## Title

Student and Teacher Translanguaging in Dual Language Elementary Mathematics
Classrooms: An Exploration of Beliefs, Responses and Functions

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## UNIVERSITY OF CALIFORNIA <br> SANTA CRUZ

# STUDENT AND TEACHER TRANSLANGUAGING IN DUAL-LANGUAGE ELEMENTARY MATHEMATICS CLASSROOMS: AN EXPLORATION OF BELIEFS, RESPONSES AND FUNCTIONS 

A dissertation submitted in partial satisfaction of the requirements for the degree of<br>DOCTOR OF PHILOSOPHY<br>in<br>EDUCATION<br>by<br>Jolene Gregory

June 2021

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#### Abstract

Student and Teacher Translanguaging in Dual Language Elementary Mathematics Classrooms: An Exploration of Beliefs, Responses and Functions

Jolene Gregory


The recent theoretical debate over the role of translanguaging in dual language programs can be described as moving from prohibition to promotion. Indeed, many researchers, theorists, and educators are now encouraging translanguaging. At the center of the debate are teachers and their beliefs, which are argued to influence their practice. Teachers' beliefs and practices are also determinant of what happens in the classroom and directly influences student learning. This study explored teacher beliefs and responses to translanguaging in dual language elementary mathematics classrooms, as well as the functions that the translanguaging served. The study design aimed to reduce inconsistencies found between beliefs and practices in previous studies and broaden understanding of translanguaging in lesser studied contexts. Each of the three chapter of the dissertation is written as a separate paper which approaches these topics from different perspectives.

The first paper explored 14 elementary dual language teachers' beliefs and responses to students' translanguaging. The findings show that the teachers held principally permissive beliefs and uniformly permitted translanguaging. The results suggest a reappraisal of the practices and purposes of translanguaging, especially as they relate to teacher beliefs.

The second paper sought to understand 14 dual language elementary teachers' beliefs and practices of their own translanguaging. It explored the functions it served and the alignment between their beliefs and practice. The findings indicate teachers' beliefs primarily recognize the academic functions of translanguaging. In contrast their practice is primarily for
social functions. This suggests a necessary reevaluation of the purposes of translanguaging and the associated pedagogical and research implications.

The third paper explores how translanguaging rates vary as a function of language of instruction, student language proficiency, location and translanguaging function. It analyzes the translanguaging practices of students and teachers in 32 elementary Spanish-English dual-language mathematics classrooms in Texas and California. The findings challenge previous research regarding teachers' response to translanguaging in dual language programs and the relationship between language proficiency and translanguaging. They also support previous research regarding translanguaging demonstrating an awareness of the linguistic capital and symbolic power that the language of power bestows and contextual patterns of translanguaging. The findings indicate that further research is required, and teacher education and dual-language programs need to approach translanguaging from a new perspective.

## Acknowledgements

I would like to recognize and thank the many people who have been a part of this endeavor. Primero, quiero reconocer mi esposo, Ranulfo, e hijos, Oliver y Orlando, quienes han brindado paciencia, sacrificio y apoyo durante los años requeridos para cumplir con este proyecto. ¡Gracias por todo! I would also like to thank my advisor, Professor Kip Téllez, for his continuous support throughout my PhD studies and research. His depth of knowledge, experience and practical approach have taught me invaluable lessons about research, mentorship, and life. In addition, I would like to thank the rest of my dissertation committee, composed of Professors Judit Moschkovich, Eduardo Mosqueda and Marco Bravo, for their encouragement and thoughtful insights which have guided me along the development of this project. My academic sisters have also been invaluable for their words of encouragement, and their service as sounding boards. Thank you Yuzhu Xia, Adria Patthoff, and Liana Balloffet! Finally, I would like to thank each and every one of the teachers that participated in this study for your trust, openness and honesty. Your willingness to open your classrooms and share your experiences and knowledge will go on to benefit many more teachers and students in the future.

## 1 Introduction

The use of more than one language between bilingual speakers in interactive speech (i.e., hybrid language), is a common yet controversial linguistic event in classrooms. The dilemmas about hybrid language use have increased particularly in dual language programs in the U.S. as they have become more common (Lindholm Leary, 2013). Disagreements over the use and purpose of hybrid language have been particularly sharp among educators and researchers. One position holds that it should be prohibited (e.g., McCarthy, 2018; Wang \& Kirkpatrick 2013); another that it should be permitted (eg. Macaro, 2005; Setati et al., 2002; Weber, 2014); and finally, that it should be promoted (e.g., García \& Kleyn, 2016; Sommerville \& Faltis, 2019). These disagreements have also extended into the terminology to describe hybrid language. A variety of terms have been proposed to refer to this practice, the most common perhaps being code-switching, or translanguaging (MacSwan, 2017). These terms are reflective of different theoretical perspectives of hybrid language.

Codeswitching (Poplack,1980; Gumperz, 1982) refers to the alternation of two languages or two different grammatical systems within a single clause, sentence, or turn. Cook (2001) adds that this switching is not at random but rather language is switched to another "according to speech function, rules of discourse, and syntactic properties of the sentence" (Cook, 2001, p. 408). Codeswitching is viewed from a monoglossic perspective which sees it as using two linguistic systems that some argue should be kept strictly separate (Del Valle, 2000; García, 2009; Wardhaugh \& Fuller, 2015). This perspective sees languages as warring or that using one language while learning another language leads to cross contamination (van Lier, 2004). From that pragmatics perspective, codeswitching is seen as a transgression (García \& Kleyn, 2016).

As originally coined by Cen Williams, translanguaging referred to the "planned and systematic use of two languages for teaching and learning inside the same lesson" (Lewis, Jones \& Baker, 2012, p. 643). In an academic context, Translanguaging in its original sense
refers to the purposeful pedagogical alternation of languages in spoken and written, receptive and productive modes where students hear or read a lesson, a passage in a book or a section of work in one language and develop their work in another. This definition has transformed to include translanguaging as the complex discursive practices that bilingual students and teachers participate in to make meaning and communicate in multilingual classrooms and other cultural and sociolinguistic contexts in which they live (García, 2009; Gort, 2015; Martínez, Hikida, \& Durán, 2015). Translanguaging is a naturally occurring phenomenon but also can be a pedagogical strategy to foster language and literacy development (Hornberger and Link, 2012) enabling bilingual students to "develop and enact standard academic ways of languaging" (García and Sylvan, 2011, p. 389). Proponents of this perspective (García, 2009; Gort, 2015) argue that bilinguals have a single linguistic repertoire and are not two monolinguals in one, also called the two solitudes assumption (Cummins, 2008). Essentially, as García and Wei (2014) indicate the difference between translanguaging, and codeswitching lies in codeswitching referring to two separate languages whereas translanguaging refers to a single new interrelated language practice.

Mathematics is an interesting context to explore hybrid language practices because most research in dual language programs has taken place in language arts classrooms (King \& Ridley, 2019; Martínez et al., 2015); therefore, looking at this topic in the mathematics classroom can provide new insights. Most of the research in mathematics classrooms of hybrid language use has been conducted outside the U.S. and although research in international contexts is informative, it is always bound by culture (Meyer et al., 2016) and cannot be separated from social and cultural dimensions (Civil, 2010). Thus, exploration of hybrid language use in U.S. mathematics classrooms has applications unique to the social and cultural dimensions in the U.S. Most research exploring hybrid language practices in mathematics utilize the term codeswitching (e.g., Adler, 1998; Bose \& Choudhury, 2010; Ferguson, 2003; Hansen-Thomas, 2009; Merritt et al., 1992; Moschkovich, 2007; Planas \& Civil, 2013; Salehmohamed \& Rowland, 2014; Setati et al., 2002). Only since 2015 has the
term translanguaging begun to appear in research exploring hybrid language practices in mathematics (e.g., Farrugia, 2017, 2018; Hansen-Thomas \& Bright, 2019; Planas, 2018; Rubinstein-Avila et al., 2015; Tavares, 2015). Although research in other fields of hybrid language practices may have emphasized differences in theoretical perspectives, the recent shift in terminology in the field of mathematics education does not emphasize differences in theoretical perspectives. Therefore, the use of each of these terms may not reflect significant differences in theoretical perspectives held by the researchers but rather reflects the academically accepted term used to describe this practice at the time these studies were published within the field. Considering this shift, I will use the term translanguaging, while recognizing that the various perspectives held, and terminology used by the teachers in this study bridge across codeswitching and translanguaging.

Due to the growing number of emergent bilingual children in U.S. schools (U.S. Department of Education, 2019), understanding teachers' beliefs surrounding bilingual students' language use, particularly translanguaging and how those beliefs influence the everyday practice of teachers' work with bilingual students and, ultimately, bilingual students' learning and academic performance is needed. Studying translanguaging in dual language programs provides an opportunity not available in other program types (i.e., English Only programs). The dual language context is one in which all participants can translanguage, therefore minimizing the pragmatic opposition to translanguaging when an interlocutor is not bilingual (McCarthy, 2018), which could confound beliefs and practices beyond the phenomenon. As a result of the pedagogical and linguistic complexity of dual language classrooms, teacher beliefs regarding the role of language or languages in the dual language classroom are particularly elaborate. However, little research of dual language teachers' beliefs about translanguaging exists (Martínez et al., 2015), despite researchers exploring pedagogical translanguaging in dual language classrooms (García \& Sylvan, 2011; Gort \& Pontier, 2013; Palmer et. al., 2014).

As a bilingual myself and as a language teacher, I am also personally intrigued by the role that translanguaging plays in students' development. I have experienced personally and professionally how translanguaging can aid in linguistic development. In my review of literature, I began to think about the how translanguaging could aid in academic development in mathematics. So, I set out in this study particularly believing that I would discover untapped academic functions and benefits for bilingual students. However, I was surprised to find that translanguaging served social functions to a greater degree than academic functions and that teachers own observation of these functions were leading to shifts in their beliefs and responses to translanguaging.

Since inconsistencies have been observed in studies of teacher's beliefs about translanguaging (Martinez et al., 2015; Nava, 2009), this study addresses some of the previous inconsistencies due to research design and elucidates the inconsistencies due to the entangled domains and cognitive dissonance of translanguaging. This dissertation takes on the form of three academic papers. Each paper explores different but highly related topics. It uses the same data set which was analyzed in three different ways to answer different sets of research questions.

### 1.1 Paper 1: Teachers' beliefs and responses to student translanguaging in dual language mathematics classrooms

The recent theoretical debate over the role of translanguaging in dual language programs can be described as moving from prohibition to promotion. Indeed, many researchers, theorists, and educators are now encouraging translanguaging. At the center of the debate are teachers and their beliefs, which are argued to influence their practice. The present study explored 14 elementary dual language teachers' beliefs and responses to students' translanguaging as well as the practice in mathematics across two states and seven different school districts. The study design aimed to reduce inconsistencies found between beliefs and practices in previous studies and broaden understanding of
translanguaging in lesser studied contexts. The findings show that the teachers held principally permissive beliefs and uniformly permitted translanguaging. The student translanguaging served primarily social functions. The results suggest a reappraisal of the practices and purposes of translanguaging, especially as the relate to teacher beliefs.

### 1.2 Paper 2: Teacher translanguaging beliefs and functions in dual language mathematics classrooms

Many researchers and theorists are promoting the academic advantages of translanguaging. However, teachers' beliefs and practices are determinant of what happens in the classroom and directly influences student learning. This study sought to understand 14 dual language elementary teachers' beliefs and practices of translanguaging, from seven school districts in California and Texas. It explored the teachers own translanguaging in mathematics, the functions it served and the alignment between their beliefs and practice. This design aimed to reduce inconsistencies found in previous studies and broaden understanding of translanguaging in lesser studied contexts. The findings indicate teachers' beliefs primarily recognize the academic functions of translanguaging. In contrast their practice is primarily for social functions. This suggests a necessary reevaluation of the purposes of translanguaging and the associated pedagogical and research implications.

### 1.3 Paper 3: Bids for Linguistic Capital through Translanguaging during Spanish and English instruction in dual language classrooms

This paper explores how translanguaging rates varies as a function of teachers' response to translanguaging, school context and student language proficiency. It analyzes the translanguaging practices of students and teachers in 32 elementary Spanish-English dual-language mathematics classrooms in Texas and California. The findings challenge previous research regarding teachers' response to translanguaging in dual language
programs and the relationship between language proficiency and translanguaging. They also support previous research regarding translanguaging demonstrating an awareness of the linguistic capital and symbolic power that the language of power bestows and contextual patterns of translanguaging. The findings indicate that further research is required, and teacher education and dual-language programs need to approach translanguaging from a new perspective.

## 2 Teachers' beliefs and responses to student translanguaging in dual language mathematics classrooms

### 2.1 Abstract

The recent theoretical debate over the role of translanguaging in dual language programs can be described as moving from prohibition to promotion. Indeed, many researchers, theorists, and educators are now encouraging translanguaging. At the center of the debate are teachers and their beliefs, which are argued to influence their practice. The present study explored 14 elementary dual language teachers' beliefs and responses to students' translanguaging as well as the practice in mathematics across two states and seven different school districts. The study design aimed to reduce inconsistencies found between beliefs and practices in previous studies and broaden understanding of translanguaging in lesser studied contexts. The findings show that the teachers held principally permissive beliefs and uniformly permitted translanguaging. The student translanguaging served primarily social functions. The results suggest a reappraisal of the practices and purposes of translanguaging, especially as the relate to teacher beliefs.

### 2.2 Introduction

Not surprisingly, teachers' beliefs have been found to influence their interpretations of new information and experiences, as well as pedagogical decisions and actions in the classroom, which then may influence student learning and achievement (Fang, 1996). Therefore, an understanding of teacher beliefs provides insight into how teachers may interpret new information and how that may influence pedagogical decisions and actions. This can be especially revealing when teachers face dilemmas such as whether to prohibit, permit or promote student translanguaging ${ }^{1}$ and what functions the translanguaging serves in their classrooms. These dilemmas about student translanguaging have increased as dual language (DL) programs in the U.S. have become more common (Lindholm Leary, 2013).

Drawing from various authors I define beliefs as time and context specific, conscious or unconscious guides for judgement and action, that draw on previous experiences and understandings (Rokeach, 1968; Schoenfeld, 1998; Skott, 2015). Teacher beliefs can be revealed through the expectations they hold for their students, and their theories surrounding the teaching and learning of a content area (Fang, 1996). Teacher beliefs regarding the role of language or languages in the dual language classroom are particularly complex and researchers have found a wide range of language ideologies and discourses manifest in the classroom both at the macro level of policy and at the micro level of student and teacher talk (e.g., Duran \& Palmer, 2014; Martinez et al.,2015). Despite the recognition that beliefs influence practice (Johnson, 1992), researchers have not found a consistent relationship between the two (Borg, 2003). At times these inconsistencies are due to cognitive dissonance (Pajares, 1992) or entangled domains (Nespor, 1987), such as those found in the dilemmas of translanguaging and the functions it serves. But they are also due to research design. Since inconsistencies have been observed in studies of teacher's beliefs about translanguaging (Martinez et al., 2015; Nava, 2009), this study addresses some of the previous inconsistencies due to research design and elucidates the inconsistencies due to the entangled domains and cognitive dissonance of translanguaging.

Often two types of research designs have been used in studies of teachers' beliefs about translanguaging. The first type of design utilized one measure such as large-scale surveys or interviews without observation of the teachers' practice (ex. Nambisian, 2014; Nava, 2009). However, Fang (1996) argues that inconsistency between studies of teachers' beliefs may result from the use of one measure and therefore indicates that these studies should include multiple measures such as classroom observation, stimulated recall, thinkaloud protocols, and focused interviews. The second type of design used multiple measures, such as surveys or interviews and classroom observations. However, studies in this design typically focus on only a few teachers in a single school/district (ex. Holdway \& Hitchcock, 2018; Martínez et al., 2015; O'Gorman Fazzolari, 2017). Considering the results from Nava's
(2009) study demonstrating the extreme variability in teachers' beliefs about translanguaging among schools in two different regions of one city, it is evident that findings from studies with a limited number of teachers or school sites might be the result of local policies and practices and do not provide a more generalizable understanding of teachers' beliefs in different contexts of influence. In addition, despite researchers exploring translanguaging in DL classrooms (García \& Sylvan, 2011; Gort \& Pontier, 2013; Palmer et. al., 2014), few studies have explored translanguaging in DL mathematics classrooms since most studies are conducted in language arts classrooms (King \& Ridley, 2019; Martínez et al., 2015). Examining teachers' beliefs about student translanguaging in the mathematics classroom can provide additional insights to those found in the language arts classroom of how beliefs and practices may vary across these contexts. Also, the DL context, one in which all participants can translanguage, minimizes the pragmatic opposition to translanguaging when an interlocutor is not bilingual (McCarthy, 2018), which could confound beliefs and practices beyond the phenomenon. However, it is also a linguistically and pedagogically complex context which permits an exploration of the many facets associated with the issue of student translanguaging. Therefore, this study uses multiple measures to examine elementary DL teachers' beliefs about and responses to their students' translanguaging and the functions it serves during mathematics in multiple school districts in California and Texas. This study explores three questions:

- What do elementary dual language teachers believe about their students' translanguaging in mathematics?
- What types of student translanguaging practices do elementary dual language teachers' permit, promote, or prohibit?
- How do teachers' beliefs about student translanguaging align with their responses to and the functions of student translanguaging?


### 2.3 Theoretical Framework

### 2.3.1 The dilemmas of translanguaging in dual language programs

The purposes of DL programs include developing proficiency in both languages, academic performance at or above grade level, and development of positive cross-cultural attitudes and behaviors (Howard et al., 2005). These programs have a dual instructional focus since language and mathematics learning are interrelated and inseparable (Brown, 2002). This dual focus leads to dilemmas surrounding translanguaging including whether to foreground the language or the subject matter content, how to develop the language and the subject matter content, and the social and political implications surrounding the choices a teacher makes regarding translanguaging (Adler, 1998). These dilemmas have led some teachers to hold beliefs supporting a stance of prohibiting translanguaging, and others permitting or promoting it.

Theories from both the field of applied linguistics as well as the field of mathematics education (Martinez et al., 2015) influence instruction in DL programs, which both play particularly important roles in forming teachers' beliefs about translanguaging. For example, in the field of applied linguistics and grounded in the language differentiation model (Volterra \& Taescher, 1978), teachers emphasize the importance of linguistic purism, and language separation (Lindholm-Leary, 2006; Martínez et al., 2015). Therefore, they hold beliefs that translanguaging interferes with language learning and thus should be prohibited in DL schools (de Jong, 2016; Martinez et al., 2015; Nambisan, 2014). Other teachers believe that the target language should be the predominant language of interaction and viewed translanguaging as unfortunate but necessary because the ideal conditions that would allow $100 \%$ use of the target language do not exist because of various factors (e.g., learner language proficiency, time pressures, etc.) (Macaro, 2005; O’Gorman Fazzolari, 2017; Setati et al., 2002). Therefore, they permit translanguaging even though they believe it should be prohibited. Strict language separation policies have also been critiqued based on empirical
evidence demonstrating the value of using the first language for second language acquisition (Moore, 2013), literacy skill development (Martínez-Álvarez, 2017) and content learning (Alvarez, 2012) leading to beliefs permitting or promoting translanguaging (Adler, 1998; Nambisan, 2014).

