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THERMODYNAMIC BEHAVIOR OF NON-STRANGE BARYONIC MATTER

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Abstract

Within the framework of relativistic mean-field theory we examine effects of the non-strange baryonic resonances on the nuclear equation of state. We assume universal coupling constants to a scalar and vector meson field. Two transcendental equations have to be solved for two unknown parameters, the baryon scalar density and the nucleon chemical potential. We solve the equations numerically using a Newton-Raphson method for two cases: all known non-strange baryon resonances included and, finally, the low mass baryon resonances and an exponentially growing continuum. We compare our result with those from relativistic ideal gas calculations and also discuss the possibility of forming metastable states, which would provide a stabilizing mechanism for nuclear fireballs.

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I. INTRODUCTION

Experiments during the last few years have shown that high energy inelastic heavy ion collisions may deposit significant amounts of "heat" into nuclei. To explore the data within the realm of local equilibrium thermodynamics, it is essential to know the equation of state for highly excited baryonic matter. To be a realistic description, the equation of state must reflect nuclear matter properties like binding and short-range repulsion. An interesting analytical model of this kind has been suggested by Walecka[1,3]). In that model two boson fields (a scalar and vector meson) were coupled to the nucleon field. By solving a linearized Dirac equation for the nucleon field, it was possible to get explicit expressions for the thermodynamical variables at finite temperatures. In ref.[1,2,3,7]) only the nucleon was considered as a possible baryonic state. However, with rising temperatures ($\geqslant$ 50 MeV) the baryon resonances, i.e. N* and Δ, will populate significantly as have been seen in relativistic Fermi gas calculations[4,5,14]). In subsequent work we will examine the effect of including explicit interactions between hadrons, in the mean field approximation, on the dependence of ultrahigh energy nuclear collisions on the asymptotic hadron spectrum. By doing so, we also hope to learn about the possible existence of metastable nuclei[9]) and about eventually occurring supercritical field phenomena.

II. SYNOPSIS OF WALECKA'S MODEL

In Appendix 1 we include infinitely many baryon fields into Walecka's model. A scalar meson (σ meson) and a vector meson (ω meson) interacts with the baryon scalar density and current, respectively. We will assume universal coupling constants for the interaction terms. To recover Walecka's model[1]) only the nucleon baryonic state should be included. For the coupling constants: $g_s^2/(4\pi\hbar c^3) = 7.27$ and $g_v^2/(4\pi\hbar c) = 10.8$ we used the values given in ref.[2]) which were obtained from nuclear matter properties. These values are close to those obtained from phase-shift analysis of nucleon-nucleon scattering data[6]). Some typical features of Walecka's model is shown in Tables 1,2,3, and 4. In Table 1 one observes that the scalar density (and thus the expectation value of the scalar meson field) increases monotonically with increasing baryon density for fixed temperature. However, as can be seen from eq. A26 and Table 2, for fixed finite low temperature, as the baryon density increases from zero to infinity, the nucleon chemical potential will start at zero, increase to a maximum value, then decrease to a minimum value, and finally, tend to infinity.

This behavior is in sharp contrast to an ideal Fermi gas, where the chemical potential, in the case considered, will increase monotonically with increasing baryon density. In Tables 3 and 4 we show the energy density and pressure as function of the baryon density and temperature. One can see that for low baryon densities ($\rho \ll 0.085\ [\mathrm{fm}^{-3}]$) and temperatures below the rest mass a good approximation to the equation of state is that of an ideal classical gas:

$$P = \rho_B \cdot T \tag{1}$$

$$\varepsilon = \rho_B\left(m_N c^2 + \frac{3}{2} T\right) \tag{2}$$

Here one sees that the energy density includes the rest energy of the nucleon ($m_N c^2 \simeq 939$ MeV). One also observes in Table 4 that for increasing baryon density the pressure becomes very high as compared to the pressure given by expression (1). The reason for this is the coupling to the vector meson which leads to a strong repulsion at short distance (high compression). In the limit when the baryon density goes to infinity the relation

$$P = \varepsilon \tag{3}$$

will be fulfilled[1]). In that limit the speed of sound will approach the speed of light in the medium.

III. INCLUSION OF DISCRETE BARYON RESONANCES

In Table 5 we display all well-established non-strange baryon resonances[8]). The spin degeneracy factor h is simply given by (2I + 1)(2J + 1). We use the formalism in Appendix 1 and 2 to include all 18 discrete resonances in the equation of state. With the baryon density ρ_B and the temperature T as known quantities one has to solve two transcendental equations for the unknown parameters: the baryon scalar density and chemical potential. In Tables 6 and 7 the calculated scalar densities and nucleon chemical potentials are displayed. Note that for a temperature of 125 MeV, there exists three branches at low baryon density. As the scalar density increases the effective masses of the resonances decreases and at some point the effective masses of the lower vacuum mass resonances will pass zero. When the effective mass of the nucleon passes zero, the $\Delta(1232)$ resonance state will be mostly populated. Due to its high statistical weight, it forms a metastable state[9]) which has a signature of a negative pressure, as can be seen in Table 9.

For very high scalar density the lower vacuum mass resonances will have negative effective masses (and thus negative contributions to the scalar density). Since the scalar density has no probabilistic meaning there is no contradiction. At this high scalar density the population is mainly in the high vacuum mass resonances. Since the upper branches occur when the scalar meson field become large, and they are associated with a low chemical potential and therefore large pair production, they seem to belong to some kind of supercritical field phenomena. At lower temperature ($T \leqslant 100$ MeV), in the baryon density interval considered, only one branch was found. In figs. 1,2 and 3 we compare the equation of state at T = 100 MeV for three different models: i) Walecka's model, ii) Walecka's model with resonances included and finally, iii) relativistic Fermi gas in chemical equilibrium.

