

UCLA

UCLA Electronic Theses and Dissertations

Title

Guided Waves in Cylindrical Structures: A Hybrid WFE/Global-Local Framework for Modeling and Defect Detection

Permalink

<https://escholarship.org/uc/item/2q59g5qf>

ISBN

9798191649160

Author

Alsaffar, Fadhel

Publication Date

2026-09-11

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

Guided Waves in Cylindrical Structures: A Hybrid WFE/Global-Local
Framework for Modeling and Defect Detection

A dissertation submitted in partial satisfaction of the
requirements for the degree Doctor of Philosophy
in Mechanical Engineering

by

Fadhel Ali Alsaffar

ABSTRACT OF THE DISSERTATION

Guided Waves in Cylindrical Structures: A Hybrid WFE/Global-Local Framework for Modeling and Defect Detection

by

Fadhel Ali Alsaffar

Doctor of Philosophy in Mechanical Engineering

University of California, Los Angeles, 2026

Professor Ajit K. Mal, Chair

The objective of this work is to develop a versatile nondestructive evaluation (NDE) tool for the design, analysis, and data generation of inspection processes for both metallic and composite pipes. The tool utilizes the waveguide finite element (WFE) method to obtain wave characteristics, including dispersion curves and global functions. The WFE solution is then used in a developed Global-Local (GL) method to model wave propagation in a coupled source-scattering problem. Commercial finite element software is used to obtain the mass and stiffness matrices, as well as to define the geometry of the waveguide and the defect. This feature provides the flexibility to study a variety of defects and multiple material systems.

The hybrid WFE/GL solver is validated through benchmark problems, energy-balance calculations, comparisons with Abaqus solutions, and experimental results for both pristine and symmetrically defected cases. This tool allows wave propagation in pipes to be studied in a more fundamental way, where the contribution of each scattered mode can be examined separately, whether reflected, transmitted, or mode-converted. The fingerprints (signals) associated with multiple types of defects, such as cracks and delaminations, can be obtained, allowing an ultrasonic signal library for these defects to be created.

The dissertation of Fadhel Ali Alsaffar is approved.

Vatsa Gandhi

Vijay Gupta

Lihua Jin

Christoph Schaal

Ajit K. Mal, Committee Chair

University of California, Los Angeles

2026

Contents

Acknowledgments	xi
Vita	xii
1 Nondestructive Evaluation in Piping Systems	1
1.1 Introduction	1
1.2 Liquid Penetrant Testing	2
1.3 Magnetic Particle Testing MPT	4
1.4 Radiographic Testing RT	5
1.5 Eddy Current Testing ECT	6
1.6 Ultrasonics Guided Waves	7
1.7 Defects in Pipes	8
1.7.1 Defects in Metallic Pipes	9
1.7.2 Corrosion	9
1.7.3 Defects in Composite Pipes	10
1.8 The Scope of This Work	12
2 Wave Propagation in Cylindrical Waveguides	14
2.1 Introduction	14
2.2 Theory	15
2.2.1 Solid Cylinder	17

2.2.2	Longitudinal Motion in Cylindrical Solid	19
2.2.3	Hollow Cylinder	20
2.3	Review of Finite Element Based Methods in Waves Propagation	23
2.4	Waveguide Finite Elements (WFE)	24
2.4.1	Formulation	26
2.4.2	Numerical Considerations and the use of Symmetry WFE Half Model	30
2.5	Physics-Informed Tracking	30
2.5.1	Motivation and tracking requirements	30
2.5.2	Conventional MAC-based mode tracking	31
2.5.3	Why individual-vector MAC becomes ambiguous for circular-pipe flex- ural modes	31
2.5.4	Mode veering and small eigengaps	33
2.5.5	Cylindrical Fourier physical signature	33
2.5.6	Complex-wavenumber continuation	35
2.5.7	Composite global assignment	36
2.5.8	Summary	36
2.6	Application	37
2.6.1	Solid Cylinder	37
2.6.2	Isotropic Pipe	38
2.6.3	Carbon-fiber composite pipe	39
2.6.4	Pipe Sandwich structure (Pipe in Pipe)	40
3	Global Local Method (GLM)	42
3.1	Introduction	42
3.2	Formulation	44
3.2.1	Source Problem	44
3.2.2	Scattering Problem	47

3.2.3	Power Flow and Wave Field Representation	49
3.2.4	Modeling Wave Attenuation with Complex Moduli	51
3.3	Nodal Force Vectors	54
3.3.1	Compact Matrix-Vector Product Form	55
4	WFE/Global–Local Method Validation and Cases	57
4.1	Validation	57
4.1.1	Transmission Problem	58
4.1.2	Solid Rod Edge Reflection Problem	59
4.1.3	Transient Response Comparison	60
4.1.4	Experimental signal Verification	66
4.2	Cases	68
4.2.1	Axial Vs Radial Vs Torsional Excitation	68
4.2.2	Woven Composite Repair	71
4.3	Computational time Consideration	72
4.4	Concluding Remarks and Future Work	73
A	Bessel Functions Review	75
B	Separation of Variables in Cylindrical Coordinates	78
	References	81

List of Figures

1.1	Typical crack detection using LPT	4
1.2	Detection of crack girth using MPT.	5
1.3	Radiography testing of small pipes.	6
1.4	Schematic of Basic Eddy current testing	7
1.5	Plain Dent [22]	10
1.6	Cracks and crack like defects [24]	11
1.7	Delamination of glass-fiber composite pipe	12
2.1	WFE model of the cylinder	27
2.2	Solid cylinder waveguide modeled in Abaqus.	37
2.3	Dispersion curves for solid cylinder with $R = 1mm$, group velocity on the right, phase velocity on the left.	38
2.4	In-plane mode shapes of $L(0, 1)$ mode at $100kHz$	38
2.5	Pipe model modeled in Abaqus.	39
2.6	Dispersion curves for aluminum pipe with $R/t = 10$ plotted against analytical solver solution (cyan color). Group velocity on the right, and phase velocity on the left.	39
2.7	In-plane mode shape for a pipe with $R/t = 10$ for $L(0, 1)$, $T(0, 1)$, and $F(1, 1)$	40
2.8	Dispersion curves of a pipe with R/t ratio of 50	40
2.9	Dispersion curves of unidirectional fiber pipe made of 4-layer laminate	41
2.10	Pipe sandwich structure made of aluminum skin and epoxy 5505 core.	41

2.11	Dispersion curves of pipe sandwich structure.	41
3.1	Global local source and scattering problems model	44
3.2	Global local source model	45
3.3	Global local scattering model	47
3.4	decay parameter of the fundamental modes of a 10mm radius pipe.	54
4.1	L(0,1) incident mode on a pipe with a 9mm radius.	58
4.2	Dispersion Curves of 90mm steel bar	60
4.3	End reflection of L(0,1) (left) and F(1,1) (right) incident modes on a solid rod	61
4.4	(a) Length ratio as a function of computation time. (b) number of flexural modes as a function of energy error.	61
4.5	(a) Abaqus pristine Model and (b) defected model	62
4.6	Dispersion Curves using linear (C3D8) elements (a) and quadratic (C3D20) elements	62
4.7	10mm radius tube with circular applied load as processed in the solver . . .	63
4.8	Input signal at the center of the source model.	64
4.9	Time and frequency response for the right going waves and the power flow of the source problem.	64
4.10	Response at the right interface between the FE and GL region for radial and axial displacements.	65
4.11	Radial and axial response at an equal distance between source and defect. .	65
4.12	Comparison of the results with and without the hi-pass filter.	66
4.13	Dispersion Curves for R=40mm pipe with the experimental data shown in group velocity curves.	67
4.14	The home-made EMAT and the experimental setup.	67
4.15	Comparison of the normalized experimental and GL solution signals at 5, 7, and 9in distance from the source.	68

4.16	The right going signals (top) and power flow (bottom) for axial, radial, and tangential excitations, respectively.	69
4.17	Total response and reflection 200mm to the left of FE model.	70
4.18	Aluminum pipe with woven carbon fiber in the middle.	71
4.19	Power flow and received signal for the full replacement case.	72
4.20	Total response and reflection 200mm to the left of FE model.	73
4.21	Computational time of linear elements as function of the length of the radius R	74
A.1	Modified Bessel functions (left) and Bessel functions (right)	77

List of Tables

4.1	Material properties of the woven carbon composite.	71
-----	--	----

Acknowledgments

All praise and thanks are due to Allah, God, the Creator for giving us the opportunity to live, learn, grow, and make a difference.

I would like to express my deep gratitude to Professor Ajit Mal for his continuous help and support throughout my program. He was always available to help with both academic and professional challenges, as well as personal difficulties. I am deeply grateful for his guidance, encouragement, and generosity.

I would also like to thank Professor Christoph Schaal. His sharp and critical thinking played an important role in making this work possible. He was not merely my co-advisor; he made sure that our relationship evolved from that of student and mentor, to colleagues, and ultimately to friendship. I am grateful to my committee members, Professor Vijay Gupta, Professor Vatsa Ghandhi, and Professor Lihua Jin, for their valuable feedback, careful review, and kind words of support.

Special thanks go to Dr. Steffen Tai and Dr. Lifu Wang for the many hours of discussion about this work and about PhD life in general. I am also thankful to my lab mates, Mattie Green and Gautham Viswaroopan, for their support and companionship. Mattie has always been a thoughtful and grounded colleague, with a remarkable ability to see things clearly and keep them in perspective.

Finally, I am deeply grateful to my wife, Zainab, who has been by my side throughout this entire journey. She has endured a great deal with patience, love, and unwavering support. May Allah reward her abundantly for her patience, sacrifice, and constant encouragement.

Vita

Education

B.S. in Mechanical Engineering, KFUPM, KSA, 2014

M.S. in Mechanical Engineering, UCLA, Los Angeles, 2020

Selected Academic and Professional Experience

Graduate Researcher, UCLA, 2021 – 2026

Teaching Fellow, UCLA 2021 – 2026

Research Specialist, Center of Excellence for Green Nanotechnologies, KSA 2015-2018

Selected Publications and Presentations

1. Green, M., Alsaffar, F., Mal, A., & Schaal, C. (2026). Rayleigh damping for approximating Lamb wave attenuation in finite element simulations. *Ultrasonics*, 108209.
2. Green, M., Alsaffar, F., Mal, A., & Schaal, C. (2025). Effect of Material Attenuation on Lamb Wave Propagation in a Solar Cell Array. *Structural Health Monitoring* 2025.
3. Alsaffar, F., Li, C. X., Jin, B. C., Mal, A., & Schaal, C. (2024, October). Investigation of Lamb Wave Characteristics in Composite Oriented Strand Board Plates. In *American Society for Composites (ASC) Annual Technical Conference* (pp. 211-221). Cham: Springer Nature Switzerland.
4. Alsaffar, F., Wang, L., Mal, A., & Schaal, C. (2023, July). Guided Waves in Pipes Versus Lamb Waves in Plates-A Convergence Analysis. In *49th Annual Review of Progress in Quantitative Nondestructive Evaluation* (Vol. 86595, p. V001T03A005). American Society of Mechanical Engineers.
5. Alsulami, A., Alharbi, M., Alsaffar, F., Alolaiyan, O., Aljalham, G., Albawardi, S., ... & Amer, M. R. (2023). Lattice transformation from 2D to quasi 1D and phonon properties of exfoliated ZrS₂ and ZrSe₂. *Small*, 19(11), 2205763.

6. Alrasheed, A., Gorham, J. M., Tran Khac, B. C., Alsaffar, F., DelRio, F. W., Chung, K. H., & Amer, M. R. (2018). Surface properties of laser-treated molybdenum disulfide nanosheets for optoelectronic applications. *ACS applied materials & interfaces*, 10(21), 18104-18112.
7. Alsaffar, F., Alodan, S., Alrasheed, A., Alhussain, A., Alrubaiq, N., Abbas, A., & Amer, M. R. (2017). Raman sensitive degradation and etching dynamics of exfoliated black phosphorus. *Scientific reports*, 7(1), 44540.

Chapter 1

Nondestructive Evaluation in Piping Systems

1.1 Introduction

Cylindrical structures form an integral part of modern life, ranging from large pipelines that transport water over hundreds of kilometers to pipelines used for the transportation of oil and gas, as well as process lines that carry liquid or gaseous products. This makes pipes and pipelines, in general, an essential part of modern infrastructure, and therefore their inspection is of paramount importance.

The term Nondestructive Evaluation (NDE) is often used interchangeably with terms such as Nondestructive Testing (NDT) and Nondestructive Inspection (NDI). Although each carries a slightly different emphasis, they share the same core meaning. They all focus on detecting defects but NDE goes further into classifying, characterizing and sizing the defect. The terminology has also evolved as the field has matured. The current definition provided by the American Society for Nondestructive Testing (ASNT) describes NDE as the profession and set of scientific inspection methods used to evaluate components, structures, and materials for flaws without causing damage, protecting the public, helping prevent failures,

reducing production costs, and keeping critical infrastructure running [1].

An interesting example of everyday NDE is buying a watermelon! First, we check its color and appearance, which serves as the visual test. Then, we tap on the surface, listening for a hollow sound—that’s the acoustic test! Finally, we analyze these tests based on our knowledge and experience to decide whether or not to buy it [2]. Just like in this example, there exists many types of NDE test methods and are used at various stages and applications. The selection of a specific nondestructive evaluation (NDE) method depends on the application, the material being inspected, the type and location of the expected defects, the geometry of the component, and the required inspection sensitivity. Since this work focuses primarily on pipes and cylindrical structures, greater emphasis will be placed on NDE techniques commonly used for these applications.

This chapter briefly reviews the most common NDE methods applicable to piping systems, including liquid penetrant testing (PT), magnetic particle testing (MT), radiographic testing (RT), Eddy-current testing (ECT), and guided-waves ultrasonic testing (GUT). All of these methods with exception of GUT are conventional methods in the sense that they scan relatively small areas or region. However, GUT has the advantage of traveling large distance and thus scanning larger areas.

1.2 Liquid Penetrant Testing

Liquid penetrant testing (LPT) is one of basic and simplest NDE tests that is used to detect surface cracks. It does not require that much training or equipment. The basic theory behind penetrant testing is based on capillary attraction. Capillary attraction is a surface-tension phenomenon that depends on surface energy and allows a liquid to be drawn into tight openings. Penetrant testing uses a brightly colored liquid with good wetting characteristics to penetrate discontinuities in nonporous solid materials. The excess liquid is then removed, and a developer powder is applied. The developer draws the penetrant out

of the discontinuity, such as a crack, and brings it to the surface, making the discontinuity visible.

A very important concept in LP is the ability to wet the surface which is measured by the contact angle. It is the angle formed where a liquid droplet meets a solid surface. It is measured through the liquid between the tangent to the liquid surface and the solid surface at the three-phase contact point. A small contact angle indicates good wetting, while a large contact angle indicates poor wetting.

Contact angle is important in penetrant testing because good wetting helps the penetrant spread across the test surface and enter surface-breaking discontinuities. However, penetration is not controlled by contact angle alone; surface tension, capillary action, cleanliness of the surface, and the geometry of the discontinuity also affect how effectively the penetrant enters an opening.

Other important penetrant characteristics include dye concentration, which affects the visibility and contrast of indications, and viscosity, which influences how easily the penetrant flows and spreads. In practice, the penetrant must also remain in contact with the surface for a sufficient dwell time so that it can enter and fill open discontinuities. Contamination, moisture, or improper cleaning can block these openings and significantly reduce inspection sensitivity.

Typically, LPT can be applied to any material that has a relatively smooth, non-porous surface. Parts with rough surface that can trap or hold the penetrant can not be tested. Porous materials such as ceramics, or fibrous materials cannot be tested as well.

In piping systems, LPT has shown to be effective specifically to piping welds. LPT is usually applied to welded carbon-steel pipes with the application a cleaner, penetrant, and developer system. Fig. 1.1 shows a typical example of circumferential crack found on the surface near welding zone and a crack found on a 316L stainless steel bend pipe[3], [4].

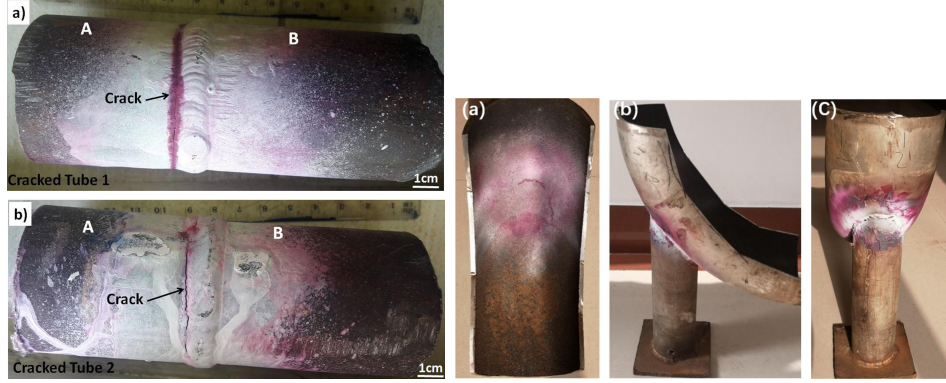


Figure 1.1: Typical crack detection using LPT

1.3 Magnetic Particle Testing MPT

Magnetic particle testing (also called magnetic particle inspection, MPI) is a very powerful technique for the inspection of near-surface cracks in ferromagnetic materials. Its working principle can be divided into three main steps: magnetizing the sample to be tested, applying fine ferromagnetic particles to flow over the surface, and detecting any surface or subsurface discontinuity that disturbs the magnetic field and produces magnetic flux leakage, resulting in the accumulation of magnetic particles around the edges of the discontinuity.

Materials, in terms of magnetization, fall into three categories: paramagnetic, diamagnetic, and ferromagnetic. Both paramagnetic and diamagnetic materials have small relative magnetic permeability, where the former has a positive value and the latter has a negative value. These two materials can be only weakly magnetized and cannot be used for MPT. Ferromagnetic materials on the other hand have high relative permeability and they retain the magnetization upon the removal of an applied magnetic field. Materials with a high content of iron, nickel, or cobalt are suitable for magnetic particle testing since they can be easily magnetized, while materials such as aluminum, brass, titanium, and austenitic stainless steel are not applicable to this technique. A high relative permeability of 100 or more is required for it to work [2].

One important application of MPT is in pipeline girth weld inspection. Cracks in these welds develop due to weld imperfections, stress corrosion cracking, or fatigue [5]. Fluoro-

rescent particles can be used to enhance the detection of accumulated particles around cracks [6]. Fig. 1.2 shows an example of crack girth detection and small tube inspection using MPT [6], [7].

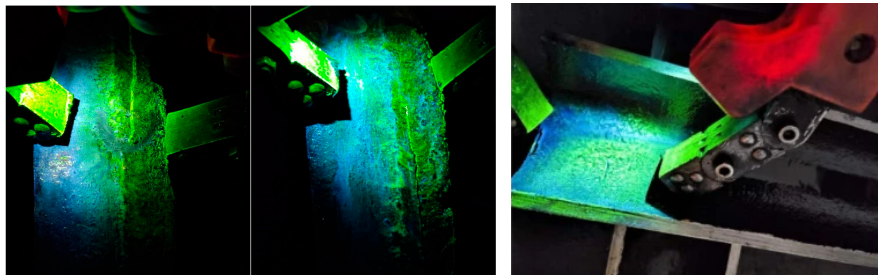


Figure 1.2: Detection of crack girth using MPT.

Magnetic particle testing can be applied during fabrication inspection, maintenance, weld repair, and failure investigations of carbon-steel piping. It is well suited for examining weld toes, heat-affected zones, girth welds, and other locations where surface cracking is expected. However, its restriction to ferromagnetic materials and its limited depth of detection mean that it is commonly combined with ultrasonic, phased-array ultrasonic, or radiographic examination when a comprehensive evaluation of pipe integrity is required.

1.4 Radiographic Testing RT

Radiography is one of the most common methods used in NDE. It is probably the only technique that provides information about both the interior and exterior of a material. Its applications in industry range from tiny electronic components to large missile parts. It employs highly penetrating X-rays, gamma rays, and other forms of radiation to scan the material without causing any damage. X-rays and gamma rays are high-energy (short-wavelength) electromagnetic waves and are typically in the range of keV to MeV, compared with 1.8–3.3 eV for visible light [2]. Gamma rays have the advantage over X-rays is that it's generated from radioactive materials which allows for portable on-site inspection and the ability to get high intensity radiation using materials like Co-60.

The basic principle of film radiography is that when the test specimen is placed between the radiation source and the film, the thicker parts of the specimen block more energy, while more energy passes through the thinner parts. The film density then varies such that thicker areas appear lighter and thinner areas appear darker. Also, for a defect to be detected, it should have an adequate change in thickness so that the intensity of the transmitted radiation differs from that in the surrounding regions.

Radiography is used in pipes at different stages: as an inspection and approval criteria during manufacturing and for in-service pipes to inspect for wall-thickness reduction or for potential cracks. Fig. 1.3 shows an schematic of scanning method and an image of scanned part.

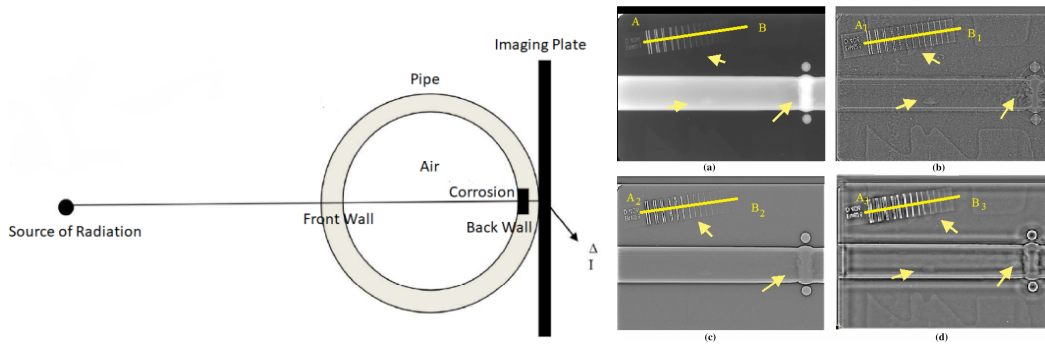


Figure 1.3: Radiography testing of small pipes.

1.5 Eddy Current Testing ECT

Eddy current testing is a method that measures a material's response to an electromagnetic field. It is based on the principle of electromagnetic induction. When a conductive material is exposed to a varying primary magnetic field, a small induced current is generated in the material, which creates a secondary magnetic field that opposes the primary field. An eddy current testing setup is achieved by using probes that are composed of two main coils: a primary coil for excitation and a pick-up coil for receiving the response. An AC current is passed through the primary coil to create the interrogating magnetic field. The pick-up coil

then measures the secondary magnetic field generated in the material due to the induced current. Fig. 1.4 shows a schematic of basic set-up and configuration of the probe [2]. It's important to note that ECU is only applicable to conductive material.

The path of the induced current in the material determines the detectability of surface and sub-surface defects. One of the measure of how deep ECT can go is the skin depth δ which determines the depth at which signal power drops to 37%. The pick-up coil 'inductor' in the probe measures the inductor's impedance Z as result of the secondary magnetic field coming from the material. Each material has specific behavior and finger print depending on it electrical conductivity, magnetic permeability and amount of solid material near the probe. These properties result on unique impedance diagram (Resistance vs inductive Reactance) as the probe is moved farther from the test specimen (lift-off distance).

Eddy current is widely used in testing wall thickness of pipes and tubes. They are widely used for heat exchanger tubes inspection as the probes are well-suited to go deep inside small diameter tubes.

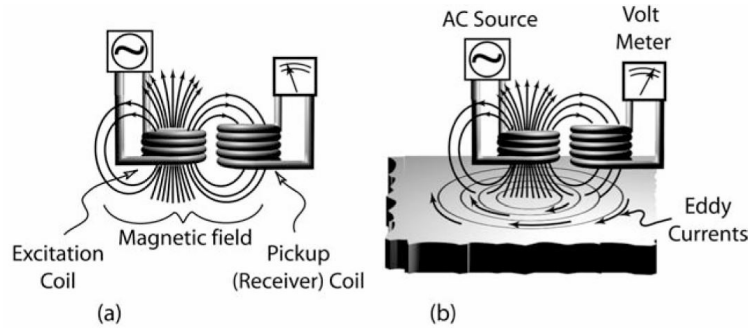


Figure 1.4: Schematic of Basic Eddy current testing

1.6 Ultrasonics Guided Waves

Ultrasonics inspection has been used for a long time for ample applications. One of their traditional uses nondestructive application field is the scan across the thickness in a pulse echo or pitch-catch configuration to inspect for defects [8]. However, it's possible for the

ultrasonics waves to be guided by the surface and propagate parallel to the guide. These types of waves are called guided waves and they have the potential to propagate for long distances and thus potentially allow to scan large areas. Worlton [9] was the first to see the potential of guided waves for nondestructive testing. He used Lamb waves to detect defects in a metallic plate. Research followed and grew over the years. Pochhammer and Chree investigated waves propagating in a free solid cylinder and derived the well-known Pochhammer-Chree dispersion equation [10]. Gazis [11] was the first to derive the equations for free propagating waves in elastic hollow cylinders (even though it contained some typographical errors that were found by other researchers). Wave propagation in pipes have been under investigation for years and have made major progress.

