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Iteration of Target Matrices in Exploratory Factor Analysis

Author
Moore, Tyler Maxwell

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Iteration of Target Matrices in Exploratory Factor Analysis

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by

Tyler Maxwell Moore

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ABSTRACT OF THE DISSERTATION

Iteration of Target Matrices in Exploratory Factor Analysis

by

Tyler Maxwell Moore

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Professor Steven P. Reise, Chair

Exploratory factor analysis (EFA) almost always involves two unique steps: 1) initial extraction of *m* orthogonal, "unrotated" factors from a covariance matrix, followed by 2) rotation of said factors to an interpretable structure. The present research is focused entirely on the second step, rotation; specifically, a new, semi-analytic rotation algorithm called iterated target rotation (ITR) is developed and tested. The goal of ITR is to improve upon basic target rotation (Hurley & Cattell, 1962) by automatically searching for a viable target matrix. Whereas basic target rotation requires a user to specify a target matrix *a priori*, ITR uses an iterative search procedure to find a viable (often ideal) target matrix. Using Monte Carlo simulation of raw data (N = 250, 500, and 1000), the performance of ITR was tested in simple, complex, and highly complex population factor structures. Further, two characteristics of the ITR algorithm itself were varied, resulting in the following four simulation condition variables: sample size (three conditions), complexity of population factor structure (four conditions), method for iterating solutions (three

conditions), and method for beginning the iterations (seven conditions). Performance was defined as the ability to accurately recover a true factor structure, as measured by the root mean-square error (RMSE) between the ITR solution and the population structure. Results suggest that, with few exceptions, ITR performs well even in highly complex structures. Limitations and future directions are discussed, and one potential improvement on the target-iteration algorithm is tested preliminarily.

The dissertation of Tyler Maxwell Moore is approved.

Peter M. Bentler

Martie G. Haselton

Catherine A. Sugar

Steven P. Reise, Committee Chair

University of California, Los Angeles

2013

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Vita

Education

2004 – Bachelor of Arts – Economics – Pomona College

2005 – Master of Science – Social and Organizational Psychology – University of Exeter

2009 – Master of Arts – Psychology – University of California, Los Angeles

2012 – Candidate of Philosophy – Psychology – University of California, Los Angeles

Distinctions

2007 – UCLA Chancellor's Prize

2008-2010 – National Science Foundation Graduate Research Fellowship (NSF GRFP)

*Employmen*t

2005-2006 – Compensation Analyst – Watson Wyatt Worldwide – New York, NY

2006-2007 – Analyst – Solid Earth, Inc. – Huntsville, AL

2007-2008 – Statistical Consultant – Teach for America – New York, NY

2008-2009 – Statistical Consultant – Millennium Wave Investments – Dallas, TX

2010-2011 – Statistical Consultant – Mauldin Management – Arlington, TX

2010-2012 – Statistical Consultant – Personnel Effects, LLC – Santa Monica, CA

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Reise, S. P., Moore, T. M., & Haviland, M. G. (2010). Bifactor models and rotations: Exploring the extent to which multidimensional data yield univocal scale scores. *Journal of Personality Assessment, 92*(6), 544-559.

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Iteration of Target Matrices in Exploratory Factor Analysis

Exploratory item-level factor analysis is a complex statistical tool used to "discover" the structure underlying a set of item responses. It plays a crucial role in scale development and revision (Briggs & Cheek, 1986; Comrey, 1988; Floyd & Widaman, 1995; Reise, Waller & Comrey, 2000), theory generation and development (Cattell, 1978; Haig, 2005; Mulaik, 1987; Preacher & MacCallum, 2003), preparation for confirmatory factor analysis (Gerbing & Hamilton, 1996; Gorsuch, 1997; van Prooijen & van der Kloot, 2001; Thompson, 2004), comparison of psychological data structures across populations (Caprara et al., 2000; Gorsuch, 1983; Schönemann, 1966), and, often inappropriately, data reduction (Crawford & Lomas, 1993; Fabrigar et al., 1999; Ford, MacCallum & Tait, 1986). This is not the place for a thorough review of factor analysis in general (see Gorsuch, 1983; Thompson, 2004), but note that, given a set of item responses, the first step of factor analysis is to extract *m* orthogonal dimensions, where *m* is determined by the researcher. These "unrotated" dimensions are not psychologically interpretable. Thus, psychologists routinely perform a "rotation of the extracted factors" to some (more meaningful) criterion.

The present research focused almost exclusively on the latter of the two steps mentioned above (factor rotation). More specifically, the present study was designed to test a new[1] rotation technique (iteration of target rotations) for potential inclusion in the long list of possibilities for researchers. The results suggest that target-iteration is especially useful in one of the most common situations encountered by researchers ("complex" structure; McDonald, 2005; Yates, 1987). In what follows, the potential strengths and weaknesses of target-iteration are explained, some of those claims are tested using Monte Carlo simulation, and the effects of modifying the

[1] But see Tucker (1944).

target-iteration method are briefly explored. First, the common factor model and some traditional approaches to factor rotation are briefly reviewed.

The Common Factor Model

The common factor model is based on the idea that an observed variable (e.g. an item response) is caused by one or more unobserved factors (e.g. personality traits) plus an error term. That is, in a raw data matrix of observed variables, each element can be represented as a sum of its causes. Specifically, for any one individual,

$$X = \Lambda f + e \tag{1}$$

Where X is a p x 1 vector of observed variables for that person,

Λ is a p x m matrix of factor loadings,

f is a m x 1 vector of factor scores for that person, and

e is a p x 1 vector of error variables.

Note that the elements of x are the only observed variables in the model; all other variables are theoretical components of the Common Factor Model itself.

Assuming a) that all variables have means of zero, b) that the error variables are uncorrelated with themselves and with the factors, and c) that the factors have unit variances, then the p x p variance-covariance[2] matrix (Σ) of observed variables can be expressed as

$$\Sigma = \Lambda \Phi \Lambda' + \Psi \tag{2}$$

where Λ is a p x m matrix of factor loadings,

Φ is a m x m matrix of correlations among the factors, and

[2] From here, I refer only to correlation matrices. Interested readers should see Anderson (1963) for details on important differences between covariance matrices and their standardized versions (correlation matrices).

Ψ is a p x p diagonal matrix of residual/unique variances of the observed variables.

Equation 2 is the starting point for common-factor analysis solutions, and is therefore crucial for understanding any topic in EFA. Before turning to the core topic of the present study (factor rotation), one should be familiar with two important decisions encountered during the first step of EFA, factor extraction.

Factor Extraction Decisions

Some important steps in the process of factor extraction are beyond the present scope, and readers are directed to Kim and Mueller (1978a, b) and Harman (1976) for more detail. One of these topics—the computational methods for estimating communalities and unrotated loadings—comprises a huge body of literature spanning decades. Another topic—the decision of how many factors to extract—is among the most contentious methodological issues in the social and biological sciences (Hoyle & Duvall, 2003; Wood, Tataryn, & Gorsuch, 1996). Nonetheless, only a few practical points from these literatures are reviewed here.

The number of factors problem

The first step of factor extraction is to determine the number of factors necessary to account for a satisfactory amount of the variance in common among the variables. What is considered "satisfactory" is subjective, and is therefore a long-standing matter of debate among researchers (see Wood, Tataryn & Gorsuch, 1996). From a scientific perspective, the question is whether the additional variance explained by extracting an additional factor is worth the loss in parsimony. Several methods exist for choosing an appropriate number of factors, including:

1. Extract the same number of factors as there are eigenvalues > 1 (Kaiser, 1960).

2. Scree Analysis. Plot the eigenvalues[3] in descending order (called a scree plot, Cattell, 1966), and search (visually) for the point at which the eigenvalues begin showing a small, negative linear trend. This is sometimes called the "elbow," and its exact location on the scree plot is subjective (but see Bentler & Yuan, 1998).
3. Specify how much of the total variance one wants to explain, and extract the number of factors necessary to do so. (Total variance explained by *m* factors is the sum of their eigenvalues divided by *m*.)
4. Parallel Analysis (PA; Horn, 1965). Generate random raw data with the same number of dimensions as the raw data whose correlation matrix is being factor analyzed. Calculate the correlation matrix of each random raw data matrix, extract the eigenvalues (using principal components extraction), calculate their averages, and overlay said averages on the original correlation matrix's scree plot. The plot of the average eigenvalues will intersect the original scree plot, representing the point at which the extraction of additional factors ceases to explain more variance than random chance.
5. Modified PA. Several variations on parallel analysis have been proposed since 1965, including improvements designed specifically for non-continuous data (such as item responses; Garrido, Abad, & Ponsoda, 2012). A potential weakness of the original PA method is that it computes eigenvalues using principal components analysis (not consistent with the common factor model). Thus, Humphreys and Ilgen (1969) suggested computing the eigenvalues with Principal-Axis Factoring (PAF), which uses the reduced correlation matrix ($\Lambda\Phi\Lambda'$ in Equation 2 of the Common Factor Model, above). Further,

[3] See Horn & Johnson (1994) for details about matrix decomposition.

Timmerman and Lorenzo-Seva (2011) suggested using Minimum Rank Factor Analysis (MRFA) to compute eigenvalues.

6. Extract factors until the reproduced correlation matrix is "similar enough" to the real correlation matrix—e.g. by calculating the average absolute differences between the two matrices' elements. "Similar enough" is subjective, but see Lawley (1940; *c.f.* Hu, Bentler, & Kano, 1992).
7. Minimum Average Partial method (MAP; Velicer, 1976). Partial out the variance explained by *m* components by subtracting the reproduced correlation matrix ($\Lambda\Phi\Lambda'$ in Equation 2) from the sample matrix, producing a matrix of partial correlations. Calculate the mean squared partial correlation. Repeat the above for $m = 1, 2, \ldots, p\text{-}1$ components, where *p* is the number of variables. The optimal *m* components will minimize the average squared partial.

Which of the above is best? Revelle and Rocklin (1979) provided an answer that is still mostly true today: "No one seems to agree which rule of thumb is best, and all agree that there exist particular data sets for which each rule will fail" (p. 404). Nonetheless, many researchers consider parallel analysis (4 and 5 in the list above) to be the best available method for determining the number of factors to extract (Hayton et al., 2004; Peres-Neto, Jackson, & Somers, 2005; also see Dinno, 2009). With an appropriate number of factors chosen, one can proceed with factor extraction.

Performance of factor extraction methods

Several methods exist for the extraction of factors from a correlation matrix (e.g. Principal-Axis, Maximum Likelihood, Least Squares; see Harman, 1976). Without going into mathematical detail, some relevant empirical findings are reviewed here. How well an extraction

method performs is influenced by many factors, including sample size, sampling bias, precision and accuracy of measurement, linearity of relationships among variables, and perhaps most importantly, characteristics of the factor model itself. Indeed, MacCallum et al. (1999), MacCallum et al. (2001), Hogarty et al. (2005), and Steger (2006) all investigated several of the above effects on factorability, generally concluding that the last effect (the factor structure itself) usually overwhelms the other effects. The obvious importance of the factor structure inspired efforts (e.g. Bentler, 1977) to develop measures of structural complexity/simplicity. Such measures are described below (see "Simple Structure").

Two of the factorability investigations cited above are notable. MacCallum et al. (1999) found that the communalities of the variables, along with the determination of the factors (how many items load per factor), were usually more important than sample size in recovering the population structure. For example, a model in which variables did not correlate highly on average (low communalities) was much more difficult to recover than a model in which variables did correlate highly (high communalities), and that effect depended very little on sample size. The same was true when comparing recoverability of a model in which factors had few simple indicators (under-determined factors) to a model in which factors had several simple indicators (over-determined factors). Sample size was again overwhelmed by factor determination. Note, however, that sample size did have an interaction effect with both communalities and factor determination, such that sample size did matter in one case: when both communalities and factor determination were low. MacCallum et al. (2001) replicated much of the above, and extended

the conclusions to more complex conditions. For example, they demonstrated that the above conclusions held even as the factor model itself departed from the common-factor model[4].

Finally, it is worth noting that most of the research on factor extraction methods has been done using product-moment correlations (of continuous variables), which are rarely appropriate for ordinal data (Pearson, 1900). To factor analyze item responses, for example, researchers will often calculate a polychoric correlation matrix, which involves estimating the correlations among hypothetical continuous variables underlying the ordinal item responses. Due to some unique characteristics of polychorics (see Mislevy, 1986; Wirth & Edwards, 2007), normal factor extraction methods intended for product-moment correlations (e.g. ML) will not always function properly when used on matrices of polychoric correlations. Research on polychoric factor extraction is therefore ongoing (e.g. Lee, Zhang, & Edwards, 2012), and one important finding is that the Ordinary unweighted Least Squares (OLS) extraction method is robust to many of the problems caused by matrices of polychorics (Forero, Maydeu-Olivares, & Gallardo-Pujol, 2009). Still, some researchers (e.g. Bock, Gibbons, & Muraki, 1998) have avoided polychoric extraction problems by directly modeling ordinal data nonlinearly (as in normal-ogive Item Response Theory models; see Embretson & Reise, 2000).

