Title
Low-frequency asymptotic analysis of seismic reflection from a fluid-saturated medium

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A HYDROLOGIC APPROACH TO BIOT'S THEORY OF POROELASTICITY

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Abstract. A simplified asymptotic representation of the reflection of seismic signal from a fluid-saturated porous medium in the low-frequency domain has been obtained.

First, the equations of low-frequency harmonic waves in a fluid-saturated elastic porous medium from the basic concepts of filtration theory has been derived. It has been verified that the obtained equations can be related to the poroelasticity model of Frenkel-Gassmann-Biot, and to the pressure diffusion equation routinely used in well-test analysis. Thus, it has been confirmed that main equations of the poroelastic and filtration theories can be derived based on the common assumptions. Moreover, the Biot's tortuosity parameter has been related to the relaxation time in dynamic Darcy's law.

Second, the reflection of a low-frequency signal from a plane interface between elastic and elastic fluid-saturated porous media has been studied. An asymptotic scaling of the frequency-dependent component of the reflection coefficient with respect to a dimensionless parameter depending on the frequency of the signal and the reservoir fluid mobility has been obtained. The dependence of this scaling on the relaxation time and tortuosity has been investigated as well.

1. Introduction

When a seismic wave interacts with a boundary between elastic and fluid-saturated media, some energy of the wave is reflected and the rest is transmitted or dissipated. It is known that both the transmission and reflection coefficients from a fluid-saturated porous medium are functions of frequency [18, 15, 35, 12]. Recently, low-frequency signals were successfully used in obtaining high-resolution images of oil and gas reservoirs [19, 20, 10] and in monitoring underground gas storage [23]. Therefore, understanding the behavior of the reflection coefficient at the low-frequency end of the seismic spectrum is of special importance.

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The main objective of this paper is to obtain a simplified asymptotic representation of the reflection of seismic signal from a fluid-saturated porous medium in the low-frequency domain. More specifically, we derive a relatively simple formula, where the frequency-dependent component of the reflection coefficient is proportional to the square root of the product of frequency of the signal and the mobility of the fluid in the reservoir. To obtain wave propagation equations, we apply a somewhat nontraditional approach by deriving them from the basic principles of the theory of filtration. This is done, in particular, to verify that both the filtration and poroelasticity theories are based on common foundation.

Indeed, both filtration theory [29, 31, 6, 3] and the theory of poroelasticity [16, 17, 7, 8, 9, 37] study, in particular, fluid flow in an elastic porous medium. The filtration theory, usually assumes steady-state or transient processes where the macroscopic transition times are significantly longer than the transition times of the local microscopic processes. The poroelasticity theory includes a model of acoustic wave propagation in fluid-saturated elastic media, where the macroscopic transition times are short and, therefore, the concept of steady-state fluid flow may be inapplicable.

To obtain a system of equations characterizing fluid and solid interaction in a macroscopically homogeneous elastic fluid-saturated porous medium, we adopt *relaxation filtration* [2, 27, 26], which employs *relaxation time* to account for the inertial and non-equilibrium effects in fluid flow, thus extending the classical Darcy's law [11, 21, 22]. Originally, Darcy's law was formulated for steady-state flow [11]. It is recognized that non-equilibrium effects are important in two-phase flow [5, 4], see also [36]. However, due to local heterogeneities, they are important in single-phase flow as well.

Further, it is demonstrated in Sections 2 and 3 that under different assumptions, the equations obtained here can be transformed either into Biot's wave equations [7, 8, 9], or into the elastic drive pressure diffusion equation [29, 25, 3].

Originally, in the derivation of the wave equations of poroelasticity, the Hamiltonian principle of least action was applied [7, 8, 9]. In order to close the system, an introduction of a parameter having dimension of density was needed. This parameter was then related to a dimensionless tortuosity factor characterizing the complex geometry of the pore space in natural rocks. There are several definitions of tortuosity in the literature, see *e.g.*, [6]. In Biot's derivation, the tortuosity factor statistically characterizes the heterogeneity of the local fluid velocity field [9]. The way this tortuosity factor and the above-mentioned relaxation time enter the equations leads to the conclusion that both are linearly related to each other. The magnitude of the relaxation time and, hence, the value of the tortuosity, affects the way the reflection coefficient depends on frequency. Since the magnitude of the tortuosity in Biot's equations ranges, in general, between one to infinity [28], it is very important to know typical tortuosity factors for different types of rock. Microscopic-scale flow modeling on pore networks [30] can provide such an estimate.

During the last fifty years, a significant effort has been spent on the investigations of attenuation of Biot's waves, see *e.g.*, [33, 34] and the references therein. It has been noticed that there must be a relation between the dependence of the attenuation on the wave frequency and the permeability of the reservoir [32]. In many cases, the attenuation coefficient can be obtained in an explicit, but quite cumbersome, form. Computation of the reflection coefficient is even more complex because it additionally requires inversion of a matrix, so even under simplifying assumptions the formula is cumbersome. In this study, we obtain a simplified asymptotic expression where the role of the reservoir fluid mobility is transparent. We focus on the simplest case of normal p-wave reflection. In addition, we assume that the grains of the solid skeleton are practically incompressible, so that all deformations of the rock and the pore space are due to the rearrangements of the grains.

The layout of the paper is as follows. In the next section, the main equations of the model are derived from the principles of filtration theory. In Section 3, the obtained relationships are compared with the Biot's equations and the pressure diffusion model. Then, in Section 5, we obtain a harmonic wave solution to the equations obtained in Section 2. A dimensionless small parameter for the asymptotic analysis is introduced in the same section. In the next section, the boundary conditions for the reflection problem are formulated. An asymptotic expression for the reflection coefficient with respect to the small parameters introduced in Section 5 is obtained in Section 6. In Section 7, we elaborate on how the relaxation time and tortuosity affect the asymptotic analysis. We end the paper with conclusions and acknowledgments.

2. Fluid-solid skeleton interaction equations

Consider a homogeneous porous medium M whose pore space is filled with a viscous fluid and the grains of the solid skeleton are displaced by an elastic wave. It is assumed that a planar p-wave is propagating along the x-axis of a fixed Cartesian coordinate system. Thus, after averaging over a plane orthogonal to x, the only non-zero component of the displacement is the x-component, and the mean displacement is one-dimensional. Due to the deformation of the skeleton, the grains are rearranged. We assume that the rearrangement occurs through elastic deformations of the cement bonds between the grains. Such an assumption is natural in many situations considered in hydrology and is quite common in the geophysical literature as well, see, *e.g.*, [12].