Peter Cawley and his colleges at Imperial college have made extensive experimental studies in pipes where they investigated various wave modes and their interaction with defects and under multiple boundary conditions such as buried steel pipes, fluid filled pipes, coated and other configurations [12], [13], [14], [15], [16]. Alleyne studied the propagation of guided waves for pipes of diameter range of 2-24in and the sensitivity to thickness reduction from corrosion [17]. It's recommended to choose lower frequencies when long range inspection is targeted to reduce wave attenuation. Barshinger studied frequency tuning and mode selection for 4in diameter steel coated pipes [18].

All of the earlier work suggests that a clear understanding of how waves propagate in pipes is essential for a guided waves NDE-based tool. It's also important for this tool to be able to capture the interaction of these waves with various potential defects, boundary conditions and material systems.

1.7 Defects in Pipes

Defects in piping structures can be very serious and costly depending on the application area and the sensitivity of the transported materials.

They're made of various materials and in variety of scales depending on the application. Inspection of these structures is very important to maintain their functionality. Guided waves have high potentials for large scale inspection due to their long range propagation distances [19]. Since each material system is vulnerable to different forms of defects, defects in metals and composites are covered in two separate sections.

1.7.1 Defects in Metallic Pipes

In this section, the common defects in metallic pipes is covered. Defects can arise from multiple factors. They can result from external interference which refers to the impact accidents with foreign objects carried by a third-party activities [20]. An example of these accidents can be a project construction or road works. Another common source of defects is improper construction and installation such as materials handling or laying heavy pipes on hard rocks which can result in dents or cracks.

Dents

Dents are characterized by their radii, depth, longitudinal and circumferential orientations. They are very common mechanical defect. Dents are not considered severe defects (especially shallow dents) as the pipe can still be used and sustain pressure. However, they act as stress concentration sites that the crack can initiate at. Fig 1.5 shows a schematic of typical plain dent on the surface of the pipe. The effect of dents increase as the radius to thickness ratio increase. Thus, thinner pipes with large diameters are more sensitive to burst failures from dents [21]. According to ASME B31.8, the allowed dent strain level is 6%.

1.7.2 Corrosion

Corrosion is an electrochemical reaction that causes material loss. Some codes and standards treat it as a local thinning of pipe wall thickness. Corrosion defects are irregular 3D material loss that are described in pipes as longitudinal, circumferential, and radial dimensions[23].

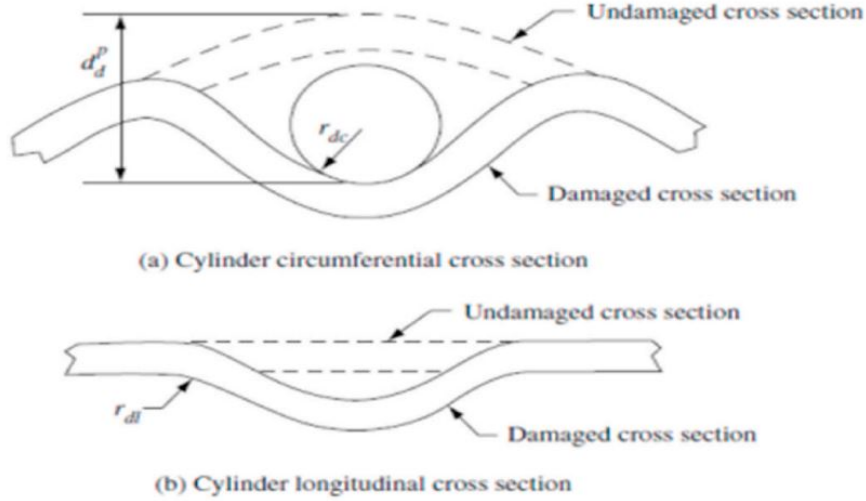


Figure 1.5: Plain Dent [22]

Corrosion failures account for about 20% of the serious accidents in liquid pipelines in the US.

Crack-like defects

A crack is a discontinuity in the materials. Fig 1.6 shows several types of crack-like defects. It's the main cause of pipeline rupture [20]. They can be a result of a corrosive environment which is a common in petrochemical plants and referred to as stress corrosion cracking. They usually appear as multiple cracks branching from the main one. . Also, welding zone can be a potential location for crack formation due to the effect of the heat affected zone.

1.7.3 Defects in Composite Pipes

Corrosion is considered one of the main drawbacks for metallic pipes especially in highly corrosive environments such as petrochemical plants and offshore application. Also, heavy weight of metals can be a drawback for cases where weight is an important factor. Fiber reinforced polymers (FRP) composite pipes provide an alternative solution to these issues. The market for FRP composite pipes is in a continuous growth and is estimated to reach USD 6.3 billion by 2030 [25]. FRP pipes have applications in oil and gas industries [26],

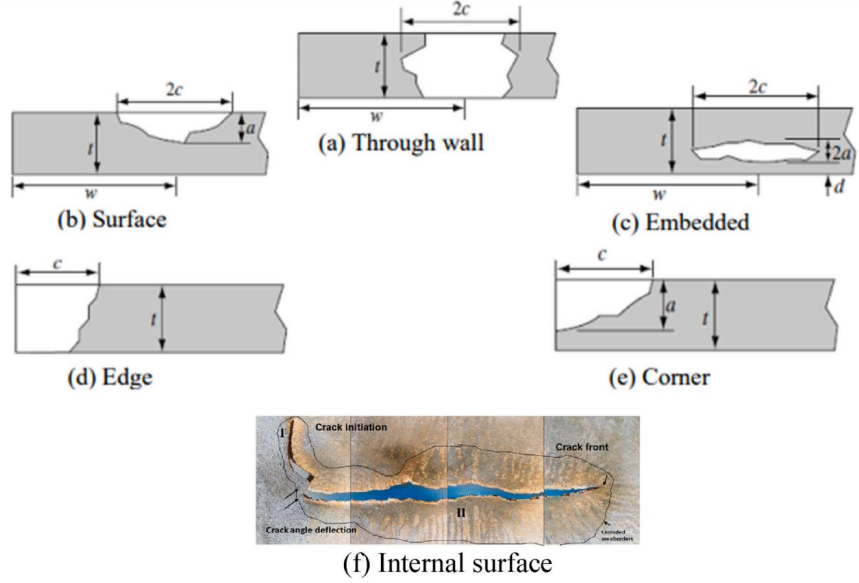


Figure 1.6: Cracks and crack like defects [24]

water transportation and sewage systems [27], and desalination plants [28]. Composites have their own drawbacks and experience different types of defects. These defects are dependent on application and the materials used. In this section, the most common defects will be discussed briefly.

Waviness

Waviness (also wrinkling) occurs due to misalignment in the fiber. It's a type of defect that occurs during the manufacturing phase of FRP. It can result in compromising the composite and reducing the stiffness.

Porosity

Porosity is another common manufacturing defect and refers to the existence of voids or air within the composite.

Delamination

Delamination is one of the most common in-service defects in composites. It's the detachment of layers within the composite. It can result from excessive stresses or low-velocity impact. It's considered a sever defect and can result in premature failure. Fig. 1.7 an example of delamination in glass fiber composite pipe [29].

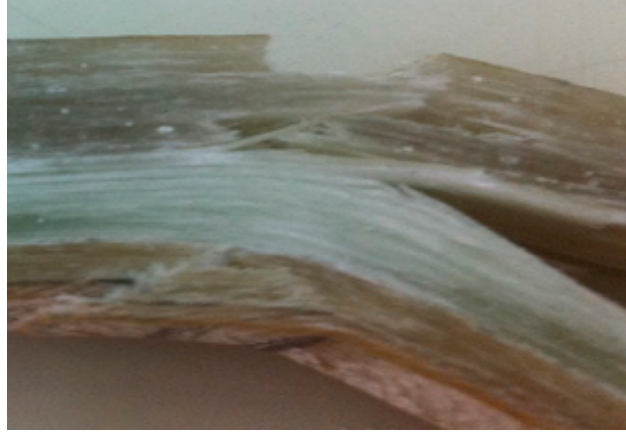


Figure 1.7: Delamination of glass-fiber composite pipe

1.8 The Scope of This Work

The objective of this work is to develop a versatile nondestructive evaluation (NDE) tool for the purpose of inspecting both metallic and composite pipes. It utilizes wave guide finite element (WFE) method to obtain waves characteristics (dispersion curves and modal displacements). The WFE solution is then used in a developed Global Local (GL) method to model wave propagation in a coupled source-scattering problem. A commercial finite element software is used to obtain the mass and stiffness matrix,the geometry of the waveguide and the defect. This feature gives the flexibility to study variety of defects and multiple material systems. The WFE solution is validated for both solid and hollow cylinders using a numerical solver based analytical solution. The plan is use this tool to study mode conversion caused by multiple defects such as cracks and delamination and create an ultrasonics library for the

fingerprint (e.g. signal) of each defect.

Chapter 2

Wave Propagation in Cylindrical Waveguides

2.1 Introduction

Studying the dynamic response of cylindrical structures is important for assessing their condition and characterizing their material and structural properties. Dynamic analysis can generally be classified into two categories based on the frequency of the applied load: low-frequency and high-frequency problems. Low-frequency problems, typically ranging from a few hertz to several hundred hertz, are commonly treated within the framework of structural dynamics, where the analysis primarily focuses on the steady-state response. In such cases, the first few vibration modes are usually of greatest interest [30]. It is worth noting that the wavelengths associated with these problems are generally much larger than the thickness of the structure.

In contrast, for high-frequency problems, typically in the kilohertz range or higher, the transient response becomes increasingly important, and higher-order modes can play a significant role in wave amplitude and propagation characteristics. Problems that fall within this category include excitation by high-frequency transducers, impact loading, and

blast-type loading.

Wave propagation in cylindrical structures is reviewed in this chapter. First, the analytical solution is derived, and the physical characteristics of the different propagating modes are discussed. In the subsequent sections, an alternative numerical approach is introduced and formulated. Finally, the application of the numerical method to isotropic and quasi-isotropic composite pipes is demonstrated.

2.2 Theory

A necessary review of Bessel functions is provided in the appendix. The Navier's equation for isotropic materials is defined by

$$(\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{u}) - \mu\nabla \times \nabla \times \mathbf{u} + \mathbf{f} = \rho\ddot{\mathbf{u}} \quad (2.1)$$

Equation (2.1) can be rewritten as (in the absence of body forces):

$$c_1^2\nabla\nabla \cdot \mathbf{u} - c_2^2\nabla \times \nabla \times \mathbf{u} - \ddot{\mathbf{u}} = 0 \quad (2.2)$$

Introducing the scalar potential functions Φ , Ψ , and χ :

$$\mathbf{u} = \nabla\Phi + \nabla \times (\hat{\mathbf{e}}_z\Psi) + \nabla \times \{\nabla \times (\hat{\mathbf{e}}_z\chi)\} \quad (2.3)$$

for the equation of motion 2.2 to be satisfied, then the sufficient conditions require:

$$\begin{aligned} \nabla^2\Phi - c_1^{-2}\ddot{\Phi} &= 0 \\ \nabla^2\Psi - c_2^{-2}\ddot{\Psi} &= 0 \\ \nabla^2\chi - c_2^{-2}\ddot{\chi} &= 0 \end{aligned} \quad (2.4)$$

where:

$$\begin{aligned}\nabla &= \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_\theta \frac{\partial}{r \partial \theta} + \mathbf{e}_z \frac{\partial}{\partial z} \\ \nabla^2 &= \frac{1}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{r \partial \theta^2} + \frac{\partial^2}{\partial z^2}\end{aligned}\tag{2.5}$$

Equation (2.4) is the wave equations written in terms of the potential functions. For harmonic disturbances of the type $e^{-i\omega t}$, the wave equation (2.4) reduces to the Helmholtz equation and can be solved using the separation of variables. So, the wave equations shown in equation 2.4 becomes:

$$\begin{aligned}\Phi(r, \theta, z, t) &= \Phi_1(r) \Phi_2(\theta) \Phi_3(z) e^{-i\omega t} \\ \Psi(r, \theta, z, t) &= \Psi_1(r) \Psi_2(\theta) \Psi_3(z) e^{-i\omega t} \\ \chi(r, \theta, z, t) &= \chi_1(r) \chi_2(\theta) \chi_3(z) e^{-i\omega t}\end{aligned}\tag{2.6}$$

These separation functions are be found to be (see Appendix B):

$$\begin{aligned}\Phi_1(r) &= A_\phi(k) J_n(\nu_1 r) + B_\phi(k) Y_n(\nu_1 r) \\ \Phi_2(\theta) &= e^{\pm i n \theta}, \quad \Phi_3(z) = e^{\pm i k z} \\ \Psi_1(r) &= A_\psi(k) J_n(\nu_2 r) + B_\psi(k) Y_n(\nu_2 r) \\ \Psi_2(\theta) &= e^{\pm i n \theta}, \quad \Psi_3(z) = e^{\pm i k z} \\ \chi_1(r) &= A_\chi(k) J_n(\nu_2 r) + B_\chi(k) Y_n(\nu_2 r) \\ \chi_2(\theta) &= e^{\pm i n \theta}, \quad \chi_3(z) = e^{\pm i k z}\end{aligned}\tag{2.7}$$

where:

$$\nu_m = \sqrt{k_m^2 - k^2}, \quad k_m = \frac{\omega}{c_m}, \quad m = 1, 2$$

where n, k are separation constants.

The displacement vectors and stress tensors in cylindrical coordinates in terms of the

potential functions Φ, Ψ, χ are:

$$\begin{aligned}
u_r &= \frac{\partial \Phi}{\partial r} + \frac{1}{r} \frac{\partial \Psi}{\partial \theta} + \frac{\partial^2 \chi}{\partial r \partial z} \\
u_\theta &= \frac{1}{r} \frac{\partial \Phi}{\partial \theta} - \frac{\partial \Psi}{\partial r} + \frac{1}{r} \frac{\partial^2 \chi}{\partial \theta \partial z} \\
u_z &= \frac{\partial \Phi}{\partial z} + \left(\frac{\partial^2}{\partial z^2} + k_2^2 \right) \chi
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
\frac{\sigma_{rr}}{\mu} &= (2k_1^2 - k_2^2) \Phi + 2 \frac{\partial u_r}{\partial r} \\
\frac{\sigma_{\theta\theta}}{\mu} &= (2k_1^2 - k_2^2) \Phi + \frac{2}{r} (u_r + \frac{\partial u_\theta}{\partial \theta}) \\
\frac{\sigma_{zz}}{\mu} &= (2k_1^2 - k_2^2) \Phi + 2 \frac{\partial u_z}{\partial z} \\
\frac{\sigma_{r\theta}}{\mu} &= \frac{1}{r} \frac{\partial u_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) \\
\frac{\sigma_{\theta z}}{\mu} &= \frac{\partial u_\theta}{\partial z} + \frac{1}{R} \frac{\partial u_z}{\partial \theta} \\
\frac{\sigma_{rz}}{\mu} &= \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}
\end{aligned} \tag{2.9}$$

2.2.1 Solid Cylinder

For a solid cylinder, the potential functions in equations 2.7 have to be defined at $r = 0$. This requires the constant B to be zero. We will study the axisymmetric cases in which we only have either torsional or longitudinal waves. For these cases, n has to be zero to eliminate the dependence on θ . The potential functions take the form:

$$\begin{aligned}
\Phi(r, z) &= A_\phi J_0(v_1 r) e^{ikz} \\
\Psi(r, z) &= A_\psi J_0(v_2 r) e^{ikz} \\
\chi(r, z) &= A_\chi J_0(v_2 r) e^{ikz}
\end{aligned} \tag{2.10}$$

Torsional Motion in Cylindrical Solid

In this section the torsional waves in a solid cylinder is found. Let's assume a cylinder that occupies the region $0 \leq r \leq a$, $0 \leq \theta \leq 2\pi$, and $-\infty < z < \infty$ and consider the axisymmetric motion such that (using equation (2.8)):

$$u_r = u_z = 0, \quad u_\theta = -\frac{\partial \Psi}{\partial r}, \quad \frac{\partial}{\partial \theta}(u_\theta) = 0 \quad (2.11)$$

The traction-free boundary condition is used at $r = a$ such that:

$$\sigma_{r\theta}/\mu = \frac{1}{r} \frac{\partial u_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) = r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) = -r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \Psi}{\partial r} \right) = 0 \quad (2.12)$$

The equation of motion is satisfied if:

$$\begin{aligned} (\nabla^2 + k_2^2)\Psi &= 0, \\ \longrightarrow \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Psi}{\partial r} \right) + \frac{\partial^2 \Psi}{\partial z^2} + k_2^2 \Psi &= 0 \end{aligned} \quad (2.13)$$

The solution of equation (2.13) that is finite at $r = 0$ is:

$$\Psi(r, z) = A(k) J_0(\nu_2 r) e^{ikz} \quad (2.14)$$

where k is a separation constant, $A(k)$ is an unknown amplitude and $\nu_2 = \sqrt{k_2^2 - k^2}$. Applying the boundary condition (2.12) give:

$$-r \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} [J_0(\nu_2 r)] \right] \Big|_{r=a} = 0 \quad (2.15)$$

This equation can be written as (differentiating rules for Bessel functions):

$$J_2(\nu_2 a) = 0 \quad (2.16)$$

This equation resembles the dispersion equation for torsional waves in solid cylinders. The

solution of the dispersion equation is:

$$\begin{aligned}\nu_2 a &= 0, 5.14, 8.42, 11.26, 14.8, 18.0, \dots \\ \nu_2 a &= x_m, \quad m = 1, 2, \dots\end{aligned}\tag{2.17}$$

where: x_m is the root of $J_2 = 0$ and it resembles the torsional mode order. For the first mode (e.g. $x_0 = 0$) it can be shown that it's nondispersive:

$$\frac{\omega^2}{c_2^2} - \frac{\omega^2}{c^2} = 0 \longrightarrow c = c_2$$

The displacement can be found to be:

$$u_\theta(r) = J_1\left(x_m \frac{r}{a}\right)\tag{2.18}$$

2.2.2 Longitudinal Motion in Cylindrical Solid

In this section, the longitudinal waves is derived. We assume axisymmetric motion (e.g. $u_\theta = \partial u / \partial \theta = 0$ and $n = 0$)

$$\mathbf{u}(r, z) = u_r(r, z)\hat{\mathbf{e}}_r + u_z(r, z)\hat{\mathbf{e}}_z$$

Since $u_\theta = 0$, we see that $\Psi = 0$.

The displacement equations become:

$$\begin{aligned}u_r(r, z) &= -\left[A_1\nu_1 J_1(\nu_1 r) - iA_3 k \nu_2 J_1(\nu_2 r)\right] e^{ikz} \\ u_z(r, z) &= \left[iA_1 k J_0(\nu_1 r) - A_3 \nu_2^2 J_0(\nu_2 r)\right] e^{ikz}\end{aligned}\tag{2.19}$$

The boundary conditions are $\sigma_{rr} = \sigma_{rz} = 0$ at $r = a$.

$$\begin{aligned}\frac{\sigma_{rr}}{\mu} = 0 &= \left[\left[(2k_1^2 - k_2^2) J_0(\nu_1 r) + 2\nu_1^2 J_0''(\nu_1 r) \right] A_1 + 2ik A_3 \nu_2^2 J_0''(\nu_2 r) \right] e^{ikz} \\ \frac{\sigma_{rz}}{\mu} = 0 &= \left[-2ik\nu_1 J_1(\nu_1 r) A_1 + (2k^2 - k_2^2) \nu_2 J_1(\nu_2 r) A_3 \right] e^{ikz}\end{aligned}\quad (2.20)$$

Using these two boundary conditions, we can write:

$$\begin{bmatrix} (2k_1^2 - k_2^2) J_0(\nu_1 r) + 2\nu_1^2 J_0''(\nu_1 r) & 2ik A_3 \nu_2^2 J_0''(\nu_2 r) \\ -2ik\nu_1 J_1(\nu_1 r) & (2k^2 - k_2^2) \nu_2 J_1(\nu_2 r) \end{bmatrix} \begin{Bmatrix} A_\phi \\ A_\chi \end{Bmatrix} = \mathbf{0} \quad (2.21)$$

$$J_0''(\nu_2 r) = \nu_2^2 \left(\frac{3}{r} J_1(\nu_2 r) - J_0(\nu_2 r) \right)$$

The determinant of equation 2.21 is the Pochhammer equation and it gives the dispersion curves for axisymmetric mode.

2.2.3 Hollow Cylinder

In this section we will derive the dispersion equations for hollow cylinders or pipes. For a hollow cylinder case, no simplification in the potential functions in 2.7 and the complete form is used.

We will study the axisymmetric cases in which we only have either torsional or longitudinal waves. For these cases, n has to be zero to eliminate the dependence on θ .

