Title
Why voters may prefer congested public clubs

Permalink
https://escholarship.org/uc/item/2rm6x4kc

Authors
Glazer, Amihai
Niskanen, Esko

Publication Date
2001-09-01
Estimation of Highway Deterioration Models by Combining Experimental Data from Different Sources

Adrian Richardo Archilla
Samer Madanat

Reprint
UCTC No. 454
The University of California Transportation Center

The University of California Transportation Center (UCTC) is one of ten regional units mandated by Congress and established in Fall 1988 to support research, education, and training in surface transportation. The UC Center serves federal Region IX and is supported by matching grants from the U.S. Department of Transportation, the California Department of Transportation (Caltrans), and the University.

Based on the Berkeley Campus, UCTC draws upon existing capabilities and resources of the Institutes of Transportation Studies at Berkeley, Davis, Irvine, and Los Angeles; the Institute of Urban and Regional Development at Berkeley; and several academic departments at the Berkeley, Davis, Irvine, and Los Angeles campuses.

Faculty and students on other University of California campuses may participate in Center activities. Researchers at other universities within the region also have opportunities to collaborate with UC faculty on selected studies.

UCTC's educational and research programs are focused on strategic planning for improving metropolitan accessibility, with emphasis on the special conditions in Region IX. Particular attention is directed to strategies for using transportation as an instrument of economic development, while also accommodating to the region's persistent expansion and while maintaining and enhancing the quality of life there.

The Center distributes reports on its research in working papers, monographs, and in reprints of published articles. It also publishes Access, a magazine presenting summaries of selected studies. For a list of publications in print, write to the address below.

DISCLAIMER

The contents of this report reflect the views of the authors, who are responsible for the facts and accuracy of the information presented herein. This document is disseminated under the sponsorship of the Department of Transportation, University Transportation Centers Program, in the interest of information exchange. The U.S. Government assumes no liability for the contents or use thereof.

The contents of this report reflect the views of the author who is responsible for the facts and accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the State of California or the U.S. Department of Transportation. This report does not constitute a standard, specification, or regulation.
Estimation of Highway Pavement Deterioration Models by Combining Experimental Data from Different Sources

Adrian Ricardo Archilla*

Samer Madanat**

*Graduate Research Assistant
**Associate Professor

Department of Civil and Environmental Engineering
University of California
Berkeley, CA 94720-1720

Reprinted from
Journal of Transportation Engineering
ASCE, September 2001

UCTC Reprint No. 454

The University of California Transportation Center
University of California at Berkeley
Estimation of Highway Pavement Deterioration Models by Combining Experimental Data from Different Sources

Adrian Ricardo Archilla\textsuperscript{1} and Samer Madanat\textsuperscript{2}

Abstract

The accurate prediction of rutting development is an essential element for the efficient management of pavement systems. In addition, progression models of highway pavement rutting can be used to study the effects of different loading levels, and thus in allocating cost responsibilities to various vehicle classes for their use of the highway system. Further, such models can be used for evaluating different strategies for design, maintenance and rehabilitation. Finally, if the models contain information about asphalt concrete mixes, they can also provide directions in the proportioning of aggregate, asphalt and air in the mix.

The objective of this paper is to demonstrate the effectiveness of the estimation of rutting models by combining the information from two data sources. The data sources considered are the AASHO Road Test and the WesTrack Road Test. Combined estimation with both data sources is used to identify parameters that are not identifiable from one data source alone. In addition, this estimation approach also yields more efficient parameter estimates.

The results presented in this paper demonstrate that joint estimation produces more realistic parameter estimates than those obtained by using either data set alone. Furthermore, joint estimation allows us to account for the effects of pavement structure, axle load configuration, asphalt concrete mix properties, freeze-thaw cycles and hot temperatures in a single model. Finally, it allows us to predict the relative contributions of rutting originating both in the asphalt concrete and in the unbound layers in the same model.

\textsuperscript{1} Graduate Student Researcher, Institute of Transportation Studies and Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720
\textsuperscript{2} Associate Professor, Institute of Transportation Studies and Department of Civil and Environmental Engineering, 114 McLaughlin Hall, University of California, Berkeley, CA 94720, Tel: 510-643-1084, Fax: 510-642-1246, email: madanat@ce.berkeley.edu
Estimation of Highway Pavement Deterioration Models
by Combining Experimental Data from Different Sources

1. Introduction

Modeling highway pavement deterioration is a critical component of pavement management systems. Pavement deterioration models provide predictions of pavement condition over time as a function of traffic, pavement characteristics and environmental factors. In asphalt concrete pavements, the deterioration manifests itself through several distress types, such as cracking, rutting, raveling, potholing, and roughness. In particular, rutting, which appears as longitudinal depressions in the wheel paths of asphalt concrete pavements, has historically been used a primary criterion of structural performance in many pavement design methods and represents a serious safety issue for road users. For these reasons, in this paper we focus on the development of rutting progression models.

The accurate prediction of rutting development is an essential element for the efficient management of pavement systems. In addition, progression models of highway pavement rutting can be used to study the effects of different loading levels, and thus in allocating cost responsibilities to various vehicle classes for their use of the highway system. Further, such models can be used for evaluating different strategies for design, maintenance and rehabilitation. Finally, if the models contain information about asphalt concrete mixes, they can also provide directions in the proportioning of aggregate, asphalt and air in the mix.

Rutting models developed to date are all limited somehow by the characteristics of the data sets used to estimate them. For example, the information in some data sets allows the identification of the effects of the structural design of the pavement (pavement layer thicknesses) but not of the effects of asphalt concrete mix characteristics and vice-versa. Different types of data have complementary characteristics that can be exploited using joint estimation.
The objective of this paper is to demonstrate the effectiveness of the estimation of rutting models by combining the information from two data sources. The data sources considered are the AASHO Road Test and the WesTrack Road Test. The AASHO Road Test (HRB, 1962) is the most comprehensive full-scale pavement test carried out to date. This data set allows the identification of the effects of layer thicknesses, freezing and thawing, and the level and number of repetitions of the loads. A total of 9,478 observations (7,035 for the original sections and 2,443 for overlaid sections) corresponding to 260 pavement sections are used for model estimation. The data set is an unbalanced panel.

WesTrack is also a full-scale experiment that consists of a specially built track in the state of Nevada. In contrast to the AASHO data set, the WesTrack data set allows the identification of the effects of asphalt concrete mix characteristics, the number of load repetitions and high air temperatures. This data set, which consists of 860 observations for 26 pavement sections, is also unbalanced.

Combined estimation with both data sources is used to identify parameters that are not identifiable from one data source alone. In addition, this estimation approach also yields more efficient parameter estimates.

The paper is organized as follows. Section 2 describes the specifications of the WesTrack and AASHO models. Section 3 presents the parameter estimation results for the individual models. Then Section 4 introduces the joint estimation approach. Section 5 presents the parameter estimation results using the joint estimation approach and analyzes the most important consequences. Section 6 concludes the paper.

2. Model specifications

This section first presents background common to the specification of the models for both data sources. Then, this common background is customized to the characteristics of each data set.
Common background

When the environmental conditions do not change appreciably, most empirical and accelerated pavement test studies exhibit a concave trend of deformation (rut depth) with respect to the number of load applications. Curve A in Figure 1 shows this trend, which illustrates how the materials harden with loading. The same increment in the number of load applications ($\Delta N$), produces a larger increment of rut depth ($\Delta RD$) near the beginning of the life of the pavement than the near the end of the life of the pavement.

