobian of this transformation is 1.}$$

Then we have

$$N_B = \int_0^{2\pi} d\theta_s \int_0^{2\pi} d\theta_c f(\theta_s - \theta_c) =$$

$$2 \int_0^{2\pi} f(\phi) d\phi \int_0^{2\pi-\phi} d\theta_c = 2 \int_0^{2\pi} f(\phi) d\phi (2\pi - \phi) = 4\pi \int_0^{2\pi} f(\phi) d\phi - 2 \int_0^{2\pi} \phi f(\phi) d\phi;$$

or if we let $\phi' = 2\pi - \phi$, $\cos(2\pi - \phi') = \cos \phi'$, $f(2\pi - \phi') = f(\phi')$,

$$N_B = 2 \int_0^{2\pi} f(\phi') d\phi' \int_0^{\phi'} d\theta_c = 2 \int_0^{2\pi} \phi' f(\phi') d\phi'.$$

Let us equate these two expressions for N_B . Thus

$$4 \int_0^{2\pi} \phi' f(\phi') d\phi' = 4\pi \int_0^{2\pi} f(\phi) d\phi,$$

$$2 \int_0^{2\pi} \phi' f(\phi') d\phi' = 2\pi \int_0^{2\pi} f(\phi) d\phi,$$

and

$$N_B = 2\pi \int_0^{2\pi} f(\phi) d\phi.$$

For N_H the variable θ_c is limited by the relation $\cos(\theta_s - \theta_c) = \frac{R_c}{r_s}$.

In the $\theta_s - \theta_c$ plane this is an integration over the shaded area shown in Fig. 3a.

Fig. 3. Region of integration over the $\theta_c - \theta_s$ plane.

This may be redrawn as illustrated in Fig. 3b; again the integral is twice the integral over one of the halves of the region.

Let $\phi = \theta_s - \theta_c$; $\theta_s = \theta_s$. Then we have

$$\begin{aligned} N_H &= \int_0^{2\pi} d\theta_s \int_{\theta_c \min}^{\theta_c \max} f(\theta_s - \theta_c) d\theta_c \\ &= \int_0^{2\pi} d\theta_s \int_0^{\cos^{-1} \frac{R_c}{r_s}} f(\phi) d\phi \\ &= 2\pi \int_0^{\cos^{-1} \frac{R_c}{r_s}} f(\phi) d\phi. \end{aligned}$$

We may now rewrite our expressions

$$N_B = \frac{1}{2} \int_0^{R_c} d\rho_c \int_0^{R_s} dr_s \int_0^{t_s} dZ_s \int_0^{2\pi} d\phi \frac{\rho_c r_s (Z_c - Z_s)}{R^3} S(\vec{r}_s) a(l)$$

and

$$\begin{aligned} N_H &= \frac{1}{2} \int_{t_s+d}^{t_s+d+t_c} dZ_c \int_{R_c}^{R_s} dr_s \int_0^{t_s} dZ_s \int_0^{\cos^{-1} \frac{R_c}{r_s}} d\phi \frac{R_c r_s [R_c - r_s \cos \phi]}{R^3} \\ &\quad \times S(\vec{r}_s) a(l). \end{aligned}$$

Let us now introduce the attenuation factor explicitly. Most often it is expressed in terms of a mean path length λ , and then $a(l) =$

$$e^{-\frac{l}{\lambda}}.$$

Accordingly, we obtain

$$N_B = \frac{1}{2} \int_0^{R_c} d\rho_c \int_0^{R_s} dr_s \int_0^{t_s} dZ_s \int_0^{2\pi} d\phi \frac{\rho_c r_s (Z_c - Z_s)}{R^3} S(\vec{r}_s) e^{-\frac{\ell}{\lambda}}$$

and

$$N_H = \frac{1}{2} \int_0^{t_s} dZ_s \int_{t_s+d}^{t_s+d+t_c} dZ_c \int_{R_c}^{R_s} dr_s \int_0^{\cos^{-1} \frac{R_c}{r_s}} d\phi \frac{R_c r_s [R_c - r_s \cos \phi]}{R^3} \times S(\vec{r}_s) e^{-\frac{\ell}{\lambda}}$$

We finally have the efficiency $N = N_H + N_B$, which is the efficiency expressed as a decimal fraction since $\int_{V_s} S dV_s = 1$.