The potential functions take the form:

$$\begin{aligned}\Phi(r, z) &= \left[A_\phi(k) J_0(\nu_1 r) + B_\phi(k) Y_0(\nu_1 r) \right] e^{ikz} \\ \Psi(r, z) &= \left[A_\psi(k) J_0(\nu_2 r) + B_\psi(k) Y_0(\nu_2 r) \right] e^{ikz} \\ \chi(r, z) &= \left[A_\chi(k) J_0(\nu_2 r) + B_\chi(k) Y_0(\nu_2 r) \right] e^{ikz}\end{aligned}\quad (2.22)$$

Torsional Waves in Hollow Cylinders

Assume a displacement of the form:

$$u_r = u_z = 0, \quad u_\theta = -\frac{\partial \Psi}{\partial r} \quad (2.23)$$

The displacement (u_θ) and torsional stress $\sigma_{r\theta}$ expressions is:

$$\begin{aligned} u_\theta &= \nu_2 \left[A_2 J_1(\nu_2 r) + B_2 Y_1(\nu_2 r) \right] e^{ikz} \\ \frac{\sigma_{r\theta}}{\mu} &= \frac{\nu_2}{r} \left[[\nu_2 r J_0(\nu_2 r) + J_1(\nu_2 r)] A_2 + [\nu_2 r Y_0(\nu_2 r) + Y_1(\nu_2 r)] B_2 \right] e^{ikz} \end{aligned} \quad (2.24)$$

The boundary condition at $r = a$ and $r = b$ is $\sigma_{r\theta} = 0$.

$$\nu_2 \begin{bmatrix} \nu_2 J_0(\nu_2 a) - \frac{2}{a} J_1(\nu_2 a) & \nu_2 Y_0(\nu_2 a) - \frac{2}{a} Y_1(\nu_2 a) \\ \nu_2 J_0(\nu_2 b) - \frac{2}{b} J_1(\nu_2 b) & \nu_2 Y_0(\nu_2 b) - \frac{2}{b} Y_1(\nu_2 b) \end{bmatrix} \begin{Bmatrix} A_2 \\ B_2 \end{Bmatrix} = \mathbf{0} \quad (2.25)$$

Longitudinal Waves in Hollow Cylinders

Assume a displacement of this form:

$$\mathbf{u}(r, z) = u_r(r, z) \hat{\mathbf{e}}_r + u_z(r, z) \hat{\mathbf{e}}_z \quad (2.26)$$

The displacements u_r and u_z are found to be:

$$\begin{aligned}
u_r(r, z) &= \frac{\partial \Phi}{r} + \frac{\partial^2 \chi}{\partial r^2}, \quad u_z(r, z) = \frac{\partial \Phi}{z} + \frac{\partial^2 \chi}{\partial z^2} + k_2^2 \chi \\
u_r(r, z) &= - \left[\nu_1 [J_1(\nu_1 r) A_\phi(k) + Y_1(\nu_1 r) B_\phi(k)] \right. \\
&\quad \left. + i k [J_1(\nu_2 r) A_\chi(k) + Y_1(\nu_2 r) B_\chi(k)] \right] e^{ikz} \\
u_z(r, z) &= \left[i k [J_0(\nu_1 r) A_\phi(k) + Y_0(\nu_1 r) B_\phi(k)] \right. \\
&\quad \left. + (k_2^2 - k^2) [J_0(\nu_2 r) A_\chi(k) + Y_0(\nu_2 r) B_\chi(k)] \right] e^{ikz}
\end{aligned} \tag{2.27}$$

The stresses σ_{rr} and σ_{rz} are found to be:

$$\begin{aligned}
\frac{\sigma_{rr}(r, z)}{\mu} &= (2k_1^2 - k_2^2) \Phi + 2 \frac{\partial u_r}{\partial r}, \quad \frac{\sigma_{rz}(r, z)}{\mu} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \\
\frac{\sigma_{rr}(r, z)}{\mu} &= \frac{1}{r} \left[\left[2\nu_1 J_1(\nu_1 r) + (2k^2 - k_2^2) r J_0(\nu_1 r) \right] A_\phi(k) \right. \\
&\quad + \left[2\nu_1 Y_1(\nu_1 r) + (2k^2 - k_2^2) r Y_0(\nu_1 r) \right] B_\phi(k) \\
&\quad + 2ik\nu_2 \left[J_1(\nu_2 r) - r\nu_2 J_0(\nu_2 r) \right] A_\chi(k) \\
&\quad \left. + 2ik\nu_2 \left[Y_1(\nu_2 r) - r\nu_2 Y_0(\nu_2 r) \right] B_\chi(k) \right] e^{ikz} \\
\frac{\sigma_{rz}(r, z)}{\mu} &= \left[- \left[2ik\nu_1 J_1(\nu_1 r) \right] A_\phi(k) - \left[2ik\nu_1 Y_1(\nu_1 r) \right] B_\phi(k) \right. \\
&\quad \left. + \left[\nu_2 (2k^2 - k_2^2) J_1(\nu_2 r) \right] A_\chi(k) + \left[\nu_2 (2k^2 - k_2^2) Y_1(\nu_2 r) \right] B_\chi(k) \right] e^{ikz}
\end{aligned} \tag{2.28}$$

The boundary conditions at the inner and outer walls are:

$$\sigma_{rr} = \sigma_{rz} = 0, \quad \text{at } r = a \text{ \& } r = b \tag{2.29}$$

These equations can be written in a matrix form as:

$$\frac{1}{ab} \begin{bmatrix} bJ_{(1+0)}(\nu_1 a) & bY_{(1+0)}(\nu_1 a) & 2ik\nu_2 bJ_{(1-0)}(\nu_2 a) & 2ik\nu_2 bY_{(1-0)}(\nu_2 a) \\ aJ_{(1+0)}(\nu_1 b) & aY_{(1+0)}(\nu_1 b) & 2ik\nu_2 aJ_{(1-0)}(\nu_2 b) & 2ik\nu_2 aY_{(1-0)}(\nu_2 b) \\ -2ik\nu_1 abJ_1(\nu_1 a) & -2ik\nu_1 abY_1(\nu_1 a) & \eta\nu_2 abJ_1(\nu_2 a) & \eta\nu_2 abY_1(\nu_2 a) \\ -2ik\nu_1 abJ_1(\nu_1 b) & -2ik\nu_1 abY_1(\nu_1 b) & \eta\nu_2 abJ_1(\nu_2 b) & \eta\nu_2 abY_1(\nu_2 b) \end{bmatrix} \begin{Bmatrix} A_\phi \\ B_\phi \\ A_\chi \\ B_\chi \end{Bmatrix} = \mathbf{0} \quad (2.30)$$

where:

$$\eta = \sqrt{2k^2 - k_2^2}$$

$$J_{(1+0)}(\nu_1 r) = 2\nu_1 J_1(\nu_1 r) + \eta J_0(\nu_1 r), \quad Y_{(1+0)}(\nu_1 r) = 2\nu_1 Y_1(\nu_1 r) + \eta Y_0(\nu_1 r)$$

$$J_{(1-0)}(\nu_2 r) = J_1(\nu_2 r) - \nu_2 r J_0(\nu_2 r), \quad Y_{(1-0)}(\nu_2 r) = Y_1(\nu_2 r) - \nu_2 r Y_0(\nu_2 r)$$

The determinant of this equation will result in the dispersion curves for the longitudinal modes in isotropic pipes.

2.3 Review of Finite Element Based Methods in Waves Propagation

Finite element method (FEM) is traditionally used to solve dynamics analyses problems. In waves propagation problems where the load frequency is very high, the wavelength becomes very small. This add an element size restriction to the FEM in which edge size has to be of the same order or smaller than the wavelength. This is required to ensure the accurate capture of all existing modes [30]. Fine mesh restrictions make large FEM problems very computational expensive. One of the well-known FE-based methods is spectral finite element (SFE) where the problem is solved in the frequency domain. This method has been applied successfully to multiply problems [30], [31], [32]. However, SFE requires a separate element

formulation and a different dynamic stiffness matrix.

2.4 Waveguide Finite Elements (WFE)

Waveguide finite element (WFE) method is finite element based approach to obtain dispersion curves and mode shapes (using eigen-values and eigen-vectors). It is used for structures with uniform cross-section along one direction where the waves propagate. The cross-section is sometimes referred to as the waveguide and it has the same physical and geometric properties along waveguide axis. The waveguide is considered the periodic unit cell and the periodic condition is used [33]. The wave characteristics of the waveguide are obtained. The waveguide can be modeled using commercial FE software to obtain the mass, stiffness, and damping matrices. These matrices are then used in the vibrational formulation that lead to the dispersion relation [34].

The main WFE formulation was developed to exploit standard finite-element models directly. Mace et al. [35] demonstrated that the wavenumbers and wave modes of a structural waveguide can be extracted by post-processing the mass and stiffness matrices of a short FE segment. An important advantage of WFE is that the underlying segment can generally be constructed using standard finite-element elements and commercial FE software. This allows geometrically complicated cross-sections, layered materials, anisotropy and damping to be incorporated without deriving new analytical wave equations. Duhamel et al. [36] extended the approach from the calculation of free waves to the forced response of finite and infinite waveguides. Once the wave basis has been computed, the structural response can be expressed using a relatively small number of traveling and evanescent waves rather than the full set of physical degrees of freedom. This reduction is particularly valuable at medium and high frequencies, where direct FE models of long structures can become computationally expensive.

The WFE formulation has subsequently been extended to various problems and more

complex waveguides. Mencik and Ichchou [37] developed a WFE formulation for guided elastodynamic structures containing an internal acoustic fluid. The method leads to a spectral problem in which the eigenvalues and eigenvectors define the propagation constants and coupled structural–acoustic wave modes.

WFE has been applied to cylindrical structures and pipes. Manconi and Mace [38] applied the method to isotropic, orthotropic and laminated cylindrical and curved panels. Their formulation used the mass and stiffness matrices of a small circumferential and axial segment and imposed phase relationships between corresponding boundaries. The resulting eigenproblem provided the dispersion characteristics and wave shapes for different circumferential orders. This demonstrated that complicated cylindrical constructions can be treated using standard FE discretizations without the need to derive specialized analytical dispersion equations.

WFE shares several characteristics with the Semi-Analytical Finite Element (SAFE) method, although the two methods construct the waveguide problem differently. In SAFE, the cross-section of a uniform waveguide is discretized and the variation in the propagation direction is assumed analytically, typically as $\exp(ikz)$ [39]. The resulting eigenvalue problem directly relates frequency, wavenumber and cross-sectional mode shape. WFE instead discretizes a finite segment in the propagation direction and imposes Bloch–Floquet or periodicity conditions between the two ends of that segment [35], [36]. SAFE can therefore be especially efficient for geometrically uniform waveguides with a well-defined propagation direction, whereas WFE is attractive when an existing conventional FE model is available or when the waveguide contains structural periodicity that is naturally described by a repeating unit cell.

The flexibility of WFE is accompanied by several numerical considerations. Because the propagation constants are obtained from an eigenvalue problem associated with a finite segment, the segment length and FE discretization must be sufficiently small to represent the wavelengths of interest. Furthermore, strongly attenuating waves may lead to eigenvalues of

very different magnitudes, which can cause numerical ill-conditioning. Waki et al. [40] examined these issues in detail and proposed more robust eigenvalue formulations for calculating free and forced wave responses. Therefore, appropriate discretization, mode classification and normalization are important when WFE is used over broad frequency ranges.

Overall, WFE provides an efficient bridge between conventional finite-element modeling and wave-based analysis. It retains the geometric and constitutive generality of FEM while replacing long sections of a waveguide by a compact basis of propagating and evanescent modes. For cylindrical structures and pipes, this capability is particularly valuable because WFE modes can be used both to calculate dispersion characteristics and as global wave functions in hybrid global–local or WFE–FE scattering formulations. In this respect, WFE and SAFE provide complementary finite-element-based techniques for obtaining the modal information required for efficient analysis of guided-wave propagation and interaction with localized structural features.

Waveguide finite element (WFE) method is finite element based approach to obtain dispersion curves and mode shapes (using eigen-values and eigen-vectors). It is used for structures with uniform cross-section along one direction where the waves propagate. The cross-section is sometimes referred to as the waveguide and it has the same physical and geometric properties along waveguide axis. The waveguide is considered the periodic unit cell and the periodic condition is used [33]. The wave characteristics of the waveguide are obtained. The waveguide can be modeled using commercial FE software to obtain the mass, stiffness, and damping matrices. These matrices are then used in the vibrational formulation that lead to the dispersion relation [34].

2.4.1 Formulation

The formulation of the WFE method starts with the traditional formulation of dynamics problems.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \quad (2.31)$$

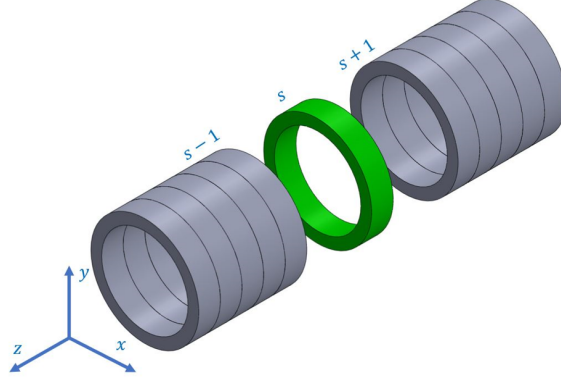


Figure 2.1: WFE model of the cylinder

Then, using separation of variables and assuming a harmonic motion of the form $e^{-i\omega t}$ the equation of motion (2.31) becomes:

$$\begin{bmatrix} \overline{\mathbf{D}}_{LL} & \overline{\mathbf{D}}_{LI} & \overline{\mathbf{D}}_{LR} \\ \overline{\mathbf{D}}_{IL} & \overline{\mathbf{D}}_{II} & \overline{\mathbf{D}}_{IR} \\ \overline{\mathbf{D}}_{RL} & \overline{\mathbf{D}}_{RI} & \overline{\mathbf{D}}_{RR} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_L \\ \mathbf{u}_I \\ \mathbf{u}_R \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_L \\ \mathbf{f}_I \\ \mathbf{f}_R \end{Bmatrix} \quad (2.32)$$

where $\overline{\mathbf{D}} = -\omega^2 \mathbf{M} + \mathbf{K}$ is the dynamic stiffness matrix. The L, I and R subscripts represent the left, interior, and right degrees of freedoms (DOFs), respectively.

Since there are no applied forces on the interior DOFs (or nodes), then we can set $\mathbf{f}_I$ to zero and reorganize the matrix as such:

$$\begin{bmatrix} \overline{\mathbf{D}}_{LL} & \overline{\mathbf{D}}_{LI} & \overline{\mathbf{D}}_{LR} \\ \overline{\mathbf{D}}_{IL} & \overline{\mathbf{D}}_{II} & \overline{\mathbf{D}}_{IR} \\ \overline{\mathbf{D}}_{RL} & \overline{\mathbf{D}}_{RI} & \overline{\mathbf{D}}_{RR} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_I \\ \mathbf{u}_L \\ \mathbf{u}_R \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_L \\ \mathbf{f}_R \end{Bmatrix} \quad (2.33)$$

The internal DOF in equation (2.33) is found as:

$$\mathbf{u}_I = -\overline{\mathbf{D}}_{II}^{-1} [\overline{\mathbf{D}}_{IL} \mathbf{u}_L + \overline{\mathbf{D}}_{IR} \mathbf{u}_R] \quad (2.34)$$

Inserting the interior DOF equation (2.34) in equation (2.33) to obtain the reduced form as

such as:

$$\begin{bmatrix} D_{LL} & D_{LR} \\ D_{RL} & D_{RR} \end{bmatrix} \begin{Bmatrix} u_L \\ u_R \end{Bmatrix} = \begin{Bmatrix} f_L \\ f_R \end{Bmatrix} \quad (2.35)$$

where:

$$\begin{aligned} D_{LL} &= \bar{D}_{LL} - \bar{D}_{LI} \bar{D}_{II}^{-1} \bar{D}_{IL}, & D_{LR} &= \bar{D}_{LR} - \bar{D}_{LI} \bar{D}_{II}^{-1} \bar{D}_{IR} \\ D_{RL} &= \bar{D}_{RL} - \bar{D}_{RI} \bar{D}_{II}^{-1} \bar{D}_{IL}, & D_{RR} &= \bar{D}_{RR} - \bar{D}_{RI} \bar{D}_{II}^{-1} \bar{D}_{IR} \end{aligned} \quad (2.36)$$

We can now insert the continuity condition and force equilibrium for the waveguide which are defined as:

$$u_R^s = u_L^{s+1}, \quad f_R^s = f_L^{s+1} \quad (2.37)$$

equation (2.37) states that the displacement of right hand side of segment s is equal to the left hand side of the next segment $(s+1)$ as shown in figure (2.1). It is important to note at this point that for the WFE to work, the left and right node locations have to be coincide because the DOFs on the right face are written in terms DOFs on the left face. When we insert the continuity condition (2.37), the reduced form (2.35), we get:

$$\begin{bmatrix} D_{LL} & D_{LR} \\ D_{RL} & D_{RR} \end{bmatrix} \begin{Bmatrix} u_L^s \\ u_L^{s+1} \end{Bmatrix} = \begin{Bmatrix} f_L \\ f_L^{s+1} \end{Bmatrix} \quad (2.38)$$

which after rearrangement in terms of upstreams and downstreams terms, we get:

$$\begin{Bmatrix} u_L^{s+1} \\ f_L^{s+1} \end{Bmatrix} = \begin{bmatrix} -D_{LR}^{-1} D_{LL} & D_{LR}^{-1} \\ D_{RL} - D_{RR} D_{LR}^{-1} D_{LL} & D_{RR} D_{LR}^{-1} \end{bmatrix} \begin{Bmatrix} u_L^s \\ f_L^s \end{Bmatrix} = T \begin{Bmatrix} u_L^s \\ f_L^s \end{Bmatrix} \quad (2.39)$$

where:

$$T = \begin{bmatrix} -D_{LR}^{-1} D_{LL} & D_{LR}^{-1} \\ D_{RL} - D_{RR} D_{LR}^{-1} D_{LL} & D_{RR} D_{LR}^{-1} \end{bmatrix}$$

Now, we introduce the phase shift (or the propagation term) $\lambda = e^{ikz} = e^{ik\Delta h}$ along the propagation direction $\hat{e}_z$. Δh is the depth of the waveguide in the propagation direction

$\hat{\mathbf{e}}_z$. The equation of a free wave propagation is then:

$$\begin{Bmatrix} u_L^{s+1} \\ f_L^{s+1} \end{Bmatrix} = \lambda \begin{Bmatrix} u_L^s \\ f_L^s \end{Bmatrix} \iff T \begin{Bmatrix} u_L^s \\ f_L^s \end{Bmatrix} = \lambda \begin{Bmatrix} u_L^s \\ f_L^s \end{Bmatrix} \quad (2.40)$$

Equation (2.40) is the eigenvalue problem and be rewritten as:

$$[\mathbf{T} - \lambda \mathbf{I}] \begin{Bmatrix} \mathbf{u}_L^s \\ \mathbf{f}_L^s \end{Bmatrix} = \mathbf{0} \quad (2.41)$$

The solution of equation (2.41) gives us the eigenvalues and eigenvectors (mode shapes). It's worth noting that there are $2n$ eigenvalues and eigenvectors, where n is the number of DOF. The wavenumbers are found from the eigenvalues by taking the natural logarithm of the eigenvalues and it can be stated as:

$$k = -\frac{i}{\Delta h} \ln(\lambda) \quad (2.42)$$

The phase and group velocities are found as such:

$$v_p = \frac{\omega}{k}, \quad v_g = \frac{\partial \omega}{\partial k} \quad (2.43)$$

Group velocity calculations involve difficulty due to numerical differentiation and this can be avoided by using the time-averaged energies and power flow:

$$v_g = \frac{P}{E_{tot}} \quad (2.44)$$

Where P is the average power flow transmitted through the cross-section of the waveguide. E_{tot} is the time average sum of the kinetic and potential energies.

$$P = \frac{1}{2} \text{Re}\{\mathbf{f}^H \dot{\mathbf{u}}\} = \frac{\omega}{2} \text{Re}\{\mathbf{f}^H \ddot{\mathbf{u}}\} \quad (2.45)$$

2.4.2 Numerical Considerations and the use of Symmetry WFE Half Model

In the previous section, we discussed the formulation of the full model. It is possible to apply symmetry and reduce the size of the problem by half. This requires applying symmetry boundary conditions by constraining the degrees of freedom about the plane of symmetry. However, it is worth noting that this results in the elimination of torsional waves. For example, applying symmetry about the y -axis requires constraining the x -components at the interface to enforce mirror boundary conditions.

2.5 Physics-Informed Tracking

2.5.1 Motivation and tracking requirements

The WFE method evaluates the wave characteristics by solving the transfer matrix eigenvalue problem at every single frequency. While this formulation results in all complex wavenumbers k and their corresponding eigenvectors (modal shapes and nodal forces), the eigenvalue solver returns the solution at an arbitrary orders and rotation at every frequency step [41], [42]. For the solution to be properly used in generating dispersion curves, or in the forced response problem, it has to be robustly tracked and sorted across the frequency domain. Although the WFE formulation provides an efficient representation of wave propagation in structures in geometry with complex cross-sections -enabled by the use of commercial FE packages-, the resulting eigenproblem can be sensitive to numerical conditioning, finite-element discretisation, and normalization and scaling of computed eigenvectors. Waki *et al.* [40] demonstrated that such numerical effects can influence both the eigensolution and the subsequent forced-response computation. These considerations become particularly important when a large number of propagating and evanescent waves are retained.

2.5.2 Conventional MAC-based mode tracking

The Modal Assurance Criterion was originally introduced as a measure of correlation between modal vectors [43]. For two modal displacement vectors ϕ_i and ϕ_j , the MAC is defined as

$$MAC_{ij} = \frac{|\phi_i^H \phi_j|^2}{(\phi_i^H \phi_i)(\phi_j^H \phi_j)}, \quad (2.46)$$

where the superscript H denotes the Hermitian transpose.

The criterion satisfies

$$0 \leq MAC_{ij} \leq 1. \quad (2.47)$$

A value close to unity indicates strong spatial correlation, while a value close to zero indicates weak correlation. A conventional mode-tracking scheme would track each mode from ω_r to ω_{r+1} by finding its maximum MAC value.

For isolated and sufficiently well-separated modes, this approach can be effective. However, it becomes problematic when eigenvalues are repeated or arbitrary rotate across frequencies [44], [45], [46].

2.5.3 Why individual-vector MAC becomes ambiguous for circular-pipe flexural modes

The principal limitation of conventional MAC tracking in the present application arises from the rotational symmetry of the circular pipe.

For a circumferential order $n > 0$, a flexural family may be represented by two angularly orthogonal solutions of the form

$$\phi_c \propto \cos(n\theta), \quad \phi_s \propto \sin(n\theta). \quad (2.48)$$

In an ideally axisymmetric pipe these two solutions belong to the same degenerate

eigenspace (e.g. share the same eigenvalue). Repeated eigenvalues do not possess a unique set of individual eigenvectors: any linear combination of eigenvectors belonging to the repeated eigenvalue is also a valid eigenvector [44].

Hence, the numerical eigensolver is free to return a rotated pair

$$\begin{bmatrix} \tilde{\phi}_1 & \tilde{\phi}_2 \end{bmatrix} = \begin{bmatrix} \phi_c & \phi_s \end{bmatrix} \mathbf{R}(\alpha), \quad (2.49)$$

where

$$\mathbf{R}(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}. \quad (2.50)$$

Equation (2.49) represents only a change of basis; the physical two-dimensional eigenspace has not changed.

Nevertheless, the MAC between one reference mode and the two rotated members is

$$\text{MAC}(\phi_c, \tilde{\phi}_1) = \cos^2 \alpha, \quad (2.51)$$

and

$$\text{MAC}(\phi_c, \tilde{\phi}_2) = \sin^2 \alpha. \quad (2.52)$$

For $\alpha = 45^\circ$,

$$\text{MAC}_{c1} = \text{MAC}_{c2} = 0.5, \quad (2.53)$$

and the individual modal assignment becomes intrinsically ambiguous.

Similarly, when $\alpha = 90^\circ$, the two numerical eigenvectors exchange their individual identities even though their combined eigenspace is unchanged.

This is not a failure of the underlying eigensolution. It is instead a consequence of

attempting to assign a unique physical identity to an individual vector within a non-unique eigenspace.

The same general issue has been recognised in modal correlation studies of repeated modes. Vinze *et al.* [45] showed that conventional MAC may provide misleading results for repeated and pseudo-repeated roots and developed a correlation measure based on vector subspaces. More recently, Xiao *et al.* [46] treated continuous-symmetry degeneracy in SAFE waveguide calculations using a rotation-invariant subspace MAC, including application to a steel pipe.

2.5.4 Mode veering and small eigengaps

A related problem occurs when two non-degenerate branches approach one another in an eigenvalue-veering region.

Perturbation theory for parameter-dependent eigenvalue problems shows that the derivative of an eigenvector contains contributions inversely related to the separation between neighbouring eigenvalues. In schematic form,

decreases, the eigenvectors can vary rapidly with the continuation parameter. Xiao *et al.* [46] discuss this behaviour in detail for SAFE dispersion tracking and show that sufficiently fine adaptive sampling is required for reliable individual-vector MAC tracking near such regions.

It is important to distinguish this phenomenon from the exact flexural degeneracy of an axisymmetric pipe. Near a mode-veering region, the individual eigenvectors remain theoretically identifiable but may rotate rapidly. In an exactly degenerate flexural subspace, the individual eigenvectors are fundamentally non-unique.

2.5.5 Cylindrical Fourier physical signature

To reduce dependence on the arbitrary Cartesian representation of the eigenvectors, the present tracking algorithm first expresses the cross-sectional displacement in cylindrical com-

ponents.

For a pipe with axis parallel to z , the radial and circumferential components may be written as

$$U_r = U_x \cos \theta + U_y \sin \theta, \quad (2.54)$$

$$U_\theta = -U_x \sin \theta + U_y \cos \theta, \quad (2.55)$$

together with the axial component U_z .

For each circumferential order $n > 0$, cosine and sine Fourier coefficients are calculated. For example, for the radial displacement,

$$a_r^{(n)} = \langle \cos(n\theta), U_r \rangle, \quad (2.56)$$

$$b_r^{(n)} = \langle \sin(n\theta), U_r \rangle. \quad (2.57)$$

Instead of using $a_r^{(n)}$ and $b_r^{(n)}$ separately, the tracking signature uses

$$S_i^{(n)} = \sqrt{|a_i^{(n)}|^2 + |b_i^{(n)}|^2}, \quad i = r, \theta, \text{ or } z \quad (2.58)$$

The resulting quantities are assembled into the modal physics-signature vector,

$$\mathbf{s} = \left[S_r^{(0)}, S_\theta^{(0)}, S_z^{(0)}, \dots, S_r^{(n)}, S_\theta^{(n)}, S_z^{(n)} \right]^T, \quad (2.59)$$

which is subsequently normalised according to

$$\hat{\mathbf{s}} = \frac{\mathbf{s}}{\|\mathbf{s}\|_2}. \quad (2.60)$$

An important property of Equation (2.58) is its invariance to rotation between the sine and cosine representations of the same circumferential order. A change in the angu-

lar orientation modifies the individual coefficients $a^{(n)}$ and $b^{(n)}$, but leaves their combined magnitude unchanged.

Consequently, the signature identifies the physical circumferential content of the mode without attempting to assign an artificial permanent identity to the two arbitrary orientations within a flexural doublet.

2.5.6 Complex-wavenumber continuation

After identifying modes with similar physical signatures, the complex wavenumber is employed as the principal discriminator between different radial branches belonging to the same wave family.

