

Toroidal Rotation Driven by the Polarization Drift

C. J. McDevitt,^{1,*} P. H. Diamond,¹ Ö. D. Gürcan,^{1,†} and T. S. Hahm²

¹*Center for Astrophysics and Space Sciences and Department of Physics, University of California at San Diego, La Jolla, California 92093-0424, USA*

²*Princeton University, Princeton Plasma Physics Laboratory, Princeton, New Jersey 08543-0451, USA*

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Starting from a phase space conserving gyrokinetic formulation, a systematic derivation of parallel momentum conservation uncovers a novel mechanism by which microturbulence may drive intrinsic rotation. This mechanism, which appears in the gyrokinetic formulation through the parallel nonlinearity, emerges due to charge separation induced by the polarization drift. The derivation and physical discussion of this mechanism will be pursued throughout this Letter.

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Turbulent momentum transport is widely recognized as a fundamental component in the description of numerous physical systems including accretion disk formation, the solar tachocline, galactic dynamics, and laboratory plasmas. The study of turbulent momentum transport in the context of strongly magnetized plasmas has recently emerged as a particularly interesting example. Recent experimental investigations have observed significant rates of toroidal rotation in the absence of external momentum sources [1–6]. The presence of this “spontaneous” or “intrinsic” toroidal rotation is inconsistent with the purely diffusive transport of toroidal momentum and hence provides a strong experimental impetus for the identification of nondiffusive components in the turbulent momentum flux.

While neoclassical theory remains a viable candidate for explaining specific manifestations of intrinsic rotation [7], flow generation induced by turbulent stresses provides the most natural candidate for many plasma regimes. From this perspective, intrinsic rotation corresponds to a specific manifestation of the more general phenomenon of mean field generation by turbulent stresses. Familiar examples of the turbulent generation of mean fields include the alpha effect in magnetic dynamo theory [8], its 3D incompressible fluid analogue, the anisotropic kinetic alpha effect [9], flow generation due to the inverse cascade of energy in 2D incompressible hydrodynamic turbulence [10,11], as well as zonal flow generation in strongly magnetized plasmas [12]. While these examples span a broad range of physical phenomena, mean field theoretical methods have proven an invaluable tool in their description. Indeed, as discussed in Table I, a concise, but by no means superficial summary of the current status of the theory of intrinsic rotation can be elucidated via a mathematical analogy with the classic mean field formulation of the alpha effect in magnetic dynamo theory.

More explicitly, the first term in Π_{EB} (see Table I), which we will refer to as a residual stress, has an ideal form for driving toroidal rotation. A simple mean field

calculation of the residual stress component of Π_{EB} demonstrates that this term is nonvanishing only when $\langle \delta E_{\parallel} \delta E_{\theta} \rangle \sim \langle k_{\parallel} k_{\theta} \rangle \neq 0$ [13], where k_{\parallel} is the wave number parallel to the equilibrium magnetic field, and θ is the coordinate associated with the shorter of the two periodic directions of a torus. This constraint, often referred to as k_{\parallel} symmetry breaking [14–16], can be understood graphically by considering plots of the radial profiles of parallel flow perturbations (computed in the fluid limit) and $\mathbf{E} \times \mathbf{B}$ flow perturbations in a sheared slab geometry. From Fig. 1(c) it is clear that in this context the k_{\parallel} symmetry breaking constraint is equivalent to the requirement that the scalar potential possess a component with odd parity about the surface defined by $x = r - r_{m,n}$, where $r_{m,n}$ is defined as $m = nq(r_{m,n})$, m and n are (respectively) the poloidal and toroidal modes numbers, and q is the safety factor. $\mathbf{E} \times \mathbf{B}$ shear [13] has been shown to break the even parity of the scalar potential about the rational surface in the absence of parallel flow, and hence induce a residual stress contribution to the momentum flux. Similarly, it has recently been shown that the breaking of up-down symmetry of the equilibrium magnetic topology is also capable of inducing a residual stress term [17]. Our focus within this Letter is on the identification of a novel mechanism which is capable of inducing a residual stress term in the absence of both $\mathbf{E} \times \mathbf{B}$ shear and broken up-down symmetry of the magnetic topology.