In general, deformations result in energy dissipation. In this paper, for simplicity, it is assumed that these energy losses are much smaller than the losses through the viscous friction in the cross-flow of the reservoir fluid. Further, we assume that the rock is "clean", so that the total mass and volume of the bonds is small relative to those of the grains. Thus, for the bulk density of the "dry" skeleton ϱ we have

$$\varrho = (1 - \phi)\varrho_g \tag{1}$$

where ϱ_g is the density of the grains and ϕ is the porosity. If we neglect the microscopic rotational motions of the grains, the mean density of momentum of a drained skeleton is given by

$$\varrho\frac{\partial u}{\partial t} = (1 - \phi)\varrho_g\frac{\partial u}{\partial t} \tag{2}$$

where u is the mean displacement of the skeleton grains in the x-direction and t denotes time.

Deformations of the skeleton change the stress field. It is natural to assume that the shear stresses are, on average, uniformly distributed in any direction orthogonal to x. In general, even uniformly distributed shear stress influences the rearrangement of the skeleton. However, the assumption of stiff grains and small-volume bonds allows us to neglect this influence. The x-component of the stress caused by a displacement u, σ_x, can be measured by the elastic forces acting on a unit (bulk) area in a plane orthogonal to x. Linear elasticity hypothesis suggests that for small displacements the total stress σ_x, and the displacement u are linearly related:

$$\sigma_x = \frac{1}{\beta}\frac{\partial u}{\partial x} \tag{3}$$

Here $\beta = 1/K$ is the drained bulk compressibility, or the inverse of the bulk modulus K. We retain subscript x in equation (3) just to emphasize that here we focus on a one-dimensional case only.

The motion of the reservoir fluid can be characterized by the superficial or Darcy velocity W measured relative to the skeleton. This means, that if we imagine a small surface element moving along with the local displacement of the grains, then the volumetric fluid flux through this surface is equal to the projection of W on the unit normal vector to the surface. The average velocity v_f of the fluid relative to the skeleton is related to the Darcy velocity by equation

$$\phi v_f = W \tag{4}$$

We assume that both the motion of the skeleton u and Darcy velocity W are just small perturbations near some equilibrium values. The same applies to the fluid pressure p.

The fluid pressure gradient has a two-fold impact on the total momentum balance in the fluid-solid system [31, 37]. First, the net force exerted by the fluid pressure on the solid grains is equal to $(1-\phi)\dfrac{\partial p}{\partial x}$. Second, since the fluid and the skeleton may move with different velocities, there is momentum transfer between the solid and the fluid through viscous friction at the pore walls. This second component is equal to $-\phi\dfrac{\partial p}{\partial x}$ and is usually called *seepage drag*. Below, we consider only small perturbations of the pressures and densities near the respective equilibrium condition. Therefore, the gravity effects, including fluctuations of the hydraulic head, can be neglected.

Thus, the total fluid pressure-related force acting on the solid skeleton is equal to $-\dfrac{\partial p}{\partial x}$. A small volume of the medium, δV, contains $\varrho\delta V$ mass of rock material and $\phi\varrho_f\delta V$ mass of fluid. Here ϱ_f is the density of the fluid. Hence, the momentum of moving fluid per unit bulk volume is

$$\phi\varrho_f\left(\frac{\partial u}{\partial t} + v_f\right) = \phi\varrho_f\frac{\partial u}{\partial t} + \varrho_f W \tag{5}$$

Thus, the momentum balance per unit bulk volume is:

$$\varrho_b\frac{\partial^2 u}{\partial t^2} + \varrho_f\frac{\partial W}{\partial t} = \frac{1}{\beta}\frac{\partial^2 u}{\partial x^2} - \frac{\partial p}{\partial x} \tag{6}$$

Here ϱ_b is the bulk density of the fluid-saturated medium:

$$\varrho_b = (1-\phi)\varrho_g + \phi\varrho_f = \varrho + \phi\varrho_f \tag{7}$$

Now, let us consider in more detail the motion of the fluid. According to Darcy's law, at steady-state conditions,

$$W = -\varrho_f\frac{\kappa}{\eta}\frac{\partial\Phi}{\partial x} \tag{8}$$

where κ is the permeability of the medium, η is the viscosity of the fluid and Φ is the flow potential [21, 22]. We consider only small perturbations near an equilibrium configuration and the Darcy velocity W is measured relative to the porous medium. Hence, the gravity term in the differential of potential Φ is replaced with a term characterizing additional pressure drop due the accelerated motion of the skeleton

$$d\Phi = \frac{dp}{\varrho_f} + \frac{\partial^2 u}{\partial t^2} dx \tag{9}$$

Darcy's law (8) is for steady-state flow. If the flow is transient, *e.g.*, due to abrupt changes in the pressure field, equation (8) may need to be modified in order to account for inertial and non-equilibrium effects. As the pressure gradient changes, the local redistribution of the pressure field does not occur instantaneously because it includes microscopic fluid flow along and between the pores. Using dimensional considerations and linearization, we replace equation (8) with a non-equilibrium relationship

$$W + \tau \frac{\partial W}{\partial t} = -\varrho_f \frac{\kappa}{\eta} \frac{\partial \Phi}{\partial x} \tag{10}$$

Here τ is a characteristic redistribution time. Such a modification was proposed by Alishaev [1, 2].

In multiphase flow, similar considerations were used to model non-equilibrium effects at the front of water-oil displacement and spontaneous imbibition [5, 4]. Some results on estimation of the relaxation time based on interpretation of experiments were reported in [27, 26, 13]. Apparently, the relaxation time is a function of pore space geometry and fluid viscosity η and compressibility β_f. Dimensional considerations suggest that $\tau = \eta\beta_f F(\kappa/L^2)$, where L is the characteristic size of an elementary representative volume of the medium and F is some dimensionless function. Time τ is apparently related to the tortuosity factor [9]. This relationship is discussed in more detail below.

Thus, summing up, we arrive at the following equation characterizing the dynamics of fluid flow

$$W + \tau \frac{\partial W}{\partial t} = -\frac{\kappa}{\eta} \frac{\partial p}{\partial x} - \varrho_f \frac{\kappa}{\eta} \frac{\partial^2 u}{\partial t^2} \tag{11}$$

The mass balances for the fluid and the solid skeleton are

$$\frac{\partial(\varrho_f \phi)}{\partial t} = -\frac{\partial\left(\varrho_f W + \phi\varrho_f \frac{\partial u}{\partial t}\right)}{\partial x} \tag{12}$$

$$\frac{\partial \varrho}{\partial t} = -\frac{\partial}{\partial x}\left(\varrho \frac{\partial u}{\partial t}\right) \tag{13}$$

For the fluid, we apply the isothermal compressibility law [24], that is, for small fluid pressure perturbation

$$\frac{d\varrho_f}{\varrho_f} = \beta_f dp \tag{14}$$

Hence, Eq. (12) can be rewritten as

$$\frac{\partial \phi}{\partial t} + \phi\beta_f \frac{\partial p}{\partial t} = -\frac{\partial W}{\partial x} - \phi\frac{\partial^2 u}{\partial x \partial t} - W\frac{\partial \varrho_f}{\partial x} - \frac{1}{\varrho_f}\frac{\partial}{\partial x}(\phi\varrho_f)\frac{\partial u}{\partial t} \tag{15}$$

Due to the smallness of the perturbations, the last two terms in equation (15) are small of a higher order and can be neglected.