For tracked branch i , let the predicted wavenumber at the next frequency be denoted by $\widehat{k}_i$. For candidate mode j , the wavenumber cost is defined as

$$C_k(i, j) = \frac{|k_j - \widehat{k}_i|}{k_{\text{scale}} + |k_j| + |\widehat{k}_i|}. \quad (2.61)$$

The cost is bounded to the interval

$$0 \leq C_k \leq 1. \quad (2.62)$$

The use of the complete complex wavenumber is significant because the same continuation quantity can be employed for propagating and evanescent modes. Furthermore, Equation (2.61) remains regular as $k \rightarrow 0$, which is important close to modal cut-on.

No global assumption such as

$$m_2 > m_1 \implies |k_{m_2}| > |k_{m_1}| \quad (2.63)$$

is imposed. Instead, each branch is followed according to its local complex dispersion trajectory.

2.5.7 Composite global assignment

The candidate-to-branch correspondence is obtained from a composite cost function of the form

$$C_{ij} = W_{\text{sig}}C_{\text{sig},ij} + W_k C_{k,ij} + W_{c_g} C_{c_g,ij} + W_n C_{n,ij} + C_{\text{dir},ij}, \quad (2.64)$$

where:

- C_{sig} is the cylindrical physics-signature cost;
- C_k is the complex-wavenumber continuation cost;
- C_{c_g} is a bounded group-velocity continuation cost;
- C_n penalises changes in circumferential order; and
- C_{dir} penalises inconsistent propagation direction.

Rather than making independent greedy decisions for individual modes, the complete cost matrix is solved as a one-to-one assignment problem. This ensures that every raw eigenvalue can be assigned to at most one tracked branch at each frequency.

Conceptually, a MAC-dominated tracker asks

Which current eigenvector most closely resembles the previous eigenvector?

The present method instead asks

Which current candidate has the correct cylindrical wave physics and represents the most consistent continuation of the complex dispersion branch?

The latter criterion is better suited to the present pipe application because it does not require the arbitrary orientation of a degenerate eigenvector to remain unchanged.

2.5.8 Summary

The mode-tracking procedure developed in here is more suited for cylindrical structures as compared to conventional MAC algorithm. MAC provides a useful measure of modal-vector correlation [43]; however, repeated roots require special consideration because the

associated individual eigenvectors are non-unique [44]. Consequently, conventional MAC can give misleading correlations for repeated modal roots, motivating subspace-based correlation methods such as SpaceMAC [45]. The same mathematical issue arises in guided-wave analysis when continuous structural symmetry creates degenerate modal subspaces; recent SAFE mode-tracking work therefore introduces rotation-invariant subspace treatment for such cases [46].

2.6 Application

Several case studies of WFE method are shown in this section for both isotropic and composite pipe. We will start with the simple solid cylinder case.

2.6.1 Solid Cylinder

In this section, a solid cylinder of radius 1mm is modeled in abaqus using C3D8 linear brick element as shown in Fig 2.2. The model has depth of $50\mu m$ along the propagation direction. The dispersion curves result and in-plane model shapes are shown in Fig (2.3) and Fig(2.4). The first three fundamental modes are shown. The red line, blue dashed line and black line are L(0,1), F(1,1) and T(0,1), respectively. The first torsional mode is known to be a non-dispersive mode.

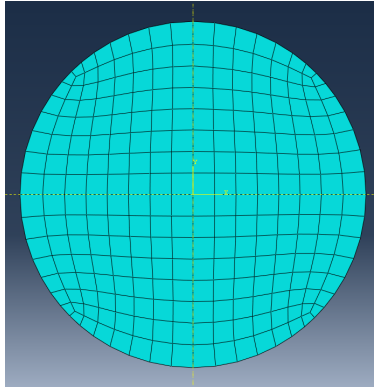


Figure 2.2: Solid cylinder waveguide modeled in Abaqus.

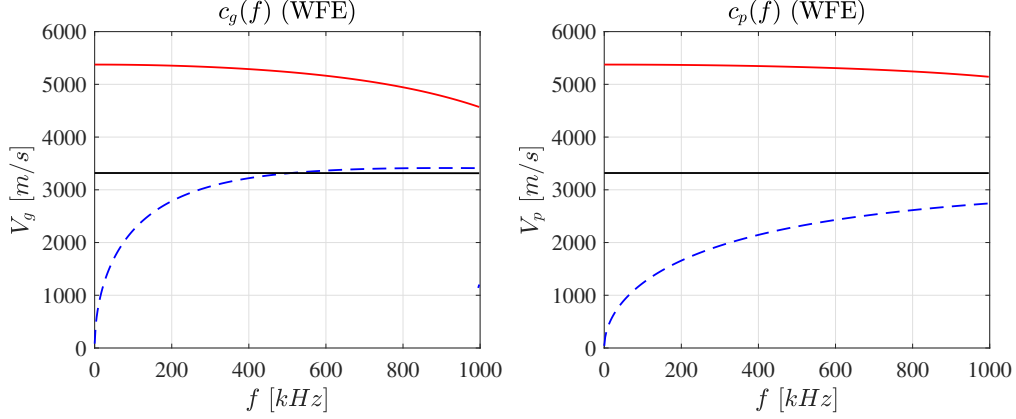


Figure 2.3: Dispersion curves for solid cylinder with $R = 1mm$, group velocity on the right, phase velocity on the left.

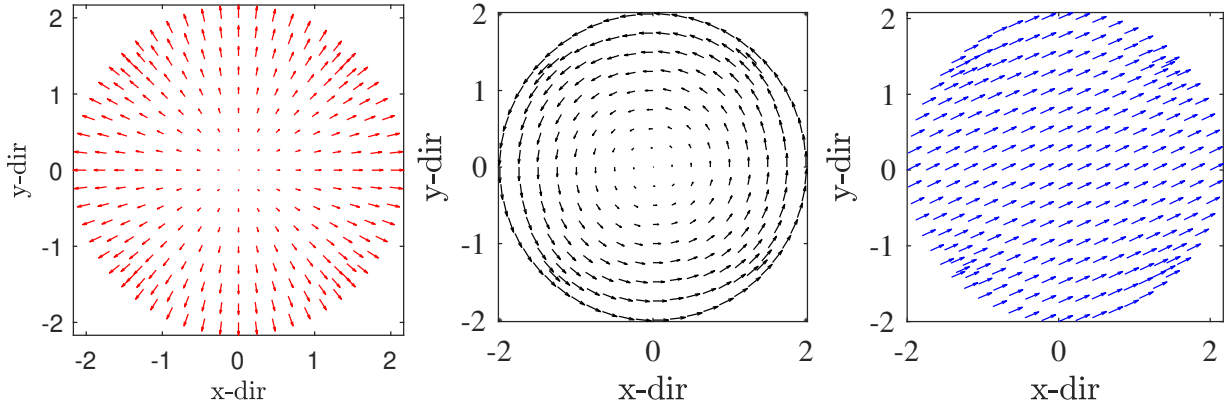


Figure 2.4: In-plane mode shapes of $L(0,1)$ mode at $100kHz$

2.6.2 Isotropic Pipe

In this section, the results for isotropic pipes with different R/t ratios are presented. A finite element model is shown in Fig 2.5 with R/t ratio of 10.

Fig 2.6 shows the dispersion curves for R/t to thickness ratio of 10 against a numerical solution of the analytical solver. The results show very good agreement between the two solutions. The in-plane modal displacements for the same pipe are shown in Fig 2.7

In Fig 2.8, a dispersion curves for an Al pipe with R/t ratio of 50. In both of these figures, it can be seen that the number of flexural modes increase as R/t ratio increase. A better way to represent the results would be to plot the velocity versus the frequency-thickness product.

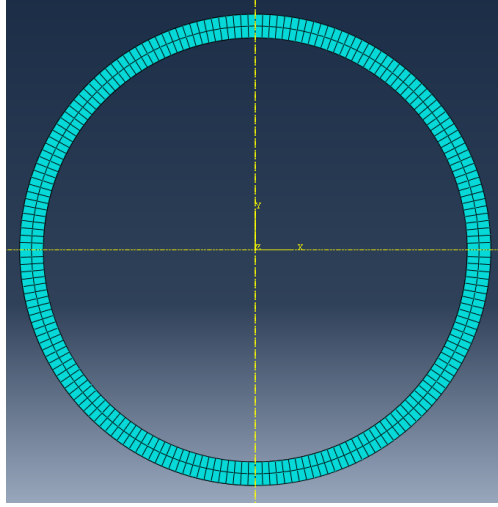


Figure 2.5: Pipe model modeled in Abaqus.

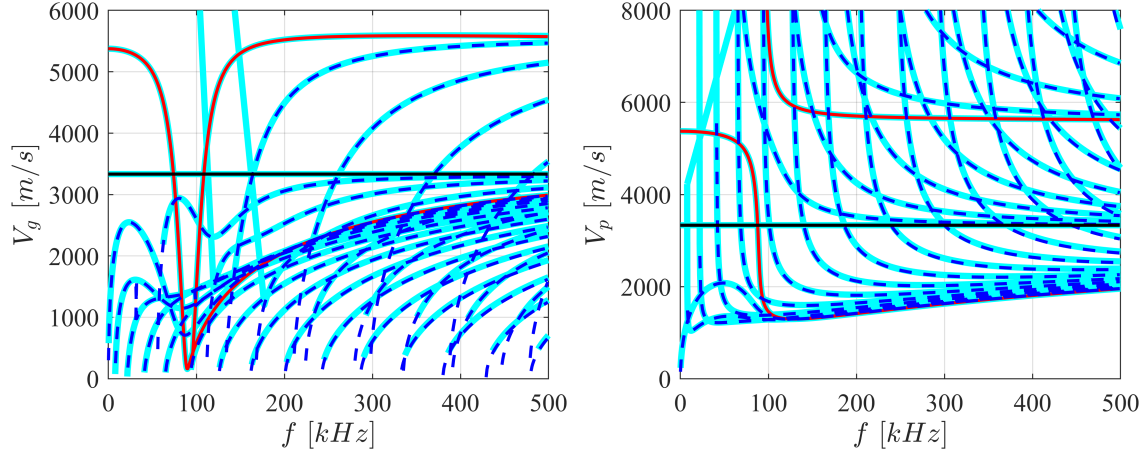


Figure 2.6: Dispersion curves for aluminum pipe with $R/t = 10$ plotted against analytical solver solution (cyan color). Group velocity on the right, and phase velocity on the left.

2.6.3 Carbon-fiber composite pipe

In this section, two cases of composite pipe are presented. The first case is for a quasi-isotropic composite pipe that is made of 4-layers unidirectional fiber lamina with the following orientation $[0\ 90]_S$. The radius of the pipe is $40mm$ and the wall thickness is $2mm$. The dispersion curves for phase and group velocities are shown in Fig 2.9.

The second case is for a plain woven carbon fiber composite with R/t ratio of 25.

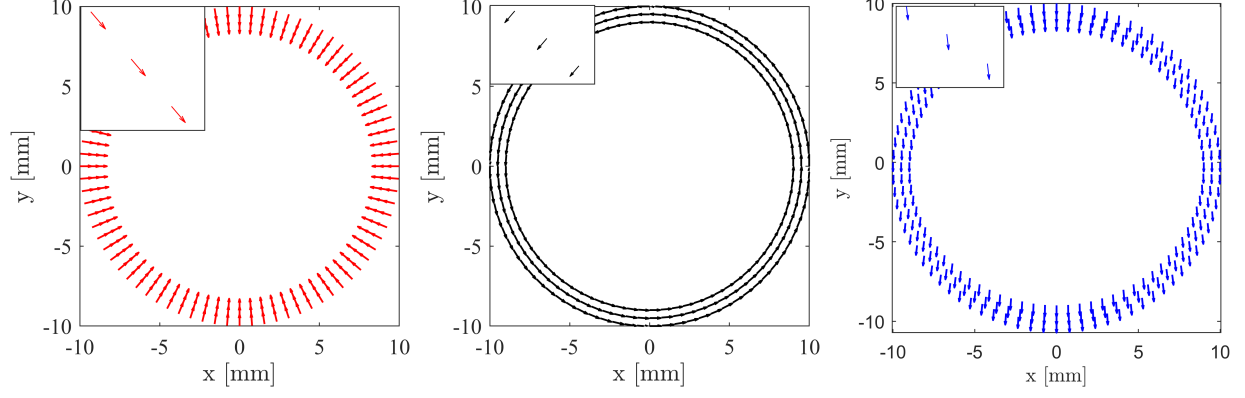


Figure 2.7: In-plane mode shape for a pipe with $R/t = 10$ for $L(0,1)$, $T(0,1)$, and $F(1,1)$

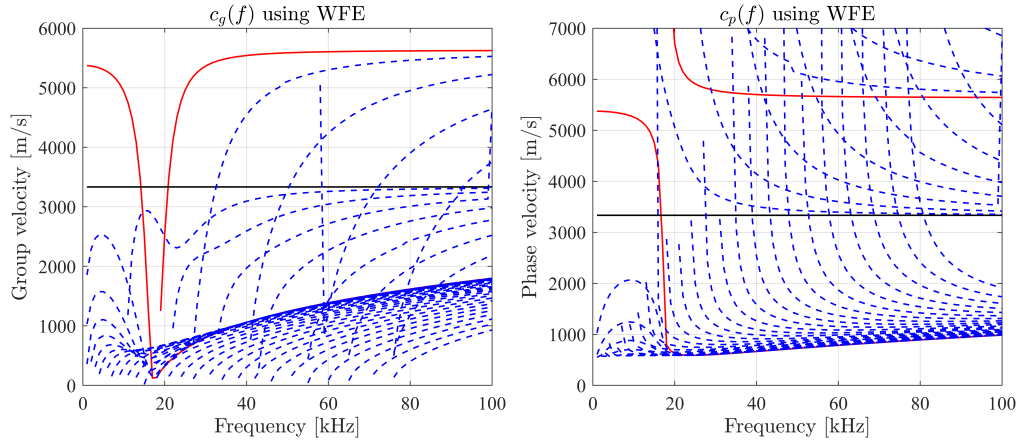


Figure 2.8: Dispersion curves of a pipe with R/t ratio of 50

2.6.4 Pipe Sandwich structure (Pipe in Pipe)

Pipe sandwich structure or pipe in pipe has received an increased interest in recent years especially for ultra deep water application due to their thermal insulation properties, high bending toughness and buckling resistance [47], [48], [49]. In this section, pipe sandwich structure is modeled and the dispersion curves results are obtained using WFE solver.

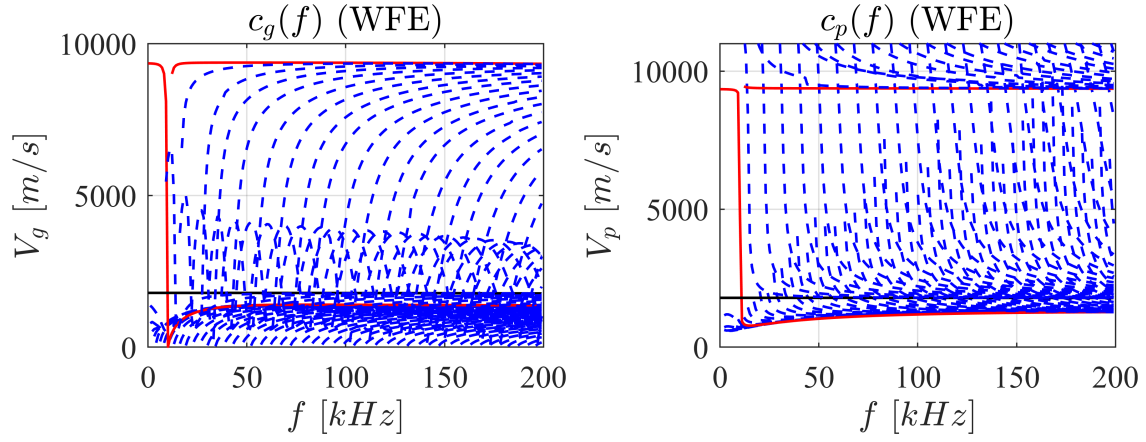


Figure 2.9: Dispersion curves of unidirectional fiber pipe made of 4-layer laminate

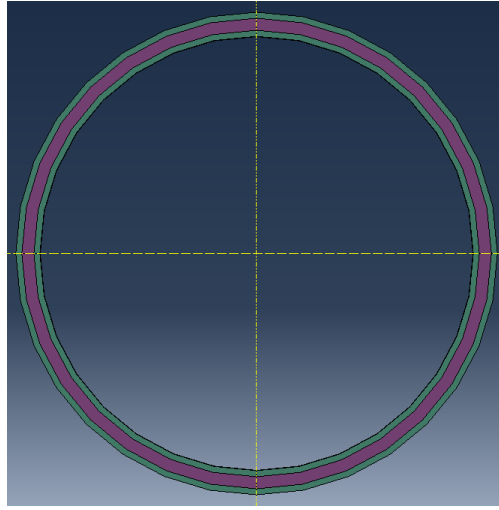


Figure 2.10: Pipe sandwich structure made of aluminum skin and epoxy 5505 core.

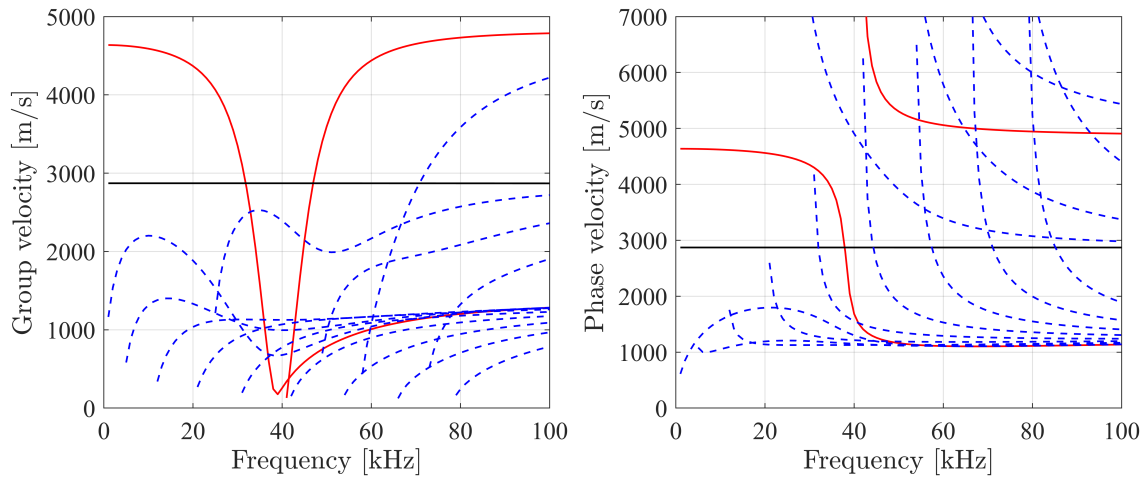


Figure 2.11: Dispersion curves of pipe sandwich structure.

Chapter 3

Global Local Method (GLM)

3.1 Introduction

The modeling of guided-wave propagation in cylindrical structures and pipes is an important tool in nondestructive evaluation (NDE), as it supports the analysis and detection of defects while also facilitating the design and development of new inspection tools. However, when a small region of geometric or material complexity is to be studied it can be computationally demanding to model this long and uniform waveguide. Examples include cracks, corrosion defects, welds, joints, local changes in wall thickness, and attachments. A conventional finite-element (FE) model is a convenient tool to represent such discontinuities accurately; however, modeling the entire propagation domain with a sufficiently fine mesh is generally inefficient, particularly at high frequencies. The global–local (GL) method provides an alternative in which the structure is separated into a finite *local* region containing the discontinuity and one or more regular *global* regions whose response is represented in terms of global wave functions. In this formulation, the computational effort associated with conventional finite elements is concentrated around the region where complex scattering takes place, while wave propagation away from the discontinuity is represented by a modal expansion.

An early and important formulation of the global–local finite-element method (GLFEM)

for elastodynamic scattering was presented by Goetschel and Dong [50]. They considered the axisymmetric scattering of elastic waves by an inclusion and combined a finite-element representation of the inhomogeneous local region with global functions obtained analytically describing the global region. This work established a central feature of the GL approach: the global functions satisfy the governing equations in the regular portion of the medium, whereas conventional finite elements are retained only where the geometry or material distribution prevents a convenient analytical representation.

The application of this concept to guided-wave scattering was developed further by Mal et al. Chang and Mal [51] analyzed Lamb-wave scattering from a rivet hole containing edge cracks using global–local finite-element method. A bounded region containing the hole and cracks was discretized using finite elements, while the exterior plate was represented analytically using Lamb-wave modes. The amplitudes of the reflected, transmitted, and evanescent modes were obtained by enforcing the appropriate displacement and traction conditions at the interfaces between the local FE region and the surrounding waveguide. Mal and Chang [52] subsequently presented a more general semi-numerical formulation for elastic-wave scattering in heterogeneous media. In their formulation, the elastodynamic field inside a finite region enclosing the scatterers is represented by finite elements, whereas the field outside this region is expanded in a suitable set of global wave functions. More recently, Tai et al. [53] extended this framework for ultrasonic guided waves in fiber–metal laminates. Their formulation combines local finite-element discretization with Lamb-wave modal expansions and incorporates both the source and scattering problems. This development is important because it demonstrates that the global–local concept is not limited to a prescribed incident mode; multiple incident modes can be added and properly applied and the resulting scattered field can be incorporated within the same computational framework.

More recent work of the global–local approach have been presented by [54] who modeled wave propagation in rail head using fully three-dimensional guided-wave scattering problems. In this work, a three-dimensional FE discretization is used for the local region while

SAFE is applied to represent the semi-infinite regular portions of the waveguide. The method was demonstrated for the interaction of guided waves with transverse defects in a rail head.

In global-local method, scattered solution is expressed directly in terms of waveguide modes, which provides a natural framework for evaluating reflection, transmission, and mode conversion coefficients, which are quantities of primary interest in guided-wave-based inspection and structural-health-monitoring applications.

This chapter is concerned with developing the formulation for global-local as it applies to cylindrical structures.

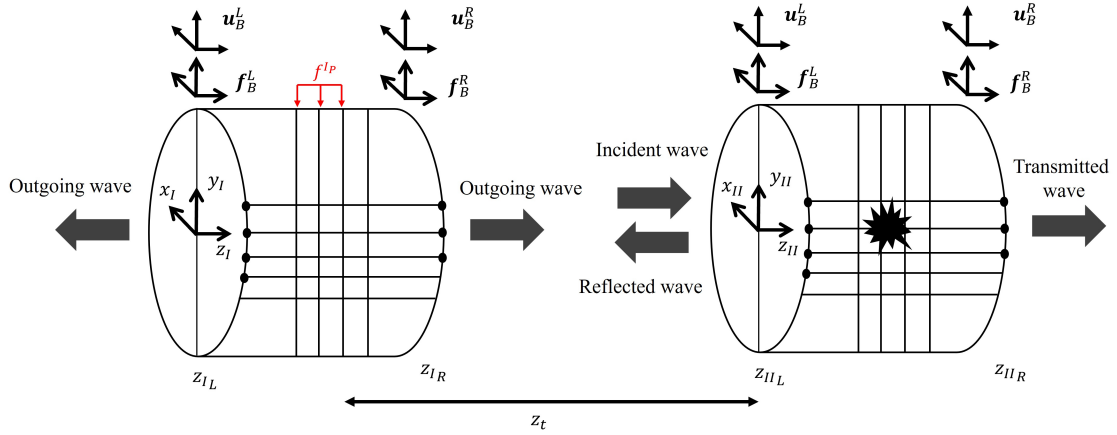


Figure 3.1: Global local source and scattering problems model

3.2 Formulation

The main formulation starts from the dynamic vibrational formulation of the system.