Within the context of gyrokinetics (see Ref. [18] for a review), the presence of this additional nondiffusive contribution to the turbulent momentum flux can be motivated by considering the general structure of the gyrokinetic equation. The first moment of the gyrokinetic equation is given by

$$\begin{aligned} \frac{\partial \langle P_{\parallel} \rangle}{\partial t} + \nabla_{\perp} \cdot \left\langle \sum_s m_s \int d^3 \bar{v} v_{\parallel} \bar{\mathbf{X}}_{\perp} \delta F_s \right\rangle \\ = \left\langle \sum_s m_s \int d^3 \bar{v} \bar{V}_{\parallel} \delta F_s \right\rangle, \end{aligned} \quad (1a)$$

TABLE I. Current status of mean field formulation of intrinsic rotation.

	α effect	Mean parallel rotation
Form of mean nonlinearity:	$\varepsilon = \langle \delta \mathbf{v} \times \delta \mathbf{B} \rangle$	$\Pi_{\text{EB}} = m_i \langle \int d^3 \bar{v} v_{\parallel} \delta F_i \delta v_r \rangle$
Fluctuation equation:	Induction equation linear in $\delta \mathbf{B}$	Gyrokinetic equation linear in δF_i
After linearization:	$\varepsilon_i \approx \alpha_{ij} B_j^{(0)} + \beta_{ijk} \frac{\partial B_i^{(0)}}{\partial x_k} + \dots$	$\Pi_{\text{EB}} \approx S_{\text{EB}} + V_c v_{\parallel}^{(0)} - \chi_{\phi} \frac{\partial v_{\parallel}^{(0)}}{\partial r}$
$(\alpha_{ij}, S) \neq 0$ if:	Velocity field lacks reflectional symmetry about a suitable origin ($\mathbf{x}' = -\mathbf{x}$)	Scalar potential lacks mirror symmetry about $r - r_{m,n} = 0$ surface
Critical feedback loop:	Toroidal field generates a poloidal field component	Sheared $\mathbf{E} \times \mathbf{B}$ flow generates a parallel flow component

where

$$\begin{aligned} \dot{\mathbf{X}} &= \hat{b} v_{\parallel} + \frac{c}{B} \hat{b} \times \nabla J_0(\lambda) \delta \phi, \\ \dot{V}_{\parallel} &= -\frac{q_s}{m_s} \hat{b} \cdot \nabla J_0(\lambda) \delta \phi. \end{aligned} \quad (1b)$$

Here, the subscript s represents the species of particle, J_0 is a Bessel function of the first kind, $\int d^3 \bar{v} \equiv 2\pi \int d\mu dv_{\parallel} B$, $\lambda \equiv k_{\perp} \rho_{\perp}$, $\mu \equiv (1/2)v_{\perp}^2/B$, $\rho_{\perp} \equiv |v_{\perp}|/\omega_{cs}$, $\omega_{cs} \equiv q_s B/(m_s c)$, $\rho_i \equiv v_{\text{thi}}/\omega_{ci}$, $v_{\text{thi}} \equiv \sqrt{T_i/m_i}$, $P_{\parallel} \equiv \sum_s m_s \int d^3 \bar{v} v_{\parallel} F_s$, and for simplicity we consider cylindrical geometry. The second term on the left-hand side (lhs) of this expression can be shown to induce momentum transport via $\mathbf{E} \times \mathbf{B}$ convection of parallel momentum (note that for simplicity we will not distinguish the parallel and toroidal directions since $B_{\phi} \gg B_{\theta}$ in tokamak plasmas), and has already been the subject of extensive study [13,19–27]. The right-hand side (rhs) of Eq. (1a) can be rewritten

$$f_{\parallel} = -\left\langle \sum_s q_s \int d^3 \bar{v} \delta F_s \hat{b} \cdot \nabla J_0(\lambda) \delta \phi \right\rangle. \quad (2)$$