With $\rho = (1-\phi)\rho_g$, Equation (13) takes on the form

$$-\frac{\partial \phi}{\partial t} + (1-\phi)\frac{1}{\varrho_g}\frac{\partial \varrho_g}{\partial t} = -\frac{1}{\varrho_g}(1-\phi)\frac{\partial \varrho_g}{\partial x}\frac{\partial u}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial u}{\partial t} - (1-\phi)\frac{\partial^2 u}{\partial x \partial t} \tag{16}$$

Again, the smallness of the perturbations implies that the first two terms on the right-hand side of the last equation can be dropped. Further on, perturbation of the grain density is a linear function of the perturbations of the stress and fluid pressure, that is

$$\frac{1}{\varrho_g} d\varrho_g = \beta_{gs} d\sigma_x + \beta_{gf} dp \tag{17}$$

where β_{gs} and β_{gf} are the respective compressibility coefficients. Thus, equation (16) can be written as

$$\frac{\partial \phi}{\partial t} = (1-\phi)\beta_{gf}\frac{\partial p}{\partial t} + (1-\phi)\left(1 + \frac{\beta_{gs}}{\beta}\right)\frac{\partial^2 u}{\partial x \partial t} \tag{18}$$

A combination of this last result with equation (15) leads to the following relationship

$$\left(1 + (1-\phi)\frac{\beta_{gs}}{\beta}\right)\frac{\partial^2 u}{\partial x \partial t} + (\phi\beta_f + (1-\phi)\beta_{gf})\frac{\partial p}{\partial t} = -\frac{\partial W}{\partial x} \tag{19}$$

Under the assumptions formulated above, the compressibility of the grains is much smaller than the compressibility of the fluid and the skeleton:

$$\beta_{gf} \ll \beta_f \quad \text{and} \quad \beta_{gs} \ll \beta \tag{20}$$

This means that the deformation occurs only through the porosity perturbations. Thus, equation (19) can be further reduced to

$$\frac{\partial^2 u}{\partial x \partial t} + \phi\beta_f \frac{\partial p}{\partial t} = -\frac{\partial W}{\partial x} \tag{21}$$

Equation (21) states that the amount of fluid volume packed into a unit bulk volume per unit time is equal to minus the divergence of the absolute fluid velocity. The fluid volume can be packed into the bulk volume because the fluid is compressible and the pressure increases, and because the porosity can also increase. Note that mathematically there is no qualitative difference between equations (19) and (21). Below, we use mass balance equation in the form (19) wherever it does not exceedingly complicate the calculations.

To summarize, we have obtained a closed system of three equations (6), (11), and (19) with three unknown functions of t and x: skeleton displacement u, fluid pressure p, and Darcy velocity W.

3. Relationship to Biot's poroelasticity and pressure diffusion equations

In this section, we demonstrate that under the assumptions formulated in Section 2 equations (6), (11), and (19) can be reduced to the system of equations obtained by Biot [7, 9], see also [14]. At the same time, neglecting the inertial terms in these equations, leads to the pressure diffusion equation used in hydrology and petroleum engineering for well test analysis, see [25, 3].

We begin with Biot's theory. We will perform the calculations using the assumption of grain incompressibility, Eq. (20). As we consider only small oscillatory deformations of the skeleton and fluctuations of the fluid flow, a "superficial" displacement of the fluid relative to the skeleton w can be introduced, so that

$$W = \frac{\partial w}{\partial t} \tag{22}$$

Note that inasmuch as w is related by Eq. (22) to the Darcy velocity of the fluid, it is different from the average microscopic fluid displacement. Substitution of (22) into equation (21) yields

$$\frac{\partial^2 u}{\partial x \partial t} + \phi\beta_f \frac{\partial p}{\partial t} = -\frac{\partial^2 w}{\partial t \partial x} \tag{23}$$

By integration in t and differentiation in x, we obtain

$$\frac{\partial p}{\partial x} = -\frac{1}{\phi\beta_f}\frac{\partial^2 u}{\partial x^2} - \frac{1}{\phi\beta_f}\frac{\partial^2 w}{\partial x^2} \tag{24}$$

Here we have utilized the assumption of the smallness of the rock-fluid system oscillations near an equilibrium configuration. Otherwise, due to the integration, equation (24) should include an unknown function of x. Now, let us substitute (22) and the result (24) in equations (6) and (11):

$$\varrho_b \frac{\partial^2 u}{\partial t^2} + \varrho_f \frac{\partial^2 w}{\partial t^2} = \left(\frac{1}{\beta} + \frac{1}{\phi\beta_f}\right)\frac{\partial^2 u}{\partial x^2} + \frac{1}{\phi\beta_f}\frac{\partial^2 w}{\partial x^2} \tag{25}$$

$$\varrho_f \frac{\partial^2 u}{\partial t^2} + \tau\frac{\eta}{\kappa}\frac{\partial^2 w}{\partial t^2} = \frac{1}{\phi\beta_f}\frac{\partial^2 u}{\partial x^2} + \frac{1}{\phi\beta_f}\frac{\partial^2 w}{\partial x^2} - \frac{\eta}{\kappa}\frac{\partial w}{\partial t} \tag{26}$$

Under the assumptions formulated above, equations (25) – (26) are equivalent to the Biot system of equations (8.34) [9]:

$$\frac{\partial^2}{\partial t^2}\left(\varrho_b u + \varrho_f w\right) = \frac{\partial}{\partial x}\left(A_{11}\frac{\partial u}{\partial x} + M_{11}\frac{\partial w}{\partial x}\right)$$

$$\frac{\partial^2}{\partial t^2}\left(\varrho_f u + m w\right) = \frac{\partial}{\partial x}\left(M_{11}\frac{\partial u}{\partial x} + M\frac{\partial w}{\partial x}\right) - \frac{\eta}{\kappa}\frac{\partial w}{\partial t}$$