As can be seen, the energy density curves for the three cases are not very different. Nonetheless, the pressure curves are entirely different due to the strong short-range repulsion in Walecka's model. However, as can be seen, the inclusion of resonances softens the equation of state considerably. Of course, the pressure of the relativistic Fermi gas is much smaller than for the other two models. It is interesting to note that the entropy curve for case ii) will show an increase when thresholds for populations in higher lying resonances are passed. The relativistic Fermi gas will have an entropy curve similar to case ii) but with slightly higher entropy for moderate baryon densities.

IV. INCLUSION OF A CONTINUUM OF RESONANCES

We will now examine the effect of including both discrete resonances and an exponentially growing continuum. We include nine discrete resonances below 1680 MeV (see Table 5) and from there on a continuum. The mass degeneracy factor we assume to be a Hagedorn spectrum:[5, 13]

$$h(s) \simeq \frac{m_N\, e^{s(m_N c^2/T_o)}}{3\, m_\pi\, s^3\, (m_N c^2/T_o)^3} \qquad s > 12\ m_\pi/m_N \qquad (4)$$

The limiting temperature $T_o = 0.958\ m_\pi c^2$ is chosen as in ref[5]; m_N and m_π is the vacuum mass of the nucleon and pion respectively. The normalization is made in a symmetric five pion mass interval centered at 1400 MeV, so as to agree with the observed non-strange baryons (see Table 5). Counting the spin-isospin degeneracy factors we have a total of 88 states available in this interval. To perform the continuum integrals we expand in modified Bessel functions[4,5]. The integration over the variable **s** was performed by the trapezoidal rule in an exponential grid up to 100 pion masses/nucleon mass. From thereon an analytical integration in terms of incomplete gamma functions was made. At lower temperature ($T \leqslant 100$ MeV), in the baryon density interval considered, the continuum will not contribute much. However, as the temperature approaches the limiting temperature, the continuum will strongly influence the thermodynamical behavior, at least at finite baryon densities. In Table 11 we show the two lower branches at a fixed temperature of 125 MeV. As can be seen by inspecting Tables 6 and 11, these two branches are only slightly modified by the presence of the continuum. The third branch, however, is very sensitive to the continuum (where most of the population lies) due to the exponential increase of the Hagedorn spectrum with mass. For this branch the usual expansion in modified Bessel functions for the continuum integrals breaks down; however, estimates from approximate analytical formulae gives a value for the scalar density around unity (and, of course, very small chemical potential). It is of particular interest to note that the metastable "$\Delta(1232)$ state" survives the inclusion of a continuum of resonances.

V. DISCUSSION

We have shown explicitly how to include infinitely many baryon fields in interaction with two isoscalar Bose fields. The introduction of new degrees of freedom, e.g. Δ and N^* will lead to a softening of the equation of state. It has been suggested[11] that for an adiabatic shock wave in a single phase system a compression of four times the equilibrium density could be obtained. The strong short-range repulsion in Walecka's model will ultimately lead to smaller compressions since the speed of sound will approach the speed of light in the medium at high compression. It is expected that heavy ion induced shock waves are extremely hot with temperatures of order 100 MeV (1.16×10^{12} °K). We have shown that for temperatures slightly higher the effective mass of the low vacuum mass baryons will be extremely small. When this happens, the thermal motion of these baryons will be highly relativistic; and thus, a rapid thermalization will occur. It is interesting to note that metastable states do occur due to the high degeneracy of the $\Delta(1232)$ state as compared to the nucleon and the small vacuum mass difference. Since our Lagrangian density does not include the coupling to the meson field in a completely correct way (some mesons are pseudoscalars), it might be interesting to examine these effects in a more appropriate coupling scheme. One can also try to incorporate various additional flavors in an analogous way as presented here.

With increasing beam energies one might be able to study the super critical field phenomena predicted in this paper. It has already been shown that existing experimental particle spectra show tails which can be interpreted in terms of equivalent temperatures of at least 50 MeV[70,12].

In conclusion, a prediction of this model is a possibility of a large anti-particle production rate when the beam energy is sufficient to produce temperatures in the neighborhood of the Hagedorn limiting temperature.

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REFERENCES

1) J. D. Walecka, Phys. Lett. 59B (1975) 109

2) S. A. Chin and J. D. Walecka, Phys. Lett. 52B (1974) 24

3) J. D. Walecka, Ann. Phys. 83 (1974) 491

4) J. L. Synge, The Relativistic Gas, North-Holland Pub. Co. Amsterdam (1957)

5) N. K. Glendenning and Y. Karant, Bull. Am. Phys. Soc. 22, (1977) and Phys. Rev. Letters 40 (1978) 374; N. K. Glendenning, Invited paper Hir Schegg, Austria, Jan. 16 (1978)(LBL-7165)

7) R. A. Freedman, Phys. Lett. 71B (1977) 369

8) Rev. of Mod. Phys. 48 (1976) S30-S32

9) G. F. Chapline, M. H. Johnson, E. Teller, and M. S. Weiss, Phys. Rev. D8 (1973) 4303

10) R. Bond, P. J. Johansen, S. E. Koonin and S.I.A. Garpman, Phys. Lett. 71B (1977) 43

11) M. Sobel, P. J. Siemens, J. P. Bondorf, and H. A. Bethe, Nucl. Phys. A251, 502 (1975)

12) G. D. Westfall, J. Gosset, P. J. Johansen, A. M. Poskanzer, W. G. Meyer, H. H. Gutbrod, A. Sandoval, and R. Stock, Phys. Rev. Letters 37, 1202 (1978)

13) R. Hagedorn in Cargèse Lectures in Physics, Vol. 6, ed. by E. Schatzman, (Gordon Breach, New York, 1973).

14) J. I. Kapusta, (LBL-6504, submitted to Phys. Rev. C).

