UC Davis

UC Davis Previously Published Works

Title

Capital controls and trade policy

Permalink

https://escholarship.org/uc/item/2r38n8q4

Authors

Lloyd, Simon P Marin, Emile A

Publication Date

2024-09-01

DOI

10.1016/j.jinteco.2024.103965

Peer reviewed

Capital Controls and Trade Policy*

Simon P. Lloyd[†]

Emile A. Marin[‡]

June 14, 2024

Abstract

How does optimal capital-flow management change with prevailing trade policies? We study the joint optimal determination of capital controls and trade tariffs in a two-country, two-good model with trade in goods and assets. Because countries are large in both markets, a country-planner can achieve higher domestic welfare by departing from free trade in addition to levying capital controls, despite the cooperative optimal allocation being efficient. However, time variation in the optimal tariff induces households to over- or under-borrow through its effects on the path of the real exchange rate. As a result, optimal capital controls are generally smaller when trade policy is constrained (i.e., by a Free-Trade Agreement), but, absent retaliation, can be larger depending on the paths of underlying fundamentals.

Key Words: Capital-Flow Management; Free-Trade Agreements; Ramsey Policy; Tariffs; Trade Policy.

JEL Codes: F13, F32, F33, F38.

^{*}Previously circulated and presented with the title: Capital Controls and Free-Trade Agreements. We are especially grateful to Giancarlo Corsetti for many helpful discussions. We also thank Laura Alfaro (discussant), Pol Antràs, Gianluca Benigno, Paul Bergin, Charles Brendon, Tiago Cavalcanti, Luca Dedola, Rob Feenstra, Rebecca Freeman, Pierre-Olivier Gourinchas, Juan Carlos Hallak (discussant), Chenyue Hu (discussant), Oleg Itskhoki, Dennis Reinhardt, Alan Taylor, Shang-Jin Wei (editor), Robert Zymek, and an anonymous referee as well as presentation attendees at the University of Cambridge, Bank of England, Money, Macro and Finance Annual Conference 2021, Royal Economic Society Annual Conference 2021, European Economic Association Annual Conference 2022, CRETE 2022, the 2022 London Junior Macro Workshop, the V Spanish Macroeconomics Network Conference, the Global Research Forum on International Macroeconomics and Finance (FRB New York), the NBER Conference on International Fragmentation, Supply Chains and Financial Frictions (Banco Central de Chile), the 9th Annual West Coast Workshop in International Finance, and the European University Institute for useful comments. Any views expressed are solely those of the authors and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee. Marin acknowledges support from the Janeway Institute at the University of Cambridge.

[†]Bank of England and Centre for Macroeconomics. Email Address: simon.lloyd@bankofengland.co.uk.

[‡]University of California, Davis. Email Address: emarin@ucdavis.edu.

1 Introduction

Trade and capital-flow management have long been key topics of macroeconomic policy and have, once more, come into sharp focus. Following at least two decades of integration (Baier and Bergstrand, 2007), the process of trade liberalization has stalled, with events like the US-China trade war contributing to substantially heightened uncertainty around world trade (Ahir, Bloom, and Furceri, 2022), a decline in the number of new regional-trade agreements and a deceleration of global value chain integration (see D'Aguanno, Davies, Dogan, Freeman, Lloyd, Reinhardt, Sajedi, and Zymek, 2021, for further discussion). Alongside this, financial liberalization too has slowed and the International Monetary Fund has partially revised their 'institutional view' to emphasize a role for managing capital flows in specific circumstances (Ghosh, Ostry, and Qureshi, 2016) opening the way for 'macroprudential' foreign-exchange interventions targeting cross-border flows (e.g., Ahnert, Forbes, Friedrich, and Reinhardt, 2020). However, despite discussions on trade and capital-flow policies both growing in prominence, academics and policymakers typically consider these measures separately. Trade-policy discussions often balance economic forces (e.g., monopoly power, comparative advantage) with political factors (e.g., de-industrialization, trade sanctions), while recent debates about capital controls have centred on their role in insulating countries from large and volatile cross-border flows.

In this paper, we provide a unifying framework to study the joint optimal determination of trade policy and capital-flow management, in a model where both instruments are driven by a common motive: to exploit a country's monopoly power in markets, often referred to as terms-of-trade manipulation. We show that active trade policy influences optimal capital-flow management through its effects on the path for the real exchange rate and, in turn, private incentives to borrow in international financial markets. This mechanism is closely related to the Harberger-Laursen-Metzler effect (Harberger, 1950; Laursen and Metzler, 1950), who argued that an appreciation of the terms of trade can improve savings in the economy, and is still prominent in policy discussions (see Eichengreen, 2019). We then extend our framework to allow for strategic interactions between countries and we assess the implications of international trade and financial arrangements for global welfare and the likelihood that trade and capital-control wars emerge.

The starting point for our analysis is a canonical two-country, two-good endowment economy model, absent nominal or financial frictions. Households make an inter-temporal consumption-savings decision and intra-temporally choose their optimal consumption bundle. In the laissez-faire or decentralized allocation, relative consumption growth across countries is proportional to the relative decrease in price levels—i.e., the rate of real exchange rate depreciation—a relationship often referred to as the Backus and Smith (1993) condition (see also Kollmann, 1995). However, households do not internalize the effect of their actions on relative prices. These pecuniary externalities, described in Geanakoplos and Polemarchakis (1986), imply that a country planner can improve on domestic welfare by manipulating the inter- and intra-temporal terms of trade—i.e., world interest rates and relative goods prices, respectively—even though the laissez-faire allocation is optimal from a global perspective. Within this setup, Costinot,

Lorenzoni, and Werning (2014), building on Obstfeld and Rogoff (1996), consider optimal capital controls which trade off the incentive to drive down the world interest rate with second-best effects on relative goods prices.¹ However, these papers rule out trade policy through an implicit or explicit Free-Trade Agreement (FTA).

Our key contribution is to relax the trade-policy constraint imposed on the planner and assess the interactions between optimal capital controls and trade tariffs within a tractable environment, using the primal approach of Lucas and Stokey (1983). We begin by analyzing the problem of a country-planner acting unilaterally to maximize domestic welfare, without retaliation from abroad. Our headline finding is that optimal capital controls are generally smaller when trade policy is constrained (i.e., by a FTA), but can be larger depending on the paths of underlying fundamentals which governs the alignment of inter- and intra- temporal incentives to manipulate the terms of trade. When the Foreign planner is allowed to retaliate, capital controls are be smaller under a FTA, regardless of the underlying path for fundamentals, due to strategic interactions across instruments.

Consider a scenario in which domestic households borrow between two periods, driven by a temporarily low endowment of the good consumed with home bias (the 'domestic good'). The planner will seek to delay aggregate consumption inter-temporally by taxing capital inflows, but also has an intra-temporal incentive to reduce consumption of the relatively expensive domestic good by levying a temporary subsidy on the second good ('foreign good'). This puts pressure on the real exchange rate to depreciate which, all else equal, results in an adjustment in optimal capital controls (see, e.g., Jeanne and Son, 2023). Intuitively, the exchange-rate depreciation encourages 'over-borrowing' by households because their domestic consumption bundle becomes cheaper today relative to the future. Therefore, a larger capital-flow tax is required to induce a constrained-efficient path for consumption. Quantitatively, this interaction between instruments can be significant; in our baseline calibration, capital controls are one-third larger when the tariff is optimally chosen.

Fluctuations in the foreign good, consumed without home bias, involve a different mix of inter- and intra-temporal incentives, which may better capture the trade-offs faced by exporting nations. When the domestic endowment of the foreign good is temporarily low, incentives to delay aggregate consumption are the same, but the optimal unilateral tariff puts pressure on the real exchange rate to appreciate. Absent further action, this incentivizes under-borrowing and therefore a smaller capital inflow tax is required.

We provide intuitive expressions for the optimal instruments based on foreign export-supply elasticities. We show that the optimal capital-inflow tax depends on the sum of trade deficits for each good, across subsequent periods, weighted by the corresponding inverse elasticity of foreign export supply—echoing findings in the optimal taxation literature (e.g., Atkinson and Stiglitz, 1980; Chari and Kehoe, 1999), where the planner taxes inelastic commodities more. Regardless

¹When domestic households borrow between two periods, the planner tends to levy capital-inflow taxes to delay consumption relative to the decentralized allocation. Capital controls introduce a wedge to the Backus-Smith condition, so consumption growth can be slower than the rate of exchange-rate depreciation. This wedge can also be understood as a measure of exchange-rate misalignment induced by the planner at the optimal allocation (e.g., Corsetti, Dedola, and Leduc, 2023).

of trade policy, the planner taxes goods more heavily when the foreign export-supply elasticity is low—since these goods will be relatively more expensive at the margin. When trade policy is optimally set, tariffs correct for the relative weighted deficits across goods varieties. Given this, capital controls depend on the deficits weighted only by the *partial* elasticity with respect to the untaxed good (i.e., the domestic good). This distinction underlies the interactions between capital controls and trade policy in our framework.

In the limiting case of unitary elasticities of inter- and intra-temporal substitution, studied in Cole and Obstfeld (1991) (henceforth 'CO'), the planner uses capital controls only in response to variation in the domestic-good endowment. In response to variation in the endowment of the foreign good, the planner only uses tariffs when trade policy can be set optimally. In contrast, when trade policy is constrained (i.e., by a FTA), the planner uses both instruments in response to variation in the endowment of either good. So, in this knife-edge case, trade policy fully substitutes for the use of capital controls.

Our analysis applies to more general environments, including where capital-flow management is driven by a demand-management motive (via an aggregate-demand externality) and where foreign-exchange interventions (FXI) are effective. When there are nominal rigidities, the planner faces an additional incentive to bring forward (delay) consumption when the economy is demand-constrained (overheating), as in Farhi and Werning (2016). Generally, within our framework, the planner targets an optimal wedge for the Backus-Smith condition, and an optimal relative-demand wedge which are interrelated in equilibrium. In an extension that allows for segmented financial markets, as in Gabaix and Maggiori (2015), we show that FXI can be used to target the same wedge as capital controls and interact similarly with trade policy.² Policy incentives and interactions also persist in small-open economies where an individual country's ability to manipulate the world interest rate disappears (although we show this is not the case with financial segmentation). In this case, in the CO limit, when inter- and intra-temporal incentives are aligned, the optimal capital-inflow tax is invariant to the size of the economy because the tax needed to address the inter-temporal margin exactly coincides with that required to address intra-temporal incentives.

Returning to our baseline model, we analyze a strategic setting with retaliation, where both country planners set policy as a mutual best response—specifically, an open-loop Nash equilibrium. Policy wars follow the same 'inverse elasticity rule'. The total wedge introduced by capital controls in the Backus-Smith condition is larger when the elasticity of inter-temporal substitution is low, whereas the total wedge in relative demand introduced by tariffs is larger when the intra-temporal elasticity of substitution between goods is low. In the Nash equilibrium, we find that capital controls tend to be smaller when trade policy is constrained by a FTA, in response to fluctuations in either goods' endowment. Interestingly, this arises because capital controls in the Home country respond to the distortion in the path for exchange rates due to tariffs levied by the Foreign planner whose effect dominates.

²Our insights on the interaction of capital-flow management and policy interventions will also apply to any policies that do not directly induce a wedge in the condition equalizing marginal-utility growth across countries, the Backus-Smith condition (e.g., monetary policy, fiscal policy).

Finally, within the strategic framework, we show that when countries optimally choose tariffs, the incentives to depart from a 'free-financial-flows agreement' (FFFA) and engage in welfare-costly capital-control wars are heightened, providing a novel argument in favor of free trade. Constraints on trade policy, like a FTA, reduce incentives for an individual country to levy capital controls (potentially either subsidies or taxes), which could prompt retaliation, because tariffs distort the path of real exchange rates over time. In short: retaining openness in trade can help to sustain financial openness.

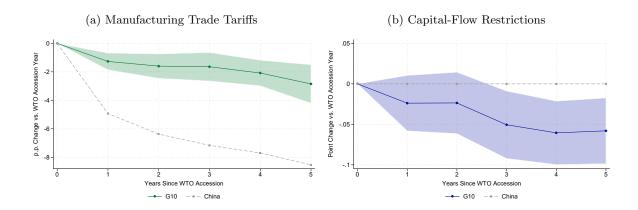
Policy in Practice. Although our analysis is theoretically grounded, our results may speak to observed policy patterns. Since being set up in 1995, advanced G10 economies acceding to the World Trade Organization (WTO) have seen average tariff rates for manufactured goods decline in the years after accession (Figure 1a), even when controlling for country- or time-specific factors. At the same time, these economies have seen their average capital-flow restrictions (measured using IMF data from Fernández, Klein, Rebucci, Schindler, and Uribe, 2016) decline relative to non-WTO members (Figure 1b). These findings are consistent with the results from our strategic framework, in which constraints on trade policy reduce incentives to levy capital-flow restrictions, and are indicative of some complementarity between trade and financial liberalization in these countries.

However, these trends are less clear-cut for developing economies acceding to the WTO, where some have seen lower tariffs alongside *tighter* capital-inflow restrictions relative to non-WTO members. This suggests that for some countries, trade and financial liberalization have been more substitutable—potentially depending on the size and income profiles of countries, a factor which our model sheds light on. In the case of China, where average tariffs fell by around 8pp in the 5 years after acceding to the WTO in 2001, there is some evidence from the People's Bank of China that restrictions on capital flows were loosened (Bank for International Settlements, 2008)—even if headline capital-inflow statistics in Figure 1b do not pick them up.