![Figure 1: Rut depth vs. cumulative number of load applications.](image)

Curve B in the same figure illustrates what happens when the environment changes to more unfavorable conditions. For example, thawing of frozen soils adversely affects the performance of unbound materials and high air temperatures affect adversely the performance of asphalt concrete mixes. In such situations, the trend in curve A is discontinued by the jumps in the increments in rut depths shown in curve B.
One can find simple expressions capable of modeling trends like that depicted by curve A. Unfortunately, similar expressions capable of modeling trends such as the one in curve B do not exist. Consequently, the rut depth for a section \( i \) at time \( t \), \( RD_{it} \), is modeled as the summation of the increments of rut depth up to time \( t \), \( \Delta RD_{is} \), in addition to a constant term indicating the rut depth immediately after pavement construction, \( c_i \).

\[
RD_{it} = c_i + \sum_{s=1}^{t} \Delta RD_{is}
\]  

(1)

In this manner, the rut depth jumps with changes in the environmental conditions can be considered in the specification of the rut depth increments. To model the increments of rut depth for each period, it is important to consider the mechanisms of rutting and the locations where deformations originate in the pavement structure. Rutting appears as consequence of densification and plastic flow in the pavement layers\(^1\). Attributing how much of the rutting is due to densification and how much is due to flow is very difficult because both mechanisms occur throughout all pavement layers in different degrees. However, near the pavement surface flow plays a more important role than densification but the importance of flow relative to densification decreases with depth. Thus, although the two mechanisms cannot be separated, one should expect different trade-offs between relevant variables near the pavement surface and deeper in the pavement structure. Consequently, the increment of rut depth for each period is modeled as the sum of an increment originating near the pavement surface and an increment originating deeper in the pavement structure. There is another reason why this is convenient. The relevant environmental effects are also different near the pavement surface and deeper in the pavement structure. Near the surface, the pavement has asphalt concrete whose performance is severely affected by high air temperatures. In contrast, the underlying layers of the pavement structure are commonly built with unbound materials that are affected by freezing and thawing. Thus, the increment of rut depth for time period \( t \) is modeled as:

\(^1\) Densification involves change of volume without change in shape whereas flow involves change in shape without change in volume.
\[ \Delta RD_i^t = \Delta RD_i^{AC} + \Delta RD_i^U \]  

Where: \( \Delta RD_i^{AC} \) is the increment in rut depth for section \( i \) in period \( t \) that originates in the asphalt concrete layer and \( \Delta RD_i^U \) is the increment in rut depth for section \( i \) in period \( t \) that originates in the underlying layers.

To model these increments of rut depth Archilla and Madanat (2000b) have used the following expressions:

\[ \Delta RD_i^U = a_i e^{b N_i^t} \Delta N_i^t \]  

\[ \Delta RD_i^{AC} = m_i e^{b N_i^t} \Delta N_i^t' \]  

Where: \( m_i \) is a function of the mix characteristics, loading, and high air temperatures and \( a_i \) is a function of the pavement layer thicknesses and a measure of freeze-thaw in the period. The expressions for \( m_i \) and \( a_i \) are presented later when the background is customized for each data set. Finally, \( b \) and \( b' \) are (negative) model parameters.

The variables \( \Delta N_i^t \) and \( \Delta N_i^t \) capture the effects of traffic loads during the loading period for rutting originating in the asphalt concrete layer and for rutting originating in the underlying layers, respectively. Similarly, the variables \( N_i^t \) and \( N_i^t \) represent the cumulative loading up to the current period \( t \). That is, they are \( N_i^t = \sum_{s=1}^{t} \Delta N_{is} \) and \( N_i^t = \sum_{s=1}^{t} \Delta N_{is}', \) respectively.

Equations (3) and (4) indicate that the increments of rutting are proportional to the loading in the period but they decrease with cumulative loading (as captured through the exponential terms). Thus, the parameters \( b \) and \( b' \) capture the hardening that occurs in the underlying layers and the asphalt concrete layer, respectively.

\( \Delta N_i^t \) and \( \Delta N_i^t' \) are defined using the concept of equivalent single axle loads, which is generally accepted in pavement engineering. The number of applications of
equivalent single axle loads is the number of 80 kN single axle loads that produce the same damage as the actual traffic loads. These are computed using a Load Equivalency Factor (LEF). The LEF for a load \( S \) gives the number of applications of single axle loads of 80-kN that produce the same damage as one passage of the load \( S \). This factor is computed as:

\[
LEF = \left( \frac{S}{80} \right)^\gamma
\]

(5)

Where: \( \gamma \) is a load equivalency coefficient that is commonly assumed to be equal to 4.

This concept is used to define \( \Delta N_{lt} \), the loading for rutting originating in the underlying layers, as follows:

\[
\Delta N_{lt} = \sum_{j=1}^{R_s} n_{sj} \left( \frac{S_j}{80} \right)^{\beta_s} + \sum_{j=1}^{R_t} n_{tj} \left( \frac{T_j}{\beta_7 80} \right)^{\beta_s}
\]

(6)

Where \( n_{sj} \) is the number of single axle load applications of load magnitude \( S_j, j=1, \ldots, R_s \); \( R_s \) is the number of different load magnitudes for single axles; \( n_{tj} \) is the number of tandem axle load applications of load magnitude \( T_j, j=1, \ldots, R_t \); and \( R_t \) is the number of different load magnitudes for tandem axles. The factor \( \left( \frac{S_j}{80} \right)^{\beta_s} \) is simply the LEF for the single axle load magnitude \( S_j \). The multiplication of the \( LEF_{sj} \) by the number of applications of load \( S_j \) in the period, \( n_{sj} \), yields the number of equivalent single axle load applications produced by single axle loads of magnitude \( S_j \). Similarly, the factor \( \left( \frac{T_j}{\beta_7 80} \right)^{\beta_s} \) is the LEF for the tandem axle load magnitude \( T_j \). In this case, the axle load \( T_j \) is standardized by a standard tandem axle load (\( \beta_7 80 \)-kN) that produces the same damage as the standard single axle load of 80-kN. The load equivalency coefficient for tandem axle loads, \( \beta_6 \), is not constrained to be the same as the load equivalency coefficient for single axles, \( \beta_s \). This is reasonable because the stress distributions produced by single and tandem axle load configurations are different deep in the pavement structure. As illustrated schematically in Figure 2, the stress distributions caused by the two loads in a tandem configuration are superimposed yielding a stress distribution that is different from the stress distribution for a single axle load.
It should be noted that the expressions for the WesTrack and AASHO Road Tests are simpler. Figure 3 shows the configuration of the truck used to load the WesTrack sections. It can be noted that there are only two single axle load magnitudes that correspond to a 53.4-kN steering axle and five 89-kN single loading axles and there is only one tandem axle load of 178-kN. For AASHO, the loading axles are either one single, two singles, one tandem, or two tandem axles whose magnitude change from section to section and the steering axle is always a single axle with a magnitude that also varies from section to section.