DESCRIPTION OF IBM 650 COMPUTER PROGRAM

In the IBM 650 program which was written to evaluate these integrals the source strength $S(r_s, \theta_s, Z_s)$ was assumed to be constant. Then, since $\int_{V_s} S dV_s = 1$, we have $S = \frac{1}{V_s}$. This is not a necessary restriction in the

program, and it could easily be modified to include any source strength that is independent of θ_s .

A description of the operation of the program follows. The program is known in our files as count 4620-02 and is written for the IBM 650 with index registers and floating-point arithmetic.

There are 2 decks of cards labeled N_B and N_H . In order to run the problem, we must give the following data to the machine:

For N_B

- In
- 1800 - lower limit on ϕ
 - 1801 - upper limit on ϕ
 - 1802 - lower limit on Z_s
 - 1803 - upper limit on Z_s
 - 1804 - lower limit on r_s
 - 1805 - upper limit on r_s
 - 1806 - lower limit on ρ_c
 - 1807 - upper limit on ρ_c
 - 1808 - $\Delta\phi$
 - 1809 - ℓ = number of intervals for ϕ ;
in fixed point (00 0000 000 ℓ)
 - 1810 - ΔZ_s
 - 1811 - k = number of intervals for Z_s ;
in fixed point (00 0000 000 k)
 - 1812 - Δr_s
 - 1813 - j = number of intervals for r_s ;
in fixed point (00 0000 000 j)
 - 1814 - $\Delta\rho_c$
 - 1815 - i = number of intervals for ρ_c ;
in fixed point (00 0000 000 i)
 - 1816 - λ = mean free path in source
 - 1817 - $S(\vec{r}_s) = \frac{1}{V} = \frac{1}{\pi R_s^2 t_s}$
 - 1818 - t_c = thickness of counter
 - 1819 - R_c = radius of counter
 - 1820 - t_s = thickness of source
 - 1821 - R_s = radius of source

- 1822 - b = distance between source and counter
- 1823 - l in fixed point (00 0000 000 l)
- 1824 - $k + 20$ in fixed point (00 0000 00 ($20 + k$))
- 1825 - $j + 40$ in fixed point (00 0000 00 ($40 + j$))
- 1826 - $i + 60$ in fixed point (00 0000 00 ($60 + i$))

For N_H

- In 1800 - lower limit on ϕ
- 1801 - R_c = radius of counter
- 1802 - lower limit on Z_s
- 1803 - upper limit on Z_s
- 1804 - lower limit on $r_s = (R_c + \Delta r_s)$
- 1805 - upper limit on r_s
- 1806 - lower limit on Z_c
- 1807 - upper limit on Z_c
- 1808 - zeros - ($\Delta\phi$ will be stored here by the program after it is determined)
- 1809 - l = number of intervals for ϕ ;
fixed point
- 1810 - ΔZ_s
- 1811 - k = number of intervals for Z_s ;
fixed point
- 1812 - Δr_s
- 1813 - j = number of intervals for r_s ;
fixed point
- 1814 - ΔZ_c
- 1815 - i = number of intervals for Z_c ;
fixed point
- 1816 through 1822 - supplied by N_B deck

1823 - l (00 0000 000 l)

1824 - $k + 20$ (00 0000 000($k + 20$))

1825 - $j + 40$ (00 0000 000 ($j + 40$))

1826 - $i + 60$ (00 0000 000 ($i + 60$))

1850 - l in floating point.

If $S(\vec{r}_s) = \frac{1}{V}$ is changed; i.e., if R_s and t_s are other than $R_s = 1.0$ inch and $t_s = 0.375$ inch, put a constant in floating-point form $= \frac{1}{2V}$ in location 0080. This should be put in following the N_B data and the N_H data.

To start the program, set the error sense switch to stop; the overflow to stop; display to distributor; half-cycle to run; programed to run; dial 70-0001-0003+ on storage-entry switches. In the read hopper place a drum-zeroing card, a load-punch routine, then the N_B deck followed by the N_B data and a transfer card to 0000. The N_H deck can be put in immediately following the transfer card, and when the program finishes computing N_B , it will automatically read the N_H deck, the N_H data following it, and another transfer card to 0000. After the decks are in the hopper, press the computer-reset button, program-start button, then start button on read-feed unit. When the machine stops on reading the last card, press end-of-file button on the read-feed unit to allow the last cards to be processed. The start button on the punch feed should also be pressed.

When the program finishes computing, it will punch one card with the following information on it in floating-point form: t_c , R_c , t_s , R_s , b , N_B , N_H , and N where $N = N_B + |N_H|$. The lights will show an instruction address of 9999 which is the end.

Copies of this program deck are available on request.