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PION CONDENSATION IN A THEORY CONSISTENT WITH BULK PROPERTIES OF NUCLEAR MATTER*

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The theory of nuclear matter at densities higher than normal has acquired particular interest in the last few years, in the context both of astrophysics and relativistic nuclear collisions. At energy densities above normal, new excitations become possible in principle, involving additional particles or field configurations than are present in the normal state. Several such excitations have been studied, known popularly as pion condensate and abnormal or density isomeric states.[1]

In general these states have been discussed in the framework of theoretical models that make little contact with the known bulk properties of nuclei, in particular their saturation energy, density and compressibility. The condensate energy has been calculated as the difference in energy between a state of the theory with and without a condensate. The large discrepancy between the theory and the properties of nuclear matter are assumed to cancel in the subtraction, yielding it is hoped, a reliable estimate of the condensate energy.

This paper makes three contributions to the theory of abnormal states of matter.

1) We have formulated and solved, in a self-consistent mean field approximation, a field theory that possesses condensate states, and reproduces at the same time the three important bulk properties of the normal state.

2) The theory is solved in its relativistically covariant form.

3) A continuous class of pion fields having a space-time dependence is constructed for which the theory can be solved.

We investigate in this paper the implications of self-consistency for the existence of condensate solutions, as well as the dependence of the condensate energy on the nuclear equation of state when the condensate energy does exist.

Formulation

Our starting point is a relativistic field theory like Walecka's[2] with potential terms as studied by Boguta and Bodmer.[3] The ingredients of the theory are a chargeless scalar and vector meson with Yukawa coupling to the neutron and proton fields. The scalar meson is responsible for binding, while the vector meson leads to repulsion at short distance and hence saturation. Nonlinear terms in the field equation for the scalar meson are introduced to reduce the high compressibility of Walecka's theory to an accepted range around $K \sim 250$ MeV. To this theory we add the pions, which are represented by a three component pseudo-scalar field. They can couple to the nucleons by

*Done in collaboration with B. Banerjee and M. Gyulassy.

pseudo-scalar or pseudo-vector coupling. We use the pseudo-vector coupling because it does not possess the unphysically large s-wave interactions of pseudo-scalar coupling, yet leads to the correct p-wave interaction.

The *Euler-Lagrange equations* derivable from the above described theory are for the scalar, vector, pi mesons and nucleons respectively,

$$(\Box + m_s^2)\sigma = g_s\, \bar{\psi}\psi - \frac{dU}{d\sigma} \tag{1}$$

$$\partial^\nu(\partial_\mu V_\nu - \partial_\nu V_\mu) = m_v^2\, V_\mu - g_v\, \bar{\psi}\, \gamma_\mu\, \psi \tag{2}$$

$$(\Box + m_\pi^2)\underset{\sim}{\pi} = g_\pi\, \partial^\mu(\bar{\psi}\gamma_5\gamma_\mu\underset{\sim}{\tau}\psi) \tag{3}$$

$$(i\not\partial - (m_N - g_s\sigma) - g_v\not V - g_\pi\, \gamma_5\gamma^\mu\underset{\sim}{\tau} \cdot \partial_\mu\underset{\sim}{\pi})\psi = 0 \tag{4}$$

$$\psi = \binom{P}{N}, \quad P = \text{4-component spinor}, \qquad \pi = (\pi_1, \pi_2, \pi_3)$$

(Slashed quantities denote $\not a \equiv \gamma_\mu a^\mu$). The form of the potential is

$$\frac{dU}{d\sigma}(\sigma) = (bm_N + cg_s\sigma)(g_s\sigma)^2 \tag{5}$$

The parameters are five in number,

$$g_\pi, \ \left(\frac{g_s}{m_s}\right), \ \left(\frac{g_v}{m_v}\right), \ b, \ c$$

The last four are used to generate a desirable equation of state in the absence of a pion condensate. One can think of three of them as being used to define the saturation energy, density and compressibility, and the fourth as determining how soft the equation of state is at high density, as defined for example, by the density at which the binding is zero. We have studied two cases, for which this density is $\sim 2\rho_0$ and $\sim 3\rho_0$.