Factor Rotation

Rotation of factors in exploratory factor analysis is an old problem (Browne, 2001; Guilford, 1974; Henson & Roberts, 2006; Jennrich, 2002; Mulaik, 2010; Sass & Schmitt, 2010; Thurstone, 1947). It is so old, in fact, that the original factor rotations were done by hand (see Comrey &

[4] "Departure from the common-factor model" is one way of saying "complexity of the population factor pattern." That is, a population factor pattern with several cross-loadings—several variables loading on more than one factor—"departs" from the common factor model.

Lee, 1992), with no help from computers. Readers interested in the progression from such "hand rotations" to the present, sophisticated computer-aided rotations are directed to Cudeck and MacCallum (2007) and Mulaik (2010). The ultimate goal of factor rotation is to identify interpretable and substantively meaningful dimensions that account for, and explain, the relationships among test items. Rotations form the basis of scale analysis and revisions (Reise, Waller & Comrey, 2000), and are thus of fundamental importance in evaluating whether an item set is consistent with a latent variable measurement model.

Transformation (rotation) matrices

Due to the indeterminacy of a factor solution (Equation 2), there are infinite ways to transform the pattern matrix (Λ) without changing the uniquenesses (diagonal elements of Ψ) or the reproduced correlation matrix (Σ). Thus, given the existence of multiple (equivalent) solutions, it is important to distinguish between the initial (unrotated) loading matrix (**A**), and the final (rotated) loading matrix (Λ). That is, for any given solution, there exists a non-singular, square matrix M such that

$$f_0 = Mf\ , \tag{3}$$

$$\Lambda_0 = \boldsymbol{A}M^{-1}\ , \tag{4}$$

$$\Phi_0 = M\Phi M'\ , \tag{5}$$

where **A** is the factor pattern matrix before rotation, and f_0, Λ_0 and Φ_0 can be substituted into Equations 1 and 2 without changing Σ or Ψ. This forms the entire basis of factor rotation. It is more common to refer to a "transformation" matrix, T, such that

$$\Lambda = \boldsymbol{A}T\ , \tag{6}$$

where, again, Λ is the rotated factor pattern matrix and **A** is the unrotated factor pattern matrix. The transformation matrix T must be non-singular, and further constraints placed on T depend on whether the rotation is orthogonal or oblique. If the rotated factors are to remain orthogonal, the only constraints placed on T are that pre-multiplying T by its transpose must equal the matrix of factor inter-correlations (Φ), which must equal the identity matrix (I):

$$T'T = \Phi = I. \tag{7}$$

If the rotated factors are to be allowed to correlate, fewer constraints are placed on T. Specifically,

$$Diag\left(T^{-1}T^{-1'}\right) = Diag(\Phi) = I. \tag{8}$$

In both the orthogonal and oblique cases of analytic rotation, the chosen transformation matrix is the one that minimizes some function of the factor pattern matrix (Λ). Details of such functions are described below.

Simple structure and interpretability

Before describing functions used in analytic rotation, some useful terminology must be introduced. When rotating a factor pattern matrix, what does one rotate *to*? The most important concept to understand is that of "simple structure," which Thurstone (1947) defined using the following five criteria (the most important being #1):

1. Each variable (e.g., row) should have at least one zero.
2. Each factor (e.g., column) should have the same number of zeros as there are factors.
3. Every pair of factors should have several variables whose factor pattern loadings are zero for one factor but not the other.

4. Whenever more than about four factors are extracted, every pair of factors should have a large portion of variables with zero weights in both factors.

5. Every pair of factors should have few variables with nonzero pattern loadings in both factors.

Some contemporary authors have used one or more of the above as guides in creating their own definitions of "simple structure." For example, Browne (2001) and Jennrich (2007) both adopt the method of counting the number of nonzero elements present in the pattern matrix. Generally speaking, the more nonzero elements a factor pattern has, the more "complex" it is—i.e., the more it departs from simple structure. A commonly used example is the existence of a cross-loading for one of the variables (rows). Assuming the simplest possible structure is one in which each row has exactly one nonzero element and each factor has at least two nonzero elements, the addition of a cross-loading is said to make the pattern more "complex." Other commonly used definitions are given by McDonald (2005) and Yates (1987).

Some mathematical definitions ("indices of factor simplicity") have also been proposed. The first (Kaiser, 1974) depended on the scale of the factors, and is therefore rarely used. Bentler (1977) proposed the Simplicity Index (S), which is a maximizeable function reaching its upper bound (of 1) when each variable loads on exactly one factor. In an effort to improve on S, Lorenzo-Seva (2003) proposed the Loading Simplicity (LS) index, which used essentially the same definition of simple structure. These mathematical definitions of factor simplicity/complexity are fundamental to (and examples of) rotation *criteria* (see below, "Factor rotations in the present study").

The reason the above terminology is important is that analytic factor rotations all have a built-in goal, which is to rotate factors to an interpretable structure. "Interpretable," in this case,

is defined by the above discussion of simple structure. Because of the indeterminacy of factor rotations, there are infinite criteria (equations) one could minimize to achieve some result. One could write an equation that, when minimized, would result in a factor pattern with large loadings on the first factor and the smallest possible loadings on all other factors. Or, one could write an equation that, when minimized, would result in the smallest possible loadings on one factor and maximally similar loadings on all other factors. The point is that one has to have some goal when creating a rotation criterion, and the description of simple structure in the above paragraph sets that goal. The analytic criteria (equations) reviewed below are all considered viable because, when minimized, they result in a factor pattern that in some way approximates simple structure.

One obvious way to test the method proposed here (iterated target rotations) is to compare its results to other common rotations (see Browne, 2001). Seven such rotations were chosen, but before describing them, it is worth clearing up a possible point of confusion between "analytic" and "semi-analytic" rotations. Whereas the seven comparison rotations chosen here are truly analytic (i.e. can be expressed as one function), the proposed method (iterated targets) is "semi-analytic." That is, although a single target rotation can be expressed as one function (and is therefore truly analytic), the overall method presented here (iterated targets) is a *series* of analytic target rotations. Each rotation in the series uses the *same* equation—the "target rotation" criterion presented in Browne (1972 a, b)—but *different* inputs for said equation. The input that is changed is the target matrix.

Factor rotations in the present study

As Sass and Schmitt (2010) point out in their review of several rotation criteria, most researchers mistakenly assume the only important choice in selecting a criterion is whether it

allows factors to correlate (oblique rotation) or not (orthogonal rotation). In fact, as those authors demonstrate, the choice of rotation can be much more complicated, because oblique criteria themselves can vary substantially in what they emphasize—e.g., some criteria attempt to get as close to simple structure as possible, whereas others are more likely to identify cross-loadings. Sass and Schmitt (2010) suggest reporting the results of multiple rotations, such that the reader can decide which is "best." This is an excellent recommendation, but even if researchers report multiple rotations, they still must choose which of the common rotations are most appropriate to the data (and produce the most consistent results). In the present study, one conclusion (elaborated below) is that, if researchers decide to report multiple factor rotations in an empirical study, target rotation should *always* be one of the methods reported.

Choosing rotations to use as examples to which the iterated targets solutions could be compared was not easy. Four review articles were used for guidance: Sass and Schmitt (2010), because of its recency; Browne (2001), because of its thoroughness; and de Winter, Dodou and Wieringa (2009) and Hogarty et al. (2005), because of their inclusion of sample size. Following the example[5] of the most recent review article (Sass and Schmitt, 2010), seven oblique rotations from the same family (Crawford-Ferguson, elaborated below) were chosen. Three of those seven rotations have names: Quartimin, Parsimax, and Factor Parsimony (Facparsim). The other four rotations do not have names, but they are still part of the Crawford-Ferguson family. An important, inevitable question is, why compare the proposed method (target iteration) to seven rotations within the same family?

First, "families" of rotation criteria tend to have one (maybe two) constants in common *within the family*, the value(s) of which determine what the rotation emphasizes. For example,

[5] Sass and Schmitt (2010) review seven rotation criteria, six of which are from the Crawford-Ferguson family.

the general criterion chosen here (Crawford-Ferguson, detailed below) includes a constant (κ), which determines how much emphasis the rotation places on finding simple structure in the variables (rows), as opposed to the factors (columns). The implication is that the Crawford-Ferguson family of rotations is not a group of individual rotations, but rather, a continuous spectrum of infinite rotations (depending on the value of κ). This is confusing, because rotations within a family tend to have names[6] (Quartimin, Parsimax, etc.), when actually, all rotations within that family use the same general rotation criterion and differ only in their values of the constant (in this case, κ). A convenient by-product of having such a general equation, with a single constant determining what type of factor structure it will "seek," is that the constant can be used as a predictor variable in a simulation study (such as the present one). In this case, the predictor variable (κ) indicates relative emphasis on row simplicity (κ = 0) versus column simplicity (κ = 1). One can cover the entire spectrum of Crawford-Ferguson possibilities by varying only κ. Conveniently, the three values of κ most relevant to the present study correspond to standard ("named") Crawford-Ferguson rotations: "Quartimin" (κ = 0) maximizes row simplicity; "Facparsim" (κ = 1) maximizes column simplicity; and "Parsimax" (see below for details on κ) maximizes both equally. Tables 1 and 2 provide examples of how the Quartimin, Parsimax, and Facparsim rotations vary in their estimations of two population structures[7].

A second reason for using only the Crawford-Ferguson criterion is that other (seemingly obvious) competitors are inappropriate. Varimax, for example, does not allow factors to

[6] Though it is confusing, there is a good reason for naming rotations. Equation 12 (below), for example, demonstrates how a *formula* can be used to set the value of the *constant* (in this case, kappa). Once a researcher demonstrates the usefulness of that formula (as opposed to a specific number) for the constant, he/she can help future researchers by naming the criterion that uses said formula (for easier organization).

[7] For simplicity, I used nine items and three factors in Tables 1 and 2, whereas the population conditions used in the present study contain twenty items and four factors.

correlate, making it theoretically unfeasible. Direct Oblimin[8], Equamax, Quartimax, and oblique Varimax all have mathematically equivalent Crawford-Ferguson rotations (Browne, 2001, Table 1). A third promising criterion (minimum entropy; McCammon, 1966) often failed to recover true population structures (as measured by RMSE) when tested with oblique factors (Browne, 2001). Finally, the Promax rotation (Hendrickson & White, 1966) comes with the drawback of using a two-stage algorithm.

The "Crawfer" criterion

As mentioned above, the Quartimin, Parsimax, and Facparsim rotations all come from the same family, meaning they all use the same equation (with only minor variation in a constant, κ). That equation is as follows:

$$f(\Lambda) = (1 - k)\sum_{i=1}^{p}\sum_{j=1}^{m}\sum_{l\neq j,l=1}^{m}\lambda_{ij}^2\lambda_{il}^2 + k\sum_{i=1}^{m}\sum_{i=1}^{p}\sum_{l\neq i,l=1}^{p}\lambda_{ij}^2\lambda_{lj}^2 \quad (9)$$

where

p = the number of variables

m = the number of factors

k = κ, the user-defined constant determining how much emphasis the criterion places on column (versus row) complexities

λ_{ij} = the factor loading corresponding to the i^{th} variable (row) in the j^{th} factor (column), and

Λ = the factor loading matrix.

[8] When used with the common default constant (Δ) value.

Equation 9 is simply a weighted sum of two similar terms—one related to row complexity, and one related to column complexity—and is called the "Crawfer" (portmanteau of "Crawford" and "Ferguson") criterion. Because it represents the entire Crawford-Ferguson family, it is the only equation necessary to describe all rotations mentioned below.

Quartimin

The Quartimin rotation minimizes the complexity of the variables (rows), and ignores complexity of the factors (columns). In other words, the Quartimin criterion sets k at its lowest possible value (0), resulting in a simplified version of Equation 9:

$$f(\Lambda) = \sum_{i=1}^{p} \sum_{j=1}^{m} \sum_{l \neq j}^{m} \lambda_{ij}^2 \lambda_{il}^2 \qquad (10)$$

Since k is set to 0, the right-hand term of Equation 9 disappears, resulting in Equation 10. In reference to the five Thurstonian criteria listed in the previous sub-section, Equation 10 is an attempt to meet criteria 3 – 5. Because it minimizes row complexity, the Quartimin rotation will always match a population structure that contains perfect simple structure (Carroll, 1953; Yates, 1987). Indeed, Sass and Schmitt (2010) go so far as to say that, assuming no cross-loading variables in the population structure, "No other rotation criterion will perform better" (p. 79).

Factor parsimony ("facparsim")

The Facparsim rotation is the opposite of the Quartimin rotation (above) insofar as it minimizes the complexity of the factors (columns) and ignores the complexity of the variables

(rows). The Facparsim criterion sets k at its highest possible value (1), resulting in another simplified version of the Crawfer criterion (Equation 9):

$$f(\Lambda) = \sum_{i=1}^{m} \sum_{j=1}^{p} \sum_{l \neq i}^{p} \lambda_{ij}^2 \lambda_{lj}^2 \qquad (11)$$

Because k is set to 0, the left-hand term of Equation 9 disappears, resulting in Equation 11.