15) A. L. Fetter and J. D. Walecka, Quantum Theory of Many-Particle Systems, McGraw-Hill Book Company, San Francisco (1971) p. 36

Table 1

Scalar density ρ_s [fm^{-3}] Walecka's model

T(MeV) \ Compression	1/2	1	2	3	4
125	0.0681	0.130	0.226	0.282	0.312
100	0.0707	0.135	0.234	0.289	0.317
50	0.0769	0.148	0.255	0.305	0.328

Table 2

Nucleon chemical potential $\mu/m_N c^2$ [dimensionless] Walecka's model

T(MeV) \ Compression					
125	0.469	0.445	0.379	0.363	0.379
100	0.578	0.527	0.430	0.406	0.419
50	0.753	0.641	0.488	0.458	0.471

Table 3

Energy density ε [MeV/fm^3] Walecka's model

T(MeV) \ Compression					
125	97.0	192.	389.	613.	882.
100	92.7	183.	369.	583.	844.
50	85.0	167.	336.	538.	789.

+Defined as baryon density/normal nuclear matter density.

Table 4
Pressure P [MeV/fm^3] Walecka's model

T(MeV) \ Compression	1/2	1	2	3	4
125	11.5	26.9	85.1	202.	383.
100	8.97	21.1	72.6	186.	366.
50	3.76	8.75	47.7	160.	339.

TABLE 5

PARTICLE	I	J^P	M(MeV)	h
p n	1/2	$1/2^+$	939.	4
Δ(1232)	3/2	$3/2^+$	1232.	16
N(1470)	1/2	$1/2^+$	1430.	4
N(1520)	1/2	$3/2^-$	1520.	8
N(1535)	1/2	$1/2^-$	1515.	4
Δ(1650)	3/2	$1/2^-$	1655.	8
N(1670)	1/2	$5/2^-$	1673.	12
N(1688)	1/2	$5/2^+$	1680.	12
N(1700)	1/2	$1/2^-$	1675.	4
Δ(1670)	3/2	$3/2^-$	1685.	16
N(1780)	1/2	$1/2^+$	1750.	4
N(1810)	1/2	$3/2^+$	1775.	8
Δ(1890)	3/2	$5/2^+$	1880.	24
Δ(1910)	3/2	$1/2^+$	1865.	8
Δ(1950)	3/2	$7/2^+$	1925.	32
N(2190)	1/2	$7/2^-$	2175.	16
N(2220)	1/2	$9/2^+$	2225.	20
Δ(2420)	3/2	$11/2^+$	2415.	48

Table 6

Scalar density $\rho_s [fm^{-3}]$ Resonances included.

Compression[+] / T(MeV)	1/2	1	2	3	4
125	0.735	0.735	0.735	0.735	0.735
125	0.431	0.428	0.410	-	-
125	0.0706	0.137	0.262	-	-
100	0.0718	0.139	0.249	0.324	0.372
50	0.0770	0.148	0.257	0.316	0.354

[+]Defined as baryon density/normal nuclear matter density.

Table 7

Nucleon chemical potential $\mu/m_N c^2$ [dimensionless]
Resonances included

Compression[+] / T(MeV)	1/2	1	2	3	4
125	0.00204	0.00407	0.00814	0.0122	0.0163
125	0.0126	0.0258	0.0594	-	-
125	0.384	0.345	0.212	-	-
100	0.539	0.478	0.346	0.270	0.237
50	0.752	0.638	0.477	0.423	0.398

[+]Defined as baryon density/normal nuclear matter density.

Table 8

Energy density ε[MeV/fm^3] Resonances included

T(MeV) \ Compression+	1/2	1	2	3	4
125	3365.	3385.	3462.	3592.	3773.
125	786.	796.	812.	-	-
125	112.	224.	475.	-	-
100	101.	200.	406.	644.	928.
50	85.4	169.	340.	548.	805.

+Defined as baryon density/normal nuclear matter density.

Table 9

Pressure P [MeV/fm^3] Resonances included

T(MeV) \ Compression+	1/2	1	2	3	4
125	143.	162.	237.	361.	536.
125	- 91.3	- 71.3	10.8	-	-
125	11.1	24.5	66.0	-	-
100	8.73	19.6	62.2	156.	309.
50	3.73	8.61	45.4	148.	306.

+Defined as baryon density/normal nuclear matter density.

Table 10

Entropy per baryon S/B Resonances included

T(MeV) \ Compression+	1/2	1	2	3	4
125	329.	165.	82.4	55.0	41.3
125	64.2	31.6	14.3	-	-
125	7.55	6.79	6.50	-	-
100	6.37	5.55	4.74	4.46	4.41
50	3.96	3.06	2.16	1.99	2.04

+Defined as baryon density/normal nuclear matter density.

Table 11

Thermodynamical properties when a continuum is included T = 125 MeV.

Compression	ρ_s [fm^{-3}]		$\mu/m_N c^2$ [dimensionless]		ε [MeV/fm^3]		p [MeV/fm^3]		S/B [dimensionless]	
1/2	0.0724	0.315	0.354	0.0269	139.	663.	10.8	-54.4	10.3	55.9
1	0.142	0.306	0.304	0.0589	287.	659.	22.9	-32.2	9.98	26.8

Figure Captions

Fig. 1 The figure shows the energy density versus the compression+ for a fixed temperature of 100 MeV. Three different models are displayed: i) Walecka's model (dashed line), ii) Walecka's model with 18 discrete resonances included (dot dashed line) and finally iii) free relativistic Fermi gas of 18 discrete resonances (dotted line).

Fig. 2 The figure shows the pressure versus the compression for a fixed temperature of 100 MeV. As in fig. 1 three different models are displayed: i) Walecka's model (dashed line), ii), Walecka's model with resonances included (dot dashed line) and finally iii) free relativistic Fermi gas of the resonances (dotted line).

Fig. 3 The figure shows the entropy per baryon versus the compression for a fixed temperature of 100 MeV. As in fig. 1 and 2 three different models are displayed: i) Walecka's model (dashed line), ii) Walecka's model with resonances included (dot dashed line) and finally iii) free relativistic Fermi gas of the resonances dotted line).