Here we emphasize that δF_s represents the distribution of gyrocenters rather than particles. An expression linking the gyrocenter distribution function to the scalar potential can be approximated by [18,28]

$$\varepsilon_{\perp} \nabla_{\perp} \cdot (n_i \nabla_{\perp} \delta \phi) = -4\pi n_0 \sum_s q_s \int d^3 \bar{v} J_0(\lambda) \delta F_s, \quad (3)$$

where $\varepsilon_{\perp} \approx \rho_s^2/\lambda_{\text{De}}^2$, $\rho_s \equiv c_s/\omega_{ci}$, $c_s \equiv \sqrt{T_e/m_i}$, λ_{De} is the electron Debye length, we have taken the limit $k_{\perp}^2 \lambda_{\text{De}}^2 \ll 1$, and we have assumed $k_{\perp}^2 \rho_i^2 < 1$, so that terms of order $\mathcal{O}(k_{\perp}^4 \rho_{\perp}^4)$ may be neglected. Thus, it is clear that even in the limit of negligible Debye length, gyrocenter quasineutrality is not satisfied. The violation of gyrocenter quasineutrality is well known to follow from the polarization drift appearing in the theory as an effective polarization shielding in the gyrokinetic Poisson equation [Eq. (3)], rather than explicitly appearing in the gyrocenter equations of motion.

Because of the violation of gyrocenter quasineutrality, Eq. (2) will in general be nonvanishing, and hence contribute to the evolution of parallel momentum. In order to motivate the form that Eq. (2) is likely to take, it is useful to briefly discuss a mathematical analogy with a dielectric medium. In particular, Eq. (3) has an approximately analogous form to the Poisson equation for a dielectric medium

[29]; i.e., the latter can be written as

$$\varepsilon \nabla^2 \delta \phi = -4\pi \sum_s q_s \int d^3 v \delta f_s. \quad (4)$$

For an electrostatic dielectric medium, f_{\parallel} can be written in terms of the Maxwell stress tensor, i.e., $f_{\parallel} = \partial \sigma_{x,\parallel} / \partial x$, where $\sigma_{x,\parallel} \equiv \varepsilon / (4\pi) E_x E_{\parallel}$, x is a radial variable, and we are considering only radial fluxes. The approximately analogous mathematical structure of the gyrokinetic Poisson equation to Eq. (4) suggests that an analogue to the Maxwell stress tensor is likely present within gyrokinetics. The derivation and analysis of this contribution to the turbulent momentum flux is pursued throughout the remainder of this Letter.

In order to derive an expression for the evolution of parallel momentum it is useful to separate the temporal and perpendicular spatial scales into a set of “fast” variables associated with the rapidly varying microturbulence, which we will denote by (\mathbf{x}_{\perp}, t) , and a set of “slow” variables, typical of equilibrium profiles, denoted by (\mathbf{X}_{\perp}, T) , where these two sets of variables should be regarded as independent [30]. This separation allows for the decomposition of the perpendicular space and time derivatives in the form: $\nabla_{\perp} \rightarrow \nabla_{\perp}^{(0)} + \varepsilon \nabla_{\perp}^{(1)}$, $\partial / \partial t \rightarrow \varepsilon \partial / \partial t + \varepsilon^2 \partial / \partial T$, with the parallel derivative ordered as $\hat{b} \cdot \nabla \sim \varepsilon \hat{b} \cdot \nabla$, where $\varepsilon \sim \rho_i / L_n$ and $L_n^{-1} \equiv -d \ln n_0 / dx$. Note that since functions of only the large scales are assumed to be uniform along magnetic field lines, there is no need to introduce an analogous decomposition in the parallel direction. Similarly, the fluctuating fields are described as follows:

$$\begin{aligned} \delta \phi &= \varepsilon \delta \phi^{(1)}(\mathbf{x}, t, \mathbf{X}_{\perp}, T) + \varepsilon^2 \delta \phi^{(2)}(\mathbf{x}, t, \mathbf{X}_{\perp}, T) + \dots, \\ F_s &= F_s^{(0)}(\mathbf{X}_{\perp}) + \varepsilon \delta F_s^{(1)}(\mathbf{x}, t, \mathbf{X}_{\perp}, T) \\ &\quad + \varepsilon^2 \delta F_s^{(2)}(\mathbf{x}, t, \mathbf{X}_{\perp}, T) + \dots, \end{aligned}$$

where $F_s^{(0)}$ is taken to be a Maxwellian. Furthermore, we may define a spatial and temporal average over the fast scales such that $\langle \delta \psi(\mathbf{x}, t, \mathbf{X}_{\perp}, T) \rangle = 0$, but functions of only slow variables are left unaltered, i.e., $\langle \psi(\mathbf{X}_{\perp}, T) \rangle = \psi(\mathbf{X}_{\perp}, T)$. Similarly, averages over the fast scales annihilate derivatives of fast variables (as well as derivatives along magnetic field lines) but commute with slow derivatives, i.e., $\langle \nabla_{\perp}^{(0)} \psi \rangle = \langle \hat{b} \cdot \nabla \psi \rangle = 0$, but $\langle \nabla_{\perp}^{(1)} \psi \rangle = \nabla_{\perp}^{(1)} \langle \psi \rangle$.

Utilizing the two-scale perturbative framework discussed above, Eq. (1a) can be simplified via an expansion in ε . Considering the second term on the lhs of Eq. (1a), this term can be simplified as

$$\begin{aligned} & \left\langle \nabla \cdot \sum_s m_s \int d^3 \bar{v} v_{\parallel} \dot{\mathbf{X}} F_s \right\rangle \\ & \approx \frac{c}{B} \nabla_{\perp}^{(1)} \cdot \left\langle \sum_s m_s \int d^3 \bar{v} v_{\parallel} \delta F_s^{(1)} (\hat{b} \times \nabla_{\perp}^{(0)} J_0^{(0)}(\lambda) \delta \phi^{(1)}) \right\rangle. \end{aligned} \quad (5)$$

Thus, the lowest order surviving term enters at $\mathcal{O}(\varepsilon^3)$, and can be recognized as describing momentum transported by $\mathbf{E} \times \mathbf{B}$ convection. Turning now to the rhs of Eq. (1a), to third order, f_{\parallel} vanishes, such that the third order momentum conservation theorem may be written

$$\frac{\partial \langle \delta P_{\parallel} \rangle}{\partial T} + \nabla_{\perp}^{(1)} \cdot \Pi_{\text{EB}}^{(2)} = 0, \quad (6)$$