Facparsim is unique among rotation criteria, because it places no emphasis on assigning variables (rows) to factors (columns), and assumes the factors each account for the same amount of variance (Crawford & Ferguson, 1970). Browne (2001), therefore, considers the Facparsim criterion to be of theoretical interest only. Nonetheless, Facparsim is included as a competing rotation in the present study for two reasons. First, the most important reason for choosing seven rotations from the same family is to cover a very wide range of possibilities with minor changes in a single equation (the Crawfer criterion, Equation 9). Facparsim is one of the extremes of Equation 9, and it would therefore require justification for non-inclusion in the present study. Second, as Gorsuch (1983) points out, the goal of exploratory factor analysis is to "simplify a factor rather than a particular variable because the interest invariably lies in learning more about the factors rather than the variables" (p. 184). As demonstrated in Tables 1 and 2, the Facparsim criterion produces reasonable solutions, especially when the population structure is complex.

Parsimax

The Parsimax rotation falls directly between the Quartimin and Facparsim rotations. It was created (Crawford & Ferguson, 1970) for the specific purpose of equalizing the influences of the

first and second terms in Equation 9—i.e., for the specific purpose of emphasizing row and column complexities equally. As in the cases of Quartimin and Facparsim, Parsimax is another version of Equation 9. Specifically,

$$f(\Lambda) = \left(1 - \frac{m-1}{p+m-2}\right)\sum_{i=1}^{p}\sum_{j=1}^{m}\sum_{l\neq j}^{m}\lambda_{ij}^2\lambda_{il}^2 + \frac{m-1}{p+m-2}\sum_{i=1}^{m}\sum_{i=1}^{p}\sum_{l\neq i}^{p}\lambda_{ij}^2\lambda_{lj}^2 \quad (12)$$

Note that Equation 12 simply replaces k with the term (m – 1)/(p + m - 2), thus guaranteeing rows (p) and columns (m) have equal weight[9]. Tables 1 and 2 demonstrate the obvious promise of the Parsimax criterion, especially when the population structure deviates from "simple." Crawford and Ferguson (1970) concluded that one of the only weaknesses of the Parsimax criterion is that it is particularly sensitive to specifying the correct number of factors. In the present study, the number of factors is assumed to be known, making Parsimax an ideal competitor with iterated targets.

Other Crawfer rotations

In addition to the three "named" Crawfer rotations described above (Quartimin, Parsimax, and Facparsim), four additional Crawfer rotations will be used. These four are currently unnamed, but correspond to kappa values of 0.2, 0.4, 0.6, and 0.8. The reason for using them is to ensure coverage of the full range of kappa values in the CF family. The Quartimin rotation covers kappa = 0, Facparsim covers kappa = 1, and Parsimax covers a variable kappa value

[9] The reason kappa does not equal 0.50 is that the number of rows is not the same as the number of columns. Thus, kappa has to adjust for $m \neq p$ when weighting the sum.

(depending on rows/columns of lambda); thus, the four "other" Crawfer rotations mentioned here cover the middle range of kappa.

Target rotation

The above rotations emphasize meeting different aspects of the simple structure criterion. That is, the researcher specifies only the relative emphasis to place on row versus column complexity (κ), and the Crawford-Ferguson family will find the solution that minimizes the weighted-sum function (some variation of Equation 9). Target rotation requires a different type of input from the researcher.

The idea of rotating to a target matrix started with Digman (1967), Hurley and Cattell (1962), and Schoenemann (1966)[10], and was expanded upon by Gruvaeus (1970), Browne (1972 a, b), and ten Berge (1977). In a target rotation, a researcher needs to specify a pattern of factor loadings *a priori*. The extracted solution is then rotated to minimize the difference between the loading matrix and the target matrix, rather than to simple structure. Note that it is possible to use a "partially specified" target matrix, meaning some of the elements in the target matrix are ignored when minimizing the criterion (below). In the present study, the only target elements used in the rotation are the zeros—i.e. the criterion minimizes the squared differences between the rotated solution and the zeros. Target rotation does not fit into the Crawford-Ferguson family, but rather, minimizes an even simpler criterion:

[10] These pioneers of target rotation referred to it as "Procrustes" rotation, which was the preferred term for several decades. Now, the term "Procrustes" is usually reserved for fully specified target rotations in the context of cross-population validation.

$$f(\mathrm{L}) = \sum_{j=1}^{m} \sum_{i \in I_j} (\lambda_{ij} - b_{ij})^2 \qquad (13)$$

where

b_{ij} = the element corresponding to the i[th] row in the j[th] column in the target matrix

$f(\mathrm{L})$ = the target rotation criterion

m = the number of factors

λ_{ij} = the factor loading corresponding to the i[th] variable (row) in the j[th] factor (column), and

$i \epsilon I_j$ indicates that only the specified elements[11] of the target matrix are used.

In other words, the target criterion [$f(\mathrm{L})$] is simply the squared error between the specified elements of the target matrix and the rotated factor pattern matrix. Note the similarities between target rotation and confirmatory factor analysis (CFA; Jöreskog, 1969): both methods require substantial input from the user, which is presumably based on some *a priori* knowledge. Indeed, the use of CFA for exploratory purposes (e.g. Ferrando & Lorenzo-Seva, 2000) is based on the same rationale as target rotation: one can use *a priori* information to aid in the exploration. A crucial difference, however, is that non-estimated loadings in a CFA must be 0; whereas, in target rotation, the algorithm simply tries to get the estimated loadings as close to 0 as possible. A large value where the user specified a 0 is an indication of a misspecification, which the user could change in a later target rotation.

Very little research has been done on the performance of target rotations. Both review articles mentioned continually above (Sass & Schmitt, 2010; Browne, 2001) avoid target rotation in their research, because target rotations require an unusually large amount of user-input (i.e., a

[11] It is possible to leave some elements of the target matrix unspecified, in which case the matrix is called a "partially specified target." This is explained further in the methods section, below.

target matrix that is at least partially specified). Further, simulation studies that do include target rotation tend to use arbitrary rules for specifying the target matrix. Asparouhov and Muthén (2009, pp. 426-428), for example, specified their target matrix based on identifiability of the solution, and they did not explain how a researcher would know *a priori* which elements should be specified (as zeros). Their results are worth noting, however: in the only condition in which target rotation was included (highly complex structure), it outperformed all other criteria.

Methods

The purpose of the present study is to test whether a series of target rotations—each "learning" from a previous target rotation in a series—can find the "best" target-rotated solution. That is, the target-iteration method in the present study is designed to solve the user-input problem of basic target rotation by automating the creation of target matrices. Further, once the algorithm establishes the "best" of its target rotations, the present study is designed to investigate how said solution compares to previously established methods of rotation.

Software

The *psych* library (Revelle, 2009, 2010) within the freeware R Statistical Package (version 2.15.0; R Development Core Team, 2012) was used for simulation of raw data matrices, rotation of some factor solutions, creation of command files to be used in other programs, reading output from other programs, and parsing said output. The freeware program called Comprehensive Exploratory Factor Analysis (CEFA; Browne et al., 2008) was used for all targeted rotations. An annotated version of the R program used below is available upon request.

Procedure

The procedure can be described in terms of the steps taken by the simulation programs after being given initial input from the user. First, the user provides the program with a population factor pattern matrix (Λ) and a matrix (Φ) of inter-correlations among the factors represented in Λ. That is, the "initial user input" is two matrices, one representing the population pattern of factor loadings, and one representing the correlations among those factors. The population factor pattern matrix could have any number of rows (each representing an item) and columns (each representing a latent factor), the only restriction being that there must be fewer columns than rows. In the present case, a 20x4 matrix suffices for all conditions.

Once the simulation program has a population factor pattern matrix, it calculates a population correlation matrix (Σ) using the common formula ($\Sigma = \Lambda\Phi\Lambda' + \Psi$). Then, it uses the sim.structure command in the *psych* library (in R) to simulate raw data and calculate a sample correlation matrix based on that raw data. (The extraction method in this study was always maximum likelihood via the factanal function in the *stats* library in R, and the rotations used were Target, Quartimin, Parsimax, Facparsim, and several others within the Crawford-Ferguson family; see below.) Using the *GPArotation* library in R, the Crawford-Ferguson rotations (Quartimin, etc.) are performed[12] immediately, yielding viable solutions to be compared to the iterated target rotation. The program then converts all of these rotated pattern matrices to target matrices to be used as starting points in series of iterated target rotations. That is, *each* Crawford-Ferguson rotation (Quartimin, etc.) is used to create a first target matrix for its own series of iterated targets. (There were seven such Crawford-Ferguson rotations used here, so each population lambda matrix yielded seven series of iterated target rotations.) With the

[12] The *GPArotation* library uses the Gradient Projection Algorithm (Jennrich, 2004; Bernaards & Jennrich, 2005; also see Browne, 1974) to estimate rotated solutions.

Crawford-Ferguson rotations completed, the simulation program uses each of them to create a target matrix.

To convert a factor solution to a target matrix, the simulation program requires the user to specify a threshold value for deciding whether a loading is "substantial" (and should therefore be estimated). That is, the user must set a threshold for whether a loading is coded as a "0" (Equation 13 will try to minimize that loading) or "?" in the target matrix. (For example, one of the thresholds used in the present study was 0.15, meaning any loading ≥ 0.15 was converted to a ? in the target matrix, and all other loadings were converted to 0.) A target rotation is then performed, and the resulting solution is used to create a new target matrix using the same rule as was used in the previous target-creation. That is, the threshold for target-creation (e.g. 0.15) does not change. Once a threshold is chosen, that threshold remains the same for all iterations (though not necessarily all *simulations*).

The above paragraph describes how an initial Crawford-Ferguson rotation is converted to an initial target matrix. Given an initial target matrix, the program then writes a data input (.inp) file and a command (.cmd) file for CEFA. The command file is the executable file necessary to run CEFA from the command prompt in the shell operating system (in this case, DOS-Basic via Windows 7). The input (.inp) file contains the information necessary for CEFA to perform a Target rotation; the most important parts of the input file are the correlation matrix (Σ) and the target matrix.

With both CEFA files written, the R program calls the command prompt (using the system command in the R *base* library), and executes the .cmd file using the CEFA core program (cefa.exe). CEFA then takes control, extracts four factors from the correlation matrix, and rotates them using the target matrix (in the .inp file). Finally, after CEFA has generated an

output (.out) file, the R program reads relevant information (Target-rotated factor loadings and standard errors) from said output file. This concludes the first iteration—i.e. one target rotation has been performed at this point in the R program. Subsequent iterations continue as described in the above two paragraphs, with one exception. Instead of using the Crawfer-rotated pattern matrix to create the target matrix, *the program now uses the output from the previous Target rotation.* This step continues until the target matrix produced is the same as one of the previous target matrices. Such a solution is said to have "converged." Preliminary simulations suggested that seven iterations is the maximum likely to be useful in the present study, so the R program stops after seven iterations (no matter what). If, after seven iterations, the target rotations are still producing new target matrices, the solution is said to be "non-converged" (i.e. no solution).

Finally, the R program compares eight solutions: the seven Crawfer rotations and the iterated-targets rotation. Note that the last solution—the target-rotated—is obtained by multiple, iterated target rotations. The criterion by which the program compares the above eight solutions is their root mean-square error (RMSE) from the population loading matrix. It is important to note that exploratory factor rotations often change the ordering of factors from the ordering used in the population matrix; for example, the ordering of the factors (columns) in the population structure might be F1-F2-F3-F4, whereas the exploratory rotation might change the order to F2-F3-F1-F4 (or any other ordering)[13]. Thus, to ensure that the RMSE's in the present study were calculated using the same factor order for both the population and exploratory solutions, the RMSE's were calculated using every possible re-ordering of the exploratory solution. Of these

[13] The reason the factors can be re-ordered in the exploratory solution is that the order of the factors does not affect the reproduced correlation matrix (Σ in Equation 2).

sixteen RMSE's (using the sixteen re-orderings of the exploratory solutions), the lowest RMSE was then taken to be the one corresponding to the most congruent re-ordering.

The above six paragraphs describe *one* simulation. Table 4 shows an example iterative progression from a random solution (i.e. the first target matrix is randomly generated[14]) to the final estimation of a population factor pattern. Note that the population structure was highly complex (twenty-six cross-loadings), and the iterative process began without any information (random target). Yet, the target-iteration procedure eventually "discovered" the true population structure using a simple rule: loadings < 0.10 were set to 0 in the updated target matrix.

In the present study, 1000 simulations (repetitions) were performed for *each* condition described below.

Simulation Conditions

Simulation studies of factor rotation methods are fairly common in the literature, providing many examples to which one can compare the present simulation conditions (below). Perhaps the most important was the large review mentioned above (Sass & Schmitt, 2010). The true population factor structures described below were heavily influenced by the choices of those authors. A major difference between the present study's method and that of Sass and Schmitt (2010), however, is that the present method varies characteristics of the target matrix, whereas Sass and Schmitt did not consider target rotations. The specifics of how target matrices were varied in the present study are explained in more detail below, but one implication is that the present study had far more simulation conditions than Sass and Schmitt (2010).

[14] Starting with a random target matrix is not a simulation condition in the present study. I use a random target matrix here to increase the number of iterations required to reach convergence (for demonstrative purposes).