Comparing the individual terms, we can establish a relationship between the relaxation time and the tortuosity factor. Namely, the relaxation time τ is related to the dynamic coupling coefficient m [9] through the inverse mobility ratio η/κ. The dynamic coupling coefficient is often expressed through the tortuosity factor T: $m = T\varrho_f/\phi$. Hence, for the tortuosity and relaxation time, we obtain the following relationship:

$$T = \tau\frac{\eta\phi}{\kappa\varrho_f} \quad \text{or} \quad \tau = T\frac{\kappa\varrho_f}{\eta\phi} \tag{27}$$

Comparison of the elastic coefficients reveals that under the assumption of isotropic porous medium and incompressible grains (the Biot-Willis coefficient $\alpha = K/H \approx 1$, and $K_u = K + K_f/\phi$), the Biot coefficients are constant and equal to

$$A_{11} = K_u \approx \frac{1}{\beta} + \frac{1}{\phi\beta_f} \quad \text{and} \quad M_{11} = M = K_u B \approx \frac{1}{\phi\beta_f} \tag{28}$$

where K_u is the undrained bulk modulus, and $B = R/H$ is Skempton's coefficient, $1/H$ being the poroelastic expansion coefficient, and $1/R$ the unconstrained specific storage coefficient.

Now, let us derive the pressure diffusion equation. Assume that the characteristic time t_D of the process is large in comparison with the relaxation time τ and the displacements of the skeleton are much smaller then the characteristic length scale of the process L:

$$t_D \gg \tau \quad \text{and} \quad u \ll L \tag{29}$$

Under this assumption, the second order time derivatives of displacement u and time derivatives of Darcy velocity W in equations (6) and (11) can be dropped:

$$\frac{\partial p}{\partial x} = \frac{1}{\beta}\frac{\partial^2 u}{\partial x^2} \tag{30}$$

$$W = -\frac{\kappa}{\eta}\frac{\partial p}{\partial x} \tag{31}$$

By integrating equation (30) in x and differentiating in t, we obtain

$$\frac{\partial^2 u}{\partial t \partial x} = \beta \frac{\partial p}{\partial t} \tag{32}$$

Formally, integration by x is defined up to a function of time. Assuming a constant pressure at infinity, this function of time also is constant. This constant is then cancelled by the differentiation with respect to t. Finally, by a substitution of equations (31) and (32) into (21), we obtain

$$\phi(\beta/\phi + \beta_f)\frac{\partial p}{\partial t} = \frac{\kappa}{\eta}\frac{\partial^2 p}{\partial x^2} \tag{33}$$

This last equation is the pressure diffusion equation routinely used in well test analysis [25, 3].

4. Plane compressional wave: an asymptotic solution

Let us consider the system of equations obtained in Section 2. We introduce the dimensionless pressure

$$P = \phi \beta_f p \tag{34}$$

and the hydraulic diffusivity

$$D = \frac{\kappa}{\phi \beta_f \eta} \tag{35}$$

Dividing equation (6) by ϱ_b and putting

$$v_b^2 = \frac{1}{\beta \varrho_b} \quad \text{and} \quad v_f^2 = \frac{1}{\phi \beta_f \varrho_b} \tag{36}$$

we obtain

$$\frac{\partial^2 u}{\partial t^2} + \frac{\varrho_f}{\varrho_b}\frac{\partial W}{\partial t} = v_b^2 \frac{\partial^2 u}{\partial x^2} - v_f^2 \frac{\partial P}{\partial x} \tag{37}$$

$$\lambda_f \frac{\partial^2 u}{\partial t^2} + W + \tau \frac{\partial W}{\partial t} = -D\frac{\partial P}{\partial x} \tag{38}$$

$$\gamma_1 \frac{\partial^2 u}{\partial x \partial t} + \gamma_2 \frac{\partial P}{\partial t} = -\frac{\partial W}{\partial x} \tag{39}$$

where

$$\lambda_f = \varrho_f \frac{\kappa}{\eta} \tag{40}$$

is the "kinematic" mobility of the fluid, and

$$\gamma_1 = 1 + (1-\phi)\frac{\beta_{gs}}{\beta} \quad \text{and} \quad \gamma_2 = 1 + (1-\phi)\frac{\beta_{gf}}{\phi\beta_f} \tag{41}$$

Clearly, λ_f has the dimension of time. The assumptions (21) imply that both dimensionless coefficients γ_1 and γ_2 are close to one. We seek a planar wave solution to the equations (37)–(39) in the form

$$u = U_s e^{i(\omega t - kx)}, \quad W = W_f e^{i(\omega t - kx)}, \quad P = P_0 e^{i(\omega t - kx)} \tag{42}$$

Substitution of Eq. (42) into (37)–(39) produces a system of algebraic equations

$$\left\{ \begin{array}{rcl} -\omega^2 U_s + i\omega \dfrac{\varrho_f}{\varrho_b} W_f & = & -v_b^2 k^2 U_s + i v_f^2 k P_0 \\ -\lambda_f \omega^2 U_s + i\tau\omega W_f & = & iDkP_0 - W_f \\ k\omega\gamma_1 U_s + i\omega\gamma_2 P_0 & = & ikW_f \end{array} \right. \tag{43}$$

Using the last equation, W_f can be eliminated from the system (43). Indeed, we get

$$W_f = -i\omega\gamma_1 U_s + \omega\gamma_2 \frac{P_0}{k} \tag{44}$$

and

$$i\omega W_f = \omega^2 \gamma_1 U_s + i\omega^2 \gamma_2 \frac{P_0}{k} \tag{45}$$

Hence,

$$\left\{ \begin{array}{rcl} -\omega^2 \left(1 - \dfrac{\varrho_f}{\varrho_b}\gamma_1\right) U_s + i\omega^2 \gamma_2 \dfrac{\varrho_f}{\varrho_b} \dfrac{P_0}{k} & = & -v_b^2 k^2 U_s + i v_f^2 k P_0 \\ -(\lambda_f - \tau\gamma_1)\omega^2 U_s + i\tau\omega^2\gamma_2 \dfrac{P_0}{k} & = & iDkP_0 + i\omega\gamma_1 U_s - \omega\gamma_2 \dfrac{P_0}{k} \end{array} \right. \tag{46}$$

Now, let us introduce two new variables

$$v = \frac{\omega}{k} \quad \text{and} \quad \xi = -\frac{iP_0}{kU_s} \tag{47}$$

Note, that v has the dimension of velocity and, in general, v is a complex quantity. The variable ξ and equation (44) relate all three amplitudes U_s, W_f, and P_0. In particular,

$$W_f = i\omega(-\gamma_1 + \gamma_2 \xi) U_s = \frac{\omega}{k}\left(-\frac{\gamma_1}{\xi} + \gamma_2\right) P_0 \tag{48}$$