Although the loading for a given section is unique, its effect on rutting originating in the asphalt concrete layer is different from its effect on rutting originating
in the underlying layers. Based on observations of simplified stress distributions near the pavement surface and at depth, we expect the effect of load to be much more pronounced at depth than near the surface of the pavement (Archilla and Madanat, 2000b). In fact, this is the main reason why loading has been considered separately for the two origins of rutting. In addition, near the surface the two loads in a tandem configuration act as two separate single axle loads. (There is no superposition of the stress distributions caused by each of the axles). From these observations the loading for rutting originating in the asphalt concrete layer is specified as follows:

\[
\Delta N_{it} = \sum_{j=1}^{R_s} n_j \left( \frac{S_j}{80} \right)^{\beta_{12}} + \sum_{j=1}^{R_t} n_j 2 \left( \frac{T_j / 2}{80} \right)^{\beta_{12}}
\]  

(7)

This expression is very similar to (6). However, there are two important differences. First, the load equivalency exponent for single axles, \(\beta_{12}\), is not assumed to be the same as \(\beta_5\), the load equivalency exponent for single axles for rutting originating in the underlying layers. As mentioned before, the a priori expectation is that \(\beta_{12}\) should be smaller than \(\beta_5\). Second, tandem axles are considered as two single axles each with half the load of the tandem axle. This is why in the second term, the load \(T_j\) is divided by 2 and the result is then standardized by 80 kN, the standard single axle load. This ratio is then raised to the \(\beta_{12}\) power, the load equivalency exponent for single axles to obtain the LEF for one of the axles of the tandem configuration. Finally, this LEF is multiplied by 2 to account for the fact that there are two single axles for each passage of a tandem axle.

To complete the specification of the two models we need expressions for \(m_{it}\) and \(a_{it}\). This is done in the following two subsections in the context of each model.

**WesTrack Model Specification**

The results of trenching studies in WesTrack indicated that almost all the rutting originated in the asphalt concrete layer (Hand, 1998). Thus, for this data source, the background presented in the previous section is simplified by considering that the rutting originating in the underlying layers is negligible. This is fortunate because all the pavement sections in WesTrack had the same structural design. Thus, even if the rutting
originating in the underlying layers had not been negligible, the related parameters (presented in the next section) could not have been identified. Clearly, the predictions of this model will be limited to rutting originating in the asphalt concrete layer.

The loading was also the same for all sections. Therefore, the load equivalency parameter in equation (7), $\beta_{12}$, cannot be identified from this data source. This does not present a problem for the model estimation since $\Delta N'_{it}$ can simply be expressed as:

$$\Delta N'_{it} = \text{constant} \times \Delta V_{it}$$  

Where $\Delta V_{it}$ is the number of vehicle passages during period t. Assuming a given value for the constant (and consequently a value for $\beta_{12}$) affects the magnitude of the coefficients multiplying $N'_{it}$ but not the predicted values of rutting. In other words if the constant is multiplied by a factor $\kappa$, the estimate of the parameter that multiplies $N'_{it}$ is divided by $\kappa$ but the rutting predictions are unaffected. The value assumed in the model is transferred from the AASHO model presented in the following subsection, which was 0.39.

To complete the model specification, we need a specification for $m_{it}$. Archilla and Madanat (2000a) have specified $m_{it}$ as a function of three asphalt concrete mix variables, high air temperatures, and loading. The three asphalt concrete mix variables are a Gradation Index ($GI$), the Voids Filled with Asphalt ($VFA$), and the in-place Air Voids ($AV$). The gradation index is a measure of the deviations of the percent passing a series of sieves from the corresponding percent passing from an associated maximum density line. The maximum density line is a line that describes gradations that theoretically achieve the maximum density for a given maximum aggregate size. The idea is that gradations that differ the most from its associated maximum density line will have a lower intrinsic resistance to rutting. A more complete description of the gradation index can be found in Archilla and Madanat (2000a). Here, it suffices to note that with this variable we try to capture the intrinsic deformability of the aggregate structure independently of the mix asphalt content and compaction.

$VFA$ is the percentage of the voids in the mineral aggregate which are filled with asphalt. Its value changes with the level of compaction and trafficking, but it approaches
a constant value near the end of the life of the pavement. The $VFA$ values we used for the estimation of the model were obtained from field samples compacted with the Superpave Gyratory Compator (AASTHO, 1997). The $VFA$ value obtained from the Superpave Gyratory Compator is an approximation to the field $VFA$ value near the end of the life of the pavement. Thus, in our model, $VFA$ represents a measure of the asphalt content near the end of the life of the pavement. $VFA$, together with an air temperature dummy variable defined below, captures the significant rutting during periods with high air temperatures that is commonly observed in mixes with excessive asphalt content. Finally, in-place air voids content is intended to capture the degree of compaction of the asphalt concrete layer immediately after construction.

The specification for $m_i$ developed by Archilla and Madanat (2000a) is

$$
m_i = (\gamma_1 + \gamma_2 GI_i) + (\gamma_3 + \gamma_4 \cdot GI_i) \left( \frac{VFA_i}{100} \right)^{r_5 + r_6 / GI_i} \cdot TempDum_i + \gamma_7 AV_i \exp(\gamma_8 N''_i)
$$

(9)

Where $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7$, and $\gamma_8$ are model parameters and $TempDum_i$ is equal to one if the mean maximum temperature in an observation period$^2$, $MeanMaxT_i$, is greater than $28.6^\circ C$ and 0 otherwise. The rationale for this threshold is explained in Archilla and Madanat (2000a). Equation (9) has three additive terms. The first term, a linear function of the gradation index for section $i$ ($GI_i$), is intended to capture the intrinsic deformability of the aggregate structure in the mix. The second term is intended to capture the interaction of high asphalt contents (high $VFA_i$) and high air temperatures ($TempDum_i = 1$). This interaction is also affected by the aggregate structure through the two linear functions on $GI_i$. Finally, the third term captures the effect of initial compaction. A pavement that is not well compacted (high $AV_i$) will tend to compact more under traffic and this will lead to more rutting. Thus, $\beta_8$ should be positive. However, the effect of initial compaction will fade away with loading. This is captured with the exponential term with a negative $\beta_9$.

In summary the specification of the WesTrack model is

$^2$ The periods between observations were about two weeks long.
With the $m_i$'s given by equation (9), $\Delta N'$ given by equation (7) (using a value for $\beta_{12} = 0.39$ from the AASHO model explained below), and where $\gamma_{10}$ replaces the constant term $c_i$ in equation (1) and $\gamma_7$ replaces the parameter $b'$ in equation (4).

**AASHO Model Specification**

In the AASHO road test, rutting originated in both the asphalt concrete layer and the underlying layers. Thus, both sources of rutting must be accounted for in the model specification. For rutting originating in the asphalt concrete layer, we would have like to use the same specification as for the WesTrack model. Unfortunately, the information we had for the asphalt concrete mixes in AASHO was very limited and therefore some simplifications were needed.

For the asphalt concrete mixes in AASHO, we only had information about the averages of the $GI$, $VFA$, and $AV$ as well as some information about their variability. In general, these averages were quite tight because all the sections were constructed according to the same specifications. The variation between sections was due mainly to construction variability, which given the experimental nature of the AASHO Road Test was relatively low. In light of the above, equation (9) was simplified for AASHO to a form:

$$m_{ii}^A = \beta_{10} + \beta_{11} \cdot TempDum_i$$

Note that in the above equation we have substituted the expressions $(\gamma_7 + \gamma_2 \cdot GI_i)$ and $(\gamma_3 + \gamma_4 \cdot GI_i) (VFA_i / 100)^{(\gamma_5 + \gamma_6 \cdot GI_i)}$ appearing in equation (9) by the parameters $\beta_{10}$ and $\beta_{11}$, respectively. This is reasonable, since $GI$ and $VFA$ were relatively similar for all sections. Note also that we have neglected the third additive term in equation (9). The reason for this is that the average of the in-place air voids for the AASHO mixes was relatively low (around 3.6 %). Therefore, for the AASHO mixes, the effect of this term is
negligible with the exception perhaps of the first one or two observations for each pavement (recall that the contribution of this term decreases with loading).