3.2.1 Source Problem

The equation of motion is:

$$M\ddot{u} + Ku = f \quad (3.1)$$

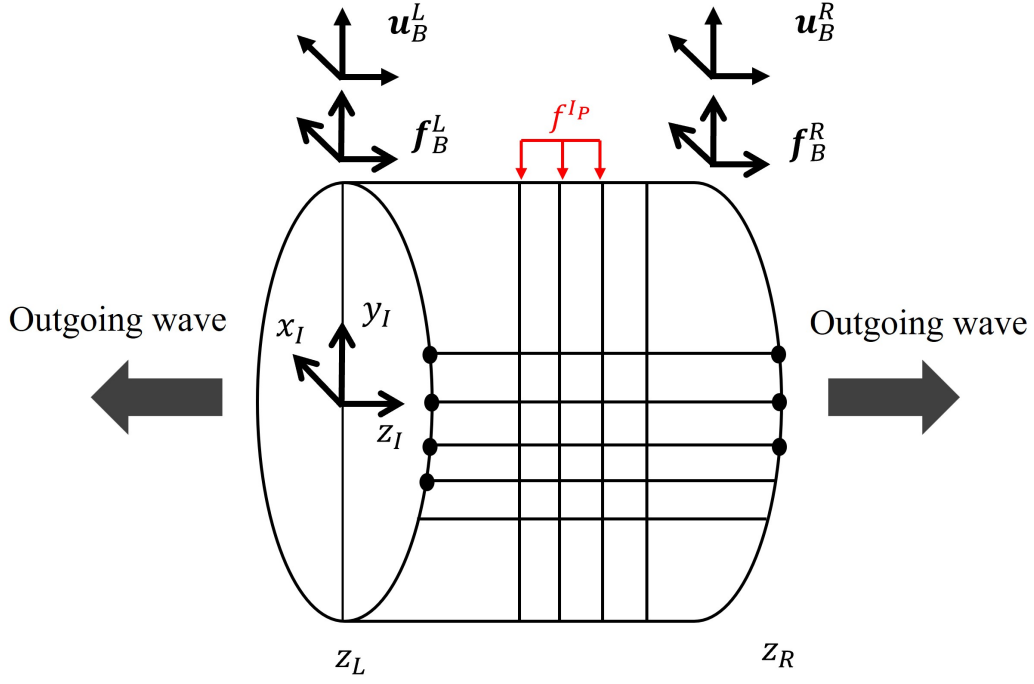


Figure 3.2: Global local source model

$$\begin{bmatrix} D_{LL} & D_{LI_P} & D_{LI_O} & D_{LR} \\ D_{I_PL} & D_{I_P I_P} & D_{I_P I_O} & D_{I_PR} \\ D_{I_O L} & D_{I_O I_P} & D_{I_O I_O} & D_{I_O R} \\ D_{RL} & D_{RI_P} & D_{RI_O} & D_{RR} \end{bmatrix} \begin{Bmatrix} u_B^L \\ \textcolor{red}{u}^{IP} \\ u^{I_O} \\ u_B^R \end{Bmatrix} = \begin{Bmatrix} f_B^L \\ \textcolor{red}{f}^{IP} \\ f^{I_O} \\ f_B^R \end{Bmatrix} \quad (3.2)$$

where L, R, I_O and I_P are nodes on the left, right and interior regions respectively. I_O are the traction free interior nodes while I_P are the interior nodes subjected to traction loads from the transducer. f^{IP} is applied load in frequency domain obtained by taking the Fourier time transform to time domain signal.

The nodal displacement and nodal forces can be found as a summation of the generated modes:

$$\begin{aligned} u_B^L &= [\overleftarrow{\mathbf{Q}}] \{ B_n^L e^{i \overleftarrow{k}_n z_L} \}, & f_B^L &= -[\overleftarrow{\mathbf{F}}] \{ B_n^L e^{i \overleftarrow{k}_n z_L} \} \\ u_B^R &= [\overrightarrow{\mathbf{Q}}] \{ B_n^R e^{i \overrightarrow{k}_n z_R} \}, & f_B^R &= [\overrightarrow{\mathbf{F}}] \{ B_n^R e^{i \overrightarrow{k}_n z_R} \} \end{aligned} \quad (3.3)$$

The $\mathbf{Q}$ and $\mathbf{F}$ are the modal displacements and nodal forces respectively. The n is the

number of each mode considered in the solution. The approach considered here, the Q and F are obtained from the WFE solution.

$$Q = \begin{bmatrix} q_x^{(1)} & q_x^{(2)} & \dots & q_x^{(N)} \\ q_y^{(1)} & q_y^{(2)} & \dots & q_y^{(N)} \\ q_z^{(1)} & q_z^{(2)} & \dots & q_z^{(N)} \\ \vdots & \vdots & \dots & \vdots \\ q_z^{(1)} & q_z^{(2)} & \dots & q_z^{(N)} \end{bmatrix}_{3j \times N} \quad F = \begin{bmatrix} f_x^{(1)} & f_x^{(2)} & \dots & f_x^{(N)} \\ f_y^{(1)} & f_y^{(2)} & \dots & f_y^{(N)} \\ f_z^{(1)} & f_z^{(2)} & \dots & f_z^{(N)} \\ \vdots & \vdots & \dots & \vdots \\ f_z^{(1)} & f_z^{(2)} & \dots & f_z^{(N)} \end{bmatrix}_{3j \times N} \quad (3.4)$$

where $3j$ represents the total number of degrees of freedom in the left boundary and N is total number of propagating modes.

We can solve for the amplitudes as:

$$\begin{aligned} B_n^L &= [\overleftarrow{Q}]^\dagger \{u_B^L\} e^{-i \overleftarrow{k}_n z_L} \\ B_n^R &= [\overrightarrow{Q}]^\dagger \{u_B^R\} e^{-i \overrightarrow{k}_n z_R} \end{aligned} \quad (3.5)$$

Plug back in the equation to get:

$$\begin{bmatrix} D_{LL} + [\overleftarrow{F}][\overleftarrow{Q}]^\dagger & D_{LI_P} & D_{LI_O} & D_{LR} \\ D_{I_PL} & D_{I_P I_P} & D_{I_P I_O} & D_{I_PR} \\ D_{I_OL} & D_{I_O I_P} & D_{I_O I_O} & D_{I_OR} \\ D_{RL} & D_{RI_P} & D_{RI_O} & D_{RR} - [\overrightarrow{F}][\overrightarrow{Q}]^\dagger \end{bmatrix} \begin{pmatrix} u_B^L \\ u^{I_P} \\ u^{I_O} \\ u_B^R \end{pmatrix} = \begin{pmatrix} 0 \\ f^{I_P} \\ 0 \\ 0 \end{pmatrix} \quad (3.6)$$

where the $[\overleftrightarrow{Q}]^\dagger$ is the pseudo-inverse of the Q matrix. The direction of the arrow indicates the direction of the wave.

The nodal forces of the left, right and interior region can be found by:

$$\begin{aligned}
f_B^L &= D_{LL} u_B^L + D_{LI_P} u^{I_P} + D_{LI_O} u^{I_O} + D_{LR} u_B^R \\
f_B^R &= D_{RL} u_B^L + D_{RI_P} u^{I_P} + D_{RI_O} u^{I_O} + D_{RR} u_B^R \\
f^{I_O} &= D_{I_O L} u_B^L + D_{I_O I_P} u^{I_P} + D_{I_O I_O} u^{I_O} + D_{I_O R} u_B^R
\end{aligned} \tag{3.7}$$

3.2.2 Scattering Problem

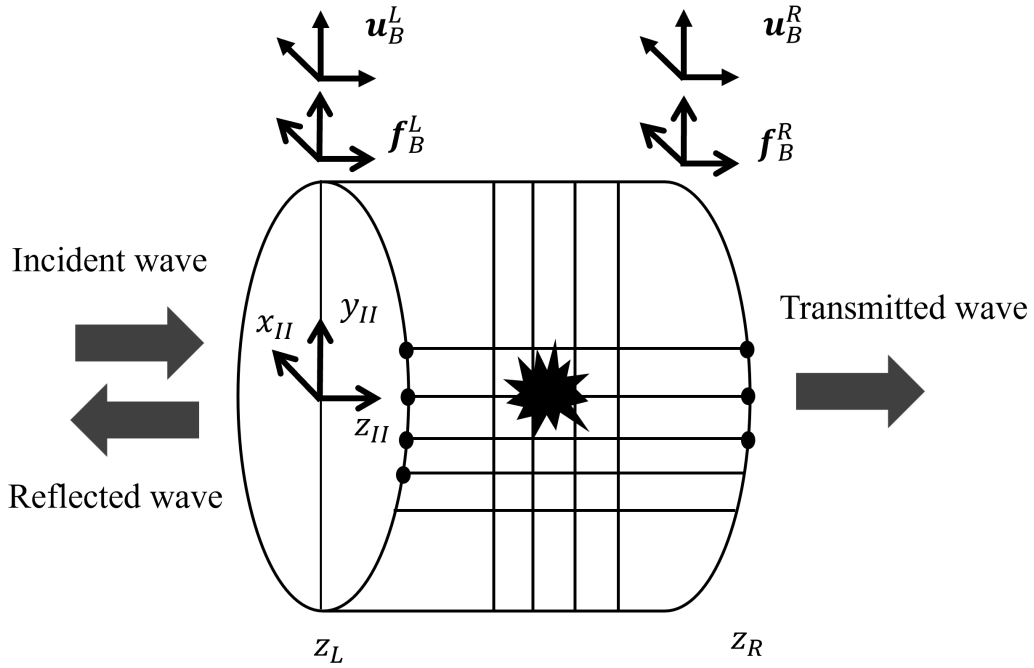


Figure 3.3: Global local scattering model

The scattering problem model is shown in in Fig.3.3. The coordinate axes x_{II} , y_{II} and z_{II} are located at the center of the left face of the pipe and at distance z_t from the source problem (not shown here). This model does not have external force, so the interior nodes do not

need to be separated into two parts like the source problem. The equation becomes:

$$\begin{bmatrix} D_{LL} & D_{LI_O} & D_{LR} \\ D_{I_O L} & D_{I_O I_O} & D_{I_O R} \\ D_{RL} & D_{RI_O} & D_{RR} \end{bmatrix} \begin{Bmatrix} u_B^L \\ u^{I_O} \\ u_B^R \end{Bmatrix} = \begin{Bmatrix} f_B^L \\ f^{I_O} \\ f_B^R \end{Bmatrix} \quad (3.8)$$

The left side of the scattering problem consists of the incident and reflected waves. The incident wave is the same as the right going wave from the source problem but with a proper phase shift to account for the distance z_t .

$$\begin{aligned} u_B^L &= [\vec{Q}] \{A_m e^{i \vec{k}_m (z_L + z_T)}\} + [\overleftarrow{Q}] \{B_n^L e^{i \overleftarrow{k}_n z_L}\} \\ f_B^L &= -[\vec{F}] \{A_m e^{i \vec{k}_m (z_L + z_T)}\} - [\overleftarrow{F}] \{B_n^L e^{i \overleftarrow{k}_n z_L}\} \end{aligned} \quad (3.9)$$

From equation (3.9), the coefficient of left propagating waves B_n^L can be found as:

$$\{B_n^L e^{i \overleftarrow{k}_n z_L}\} = [\overleftarrow{Q}]^\dagger u_B^L - [\overleftarrow{Q}]^\dagger [\vec{Q}] \{A_m e^{i \vec{k}_m (z_L + z_T)}\} \quad (3.10)$$

Equation (3.10) can be used in the f_B^L expression in equation (3.9) to obtain it in terms of the incident amplitude and the displacement on the left boundary:

$$f_B^L = ([\overleftarrow{F}][\overleftarrow{Q}]^\dagger [\vec{Q}] - [\vec{F}]) \{A_m e^{i \vec{k}_m (z_L + z_T)}\} - [\overleftarrow{F}][\overleftarrow{Q}]^\dagger u_B^L \quad (3.11)$$

Now, let's consider the right hand side of the scattering problem. On the right boundary at z_R , we have:

$$\begin{aligned} u_B^R &= [\vec{Q}] \{B_n^R e^{i \vec{k}_n z_R}\} \\ f_B^R &= [\vec{F}] \{B_n^R e^{i \vec{k}_n z_R}\} \end{aligned} \quad (3.12)$$

Similarly, after solving for the B_n^R to express the nodal force in terms of displacement

$$\begin{bmatrix} D_{LL} + [\overleftarrow{F}][\overleftarrow{Q}]^\dagger & D_{LI} & D_{LR} \\ D_{IL} & D_{II} & D_{IR} \\ D_{RL} & D_{RI} & D_{RR} - [\overrightarrow{F}][\overrightarrow{Q}]^\dagger \end{bmatrix} \begin{Bmatrix} u_B^L \\ u^I \\ u_B^R \end{Bmatrix} = \begin{Bmatrix} ([\overleftarrow{F}][\overleftarrow{Q}]^\dagger[\overrightarrow{Q}] - [\overrightarrow{F}])\{A_m e^{i\overrightarrow{k_m}(z_L+z_T)}\} \\ 0 \\ 0 \end{Bmatrix} \quad (3.13)$$

3.2.3 Power Flow and Wave Field Representation

Global Local method allows for the contribution of each mode in the global region to be separated. This allows to study of the mode conversion due to the interaction with the scatterer. In this section, the expressions for the reflected and transmitted displacement and power are shown.

Reflected Field

The modal displacement for the reflected field can be obtained from equation(3.9)

$$u_B^{ref} = [\overleftarrow{Q}]\{B_n^L e^{i\overleftarrow{k_n}z}\} \quad (3.14)$$

where $z_R^I < z < z^I I_L$.

The modal amplitude coefficient is:

$$\{B_n^L\} = \left\{ [\overleftarrow{Q}]^\dagger u_B^L - [\overleftarrow{Q}]^\dagger [\overrightarrow{Q}]\{A_m e^{i\overrightarrow{k_m}(z_L+z_T)}\} \right\} \{e^{-i\overleftarrow{k_n}z_L}\} \quad (3.15)$$

Transmitted Field

Similarly, the modal displacement for the transmitted field can be obtained from equation(3.12)

$$\mathbf{u}_B^{tr} = [\vec{Q}] \{B_n^R e^{i \vec{k}_n z}\} \quad (3.16)$$

where $z > z^I I_R$.

The modal amplitude coefficient is:

$$\{B_n^R\} = \left\{ [\vec{Q}]^\dagger \mathbf{u}_B^R \right\} \{e^{-i \vec{k}_n z_R}\} \quad (3.17)$$

Transmitted, Reflected and Dissipated Powers

The time average power for a discretized cross-section for a propagating wave is:

$$P = -\frac{1}{2} Re \{ \mathbf{F}^H \dot{\mathbf{Q}} \} = -\frac{1}{2} Re \{ -i\omega \mathbf{F}^H \mathbf{Q} \} \quad (3.18)$$

where H is the complex conjugate transpose and $\mathbf{f}, \mathbf{q}$ are the nodal force and nodal displacement vector. In our developed GLM, the $\mathbf{f}, \mathbf{q}$ are obtained from the WFE solution for either the left or right cross-section.

$$\begin{aligned} E_{tr}^n &= -\frac{1}{2} |B_n^R|^2 Re \left\{ -i\omega \overrightarrow{f_{R_n}}^H \overrightarrow{q_{R_n}} \right\} \\ E_{ref}^n &= -\frac{1}{2} |B_n^L|^2 Re \left\{ -i\omega \overleftarrow{f_{L_n}}^H \overleftarrow{q_{L_n}} \right\} \end{aligned} \quad (3.19)$$

The energy of the incident mode in the coupled source-scattering problem is defined as (in the case of no damping losses):

$$E_{inc}^n = -\frac{1}{2} |A_m|^2 Re \left\{ -i\omega \overrightarrow{f_{R_n}}^H \overrightarrow{q_{R_n}} \right\} \quad (3.20)$$

In the case where the damping is present, the dissipated power due to damping for the

propagated distance z_t is defined as [55]

$$E_{dmp}^n = E_{inc}^n \times \left[1 - e^{-2z_t \overrightarrow{k_{ni}}} \right] \quad (3.21)$$

where $\overrightarrow{k_{ni}}$ is the imaginary component of the wavenumber for the n mode. This is power dissipated in the global region. However, in the finite element region, the power dissipated due to damping is calculated using the imaginary component of the stiffness matrix. It is expressed as:

$$E_{dmpFE} = \frac{\omega}{2} U^H K_{imag} U \quad (3.22)$$

where U is the total displacement field in the FE region. The total dissipated power at every frequency step is then calculated as:

$$E_{dmp total} = E_{dmpFE} + \sum_{n=1}^N E_{dmp}^n \quad (3.23)$$

The contribution of transmitted, reflected, incident and dissipated energies at each frequency can added up to help quantify the energy error as a function of frequency. The error expression becomes

$$E_{err} = abs\left(\frac{E_{inc} - E_{tr} - E_{ref} - E_{dmp total}}{E_{inc}}\right) \times 100 \quad (3.24)$$

3.2.4 Modeling Wave Attenuation with Complex Moduli

Propagation of waves in pipe systems usually involve large travel distances during inspection especially in metallic systems. Also, for pipes made of composite materials, energy dissipation caused by the viscoelastic nature of the matrix materials become more pronounced. For these cases to be modeled reasonably, damping has to be introduced to the considered model. Since composite materials are considered, complex-valued elastic moduli are used to model the material factor as has been shown by researchers [56]. Moreover, when we have a higher frequency-thickness product, a large number of higher modes coexist at target

frequency range. The cutoff frequencies of these modes are the natural frequency of the pipe at which resonance occurs. These resonances cause unrealistic spikes in the frequency domain solution in the undamped model. The introduction of a small damping value can suppress these modes and clean up the signal. The damping model considered is the one developed by Mal [51] and is expressed in the following equation:

$$p = p_0 \left[1 + a_0 \left(\frac{f}{f_0} - 1 \right)^2 H(f - f_0) \right] \quad (3.25)$$

where $f = \omega/2\pi$ is the frequency, $H(f)$ is the Heavside step function, and a_0, f_0, p_0 are constants that determine the degree of decay. p_0 which is the first term in 3.25 represents the the dissipation effect due to internal friction and other thermodynamics factors (material attenuation). The second term in eq3.25 represents the damping caused by scattering due to inhomogeneities and fibers in case of composites. At frequency range where the wavelength is large compared to the microstructure and grain size of the material, the scattering contribution to the damping-like parameter can be neglected. In this frequency range the damping can be expressed with only p_0 and it's independent of the frequency. In this section the complex moduli are derived implementing the p value for the generalized case of orthotropic material.

For a fully orthotropic material, the stress-strain relation in Voigt notation is

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix}. \quad (3.26)$$

where C_{ij} are the 9 real independent stiffness components. Now, considering the Christoffel

equation for bulk waves propagation:

$$\Gamma_{ik} - \rho v^2 \delta_{ik} = 0, \quad \Gamma_{ik} = C_{ijkl} n_j n_l \quad (3.27)$$

where Γ, ρ, v and n are Christoffel tensor, density, bulk velocity and the unit vector along the propagation direction. Solving this equation results in these nine real velocities:

$$\begin{aligned} a_{11} &= \frac{C_{11}}{\rho}, & a_{22} &= \frac{C_{22}}{\rho}, & a_{33} &= \frac{C_{33}}{\rho}, \\ a_{44} &= \frac{C_{44}}{\rho}, & a_{55} &= \frac{C_{55}}{\rho}, & a_{66} &= \frac{C_{66}}{\rho}, \\ a_{12} &= \frac{C_{12} + C_{66}}{\rho}, & a_{13} &= \frac{C_{13} + C_{55}}{\rho}, & a_{23} &= \frac{C_{23} + C_{44}}{\rho}. \end{aligned} \quad (3.28)$$

Introducing the complex parameter p and writing it in terms of complex velocities A , we obtain this expression:

$$\begin{aligned} A_{11} &= \frac{a_{11}}{1 + ip\sqrt{\frac{a_{55}}{a_{11}}}}, & A_{22} &= \frac{a_{22}}{1 + ip\sqrt{\frac{a_{55}}{a_{22}}}}, & A_{33} &= \frac{a_{33}}{1 + ip\sqrt{\frac{a_{55}}{a_{33}}}}, \\ A_{44} &= \frac{a_{44}}{1 + ip\sqrt{\frac{a_{55}}{a_{44}}}}, & A_{55} &= \frac{a_{55}}{1 + ip}, & A_{66} &= \frac{a_{66}}{1 + ip\sqrt{\frac{a_{55}}{a_{66}}}}, \\ A_{12} &= \frac{a_{12}}{1 + ip\sqrt{\frac{a_{55}}{a_{12}}}}, & A_{13} &= \frac{a_{13}}{1 + ip\sqrt{\frac{a_{55}}{a_{13}}}}, & A_{23} &= \frac{a_{23}}{1 + ip\sqrt{\frac{a_{55}}{a_{23}}}}. \end{aligned} \quad (3.29)$$

Now, the complex moduli CC can be obtained as:

$$\begin{aligned} CC_{11} &= A_{11} \rho, & CC_{22} &= A_{22} \rho, & CC_{33} &= A_{33} \rho, \\ CC_{44} &= A_{44} \rho, & CC_{55} &= A_{55} \rho, & CC_{66} &= A_{66} \rho, \\ CC_{12} &= A_{12} \rho - CC_{66}, & CC_{13} &= A_{13} \rho - CC_{55}, & CC_{23} &= A_{23} \rho - CC_{44}. \end{aligned} \quad (3.30)$$

The complex moduli are used to introduce damping into both the WFE and GL models. Since these two methods are finite-element-based, Abaqus is used to generate the mesh and the associated mass and stiffness matrices. Two models are created in Abaqus:

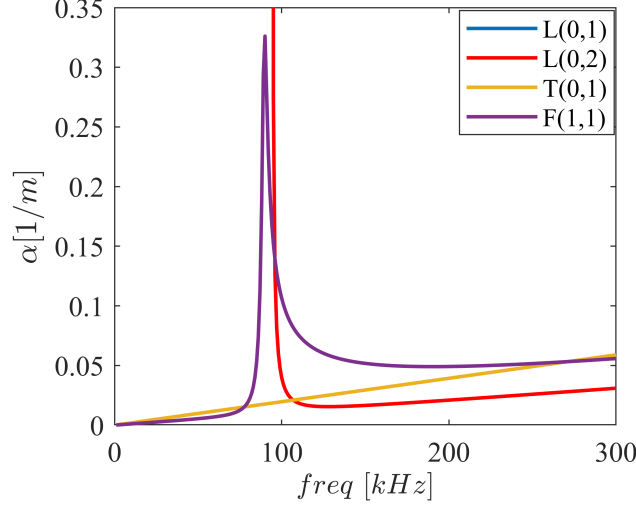


Figure 3.4: decay parameter of the fundamental modes of a 10mm radius pipe.

one using the real stiffness values and the other using the imaginary stiffness values of the complex moduli. As a result, two stiffness matrices are generated, K_{real} and K_{imag} . The final stiffness matrix used is:

$$\mathbf{K} = \mathbf{K}_{real} - i\mathbf{K}_{imag} \quad (3.31)$$

An example of the results obtained from the WFE solver after implementing the complex moduli approach with the parameter $p = 0.0005$ is shown in Fig. ?? . The decay factor α (i.e., the imaginary component of the wavenumber) for the fundamental modes is plotted against frequency. Although the parameter p itself is independent of frequency, the decay factor α is a function of frequency.

3.3 Nodal Force Vectors

In the source problem, the applied traction is applied on the interior nodes and need to be converted the nodal force vector f^{IP} . Traditional finite element approach is implemented to calculate the force vector and it's briefly discussed here. The formulation is based on the parametric mapping for the quadrilateral faces where the load is applied. Theses faces have 4 nodes in the case of linear element (C3D8) and 8 nodes in the case of quadratic (C3D20)

serendipity elements and both cases are implemented in this work. The global force vector in the spatial domain is defined as:

$$\mathbf{F} = \int_{\Gamma} \mathbf{N}^T \mathbf{N} \mathbf{t}_{node} d\Gamma \quad (3.32)$$

where $\mathbf{N}$ is the shape function, $\mathbf{t}_{node}$ is the traction and $d\Gamma$ is the area. In the isoparametric formulation, the spatial domain is expressed in the natural parent domain $[-1, 1] \times [-1, 1]$ in terms of η and ζ . The force vector becomes:

$$\mathbf{F} = \int_{-1}^1 \int_{-1}^1 \mathbf{N}^T(\eta, \zeta) \mathbf{N}(\eta, \zeta) J(\eta, \zeta) \mathbf{t}_{node} d\eta d\zeta \quad (3.33)$$

where J is the surface Jacobian that maps spatial domain to the parent domain. The implemented steps to calculate J is discussed in the following subsection.