Simulation conditions were varied in four ways. These are detailed in the outline on the following page, but can be summarized as follows. First (and most importantly), the types of population factor patterns were varied, ranging from very simple structure to highly complex structure. Second, the target-creation threshold (for converting a rotated loading to an element in a target matrix) was varied. Third, the Crawfer rotation used to create the initial target matrix was varied. Finally, the sample size (number of raw data cases generated from the population correlation matrix) was varied. More specifically, the simulation conditions were all possible combinations of the following four types:

1. Types of population factor patterns (four conditions; see Table 3):

    A. Each factor comprised exactly 5 items. Specifically, items 1-5 loaded 0.6 on factor 1, items 6-10 loaded 0.5 on factor 2, items 11-15 loaded 0.4 on factor 3, and items 16-20 loaded on factor 4 with the following loadings (respectively): 0.6, 0.55, 0.5, 0.45, 0.4. Five[15] cross-loadings of 0.25 were placed randomly in the pattern matrix.

    B. Same as above, but with 10 cross-loadings.

    C. Same as above, but with 30 cross-loadings.

    D. Number of items per factor varied. Specifically, items 1-10 loaded between 0.4 and 0.7 on factor 1, items 11-15 loaded between 0.4 and 0.7 on factor 2, items 16-18 loaded between 0.4 and 0.7 on factor 3, and items 19-20 loaded between 0.4 and 0.7 on factor 4. Twenty cross-loadings of between 0.15 and 0.35 were placed randomly in the pattern matrix.

[15] This varied slightly across simulations in this condition, but, on average, there will be five cross-loadings. That is, some individual simulations in this condition had 4 cross-loadings, some had 7, etc., but on average, there were 5.

2. Threshold for when to estimate a loading from a previous rotation:

A. Threshold = 0.05; all loadings greater than 0.05 in a given rotation were estimated (converted to ?'s) in the target matrix for the next rotation.

B. Threshold = 0.10; all loadings greater than 0.10 in a given rotation were estimated (converted to ?'s) in the target matrix for the next rotation.

C. Threshold = 0.15; all loadings greater than 0.15 in a given rotation were estimated (converted to ?'s) in the target matrix for the next rotation.

3. Crawfer rotation used to create the initial target matrix:

A. kappa = 0 (Quartimin)

B. kappa = m – 1/(p + m – 2) (Parsimax)

C. kappa = 0.20

D. kappa = 0.40

E. kappa = 0.60

F. kappa = 0.80

G. kappa = 1 (Facparsim)

4. Sample Size

A. N = 250

B. N = 500

C. N = 1000

Note that one seemingly important condition variable *not* included above is the matrix of population factor intercorrelations, phi (Φ) from Equation 2. Preliminary results suggested that the strengths of the correlations among the factors have almost no influence on the relative abilities of the rotations to approximate the population factor structure (Λ) *across conditions*.

That is, even though the rotations differ in their abilities to approximate Λ and Φ in any one condition, the relative performances of the rotations in that condition do not change if Φ is changed. An obvious question is, why simulate correlated factors at all? Although simulating orthogonal factors would simplify some of the calculations involved in rotation, correlated factors were chosen because, in reality, factors are never orthogonal. The elements of Φ therefore varied uniformly between 0 and 0.35, changing with every simulation.

The above four condition types yielded a total of 252 conditions to be simulated. The final procedural step was to detect any significant patterns among the results across the various conditions. This was accomplished using multiple least-squares regression (lm command in the *base* package of R). The dependent variable (DV) was median RMSE from each condition, and the independent variables (IVs) were the condition types: IV_1 = sample size (N); IV_2 = target-creation threshold; IV_3 = number of cross-loadings; IV_4 = kappa used for initial rotation; IV_5-IV_{10} were all possible cross-products of IV_1-IV_4. The cross-products were used to test for interactions among the variables.

Scientific goal of procedures

In summary, the present simulation study was motivated by three specific research questions:

1. Does the proposed method (iteration of target matrices) work? That is, regardless of how well target-iteration can recover a true population factor pattern matrix, does it usually converge? Further, does target-iteration at least recover a pattern matrix that fits within the requirements of exploratory factor analysis—e.g. are the communalities between 0 and 1, is the sum of the column-sums of squared loadings equal to the sum of the eigenvalues for the *m* extracted factors, and does insertion of Λ into

Equation 2 reproduce (approximately) the sample correlation matrix? This second question might seem extraneous—after all, don't communalities have to be between 0 and 1 (by definition), etc.? Unfortunately, some software routines (e.g. target rotation in the R *GPArotation* library) will produce impossible solutions in some cases, as opposed to returning an error message. Thus, it is possible that target-iterated solutions usually converge, but that the solutions are nonsensical (mathematically impossible) due to faulty software.

2. How well does the proposed method approximate true population factor structures *relative to available alternatives* (in this case, the full Crawford-Ferguson family)?

3. How and to what extent do the answers to the above questions depend on sample size (Condition 4, above), complexity of the population factor structure (Condition 1, above), and method of creating target matrices from pattern matrices (Condition 2, above)?

Results

Dependent variables

The results below are focused on one important measure of the quality of a factor solution, root mean-square error (RMSE). Figures 1 and 2 provide examples of raw RMSE distributions for two simulation conditions (1000 repetitions each, as described in the Methods section, above). Figure 1 shows only the distribution of RMSE's for the iterated-targets solutions (not the Crawford-Ferguson solutions), because the distribution of Crawford-Ferguson results looks almost identical to the distribution

shown. In other words, the results in Figure 1 are taken from a simulation condition (N = 250; Threshold = 0.10; Population Structure = 1B) in which the target-iterated solutions were *no better* than the Crawford-Ferguson solutions. The reason the target-iterated solutions do not provide any improvement over the Crawford-Ferguson solutions is that the sample size (250) is low, which is discussed further in the Main Study section, below. By contrast, the results in Figure 2 are taken from a simulation condition (N = 1000; Threshold = 0.10; Population Structure = 1D) in which the target-iterated solutions were *much better* than the Crawford-Ferguson solutions. The improvement provided by the target-iteration method is obvious in Figure 2 from the fact that the two distributions have very different means/medians.

A second dependent variable, iteration convergence rates, was intended to be included in the present analyses; however, convergence rates were so high as to prohibit meaningful analysis. Almost all iterated target solutions converged before reaching the maximum (8), and there were no meaningful patterns (hypothesized or discovered) in convergence rates across conditions. Preliminary analysis does suggest, however, that the number of iterations required for convergence can increase as sample size becomes very small. Additionally, despite the fact that convergence *rates* were too high to be meaningfully analyzed, it is worth noting two interesting phenomena suggested by the convergence patterns generally. Both of these phenomena are demonstrated in Figure 3, which shows the median RMSE (by iteration) for one simulation condition (N = 1000; Threshold = 0.10; Population Structure = 1C). First, the three functions in Figure 3 flatten out completely by the fourth iteration (labeled "TargError4" on the x-axis), indicating that iterated-targets solutions need very few iterations to converge[16]. Second, the three functions in Figure 3 end up at about the same place (rightmost points) regardless of which Crawford-Ferguson rotation was used as a starting point (leftmost points). In this particular simulation condition (30 cross-loadings), the Quartimin starting point did result in slightly worse solutions (irregularly dotted line in

[16] This remains true regardless of sample size or target-creation threshold, even in the most complex population structures.

Figure 3), but in the vast majority of conditions, the choice of starting rotation made no visible difference at all. This fact is demonstrated statistically by the regression results in Table 8, below.

Main study: Population structures 1A, 1B, and 1C

Tables 5-7 show the simulation results for 189 conditions (63 per table) outlined in the above section. Each table shows the median root mean-square error (RMSE) of the rotated factor solutions from their corresponding population structures. "Initial" solutions are the Crawford-Ferguson rotations, and "Converged" solutions are the iterated target rotations using the indicated Crawford-Ferguson rotation as a starting point (first target matrix). The only difference among Tables 5-7 is the method of generating target matrices from rotated solutions—e.g. all "Converged" solutions in Table 5 were obtained using 0.05 as the threshold for converting rotated loadings to 0's and ?'s in the target matrix. Table 6 used 0.10 as the threshold, and Table 7 used 0.15 as the threshold. Note that, because the choice of threshold has no effect on the Crawford-Ferguson rotations, the "Initial" solutions in Tables 5-7 are nearly identical for any given condition. That is, the "Initial" solution results vary within each table, but not across tables. Visual inspection of these data revealed many interesting patterns, best summarized in the regression results below.

Table 8 shows the results of the full regression model testing the condition effects (including all cross-products) on RMSE between the obtained solution and the true pattern matrix. Three of the largest effects were the main effects of sample size (N; $t = -30.12$, $p < 0.001$), number of cross-loadings (Cross; $t = 22.70$, $p < 0.001$), and threshold used to generate target matrices from solutions (Thresh; $t = 4.26$, $p < 0.001$). All three effects were in the expected direction; specifically, an increase in sample size tended to improve the solution (decrease RMSE), an increase in the number of cross-loadings tended to worsen the solution, and an increase in the

target-creation threshold tended to worsen the solution. Figure 4 shows the two largest main effects graphically. The fourth main effect (kappa value used in the initial Crawford-Ferguson rotation) was not significant, indicating that a target-iterated solution depended very little (if at all) on which Crawford-Ferguson rotation was used to obtain an initial target matrix.

Three of the six interaction effects (represented by cross-products in Table 8) were significant. The cross-product of N and target creation threshold (N*Thresh; $t = 5.20$, $p < 0.001$) was positive, indicating that the tendency of higher target-creation thresholds to produce relatively poor solutions was made even worse when N was high. Figure 5 shows this effect graphically: the irregularly dotted line (threshold = 0.15) lies above the solid line (threshold = 0.05), and the distance between those lines increases as sample size increases.

The cross-product of target-creation threshold and number of cross-loadings (Thresh*Cross; $t = 14.22$, $p < 0.001$) was positive, indicating that the tendency of higher target-creation thresholds to produce relatively poor solutions was made even worse as the number of cross-loadings increased. Put another way, the tendency of cross-loadings to worsen a solution was more pronounced when the target-creation threshold was high. Indeed, as shown in Figure 6, the number of cross-loadings tended not to matter when the target-creation threshold was low (0.05), but when the target-creation threshold was high (0.15), the addition of cross-loadings tended to worsen the solution substantially.

The cross-product of cross-loadings and kappa (Cross*Kappa; $t = -2.62$, $p < 0.01$) was negative, meaning the tendency of cross-loadings to worsen a solution was less true when the initial Crawford-Ferguson rotation had a high kappa. The effect was relatively small, making it difficult to display graphically. Suffice it to say that the choice of initial rotation (kappa) was

inconsequential unless the number of cross-loadings was high, and even then, the effect was small.

Population structure 1D

The above analyses (Table 8) explored the performance of iterated target rotation in three population structures (conditions 1A, 1B, and 1C; Table 3). The fourth condition (1D) in Table 3 is classified as "other," because it differs from conditions 1A – 1C in important ways. The purpose of including an "other" condition (1D) was to investigate whether the results from conditions 1A – 1C (Tables 5 – 7) are generalizable across a wide variety of population structures. For example, the main population factor loadings (bolded in Table 3) for conditions 1A – 1C were constant[17] across simulations. Further, the factors in conditions 1A – 1C were balanced (each factor comprised exactly five items), and most of the loadings were constant within-factor (e.g. all F1 loadings were exactly 0.60). Finally, although the cross-loadings in 1A – 1C did change locations with each simulation, they were all exactly the same size (0.25).

None of the above was true of the "other" condition (1D). Thus, if the results obtained from 1A – 1C were due largely to their unique characteristics (i.e. their results are not generalizable), one would expect the results from condition 1D to be noticeably different in some way. Table 9 shows the simulation results for population structure 1D. Note that, like Tables 5 – 7, Table 9 has horizontal levels of the conditions for N (250, 500, or 1000), but unlike Tables 5 – 7, Table 9 has columns of the conditions for threshold (0.05, 0.10, or 0.15). The 0.05-threshold condition in Table 9 can be compared to Table 5, the 0.10 condition in Table 9 can be compared to Table 6, etc.

[17] That is, the variability of loading estimates across simulations is due entirely to the random sampling of raw data; it is not due to variability in the population loadings themselves.

The results in Table 9 suggest that target-iteration will perform reliably in population structures (e.g. condition 1D) that deviate substantially from the structures (1A, 1B, and 1C) used to generate the bulk of the present study's simulation data. That is, patterns in Table 9 were consistent with those in Tables 5 – 7. First, the median RMSEs of the target-rotated solutions in Table 9 were lower than the median RMSEs of their initial rotations in almost all conditions (difference in means significant; $p < 0.01$). Second, the RMSEs of the target-rotated solutions in Table 9 were consistent with the number of cross-loadings (20) in the population structure (1D). That is, the global mean RMSE (0.053) for condition 1D (20 cross-loadings) was between[18] the means of the 10- and 30-cross-loading conditions (0.046 and 0.068, respectively). Finally, the Crawford-Ferguson kappa values (rows of Table 9) did not correlate with the RMSEs of target-iterated solutions in Table 9 (Pearson $\rho = 0.09$, n.s.), which was consistent with the non-significant effect of Kappa in Table 8. Interestingly, the median RMSEs in the Threshold = 0.05 condition (column 3 in Table 9) did appear to change with kappa to some extent ($\rho = 0.23$, $n = 21$, $p = 0.16$), perhaps suggesting that the choice of initial Crawford-Ferguson rotation began to matter when the target-creation threshold was very low (< 0.05).