We solve the field equations in the mean field approximation. The nuclear source currents on the right side are replaced by their ground state expectation values. For infinite homogeneous nuclear matter $\langle \bar{\psi}(x)\, \psi(x) \rangle$ and $\langle \bar{\psi}(x)\gamma_\mu\psi(x) \rangle$ are independent of x. Therefore the σ and V_μ fields are constants which we write as $\bar{\sigma}$ and $\bar{V}_\mu$. These values can be read from the equations of motion as

$$m_s^2\bar{\sigma} = g_s\langle \bar{\psi}\psi \rangle - \langle \frac{dU}{d\bar{\sigma}} \rangle \quad , \qquad m_v^2\, \bar{V}_\mu = g_v\, \langle \bar{\psi}\gamma_\mu\psi \rangle \tag{6}$$

As in other studies,[1] we shall also treat the pions as classical fields whose mean values have a certain space time-dependence. A charged running wave condensate

$$\langle \pi_{\pm} \rangle = \bar{\pi}_c \, e^{\pm ikx} , \quad \langle \pi_0 \rangle = 0 , \qquad kx = k_\mu x^\mu = k_0 x_0 - \underset{\sim}{k} \cdot \underset{\sim}{x} \tag{7}$$

is the functional form that has been studied previously. Boguta has pointed out to us that if we can solve our problem with the above ansatz, then it is also possible to do so for a standing wave

$$\langle \pi_{+} \rangle = \bar{\pi}_c \sin kx, \qquad \langle \pi_0 \rangle = \bar{\pi}_c \cos kx \tag{8}$$

Actually both of these are special cases of a continuous class of fields defined by

$$\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}(kx) = \sqrt{2}\, \bar{\pi}_c \, S_{\underset{\sim}{v}}(kx) \underset{\sim}{\tau} \cdot \underset{\sim}{u} \, S_{\underset{\sim}{v}}^{+}(kx) \tag{9}$$

where S_v is a unitary operator in isospin space

$$S_{\underset{\sim}{v}}(kx) = e^{-\frac{i}{2}kx \, \underset{\sim}{\tau} \cdot \underset{\sim}{v}} , \qquad \underset{\sim}{u} \cdot \underset{\sim}{v} = 0 \tag{10}$$

Here $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are unit vectors. After some algebra (9) is found to imply

$$\underset{\sim}{\pi}(kx) = \sqrt{2}\, \bar{\pi}_c \, (\underset{\sim}{u} \cos kx + \underset{\sim}{v} \times \underset{\sim}{u} \sin kx) \tag{11}$$

The Dirac equation that we have to solve is (4) with the σ, V and π fields replaced by the mean values (6, 9-11). In particular

$$\underset{\sim}{\tau} \cdot \partial_\mu \underset{\sim}{\pi} = \sqrt{2}\, \bar{\pi}_c \, k_\mu \, S_v \, \underset{\sim}{\tau} \cdot \underset{\sim}{v} \times \underset{\sim}{u} \, S_v^{+} \tag{12}$$

Under the (linear local gauge) transformation

$$\psi = S_v \, \psi_v \tag{13}$$

the Dirac equation is transformed to

$$\left[i\partial\!\!\!/ - (m_N - g_s \bar{\sigma}) - g_v \bar{V\!\!\!/} + k\!\!\!/ \underset{\sim}{\tau} \cdot \left(\frac{1}{2} \underset{\sim}{v} + \sqrt{2}\, g_\pi \, \bar{\pi}_c \, \gamma_5 \, \underset{\sim}{v} \times \underset{\sim}{u}\right) \right] \psi_v = 0 \tag{14}$$

The Dirac operator is by means of this transformation reduced to a finite matrix operator (dimension 8) with no x-dependent terms. Unlike the usual free particle Dirac equation, the spectrum is more complicated than $\sqrt{p^2 + m^2}$. In fact the eigenvalues are functions of the field strengths $\bar{\sigma}$, $\bar{V}_0$ and $\bar{\pi}_c$ and of the pion momentum $\underset{\sim}{k}$, as well as the nucleon momentum $\underset{\sim}{p}$, and the Fermi surface does not have the usual spherical symmetry but instead is cylindrically symmetric about the direction $\underset{\sim}{k}$, sometimes consisting of two unconnected regions of momentum. It can be shown that for all fields (9) the energy is degenerate.