Related Literature. Our work builds on Costinot et al. (2014) who study the role of capital controls as a means of dynamic terms-of-trade manipulation in large-open endowment economies. We depart from their assumption of free trade and, in doing so, our work combines analyses of inter-temporal incentives to manipulate the terms of trade via the use of capital controls (see e.g. Rebucci and Ma, 2020; Bianchi and Lorenzoni, 2022), with intra-temporal incentives, for which tariffs are regularly applied in practice (Broda, Limao, and Weinstein, 2008). Like us, Ju, Shi, and Wei (2013) analyze the relationship between inter- and intra-temporal trade but within a Heckscher-Ohlin model. While Jeanne (2012) and Farhi, Gopinath, and Itskhoki (2014) analyze the effects of tariffs on real exchange rates. We show that trade policy itself can give rise to incentives to levy capital controls, through their impact on real exchange rates and, in turn, incentives to over-/under-borrow.

Our analysis also contributes to the broader literature on capital controls. Notably, we show the mechanisms underpinning the interaction between optimal trade and financial policy persist in settings with aggregate-demand externalities in models with nominal rigidities (see,

Figure 1: Average Manufacturing Trade Tariffs and Capital-Flow Restrictiveness for G10 Economies Around Countries' WTO Accession



Notes: Estimated change in manufacturing tariffs (Panel a) and capital-flow restrictiveness (Panel b) in the years following accession to WTO. Manufacturing tariffs measured in % for 1990-2019 (annual frequency) from World Development Indicators (World Bank). Metric used here uses weighted mean applied tariff for manufacturing products, which measures the average of effectively applied tariffs weighted by the product import shares corresponding to each partner country. Capital-flow restrictions are a 0-1 index for 1995-2019 (Fernández et al., 2016). Estimates for G10 attained from regression of h-year policy change $y_{i,t+h} - y_{i,t-1}$, in country i at year t, on a WTO accession dummy ($WTO_{i,t} = 1$ in year of accession, 0 otherwise) and country and time fixed effects (f_i and f_t). Shaded area shows 90% confidence bands implied by standard errors, which are clustered by country. Line for China plots raw changes in tariffs and capital-flow restrictions following WTO accession in 2001.

e.g., Farhi and Werning, 2014, 2016; Schmitt-Grohé and Uribe, 2016; Marin, 2022). In addition, while we focus on a deterministic setting, we show that anticipated shocks engender preemptive policy interventions, akin to the precautionary motives driving capital controls in (see, e.g., Mendoza, 2002; Bianchi, 2011). We also draw parallels to a literature on FXI. Fanelli and Straub (2021) show that FXI and capital controls are isomorphic when there is partial segmentation in international markets, up to implementation costs. They share our focus on an endowment economy and pecuniary externalities, but, unlike us, abstract from trade policy.

The literature on trade tariffs has predominantly focused on environments with no trade in assets, albeit with a richer supply-side setup with monopolistic (and often heterogeneous) firms (see, e.g., Demidova and Rodriguez-Clare, 2009; Caliendo, Feenstra, Romalis, and Taylor, 2021). We contribute to this literature by evaluating the scope for tariffs as second-best instruments to manipulate the cost of borrowing in a dynamic setting.

Finally, our paper contributes to a growing literature assessing the joint role of trade and stabilization policies. Bergin and Corsetti (2023) study the response of monetary policy to tariff shocks. Auray, Devereux, and Eyquem (2020) study the scope for trade wars and currency wars in a New-Keynesian small-open economy model but their model features balanced trade, so there is no scope for capital controls. Jeanne (2021) studies monetary policy and the accumulation of foreign reserves, emphasizing the distinction between a demand-constrained 'Keynesian regime' and a 'classical regime' where tariffs are used to manipulate the terms of trade.

Outline. The remainder of the paper is structured as follows. Section 2 describes the model environment. Section 3 characterizes the optimal unilateral planning allocation. Section 4 discusses policy implementation, macroeconomic outcomes and model generalizations. Section 5 studies strategic cross-country interactions. Section 6 considers welfare. Section 7 concludes.

2 Basic Environment

There are two countries, Home H and Foreign F, each populated by a continuum of identical households. Time is discrete and infinite, t = 0, 1, ..., and there is no uncertainty. The preferences of the representative Home consumer are denoted by the time-separable utility function:

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t)$$

where C_t is aggregate Home consumption and u(C) is a twice continuously differentiable, strictly increasing and strictly concave function with $\lim_{C\to 0} u'(C) = \infty$. $\beta \in (0,1)$ is the discount factor. The preferences of the representative Foreign consumer are analogous, with asterisks denoting Foreign variables.

Consumers derive utility from two goods, good 1 and good 2. We denote the representative Home consumer's consumption of good 1 and good 2 by $c_{1,t}$ and $c_{2,t}$, respectively, and group them into the vector $\mathbf{c}_t = [c_{1,t} \ c_{2,t}]'$. Home aggregate consumption is defined by the aggregator $C_t \equiv g(\mathbf{c}_t)$, where $g(\cdot)$ is a function that is twice continuously differentiable, strictly increasing, concave and homogeneous of degree one. We define the Jacobian of $g(\mathbf{c}_t)$ by $\nabla g(\mathbf{c}_t) = [g_{1,t} \ g_{2,t}]'$, where $g_{i,t} = \frac{\partial g(\mathbf{c}_t)}{\partial c_{i,t}}$ for i = 1, 2, while second derivatives are written as $g_{ij,t} = \frac{\partial^2 g(\mathbf{c}_t)}{\partial c_{i,t}\partial c_{j,t}}$ for i, j = 1, 2. The aggregator for the representative Foreign consumer is written as $C_t^* \equiv g^*(\mathbf{c}_t^*)$, with analogously defined derivatives.

The Home (Foreign) consumer's period-t endowments of goods 1 and 2 are denoted by $y_{1,t}$ ($y_{1,t}^*$) and $y_{2,t}$ ($y_{2,t}^*$), respectively, and are weakly positive in all periods. Throughout, without loss of generality, we assume that Home consumers have a 'home bias' for good 1, and we therefore describe this as the 'domestic good'. Defining the time-t Home expenditure share on domestic goods as α_t , then 'home bias' implies $\alpha_t > 0.5$. Likewise, Foreign consumers prefer good 2 (the 'foreign good') and we assume $\alpha_t^* = \alpha_t$. The total world endowment of goods 1 and 2 are $Y_{1,t} \equiv y_{1,t} + y_{1,t}^*$ and $Y_{2,t} \equiv y_{2,t} + y_{2,t}^*$, respectively.

The inter-temporal budget constraint for the Home household expressed as:

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t - \mathbf{y}_t) \le 0 \tag{1}$$

where $\mathbf{p}_t = [p_{1,t} \ p_{2,t}]'$ denotes the vector of period-t world goods prices and $\mathbf{y}_t = [y_{1,t} \ y_{2,t}]'$ is the vector of Home endowments.

We define two additional quantities. First, the terms of trade is given by $S_t = p_{2,t}/p_{1,t}$ and, since good 1 is the 'domestic good' and good 2 the 'foreign good', we refer to an increase in S_t

as a deterioration of the Home terms of trade. Second, the real exchange rate is given by the ratio of consumer price indices $Q_t = P_t^*/P_t$, where $P_t^{(*)} \equiv \min_{\mathbf{c}_t^{(*)}} \{\mathbf{p}_t \cdot \mathbf{c}_t^{(*)} : g^{(*)}(\mathbf{c}_t^{(*)}) \geq 1\}$. An increase in Q_t corresponds to a depreciation of the Home real exchange rate.

Specific Functional Forms. For our numerical exercises, we use a constant relative risk aversion (CRRA) specification for per-period utility $u(C) \equiv \frac{C^{1-\sigma}-1}{1-\sigma}$, where $\sigma > 0$ denotes the coefficient of relative risk aversion. The aggregate consumption of the representative agent is given by the Armington (1969) aggregator:

$$C_t \equiv g(\mathbf{c}_t) = \left[\alpha^{\frac{1}{\phi}} c_{1,t}^{\frac{\phi-1}{\phi}} + (1-\alpha)^{\frac{1}{\phi}} c_{2,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$
(2)

where $\phi > 0$ is the elasticity of substitution between good 1 and 2. With this specification for preferences, expenditure shares are constant: $\alpha_t^{(*)} = \alpha^{(*)}$ for all t.

An interesting special case arises when $\sigma = \phi = 1$. This corresponds to the parametrization studied in Cole and Obstfeld (1991), the 'CO case', which we revisit in Section 4.3.

3 Unilateral Planning Allocation

We begin by considering an equilibrium in which the Home planner maximizes domestic welfare, while the Foreign planner is passive—i.e., does not levy taxes in response to Home policy. The optimality conditions for the representative Foreign household act as a constraint for the Home planner and are given by:

$$\beta^t u^{*\prime}(C_t^*) \nabla g^*(\mathbf{c}_t^*) = \lambda^* \mathbf{p}_t \tag{3}$$

$$\sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) = 0, \tag{4}$$

where λ^* denotes the Lagrange multiplier on the Foreign inter-temporal budget constraint.

The Home planner maximizes the discounted lifetime utility of the Home representative consumer by choosing consumption of each individual good variety $\{c_{1,t}, c_{2,t}\}_{t\geq 0}$, subject to: (i) the representative Foreign consumer's utility maximization at world prices; (ii) market clearing in each period; and (iii) any constraints imposed on trade policy. Conditions (i) and (ii) can be summarized independently of (iii), in a single implementability condition (Lucas and Stokey, 1983) described in the following lemma.

Lemma 1 (Implementability for Unilateral Planner) When the Foreign country is passive, an allocation $\{\mathbf{c}_t, \mathbf{c}_t^*\}$, together with world prices \mathbf{p}_t , form part of an equilibrium if they satisfy:

$$\sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0, \tag{IC}$$

where $\rho(C_t) \equiv u^{*\prime}(C^*(C_t))\nabla g^*(\mathbf{c}_t^*(\mathbf{c}_t))$ denotes the price of consumption at each t.

Absent constraints on trade policy, the Home planner's problem is:

$$\max_{\{c_{1,t},c_{2,t}\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t)$$
 (P-Unil-nFTA)

s.t.
$$\sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] = 0$$
 (IC)

$$C_t = g(\mathbf{c}_t)$$
 (nFTA)

where the third line (nFTA) reflects that aggregate consumption C_t can be backed out of the consumption aggregator $g(\mathbf{c}_t)$. Following Costinot et al. (2014), we assume that $\rho(g(\mathbf{c}_t))$ · $[\mathbf{c}_t - \mathbf{y}_t]$ is strictly convex to guarantee a unique solution.

3.1 Optimal Allocation

The Lagrangean associated with the planning problem is:

$$\max_{\{c_{1,t},c_{2,t}\}} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(C_t) - \mu \left\{ \sum_{t=0}^{\infty} \beta^t \boldsymbol{\rho}(C_t) \cdot [\mathbf{c}_t - \mathbf{y}_t] \right\}$$
 (5)

The first-order conditions—with respect to $c_{1,t}$ and $c_{2,t}$, respectively—are given by:

$$u'(C_t)g_{1,t} = \mu \mathcal{M}C_{1,t}^{nFTA} \tag{6}$$

$$u'(C_t)g_{2,t} = \mu \mathcal{M}C_{2,t}^{nFTA} \tag{7}$$

where μ denotes the Lagrange multiplier on the implementability constraint and $\mathcal{MC}_{i,t}^{nFTA}$ denotes the marginal cost associated with giving up a unit of good i = 1, 2:

$$\mathcal{MC}_{1,t}^{nFTA} \equiv u^{*\prime}(C_t^*)g_1^*(\mathbf{c}_t) + u^{*\prime\prime}(C_t^*)g_1^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

$$+ u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

$$\mathcal{MC}_{2,t}^{nFTA} \equiv u^{*\prime}(C_t^*)g_2^*(\mathbf{c}_t) + u^{*\prime\prime}(C_t^*)g_2^*(\mathbf{c}_t^*)\nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

$$+ u^{*\prime}(C_t^*)\frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}} \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

Equations (6) and (7) define the consumption allocations chosen by the planner. They equate the marginal benefit from a unit of good-specific consumption to its marginal cost for goods 1 and 2, respectively, for the representative Home consumer. Consider equation (6). The first term in $\mathcal{MC}_{1,t}^{nFTA}$ reflects the price for a Home household purchasing one unit of good 1. If this were the only term on the right-hand side, the planning allocation would coincide with the decentralized (or laissez-faire) allocation.

However, each additional unit purchased leads to infra-marginal changes in prices which

households do not take into account—leading to over-borrowing. In contrast, the planer internalizes how prices vary with consumption allocations, akin to a monopolist. The second term in $\mathcal{MC}_{1,t}^{nFTA}$ reflects the inter-temporal margin—how the cost of borrowing changes with aggregate consumption. The final term, captures the intra-temporal margin—how each good-specific price changes with an additional unit of good 1 consumption c_1 .