where $\nabla_{\perp}^{(1)} \cdot \Pi_{\text{EB}}^{(2)}$ is defined by Eq. (5). Equation (6) nominally provides the lowest order nontrivial expression for the evolution of parallel momentum. However, as discussed above, the residual stress component of $\Pi_{\text{EB}}^{(2)}$ vanishes for $\langle k_{\parallel} k_{\theta} \rangle = 0$. This constraint can significantly reduce the magnitude of the residual stress component of $\Pi_{\text{EB}}^{(2)}$. More specifically, for $k_{\theta} \rho_s \lesssim 1$, $\langle k_{\parallel} \rho_s k_{\theta} \rho_s \rangle = \langle (k_{\parallel}/k_{\theta}) k_{\theta}^2 \rho_s^2 \rangle \sim \langle k_{\parallel}/k_{\theta} \rangle$, which in the presence of mean $\mathbf{E} \times \mathbf{B}$ shear typically scales as $\langle k_{\parallel}/k_{\theta} \rangle \sim x_0/L_s \approx A_s \omega_{ci}^{-1} v'_E$, where v'_E is the mean $\mathbf{E} \times \mathbf{B}$ flow shear, $L_s^{-1} \equiv \text{sgn}(B_{\theta})(r/R)(1/|q|)(q'/q)$, $q \equiv rB_{\phi}/(RB_{\theta})$, and A_s is a mode dependent parameter whose magnitude based on simple models typically satisfies $L_n/L_s < |A_s| < L_s/L_n$ (see Refs. [13,31], for example). The magnitude of the radial electric field shear can be estimated by the diamagnetic term in the radial force balance equation, i.e., $v'_E \approx v_{\text{th}i} \rho_i (1 + \eta_i)/L_n^2$ where $\eta_i \equiv L_n/L_{Ti}$ and $L_{Ti}^{-1} \equiv -d \ln T_i/dx$. Thus we may estimate the spectrally averaged k_{\parallel}/k_{θ} by $\langle k_{\parallel}/k_{\theta} \rangle \approx A_s (\rho_i/L_n)^2 (1 + \eta_i)$. Hence, while a naive ordering would suggest $k_{\parallel}/k_{\theta} \sim \varepsilon$, it is clear that after averaging over the turbulence spectrum $\langle k_{\parallel}/k_{\theta} \rangle \sim \varepsilon^2$. This simple analysis suggests that the contribution to the residual stress emanating from $\Pi_{\text{EB}}^{(2)}$ often appears one order higher than would be naively anticipated. Thus, it is necessary to extend the derivation to fourth order.

At fourth order, f_{\parallel} can be approximated as (see Ref. [31] for details)

$$\begin{aligned} f_{\parallel}^{(4)} \approx & \frac{1}{4\pi} \nabla_{\perp}^{(1)} \cdot \langle \epsilon_{\perp} \delta \mathbf{E}_{\perp}^{(1)} \delta E_{\parallel}^{(2)} \rangle - \frac{1}{2} \nabla_{\perp}^{(1)} \\ & \cdot \left\langle \sum_s q_s \int d^3 \bar{v} \rho_{\perp}^2 (\nabla_{\perp}^{(0)} \delta F_s^{(1)}) \delta E_{\parallel}^{(2)} \right\rangle, \end{aligned} \quad (7)$$

where $\delta \mathbf{E} \equiv -\nabla \delta \phi$ and we have made the approximation $J_0^{(1)}(\lambda) \approx (1/4) \rho_{\perp}^2 (\nabla_{\perp}^{(1)} \cdot \nabla_{\perp}^{(0)} + \nabla_{\perp}^{(0)} \cdot \nabla_{\perp}^{(1)})$. Equation (7) provides the relevant generalization of the electrostatic Maxwell stress tensor to the gyrokinetic framework. The

presence of the first term in Eq. (7) is not surprising in light of the close analogy of the gyrokinetic Poisson equation with its counterpart in a dielectric medium, whereas the second term allows for coupling to perpendicular pressure fluctuations.

Equation (7) may be evaluated via linearization of the gyrokinetic equation. Assuming perturbations of the form $[\delta \phi(\mathbf{x}, t), \delta F_s(\mathbf{x}, t)] = \sum_{m,n} [\delta \phi_{m,n}(x), \delta F_{m,n}^{(s)}(x)] \times \exp[i(m\theta - nz/R - \omega_k t)]$, the linearized gyrokinetic equation can be written

$$\begin{aligned} \delta F_k^{(s)}(x) = & -g_k \left[\frac{c}{B} k_{\theta} \left(\frac{\partial F_s^{(0)}}{\partial x} - \frac{\partial v_{\parallel}^{(0)}}{\partial x} \frac{\partial F_s^{(0)}}{\partial v_{\parallel}} \right) - \frac{q_s}{T_s} v_{\parallel} k_{\parallel} F_s^{(0)} \right] \\ & \times J_0(\lambda) \delta \phi_k(x), \end{aligned} \quad (8)$$

where the plasma response function is given by $g_k = (\omega_k - v_{\parallel} k_{\parallel})^{-1}$, we have defined $k_{\theta} = m/r$, $k_{\parallel} = (B_{\theta}/rB)(m - nq)$, and to simplify the notation we no