+Defined as baryon density/normal nuclear matter density where we choose 0.17 [fm^{-3}] for normal nuclear matter density.

APPENDIX 1

Let us define our baryon fields over a linear space R^5 where the fifth dimension labels the vacuum masses. A specific baryon field has the notation $\psi(x,s)$, where x is a four-vector and s is defined as the vacuum mass divided by the nucleon vacuum mass. We will assume that all baryon fields interact with a neutral scalar field $\phi(x)$ and a neutral vector field $V_\lambda(x)$ with universal coupling constants. Our Lagrangian density is:

$$\mathscr{L}(x) = -\hbar c \int [\bar{\psi}(\not\partial + \xi\ (s))\ \psi]\ ds - \frac{1}{2} c^2 [(\partial_\lambda \phi)^2 + \mu^2 \phi^2]$$

$$- \frac{1}{4} F_{\lambda\rho} F_{\lambda\rho} - \frac{1}{2} m^2 V_\lambda V_\lambda + i g_v \left\{ \int \bar{\psi} \gamma_\lambda \psi\ ds \right\} V_\lambda + g_s \left\{ \int \bar{\psi}\psi\ ds \right\} \phi \quad \text{(A1)}$$

Here μ and m are inverse Compton wavelengths of the scalar and vector meson fields respectively, g_s and g_v are the universal coupling constants. The vector field V_λ has field strengths $F_{\lambda\rho}$ defined by:

$$F_{\lambda\rho} = \partial_\lambda V_\rho - \partial_\rho V_\lambda \quad \text{(A2)}$$

The function $\xi(s)$ will include both discrete and continuous baryon fields and is defined by:

$$\xi(s) = Ms \left\{ \left\{ \sum_{n=1}^{k} \delta(s - m_n/m_N) \right\} + \theta(s - m_k/m_N) \right\} \quad \text{(A3)}$$

Here M is the inverse Compton wavelength of the nucleon and m_N is the nucleon vacuum mass. It is assumed that there exists k discrete baryon fields. $\theta(x)$ is a step function. The equations of motion obtained from (A1) are

$$(\Box^2 - \mu^2)\ \phi = -\ (g_s/c^2) \int \bar{\psi}\psi\, ds \tag{A4}$$

$$\partial_\rho F_{\lambda\rho} = -\ m^2 V_\lambda + i g_v \int \bar{\psi}\gamma_\lambda \psi\ ds \tag{A5}$$

$$(\not{\partial} + \xi(s) - \frac{i g_v}{\hbar c}\gamma_\mu V_\mu - \frac{g_s}{\hbar c}\phi)\ \psi = 0 \tag{A6}$$

Equation (A6) represents infinitely many field equations one for each value of s. We will have interest in the solutions to (A4) and (A5) which corresponds to baryon and scalar densities which are independent of spatial position and time:

$$\phi = \phi_o = \frac{g_s}{\mu^2 c^2} \int \bar{\psi}\psi\ ds \tag{A7}$$

$$V_\lambda = i\delta_{\lambda 4}\ V_o = i\delta_{\lambda 4}\ \frac{g_v}{m^2} \int \bar{\psi}\gamma_\lambda \psi\ ds \tag{A8}$$

It will be convenient to define the quantities:

$$\rho_s = \int \bar{\psi}\psi\ ds \tag{A9}$$

$$\rho_B = \int \psi^\dagger \psi\, ds \tag{A10}$$

which are the baryon scalar density and baryon density respectively. Using eqs. (A7), (A8) and (A1) the linearized Lagrangian is:

$$\mathscr{L}^{o}(x) = \int [\bar{\psi}(\not{\partial} + \xi(s))\,\psi]\,ds - g_v V_o \int \psi^{+}\psi\,ds + g_s\,\phi_o \int \bar{\psi}\psi\,ds + \frac{m^2}{2} V_o^{2} - \frac{c^2\mu^2\,\phi_o^2}{2} \tag{A11}$$

which gives the linearized field equations:

$$\left(\not{\partial} + \xi(s) + \frac{g_v}{\hbar c}\gamma_4\, V_o - \frac{g_s}{\hbar c}\phi_o\right)\psi = 0 \tag{A12}$$

The Hamiltonian density is given by the prescription:

$$\mathscr{H}(x) = c\int \pi_\beta \frac{\partial \psi_\beta}{\partial x_o}\,ds - \mathscr{L}(x) \tag{A13}$$

To perform second quantization we expand the baryon field according to the functions:

$$\psi(x,s) = U^{\pm}(\mathbf{k},s)\; e^{i/\hbar\; p_\mu x_\mu} \tag{A14}$$

Using the linearized field equations (A12) we get:

$$E^{\pm}_{\mathbf{k},s} - g_v V_o = \pm\sqrt{(\hbar c\;\mathbf{k})^2 + (\hbar c\; M^*)^2} \tag{A15}$$

for the eigenvalues and an effective mass M^* defined by:

$$M^* = \xi(s) - \frac{g_s}{\hbar c}\phi_o \tag{A16}$$

A complete set of Dirac wavefunctions for a given $\mathbf{k}$ and s is given by:

$$\hbar c[\boldsymbol{\alpha} \cdot \mathbf{k} + \beta M^*] U^{\pm}(\mathbf{k},s) = (E^{\pm}_{k,s} - g_v V_o) U^{\pm} (\mathbf{k},s) \tag{A17}$$

where U^+ and U^- is unity normalized.

The Dirac field can now be second quantized and thus become an operator in the abstract occupation-number space:

$$\hat{\psi} (x,s) \frac{1}{\Omega^{\frac{1}{2}}} \sum a U^+(\mathbf{k},s) e^{i\mathbf{k}\cdot\mathbf{r}} + b^{\dagger} U^-(-\mathbf{k},s) e^{-i\mathbf{k}\cdot\mathbf{r}} \tag{A18}$$

where $a^{\dagger}$, a, $b^{\dagger}$, b is a creation-annihilation operator for particles and anti-particles respectively. Ω is the volume.