In the field of mathematics education, a growing body of educational research recommends the use of students' home languages and experiences to mediate conceptual mathematical discussions leading to beliefs permitting or promoting translanguaging (Adler, 1998; Celedón-Patichis, 2003; Khisty \& Chval, 2002; Moschkovich, 2007; Setati, 2005).

However, mastery of the specialized words, symbols, math register, discourse, and modes of argument are considered key elements of math competence and school success leading to beliefs prohibiting translanguaging (Celedón-Pattichis et al., 2010; Hansen-Thomas, 2009; Martínez et al., 2015). Teachers in DL programs, their beliefs and their practices are caught in the middle of the theoretical debates from both the applied linguistics and mathematics education fields surrounding translanguaging pedagogy (de Jong et al., 2019).

### 2.3.2 Translanguaging Functions

Translanguaging has been found to serve many functions in the classroom, including academic and social functions. Academic functions comprise making connections between informal language and formal mathematical procedures, concepts, and vocabulary (Cervetti et al., 2015; Moschkovich, 2002; Planas \& Setati, 2009). Translanguaging also connects vocabulary to students' daily life or home language, as well as when students are unfamiliar with non-mathematical vocabulary (Cervetti et al., 2015; Hansen-Thomas, 2009; Khisty \& Chval, 2002; Setati \& Barwell, 2006). Translanguaging's academic functions serve to clarify meaning, reach content-related course goals, and allow student participation (Allard, 2017).

In addition, translanguaging serves social functions such as building students' confidence (Clarkson, 2007; Domínguez, 2011; Planas \& Civil, 2013). Translanguaging also shifts voices for different audiences, and communicates subtle nuances of meaning (Martínez, 2009). For
example, in interpersonal relationships, it negotiates social distance and ingroup solidarity by accommodating to the expectation of others (Lin, 2013) and functions to achieve solidarity with those one likes or dissociate with those who they do not like or do not want to be liked by (Myers-Scotton, 2006). As such, La Page (1997) argues that someone does not necessarily adapt to the interlocutor, but to their self-image in relation to the interlocutor. The literature reporting teachers' beliefs, as well as the theoretical framework for permitting or promoting student translanguaging predominantly recognizes the academic functions over the social functions but does not explore the frequency of use for each one.

### 2.4 Methodology

### 2.4.1 Setting and Participants

This study included data from elementary teachers and students in Spanish-English DL schools in urban school districts in California and Texas. The teachers and some data were drawn from a larger study ${ }^{2}$. Thirty-two teachers participated in the larger study during the 2018-19 and 2019-20 school years with three teachers participating in both years. From this group of 32 teachers, 14 teachers were selected to explore their beliefs through semistructured interviews (see Table 1). These teachers were selected because they represented the greatest variability between interviewees based on the following criteria: grade, state, school district, school, years teaching and translanguaging rate.

## Table 1: Teachers Interviewed

| Pseudonym | Grade | State | Years Teaching |
| :--- | :---: | :---: | :---: |
| Anita | K | CA | 5 |
| Alondra | K | CA | 4 |
| Karime | 1 | CA | 3 |
| Maritza | 1 | CA | 5 |
| Yasmin | 1 | CA | 1 |
| Jessica | 2 | CA | 17 |
| Beatrix | 3 | TX | 15 |
| Ivana | 3 | CA | 1 |
| Liliana | 3 | TX | 14 |
| Marisol | 3 | TX | 10 |


| Valentina | 3 | TX | 15 |
| :--- | :---: | :---: | :---: |
| Solomon | 4 | TX | 20 |
| Graciela | 4 | TX | 5 |
| Irene | 5 | CA | 23 |

### 2.4.2 Data collection

The data included surveys, lesson videos, and semi-structured interviews. The content used from the survey included questions about the teacher background such as, years of experience, professional development experience, educational experience, and Spanish/English language proficiency. The videos ranged from 10 to 60 minutes per teacher. Following the approach taken by Brevik (2020), I only focused on those translanguaging instances recorded by the teachers' microphone. Although undoubtedly translanguaging took place in the classrooms which was not recorded by the microphones, those instances that were recorded were within the hearing of the teacher and therefore were those in which a choice was made by the teacher about how to respond to the translanguaging. The semi structured interview questions explored the teachers' perspectives of translanguaging in mathematics, and contextual and experiential factors that influenced their beliefs (see Appendix A). In addition, during the interview I asked the teachers to engage in stimulated recall (Lyle, 2003) by having them watch clips of translanguaging from their videos. This emic approach served to gain an understanding of their beliefs and interpretations of translanguaging instances in their classrooms. It also provided pertinent information about the students engaging in the translanguaging such as their language and math proficiencies. Finally, I was able to triangulate the coding of the instances through the teachers' responses to the stimulated recall.

### 2.4.3 Data analysis

Drawing on and adapting Myers-Scotton's Matrix Language Framework (2006) to identify each translanguaging instance, a psycholinguistic approach was used whereby the
language of instruction during the mathematics class was considered the Matrix language and the translanguaging instance was described as beginning with the use of an Embedded language and ending when the speaker returned to the Matrix language for a complete sentence. The quantitative coding of the videos required two rounds of coding (see Appendix B). The first round of coding aimed at identifying the translanguaging instances and the context. The second round of coding utilized codes developed deductively from previous literature and included functions (Brevik 2020; Gort \& Pontier, 2013; Reyes, 2004) and types (Gort \& Pontier, 2013; Merritt et al., 1992; Setati, 1998) of translanguaging as well as mathematical problem type and topic (NCTM Content Standards, n.d.). Codes for the semistructured interviews (see Appendix D) with the teachers were developed deductively initially based on findings from previous literature (Dopke, 1992; McMillan \& Rivers, 2011;

Nambisian, 2014; Setati et al., 2002) that were then adapted and expanded upon through an iterative inductive approach. Ten percent of the video data and semi-structured interviews was also coded by a colleague to establish agreement of the coding. In the coding of the video data, interrater agreement scores for the functions and type codes were established between $r=.92$ and $r=1.0$. In the coding of the semi-structured interviews, the interrater agreement score was $r=.80$. The scope of this paper only explored in greater detail those beliefs that were expressed by at least one third of the teachers. SPSS and $R$ were used to conduct all quantitative analysis to examine measures of central tendency.

### 2.5 Findings

### 2.5.1 Teachers' beliefs about student translanguaging

The teachers expressed beliefs associated with either academic functions or social functions, both permitting and prohibiting student translanguaging (See Table 2). A total of twenty-six different beliefs permitting student translanguaging, and fifteen different beliefs prohibiting student translanguaging were expressed. Of the six beliefs permitting student translanguaging expressed by one-third or more of the teachers, four of them were for
academic functions and two of them were for social functions. Of the three beliefs prohibiting student translanguaging expressed by one-third or more of the teachers, two of them were for academic functions and one of them was for social functions.

Table 2: Teacher beliefs about student translanguaging

| Teacher beliefs supporting student translanguaging |  |  |
| :---: | :---: | :---: |
| New Function | Belief | Number of Teachers expressing belief |
| Academic Functions |  |  |
|  | Helpful for limited proficiency learners | 10 |
| * | Helpful to scaffold learning and support thought process/idea development | 8 |
| * | Is ok because students are using previous language of learning | 5 |
| * | Increases student participation in mathematical discourse | 5 |
|  | Facilitates and ensures successful communication with the teacher or the whole class | 4 |
|  | L1 can serve as an informal needs analysis | 4 |
| * | Aids in transfer/bridging to prepare for future grade level or assessment | 4 |
|  | Helpful during certain stages of the lesson | 3 |
| * | Support teacher not breaking language of instruction | 3 |
| * | Is ok because this is not a language class | 2 |
|  | Helpful when students provide peer assistance | 2 |
| * | There is insufficient time to allow use of language of instruction | 1 |
| Social Functions |  |  |
| * | Helpful to build students' confidence or to support their comfort in the class | 9 |
| * | Students are adapting to their interlocutor | 5 |
|  | Is closer to "real world" communication | 4 |
| * | Helpful to reduce student frustration | 3 |
| * | Is ok if it is a non-academic time | 2 |
| * | Is easier for student to use than the language of instruction | 2 |
|  | Issues of identity in translanguaging | 1 |
| * | Translanguaging shows special skills |  |
| * | Translanguaging permissible because English is the dominant language of the community | 1 |
| * | Students are just resisting the language policy | 1 |
| * | Students are not encouraged/motivated to maintain the language of instruction | 1 |
| * | Translanguaging serves to resist the power of the dominant language | 1 |


| * | Students translanguage because they like to talk a lot | 1 |
| :---: | :---: | :---: |
| Teacher beliefs opposing student translanguaging |  |  |
| Academic Functions |  |  |
|  | Students should speak and think in language of instruction only in academic contexts | 6 |
|  | Teachers need to push learners' reception and production of mathematics in language of instruction | 5 |
|  | Students in grade level are up to the challenge of using language of instruction only | 3 |
|  | Target language exclusivity promotes negotiation and use of communication strategies | 2 |
|  | Should aim for an immersion type experience | 1 |
|  | Translanguaging shouldn't happen but they have no choice because of the need to prepare for standardized exams | 1 |
| * | Translanguaging reduces students' linguistic capacity in both languages | 1 |
| * | If language of instruction is students' dominant language, then they should not translanguage | 1 |
|  | Students have been "sold" on monolingual language use | 1 |
| Social Functions |  |  |
|  | Translanguaging due to laziness or connected with off-task behavior | 5 |
| * | Should push back against the dominant language | 2 |
| * | Shouldn't happen but students shouldn't be punished for it | 2 |
| * | Translanguaging is students' default response |  |
| * | Translanguaging shouldn't happen but it does because of the age/maturity of the students | 1 |
| * | Kids should be proud of speaking another language and accept being outside of their comfort zone | 1 |

### 2.5.2 Beliefs permitting student translanguaging for academic functions

I observed some examples of translanguaging for academic functions in Marisol's videos. She recorded different lessons throughout the year unlike the other teachers.

Therefore, they provide an opportunity to observe the development of a student, Sammy.
Marisol elaborated on two different examples with Sammy from the videos. In the first example the students were reviewing the addition of three-digit numbers.

## Example 1:

Teacher: Ahí va otro. [Here is another one] [Teacher writes $380+519$ on the board].
Sammy: Eight hundred and ninety-nine.

Teacher: Ok Sammy, ven, ¿qué estrategia utilizaste? [come here, what strategy did you use?] [Sammy goes to front of room and starts speaking quietly and nervously]
Sammy: Usé [l used] [Sammy pauses as if not sure how to explain]
Teacher: Bien fuerte por favor. [Nice and loud please].
Sammy: Usé [I used] number line para mi respuesta [for my answer].
Teacher: Usaste el number line. ¿Cómo? [You used the number line. How?]
Sammy: Primero puse... [First, I put...] [Student pauses]
Teacher: Házmelo [Do it for me].
Sammy: [Sammy starts drawing a number line but still seems unsure how to draw it and is muttering to himself]. And then um, five hundred and nineteen con [with] three hundred and eighty.
Teacher: Sammy, dime la estrategia que utilizaste verdaderamente [Sammy tell me the strategy you really used]. [Teacher pauses and waits for answer, but Sammy doesn't say anything]
Teacher: ¿Descompusiste el número? [Did you decompose the number?]
Sammy: No.
Teacher: Ok dime ¿qué estrategia utilizaste realmente? [Ok, tell me, what strategy did you really use?].
Sammy: I um did this, [Sammy continues drawing number line with one arch going from 380 to 899 with 519 written at the top of the arch.] and then I added those five hundred and nineteen.

Sammy was struggling with understanding the concept of how to use a number line to add numbers, based on his drawing, as well as struggling with explaining it in the language of instruction (Spanish). In the second example, two months later, the students were just beginning a unit about fractions.

## Example 2

Teacher: ¿Quién tiene otra manera de hacer un entero? [Who has another way to make a whole?] [Sammy raises his hand] Tu Sammy a ver. [You Sammy, let's see].
Sammy: You can use eight and put it, put it...oh never mind. [Sammy starts to explain but seems unsure of himself]
Teacher: Eh, no enseñanos con otra fracción [Eh, no, show us with another fraction].
Sammy: You can do it with sixths.
Teacher: A ver, ¿cuántos necesitas para hacer un entero? [Let's see, how many would you need to make a whole?]
Sammy: Seis. [Six]
Teacher: Seguro? [Are you sure?]
Sammy: Si. [Yes]
Teacher: A ver, ¿lo puedes probar? [Let's see, can you prove it?] [Sammy lines up paper sixths fractions on the whole fraction.]
Teacher: ¿Por qué piensas que tu usaste seis y ella uso ocho para hacer un entero? [Why do you think you used six and she used eight to make a whole?]
Sammy: Porque los mismos son, lo mismo hacen un entero. Entonces um, [Because the same are, the same make a whole. So um]

Teacher: Pero tu usaste seis partes y Julia uso ocho. ¿Por qué? [But you used six parts and Julia used eight. Why?]
Sammy: Porque, nosotros podríamos hacer con otro fracción, no solo con un oc...oc... [Because we could have made with another fraction, not only with an ei...ei...]
Teacher: Octavo. [Eighth]
Sammy: Octavo. [Eighth]
Teacher: Ah, ¿tú cuál usaste? [Ah and which did you use?]
Sammy: Un sexto. [A sixth.]
Teacher: ¿Y necesitas menos sextos para tener un entero? [And do you need less sixths to make a whole?] [Sammy shakes head 'no' with a confused look.]
Teacher: ¿Necesitas ocho octavos para tener un entero? [Do you need eight eighths to make a whole?] [Sammy nods head 'yes'.] ¿Cuántos sextos para tener un entero? [How many sixths to make a whole?]
Sammy: Seis. [Six]
Teacher: Seis sextos. [Six sixths]
Sammy: Si. [Yes]
Teacher: ¿Por qué a ella ocho y tu seis? [Why did she need eight and you need six?]
Sammy: Porque um...un seis hace un entero. Entonces um, podemos, podemos poner todos, todos los sextos con, en un entero $y$, también podemos, o si, también podemos poner un ocho en cima del seis para ver que, cuál es, cuál hace el entero. [Because um...a six makes a whole. So um, we can, we can put all, all the sixths with, in a whole, also we can, or if, also we can put an eight on top of the six to see that, which is, which makes the whole].
Teacher: A ver, inténtalo. [Let's see, try it.] [Sammy starts lining up the $1 / 8$ pieces on top of the $1 / 6$ pieces which are on top of the whole piece.]

In this second example, Sammy demonstrated a greater understanding of the mathematical concept than in the first example, and his Spanish proficiency had increased, but as before, the teacher permitted him to translanguage. After a few turns Sammy continued solely in the language of instruction ending with an extended turn in which he made a valid mathematical argument despite his limitations in the language. Across these two examples the teacher always permitted Sammy to use the language as he chose, with the result being Sammy's ability to understand the content and participate in the mathematical discourse.

Marisol expressed during the stimulated recall of Example 1 with Sammy that she permitted Sammy to translanguage due to both his limited linguistic and mathematical proficiency. This permitted him to draw on his complete linguistic repertoire over an extended exchange to explain his thinking. Ten out of the fourteen teachers echoed Marisol's belief that translanguaging is helpful to support 'lower' proficiency learners' understanding of the
content, and that need trumped the language policy of the program. This was the most commonly cited belief permitting student translanguaging. For example, Solomon shared that he tries to identify Spanish dominant students with ‘lower’ English proficiency and in smaller targeted groups provide support in Spanish. He justified doing this since "you want to have fidelity to the DL program, but then at the bottom line is, you know, is it serving that kid?" Likewise, Anita noted that translanguaging benefitted her students when they were less proficient in the mathematics topic they were working with, "like the subtraction, they would lean on their stronger language". The teachers referred to both linguistic and mathematic proficiency when seeing the benefit of translanguaging for these students.

Example 1 also demonstrates the second most common belief permitting student translanguaging for academic functions. In turn 10 of Example 1, Sammy begins drawing the number line and talking to himself, as he does so, to scaffold his thought process. Eight teachers expressed the belief that student translanguaging is helpful to scaffold learning and support the thought process/idea development. Another example came from Beatrix who was explaining about a student in her video who was working on solving a two-digit multiplication problem at her desk and talking to herself in English when the language of instruction was Spanish. While she was working out how to solve the problem she was saying "Now it should be eight times ten. Eight times ten is eighty, right? And eight times six?..." Beatrix explained,
"y como ahorita está quebrando el número, creo para poder resolver el problema. Pero lo estaba hablando en inglés, en su propio idioma, para que era mas fácil para ella." llike now, she is decomposing the number, I think to be able to solve the problem, but she is doing it in English, in her own language, so that it is easier for her.].

Beatrix indicated that she believed the student was scaffolding her own thoughts with translanguaging self-talk while she was solving the problem.

These examples with Sammy also demonstrate how through translanguaging Sammy was able to participate in mathematical discourse. The belief that translanguaging
allows students to participate in the mathematical discourse was expressed by five teachers.
For example, Valentina described one of her students saying,
"So, if I don't let her, like I give her extra time too, like she tries Spanish, but then she uses some words in English and I have to let her if not, she won't participate. Like they stay there quiet. Like they won't raise their hand, or they won't even try."

In Example 2, Sammy initially had a similar response and was somewhat unsure of his participation when he said, "oh never mind". But then with a little encouragement from the teacher and permission to translanguage, Sammy goes on to provide a valid argument and participate in the mathematical discourse.

Finally, while not evident in the examples with Sammy, five teachers expressed that they believed students translanguaged, in the DL context particularly, when they were talking about content that they had learned in the other target language. For example, in Ivana's class they were doing a warmup in Spanish which recycles previous content learned in English. A native Spanish speaking student was solving a problem on the board and talking to himself in English. When asked why a native Spanish speaker would be speaking English when the language of instruction was Spanish, Ivana said she believed he was speaking English because it was "the way Ms. [Teacher's last name] would have shown him in her explicit teaching." Another example in Beatrix's class involved a native Spanish speaker doing multiplication in English with unique hand signals and speaking with singsong English. Beatrix explained that she believed that the student was doing this because of a song that a teacher in first grade had taught her in English which was about multiplication. This belief was only expressed by teachers in $3^{\text {rd }}$ through $5^{\text {th }}$ grade, which are the grades where the transition from teaching math in Spanish to teaching math in English takes place. This reveals these teachers' awareness of a unique situation their students experienced that teachers in the lower grades may not necessarily experience with their students.