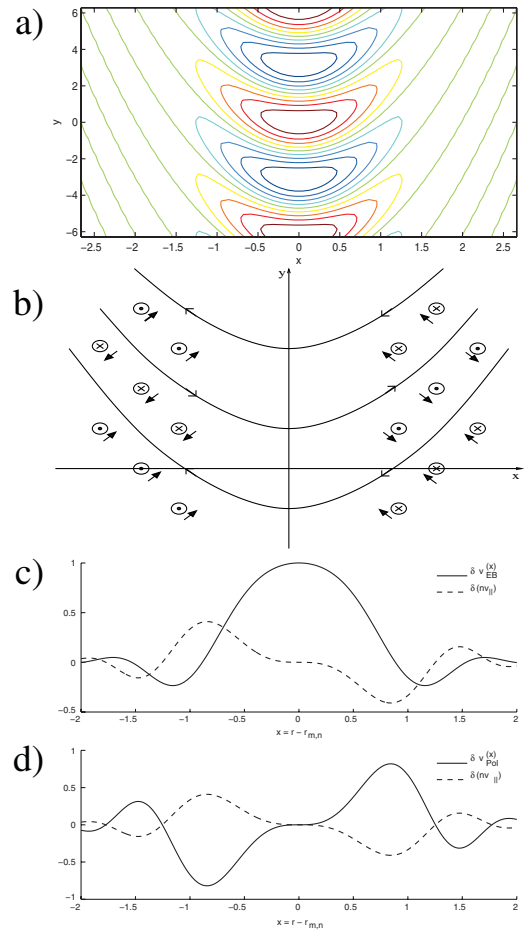


FIG. 1 (color). (a) Contours of the scalar potential in the x - y (radial-poloidal) plane for $v'_E = v_{\parallel}^{(0)} = \partial v_{\parallel}^{(0)}/\partial r = 0$. (b) Sketch of the scalar potential. The solid lines correspond to lines of maximum $\mathbf{E} \times \mathbf{B}$ flow velocity. Dark arrows indicate the direction of velocity perturbations due to the polarization drift. Circles describe the direction of parallel flow perturbations. Plots (c) and (d) describe radial eigenmodes at constant poloidal angle.

longer explicitly distinguish fast and slow derivatives. Substituting Eq. (8) into (7), and expanding g_k in the limit $v_{\text{thi}}k_{\parallel}/\omega_k < 1$, yields

$$f_{\parallel}^{(4)} = \frac{i}{4\pi} \frac{\partial}{\partial x} \left[\epsilon_{\perp} \sum_{m,n} \left(1 - \frac{\omega_{pi}^*}{\omega_k} \right) \left\langle \frac{\partial \delta \phi_{-k}}{\partial x} k_{\parallel} \delta \phi_k \right\rangle_x \right], \quad (9)$$