3.2 How Does the Optimal Allocation Vary with Trade Policy?

When trade policy is constrained, constraint (nFTA) is replaced, and the equilibrium allocation can vary. Within their two-good environment, Costinot et al. (2014) study a setting in which trade policy is constrained by a FTA. In this case, consumption allocations are Pareto efficient (from an individual-household perspective) and can be summarized by:

$$C^*(C_t) = \max_{\mathbf{c}_t, \mathbf{c}_t^*} \{ g^*(\mathbf{c}_t^*) \quad \text{s.t.} \quad \mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t \text{ and } g(\mathbf{c}_t) \ge C_t \}$$
 (8)

for some C_t , where $\mathbf{Y}_t = [Y_{1,t} \ Y_{2,t}]'$. The Pareto frontier defines efficient combinations of consumption $\{c_{1,t}, c_{2,t}\}$ for a given level of aggregate consumption C_t , which coincides with the contract curve when there are no goods-specific taxes. The Home and Foreign Pareto frontiers are defined as $\mathbf{c}(C)$ and $\mathbf{c}^*(C^*)$, which reflect individual households' optimization of consumption bundles given an aggregate consumption C and are reported in Supplementary Materials S.1.2. Therefore, the planner is restricted to choosing a sequence for aggregate consumption $\{C_t\}_{t\geq 0}$, as opposed to consumption varieties $\{c_{1,t}, c_{2,t}\}_{t\geq 0}$. We denote the planning problem when trade policy is constrained by the FTA in (8) by (P-Unil-FTA), reported in Supplementary Materials S.2.1.

In a two-country model, like the one we study, (P-Unil-FTA) amounts to a setting where tariffs are ruled out. In a more general model, however, a FTA member could have zero tariffs with countries participating in the FTA, but non-zero tariffs $vis-\acute{a}-vis$ other countries. Henceforth, for presentational ease, we refer to quantities that result from the allocation in which trade policy is constrained by (8) with the superscript FTA, and the case in which trade policy is unconstrained and set optimally with nFTA. Proposition 1 illustrates the relation between the planner's problem with and without constraints on trade policy.

Proposition 1 (Optimal Unilateral Allocations and Trade Policy) The planners' optimal allocations when trade policy is constrained by a FTA, as defined in (8), and without trade policy restrictions are related by the following condition:

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = \frac{\partial \mathcal{L}}{\partial c_1} c_1'(C) + \frac{\partial \mathcal{L}}{\partial c_2} c_2'(C)$$

The solution to (P-Unil-FTA) is defined by $\frac{d\mathcal{L}}{dC} = 0$, whereas the solution to (P-Unil-nFTA) is defined by $\frac{d\mathcal{L}}{dc_1} = \frac{d\mathcal{L}}{dc_2} = 0$.

Proof: The Pareto Frontier (8) implies $c'_1(C), c'_2(C)$ are non-zero, therefore the maximization

0.9

-■ Constrained Trade Policy
- Optimal Trade Policy

0.7

5 0.6

0.5

0.4

0.3

Figure 2: Optimal Allocations and the Pareto Frontier

Notes: Plot of optimal consumption allocations for Home consumer from capital flow taxation (i) with a constraint on trade policy from a FTA (blue circles, i.e., the Pareto frontier) and (ii) when trade policy is set optimally (green crosses) at different Home endowments. We use nine equally-spaced allocations for $y_1 \in [\alpha - 0.25, \alpha + 0.25]$, with $y_1^* = 1 - y_1$, $y_2 = 1 - \alpha$ and $y_2^* = \alpha$. Other model parameters are: $\beta = 0.96$, $\sigma = 2$, $\phi = 1.5$, and $\alpha = 0.6$. Grey/black lines denote loci of $\{c_1, c_2\}$ which attain different levels of aggregate consumption (black for C = 1, grey otherwise). Horizontal (vertical) dotted lines denote $\alpha (1 - \alpha)$, and intersect at the 'no-trade' point.

 c_2

0.5

0.6

0.7

0.8

0.4

0.2

0.1

0.3

Proposition 1 shows that when trade policy is constrained by a FTA, the unilateral planning allocation is constrained optimal. In particular, since $c_1'(C)$ and $c_2'(C)$ are positive and increasing functions, then it must be that, in general, $sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1}) = -sign(\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2}) \neq 0$ so there is an incentive to adjust consumption across varieties. In contrast, when trade policy is optimally chosen, the planner sets $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1} = \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2} = 0$, resulting in $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}C} = 0$.

Figure 2 illustrates these observations. The blue line maps the Pareto frontier: the efficient combinations of $\{c_1(C), c_2(C)\}$ for different levels of long-run aggregate consumption C, consistent with (8). When tariffs are set optimally, the planner achieves a (weakly) higher level of consumption. For $y_1 > \alpha$ —the area above the black line, where good 1 is abundant—the allocation absent a FTA is more biased towards c_1 . Whereas for $y_1 < \alpha$ —the area below the black line, where good 1 is scarce—the allocation is more biased towards c_2 . The constrained and unconstrained allocations only coincide in the case $y_1 = y_2^* = \alpha$.

4 Policy and Macro Outcomes at the Optimal Allocation

In this section, we describe the implementation of the optimal allocation and highlight how policy instruments interact. We then contrast the macroeconomic dynamics at the planning allocation, with and without free trade, to the decentralized case.

4.1 Implementation

We consider an implementation where policy instruments map directly to wedges in the Euler and relative goods demand equations. We assume households can trade in non-contingent bonds, denominated in each good variety. The Home planner can impose the same proportional tax θ_t on the gross returns to net lending in all bond markets. So, the per-period budget constraint for the Home consumer can be written as:

$$\mathbf{p}_{t+1} \cdot \mathbf{a}_{t+1} + \tilde{\mathbf{p}}_t \cdot \mathbf{c}_t = \mathbf{p}_t \cdot \mathbf{y}_t + (1 - \theta_{t-1}) (\mathbf{p}_t \cdot \mathbf{a}_t) + T_t$$

where $\tilde{\mathbf{p}}_t = \mathbf{p}_t$ when the FTA constraint is in place, \mathbf{a}_t denotes the vector of asset positions and T_t is a lump-sum rebate. Given a no-Ponzi condition, $\lim_{t\to\infty} \tilde{\mathbf{p}}_t \cdot \mathbf{a}_t \geq 0$, the first-order conditions associated with Home households' utility maximization are given by:

$$u'(C_t)g_i(\mathbf{c}_t) = \beta(1 - \theta_t)(1 + r_{i,t})u'(C_{t+1})g_i(\mathbf{c}_{t+1})$$
(9)

for i = 1, 2, where $r_{i,t} \equiv \frac{p_{i,t}}{p_{i,t+1}} - 1$ is a good-specific interest rate. Combining this with the analogous Foreign Euler equation, and using $g_{i,t}/p_{i,t} = 1/P_t$, yields the Backus and Smith (1993) condition with a wedge reflecting capital-flow taxation:

$$(1 - \theta_t) = \frac{u'(C_t)}{u'(C_{t+1})} \frac{u^{*'}(C_{t+1}^*)}{u^{*'}(C_t)} \frac{Q_t}{Q_{t+1}}$$
(10)

A tax on capital inflows (or a subsidy for outflows) is captured by $\theta_t < 0$, which can also be interpreted as a tax on current consumption relative to future consumption.

Without constraints on trade policy, the Home planner can additionally levy a proportional tax τ_t on good 2, which we refer to as a tariff, so that $\tilde{\mathbf{p}}_t = \boldsymbol{\tau}_t \cdot \mathbf{p}_t$ where $\boldsymbol{\tau}_t = [1 \ \tau_t]'$. A negative value denotes a subsidy $(\tau_t < 0)$ and is equivalent to taxing good 1 instead. The representative Home household faces a distorted price for good 2, $p_{2,t}(1+\tau_t)$, so their relative demand is given by:

$$\frac{c_{1,t}}{c_{2,t}} = \frac{\alpha}{1-\alpha} \left(\frac{1}{S_t(1+\tau_t)}\right)^{-\phi} \tag{11}$$

Alternative Instruments. While we focus on an implementation using capital controls and tariffs, the policy problem solves for the optimal wedges in equations (10) and (11). So, consistent with the public finance literature (see, e.g., Chari and Kehoe, 1999), any instruments which map to these wedges can be used. In Section 4.6, we detail an extension of the model with segmented markets and consider FXI (see, e.g., Fanelli and Straub, 2021; Bianchi and

Lorenzoni, 2022). Similarly, time-variation in the optimal relative-demand wedge may reflect manipulation of non-tariff barriers or regulation, evidenced in Broda et al. (2008).³

4.2 Interactions Between Optimal Trade and Financial Policy

To build intuition on the interactions between the capital-flow taxes and tariffs, we decompose the log of equation (10), imposing CRRA preferences, into the following two wedges:

$$\ln(1 - \theta_t) \approx -\theta_t = \underbrace{-\sigma \left(\hat{C}_t - \hat{C}_{t+1} + \hat{C}_{t+1}^* - \hat{C}_t^*\right)}_{\text{Consumption Wedge}} + \underbrace{\left(\hat{Q}_t - \hat{Q}_{t+1}\right)}_{\text{RER Wedge}}$$
(12)

where \hat{x} denotes the natural logarithm of x. The 'consumption wedge' component captures the target relative consumption growth for the planner. The 'RER wedge' reflects capital-flow taxation incentives related to the evolution of the real exchange rate Q. A higher RER wedge (corresponding to a depreciated exchange rate) implies that a larger capital-inflow tax is required to implement a given consumption allocation—so it induces over-borrowing.

Faced with a higher stream of endowments in the future, Home households will borrow to smooth consumption. However, each additional unit of consumption brought forward raises the cost of borrowing (i.e., inter-temporal margin)—so households generally over-borrow because they do not take into account the effect of their choices on prices. Then, consumption growth is too low relative to foreign, so the planner can correct this by levying capital inflow taxes $\theta_t < 0$.

The interaction between optimal capital-flow management and trade policy hinges on which good is relatively scarce. If $y_{1,t} < y_{1,t+1}$, good 1 is scarce. Then, the Home household additionally buys relatively more units of good 1 from abroad, at a time when it is relatively more expensive to do so. The optimal tariff on good 2 is negative (i.e., a subsidy) which will depreciate the real exchange rate, raising the RER wedge, and (12) implies a larger capital inflow tax is required. Inter- and intra-temporal manipulation incentives are aligned for the planner, and capital controls are larger absent a FTA. If instead $y_{2,t} < y_{2,t+1}$, good 2 is relatively scarce. In this case, the optimal tariff is positive which will appreciate the real exchange rate, lowering the RER wedge. Inter- and intra-temporal incentives are misaligned, and capital controls are smaller absent a FTA.

4.3 Optimal Instruments with Unitary Elasticities of Substitution

We first focus on the CO limit, defined by $\sigma \to 1, \phi \to 1$. We present results for general preferences in the next sub-section. In the CO limit, we can provide a sharp characterization of households' consumption behaviour, summarized in the following Lemma.

Lemma 2 (Cyclicality of Consumption in the CO Limit) In the limit as $\sigma \to 1$ and $\phi \to 1$, when there is no aggregate income variation $(Y_{i,t} = \overline{Y}_i \text{ for } i = 1, 2 \text{ and for all } t)$,

³In addition, De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) show that exporters charge variable markups over marginal costs in foreign markets, suggesting that trade agreements do not necessarily constrain relative prices.

consumption and endowments co-vary as follows.

- i. Decentralized Allocation: consumption $C_t^{(*)}$ is invariant to domestic endowments $y_{i,t}^{(*)}$.
- ii. Planning Allocation without trade policy restrictions: consumption of each good is procyclical with respect to each domestic endowment, but independent from other good endowments $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{i,t}} > 0$ and $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{j,t}} = 0$ for i, j = 1, 2 and $i \neq j$.
- iii. Planning Allocation with FTA (8): consumption is procyclical with respect to domestic endowments, $\frac{dC_t}{dy_{i,t}} > 0$ for i = 1, 2.

Proof: See Appendix A.3. \Box

Comparing (i) with (ii) or (iii) shows that households over-borrow in equilibrium. To correct the pecuniary externalities associated with this, the planner (regardless of constraints on trade policy) will choose to lean against capital inflows when income is growing, delaying consumption. The planner's specific response depends on trade-policy constraints. Absent a FTA, a separation arises at the CO limit between the optimal consumption of goods 1 and 2. When the endowment from good i grows (i.e., $y_{i,t} < y_{i,t+1}$), the planner delays consumption in that specific good i (i.e., $c_{i,t} < c_{i,t+1}$) only. However, when trade policy is constrained by a FTA, the planner delays aggregate consumption (i.e., $C_t < C_{t+1}$), if the endowment of either good is growing.

The following Proposition details the instruments chosen in the CO limit.