### 2.5.3 Beliefs permitting student translanguaging for social functions

The teachers while holding beliefs permitting translanguaging for academic functions also expressed beliefs permitting translanguaging for social functions. Nine teachers expressed the most commonly held belief that translanguaging is helpful to build students' confidence or to support their comfort in the class. In Example 2, Sammy was not confident about participating at the beginning of the example but Marisol permitted him to translanguage to express his ideas. By the end of the example Sammy confidently made a mathematical argument in front of the whole class. Marisol reflected on her response to Sammy,
> "Aunque yo les enseño en español, él sabe que puede tener la confianza de explicarlo en inglés y poquito a poquito empezó a usar el español. Y sí, cambia el comportamiento, es increíble pero sí cambia cuando les damos esa confianza." [Although I teach them in Spanish, he knows that he can feel free to explain in English and little by little he started to use Spanish. And the behavior changed, it is incredible, but it changes when we give them that confidence.]

Marisol clearly believes that by permitting Sammy to translanguage, she gave him more confidence and as a result he started trying to use Spanish more.

While the teachers expressed that they permitted translanguaging to support students with 'lower' linguistic proficiency to understand the math content, 37\% of the student translanguaging instances containing mathematics content in the classrooms of the teachers that expressed this belief did not align with the belief. This observation led to using stimulated recall of these unaligned instances and asking the teachers why they think their beliefs did not align with many of the translanguaging instances in their classrooms. Initially most teachers were surprised by this observation and were unsure how to explain it, but after thinking about it they provided a couple different explanations. One explanation was that the students were trying to adapt to their interlocutors' perceived limited proficiency in the language of instruction by using that student's dominant language. For example, Maritza replied, "I, that's a great question. I don't know, um. I'm trying to think, I'm trying to even think
like maybe did I, like who they sit with". Valentina further explained, " they're trying to help the English [speaking] students".

Another explanation was that the students were wanting to prove their linguistic skills in the other target language (English), the language of power. For example, in Irene's class (LOI = Spanish) the students were working on the floor doing calculations on their white boards, and Irene reminds the students to put commas in their numbers. Ivana, a student that Irene indicated had high proficiency in the LOI (Spanish) and low proficiency in the other target language (English) and also had low math proficiency turned to her friend, Pia, who Irene indicated had high proficiency in both languages as well as math and says, "one, two, three" and points to where she is missing commas. So, Ivana was choosing to translanguage from the language of instruction in which she had high proficiency to a language in which she had lower proficiency, despite her friend having high proficiency in the LOI to correct her mistakes in math. When asked about this, Irene responded,
"I don't know, maybe because she's an EL and she wants to prove that she can speak English...Some of the other kids, I think that that's what happens, that they're trying to prove, like, "Oh, I can speak in English just like everybody else."

Irene indicates that the motivation for the student translanguaging was to gain symbolic power using the language of power. Regardless of whether the students were translanguaging in these exceptions for the first or the second reason, both are for social functions.

The teachers when presented with evidence from their classrooms indicating practices that differed from their beliefs accepted the observations and reevaluated their beliefs. They are seen shifting their beliefs in the interview. Some teachers also acknowledged in the interviews a shift from the beliefs they held about student translanguaging when they completed their teacher credential program. For example, Irene one of the more experienced teachers shared how her beliefs have changed from when she first became a teacher,
"Well at first when I became a bilingual I teacher, I was taught that you know, stay in one language so that the kids don't do the Spanglish. So, they need to learn formal Spanish or formal English. And so, I was more strict as far as staying in one language."

She went on to share that since then her beliefs have changed due to her experience working with students and she now sees the value of permitting student translanguaging. In addition to teachers' experience working with students leading to a shift in their beliefs, conversations with their colleagues in their schools have led to a shift in their beliefs. For example, Jessica explained how her beliefs recently shifted because of conversations with colleagues,
"actually, this year we had a great discussion about it because we used to be very strict that when it is Spanish time only Spanish, and when it is English time only English. But then we were discussing, some, in a staff meeting that it is also important for the student, while they are working with friends, the classmates, to switch languages because that is the way you can support their understanding. And then we came to a kinda an agreement that sometimes we will let them if we see that the conversation is academic."

Despite previous research indicating DL teachers' beliefs prohibiting translanguaging, in this study the teachers expressed many more beliefs permitting translanguaging for academic and social functions than prohibiting it and indicated that their beliefs are shifting towards permitting translanguaging and particularly for social functions.

### 2.5.4 Beliefs prohibiting student translanguaging for academic functions

While a shift is seen in teachers' beliefs towards permitting translanguaging, they also expressed a few beliefs prohibiting student translanguaging. One belief for prohibiting student translanguaging for academic functions, expressed by six teachers, was that students should speak and think in the language of instruction only in academic contexts. They expressed that at the beginning of the year they were more flexible but as the year progressed, they had an expectation for students to only use the language of instruction. In addition, they shared that they expected students to use the language of instruction particularly during whole class time. Third, they explained that once students have been taught new content or vocabulary, the expectation is that they use only the language of
instruction. These expectations were held for a couple reasons. For example, one reason was related to expectations of what the student needed to be able to do. Graciela shared, "they need to use it because that's the academic vocabulary that is going to be on the paper, if they don't use it in the classroom, they are not going to understand." The other reason was related to expectations of how the teacher ought to teach. Yasmin explained, "we should provide them the tools to be able to express themselves fluently in one language." So, the prohibition of student translanguaging rested on expectations for not only the student but also the teacher.

Another belief for prohibiting student translanguaging for academic functions, expressed by five teachers, was because teachers felt the need to push learners' reception and production of mathematics in the language of instruction. For example, Yasmin explained, "there are some students who just say, who just don't want to practice it, but I know that they can do it, so then I push them." Jessica recognized, "we as a teacher need to push them to try and to learn in the language that is hard for them." The teachers expressed both a recognition of the students' ability to use the language of instruction as well as the challenge it may be for students to do so when they pushed them.

### 2.5.5 Beliefs prohibiting student translanguaging for social functions

Teachers also expressed beliefs prohibiting student translanguaging for social functions. Five teachers indicated that students translanguage due to laziness or connected with off-task behavior. For example, Jessica was explaining the translanguaging of the English dominant students. She said,
"when you want them to be, learn how to explain in Spanish, they get kinda lazy or just say in English. So, with those ones I am kinda more strict and say, 'You already said it in English, now you need to try to explain it in Spanish.'"

Here Jessica indicates that translanguaging indicates a socially inappropriate or undesirable attitude in the classroom and that students should adopt a different attitude of making an
effort to express themselves in the language of instruction. In both the academic functions and the social functions, teachers' beliefs lead them to prohibit translanguaging only if the student has the linguistic proficiency to participate using solely the language of instruction.

### 2.5.6 Teacher responses to student translanguaging

A total of 206 instances were observed of student translanguaging with two teachers' videos having no instances of translanguaging. The teachers prohibited student translanguaging in 6 instances. The teachers permitted student translanguaging in 197 instances and promoted it in 3 instances. As such the percentage of student translanguaging that was permitted or promoted was 33 times the percentage of student translanguaging that was prohibited. The students' translanguaging included mathematical content in 95 instances (46\%) of which the teachers permitted it in 90 instances and promoted it in 2 instances and prohibited it in 3 instances. The students' translanguaging did not include mathematical content in 109 instances (53\%) of which the teachers permitted it in 105 instances, promoted it in 1 instance and prohibited it in 3 instances. These instances typically involved small talk among students while they discussed topics such as their free time interests, preferences, lunch etc. A two-sided hypothesis test revealed that there is no difference in the teachers' responses to student translanguaging relative to it containing mathematical content $\left(\mathrm{H}_{0}: \pi_{1}=\right.$ $\pi_{2}, \mathrm{p}=0.87$ ). The translanguaging functions in instances with mathematical content revealed 68 instances for scaffolding (72\%), 5 instances for task instruction (5.3\%), 24 instances for math terminology (25.3\%), 5 instances for schema (5.3\%), 44 instances for assessment (46.3\%), 5 instances for clarifying questions (5.3\%), and 7 instances for clarifying answers (7.4\%). The translanguaging with mathematical content occurred in 5 different math problem types: 12 instances (13\%) were during word problems, 52 instances ( $55 \%$ ) were during arithmetic calculations, 8 instances ( $8 \%$ ) were with geometry, 5 instances ( $5 \%$ ) were with
measurement, 7 instances ( $7 \%$ ) were with comparing numbers, and 11 instances ( $12 \%$ ) of counting.

In three of the instances of teachers prohibiting translanguaging the students were engaged in small talk. The three instances of teachers' prohibiting translanguaging which included mathematical content occurred when the function was scaffolding, and the problem type was arithmetic calculations. As an example of this type of prohibition, the following exchange took place in a first-grade class where the language of instruction was Spanish,

## Example 3

Student: Tres por cinco igual a fifteen [Three times five equals fifteen].
Teacher: ¿Cómo se dice? [How do you say it?]
Student: Quince. [Fifteen.]

Like in this instance, the teachers did not outright say to not translanguage, but requested the students rephrase what they said in the target language. I all the cases the students were able to immediately rephrase what they said without support from the teacher or any other students.

Since over half of the translanguaging instances did not contain mathematics content, they were not serving academic functions but rather social functions. In the case of the translanguaging instances that contained mathematics content that the teachers permitted, the majority served to scaffold student learning. However, closer examination of those instances containing mathematical content revealed that the students engaging in the translanguaging often had high language proficiency in the language of instruction and thus the translanguaging was not allowing students with limited language proficiency to use their full linguistic repertoire and therefore engage with the mathematics. So, despite the arguments for a greater use of translanguaging to support students academically in the classroom by making connections between informal language and formal mathematical procedures, concepts, and vocabulary (Cervetti et al., 2015; Moschkovich, 2002; Planas \& Setati, 2009), which was observed in a few instances in this study, the findings demonstrate
that the great majority of the translanguaging was not reflective of this function and instead it served social functions.

Two principal social functions were observed. In some instances, the translanguaging served to build students' confidence. This function was observed particularly when the student had limited language proficiency like those observed with Sammy. Another social function was observed in other instances where some teachers indicated that the students were adapting to their interlocutor, however the findings indicate that the interlocutors had a high proficiency in the language of instruction. In addition, the teachers indicated that the student who was translanguaging had a low language proficiency in the other target language. Both observations seem to indicate that the translanguaging was therefore not serving the function of adapting to their interlocutor. Instead, it seems the students who were translanguaging were adapting to their self-image in relation to their interlocutor as argued by La Page (1997) by accommodating to their expectations (Lin, 2013) or to achieve solidarity with their classmates (Myers-Scotton, 2006). Also, this always took place when translanguaging to English, a higher status language. Therefore, it the students were trying to demonstrate their competence in the higher status language, as was also found by Planas and Setati (2009). The use of translanguaging bestowed symbolic power and linguistic capital on the students and solidarity with their interlocutor as Setati (2008) indicated. As such, promoting translanguaging to the home language may not be well received by students and be unproductive in accomplishing the academic functions it aims to accomplish (Allard, 2017).

In the two instances of the teachers promoting translanguaging they seemed to be trying to help the students engage more deeply with the content. In one instance Anita, a kindergarten teacher ( $\mathrm{LOI}=$ Spanish), was working on subtraction with the students and having them count in Chinese to find the difference.

## Example 4

Teacher: Vamos a contar en Chino. [Let's count in Chinese.]

All: yī, èr, sān [One, two, three.]
Teacher: ¿Cuántos les sobra? [How many do you have left?]
Students: Sān. [Three.]
Teacher: ¿Sān en español es? [Sān in Spanish is?]
Students: Tres. [Three.]
She explained, "I feel that they kind of get in the routine of counting in Spanish and it's more of like memorizing. So, I use Chinese to like kind of help them make that connection [between the word and the number of objects]". The other instance took place when Marisol was asking the student to elaborate on an explanation of how to solve a three-digit multiplication problem ( $\mathrm{LOI}=$ Spanish ).

## Example 5

Teacher: How did you figure out where to put the number when you multiplied the hundreds, by the two hundreds? When you multiplied by the two, the last one.
Student: When I multiplied two times two...
Teacher: Uh-huh, why did you put the four under the hundreds?
Student: Because if I take away these...if I take away those it would be two hundred times two. And since it is in the hundreds place, I put it in the hundreds.

The teacher by asking the student for the elaboration in English indicated a shift in the language they would be using to communicate and therefore promoted student translanguaging. Karime also shared that her school has a time set aside for translanguaging, called bridge time, but she was not observed using that time for translanguaging. She explained why, "So technically there is 15 minutes between the end of our math block dedicated for bridge time, but I honestly don't know how to approach that. They haven't given us guidance as to how that looks." So, despite beliefs supportive of translanguaging and a willingness to promote it, at least this teacher did not know how to actually engage in and promote translanguaging during this time.

### 2.6 Conclusion

This study provides insight into dual language mathematics teachers' beliefs, and responses to student translanguaging and the functions that it serves. The study of teacher
beliefs in this linguistically and pedagogically complex environment where the cognitive challenges in the classroom have grown because of new mathematics standards reveal the dilemmas that teachers face and how they respond to those dilemmas. The findings amend previous research and indicate two shifts in DL teacher beliefs as well as a shift in teachers' responses to student translanguaging, both expressed by the teachers and observed in the data. A shift was observed from previously expressed beliefs and responses prohibiting student translanguaging (Sommerville \& Faltis, 2019) to permitting it in this study. A strong consensus was also observed among the teachers regarding their beliefs for permitting student translanguaging contrary to that which was found previously (Nava, 2009). Secondly, a shift was observed in the functions that teachers believe student translanguaging serves. The teachers recognized shifts in their beliefs away from academic functions and to social functions due to their interactions in the classroom and discussions with their colleagues. In addition, the study indicates that teachers' interpretations and beliefs about student translanguaging are confounded by this complex environment and when given the opportunity to analyze student translanguaging they adapted their beliefs to be reflective of the practice they observed in their analysis. Finally, despite the preponderance of literature promoting translanguaging, the findings demonstrate a lack of strategies for promoting translanguaging in the dual language mathematics classroom both expressed by the teachers and observed in the data.

These shifts are important for a couple reasons. First, they indicate that the previous research regarding teacher beliefs about student translanguaging needs to be updated. Secondly, it shows that the driving force of these shifts seems to come from the teachers' actual experiences in the schools rather than from theoretical stances held by the fields of second language acquisition or mathematics education. This should be taken into consideration by teacher education programs as they address the use of translanguaging in coursework. However, these findings also have several limitations. This study only explored Spanish/English dual language classrooms which place different demands on teachers than
in other programs. However, this context provides the opportunity to explore this complex linguistic and pedagogical environment and provides an opportunity to explore the dilemmas of student translanguaging in a context where all can participate in translanguaging. In addition, this study explored the teachers' beliefs and practices only in mathematics. However, this was by design and offered an opportunity to gain an understanding of beliefs and practices in a subject largely unstudied. Finally, the self-reported data in the study (e.g., teachers reporting on their own beliefs) must be interpreted with caution. However, the triangulation of analyses across different data sources, as well as addressing the inconsistencies found between data sources suggests that the data provides an accurate representation of the beliefs and practices in these classrooms.

These findings have implications for future research, teacher education programs, and schools. First, researchers should explore not only which functions translanguaging is serving in the classrooms but also the frequency of use of the different functions. In addition, future research would benefit from rigorous research of teacher beliefs which is descriptive rather than prescriptive in nature and focuses particularly on the social functions of student translanguaging and how teachers are developing their beliefs about translanguaging. Secondly, teacher education programs should not only help teachers develop beliefs that demonstrate an understanding of the academic and social functions of translanguaging, but also recognize the frequency of use of the different functions. Considering the findings, this would best be accomplished through analysis of translanguaging practices in their assigned classrooms and discussions with other teachers in the school. In addition, if teachers are to promote student translanguaging, they would likely benefit from exploring specific strategies or clear models of how to use and promote it in the dual language mathematics classroom. Finally, DL schools should not only recognize the socialization that takes place through translanguaging but also acknowledge that translanguaging is taking place despite their policies. They should therefore establish policies that reflective of the functions that translanguaging serves in the classroom. While schools play an important role academically,
the socialization that takes place through translanguaging in the classroom is a vital aspect of the schooling experience and each child's linguistic and social development.

Footnote:

1. The use of more than one language between bilingual speakers in interactive speech is referred to through a variety of terms of which "code-switching" or "translanguaging" are used most often (MacSwan, 2017). While codeswitching views languages as separate systems, translanguaging views a bilingual speakers' full linguistic repertoire as an integrated system (Canagarajah, 2011), and distinct languages as merely socio-political constructions (Makoni and Pennycook, 2007). However, in the exploration of teacher's beliefs and practices in this paper I must acknowledge "the relationships between what people believe about their language (or other people's languages), the situated forms of talk they deploy, and the material effects - social, economic, environmental—of such views and use" (Makoni \& Pennycook, 2007, p. 22). Since the DL programs in this study promoted the separation of languages, and the teachers referred to the languages as distinct, a description of the beliefs and practices necessitates language representative of the socio-political constructions of distinct languages.
2. MALLI, research funded by a grant from the U.S. Department of Education, Office of English Language Acquisition, National Professional Development Program, Grant \#T365Z170070. MALLI is a professional development program that works to integrate mathematics, language, and literacy in dual language settings. It focused on developing discourse, literacy, and vocabulary strategies during mathematics instruction.

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### 2.8 Appendices

### 2.8.1 Appendix A: Interview Protocol

Teacher: $\qquad$
Date:

Thank you so much for your time today. We are investigating mathematical content and language learning through translanguaging in two-way immersion programs, and we appreciate your input and experience. Your interview responses will be shared without using your name, school name, or other identifying information, and we would be happy to share our results with you as well if you'd like.
"Would you mind if I recorded our conversation? (if using phone, turn to airplane mode. Consider backup recording on ipad or laptop in addition to phone.)"

With the recording running: (Researcher Name) interview with (Teacher Name) on (Date).
I want to begin by learning a little bit more about you, then we will talk about your school and teaching, and finally we will look at some clips from your video and talk about those.

1. Tell me about your teaching trajectory. Probes: Where did you do your credential program? What schools have you taught in? What grades? What programs? How long have you been teaching?
2. Tell me about your own language history.