To complete the AASHO model specification, we only need an expression for $a_{it}$ in equation (3). Consider first the case of fixed, favorable environmental conditions. Since the environmental conditions do not change, $a_{it} = a_i$. It is reasonable to expect that the stronger is the pavement the lower is the rutting originating in the underlying layers. As seen in equation (3), the amount of rutting is directly proportional to the value of $a_{it}$. This can be represented by a relation of the form shown in Figure 4.

![Figure 4: Anticipated relation between the $a'_t$ coefficient and the strength of a pavement.](image)

Such a relation simply indicates that the stronger the pavement, the less the accumulated rut depth for a given traffic and that in extremely strong pavements the rutting originating in the underlying layers should approach zero. The exponential function provides a way to obtain such a shape, but before giving the expression relating $a_i$ to strength, we need to define the strength. To model pavement strength, a concept similar to the structural number defined in AASHTO (1993) is used. Specifically, the strength of the pavement is modeled as:

$$RN_i = \beta_1 (T_{11} + OT_i) + \beta_2 T_{12} + \beta_3 T_{13}$$  

(12)

Where:
\( RN_i \) = resistance number for pavement \( i \) (although this is almost identical to the structural number, a different name is used to make explicit that this number is specific to rutting)

\( T_{il} \) = thickness of the asphalt concrete layer for pavement \( i \) (m);

\( OT_i \) = thickness of the asphalt concrete overlay for pavement \( i \) (m) (only for those sections that were overlaid);

\( T_{i2} \) = thickness of the granular base layer for pavement \( i \) (m);

\( T_{i3} \) = thickness of the subbase layer for pavement \( i \) (m);

\( \beta_j \) = contribution of the \( j \)th layer to the pavement resistance, where \( j = 1,2,3 \) for asphalt concrete, base, and subbase respectively.

Then, the following expression is used to relate \( a_i \) to \( RN_i \):

\[
a_i = \beta_4 e^{-RN_i} = \beta_4 e^{-(\beta_1(T_{il} + OT_i) + \beta_2 T_{i2} + \beta_3 T_{i3})}
\]

From this expression, it can be seen that as the pavement becomes more resistant, the rut depth originating in the underlying layers approaches zero asymptotically.

Most sections in the AASHO Road Test showed an evident increment in the rate of rut depth progression during the spring months. Thus, in what follows, an environmental variable is defined with the information available to include the thawing effects in our model.

The environmental information available in the database for the AASHO Road Test was very limited. Nevertheless, from the information about the maximum and minimum temperatures, a thawing index is computed with the following reasoning: Freeze will only accumulate when temperatures are below 0°C. Thus an accumulated freeze index for period \( t \) is computed as follows:

\[
\begin{align*}
AccumFze_t &= 0 \\
AccumFze_t &= \max(0, AccumFze_{t-1} - \text{MeanMin } T_t) & t = 2, ..., T_i
\end{align*}
\]
Where \( MeanMinT_t \) is the mean minimum temperature (°C) in the two week period \( t \) (in the AASHO road test there was no freezing in period 1, which is why we set \( AccumFze_i = 0 \) and where \( T_i \) is the number of observations for section \( i \).

Once the minimum temperature falls below 0 °C, freezing starts to accumulate. At some point in time the minimum temperature again exceeds 0 °C, thus reducing the accumulated freeze. When there are enough periods with temperatures above zero the accumulated freeze is exhausted and therefore the variable \( AccumFze_i \) becomes zero again.

The effect of thawing will be the greatest when there is considerable accumulated freeze from previous periods and the temperatures in the current period are substantially above zero. In such cases, there will be large amounts of water in the pavement structure with the consequent detrimental effects. Thus, a thawing index representing this interaction of cumulative freeze with temperatures above zero is defined as follows

\[
TI_t = AccumFze_i \cdot \max(MeanMaxT_t, 0) \quad \text{(with units of °C)} \tag{15}
\]

Where \( MeanMaxT_t \) is the mean maximum temperature (°C) in the two week period preceding \( t \). This thawing index will be zero when the mean maximum temperature in the period is below zero or when there is no accumulated freeze. Thus, as illustrated in Figure 5, when thawing starts, this variable starts increasing, then it reaches a maximum and it returns to zero at the end of the thawing period. Archilla and Madanat (2000b) describe other conditions that are necessary for freezing and thawing to have an effect.

Having defined the thawing index, it is now explained how it is incorporated in the model. The introduction of a correction factor for \( a_i \) when the environmental conditions change is done as follows:

\[
a_{it} = a_i e^{\beta_8 \left( \frac{TI_t}{1000} \right)} \tag{16}
\]

Where \( \beta_8 \) is a model parameter. Whenever the thawing index is zero, the new multiplicative factor \( \exp(\beta_8 TI_t/1000) \) is 1 and whenever there is thawing the factor is
greater than one implying that the pavement will rut faster during the corresponding period.

Figure 5: Thawing index computed at the AASHO Road Test.

Putting all the above together, the complete specification of the AASHO model is:

\[
RD_{it} = \beta_{i14} + \sum_{s=k}^{t} (m_{is} e^{b_{13} N_{is}} \Delta N_{is}) + \sum_{s=k}^{t} (d_{is} e^{b_{9} N_{is}} \Delta N_{is})
\]  

(17)

In the above equation, the constant term \(c_i\) of equation (1) is replaced by the parameter \(\beta_{i14}\), and the parameter \(b\) of equation (3) is replaced by the parameter \(\beta_9\). The two summations start with index \(s = k\), where \(k\) is either equal to one if the pavement has not received any overlay or equal to the index of the period in which the pavement was overlaid. It is assumed that when the pavement is overlaid, all existing rutting is
eliminated and that immediately after the overlay construction there will be some initial rutting which is similar to the initial rutting of original construction (in both cases we assume that initial rutting to be equal to $\beta_{it}$). The expressions for $m_{ht}$ and $a_{it}$ are given by equations (11) and (16), respectively. Finally, in order to account for overlays, the expressions for the cumulative loading variables are given by $N_{it} = \sum_{s=1}^{t} \Delta N_{ist}$ and

$$N'_{it} = \sum_{s=k}^{t} \Delta N'_{ist},$$

respectively. The summation in the expression for $N_{it}$ starts always at period 1 because the construction of the overlay does not affect the hardening that has occurred in the underlying layers. In contrast, the summation $N'_{it}$ starts at period $k$, the period when the pavement was overlaid. This is because when a pavement is overlaid most of the new rutting originating in the asphalt concrete layer will occur within the overlay, which has not harden. The contribution of the older asphalt concrete layer to rutting is minimal. The expressions for $\Delta N_{it}$ and $\Delta N'_{it}$ are still given by equations (6) and (7).

3. Estimation results for the individual models

This section presents the results of the parameter estimation when both models are estimated separately. The discussion of the estimation approach that follows is presented in the context of the AASHO model but it is equally applicable to the WesTrack model.