In terms of variables v and ξ defined in Equation (47), the system of equations (46) takes on the following form

$$\begin{cases} v^2\left(1-\gamma_1\frac{\varrho_f}{\varrho_b}\right)+v^2\gamma_2\frac{\varrho_f}{\varrho_b}\xi &= v_b^2+v_f^2\xi \\ (\lambda_f-\tau\gamma_1)v^2+\tau v^2\gamma_2\xi &= D\xi+\frac{i}{\omega}v^2\left(-\gamma_1+\gamma_2\xi\right) \end{cases} \tag{49}$$

or, equivalently,

$$\begin{cases} v^2\left(1-\gamma_1\frac{\varrho_f}{\varrho_b}+\xi\gamma_2\frac{\varrho_f}{\varrho_b}\right) &= v_b^2+v_f^2\xi \\ v^2\left[\lambda_f-\left(\tau-\frac{i}{\omega}\right)(\gamma_1-\gamma_2\xi)\right] &= D\xi \end{cases} \tag{50}$$

Denote

$$\tau_D=\frac{D}{v_f^2}=\frac{\kappa\varrho_b}{\eta},\qquad \gamma_v=\frac{v_b^2}{v_f^2}=\frac{\phi\beta_f}{\beta}\qquad\text{and}\qquad \gamma_\varrho=\frac{\varrho_f}{\varrho_b} \tag{51}$$

The parameters γ_v and γ_ϱ are dimensionless. Taking into account equation (40),

$$\lambda_f=\gamma_\varrho\tau_D \tag{52}$$

The dimensionless relaxation time and angular velocity are defined as

$$\theta=\frac{\tau}{\tau_D}\qquad\text{and}\qquad \varepsilon=\tau_D\omega \tag{53}$$

Dividing equations (50) by v_f^2 and putting $V=v/v_f$, we obtain

$$\begin{cases} V^2\left(1-\gamma_1\gamma_\varrho+\xi\gamma_2\gamma_\varrho\right) &= \gamma_v+\xi \\ V^2\left[\gamma_\varrho-\left(\theta-\frac{i}{\varepsilon}\right)(\gamma_1-\gamma_2\xi)\right] &= \xi \end{cases} \tag{54}$$

By eliminating V,

$$V^2=\frac{\gamma_v+\xi}{1-\gamma_1\gamma_\varrho+\gamma_2\gamma_\varrho\xi} \tag{55}$$

we obtain a quadratic equation with respect to ξ:

$$\begin{aligned} &(\gamma_2+i\varepsilon\left(-\gamma_2\gamma_\varrho+\theta\gamma_2\right))\xi^2 \\ +&\left(-\gamma_1+\gamma_2\gamma_v+i\varepsilon\left[-1+\gamma_1\gamma_\varrho+(\gamma_\varrho-\theta\gamma_1)+\theta\gamma_2\gamma_v\right]\right)\xi \\ &+\left(-\gamma_1\gamma_v+i\varepsilon\gamma_v(\gamma_\varrho-\tau\gamma_1)\right)=0 \end{aligned} \tag{56}$$

At $\varepsilon=0$, Equation (56) reduces to

$$\gamma_2\xi^2+(-\gamma_1+\gamma_2\gamma_v)\xi-\gamma_1\gamma_v=0 \tag{57}$$

The latter equation admits two *real* roots

$$\xi_0^{(1)}=\frac{\gamma_1}{\gamma_2}\qquad\text{and}\qquad \xi_0^{(2)}=-\gamma_v \tag{58}$$

By virtue of equations (20) and (41), the absolute value of the first root ξ_0^1 is close to unity, whereas the absolute value of the second one is equal to $\dfrac{\phi\beta_f}{\beta}$, that is usually larger than one. Using equation (55), we obtain two asymptotic values for the complex velocity v

$$v_0^{(1)} = 0 \quad \text{and} \quad v_0^{(2)} = v_f\sqrt{\gamma_v + \frac{\gamma_1}{\gamma_2}} \tag{59}$$

The first solution corresponds to the slow wave, whereas the second one is related to the fast wave. Naturally, this appearance of slow and fast waves is in agreements with Biot's theory [7, 9].

The exact solution to Eq. (56) can be easily obtained, but the expression is quite cumbersome. Instead, let us look for an asymptotic solution. Note that if we assume the permeability $\kappa \sim 1$ Darcy, that is $\kappa \sim 10^{-12}\ m^2$, the viscosity of the fluid $\eta \sim 1$ cP $= 10^{-3}$ Pa-s, and the bulk density of the rock $\varrho_b \sim 10^3$ kg/m^3, then $\tau_D \sim 10^{-6}$ and $\varepsilon \preceq 10^{-3}$ for frequencies ω not exceeding ~ 1 kHz. Inasmuch as γ_1 and γ_2 are of the order of unity, ε (more accurately, $i\varepsilon$) is a small parameter in equation (56) and we can look for a solution in the form

$$\xi = \xi_0 + \xi_1 i\varepsilon - \xi_2\varepsilon^2 \dots \tag{60}$$

Using notations

$$\begin{array}{lllllll} A_0 & = & \gamma_2 & \qquad & A_1 & = & -\gamma_2\gamma_\varrho + \theta\gamma_2 \\ B_0 & = & \gamma_2\gamma_v - \gamma_1 & & B_1 & = & -1 + \gamma_\varrho(1+\gamma_1) + \theta(\gamma_2\gamma_v - \gamma_1) \\ C_0 & = & -\gamma_1\gamma_v & & C_1 & = & \gamma_v(\gamma_\varrho - \theta\gamma_1) \end{array} \tag{61}$$

we obtain

$$\xi_1 = -\frac{A_1\xi_0^2 + B_1\xi_0 + C_1}{2A_0\xi_0 + B_0} \tag{62}$$

Thus, the solutions corresponding to the slow and fast waves have, respectively, the following forms

$$\xi_1^{(1)} = \gamma_v\frac{1 - \gamma_\varrho(\gamma_2\gamma_v + \gamma_1)}{\gamma_1 + \gamma_2\gamma_v} \tag{63}$$

and

$$\xi_1^{(2)} = \frac{1}{\gamma_2}\frac{\gamma_1 - \gamma_\varrho(\gamma_2\gamma_v + \gamma_1)}{\gamma_1 + \gamma_2\gamma_v} \tag{64}$$

Note, that since both $\gamma_1 \approx 1$ and $\gamma_2 \approx 1$, equations (63) and (64) can be simplified

$$\xi_1^{(1)} = \gamma_v \frac{1 - \gamma_\varrho \gamma_v - \gamma_\varrho}{1 + \gamma_v} \tag{65}$$

$$\xi_1^{(2)} = \frac{1}{\gamma_2} \frac{\gamma_1 - \gamma_\varrho \gamma_v - \gamma_\varrho}{1 + \gamma_v} \tag{66}$$

In particular, $\xi_1^{(1)}$ and $\xi_1^{(2)}$ are independent of the permeability of the formation and the viscosity of the fluid. Note that the relaxation time also disappears from the first-order approximation of ξ for both the slow and fast wave. The latter circumstance is discussed in Section 7 below.