It can be verified that the assumed space-time dependence of the pion fields is compatible with their field equations (3), and that $\bar{\pi}_c$ is constrained by the equation

$$(-k_0^2 + \underset{\sim}{k}^2 + m_\pi^2)\ \sqrt{2}\ \bar{\pi}_c = -g_\pi\ (\bar{\psi}_V\ \gamma_5\ \not{k}\ \underset{\sim}{\tau}\cdot\underset{\sim}{v}\times\underset{\sim}{u}\ \psi_V) \tag{15}$$

Now the self consistency problem can be stated. The nucleon fields depend on the mean values of the meson fields through (14). Therefore the source currents of the mesons are implicit functions of the meson mean fields themselves. Because both the momentum p^μ and the vector field $\bar{V}^\mu$ occur in the Dirac equations like $\gamma_\mu(p^\mu - g_V\ \bar{V}^\mu)$, the vector field merely produces a shift in the four vector momentum. It is not involved in the self consistency. The remaining field strengths $\bar{\sigma}$ and $\bar{\pi}_c$ are coupled and equations (6) and (15) must be solved simultaneously for their self-consistent values. The locus of solutions in the $\bar{\sigma}$, $\bar{\pi}_c$ plane are shown in Fig. 1. As can be imagined and as the figure illustrates, the occurrence of simultaneous solutions for arbitrary density, k and g_π is not assured. Rather they exist only over a limited range.

Results

We have studied condensate solutions for two sets of parameters which yield acceptable equations of state for nuclear matter. They differ in that one is softer at high density than the other but both yield acceptable values of saturation energy, density, and compressibility.

For both of these we have studied the self-consistent condensate solutions for several values of the coupling constant g_π. The non-relativistic equivalence of the pseudo-vector coupling g_π and pseudo-scalar coupling constant g_0 is $g_\pi = g_0/(2m_N)$. For the standard value $g_0^2/4\pi = 14$ this gives $g_\pi = 1.41$ fm.

In Figs. 2 and 3 we show the two equations of state without a condensate ($\pi_c \equiv 0$). In each case there are self-consistent condensate solutions some of which are shown, corresponding to several values of g_π and for $k = 1.5\ fm^{-1}$, which minimizes the energy. Since it is doubtful that there is a condensate in the normal state, an acceptable equation of state is one which, if there is a condensate at all, it occurs at densities larger than the saturation density. There are such solutions, and they occur for a narrow range of the coupling constants g_π which depends on the softness of the equation of state. Roughly these ranges are

