Title

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How Does Consumer Engagement Evolve When Brands Post Across Multiple Social Media?

ABSTRACT

Brands allocate their social media advertising across multiple platforms such as Facebook, Twitter, Instagram, and YouTube. Because consumers use multiple social media, brand communications on one platform could generate engagement within the same platform (direct effects) and potentially impact engagement with the brand on the other platforms (spillover effects). Additionally, past engagement with a post on a platform could sustain into the future, thereby improving the longevity of posts (carryover effects). These effects could also vary across platforms. Drawing on recent advertising literature, the authors propose and test differential carryover, spillover, and direct effects within and across social media. The empirical analysis indicates that these effects exist and are significant, supporting the propositions presented. The analysis provides generalizable guidelines to social media marketers on the effectiveness of the various platforms at sustaining a post and at creating direct and spillover effects across other platforms. Finally, the study also exemplifies a resource allocation model for brands to use when allocating their efforts across the various social media platforms to maximize both consumer engagement and the firm's return on social media investment.

Introduction

Consumers engage with brand communications across multiple social media platforms, such as Facebook, Twitter, and Instagram. To manage these engagements, brands employ a multi-platform social media strategy where they create and distribute branded content for and across these multiple platforms. Often consumer engagement with content posted on one of these platforms impacts engagement with the brand on the other platforms. For example, Figure 1 shows how Disney's post on Facebook or YouTube could lead to engagement with the brand's page, "handle," or content on Twitter (see Figure 1).

>>> INSERT FIGURE 1 HERE<<<

Prior research in marketing has shown that multi-channel marketing influences behavior both within a single channel as well as across channels. For example, several studies observe spillover effects of advertising across multiple media (Assael 2011; Naik and Raman 2003; Naik and Peters 2009; Sridhar et al. 2011) when brands use more than one medium to advertise. These spillovers are found to augment the direct media effects and affect multiple outcomes of brand interest such as awareness, brand value, and sales. Thus the influence of brand communications initiated in one medium carries over to other media (similar to an echo) and can have prolonged impact in the other media in the subsequent periods, i.e., past engagement affects future engagements (Hewett et al. 2016). Even though this effect has been established across media and digital ad platforms (Bruce, Murthi and Rao 2017), to the best of our knowledge, no study explores such interdependencies across multiple social media platforms in the brand's portfolio.

The direct and spillover effects due to consumer engagement with brand posts on social media emerge in multiple ways. For example, fans on Facebook can like and comment on brand-generated content. In addition, a post could generate general interest in the brand that causes some fans to interact with the brand on other platforms such as Twitter or Instagram, leading to engagement spillovers across multiple platforms. Finally, due to the carryover of past engagement into the future, some brand posts could have enduring effects that yield engagement with the post even days after the post appeared on social media. Understanding these different effects is managerially important because it has been shown that accounting for direct, spillover and carryover effects leads to more efficient allocation of resources across various media (e.g., Naik and Raman 2003; Naik and Peters 2009). These effects also provide insights into the how managers should allocate their social media posts to optimize the potential return on investment (ROI) in terms of engagement and profits—especially if engagement can be linked to downstream metrics like sales (Santini et al. 2020). Taken together, no study combines the effects of direct (consumer engagement with a brand on a particular social media post), spillover (consumers expand their engagement via their brand-related activity on other social media platforms), and carryover (consumer-driven echoes) effects as well as their impact on advertising resource allocation decisions.

In this study, we address the following questions:

(1) How does past consumer engagement with a brand's social media posts drive future engagement with the brand within the platform (carryover effects)?
(2) How does brand-generated content (posts) on one social media platform sustain engagement within the platform (direct effects) and across other platforms (spillover effects)?

(3) Given the dynamics of engagement as well as direct and spillover effects of consumer engagement, how should managers optimally allocate content across the different platforms to maximize engagement?

In sum, the study aims to help brands allocate resources by understanding how the different platforms drive consumer engagement both within and across their social media platforms by determining the direct and spillover effects in social media advertising and the efficacy of different social media platforms at sustaining engagement.