Proposition 2 (Optimal Instruments in the CO Limit) The optimal instruments for the Home unilateral Ramsey planner without constraints on trade policy, in the limit as $\sigma \to 1$ and $\phi \to 1$, are given by:

$$\theta_t^{nFTA} = 1 - \frac{1 + \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}}}{1 + \frac{c_{1,t+1} - y_{1,t+1}}{\overline{Y}_1 - c_{1,t+1}}}, \qquad \tau_t^{nFTA} = \frac{1 + \frac{c_{2,t} - y_{2,t}}{\overline{Y}_2 - c_{2,t}}}{1 + \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}}} - 1$$

In contrast, when trade policy is constrained by a FTA, as defined by (8):

$$\theta_t^{FTA} = 1 - \frac{1 + \omega_{1,t} \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}} + \omega_{2,t} \frac{c_{2,t} - y_{2,t}}{\overline{Y}_2 - c_{2,t}}}{1 + \omega_{1,t+1} \frac{c_{1,t+1} - y_{1,t+1}}{\overline{Y}_1 - c_{1,t+1}} + \omega_{2,t+1} \frac{c_{2,t+1} - y_{2,t+1}}{\overline{Y}_2 - c_{2,t+1}}}$$

where
$$\omega_{1,t} \equiv (1-\alpha) \frac{c_1^{*'}(C_t^*)}{c_{1,t}^*}$$
 and $\omega_{2,t} \equiv \alpha \frac{c_2^{*'}(C_t^*)}{c_{2,t}^*}$.
Proof: See Appendix A.4.

Consider the case with trade policy constrained by a FTA. Proposition 2 shows that the optimal capital-inflow tax at the CO limit, θ_t^{FTA} , depends on the growth of a weighted average of excess demand for both goods, with weights $\omega_{i,t}$, which depend on the foreign export-supply elasticities. In general, if households are borrowing more at time t than at t+1, $\theta_t^{FTA} < 0$ indicating an inflow tax (or a tax on borrowing at t). Tariffs are constrained to zero.

Absent a FTA, the optimal tariff τ_t^{nFTA} depends on the relative excess demand across goods (i.e., $c_{2,t} - y_{2,t}$ relative to $c_{1,t} - y_{1,t}$) weighted by the inverse elasticity of foreign export supply for each good, given by $(\overline{Y}_i - c_{i,t})^{-1}$. The lower the consumption of good i by Foreign households, the lower the price elasticity and the larger the price change when Home purchases a unit of good i. For example, if there is higher excess demand for good 2, given elasticities, a positive tariff is levied on good 2 so that households consume less of the relatively expensive good.

The corresponding optimal capital-inflow tax θ_t^{nFTA} depends on the growth rate of excess demand for good 1 only (i.e., $c_{1,t} - y_{1,t}$ relative to $c_{1,t+1} - y_{1,t+1}$) weighted by the inverse of the corresponding partial elasticity of foreign export supply. This result relies on the fact that taxes are levied on good 2 only,⁴ and reflects the separation of allocations implied by Lemma 2. If there is higher excess demand for good 1 at time t, a capital-inflow tax is levied $\theta_t^{nFTA} < 0$. A special case for the interaction between the optimal instruments arises in response to variation in the endowment of good 2.

Corollary 1 (Trade Policy as a Substitute for Capital Controls) In the limit as $\sigma \to 1$ and $\phi \to 1$, when $y_{2,t} < y_{2,t+1}$ ($y_{2,t} \ge y_{2,t+1}$), the Home unilateral planner will tax (subsidize) capital inflows $\theta_t^{FTA} \le 0$ ($\theta_t^{FTA} \ge 0$) under a FTA, as defined by (8). Absent a FTA, the capital-inflow tax is zero, $\theta_t^{nFTA} = 0$, and a tariff τ^{nFTA} is used instead.

Through the lens of our decomposition (12), this special case arises because the RER wedge moves to perfectly offset changes in the consumption wedge, so trade policy is a perfect substitute for capital controls. While optimal capital controls will be non-zero under a FTA, they will be zero when this is relaxed—replaced instead with an optimal time-varying tariff. Our numerical simulations in Section 4.5 verify that trade policy can substitute for the use of capital controls for a wider range of $\{\sigma, \phi\}$ —but only in response to good-2 endowment fluctuations, and only absent retaliation from abroad.

4.4 Optimal Instruments in the General Case

Next, we generalize the optimal instrument formulas to arbitrary preferences.

Proposition 3 (Optimal Instruments) The optimal instruments for the Home unilateral Ramsey planner without constraints on trade policy are given by:

$$\theta_t^{nFTA} = 1 - \frac{1 + (\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_{1,t}^G) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 + (\boldsymbol{\epsilon}_{t+1}^T + \boldsymbol{\epsilon}_{1,t+1}^G) \cdot [\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]}, \qquad \tau_t^{nFTA} = \frac{1 + (\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_{2,t}^G) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 + (\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_{1,t}^G) \cdot [\mathbf{c}_t - \mathbf{y}_t]} - 1$$

⁴If taxes were levied on good 1, then capital-inflow taxes would depend only on good-2 excess demand.

where $\boldsymbol{\epsilon}_{t}^{T} = -\frac{u^{*''}(C_{t}^{*})}{u^{*'}(C_{t}^{*})} \nabla g^{*}(\mathbf{c}_{t}^{*})$ and $\boldsymbol{\epsilon}_{1,t}^{G} = -\frac{1}{g_{1,t}^{*}} \frac{\partial \nabla g^{*}(\mathbf{c}_{t}^{*})}{\partial c_{1,t}^{*}}$, $\boldsymbol{\epsilon}_{2,t}^{G} = -\frac{1}{g_{2,t}^{*}} \frac{\partial \nabla g^{*}(\mathbf{c}_{t}^{*})}{\partial c_{2,t}^{*}}$ are inverse interand intra-temporal price elasticities. Instead, when constrained by the FTA, defined by (8):

$$\theta_t^{FTA} = 1 - \frac{1 + (\boldsymbol{\epsilon}_t^T + \boldsymbol{\epsilon}_t^G) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 + (\boldsymbol{\epsilon}_{t+1}^T + \boldsymbol{\epsilon}_{t+1}^G) \cdot [\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]},$$

where
$$\epsilon_t^G = -\frac{\mathrm{d}\nabla g^*(\mathbf{c}_t^*)}{\mathrm{d}C_t^*}$$
.

Once again, consider the incentives to tax capital inflows when trade policy is constrained by a FTA. Taking elasticities as given, the planner taxes inflows $(\theta_t < 0)$ if households are borrowing relatively more at time t than t + 1 (i.e., $c_t - y_t > c_{t+1} - y_{t+1}$), as before. More generally, however, the planner will tax more heavily commodities that are inelastically supplied. The inverse foreign export-supply elasticity is given by $(\epsilon_t^T + \epsilon_t^G)$ and consists of an inter-temporal term relating to the foreign elasticity of inter-temporal substitution and an intra-temporal term relating to substitution between goods varieties. Inter-temporally, the capital-inflow tax at time t is higher if either ϵ_t^T or ϵ_t^G is high (elasticity is low) relative to their t + 1 values.

Absent a FTA, the optimal tariff will be higher if the intra-temporal elasticity of supply for good 2 is lower than that for good 1 ($\epsilon_{2,t}^G > \epsilon_{1,t}^G$). The expression for the optimal capital-inflow tax now has an important difference relative to the constrained case. With optimal tariffs in play, only the partial elasticity of supply for the untaxed good 1 ($\epsilon_t^T + \epsilon_{1,t}^G$) matters instead of the total elasticity, although deficits in both goods are relevant. This difference drives the interaction between policy instruments, which we investigate numerically in the next section.

4.5 Model Simulation

To illustrate our findings, we describe two simulation scenarios which focus on fluctuations in the good-1 and good-2 endowments in turn, explaining the macroeconomic dynamics and implementation of the optimal allocations that result from each. We focus on a deterministic setting, specifying initial and terminal values for endowments, and constructing the full sequence by assuming they follow a first-order autoregressive process:

$$y_{i,t+1}^{(*)} = \left(1 - \rho_i^{(*)}\right) \overline{y}_i^{(*)} + \rho_i^{(*)} y_{i,t}^{(*)}, \quad \forall t > 0 \text{ and } i = 1, 2,$$

$$\mathbf{y}_0 = \left[y_{1,0} \ y_{2,0}\right]', \quad \mathbf{y}_0^* = \left[y_{1,0}^* \ y_{2,0}^*\right]'$$

For simplicity we assume $\rho_1 = \rho_2 = \rho_1^* = \rho_2^*$. In both scenarios, we assume a constant aggregate endowment $(Y_{1,t} = \overline{Y}_1 \text{ and } Y_{2,t} = \overline{Y}_2 \text{ for all } t)$. This is a useful benchmark in which households are perfectly able to smooth their consumption smoothing in the decentralized allocation.

Based on CRRA utility and the Armington (1969) aggregator for consumption, the model calibration is detailed in Table 1. In each scenario, we compare the decentralized allocation, the unilateral Ramsey planning allocation in which trade policy is constrained by the FTA (8), and one in which trade policy is set optimally. To focus on the dynamic implications of the

three variants in a consistent manner, we equalize the long-run equilibrium of each model by using a constant tariff for the Home country in the decentralized case and when a FTA (8) is in place—an approach that follows the New-Keynesian literature studying allocations where the steady state is first best (or constrained first best).

Table 1: Benchmark Model Calibration

Parameter	Value	Description
β	0.96	Discount factor, annual frequency
σ	2	Coefficient of relative of risk aversion
ϕ	1.5	Elasticity of substitution between goods 1 and 2
α	0.6	Share of good 1 (good 2) in Home (Foreign) consumption basket
ρ	0.8	Persistence of endowments

4.5.1 Scenario 1: Temporarily Low Endowment of Domestic Good

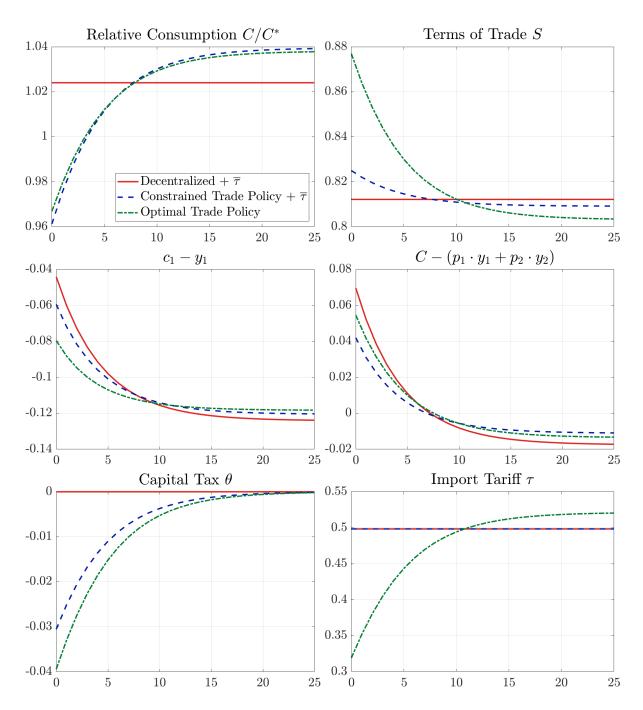
Consider a scenario in which the Home economy is recovering from a domestic downturn, or is growing more quickly than its Foreign counterpart. Specifically, the Home country's endowment of good 1 is low in the near term, and grows towards its long-run level. Denoting initial endowment values by $y_{i,0}^{(*)}$ and long-run levels by $\overline{y}_i^{(*)}$ for i=1,2, we assume that $y_{1,0}=0.9\overline{y}_1$ and $y_{2,0}=\overline{y}_2$. To ensure there is no aggregate uncertainty: $y_{1,0}^*=1-y_{1,0}$ and $y_{2,0}^*=1-y_{2,0}$.

Faced with a higher stream of endowments in the future, Home households borrow to perfectly smooth consumption in the decentralized allocation—as demonstrated by the flat line in the top-left panel of Figure 3. However, households over-borrow relative to the planning allocations, illustrating that the results from Lemma 2 apply more generally. The planner chooses a lower level of near-term aggregate consumption C, but delivers higher consumption in the long run by allocating consumption to periods when it is relatively cheaper. The optimal policy, regardless of constraints on trade policy, involves leaning against capital flows to delay consumption, as illustrated by the evolution of the balance of payments (middle right panel in Figure 3). When trade policy is constrained by a FTA, the required capital-inflow tax is around 3% in the near term and approaches zero as the endowment returns to its long-run level.

Because the Home endowment of good 1—the good consumed with home bias domestically—is initially lower, the planner has an additional incentive to restrict the excess demand for the relatively expensive good 1. As a result, when the good-1 endowment deviates from its long-run level, the planner's inter- (pertaining to the cost of borrowing) and intra-temporal (pertaining to relative goods prices) incentives to manipulate the terms of trade are aligned. The planner chooses to both delay aggregate consumption and consumption of good 1, in expectation that the future price of C and c_1 will fall (middle-left panel in Figure 3).

When trade policy is unconstrained, the planner can additionally restrict the excess demand for good 1 by lowering the tariff on good 2 in the near term, but tariffs have second-best effects on the terms of trade (and real exchange rate). The planner sets a rising path for tariffs over time—starting at around 30% (a subsidy relative to the constant tariff under a FTA) but

Figure 3: Time Profile of Optimal Allocations as the Home Endowment of Good 1 Rises in Scenario 1



Notes: Time profile for allocations in Scenario 1, simulated for 100 periods. See Table 1 for calibration details. "Constrained (Optimal) Trade Policy" refers to allocation arising from a Home planner acting unilaterally with (without) a trade-policy constraint from a FTA. The "Decentralized" and Constrained Trade Policy allocations include a constant tariff $\bar{\tau}$ to ensure that long-run allocations replicate the unconstrained case.

increasing to over 50% in the long term. This leads to a near-term depreciation of the terms of trade (upper right panel in Figure 3). Since this implies that consumption is relatively cheap for Home households, all else equal, this would lead to further over-borrowing, consistent with the Harberger-Laursen-Metzler effect. As a consequence, the capital-inflow tax is roughly one-third larger absent a FTA, at 4% in the near term—a difference that is even larger absent the constant tariff in the free-trade case.