Equation (17) is the expression of the conditional expectation function of rut depth for section $i$ at time $t$, $E(RD_{it}|X_{it}, \beta)$. This function gives expected rut depth conditional on the set of regressors $X_{it} = (1, T_{il}, T_{i2}, T_{i3}, n_{Sl1}, n_{T11}, ..., n_{SlS}, n_{T1S}, F_{Li}, R_{i}, AL_{1i}, AL_{2i}, T_{li})'$ corresponding to the $i^{th}$ observation for section $i$ and on the vector of parameters $\beta = (\beta_{1}, ..., \beta_{13}, \beta_{14.1}, ..., \beta_{14.S})'$ where $S$ is the total number of sections. The model can be expressed as the following set of regression equations:
\[ RD_{it} = E(RD_i|X_{it}, \beta) + \epsilon_{it} \quad i = 1, \ldots, S, \quad t = 1, \ldots, T_i \]  

Where \( T_i \) is the number of observations for section \( i \) and \( \epsilon_{it} \) is the error term which is assumed to have mean 0 and constant variance \( \sigma^2 \). As can be seen from equation (17) this model is nonlinear in the variables and the parameters. Moreover, the vector \( X_{it} \) contains the whole history of loading through the \( n_{Si} \) and \( n_{Ti} \). All these factors make the estimation of the model complex.

When a data set consists of observations for different pavement units through time, several methods of pooling the data can be used. Such data sets are known as panel data sets. One could estimate separate cross-section regressions (each using observations for different pavement sections at the same point in time) or separate time-series regressions (each with observations for a single pavement section over time). However, if the model parameters are constant over time and over cross-sectional units, then more efficient parameter estimates (i.e., estimates with lower variance) can be obtained if all the data are combined and a single regression is run. This is the case if all observations are the result of a single underlying deterioration process.

The simplest technique is to combine all cross-section data and time series data and perform ordinary least-squares regression on the entire data set. In the present context, this is equivalent to performing a regression using equation (18) with \( E(RD_{it}|X_{it}, \beta) \) given by equation (17) and assuming that \( \beta_{i14} = \beta_{14} \) is the same for all \( i \). The problem with this procedure is that despite the reasonableness of the assumption that all the observations are the result of a single underlying process, some unobserved heterogeneity (unobserved and persistent pavement-specific factors) is still expected among different pavement sections.

An example of the kind of unobserved heterogeneity that will be accounted for in the model estimation is the initial cross-section profile.

The advantage of a panel data set over a cross-sectional data set is that it allows the researcher greater flexibility in modeling differences in behavior across individual units (Greene 1997). The two most widely used frameworks for modeling unobserved heterogeneity are called fixed- and random-effects respectively. Both approaches assume

17
that the unobserved heterogeneity can be captured through the constant term. In the fixed effects approach, the individual effect ($\beta_{i,t}$) is taken to be constant over time and specific to the individual pavement section $i$. This approach produces consistent results (consistent as $S$, the number of sections, approaches infinity) but it is costly in terms of the number of degrees of freedom lost, because a different intercept term is required for each pavement section.

An alternative approach is the random effects specification. Since the inclusion of different constant terms ($\beta_{i,t}$) represents a lack of knowledge about the model, it is natural to view the section specific constant terms as randomly distributed across pavement sections. Specifically, it is assumed that $\beta_{i,t} = \beta_{14} + u_i$, where $u_i$ is a random disturbance characterizing the $i^{th}$ section and is constant through time with mean $E(u_i) = 0$ and constant variance equal to $\sigma^2_{u}$. With these assumptions the random-effects specification is:

$$RD_{it} = \beta_{14} + \sum_{s=k}^{t} \beta_4 e^{-(\beta_1 (T_{is} + OR_{is}) + \beta_2 T_{is} + \beta_3 T_{is})} e^{\frac{T_{is}}{(1000)}} e^{\beta_9 N_{i,t} \Delta N_{is}}$$

Without a priori ways to distinguish between individual sections, treating the intercepts as random variables is a familiar expression of the researcher's ignorance (Ruud 2000). However, the approach yields consistent parameter estimates only if the regressors are uncorrelated with the individual effects $u_i$. This can be tested using a Hausman specification test (Greene 1997).

The estimation approach for linear models can be found, for example, in Greene (1997). The estimation of the model parameters here is more complicated since the model is nonlinear in the variables and the parameters, and the panel is unbalanced (that is, there are different numbers of observations for different pavement sections). Therefore, special routines had to be programmed for estimation of the model. Details of the estimation approaches mentioned above are given in Archilla and Madanat (2000a).
Both models were estimated using the fixed- and random-effects approaches. Since they gave almost identical results, only the results of the random effects-approach are presented here. For the same reason, it was unnecessary to perform a Hausman specification test to verify if the regressors are uncorrelated with the individual effects \( u_i \). The merits of the parameter estimation results for these two models have been presented elsewhere (Archilla and Madanat 2000a, 2000b). Thus, here we discuss only the most relevant results so we can make a comparison with the joint estimation results presented later.

Table 1 shows the estimation results using the random-effects approach for the WesTrack model. During estimation, \( \gamma_1 \) and \( \gamma_3 \) were constrained to be greater than or equal to zero since there is no reason to expect a reduction in rut depth for any value of \( G_i \), and particularly when \( G_i \) equals zero. As can be observed, the constraint \( \gamma_3 \geq 0 \) is binding. It can also be observed that only seven of the other nine parameters are statistically significant at a 5% significance level. However, the joint hypothesis, that \( \gamma_8 \) and \( \gamma_9 \) are jointly equal to zero produces a \( \chi^2 \) statistic of 33.85. Since the 5 percent critical value from the chi-squared distribution with 2 degrees of freedom is 5.99, the hypothesis that \( \gamma_8 \) and \( \gamma_9 \) are jointly equal to zero is rejected.

\( \gamma_7 \) is statistically significant from zero at a 5% level and it has the correct sign. That \( \gamma_7 \) is negative indicates that the material hardens over time. In other words, the same loading increment will produce a smaller increment in rut depth for more trafficked pavements. Finally, \( \gamma_8 \) is positive, which as described in the previous section, has the expected sign.

The estimates of \( \sigma_u^2 \), 7.43 and of \( \sigma_e^2 \), 2.98 indicate that the individual effects produce more than 70% of the variance. This shows that the size of the unobserved heterogeneity is significant. Finally, the estimated standard error of the regression \( (\sqrt{\sigma_u^2 + \sigma_e^2}) \), 3.23 mm, is within the accuracy with which rut depth was measured at WesTrack.
Table 1: Parameter estimation results for the WesTrack model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Asymptotic Estimate</th>
<th>Asymptotic t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_1 )</td>
<td>8.26e-6</td>
<td>3.39</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>5.67e-6</td>
<td>6.93</td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>0.00e+0</td>
<td>0.00</td>
</tr>
<tr>
<td>( \gamma_4 )</td>
<td>1.36e-4</td>
<td>11.25</td>
</tr>
<tr>
<td>( \gamma_5 )</td>
<td>8.05e+0</td>
<td>11.97</td>
</tr>
<tr>
<td>( \gamma_6 )</td>
<td>10.88e+0</td>
<td>5.08</td>
</tr>
<tr>
<td>( \gamma_7 )</td>
<td>-2.46e-6</td>
<td>-12.82</td>
</tr>
<tr>
<td>( \gamma_8 )</td>
<td>1.63e-6</td>
<td>2.15</td>
</tr>
<tr>
<td>( \gamma_9 )</td>
<td>-2.31e-6</td>
<td>-1.32</td>
</tr>
<tr>
<td>( \gamma_{10} )</td>
<td>4.74e-1</td>
<td>0.80</td>
</tr>
</tbody>
</table>

\( \sigma^2_e = 2.98 \quad \sigma^2_v = 7.43 \)

Number of observations = 860

Table 2 shows the estimation results for the AASHO model using the random-effects approach. All but one coefficient are statistically significant at a 5% significance level and they all have the expected signs. According to these results, the asphalt concrete layer is only 1.47 \( (\beta_1/\beta_2) \) times more effective in reducing rutting than the base layer. The contribution of the base is 1.13 \( (\beta_2/\beta_3) \) times the contribution of the subbase.