3.3.1 Compact Matrix-Vector Product Form

In finite element implementations, it is computationally efficient to define a $3 \times n$ nodal coordinate matrix $\mathbf{X}_{face}$ containing the global physical coordinates and n is the number of nodes per face:

$$\mathbf{X}_{face} = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \\ z_1 & z_2 & \dots & z_n \end{bmatrix}_{3 \times n} \quad (3.34)$$

The parametric derivatives of the scalar shape functions are arranged into $n \times 1$ column vectors:

$$\frac{\partial \mathbf{N}_{\text{scalar}}}{\partial \xi} = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} \\ \frac{\partial N_2}{\partial \xi} \\ \vdots \\ \frac{\partial N_n}{\partial \xi} \end{bmatrix}_{n \times 1}, \quad \frac{\partial \mathbf{N}_{\text{scalar}}}{\partial \eta} = \begin{bmatrix} \frac{\partial N_1}{\partial \eta} \\ \frac{\partial N_2}{\partial \eta} \\ \vdots \\ \frac{\partial N_n}{\partial \eta} \end{bmatrix}_{n \times 1} \quad (3.35)$$

Using matrix multiplication, the 3×1 surface tangent vectors $\mathbf{T}_1$ and $\mathbf{T}_2$ are computed compactly as:

$$\mathbf{T}_1 = \mathbf{X}_{\text{face}} \left(\frac{\partial \mathbf{N}_{\text{scalar}}}{\partial \xi} \right) \quad (3.36)$$

$$\mathbf{T}_2 = \mathbf{X}_{\text{face}} \left(\frac{\partial \mathbf{N}_{\text{scalar}}}{\partial \eta} \right) \quad (3.37)$$

Now, the surface Jacobian represents differential area scale factor and it's defined as the norm of the cross product:

$$J(\eta, \zeta) = ||\mathbf{T}_1 \times \mathbf{T}_2|| \quad (3.38)$$

It represents the expansion/compression factor of the surface area at a given point. The integral of the force vector is evaluated numerically using Gaussian quadrature method. The force vector has this form:

$$\mathbf{F} = \sum_{i=1}^{n_{np}} \sum_{j=1}^{n_{np}} \mathbf{N}^T(\boldsymbol{\eta}_i, \boldsymbol{\zeta}_j) \mathbf{N}(\boldsymbol{\eta}_i, \boldsymbol{\zeta}_j) \mathbf{J}(\boldsymbol{\eta}_i, \boldsymbol{\zeta}_j) w_i w_j \quad (3.39)$$

where w is weight function for the Gaussian quadrature. The size of the force vector $\mathbf{F}$ for each element is $[12 \times 1]$ or $[24 \times 1]$ depending whether it's linear (C3D8) or quadratic (C3D20) element.

Chapter 4

WFE/Global–Local Method Validation and Cases

This chapter is concerned with the validation and implementation of the integrated WFE GL solver. It presents a few validation cases and show a few cases in which of interest.

4.1 Validation

In this section, the WFE-GL solver is validated using a benchmark problem for solid rod, energy-balance calculations, and by comparisson against a commercial finite element software package (Abaqus) in the transient time-domain . The first case considers a transmission problem in which a single mode is excited from the left section of the pipe and propagates toward the right. The second case examines edge reflection in a solid rod, and the results are compared with those reported in the literature. The third case considers a source problem in which a symmetric circular-ring load is applied at the middle of the pipe, generating waves that propagate in both the left and right directions. The results from the third problem are then used for the coupled source–scattering problem, in which both pristine and symmetrically defective configurations are tested and validated against Abaqus.

4.1.1 Transmission Problem

This section presents the basic simple check for transmission problem for an infinite rod and in finite hollow cylinder subjected to a single incident mode, such as $L(0,1)$, $T(0,1)$, or $F(1,1)$, and verifies the solution using the energy carried by the transmitted and reflected modes. The mode shapes corresponding to $L(0,1)$, $T(0,1)$ are individually used as incident modes propagating from left to right. The formulation used for this problem is the one mentioned for the source problem with the exception that the nodal forces and modal shapes on the right hand side of eq(3.6) are only for the mode of interest.

In Fig. 4.1, a single incident mode with unit amplitude is excited in a pipe with a radius of 10 mm. The model length is $1R$. The left- and right-pointing arrows in the legend denote the transmitted and reflected waves, respectively. It can be seen that, for all three incident modes— $L(0,1)$, $T(0,1)$, and $F(1,1)$ —the incident mode has a power flow of 1, while the power flows associated with all other reflected and transmitted modes are zero. This confirms that the incident wave is fully transmitted in the pristine configuration as expected.

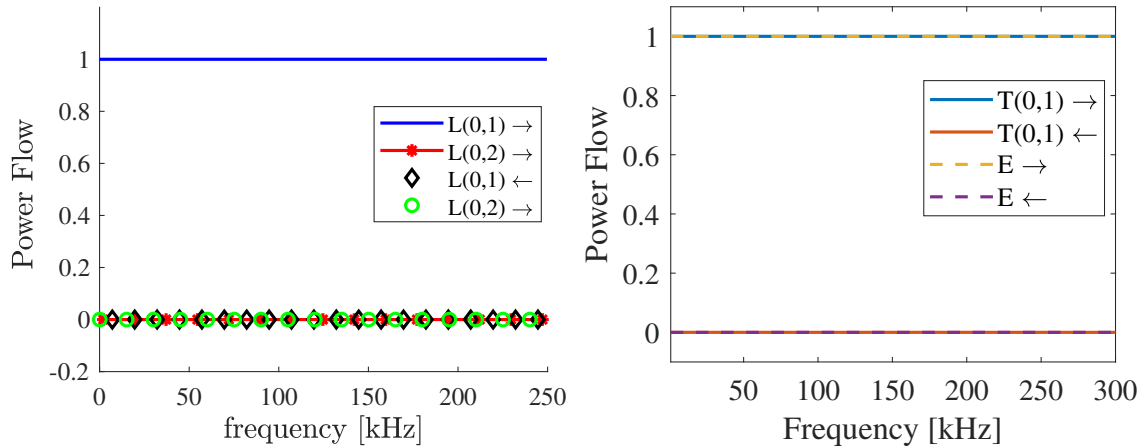


Figure 4.1: $L(0,1)$ incident mode on a pipe with a 9mm radius.

4.1.2 Solid Rod Edge Reflection Problem

The end reflection of a solid rod is one of the well-known benchmark problems and it's used here to validate the solver and is compared with solution found by other researchers [54], [57], [58]. It assumes that the solid is semi-infinite and thus there is no transmission at the right cross-section. The result of this is that the right transmission coefficient B_n^R is zero. A mode of interest with mode shape Q_m The equation becomes:

$$\begin{bmatrix} D_{LL} + [\overleftarrow{F}][\overleftarrow{Q}]^\dagger & D_{LI} & D_{LR} \\ D_{IL} & D_{II} & D_{IR} \\ D_{RL} & D_{RI} & D_{RR} \end{bmatrix} \begin{Bmatrix} u_B^L \\ u^I \\ u_B^R \end{Bmatrix} = \begin{Bmatrix} ([\overleftarrow{F}][\overleftarrow{Q}]^\dagger[\overrightarrow{Q}]_m - [\overrightarrow{F}]_m) \{A_m e^{i\overrightarrow{k}_m z_L}\} \\ 0 \\ 0 \end{Bmatrix} \quad (4.1)$$

A steel solid rod of radius $90mm$ is modeled here. The frequency step is 100 Hz and the range is $1 - 33kHz$. The chosen WFE model (global region) has 240 quadratic brick elements (C3D20) and an element edge size of $9mm$ in the propagating direction for the GL solver (local region). The cross-section mesh in both global and local finite element models are exactly the same to maintain the mesh alignment and boundary conditions at the interface. The dispersion curves is shown in Fig. 4.2. The cyan color is for the comparison with the analytical solution obtained from PCdisp [59]. It's interesting to see the behavior of L(0,3) mode in which it turns real propagating wave at around $19kHz$ and then becomes pure imaginary in the region of $20 - 21kHz$ and real afterwards. The role of L(0,3) mode is very important in the tested region here as without it the condition of complete basis is compromised and the solution becomes incorrect. The considered model included both propagating and non-propagating.

Figure 4.3 shows the power flow vs dimensionless frequency Ω of a length ratio $L/(2R)$ of 3 with L(0,1) and F(1,1) as incident waves, respectively. It has been reported in [54] that the minimum rod length ratio has to be at least $L/(2R) = 3$, for the solution to converge

where they only used real propagating modes. This length ratio can be reduced further by including the non-propagating modes in the undamped model.

An even more significant reduction of the model length can be achieved by including a small amount of damping where the ratio can be reduced to $L/(2R) = 0.05$ with a maximum error percentage of less than 0.6%. Fig 4.4 is for the end reflection of F(1,1) incident mode and it explains the effect of non-propagating modes on the error and its effect on the $L/(2R)$ ratio. Fig 4.4b illustrates the effect of the number of flexural modes (mainly non-propagating modes) on the energy error for various length ratios. It's worth mentioning that the significant reduction in the energy error occurs after including the first 16 flexural modes and these modes are the non-propagating F(1,m) family modes. The importance of this is the ability to reduce the simulation time to less than 5 minutes by using a segment length ratio of 0.05. Fig 4.6a shows this effect of time reduction which follows a quadratic increase in the computation time as length ratio (lower x-axis) or the number of modes (upper x-axis) increases which is expected for quadratic elements.

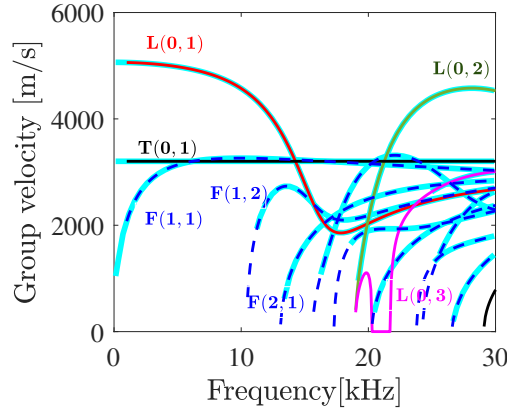


Figure 4.2: Dispersion Curves of 90mm steel bar

4.1.3 Transient Response Comparison

This section compares the solution obtained from the coupled GL source-scattering problem with the solution obtained from Abaqus. The GL problem is solved in the frequency domain,

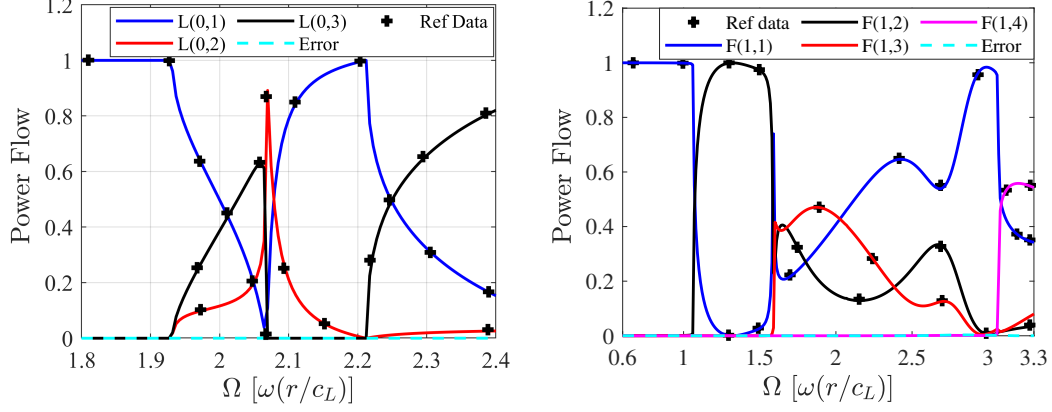


Figure 4.3: End reflection of L(0,1) (left) and F(1,1) (right) incident modes on a solid rod

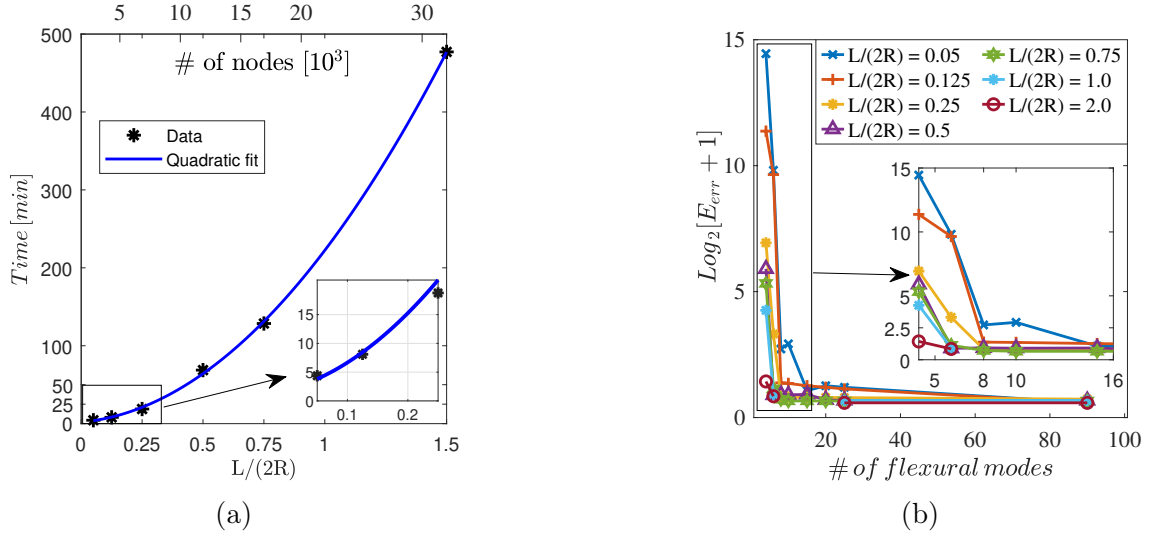


Figure 4.4: (a) Length ratio as a function of computation time. (b) number of flexural modes as a function of energy error.

after which the inverse fast Fourier transform (IFFT) is applied to obtain the transient time-domain response. The problem is solved in two stages: the source problem, described in Section 3.2.1, and the coupled scattering problem, described in Section 3.2.2.

The source problem is very similar to the transmission problem, with the exception that the excitation is generated by tractions applied to the interior finite element nodes. These node sets are defined in Abaqus during model creation, after which the input file is read by the solver in MATLAB. Numerical Gaussian integration is applied to obtain a smooth load distribution over the elements, depending on the element type used, as discussed in Section-



Figure 4.5: (a) Abaqus pristine Model and (b) defected model

(3.3). In this model, linear and quadratic brick elements (C3D8 and C3D20, respectively) were tested using approximately the same number of nodes. The quadratic elements were found to provide better results. This observation is consistent with the comparison of the dispersion curves and more importantly the modal shapes and nodal forces for each mode. Figure 4.6 shows a comparison of the dispersion curves obtained using the two element types.

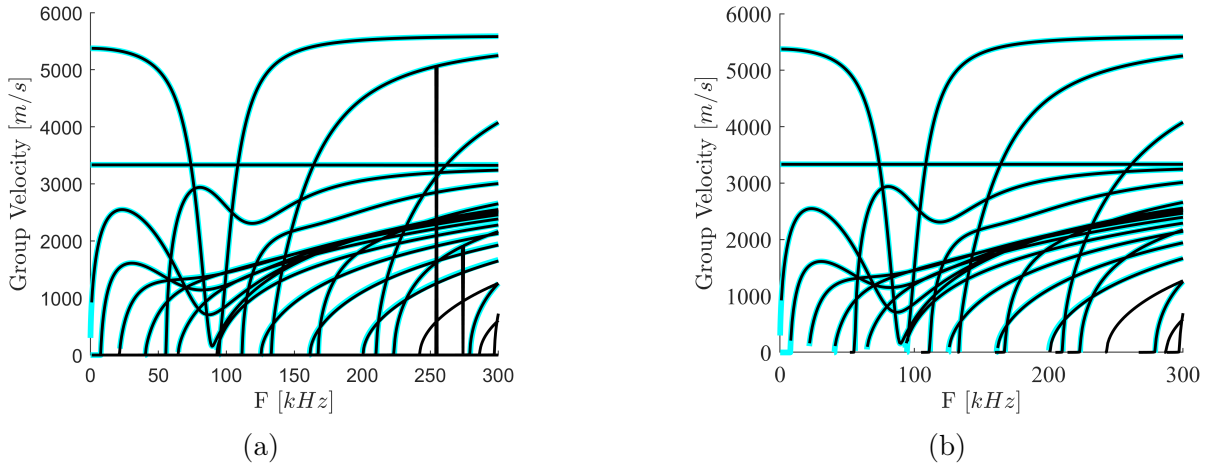


Figure 4.6: Dispersion Curves using linear (C3D8) elements (a) and quadratic (C3D20) elements

The source model is shown in Fig. 4.7. It has a radius of 10mm, a length of 10mm and a wall-thickness of 1mm. The highlighted red region represents the area where the traction is applied. The input signal is a 15-cycle Hann-windowed signal with a center frequency of

150 kHz. The input load follow a gaussian distribution in the axial direction. The applied source signal, measured at the center of the pipe, is shown in Fig. 4.8 in terms of both the applied forces and the resulting displacements. The signals are presented in both the frequency domain, corresponding to the direct GL output, and the time domain.

At the excitation points, the displacement is purely radial, with zero axial and tangential components. The excitation generates two modes, $L(0,1)$ and $L(0,2)$. As the waves propagate toward the left and right ends of the pipe, the displacement components are redistributed, and axial displacement components emerge in accordance with the characteristics of the excited modes. This behavior can be observed in Fig. 4.9, where the radial and axial displacement components are shown at the right cross-section. Finally, the modal power flow $[W]$ at the right and left ends of the finite element model for all modes are shown in Fig.4.9. It can be seen that the total transmitted power on the left and right are just the summation of $L(0,1)$ and $L(0,2)$ modes.

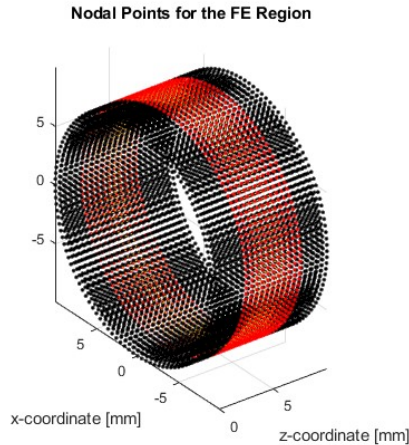


Figure 4.7: 10mm radius tube with circular applied load as processed in the solver

In the scattering model, a symmetric cut with a radial depth of 0.5 mm and an axial length of 5 mm is introduced at the center of the pipe. The scattering model is located 600 mm from the source model (i.e., $z_L = 600$ mm). The results of the coupled source-scattering problem are compared with those obtained from Abaqus using exactly the same mesh and element type. Figure 4.11 shows the time-domain response measured at a location equidistant

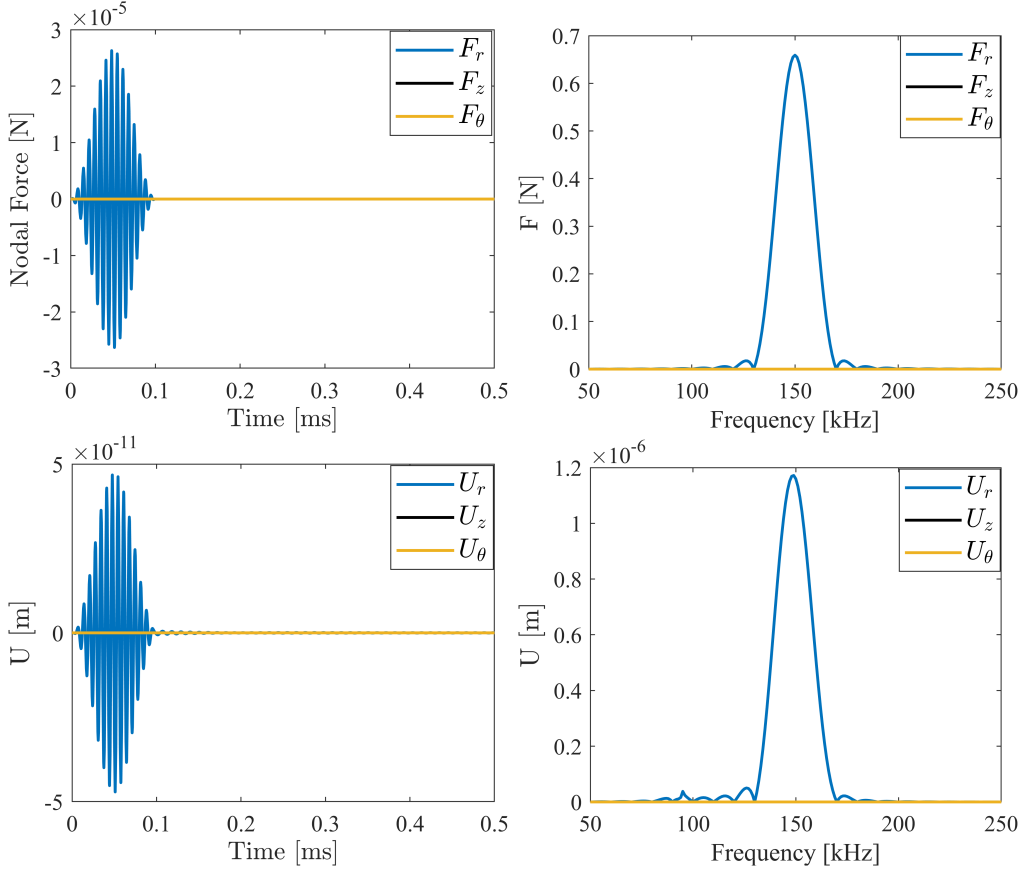


Figure 4.8: Input signal at the center of the source model.

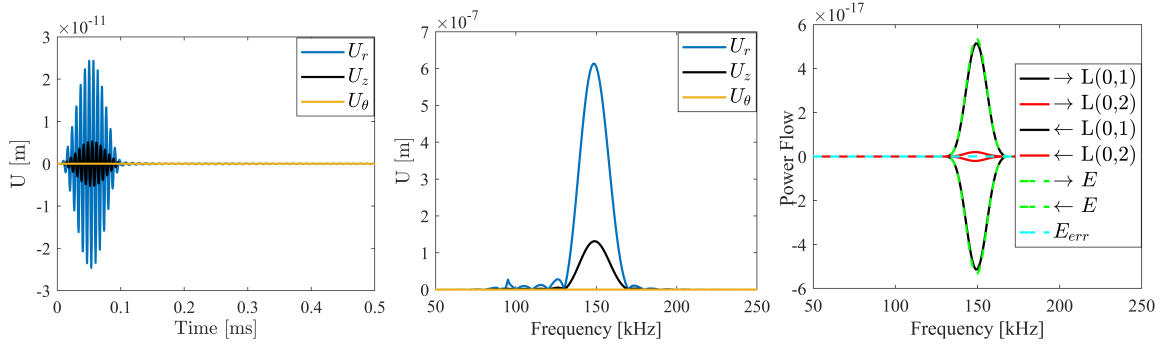


Figure 4.9: Time and frequency response for the right going waves and the power flow of the source problem.

from the source and the defect. The black curves represent the Abaqus results, while the red curves represent the GL solution. Excellent agreement is observed between the two solutions. One more check that can be done is the interface between the local finite element region with the global region. Fig. 4.10 shows the interface at the right cross-section between the FE

and GL regions.

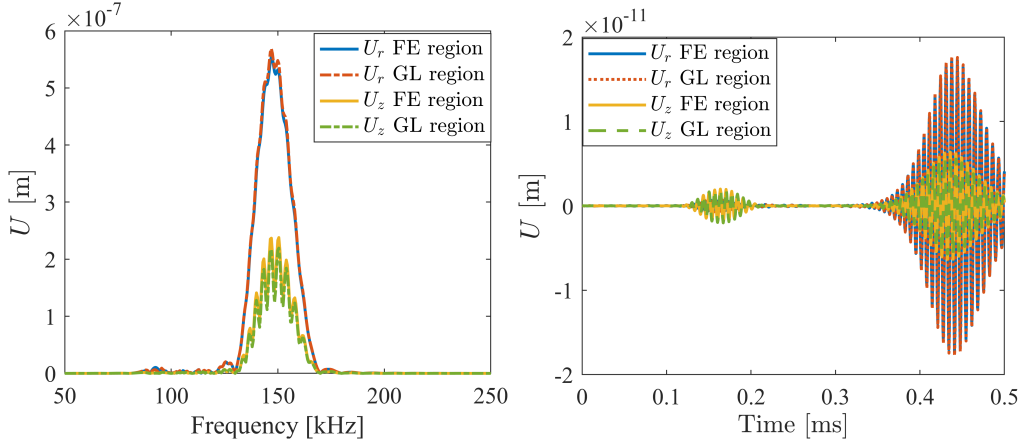


Figure 4.10: Response at the right interface between the FE and GL region for radial and axial displacements.

It is worth noting that a small amount of noise is present in the radial response of the GL solution and becomes more pronounced near the defect, as shown in Fig. 4.12. This noise is associated with the small peak observed at approximately 95 kHz in the frequency-domain response, which corresponds to the cut-on frequency of the L(0,2) mode. A high-pass filter can show the effect of removing this small noise in cleaning the signal as shown in the figure. Also, a finer mesh in the radial direction is expected to smoothen the solution.