Discussion

The above results suggest that target-iteration holds promise as an exploratory factor rotation method. It appears to be reliable (very high convergence rates) and accurate (low RMSE) even in highly complex population structures. Moreover, its tendency to detect cross-loadings does not seem to cause false identification of cross-loadings that are not there, meaning iterated target

[18] Specifically, global mean RMSE (0.053) for the 20-cross-loading 1D condition is significantly different than the global mean RMSEs of the 10- and 30-cross-loading conditions ($p < 0.01$ for both effects), but is *not* significantly different than the global mean RMSE (0.057) of the 10- and 30-cross-loading conditions *combined* .

rotation can safely be used with both complex and simple structures. Using only the above results as a guide, it is difficult to think of a situation in which iterated target rotation should not be used. It is recommended, therefore, that applied researchers always use iterated-target rotation[19] when investigating the factor structure of a set of item responses. Nonetheless, the present study had some limitations that need further investigation.

First, the number of factors in the population (four, in this case) was assumed to be known. In practice, the correct number of factors is almost never obvious from the outset, which is why the "number of factors problem" persists in the literature. Unfortunately, it is not easy to correct this problem in a simulation study. The simulation program could contain a rule for determining the number of factors, but a) the simulation results will be heavily influenced by the choice of rule, and b) it is unclear how to evaluate the results if the number of factors is wrong.

Relatedly, the number of items (20) was constant across all conditions in the present study, meaning the number of items/factor (5) was constant. The ratio items/factor is important in determining whether a factor solution is identified (Comrey & Lee, 1992; Fabrigar et al., 1999), meaning target-iteration might behave very differently when the number of items/factor is low (2 - 4). The most complex population condition in the present study (30 cross-loadings) provides a clue about how target-iteration might perform with under-identified structures. Specifically, the 30 cross-loadings resulted in very few pure indicators (items loading on only one factor), which are important for factor identification. The results in Tables 5 – 7 (30 cross loadings) suggest that the identification problem does not have a major effect on target-iteration performance.

[19] The results of the present study suggest that the choice of starting rotation does not matter; therefore, it is recommended that applied researchers start with a simple, common rotation (such as Quartimin; k = 0).

A third limitation is that RMSE was the only measure of solution accuracy used in the present study. Many alternatives to RMSE exist (see Greene, 2003; Stekler, 1987; Willmott & Matsura, 2006; Willmott et al., 1985), and have been shown occasionally to diverge from RMSE (and from each other). Mean absolute error (MAE), for example, is influenced by extreme error values far *less* than is RMSE. By using only RMSE (not MAE), one cannot be sure whether the differences in RMSE among solutions are due mainly to outliers.

A fourth limitation of the present study is that there was no simulation condition with weak factors (e.g. all loadings < 0.40). Previous research has shown that weak factor structures (specifically, small communalities) often make it difficult to extract factors (see MacCallum et al., 2001; Hogarty et al., 2005), let alone rotate them. Because population structures 1A-1C in the present study all used 0.40 as a *minimum* loading size, it is impossible to know how well iterated-target rotation would perform with weaker factor structures. Worsening the above limitation, the cross-loadings used in the present study were relatively weak (exactly 0.25) compared to the average non-cross-loading (0.50).

Finally, the "comparison" (initial) rotations used to start the iterated target rotations in this study were all from the Crawford-Ferguson family. Despite justification for using only Crawfer rotations (see "Rotations in the present study," above), the fact remains that non-Crawfer rotations (e.g. Geomin, Promax, etc.) might have yielded different results. Specifically, non-Crawfer rotations might have, a) yielded different final (iterated target) results because the initial targets would have been different, and/or b) yielded initial results that were better than the converged iterated target results. If the latter were true—i.e. if an existing analytic rotation (such as Geomin) produced better results than the converged iterated targets solution—the conclusion of the present study would be very different.

The user-input problem

One of the central goals of developing an iterated target rotation method was to capitalize on the main strength of target rotation (extreme flexibility) while avoiding the main weakness (user-input requirements). Target-Iteration still requires user-input, however; first, one must choose the initial rotation from which to start the target-iteration process, and second, one must choose the method by which target matrices are generated from previous solutions. The results of the present study (see Table 8) suggest that the choice of initial rotation is usually inconsequential, meaning one could avoid that user-input requirement by simply using any common rotation (CF-Parsimax, Promax, Geomin, etc.) as the standard initial rotation.

With the above user-input requirement eliminated, one is left with only one important user input: how should each target matrix (in a series of iterated target rotations) be generated from the solution before it in the sequence? In the present study, the decision of how to create a target matrix from a previous solution involved choosing a threshold (0.05, 0.10, or 0.15) under which all loadings were converted to zeros in the target matrix[20]. It is important to note, however, that there are many ways to generate a target matrix from a rotated solution, and using a constant threshold (of 0.10, for example) is only one of them. An alternative method is discussed in the appendix.

[20] See Conditions 2A – 2C in the "Simulation Conditions" section, above. Also see Table 4 for an example using threshold = 0.10.

Appendix

Future direction: An alternative target-creation method

One alternative to the constant-threshold method of target-creation is to use a variable-threshold method in which each row/column (or even each item) has its own threshold for whether it should be converted to a zero (or "?"). A convenient option is to use the standard errors (SE's) of the loading estimates to calculate confidence intervals around said estimates; if a loading's confidence interval contains zero, it is converted to a zero in the target matrix. Table A1 shows an example (using N = 500) of how one can go from a target-rotated solution to a new target matrix by using the 90% confidence intervals. Each element of the resulting target matrix (columns 10 - 13 in Table 10) was determined by whether that loading estimate's confidence interval contained zero. If so, that loading was coded as a zero in the target matrix.

One promising characteristic of this standard-error-based target-creation method is that it eliminates the need to specify a threshold at all. Because each loading estimate will always have a standard error, there is no need to specify any type of threshold[21], further reducing the amount of user-input (beyond covariance matrix and number of factors) required to use target-iteration. Specific strengths and weaknesses of the SE-based target-creation method in various conditions is beyond the present scope, but due to its potential as a truly input-free method, it is worth exploring whether it works at all.

There are two common statistical problems that could prove fatal to SE-based target-creation. The first is that the standard errors of factor loadings depend on sample size (N), which

[21] This comes with two caveats. First, calculating standard errors becomes much more complicated when the raw data are not continuous. CEFA, for example, will not calculate standard errors of loadings estimated from a polychoric correlation matrix (but see Lee, Zhang, & Edwards, 2012). Second, this standard-error-based method of creating targets *does* require one to specify the size of the confidence interval (90% versus 95%, etc.), but unlike the constant-threshold method, existing literature provides standards for confidence interval size.

means extremely large/small N's might produce standard errors that are so small/large that they result in target matrices of either all 0's or all ?'s. The second potential problem is that SE-based iterated target series might not converge. Because each loading has its own standard error estimate (each of which changes with each iteration), the target matrices might be so likely to change from one iteration to another that the series never settles on a solution.

With respect to the first problem (sample size), preliminary investigation using a wide variety of population structures revealed that, even at absurdly small (2 persons/item) and large (50000 persons/item) sample sizes, the standard errors remained a viable tool for target-creation. Of course, very small N's still tended to produce poor solutions, but the important point is that SE-based target-creation is useable even at the extremes of N. With respect to the second problem (convergence), preliminary investigation revealed that convergence might indeed be a problem under two conditions, a) when N is small, and b) when the population structure is very complex. Further, although not investigated here, it is almost certainly true that changing the width of the confidence interval (e.g. from 90% to 99%) would affect convergence rates. Specifically, a very narrow confidence interval would increase the probability of creating a target of all ?'s, whereas a very wide confidence interval would increase the probability of creating a target of all 0's.

To investigate the performance of the SE-based target-creation method, 1000 simulations were performed (using the conditions described on pages 21-23) in which the SE-based method was used to generate targets. Table A2 shows the median RMSE obtained from most of the conditions (some initial CF rotations omitted), and each value in Table 11 can be compared to the corresponding values in Tables 5 - 7. Without doing further analyses (e.g. Table 9), three phenomena suggested by Table 11 are worth noting. First, as mentioned above, target-iteration

using the SE-based method of target creation will encounter convergence problems if N is small and/or the population structure is complex. Second, the converged SE-based solutions tended to be more accurate (RMSE = 0.039) than the initial CF rotations (RMSE = 0.60) in all conditions[22]. Third, and most importantly, the SE-based solutions tended to be more accurate than the constant-threshold-based solutions in the main study (across all threshold conditions; Tables 5 – 7). That is, even if one considers only the best threshold condition (Table 5; threshold = 0.05), the constant-threshold method still produced a higher global RMSE (0.044) than the SE-based method[23].

Note a crucial difference between the threshold-based and standard-error-based target-creation methods: the threshold-based method ignores negative loadings (always coding them as 0), whereas the SE-based method will code a negative loading as "?" if its confidence interval does not contain 0. The threshold-based method implicitly assumes that negative loadings cannot exist without keying mistakes; therefore, if a population structure truly has negative loadings, the threshold-based method will always fail[24]. By contrast, the SE-based method is driven only by whether loadings are statistically significant; if the confidence interval of a negative loading does not include zero, the SE-based method will estimate that loading (along with the positive loadings). The threshold-based and SE-based methods will often produce different solutions based on entirely different rationales.

[22] Difference in RMSE is statistically significant (two-tailed $p < 0.001$).

[23] Two-tailed $p < 0.001$.

[24] A user would probably be alerted to this failure (as opposed to interpreting the failed solution), because the threshold-based method will often not converge when the population structure contains negative loadings. The negative loadings cause the iterated targets to "bounce" back and forth between extreme solutions (in which many loadings alternate between negative and positive).

Table 1. Quartimin-, Parsimax-, and Facparsim-rotated factor patterns using a complex population structure

	Population			Quartimin			Parsimax			Facparsim		
Item	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
1	.50	.30	.30	.62	.22	.19	.58	.26	.25	.57	.29	.29
2	.50			.58			.53			.50		
3	.50			.58			.53			.50		
4		.70			.73			.69			.67	
5		.50	.40		.51	.34		.51	.35		.51	.38
6		.30			.31			.30			.29	
7			.60			.58			.56			.54
8			.60			.58			.56			.54
9			.10									

Factor Inter-Correlations

	Population			Quartimin			Parsimax			Facparsim		
Factor	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
F1	1			1			1			1		
F2	.40	1		.56	1		.44	1		.34	1	
F3	.50	.60	1	.61	.65	1	.49	.57	1	.40	.48	1

Note. Loadings < .10 are removed; F = "Factor"; Target rotation not shown, because (in this case), it reproduces the population structure perfectly.

Table 2. Quartimin-, Parsimax-, and Facparsim-rotated factor patterns using a complex population structure

	Population			Quartimin			Parsimax			Facparsim		
Item	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
1	.50	.20	.20	.63		.17	.55		.24	.53	.11	.28
2	.50	.40		.63	.22		.56	.28		.54	.30	.11
3	.50			.61	-.15		.53			.50		
4		.80			.75			.72			.70	.12
5		.50			.47			.45			.44	
6		.20			.19			.18			.17	
7			.60			.62			.62			.61
8		.30	.50		.22	.53		.21	.55		.21	.56
9	.10	.10	.10	.13			.12		.12	.11		.13

Factor Inter-Correlations

	Population			Quartimin			Parsimax			Facparsim		
Factor	F1	F2	F3	F1	F2	F3	F1	F2	F3	F1	F2	F3
F1	1			1			1			1		
F2	.50	1		.65	1		.52	1		.43	1	
F3	.50	.50	1	.61	.52	1	.49	.52	1	.44	.42	1

Note. Loadings < .10 are removed; F = "Factor"

Table 3. Example population factor patterns using the rules described in part 1 of the "Simulation Conditions" section.