The coefficient \( \beta_7 = 1.79 \) indicates that a tandem axle load of 143.2 kN (32,220 lbs) has the same effect on rutting as an 80kN (18,000 lbs) single axle load. This is in agreement with the assumption made at the AASHO Road Test (HRB 1962) that an 18,000 lbs single axle was equivalent to a 32,000 lbs tandem axle. The coefficients \( \beta_5 = 2.87 \) and \( \beta_6 = 3.55 \) are significantly different from 4.0 (a very common assumption in pavement engineering) at a 5% significance level. This illustrates the advantage of not having presupposed, for example, a 4-power law for load equivalencies. Further, \( \beta_5 \) (single axle load) and \( \beta_6 \) (tandem axle load) are significantly different from each other at
a 5% significance level, which indicates the appropriateness of not having presupposed that $\beta_3 = \beta_6$.

Table 2: Parameter estimates of the AASHO model obtained using the random-effects approach.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Estimate</th>
<th>Asymptotic t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>7.49</td>
<td>15.33</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>5.10</td>
<td>28.24</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.51</td>
<td>28.93</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>8.46e-7</td>
<td>3.00</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>2.87</td>
<td>22.05</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>3.55</td>
<td>19.80</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>1.79</td>
<td>83.82</td>
</tr>
<tr>
<td>$\beta_8$</td>
<td>4.78e-3</td>
<td>24.59</td>
</tr>
<tr>
<td>$\beta_9$</td>
<td>-1.04e-6</td>
<td>-10.15</td>
</tr>
<tr>
<td>$\beta_{10}$</td>
<td>1.18e-5</td>
<td>27.61</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>3.89e-1</td>
<td>11.32</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>-1.20e-6</td>
<td>-24.33</td>
</tr>
<tr>
<td>$\beta_{14}$</td>
<td>1.06</td>
<td>6.51</td>
</tr>
</tbody>
</table>

$\sigma_e^2 = 4.23 \quad \sigma_\theta^2 = 8.42$

Number of observations = 9,478

The significance of $\beta_8$, the coefficient of the thawing index, shows the importance of the consideration of the environmental effects. $\beta_9$ has the expected negative sign and indicates that the underlying materials harden over time.
During estimation, the coefficients $\beta_{10}$ and $\beta_{11}$ were constrained to be greater or equal to zero since under normal conditions, the rut depth cannot decrease with loading. The constraint on $\beta_{11}$ is binding, indicating that the asphalt concrete mixes at the AASHO Road Test were not severely affected by high temperatures. Given the design characteristics of these mixes, this result is reasonable.

The estimated value of $\beta_{12}$ (0.39) was smaller than both $\beta_5$ and $\beta_6$. That is, near the pavement surface the effect of the load is not as important as it is at greater depths. Finally, $\beta_{13}$, which captures the hardening occurring in the asphalt concrete layer, also has the expected negative sign and it is of the same order of magnitude as $\beta_9$. This is surprising since the literature on material testing indicates that asphalt concrete mixes harden faster than unbound layers.

The estimates of $\sigma_u^2$, 8.42 and of $\sigma_e^2$, 4.23 indicate that the individual effects produce more than 50% of the variance. This again shows that the size of the unobserved heterogeneity is significant. The estimated standard error of the regression $(\sqrt{\sigma_u^2 + \sigma_e^2})$, 3.6 mm, is close to the accuracy with which rut depth was measured at the AASHO Road Test.

4 Joint estimation

Joint estimation is a methodology used for statistically combining different data sources. The methodology has been used, for example, by Ben-Akiva and Morikawa (1990) for combining revealed preference and stated preference data in travel demand modeling. In the area of infrastructure management, Feng-Yeu Shyr (1993) has used this methodology to develop a rail fatigue model by combining synthetic and field data. In all these cases, the different types of data have complementary characteristics that can be exploited using joint estimation. The methodology has not previously been used to model pavement performance.

The same motivation applies in our context. A model derived only from the AASHO Road Test is limited to the materials used in that test. Similarly, a model
developed from the WesTrack Road Test is limited to the layer thicknesses and axle loads used in that test. From the AASHO Road Test data set, the effect of axle loads and layer thicknesses can be identified but not the effect of the characteristics of the asphalt concrete mix. In contrast, from the WesTrack data set, the effect of the characteristics of the asphalt concrete mix can be identified but not the effect of load magnitudes or layer thicknesses.

The important features of the methodology are:

- Bias correction
- Efficiency
- Identification

Bias correction is better explained in the context of trying to estimate a model with field and experimental data. Since one is ultimately interested in predicting the pavement behavior in the field, the estimation of a correctly specified model using field data alone would yield unbiased parameter estimates. In contrast, the estimation of a properly specified model from experimental data would most likely yield biased parameter estimates (the parameters would be biased when predicting field pavement behavior). Joint estimation in this context allows the estimation of a rutting model that includes bias parameters for the experimental data source. Thus, a correction for bias can be introduced when using the model developed from experimental data to predict field behavior. This feature is not relevant in the present context because both data sources are experimental and there is no a priori reason to believe than one or the other represent field conditions better.

Statistical efficiency (lower variance of the parameter estimates) is obtained because the parameters are estimated from all the available data.

Identification is related to the complementary characteristics of both data sources already mentioned. By using joint estimation, trade-offs among attributes that are not identifiable from one of the data sources can be identified from the other data source.

The following is a brief description of the joint estimation method in the context of a rutting progression model. The two data sources are labeled A and B. Define
$RD^A =$ rut depth measurement for data source A,  
$RD^B =$ rut depth measurement for data source B,  
$x^A, x^B =$ vector of explanatory variables shared by both data sources,  
$w^A =$ vector of explanatory variables unique to the A data source,  
$z^B =$ vector of explanatory variables unique to the B data source,  
$\beta^A, \beta^B =$ vectors of model parameters for $x^A$ and $x^B$ respectively,  
$\alpha =$ vector of model parameters for $w^A$, and  
$\gamma =$ vector of model parameters for $z^B$,  

such that,  

$$E(RD^A \mid x^A, w^A) = g^A (\beta^A, x^A, \alpha, w^A)$$  

$$E(RD^B \mid x^B, z^B) = g^B (\beta^B, x^B, \gamma, z^B)$$  

where $E(RD \mid \cdot)$ is the rut depth conditional expectation function, which has a functional form $g(\cdot)$.  