We assume Jordan-Wigner type anticommutation relations:

$$\begin{aligned} \left\{ a^{(\mathbf{r})}_{\mathbf{k},s} , a^{(r')\dagger}_{\mathbf{k}',s'} \right\} &= \delta_{\mathbf{rr}'} \delta_{\mathbf{kk}'} \delta_{ss'} \\ \left\{ a^{(\mathbf{r})}_{\mathbf{k},s} , a^{(\mathbf{r}')}_{\mathbf{k}',s'} \right\} &= 0 \\ \left\{ a^{(\mathbf{r})\dagger}_{\mathbf{k},s} , a^{(\mathbf{r}')\dagger}_{\mathbf{k}',s'} \right\} &= 0 \end{aligned} \tag{A19}$$

Now in complete analogy with the work of ref. (3) we can construct the operators for the energy density, pressure and baryon density. For simplicity we only show the second quantized baryon and scalar density:

$$\hat{\rho}_B = \frac{\hat{B}}{\Omega} = \frac{1}{\Omega} \int ds \sum [a^{\dagger}a - b^{\dagger}b] \tag{A20}$$

$$\hat{\rho}_s = \frac{1}{\Omega}\int ds \sum \frac{M^*}{\sqrt{k^2+M^{*2}}}\,[a^\dagger a + b^\dagger b] \tag{A21}$$

To obtain the Grand canonincal ensemble average we use the definition[15]:

$$<\hat{O}> = \frac{Tr(e^{-\beta(\hat{H}-\mu\hat{B})}\hat{O})}{Tr(e^{-\beta(\hat{H}-\mu\hat{B})})} = \frac{\sum\sum <N_j|e^{-\beta(\hat{H}-\mu\hat{B})}\,\hat{O}|N_j>}{\sum\sum <N_j|e^{-\beta(\hat{H}-\mu\hat{B})}\,|N_j>} \tag{A22}$$

Here $\hat{O}$ is any second quantized operator. $\beta = (kT)^{-1}$ where T is the temperature and μ is the chemical potential. j denotes the set of all states with a fixed number of particles N and the sum implied in the trace is over both j and N. $\hat{H}$ and $\hat{B}$ is the Hamiltonian and baryon number operator respectively.

To perform the ensemble average we notice that the only quantities we need are the ensemble averages of the number operator for particles $\hat{N}_p = a^\dagger a$ and for antiparticle $\hat{N}_a = b^\dagger b$ which for a given s are the usual fermion distribution functions:

$$<\hat{N}_p> = \sum_{i=1}^{\infty} \frac{1}{e^{\beta(\varepsilon_i - \mu)} + 1}$$

$$<\hat{N}_a> = \sum_{i=1}^{\infty} \frac{1}{e^{\beta(\varepsilon_i + \mu)} + 1} \tag{A23}$$

Here $\varepsilon = \sqrt{k^2 + M^{*2}}\,\hbar c$ and we used the fact that $\mu_p = -\mu_a$. Since all the thermodynamical quantities only contain various combinations of the number operators, one can easily solve for them by using eqs. (A22) and (A23). We also need the assumption of chemical equilibrium which says that $\mu(s) \equiv \mu$, i.e., the chemical potentials of all the baryons are the same. To count the number of states per momentum interval we also have to include the effect of degeneracy. Let us define a general degeneracy factor $\kappa(s)$ by:

$$\kappa(s) = \left\{ \sum_{n=1}^{k} h_n \; \delta(s - m_n/m_N) \right\} + h(s) \; \theta(s - m_k/m_N) \tag{A24}$$

For the discrete case at a specific n the degeneracy is just the spin-isospin factor: $(2I + 1)(2J + 1)$. Now it is feasible to write down the ensemble averages for the scalar and baryon densities:

$$\langle\hat{\rho}_s\rangle = \frac{1}{2\pi^2} \int M^*(s) \; \kappa(s) \int \frac{k^2}{\sqrt{k^2 + M^{*2}}} \; \left\{ \langle\hat{N}_p\rangle + \langle\hat{N}_a\rangle \right\} ds \; dk \tag{A25}$$

$$\langle\hat{\rho}_B\rangle = \frac{1}{2\pi^2} \int \kappa(s) \int k^2 \; \left\{ \langle N_p\rangle - \langle\hat{N}_a\rangle \right\} ds \; dk \tag{A26}$$

Writing eq. (A7) and (A16) as:

$$M^*(\rho_s, s) = \xi(s) - \frac{g_s^2}{\hbar c^2 \mu^2} \rho_s \tag{A27}$$

we see that eqs. (A25) and (A27) are transcendental equations for ρ_s provided that the identification $\rho_s \equiv \langle\hat{\rho}_s\rangle$ can be done (mean-field approximation). To perform a calculation ρ_B and T are specified; the two transcendental eqs. (A25) and (A26) are solved by a Newton-Raphson method for the two unknowns ρ_s and μ, (or if one likes for ϕ_o and μ) and all the thermodynamical variables can then be calculated.