As social media advertising gained prominence in the marketer's portfolio, several marketing researchers began looking into the budget and brand-building implications of the use of social media in the marketing toolkit. Over the years, social media have been shown to relate to higher gross revenues and sales (Onishi and Manchanda 2012; Goh et al. 2013), higher sales due to a partner brand's social media presence (Kupfer et al. 2018) as well as to online visits and purchases (Fossen and Schweidel 2019). Research has also revealed the role of earned and owned social media in improving consumer mindset metrics and shareholder value (Colicev et al. 2018). Additionally, some studies also found that firm-generated brand communications on social media receive better reception (from consumers) than similar content delivered through traditional advertising channels like TV, print, and email (Stephen and Galak 2012; Kumar et al. 2015). Furthermore, it has been shown that user engagement on social media, in the form of likes, retweets, comments, etc. can influence offline customer behavior positively (de Vries et al. 2017; Mochon et al. 2017; Lee et al. 2018). While the importance of social media brand communications is recognized, insights into interdependencies between posts on a particular platform and cross-platform consumer engagement remain tenuous.

We differentiate from these studies by focusing on the enduring effects of past posts within a medium, the direct effects of current brand posts within the social medium, as well as the associated spillovers due to omni-social consumers (Appel et al. 2019). We use a dynamic state space model to connect brand-generated content and engagement both within and across social media platforms. We include data from 20 brands on three of the most commonly used social media platforms (by brands), namely Facebook (94%), Twitter (68%), and Instagram (54%) (Stelzner 2017) to estimate the model. The model also accounts for engagement dynamics that exist due to posts being continuously visible on a brand's social media page in the form.

First, we present a unique and easy to implement methodology for brands to measure the effect of their activities within and across multiple social media platforms. The model also allows brands to assess the prolonged effect of such content within and across the platforms. Second, because our measurements focus on user responses, it aids social media marketing managers in understanding the effectiveness of these platforms in generating user engagement in the presence of direct and spillover effects. Third, since our dataset spans 20 brands over 270 days across multiple platforms, we can provide generalizable guidelines for social media managers on the effectiveness of the various platforms at both, sustaining a post within a platform, as well as creating spillover effects across other platforms. Finally, the study also provides two resource allocation approaches for brands to use when allocating their efforts across the various social media platforms. The first approach seeks to maximize engagement, while the second approach optimizes the return on social media investment when engagement can be linked to sales.

The paper proceeds as follows. First, we develop the conceptual framework. We then proceed to describe the data, explain our empirical model and present our results, robustness checks, and alternative formulations as well as the resource allocations under the different formulations. Finally, we dicuss the managerial insights from our analysis and then conclude the paper by pointing to avenues for future research. Table 1 summarizes some of the literature stream relevant to our study and positions this research relative to the extant literature.

>>>INSERT TABLE 1 HERE<<<

Conceptual framework

Online marketing media allow for inexpensive one-to-many as well as easily scalable one-to-one interactions. These online media are often characterized as *paid*, *owned*, or *earned* or POE media (e.g., Stephen and Galak 2012, Lovett and Staelin 2016). *Paid* media usually refers to online media such as paid search advertising or paid social media advertising. *Owned* media refers to content that belongs to the organization, for example, the company's website, a brand post on its social media page. *Earned* media refers to mentions of the brand not generated by the organization, e.g., comments on Facebook by brand fans, retweets on Twitter from followers, etc. Paid and, to some extent owned media are closer to the traditional marketing methods such as advertising, branding or corporate identity. Earned media on the other hand, differ because they allow consumers to broadcast their opinions to a larger audience than was possible at any time before the Internet. In this study, we focus on owned media created and managed by the firm (brand social media pages, handles, tweets, posts, etc.) and the volume of earned media it engenders due to consumer interaction with this content in the form of likes, comments, shares, retweets, mentions, etc. Multiple studies (Dellarocras et al. 2007; Berger et al. 2010; Xiong and Bharadwaj 2014) find that volume of engagement can explain and predict sales even after controlling for other aspects of engagement such as valence. This could be because, as Godes and Mayzlin (2004) and de Vries, Gensler, and Leeflang (2012) find, most comments for brand posts on social media tend to be positive or neutral. Ilhan, Kübler and Pauwels (2018) also note that negative comments appear only 1–6% of the time.

The conceptual framework we use draws on three streams of literature: (1) direct effects of social media posts at driving engagement, (2) spillover effects across social media platforms, and (3) differential carryover effects due to the dynamics of engagement with posts. Next, we briefly elaborate on these streams of literature and discuss how they help develop our conceptual framework.