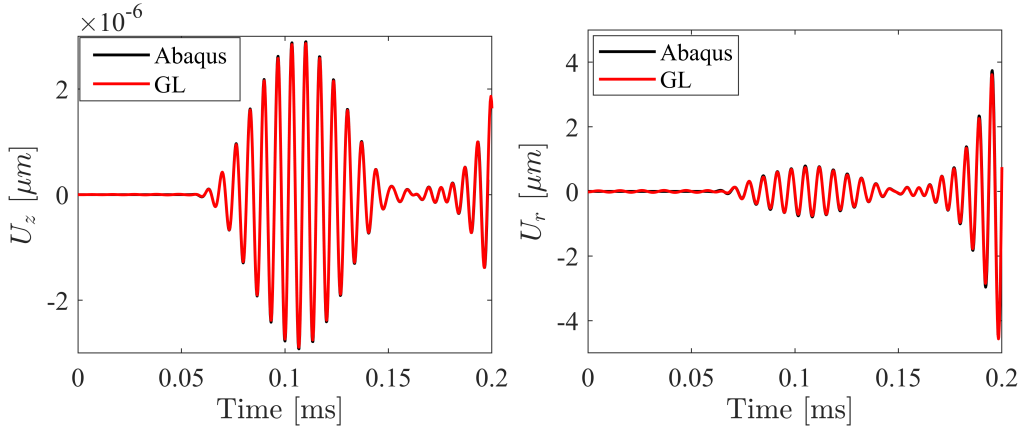


Figure 4.11: Radial and axial response at an equal distance between source and defect.

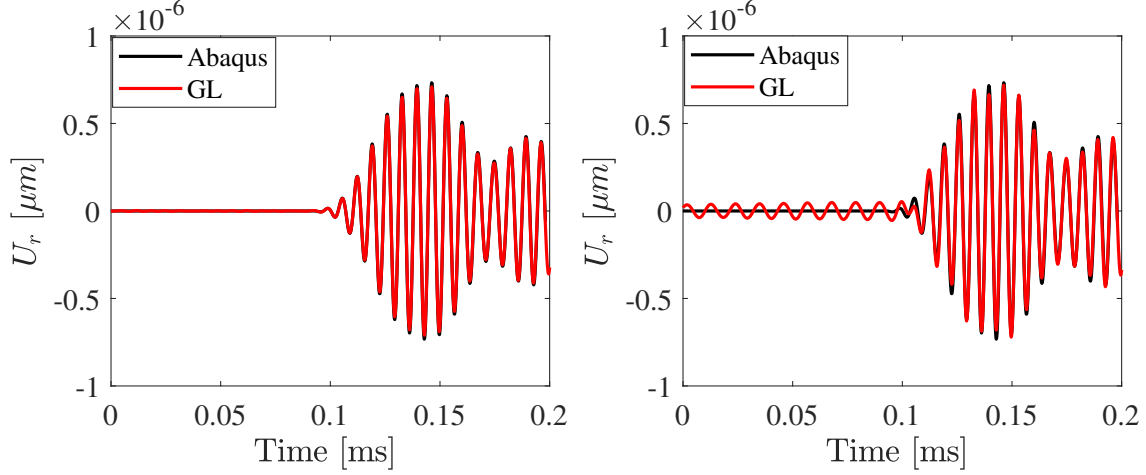


Figure 4.12: Comparison of the results with and without the hi-pass filter.

4.1.4 Experimental signal Verification

A pristine Al pipe with an outer radius of 40 mm and a wall thickness of approximately 1.6 mm is considered. The dispersion curves obtained from the WFE solver are shown in Fig. 4.13. The wavenumber plot shows the right propagating waves for all included modes. The number of modes in the group velocity curves are reduced so that the experimental group velocity data are seen clearly. The modes match well with the velocity of L(0,1) mode. A simple homemade electromagnetic acoustic transducer (EMAT) is used to excite a longitudinal wave so that the experimental results can be compared with the solution obtained from the global-local solver using a symmetric ring load. The transducer is made of enameled copper wire, permanent magnets, and a 3D-printed housing for the wires and magnets. Figure 4.14 shows the EMAT transducer and the experimental setup. A 6-cycle Hann-windowed signal with a center frequency of 60 kHz is excited. The received signal is measured using a point receiver transducer at distances of 5, 7, 9, and 11 in from the source. A signal amplifier with a gain of 25 dB is used at the receiver end. It should be noted that a more proper design would require an impedance-matching network to reduce the impedance mismatch resulting from the high inductance of the copper coils which significant amount of energy reflections at function generator-coil interface.

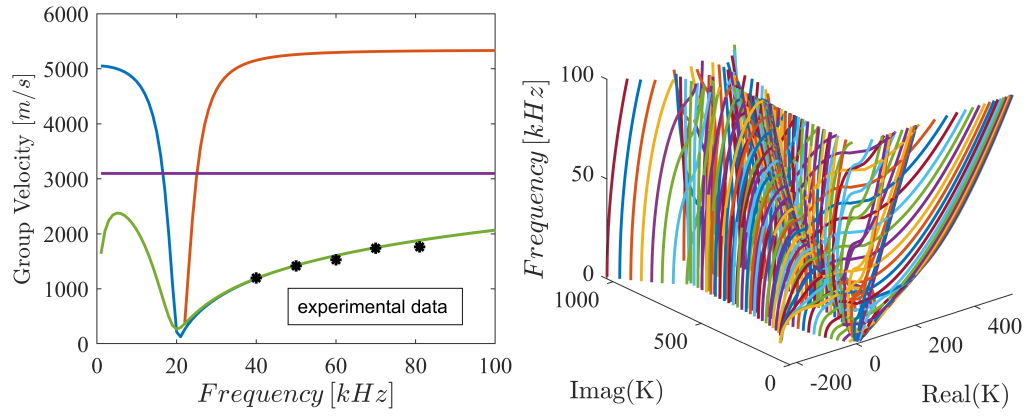


Figure 4.13: Dispersion Curves for R=40mm pipe with the experimental data shown in group velocity curves.

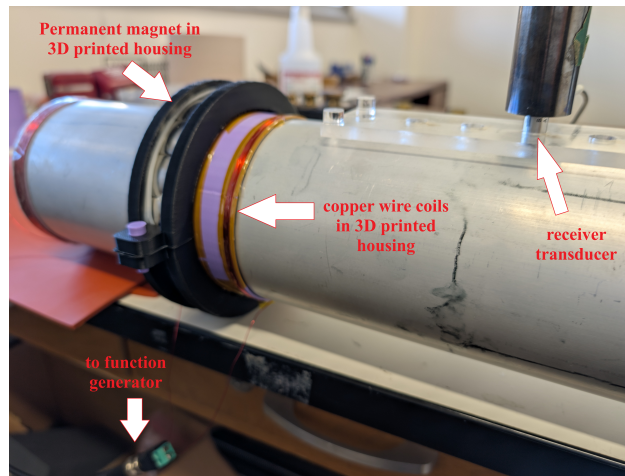


Figure 4.14: The home-made EMAT and the experimental setup.

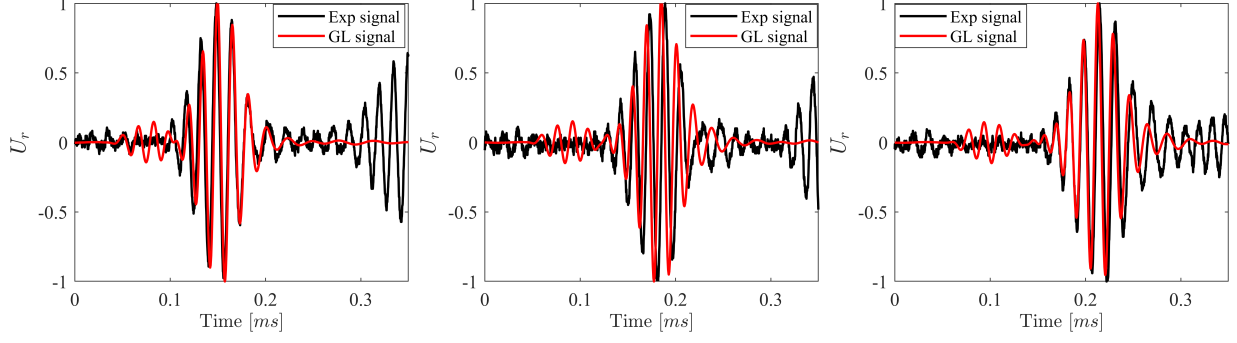


Figure 4.15: Comparison of the normalized experimental and GL solution signals at 5, 7, and 9 in distance from the source.

The signals obtained from both the solver and the experiment are normalized and compared with each other. Figure 4.15 shows the results at distances of 5, 7, and 9 in from the source, respectively. The experimental signal is noisy, this is probably because of the weak signal resulted from the impedance mismatch between the coil and the function generator. However, the arrival time of $L(0,1)$ wave packet shows very good agreement with the signal obtained from the global-local solver accross the propagation distance. The $L(0,2)$ mode cannot be clearly observed in the measurements because of the noise. In addition, the $L(0,2)$ mode is dominated by axial displacement as discussed previously, while the measurements only capture the out-of-plane displacement.

4.2 Cases

4.2.1 Axial Vs Radial Vs Torsional Excitation

In this section, a comparison is made among three symmetric source configurations: axial, radial, and torsional. The same geometry and excitation frequency considered in the previous section are used. The right-going signals and corresponding modal power flows are shown in Fig. 4.16. The power-flow results show that the axial source is dominated by the $L(0,2)$ mode, which is characterized by a large axial displacement component. As observed previously, the radial source is dominated by the $L(0,1)$ mode, with the radial displacement being the

dominant component. In contrast, the torsional source excites only the T(0,1) mode, which is characterized by purely tangential displacement.

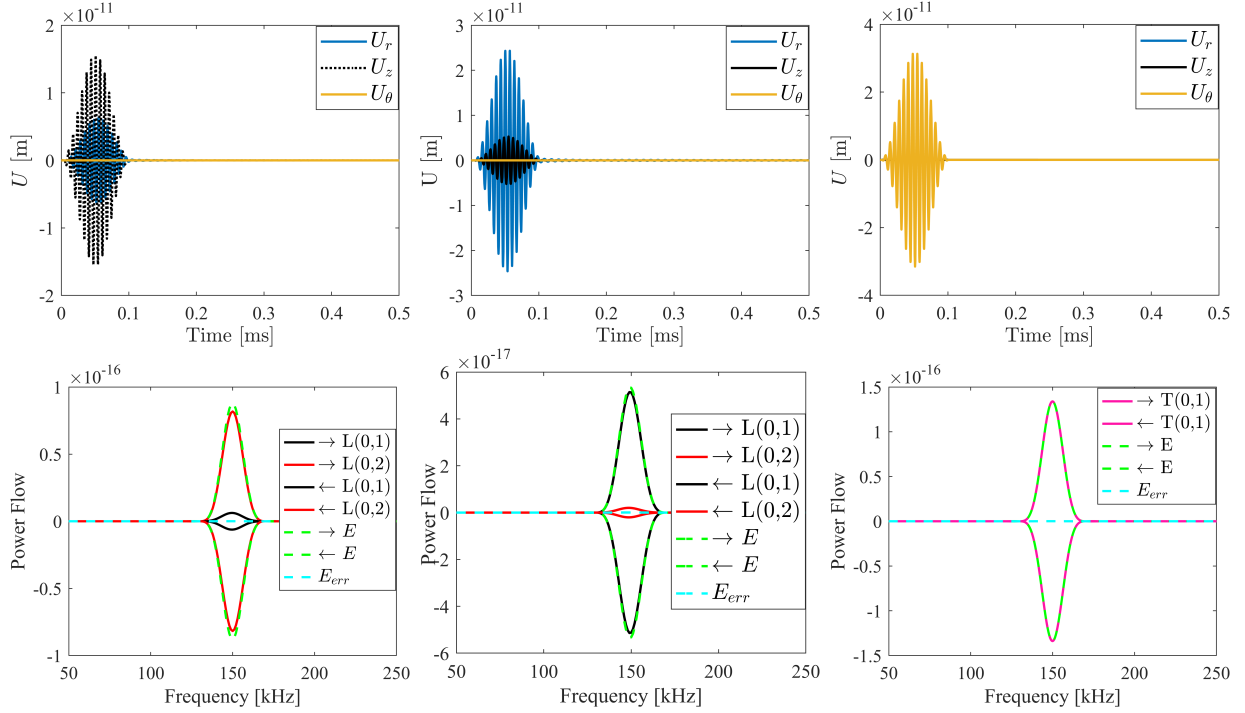


Figure 4.16: The right going signals (top) and power flow (bottom) for axial, radial, and tangential excitations, respectively.

As in the previous example, the solutions obtained for the three source configurations are introduced into the defected model for each case. Figure 4.17 shows the reflected signals measured 200 mm to the left of the defect, together with the corresponding modal power flow for all three cases. In the time-domain responses, the reflected L(0,1), L(0,2), and T(0,1) modes are highlighted in black, red, and orange, respectively.

Figure 4.17a shows the response to the axial source. Approximately 16 % of the incident power is reflected, with the reflected response dominated by the L(0,2) mode. This behavior can also be observed in the time-domain response to the left of the defect for both the U_r and U_z displacement components. The largest reflected signal occurs in the axial direction and corresponds to the L(0,2) mode.

Figure 4.17b shows the response to the radial source, which has the lowest reflected

power, approximately 2.5%. This reflected power consists of approximately 1.3% from the L(0,1) mode and 1.2% from the L(0,2) mode. The time-domain response also shows these relatively small reflected waves.

Finally, Fig. 4.17c shows the response to the torsional source, for which only the T(0,1) mode is present. This case exhibits the highest reflected power, approximately 30%. The corresponding reflection is clearly observed in the tangential displacement component, U_θ .

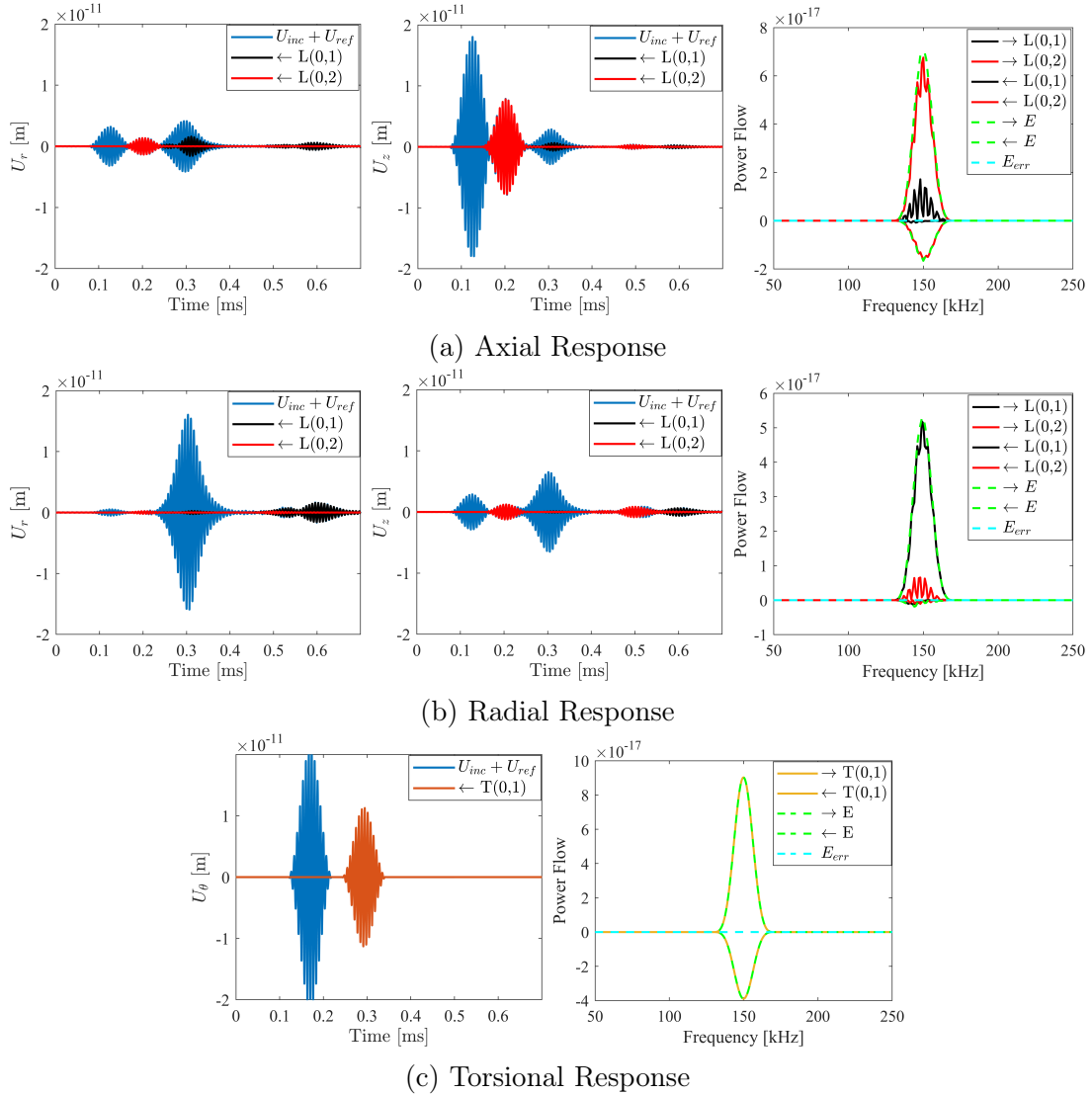


Figure 4.17: Total response and reflection 200mm to the left of FE model.

4.2.2 Woven Composite Repair

In this section, a model consisting of two materials in the finite element region is considered. The left and right ends are made of Al to maintain compatibility with the global functions, while the middle section is made of woven carbon composite. The material properties of the woven composite are shown in Table 4.1. Two cases are considered: one in which the middle section is fully replaced with a change in thickness, and another in which an additional layer is added. Fig. 4.18 shows the Abaqus model for the cases.

Table 4.1: Material properties of the woven carbon composite.

E_1	E_2	E_3	ν_{12}	ν_{13}	ν_{23}	G_{12}	G_{13}	G_{23}	ρ
60 GPa	60 GPa	9 GPa	0.05	0.30	0.30	4 GPa	3 GPa	3 GPa	1550 kg/m ³

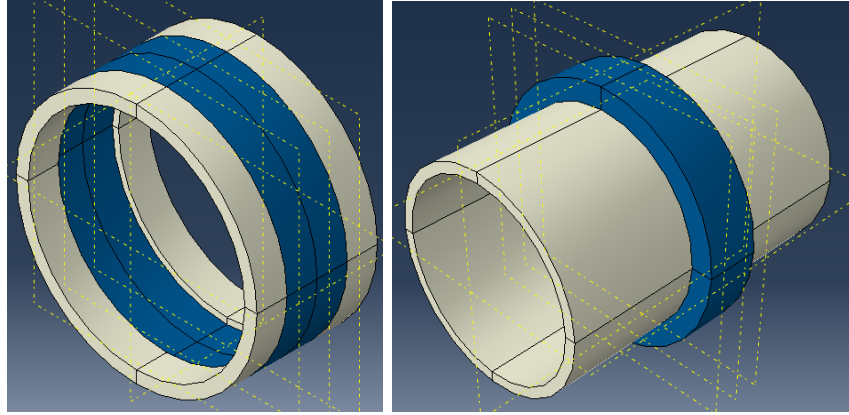


Figure 4.18: Aluminum pipe with woven carbon fiber in the middle.

Full replacement

A radial source is tested with this material system. It produces a reflected signal of approximately 20%, most of which corresponds to the L(0,1) mode, as shown in Fig. ???. In the time domain, a clear reflected signal can also be observed. This indicates that such a change in material may be detectable using a conventional transducer that measures out-of-plane displacements.

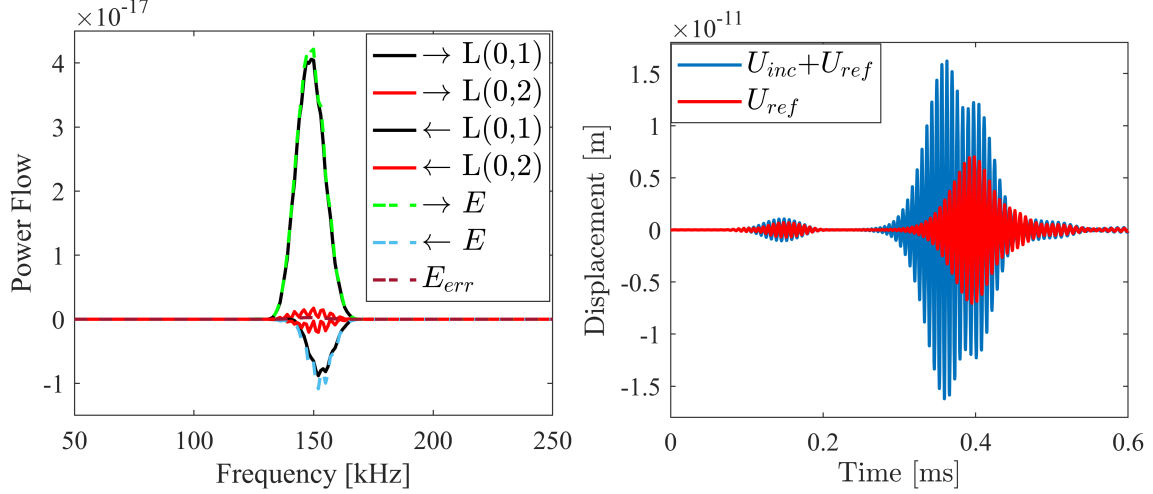


Figure 4.19: Power flow and received signal for the full replacement case.

Coating Repair Layer

For this model, radial and torsional source excitations are tested. Interestingly, the radial source is transmitted almost completely, with only a very small amount of reflection. This can be seen in Fig. 4.20a, both in the power-flow results and in the time-domain signals measured at the left (entrance) and right faces. On the other hand, the torsional source shows a much higher reflection due to the added composite layer in the middle section, as shown in Fig. 4.20b.

4.3 Computational time Consideration

One of the main advantages of the proposed approach, in addition to providing a fundamental description of all wave components, is its ability to reduce the computational time. Solving the problem in the frequency domain, where each frequency step is independent, provides the advantage of true parallel computing. Parallel computing was implemented in all the problems solved in this work. Although it was applied only on a relatively small scale because of the limited computational resources available in the lab (a personal computer and a small lab computer with 24 cores), most computations required an average of 1–2 hours, with a

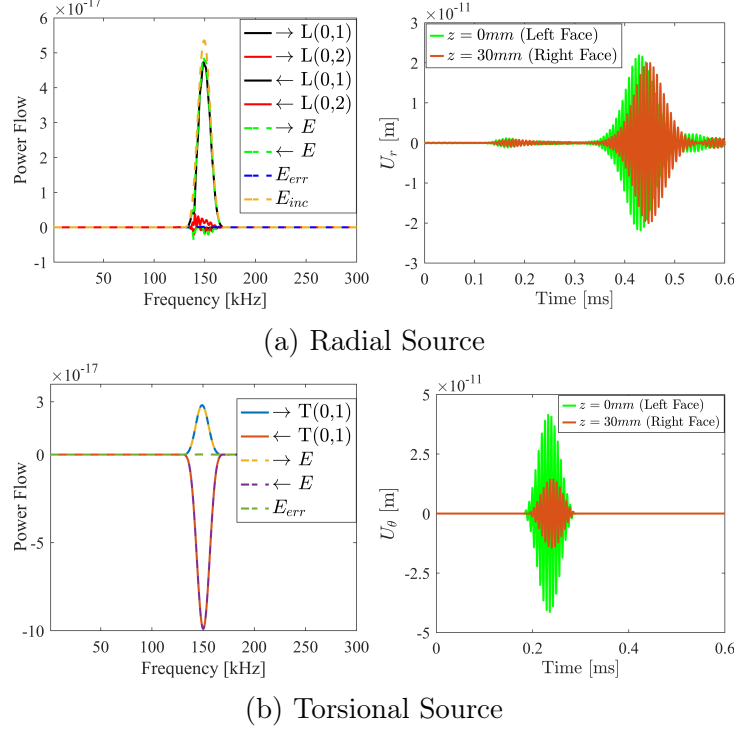


Figure 4.20: Total response and reflection 200mm to the left of FE model.

maximum computational time of approximately 6 hours for the composite repair case. These simulations were performed using quadratic elements to improve accuracy.