For convenience we will also give the expressions for the energy density ε and the pressure P:

$$\langle\hat{\varepsilon}\rangle = \frac{\hbar c}{2\pi^2}\int \kappa(s)\int \sqrt{k^2 + M^{*2}}\ \left\{\langle\hat{N}_p\rangle + \langle\hat{N}_a\rangle\right\} ds\ dk \tag{A28}$$

$$+ \frac{1}{2}\mu^2 c^2\ \phi_o^2\ + \frac{1}{2} m^2 V_o^{\ 2}$$

$$\langle\hat{P}\rangle = \frac{\hbar c}{6\pi^2}\int \kappa(s)\int \frac{k^2}{\sqrt{k^2 + M^{*2}}} \left\{\langle\hat{N}_p\rangle + \langle\hat{N}_a\rangle\right\} ds\ dk \tag{A29}$$

$$- \frac{1}{2}\mu^2 c^2\ \phi_o^{\ 2}\ + \frac{1}{2} m^2 V_o^{\ 2}$$

The entropy per baryon is given by:

$$\langle\frac{\hat{S}}{B}\rangle = \frac{\langle\hat{\varepsilon}\rangle + \langle\hat{P}\rangle}{T\ \langle\hat{\rho}_b\rangle} - \frac{\mu'}{T} \tag{A30}$$

where according to (A15)

$$\mu' = \mu + g_v\ V_o \tag{A31}$$

APPENDIX 2

We are interested in incorporating within a simple Lagrangian formalism fields whose quanta have arbitrary spin, and especially those representing the sorts of particles envisioned by the statistical bootstrap. Since the only boson fields we retain are the scalar (σ meson) ϕ and vector meson V_μ, both isoscalars, we can avoid the isospin part of the problem. The spin problem is not trivial, however. Using the Rarita-Schwinger formalism, one can formally state the problem. Let α denote the internal degrees of freedom (e.g., isospin) and $\tilde{\mu} = m_1 \ldots m_k$, $k = J-1/2$, J the spin of the particle, one finds in a plane-wave basis

$$\psi_{\tilde{\mu}\alpha}(x) = \sum_{p\lambda} [a_\alpha(p,\lambda)u_{\tilde{\mu}}(p,\lambda)f_p(x) + b^*_{-\alpha}(p,\lambda)v_{\tilde{\mu}}(p,\lambda)f^*_p(x)]$$

where $u_{\tilde{\mu}}$, $v_{\tilde{\mu}}$ are 4-component spinors satisfying

$$(\not{p}+m)\, u_{\tilde{\mu}}(p,\lambda) = 0 \qquad (\not{p}-m)\, v_{\tilde{\mu}}(p,\lambda) = 0$$

$$\not{p} \equiv \gamma\cdot p, \quad \left\{\gamma_{\lambda_i}, \gamma_{\nu_i}\right\} = 2\,\delta_{\lambda_i\nu_i}$$

and the subsidiary conditions that

$$p^{\mu_i}\, u_{\tilde{\mu}}(p,\lambda) = \gamma^{\mu_i}\, u_{\tilde{\mu}}(p,\lambda) = 0$$

To follow the formalism, we define the spinors u,v recursively, starting from ordinary Dirac spiners.

Let $\quad p_\mu(2p,2\lambda) = \bar{v}(p,\lambda)\ \gamma_\mu\ u(p,\lambda)$

Then $\quad u_\mu^{3/2}(p,\lambda) = \sum_{\lambda_1\lambda_2} (3/2,\ \lambda|1/2,\lambda_1;\ 1,\lambda_2)\ u(p,\ \lambda_1)\ e_\mu(p,\lambda_2)$

While $\quad u_{\mu_1\mu_2}^{5/2}(p,\lambda) = \sum_{\lambda_1\lambda_2} (5/2,\lambda|3/2,\lambda_1;\ 1,\lambda_2)\ u_{\mu_1}^{3/2}(p,\lambda_1)\ e_{\mu_2}(p,\lambda_2)$

etc., where $(J+1,\ \lambda|J,\lambda_1;1,\lambda_2)$ is a Clebsch-Gordan coefficient. Since we will be dealing with particles of arbitrary mass, s_i say, and for a free particle of mass $s_i, p^2 = -s_i^{\ 2}$, we will introduce the additional label s_i for species "i" replacing α used above. Thus, we may formally write

$$u^{J+1}_{\mu_1\ldots\mu_{J+1/2}}(p,s_i,\lambda) = \sum_{\lambda_1\ldots\lambda_{J+3/2}} C_{J+1}\ {}^{i}{}_{\lambda_1}\ldots{}_{\lambda_{J+3/2}}\ u(p,s_i,\lambda_1)e_{\mu_1}(p,s_i,\lambda_2)\ldots$$

$$\ldots\ e_{\mu_{J+1/2}}(p,s_i,\lambda_{J+3/2})$$

We will label each field now by the mass of its free quanta, and take for the free Lagrangian density $\mathscr{L}_0 = -\sum_i \psi_{\tilde{\mu}}(x,s_i)(\not{\partial} + s_i)\ \psi_{\tilde{\mu}}(x,s_i)$