Direct effects of social media posts

Brands use multiple social media platforms to drive consumer engagement in order to increase brand awareness, stimulate online traffic to its owned media and potentially improve sales. Several studies have shown that social media engagement not only influences the top of the consumer-brand funnel (brand awareness) but also enhances brand outcomes (e.g., sales) directly (Kumar et al. 2013; Kumar 2015; Kumar et al. 2017) or indirectly through partner brands (Kupfer et al. 2018). Kupfer et al. (2018) find that a partner brand's social media power potential, its exertion and their interaction leads to higher sales –indicating that the direct effects of social media also emerge from partnerships with strong social media brands. In addition, user engagement on social media via likes, retweets and comments can also influence customer engagement with other marketing channels in a positive manner (de Vries et al. 2017; Mochon et al. 2017; Lee et al. 2018; Fossen and Schweidel 2019). Stephen and Galak (2012) also note that the pervasiveness of social media and the ability of consumers to engage on the platform have led social media's elasticity (earned) to exceed that of traditional media.

Apart from sales, social media engagement has also been shown to improve consumer mindset metrics and shareholder value due to its effects on brand awareness and purchase intent (Colicev et al. 2018). More recently, Liu, Dzyabura and Mizik (2020) find a strong relationship between brands' portrayal on social media and consumer perceptions about brands. Even though prior research has shown that brand posts on social media have direct measurable effects on engagement, consumer perceptions, sales or firm value, no study systematically measures the direct effects of brand posts on consumer engagement within a social media platform and then compares how these effects vary across the different social media platforms. This is important because previous research in advertising shows that the effect of advertising varies by media type (e.g., for online versus offline advertising see Naik and Peters 2009; Hewett et al. 2016) as well as within media type (e.g., for the case of different types of digital media see Bruce, Murthi and Rao 2017). More recently, Voorveld, van Noort, Muntinga and Bonner (2018) and Shahbaznezhad, Dolan and Rashidirad (2021) find that the effectiveness of social media content on users' engagement is moderated by context. Voorveld et al. (2018) in their study on consumer engagement across multiple social media platforms note that these social media play different roles in the users' social media portfolio. Specifically, platforms like Facebook are more relationship focused, while Twitter focuses on communication and Instagram performs a role as a creative outlet. These differing roles change how firm content could be viewed within and across these platforms, thereby causing differences in how consumers interact and engage with the content. Similarly, using data from Facebook and Instagram, Shahbaznezhad et al. (2021) also find some evidence for differing engagement effects across these social media due to their differing "content context." Therefore, a brand should also expect that a post's ability to generate engagement varies by the social media platforms employed.

Following these studies, in this article, we determine and then compare the direct effects of brand posts at generating engagement within the social media platform. Thus, building on prior research, our conceptual framework on direct effects of social media posts leads to the following proposition.

P1: Brand posts on a social media platform (a) increase consumer engagement with the brand on that platform, and (b) the magnitudes of these increases vary by the social media platforms the brand posts on.

Spillover effects of social media posts

Unlike in its infancy, a consumer's social media activity today encompasses multiple platforms (Appel 2019). This could imply that events (brand- or user-generated) across the multiple platforms are not independent of each other. The focus of prior research on social media marketing centered primarily on studying a single social media platform and its relation to traditional media or its effect on sales. No study has examined the spillover effects of brand posts across multiple social media platforms, even though "omni-social" consumers (Appel 2019) engage with the brand across these platforms.

The concept of spillover effects is not new to marketing. Several studies have examined the spillover effects of advertising across media types. For example, Gatignon and Hanssens (1987) note that advertising effectiveness improves with higher quality of sales efforts. Naik and Raman (2003) show that television and print advertising behave synergistically and enhance the effectiveness of each other. Similarly, multiple studies measure spillover between online and offline advertising (Naik and Peters 2009), offline TV advertising and online chatter (Tirunillai and Tellis 2017), online, regional, and national advertising (Sridhar et al. 2016), generic and branded keyword search (Rutz and Bucklin 2011), firm generated content on social media and television and email advertising (Kumar et al. 2015), and paid search and display ads (Kireyev, Pauwels and Gupta 2016) amongst several others.