$$1.17 < g_\pi/\text{fm} < 1.2\ ; \qquad 1.0 < g_\pi/\text{fm} < 1.15$$

for the stiffer and softer equations of state shown. For values less than the

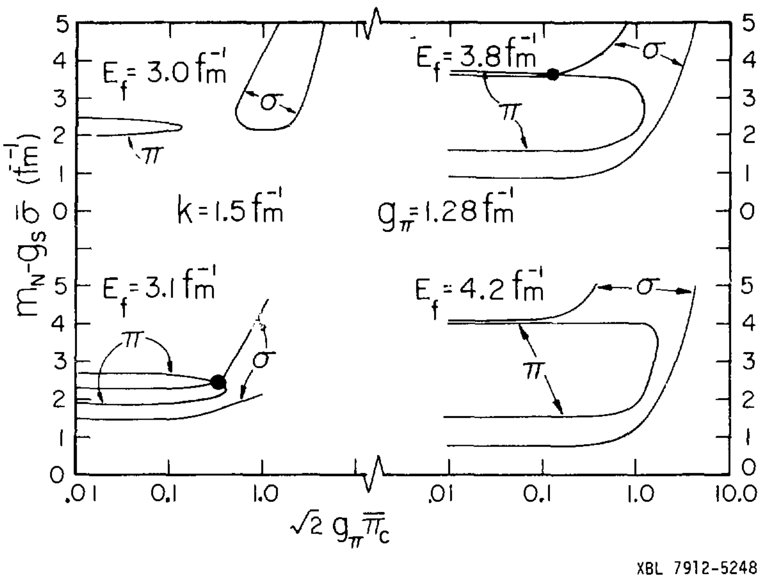

Fig. 1. Locus of solutions for σ and π equations for four Fermi energies showing the existence of common self-consistent solutions in a limited range of Fermi energy (related to density).

lower limit, solutions do not exist over the range of densities shown. Effectively for the stiffer equation of state, the condensate energy is zero. For the softer equation, the condensate energy is small and does not exceed about 10 MeV at $\rho \sim 3\rho_0$. This is in sharp disagreement with estimates based on the σ model for which the condensate energy is ~30 MeV at this density, and is a very strongly increasing function of density.

A glance at Figs. 2 and 3 reveals a strong dependence of condensate energy on modest variations in the equation of state. This implies that a reliable estimate of the condensate energy cannot be made unless the theory is consistent with the bulk properties of nuclear matter.

Summary

We have solved a relativistic field theory of nuclear matter for the self-consistent field strengths in the mean field approximation. The theory is constrained to reproduce the bulk properties of nuclear matter. We find that a weak pion condensate is compatible with this constraint. At least this is encouraging as concerns the possible existence of a new phase of nuclear matter. In contrast the Lee Wick density isomer is probably not compatible with the properties of nuclear matter.[3,4]

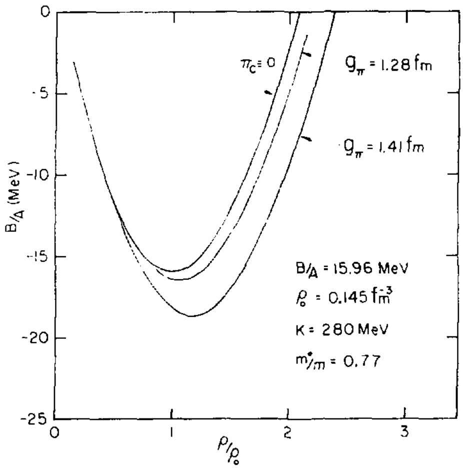

Fig. 2. Binding energy as a function of density in the absence of a pion condensate ($\pi_C \equiv 0$) and for several self-consistent condensate solutions. The coupling constants and potential parameters are $g_S/m_S = 15/m_N$, $g_V/m_V = 11/m_N$, $b = 0.004$, $c = 0008$, where $m_N = 4.77$ fm^{-1} is the nucleon mass. The pion momentum that minimizes the energy is $|\underset{\sim}{k}| = 1.5$ fm^{-1}.

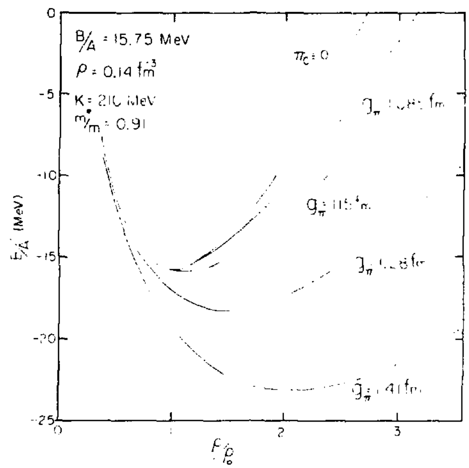

Fig. 3. As Fig. 2 but $g_S/m_S = 9/m_N$, $g_V/m_V = 5/m_N$, $b = -0.192$, $c = 2.47$.

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