

Lawrence Berkeley National Laboratory

Recent Work

Title

SCATTERING OF AN ICRF MAGNETOSONIC WAVE BY PLASMA DENSITY TURBULENCE

Permalink

<https://escholarship.org/uc/item/2qs9f6g6>

Authors

Cook, D. R.

Kaufman, ft. N.

Publication Date

1989-04-01

c.2



Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

Presented at the 8th Topical Conference on Radio Frequency
Power in Plasma, Irvine, CA, May 1-3, 1989

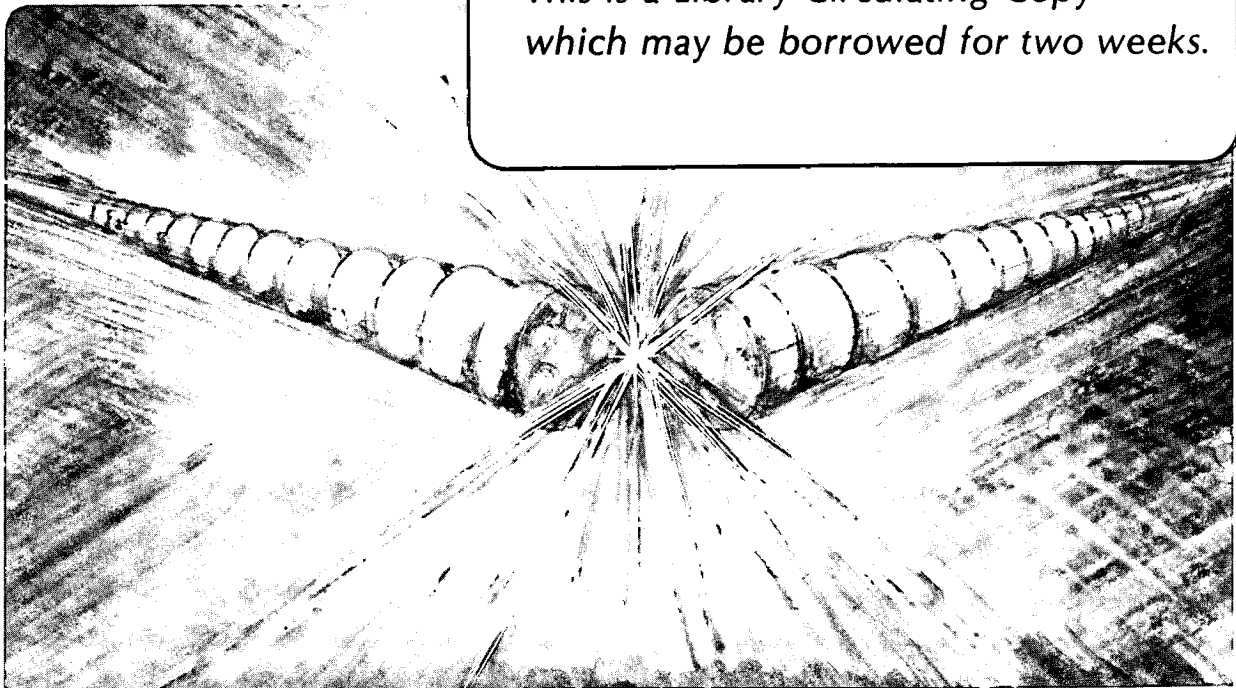
FEB 8 1989
888 7 13 1 0 1 1

Scattering of an ICRF Magnetosonic Wave by Plasma Density Turbulence

D.R. Cook and A.N. Kaufman

April 1989

TWO-WEEK LOAN COPY
*This is a Library Circulating Copy
which may be borrowed for two weeks.*



LBL-27108
c.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

**SCATTERING OF AN ICRF MAGNETOSONIC WAVE
BY PLASMA DENSITY TURBULENCE***

*Daniel R. Cook and Allan N. Kaufman
Lawrence Berkeley Laboratory and Department of Physics,
University of California, Berkeley, California 94720*

Presented at the 8th Topical Conference on Radio Frequency
Power in Plasma

1-3 May 1989

Irvine, CA 94720

Work supported by the U.S. DOE under contract No. DE-AC03-76SF00098 and by U.S. DOE Magnetic Fusion Science Fellowship, administered through Oak Ridge Associated Universities.

SCATTERING OF AN ICRF MAGNETOSONIC WAVE BY PLASMA DENSITY TURBULENCE.*

*Daniel R. Cook and Allan N. Kaufman,
Lawrence Berkeley Laboratory and Department of Physics,
University of California, Berkeley, California 94720.*

ABSTRACT

A fast ICRF magnetosonic wave, launched into a tokamak plasma, scatters off turbulent density fluctuations in the plasma edge. We use cold-fluid theory to calculate the angular distribution of the scattered wave and find it to be predominantly perpendicular to the incident wavevector for second harmonic majority heating. We calculate the mean free path and find it to be large compared to the size of tokamak devices. Therefore, scattering of ICRF magnetosonic waves by density turbulence is an utterly negligible effect.