Fossen and Schweidel (2019) explore the relationship between television advertising, social TV, online traffic and online sales. Their study identifies spillovers between television advertising and online social media chatter that help drive site visits which then lead to higher sales. Similarly, while studying viral content and its dissemination, Krijestorac, Garg, and Mahajan (2020) identify word-of-mouth as the driver of spillovers between platforms. Specifically, they find that introduction of videos to the audience of a new platform can generate word-of-mouth that leads to increases in viewership of the same video posted on another platform (prior to the new platform). These studies not only document the existence of spillovers, but also find that such spillovers could vary by media type.

Even though some prior studies discuss the role of brand- or user-generated content on social media, and multiple marketing studies have established the concept of spillovers within and across media types, no prior work has examined whether firm posts on a social media platform can generate consumer engagement with firm content across other social media platforms. As Naik and Raman (2003) also note, not accounting for spillovers or synergy between media could lead to the incorrect estimation of the overall effectiveness of a medium thereby leading to a misallocation of resources. Accordingly, we argue that firm posts on a social media platform generates consumer engagement with the firm's content on other social media platforms, leading to the following proposition.

P2: Brand posts on a social media platform (a) impact consumer engagement with the brand on other social media platforms in the brand's portfolio, and (b) the magnitudes of these spillovers vary by social media platforms.

Differential carryover effects of social media posts

Dynamic advertising models in marketing capture the instantaneous and long-term effects of advertising (Sethuraman, Tellis and Briesch 2011) using carryover effects. Carryover effects link the outcomes (goodwill, sales, awareness, engagement, or other measures) generated due to current advertising with past outcomes generated due to advertising in the previous time periods. Most studies measure a common carryover effect of advertising, even in the presence of multi-media advertising (e.g., Chintagunta & Vilcassim, 1994; Kolsarici and Vakratsas 2010; Braun and Moe 2013). However, carryover rates could vary across markets, media, and platforms.

Several studies find variations in carryover rates across markets for media that include television, print, radio, and billboards (Berkowitz, Allaway, and D'Souza 2001; Naik and Raman 2003; Sethuraman, Tellis and Briesch 2011). In many cases however, the inability to identify differential carryovers emerges from the lack of separate observations of the outcome variable by medium. For example, it is hard to identify and attribute differential carryovers for sales to advertisements on television, print, billboards if the brand only observes total sales. In the case of digital media, some studies (e.g., Breuer, Brettle and Engelen 2011; Bruce, Murthi and Rao 2017) have overcome this issue by using intermediate measures (e.g., engagement, clickthroughs, etc.) of consumer engagement with the advertisement (by digital format) to help identify these carryovers. Similar to these studies on digital advertising, different social media platforms could also exhibit differential carryover effects of consumer engagement. To the best of our knowledge, no prior study investigates how the carryover effect—a measure of the endurance of brand posts—varies across social media platforms used by the same brand. This leads us to the third proposition,

P3: Consumer engagement with a brand's posts on a social media platform (a) endures over time, and (b) the longevity of this engagement varies across social media platforms. Thus, consumer engagement across social media platforms exhibit differential carryover rates.

Data and model development

Data description

The data on a brand's behavior on social media and the consumer responses were obtained from a market research firm that collects and manages social media data for multiple brands. The marketing research firm tracked consumer engagement with brand posts twice-a-day on a daily basis and captured measures of engagement for the brand's social posts made on the date.

Consumer engagement or response data includes the number of social media likes, retweets, shares, mentions, comments, and dislikes for brand-generated content across the social media platforms. For example, when a brand makes a post on its social media brand page, a brand fan's action can be a like or dislike, retweet, share, mention, or a combination of these actions. Thus, the dataset contains information about the numbers of posts in a day (across social media platforms) and consumer responses to these posts. Unlike panel data with select consumers, the aggregated data provided by social media platforms do not reveal individual user level variables to preserve the users' anonymity. Consequently, this restricts us to tracking aggregate level user-generated actions only on the brands' owned social media pages or brand owned handles.

The social media platforms included in the dataset are Facebook, Twitter, and Instagram. The data were collected over a span of 270 days. The brands in the dataset were some of the best known and most valuable brands in the world per research by Interbrand (Interbrand 2017). The brands span multiple industries including technology companies, automobile manufacturing, digital media, e-commerce, retail, fast food, entertainment, financial services, and athletic and fashion products. In sum, the dataset contains aggregated number of social media posts and consumer responses for 20 global brands.