	1A				1B				1C				1D (other)			
	5 cross-loadings				10 cross-loadings				30 cross-loadings				20 cross-loadings			
Var	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	**.60**				**.60**				**.60**		.25		**.62**	.16	.24	
2	**.60**				**.60**	.25			**.60**			.25	**.60**			.25
3	**.60**				**.60**		.25	.25	**.60**		.25	.25	**.65**			.34
4	**.60**		.25	.25	**.60**				**.60**	.25	.25		**.65**	.33		
5	**.60**				**.60**				**.60**				**.43**		.28	
6		**.50**				**.50**				**.50**	.25	.25	**.65**	.35	.32	
7		**.50**				**.50**			.25	**.50**			**.66**			
8		**.50**			.25	**.50**		.25	.25	**.50**			**.52**		.19	.25
9		**.50**		.25		**.50**	.25			**.50**			**.69**		.28	.21
10		**.50**				**.50**			.25	**.50**	.25	.25	**.45**			
11			**.40**				**.40**		.25		**.40**			**.65**		.31
12	.25		**.40**				**.40**	.25		.25	**.40**	.25		**.41**		
13			**.40**				**.40**				**.40**	.25	.34	**.67**		
14			**.40**				**.40**		.25		**.40**	.25		**.57**		.25
15			**.40**				**.40**		.25		**.40**	.25	.20	**.46**	.21	
16				**.60**				**.60**		.25	.25	**.60**			**.56**	
17		.25		**.55**	.25	.25	.25	**.55**	.25	.25		**.55**			**.65**	
18				**.50**				**.50**		.25	.25	**.50**		.33	**.53**	
19				**.45**				**.45**			.25	**.45**			.32	**.69**
20				**.40**				**.40**		.25	.25	**.40**	.25			**.64**

Table 4. Example of perfect target iteration convergence on a complex population structure, starting from a random target matrix.

	Population structure				Random target				Target rotation 0				Target 1			
Item	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	.50		.40	.30	0	0	0	0	.38		.42	.24	?	0	?	?
2	.50	.40			?	0	0	0	.66	.12			?	?	0	0
3	.50				?	?	0	0	.48		.13		?	0	?	0
4	.50				0	0	?	0	.48		.13		?	0	?	0
5	.50		.20	.20	?	?	?	0	.41		.26	.21	?	0	?	?
6		.50		.40	?	?	?	?		.51		.35	0	?	0	?
7	.20	.50	.40		?	?	?	?	.42	.55	.22		?	?	?	0
8	.20	.50			0	0	0	0	.42	.34			?	?	0	0
9		.50		.40	0	?	?	?		.51		.35	0	?	0	?
10		.50		.30	0	?	0	0	.13	.49		.24	?	?	0	?
11	.20	.40	.50		?	?	?	?	.38	.52	.35		?	?	?	0
12	.20		.50	.40	0	?	?	0		.27	.40	.27	0	?	?	?
13		.30	.50	.30	0	?	?	0		.59	.26		0	?	?	0
14			.50		0	0	?	0		.27	.46		0	?	?	0
15		.40	.50	.20	?	?	0	?	.12	.65	.24		?	?	?	0
16	.30		.40	.50	0	0	0	0	.12	.20	.31	.44	?	?	?	?
17	.30			.50	0	0	0	?	.12			.61	?	0	0	?
18				.50	?	0	?	?		.11		.57	0	?	0	?
19			.40	.50	0	?	0	?		.33	.23	.40	0	?	?	?
20	.30	.30	.30	.50	?	0	0	?	.26	.40		.42	?	?	0	?

Note. Continued on next page.

Table 4. (Cont.)

	Target rotation 1				Target 2				Target rotation 2				Target 3			
Item	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	.47		.34	.31	?	0	?	?	.50		.41	.29	?	0	?	?
2	.53	.39			?	?	0	0	.49	.40			?	?	0	0
3	.51				?	0	0	0	.50				?	0	0	0
4	.51				?	0	0	0	.50				?	0	0	0
5	.48		.14	.23	?	0	?	?	.50		.21	.20	?	0	?	?
6		.53		.34	0	?	0	?		.51		.37	0	?	0	?
7	.21	.53	.39		?	?	?	0	.20	.50	.41		?	?	?	0
8	.23	.50			?	?	0	0	.20	.51			?	?	0	0
9		.53		.34	0	?	0	?		.51		.37	0	?	0	?
10		.53		.24	0	?	0	?		.51		.27	0	?	0	?
11	.20	.44	.50		?	?	?	0	.20	.40	.51		?	?	?	0
12	.15		.47	.37	?	0	?	?	.20		.51	.36	?	0	?	?
13		.36	.50	.20	0	?	?	?		.30	.51	.24	0	?	?	?
14			.52		0	0	?	0			.51		0	0	?	0
15		.46	.51		0	?	?	0		.40	.51	.14	0	?	?	?
16	.25		.35	.49	?	0	?	?	.30		.41	.48	?	0	?	?
17	.27			.54	?	0	0	?	.30			.51	?	0	0	?
18				.51	0	0	0	?				.50	0	0	0	?
19			.38	.46	0	0	?	?			.41	.46	0	0	?	?
20	.28	.35	.24	.46	?	?	?	?	.30	.31	.31	.47	?	?	?	?

Note. Converged solution (Target rotation 3, not shown) is identical to population structure (RMSE < .0001); population inter-factor correlations (phi) set to .20; loadings < .10 omitted; Random target generated such that each element had a 50% binary probability (0,1); Target rotation 0 is based on random target.

Table 5. Median root mean square error (RMSE) for each of the seven initial Crawfer rotations and their converged iterated-targets solutions (threshold = .05) for three sample sizes.

	N = 1000					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.035	.028	.044	.028	.087	.039
k = .133 (Parsimax)	.037	.029	.044	.029	.067	.033
k = .2	.041	.030	.046	.030	.068	.033
k = .4	.048	.034	.052	.033	.068	.033
k = .6	.054	.035	.056	.034	.068	.034
k = .8	.058	.036	.058	.035	.069	.034
k = 1 (Facparsim)	.061	.036	.060	.035	.069	.034
	N = 500					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.046	.043	.053	.043	.093	.057
k = .133 (Parsimax)	.048	.044	.052	.044	.075	.052
k = .2	.050	.045	.055	.045	.075	.052
k = .4	.058	.048	.061	.046	.075	.053
k = .6	.063	.049	.064	.047	.075	.053
k = .8	.066	.050	.066	.048	.076	.053
k = 1 (Facparsim)	.069	.050	.068	.048	.076	.053
	N = 250					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.064	.071	.070	.069	.108	.085
k = .133 (Parsimax)	.065	.071	.069	.069	.090	.082
k = .2	.066	.071	.071	.070	.090	.082
k = .4	.072	.073	.075	.072	.090	.082
k = .6	.076	.074	.078	.073	.090	.083
k = .8	.079	.075	.080	.073	.090	.083
k = 1 (Facparsim)	.081	.075	.081	.074	.090	.083

Note. Simulations = 1000; Loadings < .05 were converted to zeros in the target matrix. Loadings > .05 were converted to ? in the target matrix.

Table 6. Median root mean square error (RMSE) for each of the seven initial Crawfer rotations and their converged iterated-targets solutions (threshold = .10) for three sample sizes.

	N = 1000					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.035	.028	.043	.028	.086	.059
k = .133 (Parsimax)	.038	.028	.043	.028	.067	.045
k = .2	.041	.028	.046	.028	.067	.045
k = .4	.049	.028	.052	.028	.068	.043
k = .6	.055	.029	.056	.028	.068	.043
k = .8	.059	.029	.058	.028	.068	.042
k = 1 (Facparsim)	.061	.029	.060	.028	.068	.042
	N = 500					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.046	.040	.053	.041	.094	.069
k = .133 (Parsimax)	.048	.040	.053	.041	.075	.061
k = .2	.050	.040	.055	.041	.075	.061
k = .4	.058	.041	.061	.041	.075	.061
k = .6	.063	.041	.064	.041	.076	.060
k = .8	.066	.042	.066	.041	.076	.060
k = 1 (Facparsim)	.069	.042	.067	.042	.076	.060
	N = 250					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.064	.060	.070	.060	.107	.088
k = .133 (Parsimax)	.065	.060	.069	.060	.091	.082
k = .2	.066	.060	.070	.061	.090	.081
k = .4	.072	.061	.075	.061	.090	.082
k = .6	.077	.062	.078	.062	.090	.082
k = .8	.080	.062	.080	.062	.091	.082
k = 1 (Facparsim)	.082	.062	.081	.062	.091	.082

Note. Simulations = 1000; Loadings < .1 were converted to zeros in the target matrix. Loadings > .1 were converted to ? in the target matrix.

Table 7. Median root mean square error (RMSE) for each of the seven initial Crawfer rotations and their converged iterated-targets solutions (threshold = .15) for three sample sizes.

	N = 1000					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.035	.029	.043	.031	.087	.084
k = .133 (Parsimax)	.037	.029	.043	.030	.067	.074
k = .2	.041	.028	.046	.030	.068	.073
k = .4	.050	.028	.053	.030	.068	.073
k = .6	.055	.029	.056	.030	.068	.072
k = .8	.059	.029	.059	.030	.068	.072
k = 1 (Facparsim)	.061	.029	.060	.030	.068	.072
	N = 500					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.046	.041	.053	.043	.095	.089
k = .133 (Parsimax)	.048	.041	.052	.042	.074	.081
k = .2	.050	.041	.055	.042	.074	.081
k = .4	.058	.041	.061	.042	.075	.081
k = .6	.062	.041	.064	.042	.075	.080
k = .8	.066	.041	.066	.042	.075	.080
k = 1 (Facparsim)	.069	.041	.067	.042	.075	.080
	N = 250					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.063	.059	.071	.062	.109	.103
k = .133 (Parsimax)	.064	.059	.069	.061	.089	.095
k = .2	.066	.059	.071	.061	.089	.095
k = .4	.072	.059	.075	.061	.089	.094
k = .6	.077	.060	.079	.061	.089	.095
k = .8	.079	.060	.081	.061	.090	.095
k = 1 (Facparsim)	.081	.060	.082	.062	.090	.095

Note. Simulations = 1000; Loadings < .15 were converted to zeros in the target matrix. Loadings > .15 were converted to ? in the target matrix.

Table 8. Standardized regression coefficients and t values for four simulation condition effects and their cross-products.

Predictor	Std. effect (β)	t value	Prob.
Sample size (N)	-.70	-30.12	.000
Thresh	.10	4.26	.000
Cross	.53	22.70	.000
Kappa	.00	.21	.833
N*Thresh	.12	5.20	.000
N*Cross	-.01	-.57	.571
N*Kappa	-.01	-.41	.684
Thresh*Cross	.33	14.22	.000
Thresh*Kappa	-.04	-1.80	.074
Cross*Kappa	-.06	-2.62	.009

Note. Dependent variable is median root mean square error (RMSE); adjusted R-squared is .90; “Thresh” is the threshold used to convert factor pattern matrices to target matrices (see "Specifying a target matrix" in body); “Cross” is the number of cross-loadings.

Table 9. Median root mean square error (RMSE) for each of the seven initial Crawford-Ferguson rotations and their converged iterated-targets solutions for three sample sizes.

	N = 1000					
	Threshold = .05		Threshold = .10		Threshold = .15	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.060	.029	.059	.033	.059	.044
k = .133 (Parsimax)	.083	.038	.083	.033	.083	.042
k = .6	.100	.044	.100	.034	.099	.042
k = 1 (Facparsim)	.101	.045	.102	.034	.102	.042
	N = 500					
	Threshold = .05		Threshold = .10		Threshold = .15	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.067	.043	.066	.044	.067	.054
k = .133 (Parsimax)	.089	.050	.090	.045	.090	.052
k = .6	.104	.055	.104	.045	.105	.052
k = 1 (Facparsim)	.106	.057	.107	.045	.108	.052
	N = 250					
	Threshold = .05		Threshold = .10		Threshold = .15	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	.081	.069	.081	.063	.081	.070
k = .133 (Parsimax)	.100	.075	.100	.066	.100	.068
k = .6	.113	.080	.113	.067	.113	.068
k = 1 (Facparsim)	.115	.080	.115	.068	.115	.068

Note. Simulations = 1000; population factor structure is 1D (see Table 3).

Table A1. Target-Rotated solution, confidence intervals, and the resulting standard-error-based target matrix for a four-factor structure with twenty-eight cross-loadings and N = 500.

	Target-Rotated solution				90% confidence intervals				New target matrix			
Item	F1	F2	F3	F4	F1	F2	F3	F4	F1	F2	F3	F4
1	**.61**	.29		.29	(.54; .67)	(.19; .37)	(-.00; .04)	(.08; .50)	?	?	0	?
2	**.61**			.28	(.54; .67)	(-.01; .09)	(-.07; .12)	(.00; .54)	?	0	0	?
3	**.58**	.24		.30	(.51; .64)	(.15; .33)	(.00; .17)	(.09; .50)	?	?	?	?
4	**.53**			.34	(.45; .60)	(-.09; .03)	(-.08; .12)	(.09; .58)	?	0	0	?
5	**.66**		.38	.35	(.60; .72)	(.10; .28)	(.29; .46)	(.11; .59)	?	?	?	?
6		**.43**			(-.06; .06)	(.31; .53)	(-.01; .10)	(-.16; .13)	0	?	0	0
7		**.56**	.22		(-.10; .00)	(.44; .67)	(.10; .32)	(-.05; .32)	0	?	?	0
8	.32	**.58**			(.23; .40)	(.48; .66)	(-.09; .01)	(-.10; .01)	?	?	0	0
9	.34	**.61**	.33		(.26; .41)	(.49; .72)	(.24; .41)	(.00; .12)	?	?	?	0
10		**.48**	.33	.20	(-.01; .09)	(.37; .59)	(.24; .42)	(.06; .34)	0	?	?	?
11			**.45**		(.02; .14)	(-.03; .12)	(.36; .53)	(-.11; .02)	?	0	?	0
12			**.47**		(-.13;-.01)	(-.09; .06)	(.38; .55)	(.04; .30)	?	0	?	?
13	.30		**.36**	.34	(.22; .38)	(-.12; .02)	(.27; .45)	(.18; .50)	?	0	?	?
14	.34	.27	**.41**	.25	(.26; .40)	(.17; .36)	(.33; .48)	(.13; .36)	?	?	?	?
15			**.38**		(-.10; .03)	(-.08; .09)	(.29; .47)	(-.07; .07)	0	0	?	0
16				**.58**	(-.13; .05)	(-.08; .03)	(-.07; .03)	(.48; .67)	0	0	0	?
17			.26	**.54**	(-.03; .08)	(.03; .24)	(.16; .35)	(.43; .63)	0	?	?	?
18		.20		**.43**	(-.04; .08)	(.09; .30)	(.03; .22)	(.32; .54)	0	?	?	?
19		.22	.32	**.39**	(-.05; .06)	(.11; .32)	(.22; .40)	(.27; .50)	0	?	?	?
20				**.46**	(-.07; .05)	(.00; .16)	(-.05; .08)	(.36; .55)	0	?	0	?