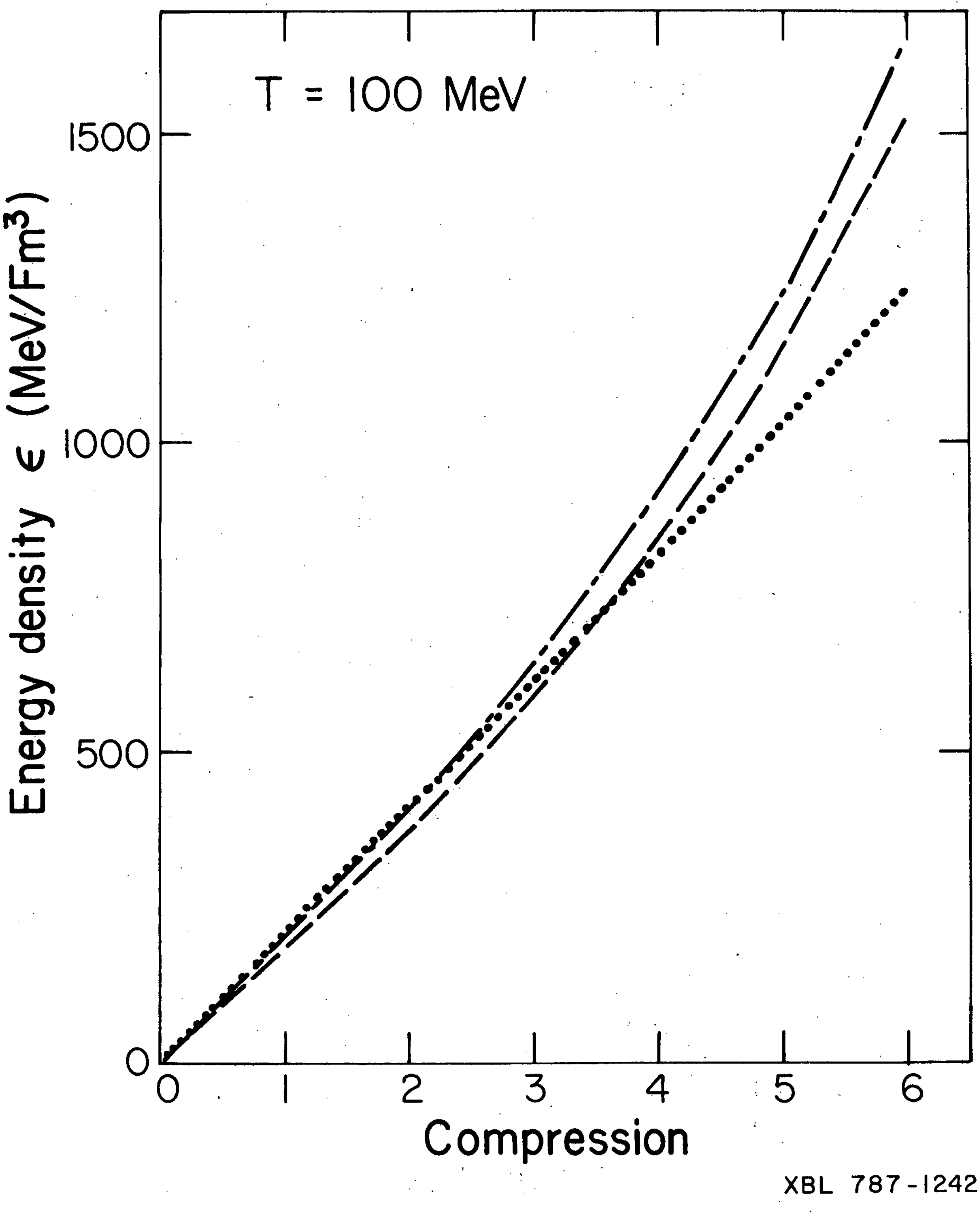
and for the interaction, we take two terms:

$$\mathscr{L}_v = ig_v \sum_i \bar{\psi}^i_{\tilde{\mu}}\ \gamma_\nu\ \psi^i_{\tilde{\mu}} V_\nu\quad ,\ \mathscr{L}_s = g_s \sum_i \bar{\psi}^i_{\tilde{\mu}}\ \ \psi^i_{\tilde{\mu}}\ \ \phi$$

where we have explicitly assumed a universal coupling to V_ν and ϕ. Upon inserting the expressions for $\bar{\psi}_{\tilde{\mu}}(s_i)$, we become interested in the effective interaction

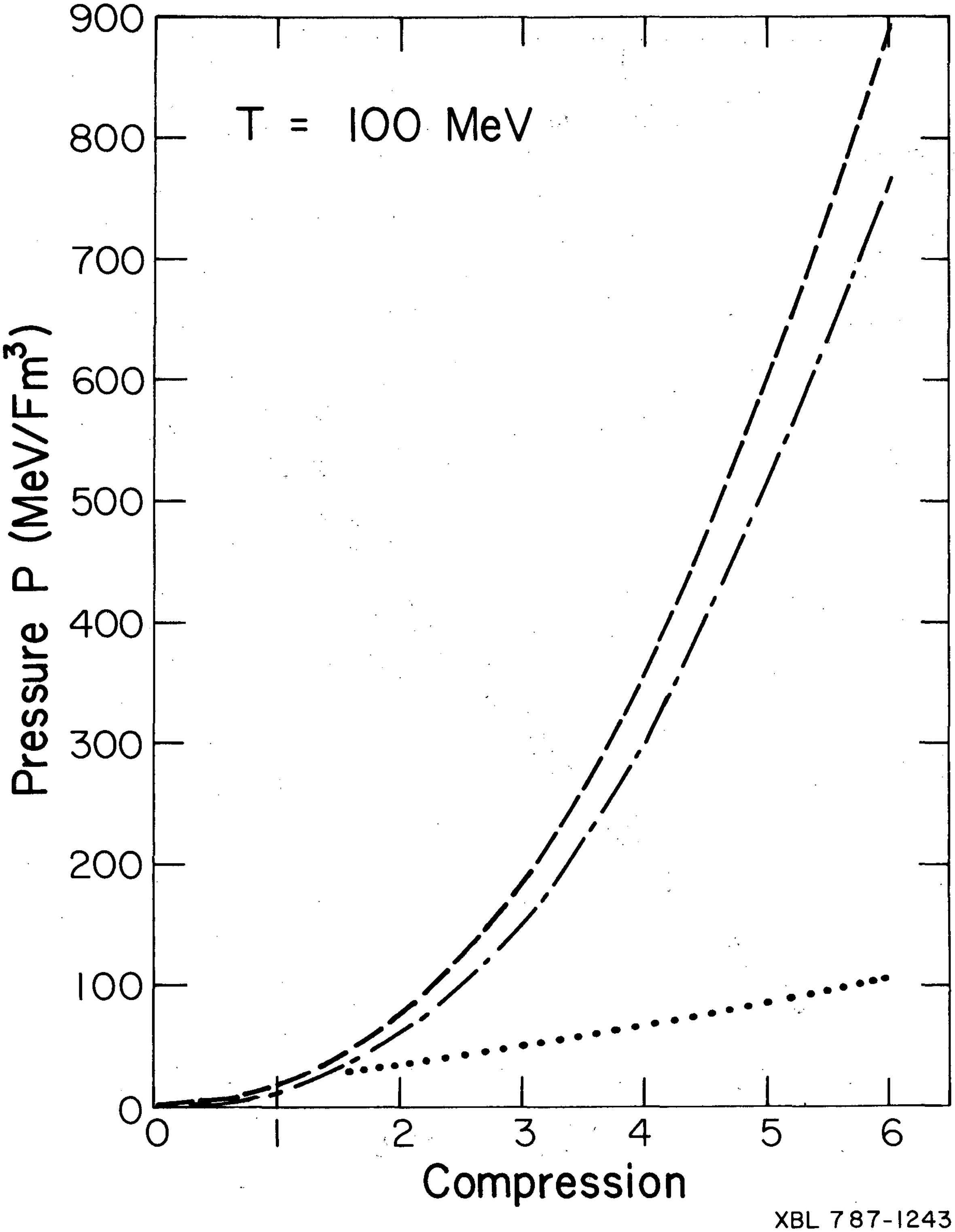
$$\mathscr{L}_v^{eff} = ig_v \sum_i c_i \bar{\psi}^i \gamma_\mu \psi^i V_\mu \qquad \mathscr{L}_s^{eff} = g_s \sum_i c_i' \bar{\psi}^i \psi^i \phi$$