Figure S1 in Supplementary Materials S.2.2 details the decomposition of the optimal inflow tax according to (12). Since inter- and intra-temporal incentives are aligned, the consumption and RER wedges move in the same direction. While capital controls are predominantly driven by the consumption wedge, the difference across regimes is driven by differences in the RER wedge.

4.5.2 Scenario 2: Temporarily Low Endowment of Foreign Good

Next, consider the case in which the Home endowment of the foreign good (good 2) starts at a low value relative to its long-run level. We assume that $y_{2.0}^* = 1.1\overline{y}_2^*$ and $y_{1.0}^* = \overline{y}_1^*$.

As in scenario 1, households borrow in the near term, anticipating their endowment will increase in the future. However, the net supply of good 1 that Home sells abroad rises, because c_1 falls while y_1 is unchanged. The planner wants to delay aggregate consumption C intertemporally, but has an incentive to act monopolistically and drive up the price of good 1 (intratemporally), middle-left panel in Figure 4. When policy is constrained by a FTA, the planner levies a capital-inflow tax in the near term, which implies disproportionately lower consumption of good 1, trading off inter- and intra-temporal incentives to manipulate the terms of trade.

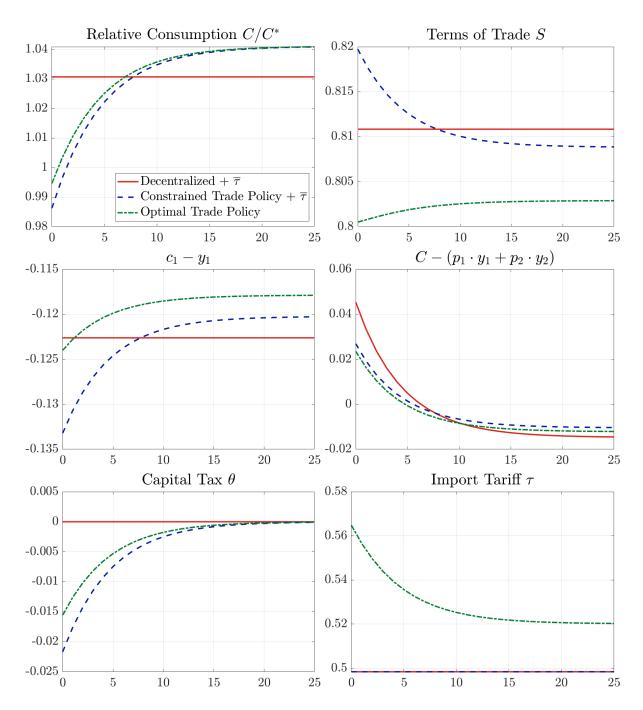
Absent free trade, the planner additionally levies a relatively high tariff in the near term to increase Home demand for good 1, c_1 , and drive up its relative price. The declining path for tariffs, all else equal, implies that the terms of trade will depreciate over time (upper right panel in Figure 4)—making Home consumption relatively expensive in the near-term and discouraging borrowing. As a result, time-varying tariffs act as a substitute for the capital-inflow tax, which is lower in the no-FTA case at 1.5% as opposed to over 2% on impact. At the CO limit (Corollary 1), the capital-inflow tax is about 1% on impact with a FTA in place, but falls to zero when tariffs are optimally chosen.

Figure S2 in Supplementary Materials S.2.2 plots the corresponding decomposition (12). In contrast to scenario 1, absent a FTA, the RER wedge moves in the opposite direction to the consumption wedge. This reflects the fact that, generally, when inter- and intra-temporal incentives are misaligned, high near-term tariffs appreciate the real exchange rate disincentivizing consumption and trade policy can partly (or fully in the CO limit) substitute for capital-flow management.

4.5.3 Robustness

Before proceeding, we summarize some additional considerations pertaining to our simulations.

Figure 4: Time Profile of Optimal Allocations as the Foreign Endowment of Good 2 Falls in Scenario 2



Notes: Time profile for allocations in Scenario 2, simulated for 100 periods. See Table 1 for calibration details. "Constrained (Optimal) Trade Policy" refers to allocation arising from a Home planner acting unilaterally with (without) a trade-policy constraint from a FTA. The "Decentralized" and Constrained Trade Policy allocations include a constant tariff $\bar{\tau}$ to ensure that long-run allocations replicate the unconstrained case.

Comparative Statics. Two parameters influence the size of inter- and intra-temporal motives: the respective elasticities of substitution (see Proposition 3). We explore these comparative statics in Supplementary Materials S.2.3. When the elasticity of inter-temporal substitution $1/\sigma$ is low (i.e., σ is high), the planner levies larger capital controls in an attempt to reallocate consumption inter-temporally. When the intra-temporal (trade) elasticity ϕ is low, the planner sets larger tariffs. These factors influence the quantitative difference in optimal capital controls when trade policy is constrained vs. unconstrained. In our baseline simulation of scenario 1, in Figure 3, optimal capital controls are about one-third larger when trade policy is set optimally. When the intra-temporal elasticity is halved ($\phi = 0.75$), the increase in tariffs leads to an even larger increase in capital controls—by around 50%.

Commitment and Time Consistency. As in Lucas and Stokey (1983) and Costinot et al. (2014), the optimal policy is time consistent if the planner at time 0 has a full maturity structure of debt instruments available to them. The planner can structure debt in such a way so as to induce future planners to stick to the consumption plan. In Supplementary Materials S.2.4, we illustrate that this reasoning continues to hold with multiple goods and when trade policy is unconstrained.

Anticipated Changes in Endowments. The mechanisms that underpin optimal allocations in scenarios 1 and 2 also carry over to instances in which changes in endowments are anticipated. In these cases, optimal trade and financial policy involves preemptive action in advance of the shock itself—similar to Mendoza (2002) and Bianchi (2011). For instance, when a temporary fall in the good-1 endowment is anticipated, the Home planner will subsidize capital-inflows, only one period prior to the shock, to facilitate borrowing and help smooth consumption. Absent a FTA, the planner only employs tariffs to subsidize consumption of the relatively abundant good contemporaneously to the fall in endowment. We discuss this further in Supplementary Materials S.2.5.

Ruling out Capital Controls. We also consider a setting where the planner optimally chooses tariffs while capital controls are ruled out by a Free Financial Flows Agreement (FFFA). This case serves both as a useful benchmark to evaluate the welfare consequences of policy interventions, but also illustrates how tariffs can be used as a second-best instrument to manipulate the cost of borrowing over time. The key takeaway is that when inter- and intra-temporal motives are aligned (e.g. scenario 1), the variation in optimal tariffs is smaller under a FFFA—since they lead to inefficiently high borrowing—but larger when incentives are misaligned (e.g. scenario 2) since they correct existing borrowing inefficiencies. Full details in Supplementary Materials S.2.6.

4.6 Generality of Results

Next, we briefly discuss how our results apply to more general environments.

Production, Nominal Rigidities and Aggregate-Demand Externalities. We first consider a variation of the model where policy is driven by alternative *incentives*, specifically demand management. We extend the model to feature production of non-traded goods (denoted with subscript NT), endogenous labor supply and nominal-wage rigidities. Non-traded goods are produced with a linear production technology $y_{NT,t} = A_t L_t$ under perfect competition (where A_t denotes productivity and L_t labor). The associated firm maximization yields $p_{NT,t} = \frac{w_t}{A_t}$, where we assume the nominal wage is perfectly rigid $w_t = \overline{w}$, and $c_{NT,t} = y_{NT,t}$ in equilibrium. A full model exposition is presented in Supplementary Materials S.3.1.

In this setting, the marginal benefit to the unilateral Home planner from a unit of tradable consumption $c_{T,t}$ can be written as: $u'(C_t)g_{T,t}\left(1+\frac{\omega}{1-\omega}\tau_t^L\right)$, where $g_{T,t}$ is the derivative of the aggregate consumption aggregator with respect to the tradable good, ω represents the expenditure share on non-tradable goods, and τ_t^L is the labor wedge, given by:

$$\tau_t^L = 1 + \frac{1}{A_t} \frac{v_{L,t}}{u'(C_t)g_{T,t}}$$

where $\nu_{L,t}$ represents the marginal disutility of labor supply for the household at time t. It is positive when the economy is demand constrained and households are involuntarily unemployed. The marginal benefit of a unit of tradable consumption is higher when the economy is demand constrained, generating an additional incentive for a planner to bring forward consumption.

Returning to the planner's problem, the implementability constraint (Lemma 1) is unchanged. Absent constraints on trade policy, the first-order conditions with respect to goods 1 and 2 are given by:

$$u'(C_t)g_{1,t}\left(1 + \frac{\omega}{1 - \omega}\tau_t^L\right) = \mu \mathcal{MC}_{1,t}$$
$$u'(C_t)g_{2,t}\left(1 + \frac{\omega}{1 - \omega}\tau_t^L\right) = \mu \mathcal{MC}_{2,t}$$

Moreover, tariffs affect the path of the exchange rate for tradables in the same way as in the baseline setup. As such, faced with constrained demand and unemployment ($\tau_t^L > 0$), the planner brings consumption forward with an optimal mix of capital-inflow subsidy or an import subsidy which puts pressure on the exchange rate to depreciate, as in the baseline model.

Segmented Markets and Quantity Interventions. We also consider how our results generalize when alternative *instruments* are used to deliver the optimal allocation. Importantly, we show that a similar outcome can be achieved if the planner uses quantity interventions (e.g., open-market operations or FXI) in place of capital controls. Consider an extension of the model with non-traded goods and segmented financial markets, detailed in Supplementary Materials S.3.2. There is a single asset in each economy, denominated in units of the domestic non-traded good, which households trade with financial intermediaries—where positions are denoted by

 a_{t+1} and a_{t+1}^{I} , respectively.⁵ The intermediation problem implies one additional equilibrium condition:

$$\left[R_{NT,t}^{*-1} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} - R_{NT,t}^{-1} \right] = \Gamma a_{t+1}^{I}$$
(13)

where $R_{NT,t}^{(*)}$ is the cost of borrowing in the Home (Foreign) and $\mathcal{E}_t = p_{NT,t}^*/p_{NT,t}$. The parameter Γ captures how binding limits to arbitrage are, with $\Gamma \to 0$ being the frictionless limit.

The planner can take a position a_{t+1}^G in domestic assets, financed by selling foreign assets, and market clearing requires: $a_{t+1} + a_{t+1}^G + a_{t+1}^I = 0$. Through this, the planner can affect intertemporal consumption smoothing via the balance sheet of financial intermediaries a_{t+1}^I . If $a_{t+1}^G = 0$, (13) indicates that when households are borrowing ($a_{t+1} < 0$) they face higher borrowing costs ($R_{NT,t+1}$ rises) because financiers facing limits to arbitrage must be compensated for taking the opposite position ($a_{t+1}^I > 0$). Planner intervention (e.g., in the form of FXI) can reduce the size of imbalances that need to be intermediated and so the spread narrows. Proposition C2 in Supplementary Materials S.3.2 formalizes that FXI can target a similar allocation to capital controls when markets are segmented.

Moreover, even as countries become small in financial markets (discussed next), the incentive for the planner to manipulate borrowing remains when there are intermediation frictions and profits are not fully rebated to Home households.

Country Size. Finally, we describe how our results generalize with respect to country size. Within our two-country model, countries are large in goods and financial markets. So planners internalize the effects of domestic allocations on both goods prices and the world real interest rate. Supplementary Materials S.3.3 details a small-open economy setting, with $N \to \infty$ foreign countries. While there are a range of outcomes in the small-open economy setting, an interesting knife-edge case arises when $\sigma = \phi = 1$. Here, the required size of capital controls for inter- and intra-temporal incentives is the same: in scenario 1, as $N \to \infty$, the optimal size of capital controls in both the FTA and no-FTA case is unchanged. Moreover, even though the optimal tariff falls, it is always non-zero since Home goods are scarce. Moving away from this limiting case, when $\sigma > \phi$, the size of capital controls will fall as N rises since the inter-temporal motive dominates, while the opposite is true for $\sigma < \phi$. In scenario 2, consistent with Corollary 1, the optimal capital-inflow tax is 0 absent an FTA.

5 Strategic Planning Allocation

We next consider an open-loop Nash equilibrium, where each planner chooses allocations taking the other's tax sequence $\{\theta_t^{(*)}, \tau_t^{(*)}\}$ as given, where τ_t^* denotes Foreign tariffs levied on good 1. Players cannot observe the actions of their opponents and therefore do not respond optimally

⁵Since the model is deterministic, the exact denomination does not affect the spanning properties of the asset. Without a non-traded good, trade in real bonds would imply interest-rate equalization by the law of one price.

to each others' change in strategy (see, e.g., Fudenberg and Levine, 1988). The strategic equilibrium counterpart to equations (6)-(7) indicate that the ratio of marginal costs from bringing forward a unit of consumption of each good, in each country, should be proportional to the bargaining power of each country.