Note. F = Factor; loadings < .20 not shown in target-rotated solution; population factor structure (not shown) was similar to condition 1a in Table 3 (but with a different cross-loading pattern).

Table A2. Median root mean square error (RMSE) for four initial Crawford-Ferguson rotations and their converged iterated-targets solutions for three sample sizes.

	N = 1000					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	0.035	0.028	0.044	0.028	0.088	0.044
k = .133 (Parsimax)	0.038	0.029	0.044	0.028	0.068	0.035
k = .6	0.054	0.029	0.056	0.028	0.070	0.035
k = 1 (Facparsim)	0.060	0.029	0.060	0.028	0.070	0.035
	N = 500					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	0.046	0.042	0.053	0.040		
k = .133 (Parsimax)	0.048	0.042	0.052	0.040	Not applicable	
k = .6	0.063	0.042	0.064	0.041	Convergence < 90%	
k = 1 (Facparsim)	0.069	0.042	0.068	0.041		
	N = 250					
	Cross-Loadings = 5		Cross-Loadings = 10		Cross-Loadings = 30	
Initial Rotation	Initial	Converged	Initial	Converged	Initial	Converged
k = 0 (Quartimin)	0.063	0.059				
k = .133 (Parsimax)	0.064	0.059	Not applicable		Not applicable	
k = .6	0.076	0.060	Convergence < 90%		Convergence < 90%	
k = 1 (Facparsim)	0.081	0.059				

Note. Simulations = 1000; standard errors of factor loading estimates were used to create target matrices from rotated solutions; if a loading's 90% confidence interval included zero, it was converted to zero in the target matrix; otherwise, it was converted to a ?.

Figure 1. Histogram of raw simulation results (RMSE of converged iterated-target solutions) for one simulation condition: N = 250; population structure = 1B (10 cross-loadings); target-creation threshold = 0.10.

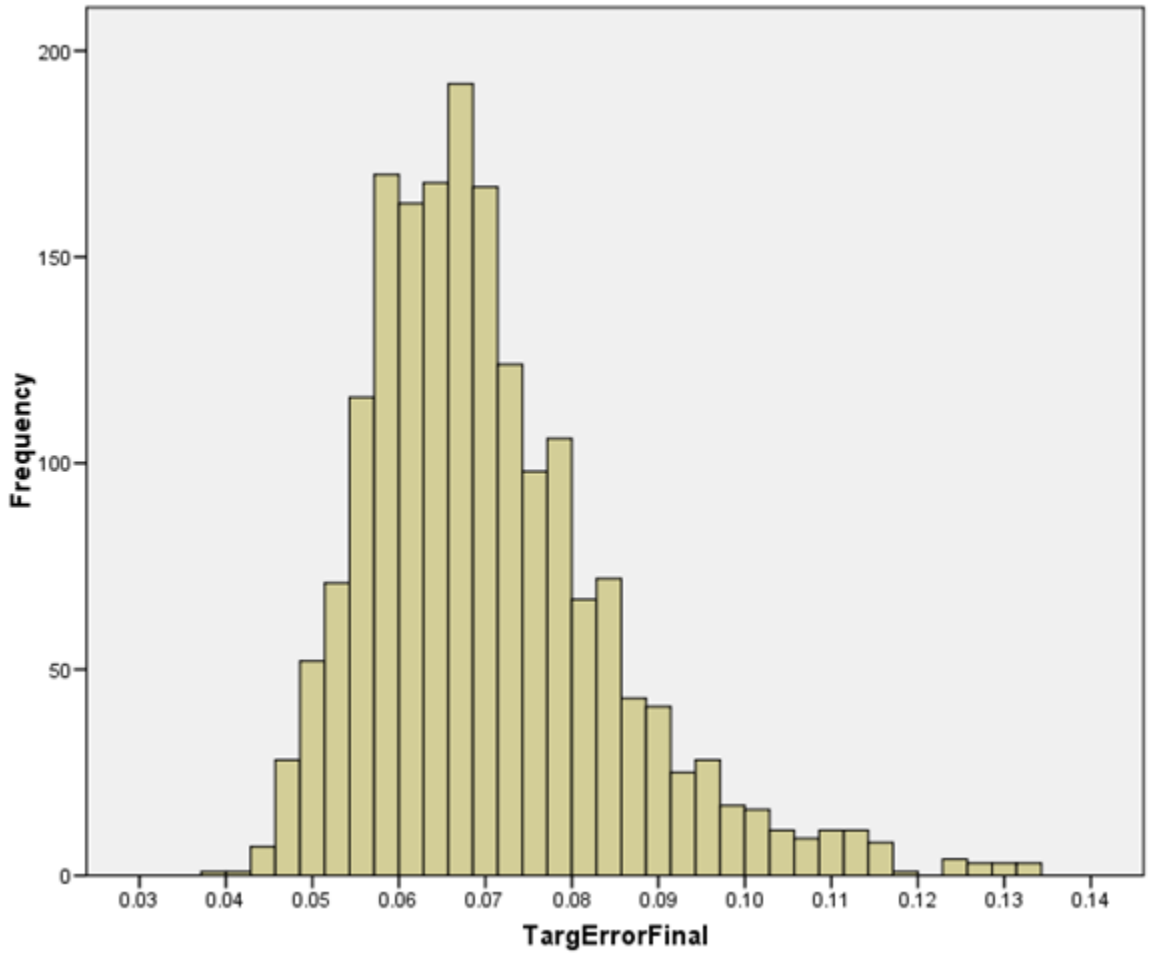


Figure 2. Histograms of raw simulation results (RMSE of Crawford-Ferguson and converged iterated-target solutions) for one simulation condition: N = 1000; population structure = 1D; target-creation threshold = 0.10.

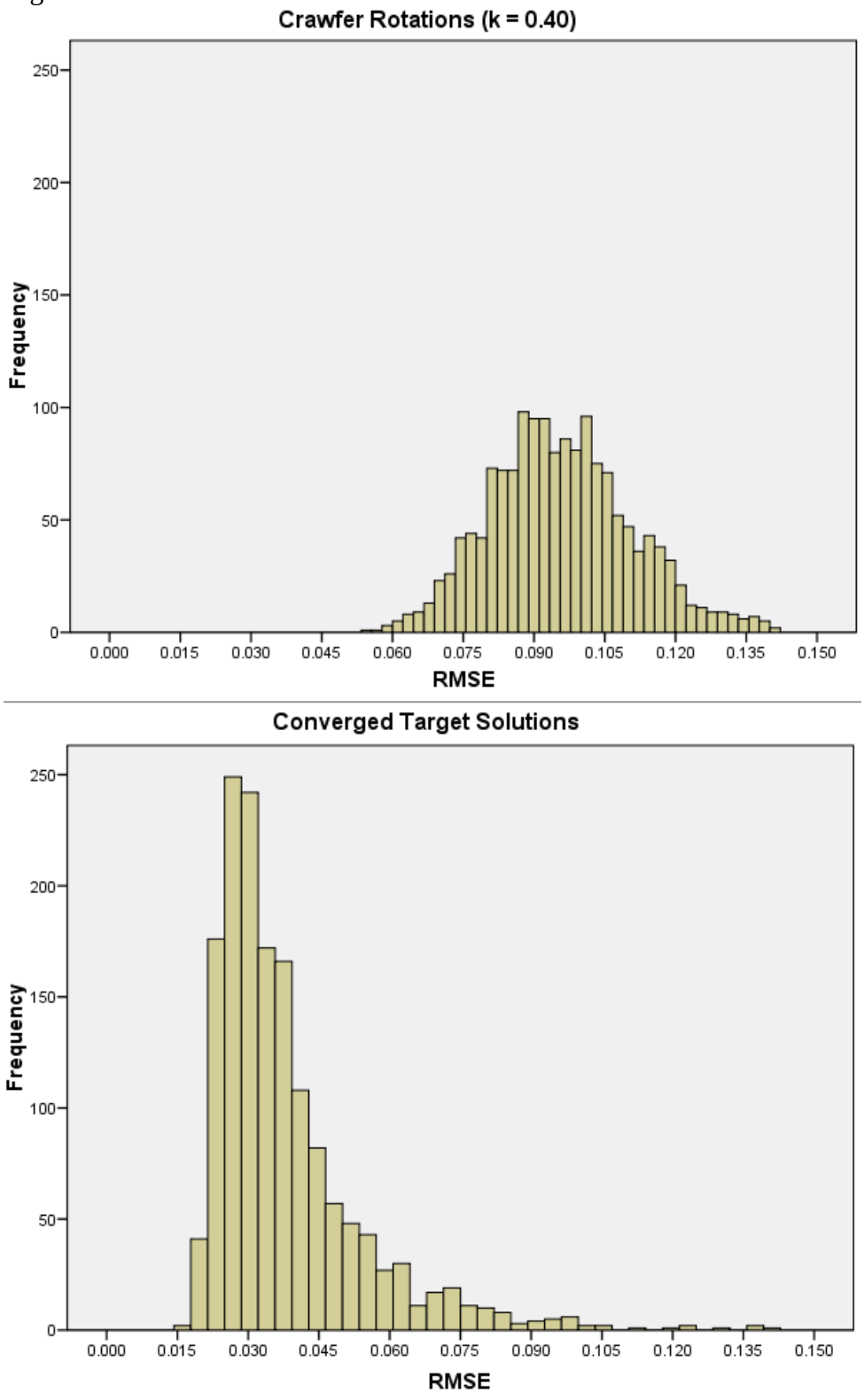


Figure 3. Median RMSE by iteration of converged iterated-target solutions for one simulation condition: N = 1000; population structure = 1C; target-creation threshold = 0.05.

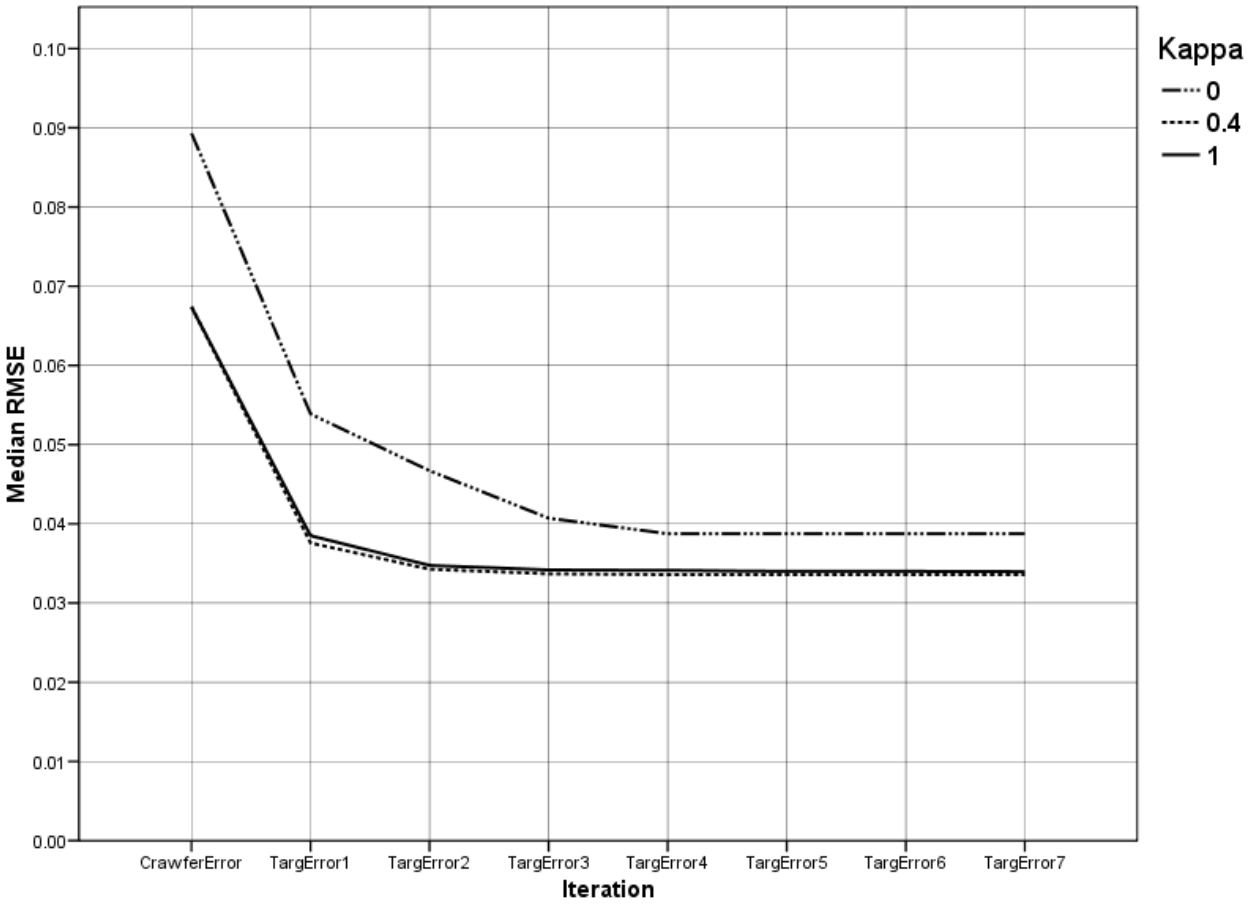


Figure 4. Distributions of median root mean-square error, by sample size and number of cross-loadings (population factor structure complexity).