INTRODUCTION

The incident magnetosonic wavevector has $k_{\parallel} \ll k_{\perp}$ and experimental data indicate that the density turbulence has $k_{\parallel} \approx 0$. We therefore in our model let $k_{\parallel} = 0$ for all wavevectors, incident, scattered, and turbulent. The scattering occurs in the plane perpendicular to B_0 . Experimental data indicate that the correlation length of the density turbulence is small compared to the magnetosonic wavelength. The density turbulence fluctuates slowly compared to the frequency of the magnetosonic wave, so the turbulent scatterers look stationary to the incident wave, and the scattering is elastic. In the wave field quantities which follow, an $e^{i\omega t}$ time dependence is to be understood. We found many useful ideas which helped us with this calculation in Ishimaru's fine book, *Wave Propagation and Scattering in Random Media*¹.

**Work supported by US DOE under contract No. DE-AC03-76SF00098 and by US DOE Magnetic Fusion Science Fellowship, administered through Oak Ridge Associated Universities.*

We begin with a few basic symbols and relations:

$$\begin{aligned}
 \psi(x, y) &\equiv \tilde{B}(x, y)/B_o, & \mu(x, y) &\equiv n_t(x, y)/n_o, & \mathbf{k}' &\equiv \mathbf{k}_{\text{scat}} - \mathbf{k}_o, \\
 \psi(\mathbf{x}) &= \psi_{\text{inc}}(\mathbf{x}) + \psi_{\text{scat}}(\mathbf{x}), & n &= n' + \tilde{n}, & \mathbf{k}_{\text{scat}} &\equiv k_o \hat{r}, \\
 \psi_{\text{inc}}(\mathbf{x}) &= \psi_o e^{i\mathbf{k}_o \cdot \mathbf{x}}, & n' &\equiv n_o + n_t, & k_o^2 &\equiv \omega^2/C_A^2, \\
 C_A^2 &\equiv B_o^2/4\pi M_i n_o, & \Omega_i &\equiv eB_o/M_i c, & \lambda_o &\equiv 1/k_o,
 \end{aligned}$$

$$\text{Re} \left[e^{-i\omega t} \tilde{B}(x, y) \right],$$

$n_o,$

$\tilde{n},$

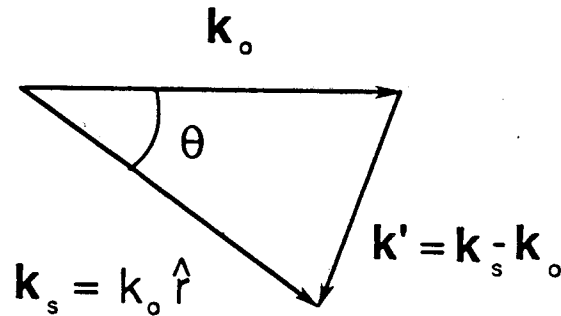
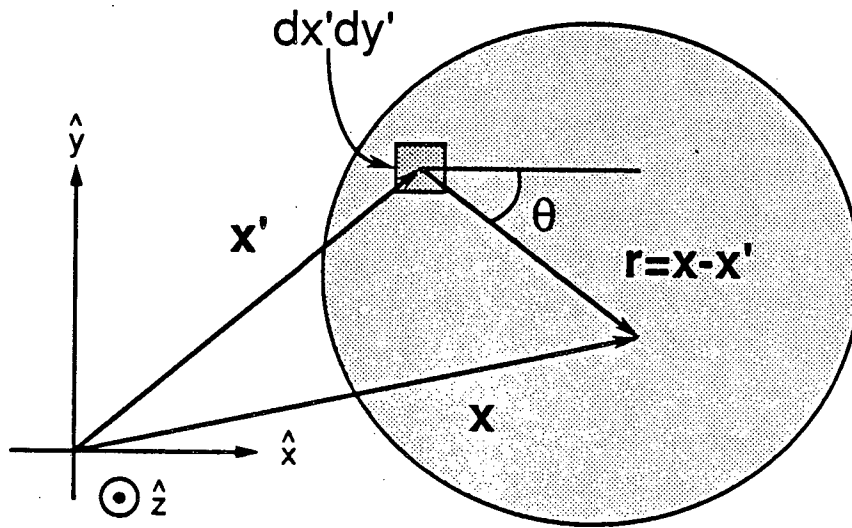
$n_t,$

magnetic field of the wave

uniform background density

density variation due to wave

turbulent density fluctuation



From the equations of two-component cold fluid theory, we obtain the following wave equation:

$$(\nabla^2 + k_o^2)\psi(x, y) = \left\{ -k_o^2\mu + \left[\nabla\mu - i\frac{\omega}{\Omega_i}(\hat{z} \times \nabla\mu) \right] \cdot \nabla \right\} \psi(x, y)$$