Probes: What do you consider to be your first language? What language do you feel more proficient or comfortable speaking?
3. Tell me about your own schooling/math trajectory.

Probes: Where did you go to elementary, middle, and high school? What language did you learn math in? Was math easy or difficult for you? Why? Describe your math teachers.

## Now I would like to talk about your school and teaching

4. Do you teach math in English or Spanish? How does that change in different grade levels at your school?
5. What is your school language policy?

Probes: What is their policy about language mixing? Is there an official and an unspoken policy? What is it?
6. What is your language policy in your classroom?

Probes: Do you allow your students to mix languages during math? Why? Do you mix languages during math? Why? What do you think will be the main difficulty/difficulties that you would face in trying to use only the language of instruction in a mathematics lesson?
7. Do your students switch between languages during your math classes?

Probes: When (small groups, between peers, for brainstorming, talk about things outside of content)? Why do you think they do it? Could you give me some examples? Do you think that changes when the language of instruction changes?

- Follow up to "Yes": Are there any particular instructional purposes or moments in which you encourage/allow students to mix languages? Could you give me some examples?
Probes: Is there a difference between when you are doing a conceptual and a procedural lesson? What is the greatest motivation for them switching languages cognitive or sociopolitical reasons?

8. Do you ever switch between languages during math?

Probes: When? Why? Do you think that changes when the language of instruction changes?

- Follow up to "Yes": Are there any particular instructional purposes or moments in which you mix languages? Could you give me some examples?
Probes: Is there a difference between when you are doing a conceptual and a procedural lesson? What is the greatest motivation for you switching languages cognitive or sociopolitical reasons?

9. What other PD have you participated in focused on teaching math?

Probes: When? Why? Who did it?

## Now I would like to look at the clips from your video

10. Before we begin can you tell me about this lesson, the learning objective, where it came in the unit sequence etc.

## Watch translanguaging clip. Clips used:

11. Describe what you see happening here.

Probes: What is interesting or stands out to you? What do you wonder about what you watched? Did you notice any language switching? Why do you think this switching is taking place? Did anything you noticed during the lesson cause you to act differently than you had planned?
12. Can you tell me about this student?

Probes: What is this student's linguistic background (Oral, written, informal, academic, math proficiency, other important info to know about the student)? How does students' linguistic background influence the languages of instruction and the mixing of languages?

## Repeat with next clips

This has been extremely valuable, and I have a lot to work with here. Before we wrap up, l'd like to give you a chance to add anything you think is relevant to what we have talked about today.

Thank you so much for sharing your time and insight with me today.
2.8.2 Appendix B: Video coding scheme

| Primary Code (CODE) | Quantitative Description |
| :---: | :---: |
|  | CODING ROUND 1 |
| School Year (YEAR) | $0=2018-19,1=2019-20$ |
| Teacher ID (TID) |  |
| Video \# (VID) |  |
| State (STATE) | 0 = California, 1 = Texas |
| Time Stamp (TIME) |  |
| Duration of instance (in seconds) (DUR) |  |
| Translanguaging Rate (RATE) | Instances/minute |
| Language of Instruction (LOI) | 0 = English, 1 = Spanish |
| School Code (SCHL) | 1-16 |
| School District (DIST) | 1-9 |
| Interviewed (INT) | $0=$ No, $1=$ Yes |
| Translanguager (TLER) | $0=$ Student, 1 = Teacher, $2=$ Both teacher and student $3=$ Multiple Students |
| Grouping (GROUP) | 1= Whole Class, $2=$ Small group, 3 = Alone |
| Teacher response (TRESP) | $0=$ Prohibiting, 1 = Permitting, $2=$ Promoting |
| Grade (GRADE) |  |
| Mathematical Content (MATHCON) | $0=$ No, $1=$ Yes |
| Transcription (TRAN) |  |
|  | CODING ROUND 2 |
| Functions |  |
| Academic -Scaffolding (SCAF) | $0=$ Not present, $1=$ Present - Translanguaging used to offer guidance, explain/expand a teaching point, bridges communication gaps, reduces ambiguity, or offer translation for students' lack of comprehension in the target language. |
| Academic - <br> Metalinguistic Explanation (METAEX) | $0=$ Not present, $1=$ Present - Translanguaging used to focus on linguistic forms through explicit explanations. |
| Academic - Task Instruction (TASK) | $0=$ Not present, $1=$ Present - Translanguaging used to give task instructions for an activity or procedure. |
| Academic Mathematical Terminology (MATHTERM) | $0=$ Not present, $1=$ Present - Translanguaging used to provide new mathematical terminology or vocabulary clarification |
| Academic - Nonmathematical Terminology (XMATHTERM) | $0=$ Not present, 1 = Present - Provide non-mathematical vocabulary that students most likely do not already possess |
| Academic - Other Domains (DOMAINS) | $0=$ Not present, $1=$ Present - Translanguaging is used to refer to another domain about a matter relevant to the target language topic |
| Academic - Schema (SCHEMA) | $0=$ Not present, 1 = Present - Teacher explains something that adds to student's existing knowledge |


| Academic - <br> Assessment (ASSESS) | $0=$ Not present, 1 = Present - Asks a student to demonstrate what they know |
| :---: | :---: |
| Management Classroom Management (MANAGE) | $0=$ Not present, $1=$ Present - Translanguaging is used to manage students' behavior in the classroom, lack of student concentration, talk, or misconduct |
| Communicative - Affirm (AFFIRM) | $0=$ Not present, 1 = Present - Affirm a speaker's statement or provide positive reinforcement |
| Communicative Clarify - Ask question to clarify what was said (CLARQ) | $0=$ Not present, 1 = Present - Listener doesn't hear or understand what the speaker said, or listener isn't sure he/she has understood the information correctly and is checking her understanding of what the speaker said. |
| Communicative Clarify - Answer question (CLARA) | $0=$ Not present, $1=$ Present - Responds to a question indicating a misunderstanding of the information |
| Mathematics |  |
| Problem Type (PROB) | Word problem $=1$, Arithmetic Calculation $=2$ Geometry $=3$, Measurement $=4$, Comparing numbers $=5,6=$ Counting |

### 2.8.3 Appendix C: Interview coding scheme

| Code Category | Subcategory (CODE) | Quantitative Description |
| :---: | :---: | :---: |
| Student <br> Background | Student Name (SNAME) |  |
|  | Student Home Language (SHOMEL) | $\begin{aligned} & 0=\text { English, } 1 \text { = Spanish, } 2 \\ & =\text { Other } \end{aligned}$ |
|  | Student Math Proficiency (SMAP) | $\begin{aligned} & 1 \text { = Low, } 2 \text { = Average, } 3= \\ & \text { High } \end{aligned}$ |
|  | Student Spanish Language Proficiency (SSLP) | $\begin{aligned} & 1 \text { = Low, } 2 \text { = Average, } 3= \\ & \text { High } \end{aligned}$ |
|  | Student English Language Proficiency (SELP) | $\begin{aligned} & 1=\text { Low, } 2 \text { = Average, } 3= \\ & \text { High } \end{aligned}$ |
|  | Student Gender (SGEN) | 0 = Male, 1 = Female |
|  | Other |  |
| Teacher Background | Teacher Birthplace (BIRTH) | $0 \text { = U.S., } 1 \text { = Other }$ <br> Spanish speaking country |
|  | K-12 Math Experience |  |
|  | Math PD Attended |  |
| School context | Language Shift (SHIFT) | What grade language of instruction changes in mathematics |
|  | Patterns of Translanguaging (PATTL) | When teacher sees translanguaging happening frequently |
|  | Point in lesson (POINT) | $\begin{aligned} & 1=\text { Beginning, } 2=\text { Middle, } \\ & 3=\text { End, } 4 \text { = Review } \end{aligned}$ |
| Beliefs permitting student Translanguaging | Facilitates and ensures successful communication with the teacher or the whole class | $0=$ No, 1 = Yes |
|  | Helpful when students provide peer assistance | $0=$ No, 1 = Yes |
|  | Helpful for lower-proficiency learners | $0=$ No, $1=$ Yes |
|  | Helpful during certain stages of the lesson (SSS4) | $0=$ No, $1=$ Yes |
|  | L1 can serve as an informal needs analysis | $0=\mathrm{No}, 1=\mathrm{Yes}$ |
|  | Closer to "real world" communication | $0=$ No, $1=$ Yes |
|  | Issues of identity in Translanguaging | $0=$ No, $1=$ Yes |
|  | Students adapting to interlocutor | $0=$ No, $1=$ Yes |
|  | To support teacher not breaking language of instruction | $0=$ No, $1=$ Yes |
|  | Scaffolding learning by using students' dominant language to support thought process/idea development | $0=\mathrm{No}, 1=\mathrm{Yes}$ |
|  | To build students' confidence or support their comfort in the class | $0=$ No, 1 = Yes |
|  | Students are using previous language of learning | $0=$ No, 1 = Yes |
|  | Helpful to aid in transfer/bridging to prepare for future grade or assessment | $0=$ No, $1=$ Yes |


|  | It is a non-academic time | $0=$ No, $1=$ Yes |
| :---: | :---: | :---: |
|  | Helpful to increase student participation in mathematical discourse | $0=\mathrm{No}, 1$ - Yes |
|  | Helpful to reduce frustration | $0=$ No, $1=$ Yes |
|  | It is easier | $0=$ No, $1=$ Yes |
|  | This is not a language class | $0=$ No, $1=$ Yes |
| Beliefs prohibiting student Translanguaging | Due to laziness or connected with offtask behavior | $0=$ No, 1 = Yes |
|  | Students should speak and think in LOI only | $0=$ No, $1=\mathrm{Yes}$ |
|  | Should aim for an immersion-type experience | $0=$ No, $1=\mathrm{Yes}$ |
|  | Students in grade level are up to the challenge of using LOI only | $0=$ No, 1 = Yes |
|  | Target language exclusivity promotes negotiation and use of communication strategies | $0=$ No, 1 = Yes |
|  | Students have been "sold" on monolingual language use | $0=$ No, 1 = Yes |
|  | Shouldn't happen but have no choice because of need to prepare for standardized exams | $0=$ No, $1=\mathrm{Yes}$ |
|  | Need to push learners' reception and production of mathematics in LOI | $0=$ No, $1=\mathrm{Yes}$ |
|  | Push back against the dominant language | $0=$ No, $1=\mathrm{Yes}$ |
|  | Shouldn't happen but students shouldn't be punished for it | $0=$ No, 1 = Yes |

## 3 Teacher translanguaging beliefs and functions in dual language mathematics classrooms


#### Abstract

3.1 Abstract

Many researchers and theorists are promoting the academic advantages of translanguaging. However, teachers' beliefs and practices are determinant of what happens in the classroom and directly influences student learning. This study sought to understand 14 dual language elementary teachers' beliefs and practices of translanguaging, from seven school districts in California and Texas. It explored the teachers own translanguaging in mathematics, the functions it served and the alignment between their beliefs and practice. This design aimed to reduce inconsistencies found in previous studies and broaden understanding of translanguaging in lesser studied contexts. The findings indicate teachers' beliefs primarily recognize the academic functions of translanguaging. In contrast their practice is primarily for social functions. This suggests a necessary reevaluation of the purposes of translanguaging and the associated pedagogical and research implications.


### 3.2 Introduction

Dual language (DL) teachers regularly face dilemmas in the DL mathematics classroom, including how to develop the language and the subject matter content, whether to foreground the language or the subject matter content, whether to prohibit, permit or promote translanguaging ${ }^{1}$ and the social and political implications surrounding the choices they make regarding translanguaging (Adler, 1998). Teachers respond to these dilemmas based on a variety of beliefs regarding translanguaging and the role of language in these linguistically and pedagogically complex classrooms.

Drawing from various authors I define beliefs as time and context specific, conscious or unconscious guides for judgement and action, that draw on previous experiences and understandings (Rokeach, 1968; Schoenfeld, 1998; Skott, 2015). Teacher beliefs can be
revealed through the expectations they hold for their students, and their theories surrounding the teaching and learning of a content area (Fang, 1996). Teacher beliefs regarding the role of language or languages in the dual language classroom are particularly complex and researchers have found a wide range of language ideologies and discourses manifest in the classroom both at the macro level of policy and at the micro level of student and teacher talk (e.g., Duran \& Palmer, 2014; Martinez et al.,2015).

Since teacher beliefs influence their interpretations of situations and information, as well as their decisions and actions in the classroom, those beliefs thereby influence student learning and achievement. This leads to a cyclical relation among teacher beliefs, classroom pedagogy, and student learning and achievement with each factor influencing the others (Fang, 1996; Pajares, 1992). With the increasing number of dual language programs in the U.S. (Lindholm Leary, 2013) and emerging bilingual children in U.S. schools (U.S. Department of Education, 2019), research on teachers' beliefs and practices of translanguaging aids in comprehending this cyclical relationship. In addition, it informs teacher educators' efforts to adapt teacher education programs to better serve teachers of bilingual students in developing the necessary access and equity focused critical perspective amid social and political controversies (Téllez \& Varghese, 2013).

A review of the existing research suggests that a deeper understanding of teacher beliefs and practices of translanguaging might benefit from research designs which include less studied contexts and the use of multiple measures with a larger and broader sample. Most research on translanguaging has taken place in language arts classrooms; with limited research conducted in mathematics classrooms (King \& Ridley, 2019; Martínez et al., 2015). Therefore, mathematics provides an alternative perspective for exploring this topic and can provide new insights. Most of the research in mathematics classrooms has been conducted outside the U.S. and although research in international contexts is informative, translanguaging is always bound by culture (Meyer et al., 2016) and cannot be separated from social and cultural dimensions (Civil, 2010). Thus, exploration of teachers' beliefs and
practices in U.S. mathematics classrooms has applications unique to the social and cultural dimensions in the U.S. Also, the DL context is one in which all participants can translanguage and therefore reduces pragmatic opposition to translanguaging when an interlocutor is not bilingual (McCarthy, 2018) which could confound beliefs and practices beyond the phenomenon. However, it is also a linguistically and pedagogically complex context which permits an exploration of the many facets associated with the issue of teacher translanguaging.

Those studies exploring teachers' beliefs and practices of translanguaging in the U.S. have often used two types of research designs which limit their application in teacher education programs and future research. The first type of design involved large scale surveys or interviews without observation of the teachers' practice (ex. Nambisian, 2014; Nava, 2009). As a result, inconsistencies in teachers' beliefs were found between studies. Fang (1996) argues that inconsistency between studies may result from the use of a single measure and threats from construct validity. Therefore Fang (1996) suggests that studies of teacher beliefs include multiple measures such as classroom observation, stimulated recall, think-aloud protocols, and focused interviews. The other type of design involved surveys or interviews and observations but included few teachers and/or was in a single school (ex. Holdway \& Hitchcock, 2018; Martinez et al., 2015; O'Gorman Fazzolari, 2017). Because the results from Nava's (2009) study show extreme variability in teachers' beliefs about translanguaging among schools in two different regions of one city, a larger and broader sample of participants would provide a more generalizable understanding of teachers' beliefs in different contexts.

In consideration of the previous research, I have conducted a study to examine dual language elementary teachers' beliefs and practices of translanguaging during mathematics across multiple schools to provide answers to the following research questions:

- What kinds of translanguaging practices do elementary dual language teachers participate in, with what frequency, with what interaction patterns, and for what purposes?
- What do they believe about their own translanguaging in mathematics?
- How do teachers' beliefs about their translanguaging align with the teacher translanguaging functions observed in elementary dual language mathematics classrooms?


### 3.3 Theoretical framework

### 3.3.1 Teacher beliefs about translanguaging in dual language programs

DL programs strive to develop students' linguistic proficiency in two languages simultaneously with academic performance at or above grade level and develop positive cross-cultural behaviors and attitudes (Howard et al., 2005). In these programs, mathematics and language learning are inseparable and interrelated (Brown, 2002) so teachers must focus on developing both simultaneously. As such, mathematics instruction in DL programs is grounded in theories from both the field of mathematics education as well as the field of applied linguistics (Martinez et al., 2015) and these theories influence teacher beliefs about translanguaging.

Teachers' beliefs about instruction and translanguaging are influenced by the language differentiation model (Volterra \& Taescher, 1978). The language differentiation model (Volterra \& Taescher, 1978) suggests that use of the first language interferes with and causes a disuse of the second language and therefore impedes the development of the second language. Therefore, DL teachers promote linguistic 'purism', only using the target language for the given time of day or subject and believe that translanguaging should not happen (de Jong, 2016; Nambisan, 2014; Martínez et al., 2015). In addition, Sommerville \& Faltis (2019) indicate that the standard language variety is emphasized by DL teachers because it is seen as the proper language necessary for academic performance, particularly
in mathematics. Mastery of the specialized words, symbols, math register (Halliday \& Martin, 1993, Pimm 1987) discourse (Gee, 1996), and modes of argument such as precision, brevity, and logical coherence (Forman, 1996) are considered necessary in mathematics so as to be able to participate in mathematics discourse communities (Roth \& Tobin, 2007; Solomon, 2009). They are also seen as essential elements of mathematical competence and school success (Celedón-Pattichis et al., 2010; Martínez et al., 2015). Grounded in these theories, teachers feel guilty or uncomfortable with their own translanguaging in class (Kirkpatrick, 2014) because they recognize their role as language models for their students (Baker, 2009) and may also hold deficit beliefs that their translanguaging demonstrates their (own)? incompetence in one or both languages (Cloud et al., 2000; Gumperz, 1982) despite substantial evidence to the contrary (MacSwan, 1999; Poplack, 2000). Palviainen et al. (2016) also found that some teachers believe certain translanguaging practices, such as direct translation, or co-languaging (García, 2009), leads to students passively waiting for content to be presented in their native language instead of learning content in the target language (Lewis et al., 2012). As a result of these beliefs, some? teachers do not engage in translanguaging.

These strict language separation policies have also been critiqued based on empirical evidence demonstrating the value of using the first language for second language acquisition (Moore, 2013), literacy skill development (Martínez-Álvarez, 2017), content learning (Alvarez, 2012) and to mediate conceptual mathematical discussions (Khisty \& Chval, 2002; Moschkovich, 2007; Setati, 2005).Teachers who recognize the multiple functions of translanguaging permit restricted use in certain circumstances or even promote free use of translanguaging in their classrooms. For example, some dual language teachers permit and engage in translanguaging because they believe translanguaging is needed to support students' conceptual understanding when the pedagogical focus of the subject matter outweighed that of the language, for behavior management, and in interpersonal, affective interactions with students (Cahyani et al., 2018; Martínez et. al., 2015). They also believe it
helps students to prepare for standardized testing (Henderson \& Palmer, 2015) and meet individual student's needs which enables expressions of complex ideas (Sommerville \& Faltis, 2019; Weber, 2014). Teachers that hold these beliefs engage in translanguaging.