To see this, let $\tau_t^* \equiv [(1 + \tau_t^*)^{-1} \ 1]'$ denote the vector of Foreign goods-specific tariffs. The Home planning problem, accounting for the optimal response by the Foreign planner, is then:

$$\max_{\{\mathbf{c}_t\}} \quad \sum_{t=0}^{\infty} \beta^t u(C_t) \tag{P-Nash-nFTA}$$

s.t.
$$\sum_{t=0}^{\infty} \left[\prod_{s=0}^{t-1} (1 - \theta_s^*) \right] \beta^t u^{*'}(C_t^*) \boldsymbol{\tau}_t^{*-1} \cdot \nabla g^*(\mathbf{c}_t^*) \cdot [\mathbf{c}_t - \mathbf{y}_t] \le 0$$
 (IC-Nash-nFTA)
$$C_t \equiv g(\mathbf{c}_t)$$
 (nFTA)

which differ from to the unilateral problem (P-Unil-nFTA) by the additional terms in the implementability constraint reflecting the Foreign capital-flow tax θ_t^* and tariff τ_t^* .

5.1Optimal Strategic Allocation

Problem (P-Nash-nFTA) yields the optimality conditions:

$$u'(C_t)g_1(\mathbf{c}_t) = \mu \hat{\mathcal{M}} \mathcal{C}_{1,t}^{nFTA}$$

$$u'(C_t)g_2(\mathbf{c}_t) = \mu \hat{\mathcal{M}} \mathcal{C}_{2,t}^{nFTA}$$

$$(14)$$

$$u'(C_t)g_2(\mathbf{c}_t) = \mu \hat{\mathcal{M}} \mathcal{C}_{2,t}^{nFTA} \tag{15}$$

where μ denotes the Lagrange multiplier on the implementability constraint and:

$$\hat{\mathcal{M}}\mathcal{C}_{1,t}^{nFTA} \equiv u^{*\prime}(C_{t}^{*})(1+\tau_{t}^{*})^{-1}g_{1}^{*}(\mathbf{c}_{t}^{*}) + u^{*\prime\prime}(C_{t}^{*})g_{1}^{*}(\mathbf{c}_{t}^{*})\boldsymbol{\tau}_{t}^{*-1} \cdot \nabla g^{*}(\mathbf{c}_{t}^{*}) \cdot [\mathbf{c}_{t} - \mathbf{y}_{t}]$$

$$+ u^{*\prime}(C_{t}^{*})\boldsymbol{\tau}_{t}^{*-1} \cdot \frac{\partial \nabla g^{*}(\mathbf{c}_{t}^{*})}{\partial c_{1,t}} \cdot [\mathbf{c}_{t} - \mathbf{y}_{t}]$$

$$\hat{\mathcal{M}}\mathcal{C}_{2,t}^{nFTA} \equiv u^{*\prime}(C_{t}^{*})g_{2}^{*}(\mathbf{c}_{t}^{*}) + u^{*\prime\prime}(C_{t}^{*})g_{2}^{*}(\mathbf{c}_{t}^{*})\boldsymbol{\tau}_{t}^{*-1} \cdot \nabla g^{*}(\mathbf{c}_{t}^{*}) \cdot [\mathbf{c}_{t} - \mathbf{y}_{t}]$$

$$+ u^{*\prime}(C_{t}^{*})\boldsymbol{\tau}_{t}^{*-1} \cdot \frac{\partial \nabla g^{*}(\mathbf{c}_{t}^{*})}{\partial c_{2,t}} \cdot [\mathbf{c}_{t} - \mathbf{y}_{t}]$$

The Foreign planner undertakes an analogous maximization. Combining the optimality conditions of the Home and Foreign planners yields the equilibrium allocation, summarized in the following proposition.

Proposition 4 (Capital Controls and Tariff Wars) In a Nash equilibrium where each country chooses optimal capital controls $\{\theta_t, \theta_t^*\}$ and tariffs $\{\tau_t, \tau_t^*\}$ for all $t \geq 0$, the allocations $\{\mathbf{c}_t, \mathbf{c}_t^*\}$ satisfy:

$$\frac{\hat{\mathcal{M}}\mathcal{C}_{1,t}^{nFTA}}{\hat{\mathcal{M}}\mathcal{C}_{1,t}^{*nFTA}} = \alpha_{1,0}^{nFTA} \qquad \frac{\hat{\mathcal{M}}\mathcal{C}_{2,t}^{nFTA}}{\hat{\mathcal{M}}\mathcal{C}_{2,t}^{*nFTA}} = \alpha_{2,0}^{nFTA}$$

$$(16)$$

where

$$\alpha_{i,0}^{nFTA} \equiv \frac{\hat{\mathcal{MC}}_{i,0}^{nFTA}}{\hat{\mathcal{MC}}_{i,0}^{*nFTA}} \quad for \ i=1,2$$

Proof: See Appendix A.6.

The ratio of the marginal cost of a unit of consumption for the planner across countries, for each good variety, is equal to a constant. The constants $\{\alpha_{i,0}^{nFTA}\}$ reflect the bargaining power of the Foreign country relative to the Home with respect to each good and depend on initial conditions. The interpretation of the marginal cost terms is consistent with that in Section 3. Similar arguments to Lemma 2 can be used to evaluate over-borrowing externality in this environment, but to streamline our discussion we focus on a numerical investigation next.

5.2 Numerical Simulations

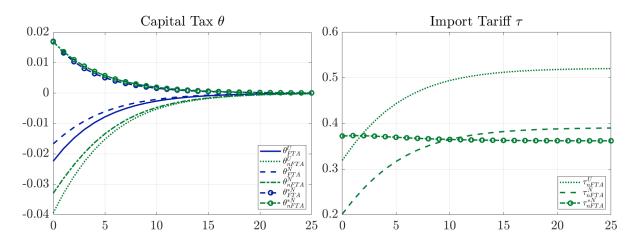
We revisit the two scenarios from Section 4.5 to assess how strategic interactions affect macroeconomic allocations and policy outcomes. The headline result is that capital controls are smaller in both scenarios when trade policy is constrained by a FTA.

Scenario 1. Figure 5 presents the optimal capital-inflow taxes and tariffs for scenario 1, comparing the cases where trade policy is constrained (blue) and unconstrained (green) in the strategic (dashed) and unilateral (solid) settings. As in the unilateral case, Home households over-borrow in the decentralized setting. In the strategic setting, the Home planner will delay consumption using a capital-inflow tax, while the Foreign planner brings forward consumption with a capital-inflow subsidy, regardless of trade policy constraints. The required capital-inflow tax set by the Home planner in the strategic setting is smaller than that in the unilateral case, since Foreign policy is helps to tilt near-term consumption to Foreign households.

The Home planner still has an incentive to delay consumption of good 1, so—unconstrained by a FTA—sets an increasing path for tariffs today. However, while the Home planner's interand intra-temporal incentives are aligned, they are opposed for the Foreign planner, so Foreign tariffs decline somewhat over time. Overall, since the Home tariff varies more, trade policy implies a relative depreciation of the terms of trade. So, as in the with the unilateral case, the capital-inflow tax is larger absent a FTA, consistent with the real-world findings in Figure 1, which suggests that joining the WTO supported reductions in capital controls.

Scenario 2. Figure 6 presents the corresponding figures for scenario 2. Here, strategic interactions make a difference in the outcome. In the Foreign country, where good 2 is relatively abundant in the near term, the planner seeks to increase the price of good 2 and does so by setting a declining path for tariffs on good 1. Because the Foreign country is large in the market for good 2, this effect dominates the Home planner's incentive to manipulate relative prices. Consequently, the Home planner faces a real exchange rate depreciation (relative to the constrained trade-policy case) which encourages Home households to borrow further—requiring a larger capital-inflow tax—changing the result relative to the unilateral case.

Figure 5: Optimal Capital-Inflow Taxes and Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 1



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home, solid lines). 'N' denotes Nash outcome for Home (dashed lines) and Foreign (dashed lines with circle markers).

Comparative Statics. As in the unilateral case, the size of capital controls and tariffs depend on the values of the inter- and intra-temporal elasticities of substitution. Lower values for the elasticity of inter-temporal substitution $1/\sigma$ are associated with capital-control wars, while lower trade elasticities ϕ are associated with tariff wars, discussed further in Supplementary Materials S.4.4.

6 Welfare and Policy Games

Finally, we analyze the welfare costs of capital controls and trade tariffs and show that capital-control wars are less likely to emerge when a FTA (8) is in place.

6.1 Global-Cooperation Benchmark

As a starting point, consider the world planning problem maximizing joint (world) welfare:

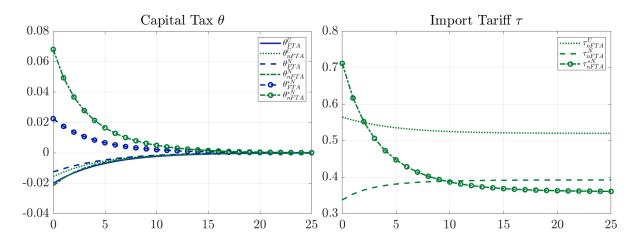
$$\max_{\{\mathbf{c}_t, \mathbf{c}_t^*\}} \quad \sum_{t=0}^{\infty} \beta^t \left[u(g(\mathbf{c}_t)) + \kappa u(g^*(\mathbf{c}_t^*)) \right]$$
 (P-Coop)

s.t.
$$\mathbf{c}_t + \mathbf{c}_t^* = \mathbf{Y}_t$$
 (RC)

$$\mathbf{c} = \mathbf{c}(C), \quad \mathbf{c}^* = \mathbf{c}^*(C)$$
 (FTA)

where κ is the relative weight attributed to Foreign welfare. By the First Welfare Theorem, since there are no frictions in the global economy, the cooperative first-best allocation coincides with the laissez-faire equilibrium (no intervention). Therefore, relaxing constraints on trade policy will have no impact on the global planning allocation—as the following Proposition summarizes.

Figure 6: Optimal Capital-Inflow Taxes and Tariffs for Home and Foreign in the Nash Equilibrium for Scenario 2



Notes: Optimal capital controls and taxes. 'U' subscript denotes unilateral optimal policy result (for Home, solid lines). 'N' denotes Nash outcome for Home (dashed lines) and Foreign (dashed lines with circle markers).

Proposition 5 (Global Cooperation Allocation) In the cooperative allocation resulting from (P-Coop), there exists κ such that no capital controls are optimal, $\theta_t = \theta_t^* = 0$. Relaxing the FTA, as defined in (8), will have no effect on this allocation since $\tau_t = \tau_t^* = 0$.

Proof: See Appendix A.7. \Box

The weight κ which ensures no intervention is optimal is given by the ratio of Lagrange multipliers on households' budget constraints, determined by their initial assets, and is equal to 1 in the symmetric case. Since the cooperative outcome is first best, Corollary 2 follows.

Corollary 2 (Negative Spillovers) Any policy intervention which improves welfare in one country necessarily reduces global welfare by disproportionately worsening welfare in the other.

Proof: Follows directly from Proposition 5.

To analyze the global welfare implications, we revisit scenarios 1 and 2 and consider the allocations arising in both the unilateral and strategic settings, with and without the FTA and the FFFA. We compare welfare by assessing consumption-equivalent variation relative to the global-cooperative allocation (i.e., one of no intervention) in Table S1 of Supplementary Materials S.4.5. Country-level and global welfare costs from policy wars are disproportionately larger when countries depart from the FTA since introducing distortions along the intra-temporal margin will exacerbate over-/under-borrowing through the impact of tariffs on the real exchange rate.

6.2 Trade-Policy Constraints and Prospects for Capital-Control Wars

But can commitment to a trade policy constraint, like a FTA, discourage costly capital-control wars in the first place? To answer this question, we consider a dynamic setting in which country

planners begin in an equilibrium without capital controls (FFFA), either with a trade-policy constraint (FTA) in place or with optimal tariffs instead. We assess the incentive for the Home planner to deviate from the FFFA and levy capital controls.

To do this we assume that the Foreign planner initially sets no tariffs in the FTA case and the optimal tariff absent a FTA. They assume that the Home planner will adopt the same trade policy (i.e., either none or the optimal) and will never levy capital controls. The Home planner deviates and sets capital-flow taxes (and tariffs in the unconstrained case) assuming the Foreign planner remains passive. However, we allow the Foreign planner to retaliate after \bar{t} periods by re-optimizing and choosing capital controls (and tariffs in the no FTA case)—a 'Grim Trigger' strategy (see, e.g., Friedman, 1971). For our experiments, we use $\bar{t}=5$, but this parameter does not have important implications for the economics. At this stage, the game is the same as in the strategic allocation: planners choose mutual best responses.