where the $\bar{\psi}^i$ refer in tree approximation to fields transforming as Dirac fields. We take for an ansatz that $c_i = c_i' = \rho(s_i)$ where in thermal equilibrium $\rho(s_i)$ is the level density given by Hagedorn.



T = 100 MeV
Energy density ϵ (MeV/Fm³)
1500
1000
500
0
0
1
2
3
4
5
6
Compression
XBL 787-1242

Fig. 1



T = 100 MeV
Pressure P (MeV/Fm^3)
900
800
700
600
500
400
300
200
100
0
0
1
2
3
4
5
6
Compression
XBL 787-1243

Fig. 2

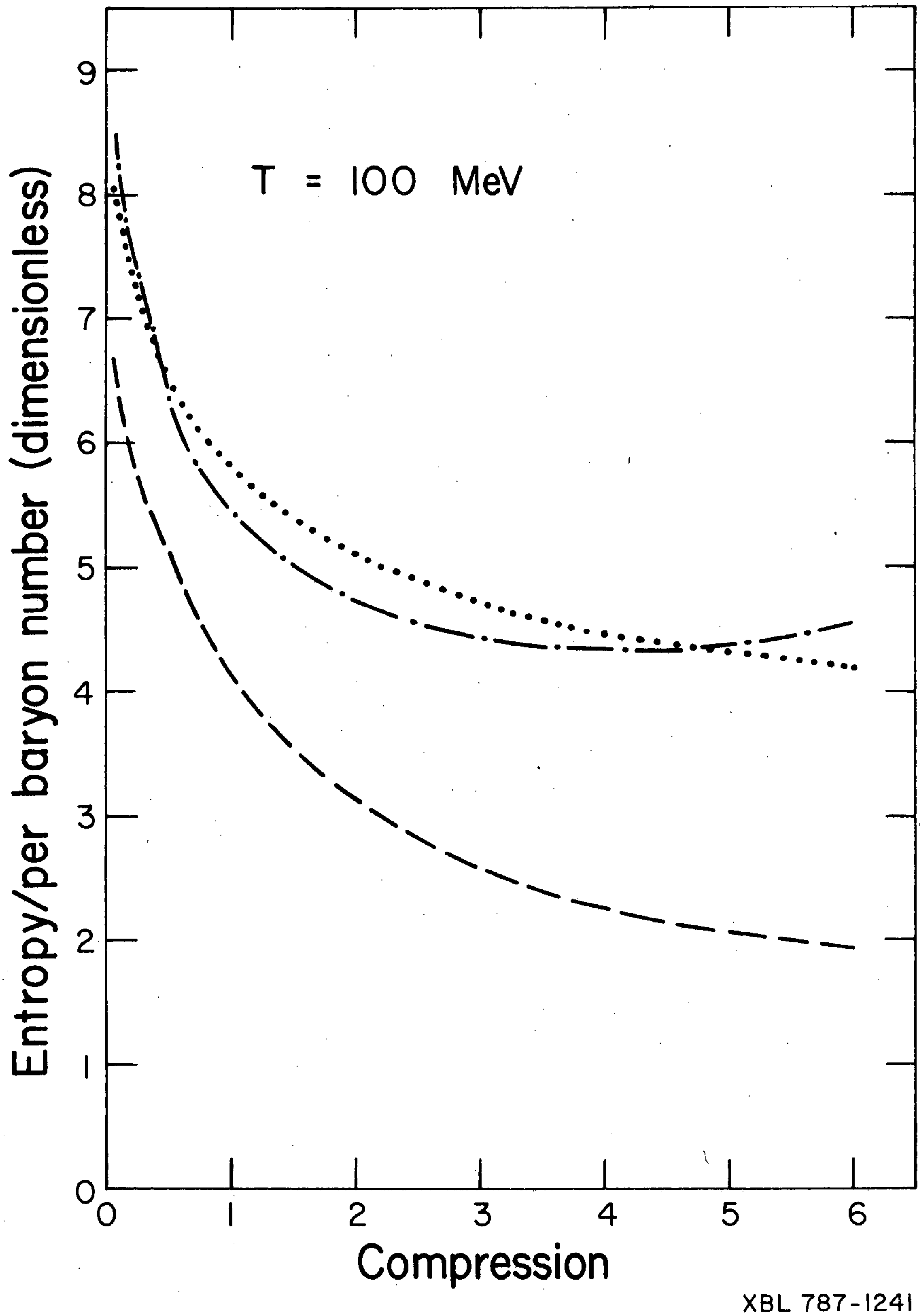


Fig. 3

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