### 3.3.2 The functions of translanguaging in mathematics classrooms

Translanguaging has been found to serve social functions (Giles \& Ogay, 2007; Lin, 2013; Martínez, 2009, Myers-Scotton, 1995; Sommerville \& Faltis, 2019), linguistic functions (Cenoz, 2017; Creese \& Blackledge, 2010; Gort \& Sembiante, 2015; Lin, 2013) and academic functions (García, 2013; Gort \& Pontier, 2013; Palmer et al., 2014;) amongst others. Studies in mathematics classrooms have identified a variety of academic functions of translanguaging including connecting informal language with formal mathematical procedures, concepts, and vocabulary (Cervetti et al., 2015; Moschkovich, 2002; Planas \& Setati, 2009). Studies also found that it is often used at specific times in a lesson, for example, in connecting vocabulary to students' daily life or home language as well as when students are unfamiliar with non-mathematical vocabulary (Cervetti et al., 2015; HansenThomas, 2009; Khisty \& Chval, 2002; Salehmohamed \& Rowland, 2014; Setati \& Barwell, 2006), when students do not quite understand the concept or when a concept is new (Planas \& Setati, 2009; Setati, 1998), and during exploratory talk (Adler, 2001; Parvanehnezhad \& Clarkson, 2008; Setati et al., 2002).

Social functions of translanguaging in mathematics classrooms have been recognized to a much lesser degree. Primarily, translanguaging in mathematics classrooms functions socially to build students' confidence (Clarkson, 2007; Domínguez, 2011; Planas \& Civil, 2013). However, in other content areas and outside the classroom, it has been found to function to communicate subtle nuances of meaning and shift voices for different audiences (Martínez, 2009). In interpersonal relationships, it accommodates to the expectation of others and thus negotiates social distance and ingroup solidarity (Lin, 2013). This allows the speaker to associate or dissociate with those one likes, dislikes or does not want to be liked
by (Myers-Scotton, 2006). As a result, La Page (1997) indicates that a person does not adapt to their interlocutor, but to their self-image in relation to the interlocutor. Previous research of the advantages of teacher translanguaging predominantly recognizes the academic functions and social benefits to the student, rather than how teachers use translanguaging to associate or dissociate with their students. In addition, previous research has not explored the frequency of the use of each of these academic and social functions.

### 3.4 Methodology

### 3.4.1 Setting and Participants

This study included participants and data from a larger study ${ }^{2}$ with elementary teachers in Spanish-English dual language schools from various urban school districts in California and Texas. Thirty-two teachers participated in the larger study. From this group of teachers, 14 teachers were selected to explore their beliefs through semi-structured interviews (see Table 1). The selection of these teachers was aimed at providing the greatest variability between interviewees considering the following criteria: grade, state, school district, years teaching and translanguaging rate.

Table 3: Teachers Interviewed

| Pseudonym | Grade | State | Years Teaching |
| :--- | :---: | :---: | :---: |
| Anita | K | CA | 5 |
| Alondra | K | CA | 4 |
| Karime | 1 | CA | 3 |
| Maritza | 1 | CA | 5 |
| Yasmin | 1 | CA | 1 |
| Jessica | 2 | CA | 17 |
| Beatrix | 3 | TX | 15 |
| Ivana | 3 | CA | 1 |
| Liliana | 3 | TX | 14 |
| Marisol | 3 | TX | 10 |
| Valentina | 3 | TX | 15 |
| Solomon | 4 | TX | 20 |
| Graciela | 4 | TX | 5 |
| Irene | 5 | CA | 23 |

### 3.4.2 Data collection

The data included videos of lessons, surveys, and semi-structured interviews. The videos ranged from 10 to 60 minutes per teacher. Following the approach taken by Brevik (2020), focus was on those instances picked up by the teachers' microphone. The survey conducted by MALLI included questions about instructional setting, teaching practices, mentoring, teacher experiences, and teacher background. However, only the section on teacher background was used. In particular, the questions included teacher language proficiency in Spanish and English, schooling experience, language experience, teaching experience, and Professional Development experience for ELL/Bilingual students. Finally, the semi-structured interviews involved open ended questions about the teachers' perspectives of translanguaging in mathematics, and contextual and experiential factors that influence their beliefs (see Appendix A). In addition, during the interview, the teachers engaged in stimulated recall (Lyle, 2003) of their interpretations of their translanguaging by having them watch clips from their videos. This emic approach (Pike, 1967) served to reveal the participants interpretation of the translanguaging instances and their beliefs about translanguaging as well as to triangulate the coding of the instances. The interviews were audio recorded and later transcribed for analysis.

### 3.4.3 Data analysis

The quantitative coding of the videos involved two rounds of coding (see Appendix B). The first round of coding aimed at identifying the context and the translanguaging instances. Each instance was identified by drawing on Myers-Scotton (2006) Matrix Language Framework where the target language during the mathematics class was the Matrix language and the translanguaging instance was described as beginning with the use of an Embedded language and ending when the speaker returns to the Matrix language. In the second round of coding, codes were developed deductively based on codes from previous literature. The code categories included translanguaging functions (Brevik 2020;

Gort \& Pontier, 2013; Reyes, 2004)and types (Gort \& Pontier, 2013; Merritt et al., 1992; Salemohamed \& Rowland, 2014; Setati, 1998); as well as problem type and topic (NCTM Content Standards, n.d.).Codes for the semi-structured interviews with the teachers were developed deductively initially based on codes from previous literature(McMillan \& Rivers, 2011; Nambisian, 2014; Setati et al., 2002) and then adapted and expanded upon through an iterative inductive approach(see Appendix C). In addition, codes were created for data from the surveys. Ten percent of the video data and semi-structured interviews was also coded by a colleague to establish agreement of the coding. In the video data, interrater agreement scores for the functions and type codes were established between $r=.92$ and $r=1.0$. In the coding of the semi-structured interviews, the interrater agreement score was $\mathrm{r}=.80$. Alignment between the beliefs and translanguaging patterns were analyzed only for beliefs expressed by two or more of the teachers. SPSS and R were used to conduct the quantitative analysis to examine measures of central tendency.

### 3.5 Findings

### 3.5.1 Teachers' translanguaging practices

The teachers did not frequently translanguage in their videos. Of the 223 instances of translanguaging in the videos, 16 instances were of teachers' translanguaging by themselves and 8 were the teacher together with a student. Ninety-two percent of the translanguaging took place in schools that had a strict language policy for teachers and in $58 \%$ of the instances the teachers indicated that they held a strict language policy for their own translanguaging. All teacher translanguaging occurred when the language of instruction was Spanish. Fifty-four percent took place during review lessons. Ninety-six percent of the instances took place in whole group setting. The predominant problem types where teacher
translanguaging took place were arithmetic calculation (54\%) and word problems (42\%). Fifty percent of the instances were regulatory and $46 \%$ were explanatory.

The teacher translanguaging included mathematical content in 38\% of the instances, of which 2 instances were for scaffolding, 5 instances were for mathematical terminology, one was for a clarifying answer, and one was for non-mathematical terminology. An example of the scaffolding that took place occurred in Graciela's class when the students were trying to find the difference between the height of two buildings. One of the students was confused as to what he was supposed to be doing (See Example 1).

## Example 1:

Teacher: ¿Cuál es el plan Ethan? ¿Qué tienes que hacer para conseguir la diferencia de altura? [What is the plan, Ethan? What do you have to do to find the difference in height?]
Student: With the, what is it called? um, I think its menos [minus].
Teacher: Exactly when you find the difference if I have five pencils, but you have seven, how many more pencils do you have than me?
Student: [can't hear what he says]
Teacher: You found the difference between the pencils. Now you, vas a encontrar la diferencia entre la cantidad de metros que tienen ambos edificios. [are going to find the difference between the amount of meters that both buildings have].

The student seemed unsure of what the word diferencia [difference] meant and the exchange confirmed his understanding and allowed him to continue with the activity. Teacher translanguaging also occurred with mathematical terminology particularly related to different mathematical procedures (ex. base ten, repeated addition, circle diagrams). It is unlikely that these terms were part of the students' everyday home language. So, the fact that these terms were being used in English indicates that the terms were most likely taught to the students in English previously, despite the class being in Spanish. Sixty-three percent of the instances did not involve mathematical content, of which five were for task instruction, six were for classroom management and four were to affirm students.

### 3.5.2 Teachers' beliefs supporting teacher translanguaging

The teachers expressed beliefs both supporting and opposing teacher translanguaging. However, they expressed beliefs supporting teacher translanguaging 1.6 times more frequently than those opposing teacher translanguaging. A total of thirteen different beliefs supporting teacher translanguaging were expressed, with a range of one to five teachers expressing the same belief $(\mu=1.8, \sigma=1.1)$. Six of those beliefs were noted in previous literature (McMillan \& Rivers, 2011; Nambisian, 2014; Setati et al., 2002) while seven of the beliefs were developed inductively (See Table 2). Ten of the beliefs served academic functions while three of them served social functions.

Table 4: Teacher beliefs about teacher translanguaging

| Inductive Code | Belief | Number of Teachers |
| :---: | :---: | :---: |
| Academic Function permitting/promoting teacher translanguaging |  |  |
|  | Translanguaging is used to facilitate and ensure successful teacher-student communication and content comprehension and is not a reflection of their language proficiency in the language of instruction | 5 |
|  | Translanguaging aids in transfer/bridging to prepare students for future grades/assessments | 3 |
|  | Translanguaging is useful for beginning stages of the lesson/year | 2 |
|  | Translanguaging is useful for small group differentiation | 2 |
| * | Translanguaging increases student participation in mathematical discourse | 2 |
|  | Helpful for teaching vocabulary | 1 |
|  | Monolingual speech is not appropriate given previous learning experiences | 1 |
| * | There is not an appropriate translation | 1 |
| * | My non-dominant language limits me | 1 |
| Social Function permitting/promoting teacher translanguaging |  |  |
|  | Translanguaging is "natural" for bilinguals to use | 2 |
| * | It is easier or more efficient | 1 |
| * | To reduce student frustration | 1 |
| Academic Function prohibiting teacher translanguaging |  |  |
|  | Teacher's not translanguaging encourages target language use | 4 |
|  | More negotiation of meaning occurs if language use is monolingually | 3 |
| * | Translanguaging is not acceptable in whole group discussion | 2 |
| Social Function prohibiting teacher translanguaging |  |  |
|  | The school language policy should be followed | 6 |


|  | Teacher translanguaging will lead to overuse of <br> translanguaging by students | 1 |
| :---: | :--- | :---: |
| ${ }^{*}$ | Not translanguaging because speaking native language | 1 |
| ${ }^{*}$ | Not appropriate during instruction | 1 |

## Academic functions

The most commonly shared beliefs permitting or promoting teacher translanguaging served academic functions. One came from five of the fourteen teachers who expressed that translanguaging is intentionally used to facilitate and ensure successful teacher student communication and content comprehension and is not a reflection of their language proficiency in the language of instruction. For example, Solomon explained that he doesn't translanguage because "I can explain this better in Spanish or my comfort level is better in Spanish, so that's not what my language background is. That's 'yeah it would only be to facilitate student understanding' and it will be a very conscious decision to do so". This type of translanguaging can be observed in Example 1 above from Graciela's class. Graciela indicated in the survey and reiterated in the interview that her English proficiency was significantly lower than her Spanish proficiency. So, it is quite clear that the teacher is translanguaging to a language that she has lower proficiency and comfort in. She does this to support the student's understanding and ability to participate in the activity. Likewise, Valentina shared, "So if I have an English speaker and he asked me a question in English, I don't know what, I mean, I should respond in Spanish because I'm teaching that subject in Spanish, but I feel like more comfortable doing it in English, so that way to make just to make sure that he understands what you know the answer is." An example of this occurred in Marisol's class which was in Spanish when she was asking a student to elaborate on their explanation.

## Example 2:

Student: Can you say it in English?
Teacher: How did you figure out where to put the number when you multiplied by the hundreds, by the two hundreds? When you multiplied by the two, the last one.
Student: When I multiplied two times two...

Teacher: Uh-huh, why did you put the four under the hundreds?
Student: Because if I take away these...if I take away those it would be two hundred times two. And since it is in the hundreds place, I put it in the hundreds.

In this example it would have been quite inappropriate for the teacher to continue to ask in Spanish even when the student was indicating a misunderstanding of the question. Therefore, the socially appropriate response was in English and the teacher understood that this was also necessary because the student was not understanding what she was asking. In both these examples, the teachers indicate that they are very aware that they are switching languages and make this conscious decision when they recognize that it is necessary for the student to understand what they are saying and emphasize that the student's limitations in the language is the driving force for this decision. Despite this belief being held by many teachers, only two instances (8\%) of teachers translanguaging to facilitate communication were observed. Neither of these instances occurred with teachers who had expressed this belief. Therefore, while this was the most commonly expressed belief for permitting or promoting teacher translanguaging, it does not seem to explain the primary purpose of teacher translanguaging.

Three of the teachers also expressed that translanguaging aids in transfer/bridging to prepare students for future grades/assessments. For example, Karime explained that she used translanguaging sometimes in her class because her students get assessed in their native language, and "sometimes it's hard to bridge those two like math terms, right? Like when we're talking about decenas and then we're talking about tenths [tens], like it's hard for those students who are taking the assessment in English and have never heard these words in English before." Two of the three teachers who expressed this belief engaged in translanguaging of math terms particularly for mathematical procedures. Those instances represented $56 \%$ of the translanguaging that occurred with mathematical content and therefore seem to indicate that this is the primary purpose of teacher translanguaging for academic functions. In addition, five of the six teachers who translanguaged were in the
upper elementary grades. Therefore, this may indicate that as students get into higher grades the need to prepare students linguistically for future grades/assessments becomes more urgent and teachers are more aware of their needs to use the other target language. So, there was alignment between this belief and the teacher's practice. However, this type of translanguaging only represents $21 \%$ of the total teacher translanguaging and therefore does not explain the majority of teacher translanguaging taking place in these classrooms.

Two teachers indicated they believed that translanguaging is useful for beginning stages of the lesson/semester. For example, Anita explained, "And I only do it at the beginning of the year with safety stuff like in line and washing hands stuff like that. I really try not to do that like towards the end of the year." However, the translanguaging observed in these lessons was recorded more than halfway through the year and $54 \%$ of it took place in a review lesson. Therefore, teachers were actually observed translanguaging at the end of the unit or later in the year as they were doing a review lesson contrary to what was expressed in this belief.

Two teachers also expressed that translanguaging is useful for small group differentiation. For example, Jessica shared, "Well maybe if I see a kid so frustrated and doesn't get it, maybe I will try to say it in English, but not as the whole class." However, 96\% of the translanguaging was observed in the whole class setting. So, while it may be true that translanguaging can be useful for small group differentiation, this was not the primary use that teachers were making of it.

Two teachers also shared that they believed translanguaging increases student participation in mathematical discourse. For example, Ivana explained that when the class is discussing some mathematical concept and one student is not understanding and able to participate in the discussion, she will translanguage to involve the student in the conversation. She said, "But if a student is just looking at me blank faced in English, then I will speak to them in Spanish. And then that gives them the freedom of speaking back to me in Spanish." This was observed in Example 1 in Graciela's class and Example 2 from

Marisol's class where both teachers' use of English allowed the students to not only understand the question that was being asked, but also to continue in English to elaborate on how to solve the problem and thus participate in mathematical discourse. While this was observed in these cases, they are the only ones and represent $8 \%$ of the instances of teacher translanguaging.

## Social Functions

Beliefs for permitting or promoting teacher translanguaging for social functions were expressed as well but by significantly fewer teachers. Two teachers shared that they believe translanguaging should be permitted because it is "natural" for bilinguals to translanguage. For example, Valentina revealed "and you switch when sometimes they ask you a question in English and then you just naturally respond to them in English. It just comes out right?". So contrary to what the teachers expressed in the most commonly expressed belief where translanguaging was a conscious decision, here the teachers were expressing that translanguaging is a "natural" and unconscious response. An example of this type of translanguaging was observed in Marisol's classroom when she was asking a student to explain how they solved a problem.

## Example 3:

Teacher: ¿Puedes venir y enseñarme que estrategia utilizaste? Ven. ¿Qué estrategia utilizaste? [Can you come and show me what strategy you used? Come. What strategy did you use?]
Student: In English?
Teacher: English or... Ingles o español. En lo que sientes mas confiado. [English or Spanish. In what you feel most confident.]

Here we observe Marisol using English at the beginning of her response to the students' question in English, but she very quickly returns to Spanish, the language of instruction and corrects what she says in Spanish. This response to her translanguaging indicates that her speaking English initially was an unconscious and intuitive response, but she did not consider it to be appropriate, so she continues in English and corrects what she said. Despite
indications of this being viewed as intuitive and therefore should be permitted, this example demonstrates a contradiction to that belief. Valentina further explained that she responds unconsciously with translanguaging in both languages, but it is much less common in English for her. She qualified her belief of translanguaging being a "natural" response more in her first/dominant language and not so much in her second language. This intuitive response using the language from the previous utterance was observed in $62 \%$ of the instances of teacher translanguaging, however the prompt return to the language of instruction in these instances demonstrates a misalignment between the belief and practice.

### 3.5.3 Teachers' beliefs opposing teacher translanguaging

A total of seven different beliefs opposing teacher translanguaging were expressed, with a range of one to six teachers expressing the same belief ( $\mu=2.6, \sigma=1.8$ ). Four of those beliefs were noted in previous literature (McMillan \& Rivers, 2011; Nambisian, 2014; Setati et al., 2002) while three of the beliefs were developed inductively (See Table 2). Three of the beliefs served academic functions while four served social functions.

## Academic functions

Four teachers expressed that they limit their own translanguaging because they believe it encourages the use of the target language. For example, Liliana explained the benefits of maintaining the target language, "I try to stay in language, because I want them to learn the academic vocabulary and to practice it you know." While Maritza argues the consequences of not maintaining the target language saying,
"I've seen a lot of teachers at my school site that just kind of, let it slide, and it really ends up affecting the student because they lose a lot of the language because it's easier for them to speak in English. But I, in my classroom at least, I really strive for it."

To explore if this belief is reflected in the classroom practice a correlation between the teacher translanguaging frequency and the overall translanguaging rate in the classroom
revealed that there was a statistically significant correlation $(r=0.56, p=.04)$. So, it does appear that the amount of teacher translanguaging is related to the amount of student translanguaging.