We present the paths for instruments and consumption, in both the constrained and unconstrained cases for both scenarios in Appendix S.4.6. In all cases, the Home planner attains a higher consumption level in the first \bar{t} periods, which comes at the cost of Foreign consumption. After \bar{t} periods, allocations coincide with the Nash outcome. Using these simulations, we calculate the consumption-equivalent welfare gains for the Home and Foreign country to contrast how the incentive to levy capital controls varies with trade policy constraints. Table S2 demonstrates that a (credible) commitment to a FTA (8) reduces the incentive for a country to deviate and levy capital controls. In scenario 1, levying capital controls increases Home welfare by 0.134% with a FTA in place compared to 0.188% without one; in scenario 2, the welfare gains are over three-times larger without a trade-policy constraint. Moreover, the costs for countries that do not deviate are significantly larger when trade policy is unconstrained. In scenario 1, Foreign losses are over 50% larger without a constraint; in scenario 2, the losses are over three-times larger. Intuitively, if the initial equilibrium is one with competition over tariffs, countries face a distorted path for aggregate consumption and, therefore, the welfare gains from levying capital controls are larger.

7 Conclusion

In this paper, we provide a unified framework for the analysis of capital-flow management and trade policy. We show that introducing tariffs distorts the cost of borrowing over time which, in turn, gives rise to a novel motive for managing capital flows. When tariffs are optimally chosen without retaliation from abroad, whether optimal capital controls are larger or smaller depends on whether the inter- and intra-temporal incentives to manipulate the terms of trade are aligned or misaligned. From a technical perspective, we show this is because, when tariffs are optimally chosen, optimal capital controls depend only on the partial elasticity of foreign export supply, with respect to the domestic (untaxed) good, as opposed to the total elasticity which matters when trade policy is ruled out by a FTA.

Allowing the Foreign planner to retaliate, our simulations suggest that capital controls are always smaller under a FTA. This chimes with the experience of G10 economies since countries'

accession to the WTO from 1995 onwards. Finally, we conduct a policy experiment and show that commitment to a FTA can reduce incentives to levy capital controls. Since capital-control wars are costly for global welfare, our analysis highlights a novel argument in favor of FTAs: retaining openness in trade can help to sustain financial openness.

An important step for future research is to investigate the role of uncertainty and incomplete markets. While policy cannot improve upon the cooperative allocation absent additional frictions when financial markets are complete, this is not the case with financial-market incompleteness. Additionally, the interaction between capital-flow taxes and tariffs will then depend on the currency denomination of debt which can alter the balance between inter- and intratemporal incentives facing the planner due to the desire to the inflate away debt obligations.

References

- Ahir, H., N. Bloom, and D. Furceri (2022): "The World Uncertainty Index," Working Paper 29763, National Bureau of Economic Research.
- Ahnert, T., K. Forbes, C. Friedrich, and D. Reinhardt (2020): "Macroprudential FX Regulations: Shifting the Snowbanks of FX Vulnerability?" *Journal of Financial Economics*.
- ARMINGTON, P. S. (1969): "A Theory of Demand for Products Distinguished by Place of Production," *IMF Staff Papers*, 16, 159–178.
- ATKINSON, A. AND J. STIGLITZ (1980): Lectures on Public Economics Updated edition, Princeton University Press, 1 ed.
- Auray, S., M. B. Devereux, and A. Eyquem (2020): "Trade Wars, Currency Wars," NBER Working Papers 27460, National Bureau of Economic Research, Inc.
- Backus, D. and G. Smith (1993): "Consumption and real exchange rates in dynamic economies with non-traded goods," *Journal of International Economics*, 35, 297–316.
- Backus, D. K. and P. J. Kehoe (1989): "On the denomination of government debt: A critique of the portfolio balance approach," *Journal of Monetary Economics*, 23, 359–376.
- Baier, S. L. and J. H. Bergstrand (2007): "Do free trade agreements actually increase members' international trade?" *Journal of International Economics*, 71, 72–95.
- Bank for International Settlements (2008): "Financial Globalisation and Emerging Market Capital Flows," No. 44.
- Bergin, P. R. and G. Corsetti (2023): "The macroeconomic stabilization of tariff shocks: What is the optimal monetary response?" *Journal of International Economics*, 143, 103758.
- BIANCHI, J. (2011): "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101, 3400–3426.
- BIANCHI, J. AND G. LORENZONI (2022): "The prudential use of capital controls and foreign currency reserves," in *Handbook of International Economics: International Macroeconomics, Volume 6*, ed. by G. Gopinath, E. Helpman, and K. Rogoff, Elsevier, vol. 6 of *Handbook of International Economics*, 237–289.

- Broda, C., N. Limao, and D. E. Weinstein (2008): "Optimal Tariffs and Market Power: The Evidence," *American Economic Review*, 98, 2032–2065.
- Caliendo, L., R. C. Feenstra, J. Romalis, and A. M. Taylor (2021): "A Second-best Argument for Low Optimal Tariffs," NBER Working Papers 28380, National Bureau of Economic Research, Inc.
- CHARI, V. V. AND P. J. KEHOE (1999): "Optimal Fiscal and Monetary Policy," NBER Working Papers 6891, National Bureau of Economic Research, Inc.
- Cole, H. L. and M. Obstfeld (1991): "Commodity trade and international risk sharing: How much do financial markets matter?" *Journal of Monetary Economics*, 28, 3–24.
- CORSETTI, G., L. DEDOLA, AND S. LEDUC (2023): "Exchange Rate Misalignment and External Imbalances: What is the Optimal Monetary Policy Response?" *Journal of International Economics*, 114, 103771.
- Costinot, A., G. Lorenzoni, and I. Werning (2014): "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation," *Journal of Political Economy*, 122, 77–128.
- D'AGUANNO, L., O. DAVIES, A. DOGAN, R. FREEMAN, S. LLOYD, D. REINHARDT, R. SAJEDI, AND R. ZYMEK (2021): "Global value chains, volatility and safe openness: is trade a double-edged sword," Bank of England Financial Stability Papers 46, Bank of England.
- DE LOECKER, J., P. GOLDBERG, A. KHANDELWAL, AND N. PAVCNIK (2016): "Prices, Markups, and Trade Reform," *Econometrica*, 84, 445–510.
- Debortoli, D., R. Nunes, and P. Yared (2021): "Optimal Fiscal Policy without Commitment: Revisiting Lucas-Stokey," *Journal of Political Economy*, 129, 1640–1665.
- Demidova, S. and A. Rodriguez-Clare (2009): "Trade policy under firm-level heterogeneity in a small economy," *Journal of International Economics*, 78, 100–112.
- EGOROV, K. AND D. MUKHIN (2023): "Optimal Policy under Dollar Pricing," American Economic Review, 113, 1783–1824.
- EICHENGREEN, B. (2019): "Trade Policy and the Macroeconomy," IMF Economic Review, 67.
- Fanelli, S. and L. Straub (2021): "A Theory of Foreign Exchange Interventions," *Review of Economic Studies*, 88, 2857–2885.
- FARHI, E., G. GOPINATH, AND O. ITSKHOKI (2014): "Fiscal Devaluations," *The Review of Economic Studies*, 81, 725–760.
- FARHI, E. AND I. WERNING (2014): "Dilemma not Trilemma? Capital Controls and Exchange Rates with Volatile Capital Flows," *IMF Economic Review (Special Volume in Honor of Stanley Fischer)*, 62, 569–605.
- ———— (2016): "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities," *Econometrica*, 84, 1645–1704.
- FERNÁNDEZ, A., M. W. KLEIN, A. REBUCCI, M. SCHINDLER, AND M. URIBE (2016): "Capital Control Measures: A New Dataset," *IMF Economic Review*, 64, 548–574.

- FRIEDMAN, J. W. (1971): "A Non-cooperative Equilibrium for Supergames," Review of Economic Studies, 38, 1–12.
- FUDENBERG, D. AND D. K. LEVINE (1988): "Open-loop and closed-loop equilibria in dynamic games with many players," *Journal of Economic Theory*, 44, 1–18.
- GABAIX, X. AND M. MAGGIORI (2015): "International Liquidity and Exchange Rate Dynamics," Quarterly Journal of Economics, 130, 1369–1420.
- Geanakoplos, J. and H. Polemarchakis (1986): "Existence, Regularity, and Constrained Suboptimality of Competitive Allocations when the Asset Market Is Incomplete," in *Essays in Honor of Kenneth Arrow*, ed. by W. Heller, R. Starr, and D. Starrett, Cambridge University Press, vol. 3, 65–95.
- GHOSH, A. R., J. D. OSTRY, AND M. S. QURESHI (2016): "When Do Capital Inflow Surges End in Tears?" The American Economic Review, 106, 581–585.
- HARBERGER, A. C. (1950): "Currency Depreciation, Income, and the Balance of Trade," *Journal of Political Economy*, 58, 47–60.
- HEATHCOTE, J. AND F. PERRI (2016): "On the Desirability of Capital Controls," *IMF Economic Review*, 64, 75–102.
- Jeanne, O. (2012): "Capital Account Policies and the Real Exchange Rate," in *NBER International Seminar on Macroeconomics 2012*, National Bureau of Economic Research, Inc, NBER Chapters, 7–42.
- Jeanne, O. and J. Son (2023): "To What Extent are Tariffs Offset by Exchange Rates?" working paper.
- Ju, J., K. Shi, and S.-J. Wei (2013): "On the Connections between Intra-temporal and Intertemporal Trades," in *NBER International Seminar on Macroeconomics 2013*, National Bureau of Economic Research, Inc, NBER Chapters, 36–51.
- Kollmann, R. (1995): "Consumption, real exchange rates and the structure of international asset markets," *Journal of International Money and Finance*, 14, 191–211.
- LAURSEN, S. AND L. A. METZLER (1950): "Flexible Exchange Rates and the Theory of Employment," The Review of Economics and Statistics, 32, 281–299.
- Lucas, R. E. and N. L. Stokey (1983): "Optimal fiscal and monetary policy in an economy without capital," *Journal of Monetary Economics*, 12, 55–93.
- Marin, E. (2022): "The Hegemon's Dilemma," Mimeo.
- MENDOZA, E. G. (2002): "Credit, Prices, and Crashes: Business Cycles with a Sudden Stop," in *Preventing Currency Crises in Emerging Markets*, National Bureau of Economic Research, Inc, NBER Chapters, 335–392.

Obstfeld, M. and K. S. Rogoff (1996): Foundations of International Macroeconomics, vol. 1 of MIT Press Books, The MIT Press.

REBUCCI, A. AND C. MA (2020): "Capital Controls: A Survey of the New Literature,".

SCHMITT-GROHÉ, S. AND M. URIBE (2016): "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 124, 1466–1514.

Appendix

A.1 Proof to Lemma 1

Foreign households maximize their discounted lifetime utility subject to their inter-temporal budget constraint, given world prices \mathbf{p}_t :

$$\max_{\{\mathbf{c}_t\}} \quad U_0^* = \sum_{t=0}^{\infty} \beta^t u^*(g^*(\mathbf{c}_t)) \quad \text{s.t.} \quad \sum_{t=0}^{\infty} \mathbf{p}_t \cdot (\mathbf{c}_t^* - \mathbf{y}_t^*) \le 0$$

The first-order condition is given by (3) where λ^* is the Lagrange multiplier on the Foreign inter-temporal budget constraint (4). Lemma 1 follows by substituting (3) into (1).

A.2 Proof to Proposition 1

Taking a total derivative of $\frac{d\mathcal{L}}{dC}$ implies: $\frac{d\mathcal{L}}{dC} = \frac{\partial \mathcal{L}}{\partial c_1} c'_1(C) + \frac{\partial \mathcal{L}}{\partial c_2} c'_2(C)$. Substituting optimality conditions under the FTA into the left-hand side, and those from without an FTA into the right-hand side, verifies the equality.

Next, we prove that (P-Unil-FTA) is a constrained version of (P-Unil-nFTA). The solution to (P-Unil-FTA) satisfies $\frac{d\mathcal{L}}{dC} = 0$ at the (constrained) optimal allocation. Since $c_1'(C)$ and $c_2'(C)$ are positive and increasing functions detailed in Appendix S.1.2, generally $sign(\frac{d\mathcal{L}}{dc_1}) = -sign(\frac{d\mathcal{L}}{dc_2})$ indicating an incentive to adjust consumption across varieties remains at the constrained-optimal allocation. The solution to (P-Unil-nFTA) given by (6) and (7) implies $\frac{d\mathcal{L}}{dc_1} = \frac{d\mathcal{L}}{dc_2} = 0$ which implies aggregate consumption is (unconstrained) optimal as well. Formally, denote:

$$\overline{C} = \{C : \max \ \mathcal{L}(C) \mid c_1(C), c_2(C) \text{ on Pareto frontier}\},$$
(A1)

where \overline{C} is a scalar because \mathcal{L} is strictly concave in the region of interest. Then note that $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1|c_1(\overline{C}),c_2(\overline{C})}, \frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2|c_1(\overline{C}),c_2(\overline{C})} \neq 0$. If, $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1|c_1(\overline{C}),c_2(\overline{C})} > 0$, then $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2|c_1(\overline{C}),c_2(\overline{C})} < 0$ and there exists an ϵ perturbation such that a $c_1(\overline{C}) \pm \epsilon, c_2(\overline{C}) \pm \epsilon$ are preferred. The same is true for $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_1|c_1(\overline{C}),c_2(\overline{C})} < 0$, and $\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}c_2|c_1(\overline{C}),c_2(\overline{C})} > 0$.