Since, in principle, the parameter estimation is always the result of the optimization of some objective function, the true parameters $\beta$ can be estimated by forming the joint objective function as the sum of the objective functions for the individual data sources and optimizing it with respect to all the parameters$^3$. The estimation also produces estimates of the parameters $\alpha$ and $\gamma$ of the variables not shared by both data sources. When using Generalized Least Squares (GLS), the objective function is a weighed sum of square residuals and the optimization is a minimization problem.  

Before applying joint estimation to the AASHO and WesTrack data sources, it is necessary to examine their compatibility. Archilla and Madanat (2000a) compared factors such as rut depth measurement technology, aggregates, asphalt type, load magnitude, tire inflation pressures, target vehicles' speeds, and traffic wander in both tests.  

---  

$^3$ The summation of objective functions is justified because it is assumed (and reasonable) that the error terms for the two data sources are independent.
From all the factors above, it was concluded that although there are some differences between the two tests, none of them seems to be so influential as to render the deterioration mechanisms of the asphalt concrete layer in both tests different. In addition to the aforementioned variables, there are many other factors such as tire types, aggregate mineralogy, etc, that also influence the deterioration mechanism but cannot be accounted for in the models. If all these factors have small influences, they will be captured by the error term assumptions. Thus, although one test may be biased with respect to the other, it is expected that the bias is smaller than the precision with which the parameters in common between the two models can be estimated. From the above observations, it is reasonable to assume that both data sources are compatible for joint estimation.

5. Joint estimation results

The specifications of the models to be estimated jointly are the same specifications given before. These were presented in equations (10) and (17).

Given the compatibility of the two data sources, two parameters are shared by these models. However, only one of them can be identified from both models. The parameter $\beta_{12}$ is common to both specifications but it can only be identified from the AASHO data source (recall that to estimate the WesTrack model this parameter was set equal to 0.39). In addition, the parameter $\beta_{13}$ in the AASHO model is equivalent to the parameter $\gamma$ in the WesTrack model. This parameter measures the hardening of the asphalt concrete layer.

As mentioned before, of the three main features of joint estimation, identification is the most relevant for this research. Joint estimation allows the simultaneous identification of the effects of layer thicknesses, asphalt concrete mix characteristics, high temperatures, and thawing.

Statistical efficiency is obtained because the parameters are estimated from all the available data. However, the models used for joint estimation are a constrained
version of the models estimated individually. This is because the parameters $\beta_{13}$ and $\gamma$ are constrained to have the same value and no bias parameters are specified. This may result in some parameters having higher standard errors when the models are estimated jointly.

Table 3 presents the joint estimation results using the random-effects approach. Many of the interpretations are similar to the ones given before. Thus, only some points are elaborated here.

The first important difference that can be noted with the results from the individual estimations are the changes in the material hardening parameters $\beta_9$ and $\beta_{13}$ ($\gamma$). Table 4 shows the estimated parameters for three different cases. The values shown in the first and second columns are the estimates of parameters $\beta_9$ and $\beta_{13}$ when the AASHO model is estimated individually excluding and including the overlay information, respectively. The values shown in the third column are the estimates of $\beta_9$ and $\beta_{13}$ from joint estimation approach. When the AASHO model was estimated without the overlay information, the estimated value of $\beta_9$ was zero, which means that there was no hardening in the underlying layers. This is not satisfactory since the results of laboratory experiments indicate that the unbound materials commonly used in the underlying layers harden with loading. Consequently, there seems to be an overestimation of the effects of thawing during the second thawing period for the overlaid sections. When the model was estimated with the overlay information, the estimated value of $\beta_9$ was almost the same as the estimated value of $\beta_{13}$. This is not very satisfactory either because results from laboratory experiments show that asphalt concrete material harden faster than unbound materials. In this case, the hardening of the underlying layers may have been overestimated and consequently the effects of thawing during the second thawing period may have been underestimated.

When the AASHO model is estimated jointly with the WesTrack model, the estimated value of $\beta_{13} = \gamma$ falls within values for the AASHO model estimated individually with and without the overlay information that is intuitively correct. Another important benefit is that the information about the rutting of the asphalt concrete layer in
WesTrack, allows a better allocation of how much rutting is originating in the underlying layers and how much rutting is originating in the asphalt concrete layer in the AASHO model. The hardening parameters play an important role in this allocation. After joint estimation, the estimated value of $\beta_{l3}$ is now about ten times larger than the estimated value of $\beta_9$. This is consistent with the observation above that asphalt concrete materials harden faster than unbound materials.

The estimated values of $\beta_1$, $\beta_2$, $\beta_3$ are higher than before, but these parameters are linked to $\beta_4$, which is also higher. The ratio of the estimated values of $\beta_1$ and $\beta_2$ is now 1.95, higher than the previously estimated ratio of 1.47. However, the ratio $\beta_2/\beta_3$ is only changed from 1.13 to 1.11. This is again reasonable since the WesTrack data set contains better information about the behavior of asphalt concrete mixes. Thus, with joint estimation one can expect changes in the estimated effectiveness of a unit thickness of asphalt concrete material with respect to the estimated effectiveness of a unit thickness of the base and subbase materials. However, the relative effectiveness of the base and subbase materials should be unaffected, which was the case here.

The estimated values of $\beta_5$ and $\beta_6$ are lower than before. Again, they are both significantly different from 4.

With respect to the WesTrack model, $\gamma_8$ is significantly different from zero at a 5% significance level but $\gamma_9$ is not. The joint hypothesis that both coefficients are equal to zero produces a $\chi^2$ statistic of 36.6. Since the 5 percent critical value from the chi-squared distribution with 2 degrees of freedom is 5.99, the hypothesis that $\beta_8$ and $\beta_9$ are jointly equal to zero is rejected. Thus, the joint estimation results reinforce the importance of the variable “in-place air voids” in the model specification.
Table 3: Joint estimation results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>AASHO Model</th>
<th>WesTrack Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameter estimate</td>
<td>Asymptotic t-statistic</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>9.28</td>
<td>16.2</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>4.77</td>
<td>23.3</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>4.29</td>
<td>24.0</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>1.71e-6</td>
<td>3.3</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>2.44</td>
<td>25.2</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>2.86</td>
<td>24.5</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>1.68</td>
<td>61.0</td>
</tr>
<tr>
<td>$\beta_8$</td>
<td>4.26e-3</td>
<td>23.7</td>
</tr>
<tr>
<td>$\beta_9$</td>
<td>-2.28e-7</td>
<td>-3.7</td>
</tr>
<tr>
<td>$\beta_{10}$</td>
<td>1.70e-5</td>
<td>22.5</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>0.00</td>
<td>- -</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>0.56</td>
<td>12.1</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>-2.11e-6</td>
<td>-22.1</td>
</tr>
<tr>
<td>$\beta_{14}$</td>
<td>0.76</td>
<td>4.6</td>
</tr>
</tbody>
</table>

$\sigma^2_e = 5.81$ $\sigma^2_u = 8.18$ $\sigma^2_e = 3.00$ $\sigma^2_u = 7.59$

Number of observations = 9504 Number of observations = 860

Total number of observations = 10364
Table 4: Comparison of estimates from three different estimations.