We make the Rayleigh-Born Approximation on the wavefunction in the above equation by letting $\psi_{scat} \ll \psi_o$. Next we perform a far field approximation on the Green's Function to obtain the following scattered wavefunction:

$$\psi_s(\mathbf{x}) = \sqrt{\frac{k_o}{8\pi}} \left(i\hat{k}_o - \frac{\omega}{\Omega_i} \hat{z} \times \hat{k}_o \right) \cdot \nabla_{\mathbf{x}} \int d^2\mathbf{x}' \mu(\mathbf{x}') \frac{e^{ik_o[\hat{k}_o \cdot \mathbf{x}' + |\mathbf{x} - \mathbf{x}'|]}}{|\mathbf{x} - \mathbf{x}'|^{\frac{1}{2}}}$$

The spectral density $S(\mathbf{k}'; \mathbf{x}')$ is the Wigner Function of the relative density turbulence, $\mu(\mathbf{x}')$. It represents the *frequency-integrated wavevector content* of the relative density turbulence at position \mathbf{x}' . The $\langle \rangle$ indicates a time average over a time which is long compared to the timescale of the density turbulence.

$$S(\mathbf{k}'; \mathbf{x}') \equiv \int d^2\mathbf{s} e^{-i\mathbf{k}' \cdot \mathbf{s}} \left\langle \mu(\mathbf{x}' + \frac{1}{2}\mathbf{s}) \mu(\mathbf{x}' - \frac{1}{2}\mathbf{s}) \right\rangle$$

We construct the Wigner Function of the scattered wave function and use it to obtain the following expression for the scattered wave intensity:

$$\langle |\psi_s(\mathbf{x})|^2 \rangle = \frac{k_o^3}{8\pi} \int \frac{d^2\mathbf{x}'}{r} g(\theta) S(\mathbf{k}'; \mathbf{x}') |\psi_o|^2, \quad g(\theta) \equiv \cos^2\theta + \left(\frac{\omega}{\Omega_i}\right)^2 \sin^2\theta$$

The strength of the density turbulence, $\langle \mu^2(\mathbf{x}') \rangle$ is related to the spectral density and to the wavenumber spread k_n of the density turbulence as follows:

$$\langle \mu^2(\mathbf{x}') \rangle = \int \frac{d^2k'}{(2\pi)^2} S(\mathbf{k}'; \mathbf{x}') \equiv \frac{k_n^2}{(2\pi)^2} S(\mathbf{k}' \rightarrow 0; \mathbf{x}'),$$

$$k_n^2 \equiv \int d^2k' \frac{S(\mathbf{k}'; \mathbf{x}')}{S(\mathbf{k}' \rightarrow 0; \mathbf{x}')$$

We obtain the scattering cross section density from the scattered wave intensity and use it to construct the mean free path,

$$\frac{1}{l(\mathbf{x}')} \equiv \int d\theta \sigma(\mathbf{x}'; \theta),$$

which may be expressed in terms of the wavenumber spread k_n of the density turbulence as follows:

$$\frac{\lambda_o}{l} = \frac{\pi^2}{2} \left[1 + \left(\frac{\omega}{\Omega_i} \right)^2 \right] \langle \mu^2 \rangle \left(\frac{k_o}{k_n} \right)^2$$

Using Ritz *et al.*² data from TEXT we obtain the following:

$k_o \approx 0.1 \text{ cm}^{-1}$,	at the plasma edge
$k_n \approx 6 \text{ cm}^{-1}$,	spread in turbulent wavenumbers
$\mu \approx 0.2$,	turbulence strength at the plasma edge
$\omega/\Omega_i = 2$,	second harmonic heating
$\lambda_o/l \approx 10^{-4}$,	from above equation
$l \approx 1 \text{ km}$,	mean free path

CONCLUSION

Two dimensional cold-fluid theory predicts a mean free path large compared to the size of tokamak devices. Scattering of incident ICRF magnetosonic waves by turbulent density fluctuations is utterly negligible.

REFERENCES

1. A. Ishimaru, Wave Propagation and Scattering in Random Media (Academic Press, N.Y., 1978), p. 329.
2. C.P. Ritz *et al.*, Nucl. Fusion 27, 1125 (1987).

LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
1 CYCLOTRON ROAD
BERKELEY, CALIFORNIA 94720