A.3 Proof to Lemma 2

(i) The decentralized allocation is given by:

$$u'(C_t)g_1(\mathbf{c}_t) = \mu u^{*'}(C_t^*)g_1^*(\mathbf{c}_t), \qquad u'(C_t)g_2(\mathbf{c}_t) = \mu u^{*'}(C_t^*)g_2^*(\mathbf{c}_t)$$

Evaluating the equations above as $\sigma \to 1$ and $\phi \to 1$ yields: $\frac{\alpha}{c_{1,t}} = \mu \frac{1-\alpha}{c_{1,t}^*}$ and $\frac{\alpha}{c_{2,t}} = \mu \frac{1-\alpha}{c_{2,t}^*}$, which can be rewritten using market clearing as: $\frac{\alpha}{c_{1,t}} = \mu \frac{1-\alpha}{Y_{1,t}-c_{1,t}}$ and $\frac{\alpha}{c_{2,t}} = \mu \frac{1-\alpha}{Y_{2,t}-c_{2,t}}$. Keeping total world endowment of each good constant $(Y_{i,t} = \overline{Y_i} \text{ for } i = 1, 2, \text{ for all } t)$, consumption of each good variety is constant at the decentralized allocation.

(ii) Evaluate equations (6) and (7) as $\sigma \to 1$ and $\phi \to 1$. In this case, (6) can be written as:

$$\frac{\alpha}{c_{1,t}} = \mu(1-\alpha)\frac{1}{c_{1,t}^*} \left\{ 1 + \left[(1-\alpha)\frac{1}{c_{1,t}^*}(c_{1,t} - y_{1,t}) + \alpha\frac{1}{c_{2,t}^*}(c_{2,t} - y_{2,t}) \right] + \left[\left(\frac{1}{c_{1,t}^*} - (1-\alpha)\frac{1}{c_{1,t}^*} \right) (c_{1,t} - y_{1,t}) - \alpha\frac{1}{c_{2,t}^*}(c_{2,t} - y_{2,t}) \right] \right\}$$

The inter- and intra-temporal partial price adjustments on good 1 partly cancel out, but the inter- and intra-temporal price adjustments on good 2 fully cancel out. This then simplifies to:

$$\frac{\alpha}{c_{1,t}} \left(\mu \frac{1 - \alpha}{\overline{Y}_1 - c_{1,t}} \right)^{-1} = 1 + \frac{c_{1,t} - y_{1,t}}{\overline{Y}_1 - c_{1,t}},$$

Analogously, (7) simplifies to:

$$\frac{1-\alpha}{c_{2,t}} \left(\mu \frac{\alpha}{\overline{Y}_2 - c_{2,t}} \right)^{-1} = 1 + \frac{c_{2,t} - y_{2,t}}{\overline{Y}_2 - c_{2,t}}$$

The left-hand side of these last two expressions are decreasing in $c_{1,t}$ and $c_{2,t}$, respectively.

Similarly, the right-hand side is increasing in $c_{i,t}$ and decreasing in $y_{i,t}$ (for i=1,2, respectively), but $y_{j,t}$ does not feature in the first-order condition for $c_{i,t}$ for $j \neq i$. From this it follows that $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{i,t}} > 0$ and $\frac{\mathrm{d}c_{i,t}}{\mathrm{d}y_{j,t}} = 0$, verifying the lemma.

(iii) Evaluate (S4) as $\sigma \to 1$ and $\phi \to 1$ and simplifying:

$$C_{t}^{-1} = \mu \left[\frac{(1-\alpha)C_{t}^{*}}{c_{1,t}^{*}} c_{1}'(C_{t}) + \frac{\alpha C_{t}^{*}}{c_{2,t}^{*}} c_{2}'(C_{t}) \right] C_{t}^{*-1} \left\{ 1 - \left[\left(\frac{(1-\alpha)C_{t}^{*}}{c_{1,t}^{*}} c_{1}^{*\prime}(C_{t}^{*}) \right) \frac{c_{1,t} - y_{1,t}}{c_{1,t}^{*}} + \left(\frac{\alpha C_{t}^{*}}{c_{2,t}^{*}} c_{2}^{*\prime}(C_{t}^{*}) \right) \frac{c_{2,t} - y_{2,t}}{c_{2,t}^{*}} \right] \right\}$$

As in Costinot et al. (2014), taking the total derivative of this with respect to \mathbf{y}_t , yields $\mathrm{d}C_t > 0$ if $\sum_i \frac{\mathrm{d}u'(C_t^*)\nabla g_{i,t}^*}{\mathrm{d}C_t}\mathrm{d}y_{i,t} > 0$. Evaluating this in the CO limit yields:

$$1 + \frac{P_t C_t}{P_t^* C_t^*} > \frac{2\alpha - 1}{\alpha} \tag{A2}$$

The right-hand side attains a maximum at $\alpha = 1$, so the inequality is trivially satisfied.

A.4 Proof to Proposition 2

This follows from the above after substituting for CRRA per-period utility and the Armington aggregator as $\sigma \to 1$ and $\phi \to 1$.

A.5 Proof to Proposition 3

nFTA: Rearranging (6), combining with equation (10), and using $Q_t = \frac{g_{1,t}}{g_{1,t}^*}$, yields the optimal capital-inflow tax:

$$\theta_{t}^{nFTA} = 1 - \frac{1 - \left(\frac{u^{*''}(C_{t}^{*})}{u^{*'}(C_{t}^{*})}\nabla g^{*}(\mathbf{c}_{t}^{*}) + \frac{1}{g_{1,t}^{*}}\frac{\partial \nabla g^{*}(\mathbf{c}_{t}^{*})}{\partial c_{1,t}^{*}}\right) \cdot [\mathbf{c}_{t} - \mathbf{y}_{t}]}{1 - \left(\frac{u^{*''}(C_{t+1}^{*})}{u^{*'}(C_{t+1}^{*})}\nabla g^{*}(\mathbf{c}_{t+1}^{*}) + \frac{1}{g_{1,t+1}^{*}}\frac{\partial \nabla g^{*}(\mathbf{c}_{t+1}^{*})}{\partial c_{1,t+1}^{*}}\right) \cdot [\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]}$$

Using Home and Foreign relative-demand expressions, the optimal tariff can then be expressed as $1 + \tau_t^{nFTA} = \frac{g_{2,t}/g_{1,t}}{g_{2,t}^2/g_{1,t}^2}$. Then, using equations (6) and (7):

$$\tau_t^{nFTA} = \frac{1 - \left(\frac{u^{*''}(C_t^*)}{u^{*'}(C_t^*)} \nabla g^*(\mathbf{c}_t^*) + \frac{1}{g_{2,t}^*} \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{2,t}^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 - \left(\frac{u^{*''}(C_t^*)}{u^{*'}(C_t^*)} \nabla g^*(\mathbf{c}_t^*) + \frac{1}{g_{1,t}^*} \frac{\partial \nabla g^*(\mathbf{c}_t^*)}{\partial c_{1,t}^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]} - 1$$

FTA: Rearranging equation (S4) using $Q_t^{-1} = -\frac{dC_t^*}{dC_t} = \nabla g^*(\mathbf{c}_t^*)\mathbf{c}_t'(C_t) = -\nabla g^*(\mathbf{c}_t^*)\mathbf{c}_t^{*\prime}(C_t)$ yields:

$$\frac{u'(C_t)}{\mu u^{*'}(C_t^*)}Q_t = 1 - \left(\frac{u^{*''}(C_t^*)}{u^{*'}(C_t^*)}\nabla g^*(\mathbf{c}_t^*(C_t)) + \frac{d\nabla g^*(\mathbf{c}_t^*(C_t))}{dC_t^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]$$

Combining this with equation (10) yields:

$$\theta_t^{FTA} = 1 - \frac{1 - \left(\frac{u^{*''}(C_t^*)}{u^{*'}(C_t^*)}\nabla g^*(\mathbf{c}_t^*(C_t)) + \frac{d\nabla g^*(\mathbf{c}_t^*(C_t))}{dC_t^*}\right) \cdot [\mathbf{c}_t - \mathbf{y}_t]}{1 - \left(\frac{u^{*''}(C_{t+1}^*)}{u^{*'}(C_{t+1}^*)}\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1})) + \frac{d\nabla g^*(\mathbf{c}_{t+1}^*(C_{t+1}))}{dC_{t+1}^*}\right) \cdot [\mathbf{c}_{t+1} - \mathbf{y}_{t+1}]}$$

Substituting the definitions for elasticities yields the expressions in Proposition 3.

A.6 Proof to Proposition 4

We derive mutual best responses, for each good. Dividing (14) by its t+1 analogue yields:

$$\frac{C_t^{-\sigma}g_{1,t}}{C_{t+1}^{-\sigma}g_{1,t+1}} = \frac{1}{1-\theta_t^*} \frac{\hat{MC}_{1,t}}{\hat{MC}_{1,t+1}}$$

Introduce $1 - \theta_t$ using the Home Euler (9) and substitute out $\frac{1}{1 - \theta_t^*}$ using the Foreign Euler

equation. This yields the expression for the optimal tax on capital flows levied by Home:

$$1 + \sigma C_{t}^{*-1} \left[g_{1,t}^{*}(c_{1,t} - y_{1,t}) + g_{2,t}^{*}(1 - \tau_{t}^{*})^{-1}(c_{2,t} - y_{2,t}) \right] - \frac{1}{g_{1,t}^{*}} \left[g_{11,t}^{*}(c_{1,t} - y_{1,t}) + g_{21,t}^{*}(1 - \tau_{t}^{*})^{-1}(c_{2,t} - y_{2,t}) \right]$$

$$1 - \theta_{t} = \frac{1}{1 + \sigma C_{t+1}^{*-1} \left[g_{1,t+1}^{*}(c_{1,t+1} - y_{1,t+1}) + g_{2,t+1}^{*}(1 - \tau_{t+1}^{*})^{-1}(c_{2,t+1} - y_{2,t+1}) \right] - \frac{1}{g_{1,t+1}^{*}} \left[g_{11,t+1}^{*}(c_{1,t+1} - y_{1,t+1}) + g_{21,t+1}^{*}(1 - \tau_{t+1}^{*})^{-1}(c_{2,t+1} - y_{2,t+1}) \right]$$
(A3)

Abroad, following the analogous steps as for (A3) yields the expression for the optimal tax on capital flows levied by the Foreign country $(1 - \theta_t^*)$. Finally, combine (A3) and the expression for $1 - \theta_t^*$ and substitute out τ_t and τ_t^* :

$$C_{t}^{*-\sigma}g_{1,t}^{*} + \sigma C_{t}^{*-\sigma-1}g_{1,t}^{*} \left[g_{1,t}^{*}(c_{1,t} - y_{1,t}) + g_{1,t}^{*}S_{t}(c_{2,t} - y_{2,t}) \right] - C_{t}^{*-\sigma} \left[g_{11,t}^{*}(c_{1,t} - y_{1,t}) + g_{21,t}^{*} \frac{g_{1,t}^{*}}{g_{2,t}^{*}} S_{t}(c_{2,t} - y_{2,t}) \right] - C_{t}^{-\sigma}g_{1,t} + \sigma C_{t}^{-\sigma-1}g_{1,t} \left[g_{1,t}(c_{1,t}^{*} - y_{1,t}^{*}) + g_{1,t}S_{t}(c_{2,t}^{*} - y_{2,t}^{*}) \right] - C_{t}^{-\sigma} \left[g_{11,t}(c_{1,t}^{*} - y_{1,t}^{*}) + g_{21,t} \frac{g_{1,t}}{g_{2,t}} S_{t}(c_{2,t}^{*} - y_{2,t}^{*}) \right] \right]$$

The constant $\alpha_{1,0}$ is given by the same equation evaluated at t=0. Analogous steps for the good-2 first-order conditions (Home and Foreign) yield the second equilibrium condition.

A.7 Proof to Proposition 5

When trade policy is constrained by a FTA, the optimal cooperative allocation satisfies:

$$u'(g(\mathbf{c}_t)) + \kappa u'(g(\mathbf{c}_t^*)) \frac{\mathrm{d}C^*}{\mathrm{d}C} = 0$$
(A4)

where $\frac{\mathrm{d}C_t^*}{\mathrm{d}C_t} = -\frac{P_t}{P_t^*}$, yielding the decentralized condition (10) with $\kappa = \frac{u'(g(\mathbf{c}_{t-1}))}{u'(g(\mathbf{c}_{t-1}^*))} \frac{P_{t-1}^*}{P_{t-1}}$ implying $\theta_t = 0$. Removing constraints on trade policy does not change the optimal allocation (since goods taxes are zero at the optimal). We get two first-order conditions,

$$u'(g(\mathbf{c}_t))g_1 + \kappa u'(g(\mathbf{c}_t^*))g_1^* \frac{dc_1^*}{dc_1} = 0, \quad u'(g(\mathbf{c}_t))g_2 + \kappa u'(g(\mathbf{c}_t^*))g_2^* \frac{dc_2^*}{dc_2} = 0$$
 (A5)

Note that $\frac{g_1}{g_1^*} = \frac{dC}{dc_1} \frac{dc_1^*}{dC^*} = \frac{dC}{dC^*} \frac{dc_1^*}{dc_1} = -\frac{dC}{dC^*}$, therefore both of the above conditions imply (A4), as in the FTA case.