<table>
<thead>
<tr>
<th></th>
<th>AASHO No Ovl</th>
<th>AASHO Ovl.</th>
<th>Joint Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_9$</td>
<td>0.00</td>
<td>-1.04 x 10^{-6}</td>
<td>-2.28 x 10^{-7}</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>-2.21 x 10^{-6}</td>
<td>-1.20 x 10^{-6}</td>
<td>-2.11 x 10^{-6}</td>
</tr>
<tr>
<td>$\beta_{1}/\beta_{2}$</td>
<td>1.47</td>
<td>1.95</td>
<td></td>
</tr>
<tr>
<td>$\beta_{2}/\beta_{3}$</td>
<td>1.13</td>
<td>1.11</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6 shows the observed and predicted rut depth values for two of the sections at the AASHO Road Test. The values after the word "Design" in the section titles indicate the thicknesses in meters of the asphalt concrete, base, and subbase layers. For overlaid sections, the thickness of the asphalt concrete overlay plus a '+' sign appears before the original asphalt concrete thickness. The axle loads are expressed in kN (KiloNewtons) and S or T indicate that the pavement was loaded with a single or tandem axle respectively.

It can be observed that the model replicates the rutting trends quite well for both section 257, which was not overlaid, and for section 303, which was overlaid. The dashed line in the figure shows how much of the rutting is attributed to the asphalt concrete layer. The difference between the two lines in the figure represents the rutting attributed to the underlying layers. It can be seen that most of the rutting attributed to the underlying layers occurs during the thawing periods. For these two sections, about 50% of the rutting is attributed to each source.
Figure 6: Predicted total rut depth and rut depth originating in the asphalt concrete layer vs. time with the parameter estimates from the joint estimation for two AASHO sections.

Figure 7 shows the observed vs. predicted rut depth values in eight of the 26 sections in WesTrack. It can be observed in these figures that in most cases, the model replicates the pavement behavior well. This point deserves particular emphasis because the pavement behavior is replicated well over a wide range of loads and pavement structures, for asphalt concrete mixes of different characteristics and in two different environments.
Figure 7: Observed and predicted rut depth vs. time for eight WesTrack sections. The lowest end of the load range deserves particular attention.

The lowest loads are usually ignored in the calculations of ESALs, a consequence of the fourth power ordinarily assumed for load equivalencies. However, the estimated value of $\beta_{l2} = 0.56$ indicates that even small loads produce non-negligible rutting in the asphalt concrete layer. This can be seen in Figure 8 for two AASHO sections with axle loads of 27 kN. Although the axle loads were too small relative to the pavement structures to produce any significant rutting in the underlying layers during the thawing periods, they produced significant rutting in the asphalt concrete layer (about 70 to 80% of the rutting is attributed to this source). It must be noted that the 0.56 value most likely accounts for the effects of tire pressures, which for AASHO and in general are positively correlated with the loads. Near the surface, the stresses are determined by the tire contact pressures more than by the loads. The lowest load applied at the AASHO Road Test was 9 kN. Even this small load produced a rut depth of about 5 mm at the end of the test for
most sections. For passenger cars, the model may give an upper bound on their effect since these vehicles have a smaller width and smaller tire imprints than trucks. Thus, more vehicles passages are likely to be necessary to obtain the same coverage produced by trucks. In addition, the cars tire inflation pressures are lower than for trucks.

As mentioned before, of the three main features of joint estimation, the most relevant in this context is identification. After having estimated the model jointly from both data sources, the following model can be used for the estimation of rut depths:

\[ RD_{lt} = \beta_{14} + \sum_{s=k}^{t} \beta_4 e^{-\left( \beta_1 (T_{lt} + OT_{lt}) + \beta_2 T_{lt} + \beta_3 T_{lt} \right)} e^{\frac{\beta_4}{1000} N_{lt}} e^{\beta_5 N_{lt}} \Delta N_{lt} \]

\[ + \sum_{s=k}^{t} m_{ls} e^{\gamma N_{lt}} \Delta N_{lt} \]

Where all the terms are as previously defined. Notice that instead of using \( m^4_{lt} \) we can now use the more complete term \( m_{lt} \) in equation (9), which accounts for asphalt concrete mix characteristics.
Figure 8: Predicted total rut depth and rut depth originating in the asphalt concrete layer vs. time with the parameter estimates from the joint estimation for two AASHO sections trafficked with 27 kN loads.

6. Discussion

Even after using joint estimation, the model still has some limitations. As pointed out in the previous two chapters, the experimental data may not represent the true deterioration mechanism of in-service pavements because of differences in factors such as traffic wander, traffic speed, and material aging. In addition, the results are limited to the subgrade, subbase, and base materials used at the AASHO and WesTrack road tests. Further, it is also limited to asphalt concrete mixes with a nominal maximum aggregate size of 19 mm and to asphalt cements PG64-22.

Another factor not included in the model is the effect of cracking. When a pavement cracks, it allows the ingress of water from the pavement surface to the underlying layers with the consequent detrimental effects on rutting performance. Unfortunately, the information regarding severe cracking at the AASHO road test was limited because either the sections were taken out of the test or they were overlaid. A similar situation occurred in WesTrack. It should be noted however that the effects of cracking on rutting performance are most significant only for regions with intense precipitation and poor maintenance. Since the problem is essentially the same as with thawing, which is the presence of a considerable amount of water in the underlying layers interacting with traffic loads, it could be taken into account in a similar fashion as the thawing index. Further, if preventive maintenance such as crack sealing is performed timely, the problem of cracking can be greatly reduced.

Most of the limitations mentioned above are common to all existing empirical models. In addition, some of these limitations are not so severe, particularly in a pavement management context where the predictions are limited to a few years. For example, the subgrade material at the AASHO road test is representative of weak subgrades that are affected by frost. For stronger subgrades, one can bound the prediction of rutting by using the complete specification for an upper bound and the
specification for rutting in the asphalt concrete mix for a lower bound. In addition, if no source of water is present and there is no freeze-thaw cycles, an estimate of rutting in the asphalt concrete mix may be all that is necessary since most of the rutting will occur in that layer. Finally, the asphalt cement used in the mix does not play a role in the rutting performance for pavement sections in very cold regions, unless very soft asphalt is used to avoid cracking problems.

To use the model for prediction, the user needs to have the loading and environmental information in two-week periods. The interval between observations at the AASHO Road test was 2 weeks. In WesTrack the interval between observations was variable but on average was also approximately 2 weeks. Nowadays, to obtain the required environmental information for two-week periods should pose no problem, at least in the United States. For example, the LTPP database (Ostrom et. al., 1997) contains the information for thousands of weather stations throughout the United States and Canada. The loading information is also simple to obtain for routes with weigh in motion (WIM) scales. But even without the detailed information from WIM scales, the analyst can usually obtain some information about the traffic distribution, at least seasonally. Thus, an estimate of the total loading can be prorated accordingly. The use of 26 periods for every year may seem a burden for practitioners. However, this is necessary to obtain precise estimates of rut depth. Most mechanistic-empirical models require at least the same level of detail to have some merit. In addition, this model can be programmed in a spreadsheet, which is still much simpler than the computation of strains and stresses in a mechanistic-empirical approach.

As already mentioned, the extension of the model to include the effects of overlays greatly expands its scope of application. Another major consequence of including the information for overlays is the different estimates of the parameters representing the hardening of the materials (β9 and β13). Precise estimates for these parameters are considered important because of the short period of time during which the experiment took place. If one wants to predict the pavement behavior over longer periods, it is essential to model material hardening correctly.
7. References


35