Three teachers also expressed that more negotiation of meaning occurs if language use is monolingual and thus limit their own translanguaging. For example, Anita who teaches kindergarten explained that she uses translanguaging at the beginning of the year when the kids don't understand, but then later in the year she stops "because I really want them to understand what l'm saying in Spanish, to not just automatically translate or wait for the translation." While this may be true to some degree, Examples 1 and 2 from Graciela's and Marisol's classrooms indicate that the translanguaging did allow the students to engage with the meaning of not only the language in the exchange but also the mathematical content. Therefore, it is possible for negotiation of meaning to take place either monolingually or through translanguaging.

Finally, two teachers expressed that translanguaging is not acceptable in whole group discussion. Neither teacher who expressed this belief participated in translanguaging, however, as previously mentioned, $96 \%$ of the teacher translanguaging that occurred took place in whole group settings. Therefore, for these teachers, this may be true and represents what occurs in practice in their classrooms. However, this is not representative of the majority of teacher translanguaging.

## Social Functions

Six teachers expressed that they prohibit teacher translanguaging because the school language policy should be followed. For example, Jessica shared that she followed the principal's policy saying, "I am kinda strict with the rules from the principal, so if she says no then no." Whereas Beatrix explained that the policy came from the district. She said, "pero hay ciertos minutos para cada materia y se tienen que seguir según el curriculum que tiene el distrito. [but there are a certain number of minutes for each content area that have to be
followed according to the curriculum that the district has.]" However, of the six teachers that expressed this belief, two were observed translanguaging and their school had a strict language policy for teachers. In addition, 92\% of teacher translanguaging took place in schools that had strict language policies for teachers. Therefore, while teachers may express this view, teachers do participate in translanguaging contrary to this belief.

### 3.6 Conclusion

This study reveals how teachers' beliefs and practices respond to the dilemmas they encounter about their own use of translanguaging in the linguistic and pedagogic complexity of the dual language mathematics classroom. The findings serve to amend previous research and reveal the relationship between dual language mathematics teachers' beliefs about their own translanguaging and the various functions that it serves. The study builds on previous research by using multiple measures with a broader sample of teachers and exploring the functions of translanguaging in mathematics, a less studied content area. Although I found that teacher translanguaging was infrequent, I was able to identify trends in how teachers use translanguaging in the classroom which oftentimes differed from what they expressed in their beliefs.

The teachers expressed beliefs which centered primarily around the academic functions for both permitting/promoting and prohibiting teacher translanguaging. The primary teacher belief prohibiting teacher translanguaging was to encourage the use of the target language. While a correlation was found between teacher and student translanguaging rate, it was not possible to determine that teachers restricted translanguaging caused less student translanguaging. In contrast, the most commonly held belief permitting/promoting translanguaging for academic functions; was that it aids in communication. However, this was rarely observed in the translanguaging instances. Two other academic functions were expressed in teacher beliefs that were observed in the videos. Those were that teacher
translanguaging aided in transfer/bridging and increases student participation in mathematical discourse. These findings indicate that teacher translanguaging supports students academically in the classroom by making connections between informal language and formal mathematical procedures, concepts, and vocabulary (Cervetti et al., 2015; Moschkovich, 2002; Planas \& Setati, 2009). However, since teacher translanguaging did not include mathematical content in most of the instances it primarily served a social function.

The social functions of translanguaging while less frequently identified in the teachers' beliefs played roles contrary to the trends identified in the beliefs. Although rarely recognized in their beliefs, teachers were observed responding in the language of the previous utterance in most of the instances. A couple of teachers recognized this social function as being a "natural" response that should be permitted. However, the teachers prompt return to the language of instruction reveals contrary beliefs also. A similar paradox in beliefs surrounding the social functions of translanguaging were found by Setati (2005). In contrast, the most commonly recognized belief of prohibiting teacher translanguaging for a social function was to follow the school language policy. However, the highest translanguaging rates were observed with teachers in schools that had a strict policy against translanguaging. A similar paradox in beliefs surrounding the social functions of translanguaging was previously found (Setati, 2005) and teacher codeswitching was identified as serving primarily the social functions (Setati, 2008). The findings indicate that teachers would benefit from a greater understanding of the social functions that their translanguaging does and does not serve in their classrooms.

In summary, the teachers expressed beliefs about teacher translanguaging very similar to those that have been found in previous studies (Henderson \& Palmer, 2015; Lewis et al., 2012; McMillan \& Rivers, 2011; Nambisian, 2014; O'Gorman Fazzolari, 2017; Palviainen et al., 2016; Setati et al., 2002). However, their actual translanguaging practice differed from seven of the ten beliefs that were examined. These findings reveal that the complexity of their environment confounds teachers' beliefs and practices surrounding the
functions of translanguaging. Yet, those that did not differ reveal focus areas for future research and practice. These findings are important for three reasons. First, since contradictions between teachers' beliefs and practice are commonly found, (Borg, 2003) an understanding of the practice cannot be based solely on teachers' expressed beliefs and must include multiple measures like Fang (1996) indicates. Second, while previous research has indicated the academic benefits of student translanguaging (Cervetti et al., 2015; Moschkovich, 2002; Planas \& Setati, 2009), this study indicates similar benefits for students from teacher translanguaging which needs to be recognized and studied further. Third, they indicate that previous research of translanguaging needs to be updated to recognize the social functions it serves in the classroom.

These findings have limitations associated with the context in which this study took place and the use of self-reported data. This study only explored Spanish/English dual language classrooms which place different demands on teachers than in other programs. However, this context provides the opportunity to explore this complex linguistic and pedagogical environment and provides an opportunity to explore the dilemmas of student translanguaging in a context where all can participate in translanguaging. In addition, this study explored the teachers' beliefs and practices only in mathematics and may not be representative of other content areas. However, this was by design and offered an opportunity to gain an understanding of beliefs and practices in a subject largely unstudied. Finally, some of the data was self-reported and as such is inherently subjective. However, due to the triangulation across different data sources as well as the discussion with the teachers of inconsistencies found between data sources, I am confident that the data provides an accurate representation of the beliefs and practices in these classrooms.

This study has implications for teacher education and future research. The teachers' beliefs about their own translanguaging indicate that they need to develop greater awareness of their own practice and the functions that their translanguaging serves, and the advantages it affords. Teacher education programs and professional development programs should not
only introduce a theoretical foundation of translanguaging to their teacher candidates, but also provide opportunities for those teachers to analyze their own translanguaging practice and consider how their practice is or is not reflective of their beliefs. This would also provide them an opportunity to expand their understanding of the functions that translanguaging serves in their classrooms. In addition, since teachers engaged in limited translanguaging, and yet it was found to serve both academic and social functions, teacher education programs and professional development programs could consider how to support their teachers with specific strategies of translanguaging to help them engage in translanguaging aligned those functions. In addition, schools should consider amending policies regarding translanguaging to be reflective of the social and academic functions it serves. Considering the dominance of social functions of translanguaging in practice and yet the limited number of studies exploring and promoting the social functions, future research of teacher translanguaging would benefit from studies using multiple measures to explore both beliefs and practice in different content areas, and particularly focused on the social functions of translanguaging. Both teacher education and research, must recognize and apply the primary functions of translanguaging in manners that "foster language practices that approximate authentic interaction contexts" (Gort \& Pontier, 2013). If future research, teacher education programs and schools are to support teachers in doing this, a shift in focus is necessary from academic functions to social functions of translanguaging as well as an analysis of the authenticity of some academic functions.

Note:

1. The use of more than one language between bilingual speakers in interactive speech is referred to through a variety of terms of which "code-switching" or "translanguaging" are used most often (MacSwan, 2017). While codeswitching views languages as separate systems, translanguaging views a bilingual speakers' full linguistic repertoire as an integrated system (Canagarajah, 2011), and distinct languages as merely socio-political constructions (Makoni and Pennycook, 2007). However, in the exploration of teacher's beliefs and practices in this paper I must acknowledge "the relationships between what people believe about their language (or other people's languages), the situated forms of talk they deploy, and the material effects- social, economic, environmental-of such views and use" (Makoni \& Pennycook, 2007, p. 22). Since the DL programs in this study promoted the
separation of languages, and the teachers referred to the languages as distinct, a description of the beliefs and practices necessitates language representative of the socio-political constructions of distinct languages.
2. MALLI, research funded by a grant from the U.S. Department of Education, Office of English Language Acquisition, National Professional Development Program, Grant \#T365Z170070. MALLI is a professional development program that works to integrate mathematics, language, and literacy in dual language settings. It focused on developing discourse, literacy, and vocabulary strategies during mathematics instruction.

### 3.7 References

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### 3.8 Appendices

### 3.8.1 Appendix A: Interview Protocol

Teacher: $\qquad$
Date:

Thank you so much for your time today. We are investigating mathematical content and language learning through translanguaging in two-way immersion programs, and we appreciate your input and experience. Your interview responses will be shared without using your name, school name, or other identifying information, and we would be happy to share our results with you as well if you'd like.
"Would you mind if I recorded our conversation? (if using phone, turn to airplane mode. Consider backup recording on ipad or laptop in addition to phone.)"

With the recording running: (Researcher Name) interview with (Teacher Name) on (Date).
I want to begin by learning a little bit more about you, then we will talk about your school and teaching, and finally we will look at some clips from your video and talk about those.
13. Tell me about your teaching trajectory.

Probes: Where did you do your credential program? What schools have you taught in? What grades? What programs? How long have you been teaching?
14. Tell me about your own language history.

Probes: What do you consider to be your first language? What language do you feel more proficient or comfortable speaking?
15. Tell me about your own schooling/math trajectory.

Probes: Where did you go to elementary, middle, and high school? What language did you learn math in? Was math easy or difficult for you? Why? Describe your math teachers.

## Now I would like to talk about your school and teaching

16. Do you teach math in English or Spanish? How does that change in different grade levels at your school?
17. What is your school language policy?

Probes: What is their policy about language mixing? Is there an official and an unspoken policy? What is it?
18. What is your language policy in your classroom?

Probes: Do you allow your students to mix languages during math? Why? Do you mix languages during math? Why? What do you think will be the main difficulty/difficulties that you would face in trying to use only the language of instruction in a mathematics lesson?
19. Do your students switch between languages during your math classes?

Probes: When (small groups, between peers, for brainstorming, talk about things outside of content)? Why do you think they do it? Could you give me some examples? Do you think that changes when the language of instruction changes?

- Follow up to "Yes": Are there any particular instructional purposes or moments in which you encourage/allow students to mix languages? Could you give me some examples?
Probes: Is there a difference between when you are doing a conceptual and a procedural lesson? What is the greatest motivation for them switching languages cognitive or sociopolitical reasons?

20. Do you ever switch between languages during math?

Probes: When? Why? Do you think that changes when the language of instruction changes?

- Follow up to "Yes": Are there any particular instructional purposes or moments in which you mix languages? Could you give me some examples?
Probes: Is there a difference between when you are doing a conceptual and a procedural lesson? What is the greatest motivation for you switching languages cognitive or sociopolitical reasons?

21. What other PD have you participated in focused on teaching math?

Probes: When? Why? Who did it?

## Now I would like to look at the clips from your video

22. Before we begin can you tell me about this lesson, the learning objective, where it came in the unit sequence etc.

## Watch translanguaging clip. Clips used:

23. Describe what you see happening here.

Probes: What is interesting or stands out to you? What do you wonder about what you watched? Did you notice any language switching? Why do you think this switching is taking place? Did anything you noticed during the lesson cause you to act differently than you had planned?
24. Can you tell me about this student?

Probes: What is this student's linguistic background (Oral, written, informal, academic, math proficiency, other important info to know about the student)? How does students' linguistic background influence the languages of instruction and the mixing of languages?

## Repeat with next clips

This has been extremely valuable, and I have a lot to work with here. Before we wrap up, l'd like to give you a chance to add anything you think is relevant to what we have talked about today.

Thank you so much for sharing your time and insight with me today.

### 3.8.2 Appendix B: Video coding scheme

| Primary Code (CODE) | Description |
| :---: | :---: |
|  | CODING ROUND 1 |
| School Year (YEAR) | $0=2018-19,1=2019-20$ |
| Teacher ID (TID) |  |
| Video \# (VID) |  |
| State (STATE) | 0 = California, 1 = Texas |
| Time Stamp (TIME) |  |
| Duration of instance (in seconds) (DUR) |  |
| Translanguaging Rate (RATE) | Instances/minute |
| Language of Instruction (LOI) | 0 = English, 1 = Spanish |
| School Code (SCHL) | 1-16 |
| School District (DIST) | 1-9 |
| Interviewed (INT) | $0=$ No, $1=$ Yes |
| Translanguager (TLER) | $0=$ Student, 1 = Teacher, $2=$ Both teacher and student $3=$ Multiple Students |
| Grouping (GROUP) | 1 Whole Class, $2=$ Small group, 3 = Alone |
| Teacher response (TRESP) | $0=$ Prohibiting, $1=$ Permitting, $2=$ Promoting |
| Grade (GRADE) |  |
| Mathematical Content (MATHCON) | $0=$ No, $1=$ Yes |
| Transcription (TRAN) |  |
|  | CODING ROUND 2 |
| Functions |  |
| Academic -Scaffolding (SCAF) | $0=$ Not present, $1=$ Present - Translanguaging used to offer guidance, explain/expand a teaching point, bridges communication gaps, reduces ambiguity, or offer translation for students' lack of comprehension in the target language. |
| Academic - <br> Metalinguistic <br> Explanation (METAEX) | $0=$ Not present, $1=$ Present - Translanguaging used to focus on linguistic forms through explicit explanations. |
| Academic - Task Instruction (TASK) | $0=$ Not present, $1=$ Present - Translanguaging used to give task instructions for an activity or procedure. |
| Academic Mathematical Terminology (MATHTERM) | $0=$ Not present, $1=$ Present - Translanguaging used to provide new mathematical terminology or vocabulary clarification |
| Academic - Nonmathematical Terminology (XMATHTERM) | $0=$ Not present, 1 = Present - Provide non-mathematical vocabulary that students most likely do not already possess |
| Academic - Other Domains (DOMAINS) | $0=$ Not present, $1=$ Present - Translanguaging is used to refer to another domain about a matter relevant to the target language topic |
| Academic - Schema (SCHEMA) | $0=$ Not present, 1 = Present - Teacher explains something that adds to student's existing knowledge |


| Academic - <br> Assessment (ASSESS) | $0=$ Not present, 1 = Present - Ask a student to demonstrate what she knows |
| :---: | :---: |
| Management Classroom Management (MANAGE) | $0=$ Not present, $1=$ Present - Translanguaging is used to manage students' behavior in the classroom, lack of student concentration, talk, or misconduct |
| Communicative - Affirm (AFFIRM) | $0=$ Not present, 1 = Present - Affirm a speaker's statement or provide positive reinforcement |
| Communicative Clarify - Ask question to clarify what was said (CLARQ) | $0=$ Not present, $1=$ Present - Listener doesn't hear or understand what the speaker said, or listener isn't sure he/she has understood the information correctly and is checking her understanding of what the speaker said. |
| Communicative Clarify - Answer question (CLARA) | $0=$ Not present, $1=$ Present - Responds to a question indicating a misunderstanding of the information |
| Type |  |
| Reformulation, Paraphrase (REFORM) | $0=$ Not present, 1 = Present Translanguaging reformulated utterances into a different language, without adding any new information |
| Content of Activity Explanatory (EXPLAN) | $0=$ Not present, $1=$ Present Translanguaging used non-LOI and was not necessarily reformulating or translating what she had just said but instead progressed with the discourse to present new concepts, illustrate facts and elaborate on them |
| Content of Activity Regulatory (REGUL) | $0=$ Not present, $1=$ Present Translanguaging used non-LOI and was not necessarily reformulating or translating what she had just said but instead progressed with the discourse to control or to attract the attention of students, and for student behavioral regulation in general |
| $\begin{array}{\|l} \hline \text { Translation - Direct } \\ \text { Translation (DIRTRAN) } \\ \hline \end{array}$ | $0=$ Not present, $1=$ Present Translanguaging is translation of an utterance as is into a different language |
| Mathematics |  |
| Problem Type (PROB) | Word problem $=1$, Arithmetic Calculation $=2$ Geometry $=3$, <br> Measurement $=4$, Comparing numbers $=5$, Counting $=6$ |
| Math Topic (TOPIC) | Adding fractions or understanding area, for example. |

### 3.8.3 Appendix C: Interview coding scheme

| Code Category | Subcategory (CODE) | Description |
| :---: | :---: | :---: |
| Student Background | Student Name (SNAME) |  |
|  | Student Home Language (SHOMEL) | 0 = English, 1 = Spanish, 2 = Other |
|  | Student Math Proficiency (SMAP) | $\begin{aligned} & 1 \text { = Low, } 2 \text { = Average, } 3 \\ & =\text { High } \end{aligned}$ |
|  | $\begin{aligned} & \text { Student Spanish Language Proficiency } \\ & \text { (SSLP) } \end{aligned}$ | $\begin{aligned} & 1 \text { = Low, } 2 \text { = Average, } 3 \\ & =\text { High } \end{aligned}$ |
|  | Student English Language Proficiency (SELP) | $\begin{aligned} & 1 \text { = Low, } 2 \text { = Average, } 3 \\ & =\text { High } \end{aligned}$ |
|  | Student Gender (SGEN) | 0 = Male, 1 = Female |
|  | Other |  |
| Teacher Background | Teacher Birthplace (BIRTH) | 0 = U.S., 1 = Other Spanish speaking country |
|  | K-12 Math Experience |  |
|  | Math PD Attended |  |
| School context | School Language Policy (LPSCHL) | Strict for teachers and students $=0$, Strict for teachers, Flexible for students $=1$, Strict for students, flexible for teachers $=2$, Flexible for teachers and students = 3 |
|  | Teacher Language Policy (LPTEACH) | Strict for teachers and students $=0$, Strict for teachers, Flexible for students $=1$, Strict for students, flexible for teachers $=2$, Flexible for teachers and students = 3 |
|  | Language Shift (SHIFT) | What grade language of instruction changes in mathematics |
|  | Patterns of Translanguaging (PATTL) | When teacher sees translanguaging happening frequently |
|  | Point in lesson (POINT) | 1 = Beginning, $2=$ <br> Middle, 3 = End, 4 = <br> Review |
| Beliefs support student Translanguaging | Translanguaging is used to facilitate and ensure successful teacher-student communication and content comprehension and is not a reflection of their language proficiency in the language of instruction | $0=$ No, 1 = Yes |


|  | Translanguaging aids in transfer/bridging to prepare students for future grades/assessments (ST8) | $0=$ No, $1=$ Yes |
| :---: | :---: | :---: |
|  | Translanguaging is useful for beginning stages of the lesson/year (ST5) | $0=$ No, $1=$ Yes |
|  | Translanguaging is useful for small group differentiation (ST7) | $0=$ No, 1 = Yes |
|  | Translanguaging increases student participation in mathematical discourse (ST11) | $0=$ No, $1=$ Yes |
|  | Helpful for teaching vocabulary (ST2) | $0=$ No, 1 = Yes |
|  | Monolingual speech is not appropriate given previous learning experiences (ST4) | $0=$ No, $1=$ Yes |
|  | There is not an appropriate translation (ST10) | $0=$ No, 1 = Yes |
|  | My non-dominant language limits me (ST14) | $0=$ No, 1 = Yes |
|  | Translanguaging is "natural" for bilinguals to use (ST3) | $0=$ No, 1 = Yes |
|  | It is easier or more efficient (ST9) | $0=$ No, $1=$ Yes |
|  | To reduce student frustration (ST12) | $0=$ No, $1=$ Yes |
| Beliefs oppose student | Teacher's not translanguaging encourages target language use (OT4) | $0=$ No, $1=$ Yes |
| Translanguaging | More negotiation of meaning occurs if language use is monolingually (OT1) | $0=$ No, 1 = Yes |
|  | Translanguaging is not acceptable in whole group discussion (OT5) | $0=$ No, 1 = Yes |
|  | The school language policy should be followed (OT2) | $0=$ No, $1=$ Yes |
|  | Teacher translanguaging will lead to overuse of translanguaging by students (OT3) | $0=$ No, 1 = Yes |
|  | Not translanguaging because speaking native language (OT6) | $0=$ No, 1 = Yes |
|  | Not appropriate during instruction (OT7) | $0=$ No, 1 = Yes |

# 4 Bids for Linguistic Capital through Translanguaging during Spanish and English instruction in dual language classrooms 


#### Abstract

4.1 Abstract

This paper explores how translanguaging rates varies as a function of teachers' response to translanguaging, school context and student language proficiency. It analyzes the translanguaging practices of students and teachers in 32 elementary Spanish-English dual-language mathematics classrooms in Texas and California. The findings challenge previous research regarding teachers' response to translanguaging in dual language programs and the relationship between language proficiency and translanguaging. They also support previous research regarding translanguaging demonstrating an awareness of the linguistic capital and symbolic power that the language of power bestows and contextual patterns of translanguaging. The findings indicate that further research is required, and teacher education and dual-language programs need to approach translanguaging from a new perspective.