Using equation (55), we obtain that

$$v^{(1)} = \pm v_b \sqrt{\frac{i\varepsilon}{\gamma_1 + \gamma_2 \gamma_v} + \dots} \tag{67}$$

and

$$v^{(2)} = \pm v_f \sqrt{\gamma_v + \frac{\gamma_1}{\gamma_2}} + v_f V_1 i\varepsilon + \dots \tag{68}$$

where V_1 is the first coefficient of the expansion of V in the powers of $i\varepsilon$. The last two equations, in a combination with equation (63), imply that

$$k^{(1)} = \pm \frac{1}{\tau_D v_b} \sqrt{\gamma_1 + \gamma_2 \gamma_v} \sqrt{-i\varepsilon} + \dots \tag{69}$$

$$k^{(2)} = \pm \frac{1}{\tau_D v_f} \frac{1}{\sqrt{\gamma_v + \frac{\gamma_1}{\gamma_2}}} \varepsilon + \dots \tag{70}$$

The imaginary part of k must be negative, therefore, from (69) we infer that

$$k^{(1)} = \frac{1}{\tau_D v_b} \sqrt{\gamma_1 + \gamma_2 \gamma_v} \, \frac{1 - i}{\sqrt{2}} \sqrt{\varepsilon} + \dots \tag{71}$$

and, respectively,

$$v^{(1)} = v_b \sqrt{\frac{1}{\gamma_1 + \gamma_2 \gamma_v}} \, \frac{1 + i}{\sqrt{2}} \sqrt{\varepsilon} + \dots \tag{72}$$

By virtue of equations (58) and (48)

$$W_f = -i\omega(\gamma_1 - \gamma_2 \xi) U_s \tag{73}$$

Furthermore, using equations (60), we get for the fast wave

$$W_f^{\text{fast}} = -\varepsilon \omega \gamma_2 \xi_1^{(2)} U_s^{\text{fast}} + \dots \tag{74}$$

The right-hand side of the last equation is small with respect to ε. That means that at low frequencies, the fast wave is actually a coherent oscillation of the skeleton and the fluid. At the same time, for the slow wave we obtain a finite nonzero limit if the Darcy velocity amplitude is compared with the amplitude of the time-derivative of the displacement

$$W_f^{\text{slow}} = -i\omega\left(\gamma_1 + \gamma_2\gamma_v\right) U_s^{\text{slow}} + \dots \tag{75}$$

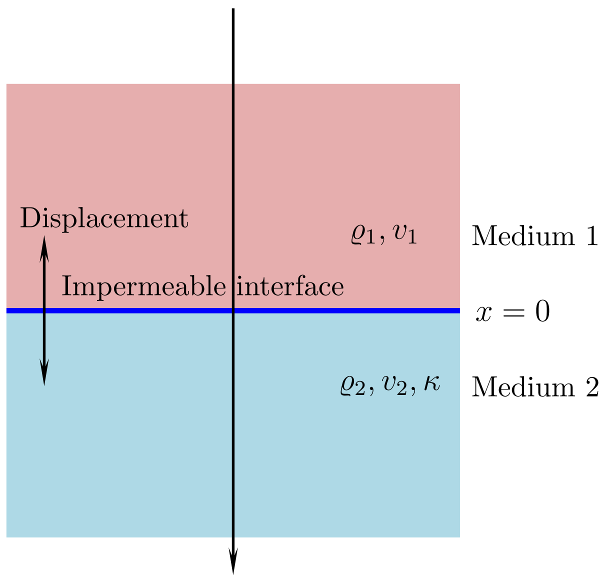

FIGURE 1. One dimensional propagation of a low-frequency disturbance perpendicular to the impermeable interface between medium M_1 and porous, permeable solid M_2 fully saturated with a liquid.

5. Boundary conditions

Consider a normal incidence of a compressional elastic wave upon a plane interface $x = 0$ separating media M_1 and M_2 occupying half-spaces $x < 0$ and $x > 0$, respectively, see **Figure 1**. Their properties are characterized by the bulk densities ϱ_i and the speeds of sound v_i, $i = 1, 2$. The medium M_2 is porous and saturated by a fluid, that is, it consists of a solid skeleton and fluid-filled pore space. We assume that the boundary between the media is impermeable for fluid flow and the permeability of medium M_2 is characterized by a coefficient κ. To calculate the reflection coefficient, boundary conditions at the interface between the media, *i.e.*, at $x = 0$, must be formulated.

Under the assumptions of Section 3, and neglecting the heterogeneities of the materials, we can assume that the displacements of the solid particles composing the media are parallel to x, and so is the flux of the fluid in the pore space. There is an important difference between the fluid and solid motion. The solid particles move more or less coherently near the respective equilibrium positions, whereas fluid particles move in a much more dispersed manner implied by the complexity of the pore space geometry. Only the mean volumetric flux or Darcy velocity of the moving fluid is parallel to x. This quantity is the result of averaging the microscopic fluid velocity field over a representative volume. In the case under consideration, such an averaging can be performed over a plane $x = \text{Const} > 0$.

Denote by u_1 and u_2 the displacements of the solid particles in media M_1 and M_2, respectively.

First, the continuity of the displacements and microscopic stresses requires that

$$u_1\big|_{x=0} = u_2\big|_{x=0} \tag{76}$$

$$-\frac{1}{\beta_1}\frac{\partial u_1}{\partial x}\bigg|_{x=0} = -\frac{1-\phi}{\beta_2}\frac{\partial u_2}{\partial x}\bigg|_{x=0} + \phi p\big|_{x=0} \tag{77}$$

Zero fluid flux through the boundary implies

$$W_f\big|_{x=0} = 0 \tag{78}$$

Boundary conditions (76)–(78) will be used in the next section for investigation of the reflection coefficient.