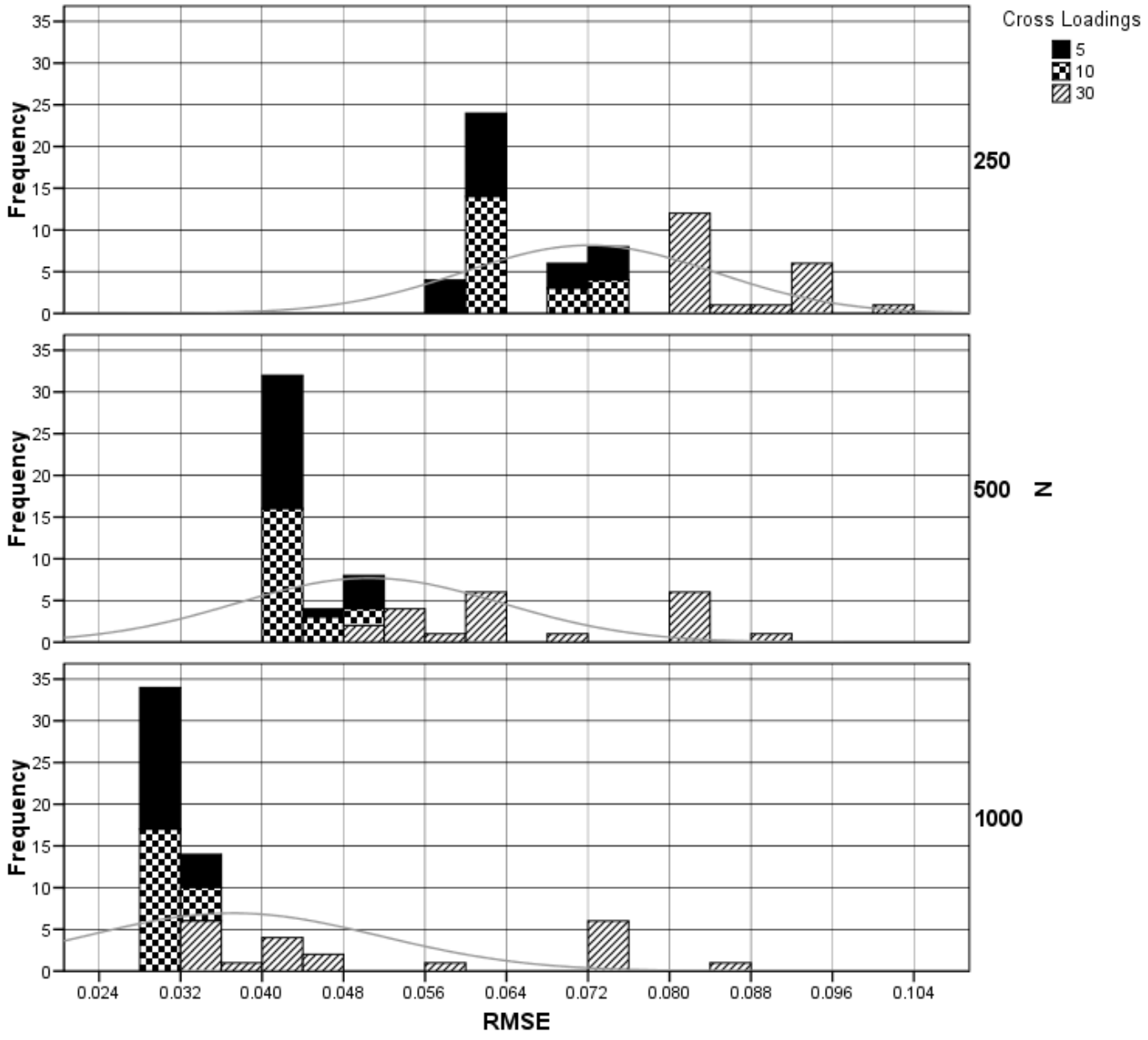


Figure 5. Demonstration of interaction between target-creation threshold and sample size using predicted values of RMSE from regression model.

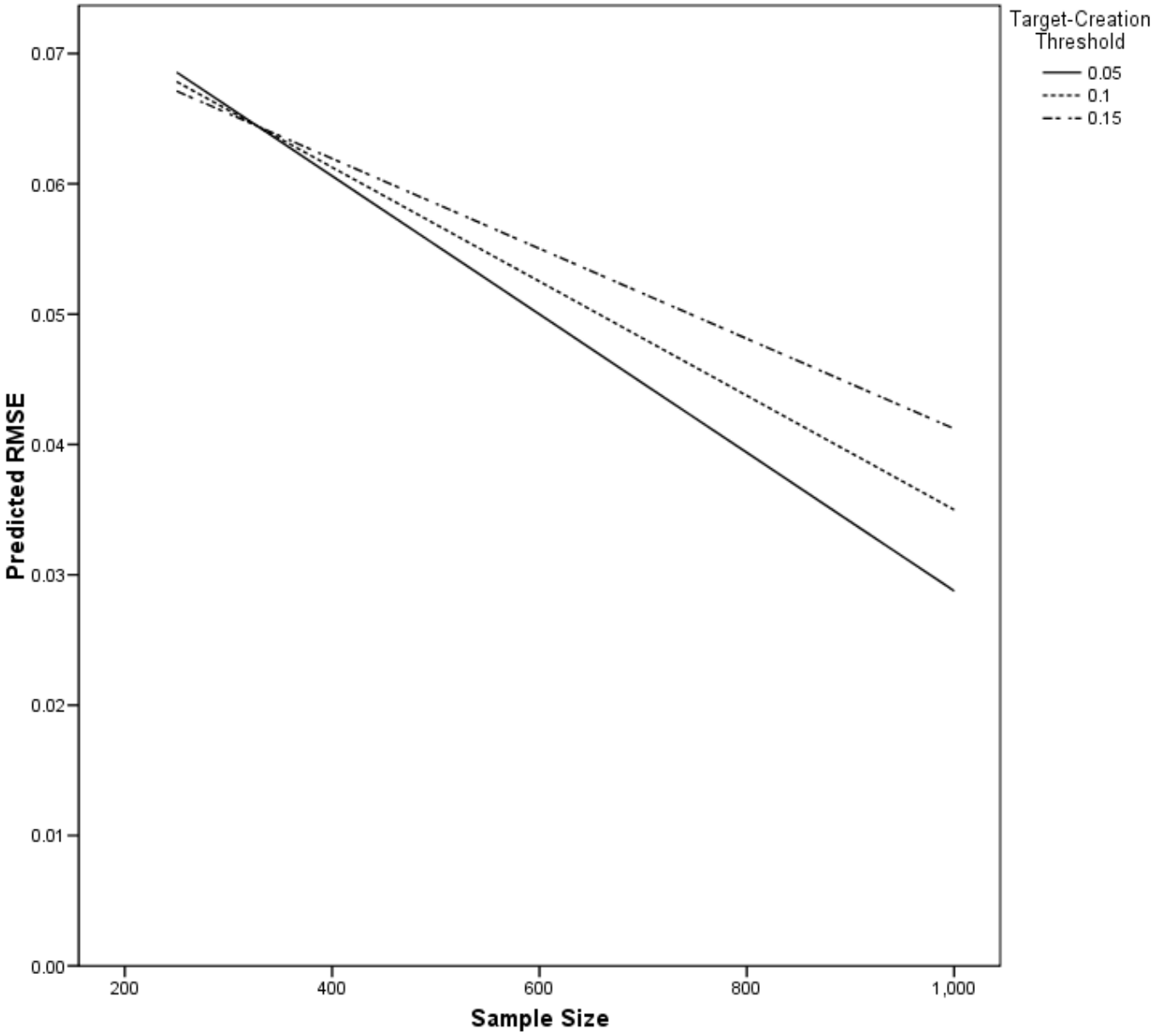
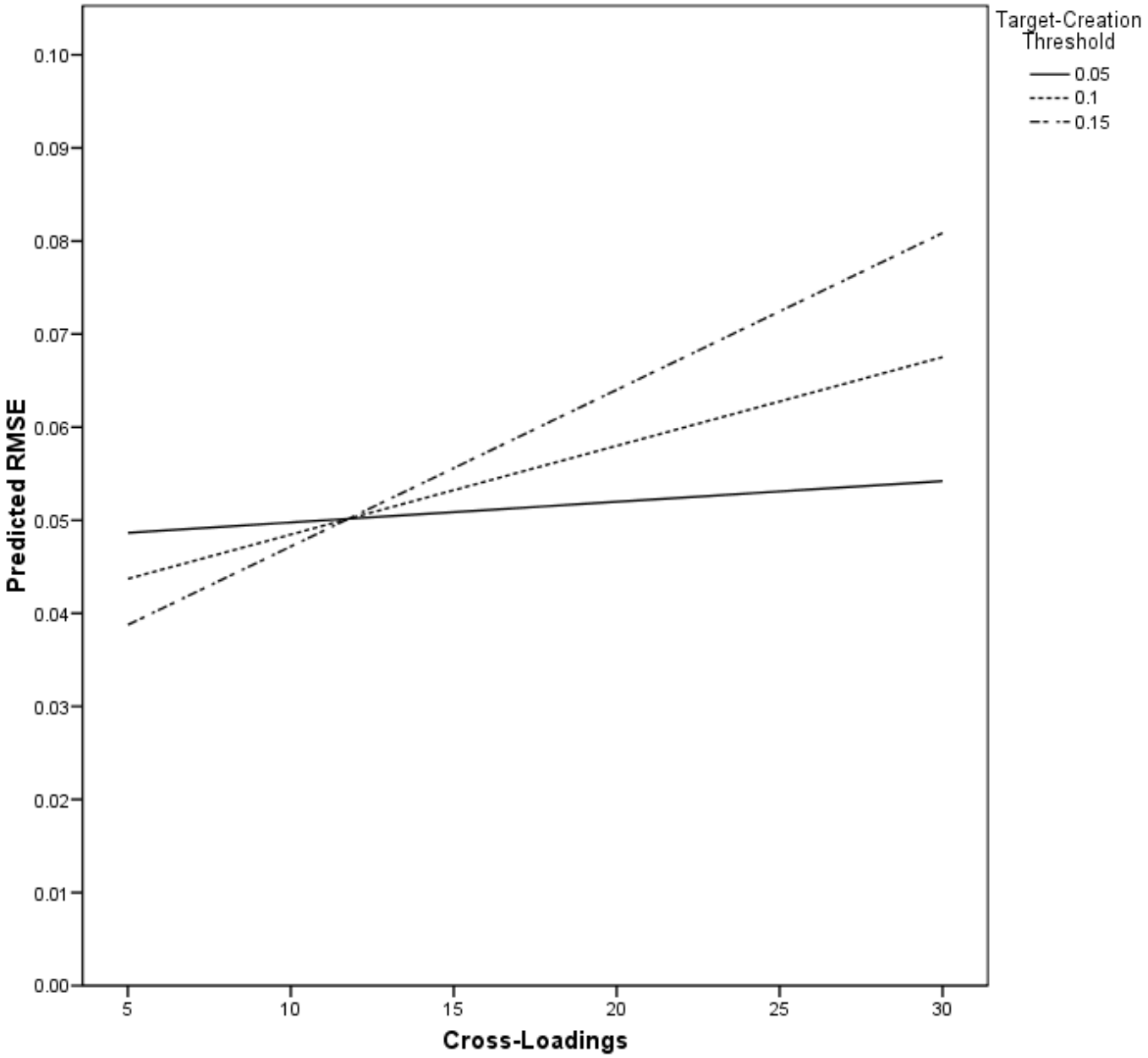


Figure 6. Demonstration of interaction between target-creation threshold and number of cross-loadings using predicted values of RMSE from regression model.



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