### 4.2 Introduction

In light of the increasing numbers of emerging bilingual children in U.S. schools (U.S. Department of Education, 2019), research on teacher and student language use, and the effects of language use on students' learning has become more urgent. One practice that is being promoted to support student learning is translanguaging (Garcia \& Wei, 2015; Moschkovich, 2007; Palmer et al., 2014; Planas, 2018). Translanguaging has been shown to serve a variety of social functions (Giles \& Ogay, 2007; Lin, 2013; Martínez, 2009, MyersScotton, 1995; Sommerville \& Faltis, 2019), linguistic functions (Cenoz, 2017; Creese \& Blackledge, 2010; Gort \& Sembiante, 2015; Lin, 2013) and academic functions (García, 2013; Gort \& Pontier, 2013; Palmer et al., 2014;). Despite the recognition of many functions of translanguaging, previous research has found that teachers prohibited translanguaging
(Sommerville \& Faltis, 2019). However, it is not clear if students were permitted to translanguage, with what frequency and/or in what contexts they would tend to use translanguaging for each of these functions.

Research and the promotion of translanguaging in contexts where not all participants are bi/multilingual in the same languages face certain pragmatic opposition (McCarthy, 2018) since not all interlocutors can engage with one another through translanguaging. However, dual language (DL) programs, which have been increasing in number in the U.S. (Lindholm Leary, 2013), provide an opportunity to explore translanguaging without this pragmatic opposition and to explore it from two different perspectives, when instruction takes place in each language. Despite researchers exploring translanguaging in DL classrooms (García \& Sylvan, 2011; Gort \& Pontier, 2013; Palmer et. al., 2014), few studies have explored it in mathematics classrooms since most studies are conducted in language arts classrooms (King \& Ridley, 2019; Martínez et al., 2015). Because of this, mathematics is an interesting context to explore language use and can provide new insights.

Dual language mathematics classrooms are linguistically and pedagogically complex environments where dilemmas surround the use of translanguaging and as a result the practice in the classroom. Teachers face dilemmas such as how to develop the language and the subject matter content, whether to foreground the language or the subject matter content, as well as the social and political implications surrounding students' and teachers' language choices and opportunities that exist in the classroom (Adler, 1998). Included in these dilemmas is whether a strict language policy or a translanguaging policy should be implemented in DL classrooms. One position holds that translanguaging ${ }^{1}$ should be prohibited (e.g., McCarthy, 2018; Wang \& Kirkpatrick 2013); another that it should be permitted (e.g., Macaro, 2005; Setati et al., 2002; Weber, 2014); and third, that it should be promoted (e.g., García \& Kleyn, 2016; Sommerville \& Faltis, 2019). Some researchers argue that it is unfair to ask students to not use their full linguistic repertoire (García \& Kleyn, 2016; Otheguy et al., 2015) and assume or suggest that if students were permitted to use their
complete linguistic repertoire, they would do so. However, the use of translanguaging in the classroom has been shown to be much less straightforward and other factors such as the language status (Planas \& Setati, 2009) influence linguistic practices as well in the classroom.

In consideration of previous research, translanguaging studies require consideration of cultural embeddedness (Meyer et al., 2016) and cannot be separated from social and cultural dimensions (Civil, 2010). Therefore, further research is necessary within the U.S., since most research on translanguaging in mathematics has been conducted outside of the U.S. As such, this study examines elementary Spanish-English DL students' translanguaging during mathematics in multiple school districts in California and Texas, two states with many DL programs. It builds on previous research that qualitatively examined student translanguaging in mathematics classrooms by providing a broader sample across a variety of contexts and therefore generalizable to a larger population. This research is guided by three questions:

- How does translanguaging rate vary as a function of teachers' responses to translanguaging in U.S. Spanish-English dual language elementary schools?
- How does translanguaging rate vary as a function of school context in U.S. Spanish-English dual language elementary schools?
- How does translanguaging rate vary as a function of student Spanish and English language proficiency in U.S. Spanish-English dual language elementary schools?


### 4.3 Theoretical framework

### 4.3.1 Language and mathematics learning in dual language programs

Dual language programs strive to develop students' linguistic proficiency in two languages simultaneously with academic performance at or above grade level and develop
positive cross-cultural behaviors and attitudes (Howard et al., 2005). Since mathematics and language learning are inseparable and interrelated (Brown, 2002), DL programs must focus on developing both simultaneously. As such, mathematics instruction in DL programs is grounded in theories from both the field of education (Martinez et al., 2015) as well as the field of applied linguistics. The language differentiation model (Volterra \& Taescher, 1978) particularly influences language policy, instruction and translanguaging. Drawing on the language differentiation model (Volterra \& Taescher, 1978), DL teachers promote linguistic purism and language separation because they indicate the use of the first language interferes with and causes a disuse of the second language, and therefore impedes the development of the second language (Nambisan, 2014; Lindholm-Leary; 2006; Martínez et al., 2015; TorresGuzmán, 2007). The standard language variety (Sommerville \& Faltis, 2019) is also emphasized by DL teachers as it is seen as the proper language necessary for academic performance, particularly in mathematics. Mastery of the specialized words, symbols, math register, (Halliday \& Martin, 1993, Pimm 1987) discourse (Gee, 1996) and modes of argument such as precision, brevity, and logical coherence (Forman, 1996) are considered necessary in mathematics so as to be able to participate in mathematics discourse communities (Roth \& Tobin, 2007; Solomon, 2009) and are seen as essential elements of mathematical competence and school success (Celedón-Pattichis et al., 2010; Lemke, 1990; Martínez et al., 2015) and thus translanguaging may be discouraged. These theories have both led to language separation policies in DL mathematics classrooms.

However, strict language separation policies have also been critiqued based on empirical evidence demonstrating the value of using the first language for second language acquisition (Moore, 2013) and that asking bilingual children to perform with less than half their repertoire is unfair and not supported by research on second language development (García \& Kleyn, 2016). In addition, translanguaging has been found to aid in literacy skill development (Martínez-Álvarez, 2017) and content learning (Alvarez, 2012). Some researchers also promote an unrestricted use of translanguaging in the classroom from a transformative
stance. They argue for using a child's full linguistic repertoire to transform language hierarchies in schools (García \& Kleyn, 2016). Sommerville and Faltis (2019) likewise argue that translanguaging in schools that promote language separation can be understood as a tactic (de Certeau, 1984) that challenges the traditional language separation policy.

Studies promoting the use of translanguaging suggest that the direction of translanguaging would be from the language of instruction to the home language because it is associated with students being more relaxed and making connections to the mathematical procedures, concepts, and vocabulary they are learning, which enhances their understanding (Bose \& Choudhury, 2010; Planas \& Civil, 2013, Salehmohamed \& Rowland, 2014; Setati, 2005; Setati et al., 2002; Tavares, 2015). However, research in contexts outside of the U.S. has revealed that this is not always the case (ex. Setati, 2008; Planas \& Setati, 2009).

### 4.3.2 Power and translanguaging in mathematics

Teachers are in a position to empower or disempower a certain type of language use either indirectly or directly. As part of classroom management, teachers act as socializing agents into the "prevailing accepted patterns" of multilingualism (Merritt et al., 1992). This accepted pattern can include ideologies from beyond the classroom, such as school or regional patterns, but for classroom management it means that teachers are determining when and how translanguaging is accepted. Previous research on how power structures influence translanguaging and how translanguaging is used to wield power has also explored the legitimacy that languages receive (Planas \& Civil, 2013; Setati, 2005), the boundaries that are drawn between languages (Canagarajah, 2011; Farrugia, 2018), contextual influences on translanguaging (Merritt et al., 1992; Meyer et al., 2016; Setati \& Adler, 2000; Planas \& Civil, 2013), and classroom management and power dynamics (Bose \& Choudhury, 2010; Merritt et al., 1992; Zahner \& Moschkovich, 2011; Salehmohamed \& Rowland, 2014; Setati, 2005; Sommerville \& Faltis, 2019).

Research on translanguaging has revealed that language status influences translanguaging. The use of one language in mathematics may be preferred over the home language if it provides access to social goods, and social and economic power. For example, Setati (2008) found that despite recognizing the value of learning mathematics in a language that students understood better, the teachers and students in South Africa preferred using English during mathematics because they are aware of the "linguistic capital of English and the symbolic power it bestows on those who can communicate in it" (p.106) and because the home language did not have the necessary vocabulary. Likewise, Planas and Setati (2009) indicate, that in Catalonia, Spanish is a low status language and therefore in whole class settings the Spanish speaking immigrant students from Latin America rarely translanguaged to Spanish, their home language, because it had lower status than Catalan, the dominant societal language, even if it was common for them to translanguage to Spanish in small group settings. It is possible that even when the language of instruction is students' home language, they may choose to translanguage to the other target language in DL classrooms when it is the language of power. This idea is grounded in the concept of a societal linguistic market (Bourdieu, 1977, p. 654), where languages cooperate and compete with each other depending on the relative value of each language in a given context, leading to a language of power. A language of power is a language through which social structure and social action are accomplished and domination is reproduced (Clegg, 1987).

### 4.4 Methodology

### 4.4.1 Setting and Participants

Because I am interested in the real-world use of translanguaging, I use naturalistic inquiry (i.e., non-manipulative) (Patton, 2002) and study translanguaging practices that take place in Spanish-English DL elementary mathematics classrooms. Some of the data in this study comes from a larger study ${ }^{2}$. The schools were in nine different urban school districts in

California and Texas. Thirty-two teachers participated in this study during the 2018-19 and 2019-20 school years with three teachers participating in both years.

### 4.4.2 Data Collection and Coding

The dataset was compiled through lesson videos, and semi-structured interviews which included measures of instructional setting including language of instruction, state, grouping of students, math problem, school and teacher language policy, and student language proficiency for the translanguaging instances identified in the videos.

The videos ranged from 10 to 60 minutes per teacher in accordance with the grade level they taught, for a total of 20 hours and 31 minutes of classroom videos. Each teacher wore a microphone to pick up their voices and the voices of the children near them. Following the approach taken by Brevik (2020) I focused on only those translanguaging instances recorded by the teachers' microphone. Although I recognize that not all translanguaging instances in the classroom were detected by the microphone, those that were recorded were within the hearing of the teacher and a choice was made by the teacher about how to respond to the translanguaging. I analyzed the translanguaging instances in these lesson videos with a focus on the teachers' response to the translanguaging instances.

I conducted semi-structured interviews (Kvale, 1996) with fourteen of the thirty-two teachers (See table 1). These teachers were selected to provide the greatest variability between interviewees based on the following criteria: video translanguaging rate, state, school district, and language of instruction. This allows for exploration of the previously described relationship between translanguaging and the context (Borg, 2003; Brown \& Cooney, 1982; Nava, 2009; Skott, 2015), social interactions (Flores, 2001) and the political affordances and constraints of teachers' situations (Windshitl, 2002).

Table 5: Teachers Interviewed

| Pseudonym | Grade | State | Years Teaching |
| :--- | :---: | :---: | :---: |
| Anita | K | CA | 5 |
| Alondra | K | CA | 4 |
| Karime | 1 | CA | 3 |


| Maritza | 1 | CA | 5 |
| :--- | :---: | :---: | :---: |
| Yasmin | 1 | CA | 1 |
| Jessica | 2 | CA | 17 |
| Beatrix | 3 | TX | 15 |
| Ivana | 3 | CA | 1 |
| Liliana | 3 | TX | 14 |
| Marisol | 3 | TX | 10 |
| Valentina | 3 | TX | 15 |
| Solomon | 4 | TX | 20 |
| Graciela | 4 | TX | 5 |
| Irene | 5 | CA | 23 |

The coding of the videos aimed to identify the translanguaging instances, teacher ID, state, time stamp of the instance, duration of video, duration of translanguaging instance, language of instruction, translanguager, student grouping, teacher response to translanguaging, grade level, state, mathematics problem type as well as a simple transcription of the translanguaging instance. I drew on Myers-Scotton (2006) Matrix Language Framework to identify each instance. In this framework, Myers-Scotton (2006) identifies an Embedded language (or guest language) which is inserted in the Matrix language. Although Myers-Scotton (2006) uses grammar to determine the Matrix language, I took a sociolinguistic approach and used the context to determine the Matrix language. I considered the target language during the mathematics class as the Matrix language and the translanguaging instance was described as beginning with the use of an Embedded language and ending when the speaker returned to the Matrix language. In some circumstances the speaker switched back and forth between both the Matrix language and the Embedded language. In this case, an instance began with the first use of the Embedded language and ended when they switched back to the Matrix language for a complete sentence. Codes from the semi-structured interviews with the teacher included were then added to each coded translanguaging instance (See Appendix A for a complete description of each code). The dataset included 447 translanguaging instances after extreme outliers (RATE $>0.254$ ) were removed.

### 4.4.3 Variables in the Models

Question Predictors. The Language of instruction (LOI) is a dichotomous nominal variable coded $0=$ English, $1=$ Spanish. The mean for LOI is 0.82 (sd = 0.37). The student's English language proficiency (SELP) and Spanish language proficiency (SSLP) are ordinal variables that range from 1 to 3 (low to high), based on the teacher reported level of student language proficiency. The mean for SELP is 2.47 ( $s d=0.89$ ). The mean for SSLP is $2.42(s d=0.77)$. Outcome Variable. Teachers' response (TRESP) is an ordinal variable coded $0=$ prohibit, $1=$ permit, $2=$ promote. The mean of TRESP is 1.22 (sd $=0.48$ ). Translanguaging rate (RATE) was calculated by dividing the duration of the translanguaging instance by the length of the lesson video in seconds. The mean for RATE is 0.08 ( $s d=0.06$ ).

Control Predictors. The analyses included control predictors to account for the political and classroom context variables. State (STATE) is a dichotomous nominal variable coded $0=$ California, $1=$ Texas. The mean for STATE is 0.64 (sd = 0.48). Student grouping (GROUP) is a dichotomous nominal variable coded $1=$ Whole Class, $2=$ Small Group. The mean for

GROUP is 1.44 ( $\mathrm{sd}=0.5$ ).
Table 6: Descriptive statistics of all variables

| Variable | Description | N | mean | sd | min. | max. |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| RATE | Translanguaging rate <br> (translanguaging duration in <br> seconds/length of video in <br> seconds) | 447 | 0.08 | 0.06 | 0 | 0.254 |
| TRESP | Teacher response to <br> translanguaging 0 = prohibit, 1 <br> permit, 2 = promote | 445 | 1.22 | .48 | 0 | 2 |
| LOI | Language of instruction 0 $=$ <br> English, 1 = Spanish | 447 | 0.82 | 0.39 | 0 | 1 |
| GROUP | Student grouping 0 = Whole <br> Class, 1 = Small Group | 440 | 1.44 | 0.5 | 1 | 2 |
| STATE | State 0 = California, 1 = Texas | 447 | 0.64 | 0.48 | 0 | 1 |
| SSLP | Student Spanish Language <br> Proficiency 1 = Low, 2 = <br> Average, 3 = High | 104 | 2.42 | 0.77 | 1 | 3 |
| SELP | Student English Language <br> Proficiency 1 = Low, 2 = <br> Average, 3 = High | 101 | 2.47 | 0.9 | 1 | 3 |

### 4.4.4 Data analysis

Analysis identified extreme outliers with a translanguaging rate > 0.254 (2 sd), which were removed from the dataset. Data analysis involved OLS Regression to examine the main effects of how translanguaging rate varies as a function of teachers' response to translanguaging, how translanguaging rate varies as a function of school context and how translanguaging rate varies as a function of students' language proficiency in English and Spanish when the language of instruction was Spanish. It was not possible to examine the relationship between translanguaging rate and students' language proficiency when the language of instruction was English ( $\mathrm{n}=17$ ) or the relationship between translanguaging rate and teachers' language proficiency in English $(\mathrm{n}=40)$ and Spanish $(\mathrm{n}=42)$ due to the limited number of instances in the sample. An OLS regression was run with the predictor variables and correlations were examined between the categorical variables.