where $\langle \dots \rangle_x \equiv \int_{-\infty}^{\infty} dx (\dots)$, we have neglected additional finite Larmor radius corrections, and only considered transport in the radial direction. Before proceeding further it is useful to note that in contrast to the residual stress contribution arising from $\Pi_{\text{EB}}^{(2)}$, where $\langle k_{\parallel} k_{\theta} \rangle \neq 0$ is required, $f_{\parallel}^{(4)}$ requires $\langle \delta E_r \delta E_{\parallel} \rangle \sim \langle k_{\parallel} k_r \rangle \neq 0$. Within quasilinear theory, the latter constraint can be seen to be satisfied via general considerations of the linear drift wave eigenmode. The radial wave number can be defined as $k_r(x) \equiv -i\partial/\partial r \ln \delta \phi_k(x)$. Energy is required to propagate away from the rational surface [32]. Since drift waves are backward waves, the radial phase velocity $\omega_k/k_r(x)$ must point toward the rational surface. This constraint is manifested by a ‘‘crescent’’ shape of the contours of the scalar potential in Fig. 1(a). The presence of this curved topology can be easily seen to bend perturbations of $\delta \mathbf{v}_{\text{Pol}}$ in the radial direction [see Fig. 1(b)]. Since inward $\delta v_{\text{Pol}}^{(r)}$ can be seen to be correlated with positive $\delta(nv_{\parallel})$ [into the page in Fig. 1(b)], and outward $\delta v_{\text{Pol}}^{(r)}$ correlates with negative $\delta(nv_{\parallel})$, net positive momentum is transported inward, for this simple example.

In order to further simplify $f_{\parallel}^{(4)}$, it is useful to utilize the radial eigenmodes of the underlying modes, i.e., $\delta \phi_k(x) = \sum_l a_l H_l(\sqrt{l} \mu_k x) \exp(-\frac{l}{2} \mu_k x^2)$, where $\rho_i \mu_k \equiv (v_{\text{thi}} |k_{\theta}| / \omega_k) |L_s|^{-1}$ and outgoing wave boundary conditions have been utilized to select the sign of the effective radial wave number $k_r(x) = \mu_k x$ [32]. Substituting this expression into Eq. (9) (only considering $l = 0$ modes), yields

$$f_{\parallel}^{(4)} \approx \frac{-1}{4\pi} \frac{\partial}{\partial x} \left[\epsilon_{\perp} \sum_{m,n} \left(1 - \frac{\omega_{pi}^*}{\omega_k} \right) k_y \frac{\text{Re}[\mu_{-k}]}{L_s} \langle x^2 |\delta \phi_k|^2 \rangle_x \right], \quad (10)$$

where $\text{Re}[\mu_{-k}] = -\text{Re}[\mu_k]$, $\omega_{pi}^* \equiv -(1 + \eta_i) \omega_e^* / \tau$, $\omega_e^* \equiv k_{\theta} \rho_s (c_s / L_n)$, and $\tau \equiv T_e / T_i$. Comparing the magnitude of Eq. (10) with the residual stress component of Π_{EB} (see Ref. [13] for example), which we denote as S_{EB} , yields

$$\frac{f_{\parallel}^{(4)}}{\partial S_{\text{EB}} / \partial x} \sim \tau \frac{L_n}{L_s} \left(\frac{\tau}{1 + \eta_i} \right) \left(\frac{\omega_k}{\omega_e^*} \right)^2 \frac{\omega_k}{\Delta \omega}, \quad (11)$$

where we have approximated the radial electric field by $v'_E \approx v_{\text{thi}} \rho_i (1 + \eta_i) / L_n^2$, and assumed the radial extent of the mode to be set by the points of strong ion Landau damping (i.e., $v_{\text{thi}} k_{\parallel} = \omega_k$). While L_n / L_s is typically

small, $\omega_k / \Delta \omega$ is typically greater than one, so that *neither* of these terms should be considered negligible *a priori*.

In this Letter, a novel mechanism for driving intrinsic rotation has been derived. This mechanism, which arises from the parallel nonlinearity within the gyrokinetic framework, does not require mean $\mathbf{E} \times \mathbf{B}$ shear, and is thus likely to be active in a wide range of plasma regimes. The role of this polarization induced residual stress in the developing theory of intrinsic rotation will be discussed in a future publication.

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*cmcdevitt@ucsd.edu

†Current address: Laboratoire de Physique des Plasmas, Ecole Polytechnique, CNRS, 91128 Palaiseau Cedex, France.

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