6. Reflection coefficient

To calculate the reflection coefficient, we have to equate the sum of incident and reflected displacements in medium M_1

$$u_1 = U_1 e^{i(\omega t - k_1 x)} + RU_1 e^{i(\omega t + k_1 x)} \tag{79}$$

with the sum of slow and fast waves transmitted into medium M_2

$$p = \frac{1}{\phi\beta_f}P_0^s e^{i(\omega t - k_s x)} + \frac{1}{\phi\beta_f}P_0^f e^{i(\omega t - k_f x)} \tag{80}$$

$$u_2 = \frac{1}{1-\phi}U_2^s e^{i(\omega t - k_s x)} + \frac{1}{1-\phi}U_2^f e^{i(\omega t - k_f x)} \tag{81}$$

using boundary conditions (76), (77) and (78). We assume zero attenuation in medium M_1, therefore $k_1 > 0$ is real and $\omega k_1 = v_1$ is the p-wave velocity in this medium.

Utilizing the first equation (48), we obtain

$$\begin{cases} (1+R)U_1 &= U_2^s &+ U_2^f \\ \dfrac{ik_1}{\beta_1}(1-R)U_1 &= \dfrac{ik_2^s}{\beta_2}U_2^s &+ \dfrac{ik_2^f}{\beta_2}U_2^f \\ & &+ \dfrac{P_0^f + P_0^s}{\beta_f} \\ 0 &= i\omega(-\gamma_1+\gamma_2\xi^s)U_2^s &+ i\omega(-\gamma_1+\gamma_2\xi^f)U_2^f \end{cases} \tag{82}$$

Further, by virtue of equation (47), we get

$$\begin{cases} -(1+R)U_1 + U_2^s + U_2^f = 0 \\ -\dfrac{k_1}{\beta_1}(1-R)U_1 + k_2^s\left(\dfrac{1}{\beta_2}+\dfrac{\xi^s}{\beta_f}\right)U_2^s + k_2^f\left(\dfrac{1}{\beta_2}+\dfrac{\xi^f}{\beta_f}\right)U_2^f = 0 \\ (\gamma_1-\gamma_2\xi^s)U_2^s + (\gamma_1-\gamma_2\xi^f)U_2^f = 0 \end{cases} \tag{83}$$

Note that in medium M_1 we have

$$k_1 = \frac{1}{v_1}\omega \tag{84}$$

where v_1 is the speed of sound, *i.e.*, a characteristic of the medium, which does not depend on the frequency.

Dividing through by U_1 and putting $Z_1 = R$, $Z_2 = U_2^s/U_1$, and $Z_3 = U_2^f/U_1$, we obtain the following system of equations

$$\begin{cases} -Z_1 + Z_2 + Z_3 = 1 \\ \omega Z_1 + v_1 k_2^s\left(\dfrac{\beta_1}{\beta_2}+\xi^s\dfrac{\beta_1}{\beta_f}\right)Z_2 + v_1 k_2^f\left(\dfrac{\beta_1}{\beta_2}+\xi^f\dfrac{\beta_1}{\beta_f}\right)Z_3 = \omega \\ (\gamma_1-\gamma_2\xi^s)Z_2 + (\gamma_1-\gamma_2\xi^f)Z_3 = 0 \end{cases} \tag{85}$$

Hence, using equations (70) and (69) and notation (53), the system of equations (85) can be presented in the following asymptotic form

$$\begin{cases} -Z_1 + Z_2 + Z_3 = 1 \\ \sqrt{\varepsilon}Z_1 + A_{22}Z_2 + A_{23}\sqrt{\varepsilon}Z_3 = \sqrt{\varepsilon} \\ \left(A_{32}^{(1)} + A_{32}^{(2)}i\varepsilon\right)Z_2 + A_{33}i\varepsilon Z_3 = 0 \end{cases} \tag{86}$$

The expressions for the coefficients A_{ij} can be obtained from the asymptotic formulae (60), (63), (64), (70), and (71):

$$A_{22} = \frac{v_1}{v_b}\sqrt{\gamma_1+\gamma_2\gamma_v}\;\gamma_s\frac{1-i}{\sqrt{2}} \tag{87}$$

$$A_{23} = \frac{v_1}{v_f}\sqrt{\frac{\gamma_2}{\gamma_1+\gamma_2\gamma_v}}\;\gamma_f \tag{88}$$

$$A_{32}^{(1)} = \gamma_1+\gamma_2\gamma_v \tag{89}$$

$$A_{32}^{(2)} = -\gamma_2\gamma_v\frac{1-\gamma_\varrho(\gamma_2\gamma_v+\gamma_1)}{\gamma_1+\gamma_2\gamma_v} \tag{90}$$

$$A_{33} = -\frac{\gamma_\varrho\gamma_1-\gamma_1+\gamma_\varrho}{\gamma_1+\gamma_2\gamma_v} \tag{91}$$

Here we used the notations

$$\gamma_s = \beta_1\left(\frac{1}{\beta_2}-\gamma_v\frac{1}{\beta_f}\right) \quad \text{and} \quad \gamma_f = \beta_1\left(\frac{1}{\beta_2}+\frac{\gamma_1}{\gamma_2}\frac{1}{\beta_f}\right) \tag{92}$$

From the last equation (86),

$$Z_2 = -\frac{A_{33}}{A_{32}^{(1)}}i\varepsilon Z_3+\ldots \tag{93}$$

This means that at low frequencies (*i.e.*, at $\varepsilon\to 0$), the slow wave displacement is scaled with the velocity of fast displacement and, therefore, is one order of magnitude smaller. In other words, the slow part of the signal practically does not propagate and is mostly responsible for the reflection.

Substitution of (93) into the first two equations (86) yields

$$\begin{cases} -Z_1 + \left(1-\dfrac{A_{33}}{A_{32}^{(1)}}i\varepsilon\right)Z_3 = 1 \\ \sqrt{\varepsilon}Z_1 + \left(A_{23}\sqrt{\varepsilon}-A_{22}\dfrac{A_{33}}{A_{32}^{(1)}}i\varepsilon\right)Z_3 = \sqrt{\varepsilon} \end{cases} \tag{94}$$

Cancelling the $\sqrt{\varepsilon}$ in the second equation (94) and dropping terms of the order higher than $\sqrt{\varepsilon}$, we obtain that

$$Z_3 = Z_1+1 \tag{95}$$

Consequently,

$$Z_1 = \frac{1-A_{23}+A_{22}\dfrac{A_{33}}{A_{32}^{(1)}}i\sqrt{\varepsilon}}{1+A_{23}-A_{22}\dfrac{A_{33}}{A_{32}^{(1)}}i\sqrt{\varepsilon}} \tag{96}$$