The linear regression model was:
RATE $=\beta_{0}+\beta_{1}$ LOI $+\beta_{2}$ GRADE $+\beta_{3}$ STATE $+\beta_{4}$ GROUP $+\beta_{5}$ TRESPPROMOTE + $\beta_{6}$ TRESPPROHIBIT $+\beta_{7}$ SSLP $+\beta_{8}$ SELP $+\varepsilon$

In consideration of the previous research indicating that students' language use reflects an awareness of the linguistic capital and symbolic power a language bestows leading to lesser translanguaging to a low status language and particularly in whole class settings (Planas \& Setati, 2009; Setati, 2008), I hypothesize that if, as has been previously suggested in literature, bilingual speakers engage in translanguaging in order to use their full linguistic repertoire, I would expect to find that the translanguaging rate is similar in both languages and contexts in these DL classrooms. However, if bilingual speakers engage in translanguaging as part of a linguistic marketplace and the language of power influences bilingual speakers translanguaging practices, I expect to see a higher translanguaging rate
when the language of instruction is not the language of power, in other words when it is Spanish, assuming English is the language of power in the U.S. In addition, I would expect to see a differential translanguaging rate in different contexts.

### 4.5 Findings

Model 1 and 2 serve as the control models of language of instruction and student English proficiency without covariates, and Models 3 and 4 include covariates of the school context and the student English proficiency. In examining the overall explanatory value of each model using $R^{2}$, it is evident that the inclusion of covariates increases the explanatory value in Models 3 and 4. Model 4 has the greatest explanatory value ( $R^{2}=.80$ ) and examines the main effects of both the school context and the student English proficiency on translanguaging rate.

The parameter estimates indicate that the translanguaging rate on average was .48 standard deviations higher ( $\beta=.029,95 \% \mathrm{Cl}[.015, .062], \mathrm{p}<.000$ ) when the language of instruction was Spanish than when it was English. Secondly, they indicate that the translanguaging rate on average was 16 standard deviations lower ( $\beta=-.010,95 \% \mathrm{Cl}[-.012$, -.008], p < .000) for each successive grade higher. Third, they indicate that the translanguaging rate on average was .85 standard deviations higher ( $\beta=.051,95 \% \mathrm{Cl}[.043$, .059], $\mathrm{p}<.000$ ) in Texas than in California. In addition, they indicate that the translanguaging rate was .15 standard deviations higher ( $\beta=.009,95 \% \mathrm{CI}[.002, .017], \mathrm{p}<.000$ ) when the students were in small groups than when they were in whole class. Fifth, they indicate that the translanguaging rate on average was .7 standard deviations lower ( $\beta=-.042,95 \% \mathrm{CI}[-$ $.060,-.002], \mathrm{p}$ < .05) when the teacher response was promoting translanguaging as compared to permitting and prohibiting translanguaging. Strong correlations were observed between teacher response of promoting and language of instruction of English $(R=.579, p<$ .01). Finally, they indicate that the translanguaging rate on average was .05 standard deviations lower ( $\beta=-.003,95 \% \mathrm{Cl}[-.007, .000], \mathrm{p}<.089$ ) for each unit increase in student

English proficiency. Student Spanish proficiency was not found to be a statistically significant predictor of translanguaging rate. The final model of how translanguaging rate varies as a function of school context and student English proficiency is:

```
RATE = 0.37 + 0.029(LOI) - 0.01(GRADE) + 0.51(STATE) + 0.009(GROUP) -
0.042(TRESPPROMOTE) - 0.029(TRESPPROHIBIT) - 0.003(SELP) +\varepsilon
```


## Table 7: OLS Regression Models Predicting Translanguaging Rate

|  | Model 1 | Model 2 | Model 3 | Model 4 |
| :---: | :---: | :---: | :---: | :---: |
| Fixed Effects | Coef. (SE) | Coef. (SE) | Coef. (SE) | Coef. (SE) |
| Intercept | .018* (.005) | .095*** (.009) | .085*** (.009) | .037** (.012) |
| School Context |  |  |  |  |
| LOI - Spanish | . 078 (.006) |  | .014* (.006) | .029*** (.007) |
| Grade |  |  | -.010*** (.001) | -.010*** (.001) |
| Texas |  |  | .059*** (.004) | .051*** (.004) |
| Small Group |  |  | -.014*** (.003) | .009* (.004) |
| TRESP - Promote |  |  | -.040*** (.005) | -.042*** (.009) |
| TRESP - Prohibit |  |  | . 005 (.010) | -. 029 (.016) |
| Student Language Proficiency |  |  |  |  |
| English |  | -.009* (.004) |  | -. 003 (.002) |
| Random Effects |  |  |  |  |
| SE | . 047 | . 032 | . 035 | . 015 |
| $\mathrm{R}^{2}$ | . 29 | . 07 | . 62 | . 80 |
| F | 177.744 | 6.899 | 118.095 | 53.154 |
| p | . 000 | . 010 | . 000 | . 000 |

### 4.6 Discussion

The location of the schools was the largest predictor of translanguaging rate. Different factors may be playing into how the location influences translanguaging rate. One factor may be different perspectives of bilingualism and bilingual practices in each state. These perspectives may be the result of differing historical legislation despite new legislation being in place at the time of the study. For example, Prop 227, in California, hampered bilingual instruction and thus devalued bilingualism in the general society, previous to Prop 58, whereas Texas has had a longstanding support of bilingual instruction and therefore bilingualism may be viewed more positively in the general society. Setati (2008) indicates similar findings with legislation determining different languages of instruction at the elementary and secondary level resulting in the teachers adopting perspectives reflective of
that legislation. Another factor may be the proximity of the schools to the Mexico-U.S. border and the unique culture that exists in the borderlands that is more receptive of translanguaging (Esquinca et al, 2014). The schools in Texas were much closer to the border than the schools in California and thus the translanguaging rate may have been influenced by this factor. Further exploration of these factors requires greater study; however, it is evident that sociopolitical contexts play a significant role in the translanguaging taking place in mathematics classrooms across the U.S.

The second most influential predictor was the teacher promoting translanguaging which was associated with a lower translanguaging rate than when the teacher permitted or prohibited translanguaging. This is an unexpected finding since it has been previously suggested that if teachers were to promote translanguaging students would use their complete linguistic repertoire (García \& Kleyn, 2016; García \& Sylvan, 2011; Gort \& Pontier, 2013; Otheguy et al., 2015; Palmer et. al., 2014); however, quite the opposite was found. This may be explained by the findings of how language of instruction and student language proficiency predict the translanguaging rate. Language of instruction is a significant predictor of translanguaging rate with a much higher translanguaging rate when the language of instruction was Spanish. While a much less significant predictor of translanguaging rate is the student English proficiency, it is noteworthy that the translanguaging rate is lower with each unit increase in English proficiency. Taken together this means that translanguaging rate increases when the language of instruction is Spanish and with students who have lower proficiency in English. This therefore indicates that translanguaging does not just occur in the direction of the home language like has been found previously (Bose \& Choudhury, 2010; Planas \& Civil, 2013, Salehmohamed \& Rowland, 2014; Setati, 2005; Setati et al., 2002; Tavares, 2015) and that other factors may be influencing translanguaging practices like other studies have found (ex. Setati, 2008; Planas \&Setati, 2009). Setati (2008) and Planas and Setati (2009) both indicate that translanguaging is used as bids to gain capital or status through the use of the language of power. Therefore, since teachers promoting
translanguaging is strongly correlated with the language of instruction in English, translanguaging in this context would be away from the language of power and thus was opposite of how students were using translanguaging to gain symbolic power. As a result, teachers promoting translanguaging was associated with a lower translanguaging rate.

Finally, the grouping of students is a significant predictor of translanguaging rate, with higher translanguaging rate in whole class settings. This finding is unexpected as translanguaging was previously associated with small group settings (Planas \& Setati, 2009). It can be explained by the previous predictors which indicate a higher translanguaging rate when translanguaging towards the language of power and for students with lower proficiency in the language of power. These public displays of the use of the language of power may provide assumed greater benefits in how the student believes they are viewed by others and their association with them (Lin, 2013; Myers-Scotton, 2006) since a larger number of people are able to witness these displays than in a smaller group.

### 4.7 Conclusion

Amid the debates surrounding the use of translanguaging in linguistically and pedagogically complex environments like those found in DL programs, teachers are tasked with fostering "language practices that approximate authentic interactional contexts" (Gort \& Pontier, 2013). This study set out to understand what those authentic translanguaging practices look like in dual-language mathematics classrooms. The findings challenge previous research and theories.

First, they challenge research and theory that indicate that teachers in DL programs prohibit translanguaging and thus imply that if teachers were to permit or promote translanguaging, particularly for language minority students, then students would engage in using their full linguistic repertoire (e.g., García \& Kleyn, 2016; Sommerville \& Faltis, 2019).This study found quite the opposite, in that teachers almost unilaterally were permitting
and promoting translanguaging, yet the students were limiting their own translanguaging to Spanish, while much more freely translanguaging to English. In addition, this study found that when teachers were more permissive and promoting translanguaging, the translanguaging rate dropped. These findings demonstrate that promoting translanguaging will not necessarily have the effect indicated by Garcia and Kleyn (2016).

Secondly, they challenge recent research that has indicated a lack of a causal relationship between language proficiency and translanguaging (e.g., Martínez, 2010). A relationship was found of a different nature than that which was explored previously, that translanguaging occurs to compensate for a bilingual speaker's lack of competence in one or both languages (Cloud et al., 2000; Zentella, 1997). In fact, this study found that despite language competence, the students would translanguage to English, the higher status language as they bid for linguistic capital, like Planas and Setati (2009) found. As such, it also contradicts previous arguments for the power of translanguaging to transform language hierarchies in schools (García \& Kleyn, 2016) since the students used translanguaging to reify those hierarchies.

Third, they challenge previous research indicating that students tend to translanguage less in whole class situations (Planas \& Setati, 2009) as this study found that students translanguage less in small group settings. This finding in combination with the finding that language status and bids for linguistic capital drive translanguaging indicate that the display of linguistic capital is much more common in public settings. In addition, the study demonstrated variability in translanguaging practice across state lines with greater acceptance and participation in translanguaging in Texas. As such these public displays of linguistic capital vary and are influenced by socio-political contexts.

This study has some limitations. First, this study only explored Spanish/English dual language classrooms which place different demands on teachers than in other programs. However, this context provides the opportunity to explore this complex linguistic and pedagogical environment and provides an opportunity to explore the dilemmas of student
translanguaging in a context where all can participate in translanguaging. In addition, this study explored student translanguaging practices only in mathematics and may not be representative of other content areas. However, this was by design because it provided an opportunity to gain an understanding of student translanguaging practices in a content area that is largely unstudied in U.S. classrooms. Secondly, due to the limited number of translanguaging instances by students when the language of instruction was English and by teachers, the relationships between language proficiency and translanguaging rate were only explored for students when the language of instruction was Spanish. Therefore, the findings regarding language proficiency cannot be applied to classrooms where the language of instruction is English and for teachers translanguaging practices. Third, the teacher reported data in the study (e.g., student language proficiency) must be interpreted with caution. However, I recognize that language proficiency can be measured in different ways and all have limitations. The limitations from this means of measuring language proficiency were addressed through triangulation of analyses across different data sources, as well as addressing the inconsistencies found between data sources. This suggests that the data provides an accurate representation of the student language proficiencies. Finally, a random sample for the translanguaging instances was not obtained due to the inability to identify all speakers in the videos. However, efforts were made to sample across the greatest variability in order to approximate a random sample.

The findings from this study have important implications for practice, policy and further research. Due to the contradictions found in this study to previous theory and research, teacher preparation programs and DL programs may be better served by trying to understand students' translanguaging, rather than trying to wrangle their translanguaging practices into alignment with theories of what they should be doing. They may then be able to draw on the authentic functions of students' translanguaging in a way that prepares and empowers students to engage with the societal linguistic market. In addition, further research is necessary to deepen understanding of the relationship between teachers' response,
language proficiency, and language status with students' translanguaging practice, particularly during English instruction in dual-language mathematics classrooms. Also, in light of findings indicating translanguaging serving a social function, further research is necessary to explore the relationships between student translanguaging and academic as well as social-emotional outcomes for students in DL programs. As research, theory and practice support the social functions of translanguaging, students will benefit in and beyond the classroom.

Note:

1. The use of more than one language between bilingual speakers in interactive speech is referred to through a variety of terms of which "code-switching" or "translanguaging" are used most often (MacSwan, 2017). While codeswitching views languages as separate systems, translanguaging views a bilingual speakers' full linguistic repertoire as an integrated system (Canagarajah, 2011), and distinct languages as merely socio-political constructions (Makoni and Pennycook, 2007). However, in the exploration of teacher's beliefs and practices in this paper I must acknowledge "the relationships between what people believe about their language (or other people's languages), the situated forms of talk they deploy, and the material effects- social, economic, environmental-of such views and use" (Makoni \& Pennycook, 2007, p. 22). Since the DL programs in this study promoted the separation of languages, and the teachers referred to the languages as distinct, a description of the beliefs and practices necessitates language representative of the socio-political constructions of distinct languages.
2. MALLI, research funded by a grant from the U.S. Department of Education, Office of English Language Acquisition, National Professional Development Program, Grant \#T365Z170070. MALLI is a professional development program that works to integrate mathematics, language, and literacy in dual language settings. It focused on developing discourse, literacy and vocabulary strategies during mathematics instruction.

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### 4.9 Appendix

### 4.9.1 Appendix A: Video coding scheme

| Primary Code | Description |
| :---: | :---: |
| Teacher ID (TID) | 1-32 |
| State (STATE) | $0=$ California, 1 = Texas |
| Time Stamp (TIME) |  |
| Duration of instance (DURTL) | In seconds |
| Duration of Video (LENGTH) | In seconds |
| Translanguaging Rate (RATE) | Sum of the duration of translanguaging instances for the teacher/duration of video |
| Language of Instruction (LOI) | 0 = English, 1 = Spanish |
| Translanguager (TLER) | $0=$ Student, 1 = Teacher, $2=$ Both teacher and student $3=$ 2+ Students |
| Grouping (GROUP) | 0 = Whole Class, 1 = Small group, 2 = Pair, 3 = Alone |
| Teacher response (TRESP) | 0 = Prohibiting, 1 = Permitting, 2 = Promoting |
| Grade (GRADE) | 0-7 |
| Transcription (TRAN) |  |
| Problem Type (PROB) | Word problem $=1$, Arithmetic Calculation $=2$ Geometry $=3$, <br> Measurement $=4$, Comparing numbers $=5$, Counting $=6$ |
| School Language Policy (LPSCHL) | 0 = Strict, 1 = Flexible |
| Teacher Language Policy (LPTEACH) | 0 = Strict, 1 = Flexible |
| Student Spanish Language Proficiency (SSLP) | 1 = Intermediate, 2 = Advanced, 3 = Distinguished, 4 = Superior |
| Student English <br> Language Proficiency (SELP) | 1 = Intermediate, 2 = Advanced, 3 = Distinguished, 4 = Superior |

## 5 Conclusion

This study reframes the conversation about the purposes, practices, and policies of translanguaging in the dual language elementary mathematics classroom in the U.S. It unravels some of the inconsistencies found in previous research due to the entangled domains and cognitive dissonance that teachers experience with translanguaging in their linguistically and pedagogically complex classrooms. It also provides a more rigorous research design using multiple measures and a broader sample than those found in previous research. As such it provides insight into dual language mathematics teachers' beliefs, and responses to student translanguaging. It reveals how teachers' beliefs and practices respond to the dilemmas they encounter about their own use of translanguaging. Finally, it reveals the authentic functions and frequency of translanguaging practices in dual language mathematics classrooms. The findings from all three of these papers have joint implications for future research, teacher education programs, and schools.

The findings indicate that the previous research regarding teacher beliefs and practices about translanguaging needs to be regularly updated as beliefs and practices have shifted and will most likely continue to shift. There are several implications for future research regarding what needs further exploration, and how that research should be carried out. First, while creating a list of the diverse functions that translanguaging can serve in the classroom is informative, that knowledge can be deepened by an understanding of the frequency of use of those functions, the relationship between them and the variation that occurs across teachers and students. In addition, further research is necessary to deepen understanding of the relationship between teachers' response, language proficiency, and language status with translanguaging practice, particularly during English instruction in dual-language mathematics classrooms. Also, considering findings indicating translanguaging serving primarily a social function, further research is necessary to explore the relationships between translanguaging and academic as well as social-emotional outcomes for students in DL programs. In addition,
further research is necessary to explore how teachers are developing their beliefs about translanguaging and the mechanisms that are at work in discussions with colleagues and analysis of their own classrooms. Secondly, to understand the authentic use of translanguaging, future research of translanguaging in dual language mathematics classrooms would benefit from rigorous descriptive rather than prescriptive research of beliefs and practices, using multiple measures and larger samples across the different contexts in the U.S. This research could then better inform teacher education programs as they develop teachers' understandings of the authentic and complex translanguaging practices in the classroom. It could also inform schools as they design policies around translanguaging to be reflective of these practices and valuing of their contribution to students' academic, linguistic, and social development.

The implications for teacher education programs include how teachers' beliefs and practices about can effectively be developed, and what aspects of translanguaging teachers need to come to understand. First, the findings show that the driving force of the shifts in teachers' beliefs and practices seems to come from the teachers' actual experiences in the schools rather than from theoretical stances held by the fields of second language acquisition or mathematics education or policies held by the school. Therefore, as teacher education programs work to develop teachers understanding and responses to translanguaging, I would advise them to consider utilizing classroom observation and discussions with other teachers as an essential tool in this process. This would best be accomplished through analysis of translanguaging practices in their assigned classrooms and discussions with other teachers in the school where they are familiar with the students and the context. In addition, if teachers are to promote translanguaging, they would likely benefit from exploring specific strategies or clear models of how to use and promote authentic practices in the dual language mathematics classroom. Secondly, teacher education programs should not only introduce a theoretical foundation of translanguaging but help teachers develop beliefs that demonstrate an understanding of the academic and social functions of translanguaging, the
frequency of use of the different functions, the relationship between those functions, and the variation in functions across teachers and students.

Finally, DL schools should not only recognize the socialization that takes place through translanguaging but also acknowledge that translanguaging is taking place despite their policies. This understanding is dependent on both an analysis of their philosophical and theoretical foundation for their policies and how that is reflective of their own or others' rigorous research exploring translanguaging practice. Through this process they can ground their policies in authentic practices which are reflective of the functions that translanguaging serves in the classroom and in students' academic, linguistic, and social development.

Due to the contradictions found in this study to previous theory and research, teacher preparation programs and DL programs may be better served by trying to understand students' translanguaging, rather than trying to wrangle their translanguaging practices into alignment with theories of what they should be doing. Both teacher education and research, must recognize and apply the primary functions of translanguaging in manners that "foster language practices that approximate authentic interaction contexts" (Gort \& Pontier, 2013). If future research, teacher education programs and schools are to support teachers in doing this, a shift in focus is necessary from academic functions to social functions of translanguaging as well as an analysis of the authenticity of some academic functions. Teachers may then be able to draw on the authentic functions of students' translanguaging in a way that prepares and empowers students to engage with the societal linguistic market.

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