However, solutions obtained using linear elements can still provide reasonable results and can be useful for informing design and analysis decisions, while requiring only a few minutes of computational time. Figure 4.21 shows the computational time using linear elements for a pipe with a radius of 10 mm as a function of the model length, expressed in units of R . An equivalent model was also run using the Abaqus/Explicit solver, for which the computational time exceeded 200 min, corresponding to a minimum computational speedup of approximately $3\times$.

4.4 Concluding Remarks and Future Work

This work implemented a hybrid WFE/GL framework for modeling 3D wave propagation in pipes. The solution is validated against benchmark problem, commercial FEA software, and

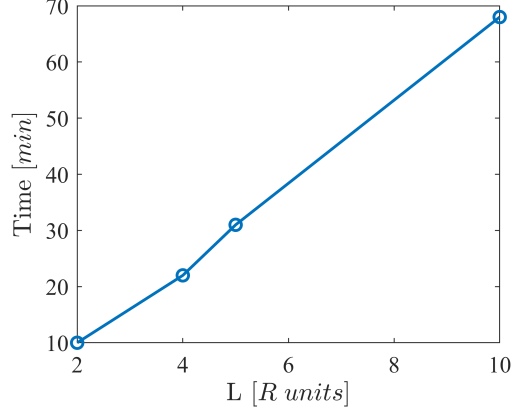


Figure 4.21: Computational time of linear elements as function of the length of the radius R an experimental signal. The technique provide a platform for testing more material system under various configurations.

Future work can include an improvement of the WFE solver by implementing the rotational symmetry and converting it into a practically a 1D problem [60]. However, this would require 3D reconstruction of mode shapes and nodal forces to match with the 3D GL model. Improved data storage and handling are also important, since the size of the global functions required by the GL solver can be very large.

Appendix A

Bessel Functions Review

The Bessel equation takes the following form:

$$x^2 y'' + xy' \pm (x^2 \mp n^2)y(x) = 0 \quad (\text{A.1})$$

Assume that x is real and positive and n is a positive integer. The two linearly independent series solutions about $x = 0$ are denoted by $J_n(x)$ (the Bessel function of the first kind) and $Y_n(x)$ (the Bessel function of the second type) or by $I_n(x)$ (the modified Bessel function of the first kind) and K_n (the modified Bessel function of the second kind). $J_n(x)$ is finite at $x = 0$ and has series representation:

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{x}{2}\right)^{2m}}{m!(m+n)!} \quad (\text{A.2})$$

and further note that:

$$Y_n(x) = \frac{2}{\pi} \ln(x) + O(1), \quad \text{as } x \rightarrow 0 \quad (\text{A.3})$$

So that $Y_n(x)$ has a logarithmic singularity at $x = 0$ For large x :

$$\begin{aligned} J_n(x) &= \sqrt{\frac{2}{\pi x}} \left[\cos \left(x - \frac{n\pi}{2} - \frac{\pi}{4} \right) - \frac{4n^2 - 1}{8x} \sin \left(x - \frac{n\pi}{2} - \frac{\pi}{4} \right) + O(x^{-2}) \right] \\ Y_n(x) &= \sqrt{\frac{2}{\pi x}} \left[\sin \left(x - \frac{n\pi}{2} - \frac{\pi}{4} \right) + O(x^{-1}) \right] \end{aligned} \quad (\text{A.4})$$

The Hankel functions $H_n^{(1)}(x), H_n^{(2)}(x)$ are defined by the equations:

$$\begin{aligned} H_n^{(1)}(x) &= J_n(x) + i Y_n(x) \\ H_n^{(2)}(x) &= J_n(x) - i Y_n(x) \end{aligned} \tag{A.5}$$

Note that Hankel functions $H_n^{(1)}(x), H_n^{(2)}(x)$ form an alternative set of linearly independent solution of Bessel equation and that both have logarithmic singularities at $x = 0$.

Also, for large values of x Hankel functions are approximates by:

$$\begin{aligned} H_n^{(1)}(x) &= \exp \left[i \left(x - \frac{n\pi}{2} - \frac{\pi}{4} \right) \right] + O(x^{-3/2}) \\ H_n^{(2)}(x) &= \exp \left[-i \left(x - \frac{n\pi}{2} - \frac{\pi}{4} \right) \right] + O(x^{-3/2}) \end{aligned} \tag{A.6}$$

The recurrence relations for Bessel functions:

$$\begin{aligned} J_{n-1}(x) + J_{n+1}(x) &= 2n J_n(x)/x \\ J_n(x) - J_{n+1}(x) &= 2J'_n(x) \\ \{x^n J_n(x)\}' &= x^n J_{n-1}(x) \\ \{x^{-n} J_n(x)\}' &= -x^{-n} J_{n+1}(x) \\ J'_0(x) &= -J_1(x) \\ J''_0(x) &= x^{-1} J_1(x) - J_0(x) \end{aligned} \tag{A.7}$$

Differentiation

$$\begin{aligned} \frac{d}{dx} J_n(x) &= \frac{1}{2} (J_{n-1}(x) - J_{n+1}(x)) = \frac{n}{x} J_n(x) - J_{n+1}(x) \\ \frac{d}{dx} (x^n J_n(x)) &= x^n J_{n-1}, \quad \frac{d}{dx} (x^{n-1} J_n(x)) = -x^{-1} J_{n+1} \end{aligned} \tag{A.8}$$

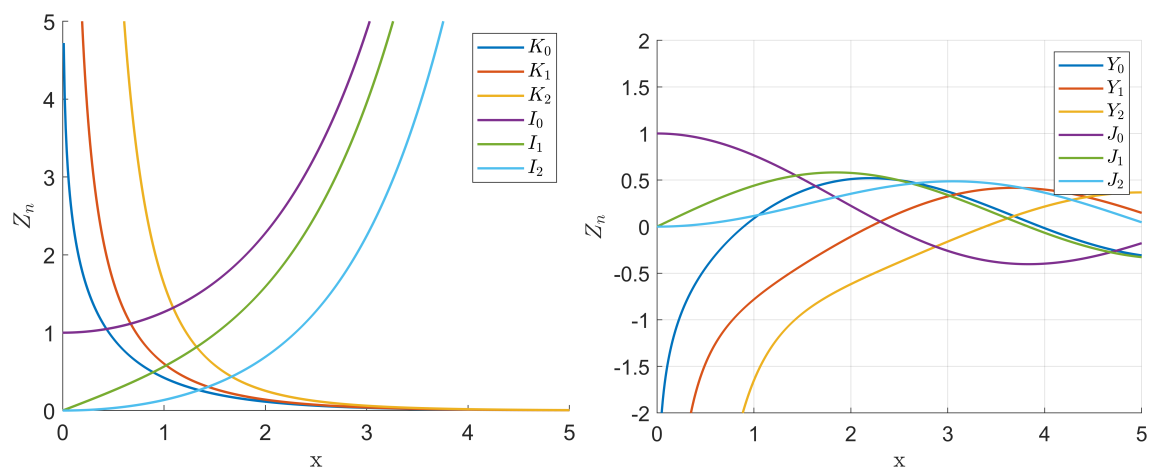


Figure A.1: Modified Bessel functions (left) and Bessel functions (right)

Appendix B

Separation of Variables in Cylindrical Coordinates

The wave equations are:

$$\begin{aligned}\nabla^2\Phi - c_1^{-2}\ddot{\Phi} &= 0 \\ \nabla^2\Psi - c_2^{-2}\ddot{\Psi} &= 0 \\ \nabla^2\chi - c_2^{-2}\ddot{\chi} &= 0\end{aligned}\tag{B.1}$$

Take the second equation as an example and use the separation of variables:

$$\begin{aligned}\Psi(r, \theta, z, t) &= \Psi_1(r)\Psi_2(\theta)\Psi_3(z)T(t) \\ T(t) &= e^{-i\omega t}\end{aligned}\tag{B.2}$$

Plug equation (B.2) in (B.1) to get:

$$\begin{aligned}\frac{1}{r} [\Psi'_1 + r\Psi''_1] \Psi_2\Psi_3T + \frac{1}{r^2}\Psi_1\Psi''_2\Psi_3T + \Psi_1\Psi_2\Psi''_3T - c_2^{-2}\Psi_1\Psi_2\Psi_3T'' &= 0 \\ \frac{1}{r} [\Psi'_1 + r\Psi''_1] \frac{1}{\Psi_1} + \frac{1}{r^2} \frac{\Psi''_2}{\Psi_2} + \frac{\Psi''_3}{\Psi_3} &= c_2^{-2} \frac{T''}{T} = -k_2^2\end{aligned}\tag{B.3}$$

from equation (B.3) we see that:

$$\begin{aligned}
c_2^{-2} \frac{T''}{T} &= -k_2^2 \\
\frac{\Psi_3''(z)}{\Psi_3(z)} &= -k^2 \\
\frac{1}{r} [\Psi_1' + r\Psi_1''] \frac{1}{\Psi_1} + \frac{1}{r^2} \frac{\Psi_2''}{\Psi_2} &= k^2 - k_2^2 = -\nu_2^2
\end{aligned} \tag{B.4}$$

where:

$$\nu_2 = \sqrt{k_2^2 - k^2}$$

equation B.3 can be rewritten as:

$$r [\Psi_1' + r\Psi_1''] \frac{1}{\Psi_1} + r^2 k_r^2 = -\frac{\Psi_2''}{\Psi_2} \tag{B.5}$$

It's clear that

$$\frac{\Psi_2''(\theta)}{\Psi_2(\theta)} = -n^2 \tag{B.6}$$

where n is a constant. The solution of the second order ordinary differential equations B.5, B.6 gives:

$$\begin{aligned}
\Psi_2(\theta) &= D_1 e^{in\theta} + D_2 e^{-in\theta} \\
\Psi_3(z) &= e^{\pm i k_z z}
\end{aligned} \tag{B.7}$$

Since $\Psi_2(2\pi + \theta) = \Psi_2(\theta)$, then n has to be an integer number.

Rearrange the second line of (B.5) to get:

$$r^2 \Psi_1'' + r \Psi_1' + r^2 (k_r^2 - n^2) \Psi_1 = 0, \quad \text{for } k_2^2 > k^2 \tag{B.8a}$$

$$r^2 \Psi_1'' + r \Psi_1' - r^2 (k_r^2 + n^2) \Psi_1 = 0, \quad \text{for } k_2^2 < k^2 \tag{B.8b}$$

These are well-known differential equations and their solutions are given by Bessel

functions (B.8a) and modified Bessel functions (B.8b):

$$\Psi_1(r) = R_1 J_n(k_r r) + R_2 Y_n(k_r r)$$

$$\text{Or} \tag{B.9}$$

$$\Psi_1(r) = R_1 H_n^{(1)}(k_r r) + R_2 H_n^{(2)}(k_r r)$$

for Bessel functions, and:

$$\Psi_1(r) = R_1 I_n(k_r r) + R_2 K_n(k_r r) \tag{B.10}$$

for modified Bessel functions.

References

- [1] American Society for Nondestructive Testing. “What is nondestructive testing?” Accessed: Aug. 15, 2026. [Online]. Available: <https://www.asnt.org/what-is-nondestructive-testing>
- [2] P. J. Shull, *Nondestructive evaluation: theory, techniques, and applications*. CRC press, 2002.
- [3] A. Ahmadi, M. shayegani Akmal, A. Pasha, and S. Yareie, “Failure analysis of cracked 2.25 cr–1.0 mo steel tubes of an oil refinery boiler,” *Engineering Failure Analysis*, vol. 110, p. 104435, 2020.
- [4] L. Chen et al., “Failure analysis of stress corrosion cracking of 316l stainless steel bend pipe in the atmospheric tower,” *Engineering Failure Analysis*, vol. 174, p. 109519, 2025.
- [5] Q. Feng, R. Li, B. Nie, S. Liu, L. Zhao, and H. Zhang, “Literature review: Theory and application of in-line inspection technologies for oil and gas pipeline girth weld defection,” *Sensors*, vol. 17, no. 1, p. 50, 2016.
- [6] L. Li, X. Yan, L. Zhu, G. Wu, and S. Zhang, “Failure analysis of girth weld cracking in gas transmission pipelines subjected to ground subsidence and traffic loads,” *Materials*, vol. 17, no. 22, p. 5495, 2024.
- [7] J. Wang, Y. Li, X. Kuang, Y. Ma, J. Luo, and Y. Long, “Failure analysis of 3cr oil pipe body fracture,” *Scientific Reports*, vol. 15, no. 1, p. 25975, 2025.
- [8] M. V. Felice and Z. Fan, “Sizing of flaws using ultrasonic bulk wave testing: A review,” *Ultrasonics*, vol. 88, pp. 26–42, 2018.
- [9] D. Worlton, “Ultrasonic testing with lamb waves,” General Electric Co., Hanford Atomic Products Operation, Richland, Wash., Tech. Rep., 1956.
- [10] C. Chree, “Longitudinal vibrations of a circular bar,” *Quart. J. Pure Appl. Math.*, vol. 21, pp. 287–298, 1886.
- [11] D. C. Gazis, “Three-dimensional investigation of the propagation of waves in hollow circular cylinders. i. analytical foundation,” *The journal of the Acoustical Society of America*, vol. 31, no. 5, pp. 568–573, 1959.
- [12] E. Leinov, M. J. Lowe, and P. Cawley, “Ultrasonic isolation of buried pipes,” *Journal of Sound and Vibration*, vol. 363, pp. 225–239, 2016.

- [13] S. Mariani, S. Heinlein, T. Vogt, P. Cawley, et al., “Performance of a guided wave pipe monitoring system over extended periods of field operation,” *Mater. Eval*, vol. 81, no. 8, pp. 36–46, 2023.
- [14] E. C. Rodgers, S. Mariani, and P. Cawley, “The use of circumferential guided waves to monitor axial cracks in pipes,” *Structural Health Monitoring*, vol. 22, no. 4, pp. 2609–2625, 2023.
- [15] E. Leinov, M. J. Lowe, and P. Cawley, “Investigation of guided wave propagation and attenuation in pipe buried in sand,” *Journal of Sound and Vibration*, vol. 347, pp. 96–114, 2015.
- [16] R. Long, M. Lowe, and P. Cawley, “Attenuation characteristics of the fundamental modes that propagate in buried iron water pipes,” *Ultrasonics*, vol. 41, no. 7, pp. 509–519, 2003.
- [17] D. Alleyne, B. Pavlakovic, M. Lowe, and P. Cawley, “Rapid, long range inspection of chemical plant pipework using guided waves,” *Review of Progress in Quantitative Nondestructive Evaluation: Volume 20*, vol. 20, pp. 180–187, 2001.
- [18] J. Barshinger, J. L. Rose, and M. J. Avioli Jr, “Guided wave resonance tuning for pipe inspection,” *J. Pressure Vessel Technol.*, vol. 124, no. 3, pp. 303–310, 2002.
- [19] P. Cawley, “Guided waves in long range nondestructive testing and structural health monitoring: Principles, history of applications and prospects,” *NDT & E International*, p. 103 026, 2023.
- [20] S. Vishnuvardhan, A. R. Murthy, and A. Choudhary, “A review on pipeline failures, defects in pipelines and their assessment and fatigue life prediction methods,” *International Journal of Pressure Vessels and Piping*, vol. 201, p. 104 853, 2023.
- [21] J. Fowler, “Criteria for dent acceptability in offshore pipeline,” in *Offshore Technology Conference*, OTC, 1993, OTC–7311.
- [22] K. Escoe, *Piping and pipelines assessment guide*. Elsevier, 2006, vol. 1.
- [23] G. Qin and Y. F. Cheng, “A review on defect assessment of pipelines: Principles, numerical solutions, and applications,” *International journal of pressure vessels and piping*, vol. 191, p. 104 329, 2021.
- [24] A. Api, *579-1/asme ffs-1: Fitness-for-service*, 2007.
- [25] M. Waqar, A. M. Memon, M. Sabih, and L. M. AlHems, “Composite pipelines: Analyzing defects and advancements in non-destructive testing techniques,” *Engineering Failure Analysis*, p. 107 914, 2023.
- [26] W. Badeghaish, M. Noui-Mehidi, and O. Salazar, “The future of nonmetallic composite materials in upstream applications,” in *SPE Gas & Oil Technology Showcase and Conference*, SPE, 2019, D021S014R004.
- [27] J. LeBlanc and M. Sternisha, “Fiberglass pipe is helping solve the world’s drinking water shortage,” *Pipelines 2016*, pp. 1589–1599, 2016.

- [28] A. Chakraverty, S. Dash, H. Maharana, S. Beura, and U. Mohanty, “A novel investigation on durability of gre composite pipe for prolonged sea water transportation,” *Composites Communications*, vol. 17, pp. 42–50, 2020.
- [29] E. Mahdi Saad, S. Gowid, and J. J. Cabibihan, “Rupture of an industrial gfrp composite mitered elbow pipe,” *Polymers*, vol. 13, no. 9, p. 1478, 2021.
- [30] S. Gopalakrishnan, A. Chakraborty, and D. R. Mahapatra, *Spectral finite element method: wave propagation, diagnostics and control in anisotropic and inhomogeneous structures*. Springer Science & Business Media, 2007.
- [31] S. Finnveden, “Spectral finite element analysis of the vibration of straight fluid-filled pipes with flanges,” *Journal of Sound and vibration*, vol. 199, no. 1, pp. 125–154, 1997.
- [32] P. Shorter, “Wave propagation and damping in linear viscoelastic laminates,” *The Journal of the Acoustical Society of America*, vol. 115, no. 5, pp. 1917–1925, 2004.
- [33] D. J. Mead, “Wave propagation and natural modes in periodic systems: I. mono-coupled systems,” *Journal of Sound and Vibration*, vol. 40, no. 1, pp. 1–18, 1975.
- [34] C. Schaal and A. Mal, “Dispersion of guided waves in composite laminates and sandwich panels,” in *Health Monitoring of Structural and Biological Systems 2015*, SPIE, vol. 9438, 2015, pp. 137–148.
- [35] B. R. Mace, D. Duhamel, M. J. Brennan, and L. Hinke, “Finite element prediction of wave motion in structural waveguides,” *The Journal of the Acoustical Society of America*, vol. 117, no. 5, pp. 2835–2843, 2005. DOI: 10.1121/1.1887126
- [36] D. Duhamel, B. R. Mace, and M. J. Brennan, “Finite element analysis of the vibrations of waveguides and periodic structures,” *Journal of Sound and Vibration*, vol. 294, no. 1–2, pp. 205–220, 2006. DOI: 10.1016/j.jsv.2005.11.014
- [37] J.-M. Mencik and M. N. Ichchou, “Wave finite elements in guided elastodynamics with internal fluid,” *International Journal of Solids and Structures*, vol. 44, no. 7–8, pp. 2148–2167, 2007. DOI: 10.1016/j.ijsolstr.2006.06.048
- [38] E. Manconi and B. R. Mace, “Wave characterization of cylindrical and curved panels using a finite element method,” *The Journal of the Acoustical Society of America*, vol. 125, no. 1, pp. 154–163, 2009. DOI: 10.1121/1.3021418
- [39] A. Marzani, E. Viola, I. Bartoli, F. Lanza di Scalea, and P. Rizzo, “A semi-analytical finite element formulation for modeling stress wave propagation in axisymmetric damped waveguides,” *Journal of Sound and Vibration*, vol. 318, no. 3, pp. 488–505, 2008. DOI: 10.1016/j.jsv.2008.04.028
- [40] Y. Waki, B. R. Mace, and M. J. Brennan, “Numerical issues concerning the wave and finite element method for free and forced vibrations of waveguides,” *Journal of Sound and Vibration*, vol. 327, no. 1–2, pp. 92–108, 2009. DOI: 10.1016/j.jsv.2009.06.005
- [41] B. R. Mace, D. Duhamel, M. J. Brennan, and L. Hinke, “Finite element prediction of wave motion in structural waveguides,” *The Journal of the Acoustical Society of America*, vol. 117, no. 5, pp. 2835–2843, 2005.

- [42] Y. Waki, B. Mace, and M. Brennan, “Free and forced vibrations of a tyre using a wave/finite element approach,” *Journal of Sound and Vibration*, vol. 323, no. 3-5, pp. 737–756, 2009.
- [43] R. J. Allemang and D. L. Brown, “A correlation coefficient for modal vector analysis,” in *Proceedings of the 1st International Modal Analysis Conference*, Orlando, Florida, USA, 1982, pp. 110–116.
- [44] I. U. Ojalvo, “Modal sensitivity for structural systems with repeated frequencies,” in *Sensitivity Analysis in Engineering*, NASA Technical Reports Server, Document ID 19870009435, NASA Langley Research Center, 1987. [Online]. Available: <https://ntrs.nasa.gov/citations/19870009435>
- [45] P. M. Vinze, R. J. Allemang, and A. W. Phillips, “Developing a correlation criterion (spacemac) for repeated and pseudo-repeated modes,” in *Topics in Modal Analysis & Testing, Volume 8*, ser. Conference Proceedings of the Society for Experimental Mechanics Series, B. Dilworth and M. Mains, Eds., Springer, 2021, pp. 189–200. DOI: 10.1007/978-3-030-47717-2_18
- [46] D. Xiao, Z. Sharif-Khodaei, and M. H. Aliabadi, “Rigorous foundations of adaptive mode tracking in single-parametric hermitian eigenvalue problems: Existence theorems, error indicators, and application to safe dispersion analysis,” *arXiv preprint arXiv:2603.13193*, 2026. arXiv: 2603.13193. [Online]. Available: <https://arxiv.org/abs/2603.13193>
- [47] Y. Yang, F. Wan, F. Guan, H. Tian, and W. Chen, “Buckling behavior of sandwich pipe under external pressure and lateral load,” *International Journal of Pressure Vessels and Piping*, vol. 207, p. 105114, 2024.
- [48] P. S. Raju, A. Sharma, and A. Gopichand, “Modeling and simulation of sandwich pipes for ultra deepwater applications,” in *AIP Conference Proceedings*, AIP Publishing, vol. 2943, 2023.
- [49] K. Arjomandi and F. Taheri, “Bending capacity of sandwich pipes,” *Ocean Engineering*, vol. 48, pp. 17–31, 2012.
- [50] D. B. Goetschel, S. B. Dong, and R. Muki, “A global-local finite element analysis of axisymmetric scattering of elastic waves,” *Journal of Applied Mechanics*, vol. 49, no. 4, pp. 816–820, 1982. DOI: 10.1115/1.3162622
- [51] Z. Chang and A. K. Mal, “Scattering of lamb waves from a rivet hole with edge cracks,” *Mechanics of Materials*, vol. 31, no. 3, pp. 197–204, 1999. DOI: 10.1016/S0167-6636(98)00060-X
- [52] A. Mal and Z. Chang, “A semi-numerical method for elastic wave scattering calculations,” *Geophysical Journal International*, vol. 143, no. 2, pp. 328–334, 2000. DOI: 10.1046/j.1365-246X.2000.01244.x
- [53] S. Tai, F. Kotobuki, L. Wang, and A. K. Mal, “Modeling ultrasonic elastic waves in fiber-metal laminate structures in presence of sources and defects,” *Journal of Non-destructive Evaluation, Diagnostics and Prognostics of Engineering Systems*, vol. 3, no. 4, p. 041102, 2020. DOI: 10.1115/1.4046946

- [54] A. Spada, M. Capriotti, and F. Lanza di Scalea, “Global–local model for three-dimensional guided wave scattering with application to rail flaw detection,” *Structural Health Monitoring*, vol. 21, no. 2, pp. 370–386, 2022.
- [55] L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of acoustics*. John Wiley & sons, 2000.
- [56] A. K. Mal, Y. Bar-Cohen, and S.-S. Lih¹, “Wave attenuation in fiber-reinforced,” *M3D: mechanics and mechanisms of material damping*, no. 1169, p. 245, 1992.
- [57] F. Benmeddour, F. Treyssède, and L. Laguerre, “Numerical modeling of guided wave interaction with non-axisymmetric cracks in elastic cylinders,” *International journal of Solids and Structures*, vol. 48, no. 5, pp. 764–774, 2011.
- [58] R. Gregory and I. Gladwell, “Axisymmetric waves in a semi-infinite elastic rod,” *The Quarterly Journal of Mechanics and Applied Mathematics*, vol. 42, no. 2, pp. 327–337, 1989.
- [59] F. Seco and A. R. Jiménez, “Modelling the generation and propagation of ultrasonic signals in cylindrical waveguides,” *Ultrasonic waves*, vol. 28, 2012.
- [60] E. Manconi and B. R. Mace, “Wave characterization of cylindrical and curved panels using a finite element method,” *The Journal of the Acoustical Society of America*, vol. 125, no. 1, pp. 154–163, 2009.