Again, retaining only the terms of the order $\sqrt{\varepsilon}$, we finally obtain

$$Z_1 = \frac{1 - A_{23}}{1 + A_{23}} + \sqrt{2}\frac{\tilde{A}_{22}A_{33}}{A_{32}^{(1)}}\frac{1}{(1 + A_{23})^2}(1 + i)\sqrt{\varepsilon} \tag{97}$$

where

$$\tilde{A}_{22} = \frac{v_1}{v_b}\sqrt{\gamma_1 + \gamma_2\gamma_v}\,\gamma_s \tag{98}$$

Analysis of the expression (88) yields that in practicalsituations, the coefficient A_{23} is greater than one. Therefore, the frequency-independent component of the reflection coefficient is negative. The frequency-dependent component of the reflection has the same sign as $\tilde{A}_{33}$. The latter is positive if and only if

$$\gamma_\varrho < \frac{\gamma_1}{1 + \gamma_1} \tag{99}$$

The right-hand side of the last inequality is approximately equal to 0.5. Hence, roughly speaking, $\tilde{A}_{33}$ is positive when the fluid density is at least twice less than the bulk density of the saturated medium. In such a case the maximum of the absolute value of the reflection coefficient is attained at $\varepsilon = 0$. At the same time, for dense fluids, the first-order term of the asymptotic expansion, which is proportional to the square root of ε, may vanish and the first frequency-dependent term will be linear. In this case, the tortuosity coefficient becomes an important factor.

In the original variables (51), equation (97) takes on the form

$$R = \frac{1 - A_{23}}{1 + A_{23}} + \sqrt{2}\frac{\tilde{A}_{22}A_{33}}{A_{32}^{(1)}}\frac{1}{(1 + A_{23})^2}(1 + i)\sqrt{\frac{\kappa\varrho_b}{\eta}\omega} \tag{100}$$

Note that the last equation relates the reflectivity to the frequency through the factor of $\tau_D = \dfrac{\kappa\varrho_b}{\eta}$ having the dimension of time. It involves a property of the rock, the permeability coefficient, a property of the fluid, the viscosity, and a property of the coupled fluid-rock system, the bulk density. Note that the frequency scaling proposed here is similar to but not the same as the scaling introduced in [18].

7. The role of relaxation time and tortuosity

The asymptotic calculations presented above show that the dimensionless parameter θ, related to both relaxation time and tortuosity factor, disappears from the first-order terms. However, if θ is large, then some expansions obtained in Sections 4 and 6 have to be reviewed. Practically, the range of frequencies is limited by the specifications of the available tools. Therefore, it may happen that within the range of frequencies available for analysis the product $\theta\varepsilon$ is not negligibly small, and the theoretical passage to the limit as $\varepsilon \to 0$ should be replaced with analysis at some intermediate finite values of ε. In such a case, the asymptotic analysis must be performed differently. In this section, we consider two examples of such analysis.

First, let us assume that within the range of available frequencies, the parameter $\varepsilon\theta$ is of the order of one. In original variables, this condition is equivalent to

$$\omega \sim \frac{1}{\tau} \tag{101}$$

Regrouping the coefficients in the Equation (56) and dividing through by $1 + i\theta\varepsilon$, we obtain

$$(A_0 + A_1^\theta i\varepsilon)\xi^2 + (B_0 + B_1^\theta i\varepsilon)\xi + C_0 + C_1^\theta i\varepsilon = 0 \tag{102}$$

where the coefficients with zero indices are the same as in equation (61), and

$$\begin{aligned} A_1^\theta &= -\frac{\gamma_2\gamma_\varrho}{1 + i\theta\varepsilon} \\ B_1^\theta &= \frac{-1 + \gamma_\varrho(1 + \gamma_1)}{1 + i\theta\varepsilon} \\ C_1^\theta &= \frac{\gamma_v\gamma_\varrho}{1 + i\theta\varepsilon} \end{aligned} \tag{103}$$

Hence, the frequency-independent zero-terms of asymptotic expansions of the solutions ξ are the same as in Equation (58). To calculate the first order coefficients, we note that formally the coefficients (103) are equal to the respective coefficients in Equations (61) evaluated at $\tau = 0$ and divided by $1 + i\theta\varepsilon$. This fact, in conjunction with the observation that the asymptotic expansion of the reflection coefficient (100) does not depend on τ, significantly simplifies the calculations. Indeed, for the first-order coefficients of asymptotic expansion for ξ we can reuse equations (63) and (64) if we put there $\tau = 0$ and multiply the right-hand sides by an additional factor of $\dfrac{1}{1 + i\theta\varepsilon}$. Clearly, the calculations for the first order terms of expansions of v and k can be carried out in a similar manner. The final result is that the reflection coefficient in the asymptotic expression (100) takes on the form

$$R = \frac{1 - A_{23}}{1 + A_{23}} + 2\frac{A_{22}A_{33}}{A_{32}^{(1)}}\frac{1}{(1 + A_{23})^2}\sqrt{i - \theta\varepsilon}\sqrt{\frac{\kappa\varrho_b}{\eta}\omega} \tag{104}$$

Thus, in a case where $\tau\omega = O(1)$, the relaxation time and tortuosity affect both the amplitude and the phase shift of the reflected signal.

Now, consider another extreme situation where $\theta \gg 1$, so that after a division of equation (56) by θ all terms with θ in the denominator can be neglected. In such a case, we obtain a quadratic equation

$$i\varepsilon(A_0\xi^2 + B_0\xi + C_0) = 0 \tag{105}$$

The latter implies that the frequency dependence of ξ (and, therefore, of the reflection coefficient as well) vanishes. This conclusion means that at a very large relaxation time (or, equivalently, at a very large tortuosity), the inertial term in equation (38) makes the dissipation term on the right-hand side unimportant. Consequently, the fluid-saturated medium acts as an elastic composite medium and we arrive at a classical frequency-independent elastic wave reflection.

8. Conclusions

Equations of elastic waves propagation in fluid-saturated porous media have been obtained form the basic principles of hydrology. It has been demonstrated, that under different assumptions, these equations can be reduced either to Biot's poroelasticity model or to the pressure diffusion equation. The tortuosity factor entering Biot's equations has been expressed through the relaxation time from the dynamic version of Darcy's law. This result can be used for evaluating tortuosity from a macroscopic flow experiment. The low-frequency asymptotic behavior of the reflection of a planar seismic signal from an interface between an elastic medium and fluid-saturated porous medium has been investigated. The frequency-dependent component of the reflection coefficient has been scaled with the square root of the characteristic time, which depends on the reservoir fluid mobility. The dependence of this characteristic time properties of the medium and the fluid has been investigated.

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