For each social media platform, the dataset has information on the date and number of brand post (firm-generated) within the specific platform, as well as the total number of consumer responses (e.g., such as likes, retweets) that the posts generated within the specific social media. The unit of observation is the number of posts in a day and various types of engagement observable during that day. The dataset does not contain information on the actual post content or the text of the consumer comments. The data indicates that participation on a social media platform and the resultant daily postings vary significantly across brands. In total, across the 20 brands and three social media, we obtained 2390 observations. We provide a brief summary of the data across all brands and social media in (Table 2A).

>>>INSERT TABLE 2A and 2B HERE<<<

Consumer engagement

Consistent with the previous marketing literature (Godes and Mayzlin 2009; Duan et al. 2008; Liu 2006), we use metrics such as likes, shares, retweets, mentions, and comments, to measure consumer engagement with a social media post. The specific metric varies by the platform the brand used to make the post. We provide a short overview of consumer engagement by platform below.

Table 2A provides the descriptive statistics for the posts, and engagements by media across all brands. For each of the social media discussed, we provide the average weights of the principal components analysis, and the average of the variance explained by the first factor in Table 2B. We note that the PCA assigns weights to each of the engagement metrics based on their "importance" in explaining the variance in engagement. These weights, as shown in the table, vary by type of engagement and platform.

Facebook (FB) The data contains measures of the number of brand posts on Facebook as well as daily consumer engagement in the form of new likes, shares, number of comments, and mentions tied to the brand's Facebook page. We combine the multiple engagement metrics to derive a single consumer engagement measure that incorporates all observed engagement on the brand's page using a principle component analysis (PCA) approach. We use the first factor from the PCA to construct the focal engagement variable. We call this factor *FBE*. *FBE* is the linear combination of the engagement measures, and it averages 8989.10 per day across all brands.

Twitter (TW) On Twitter, we use the number of tweets sent out by a brand as a measure for brand posts. Consumer engagement metrics include retweets, replies, and mentions tied to the brand's Twitter handle. Following a similar approach using PCA analysis, we combine the various engagements to yield a one-factor representation of Twitter engagement (*TWE*). The resulting mean of *TWE* is 769.68 across all brands.

Instagram (IN) Engagement is measured using data on likes, shares, and comments tied to the brand's Instagram page. Brand posts measure the brand's activity (posts) on the platform. Similar to the descriptions above, we conduct PCA for the engagement measures for each brand to derive one factor that represents the level of engagement with the brand's posts. The average value for IN engagement (*INE*) is 101304.55.

Endogeneity

Endogeneity may arise in the data if a firm's social media postings vary in response to consumer engagement. This could lead to biases in the estimation of the impact of posts on consumer engagement. To correct for this endogeneity in a focal brand's social media posts, we adopt the control function approach (Petrin and Train 2010; Woolridge 2015). A variable qualifies as a control function if its inclusion in the estimation procedure renders the endogenous variable "suitably exogenous" (Woolridge 2015). Following Sridhar et al. (2016), Rutz and Watson (2019), and Bayer et al. (2020) and the procedure outlined in Woolridge (2015) we construct the control function by relating social media posts within the platform by other brands (j) to the posts of the focal brands (i) at every time t. Thus, we use the posts of the other brands as instruments to determine new variables, notably the residuals of Equations (1)–(3), that when included in the focal model render the engagement as independent of the focal brand posts. Therefore, this procedure, when applied to each brand's social media posts across the three platforms, allows us to construct new social media post residuals that we include in the focal models described in the next section.

$$\ln\left(FBP_{it}\right) = \sum_{j=1, j\neq i}^{J} \beta_{Fj} \ln\left(FBP_{jt}\right) + \epsilon_{1it} \tag{1}$$

$$\ln\left(TWP_{it}\right) = \sum_{j=1, j\neq i}^{J} \beta_{Tj} \ln\left(TWP_{jt}\right) + \epsilon_{2it} \tag{2}$$

$$\ln\left(INP_{it}\right) = \sum_{j=1, j\neq i}^{J} \beta_{Ij} \ln\left(INP_{jt}\right) + \epsilon_{3it} \tag{3}$$

where FBP_{it}, TWP_{it} and INP_{it} are the brand's Facebook, Twitter and Instagram posts. Estimating (1)-(3) using OLS yields the predicted residuals, $\widehat{FBP_{resid_{it}}}$, $\widehat{TWP_{resid_{it}}}$, $\widehat{INP_{resid_{it}}}$. Including these residuals in our focal model, described in the next section, mitigates the potential endogeneity because the retained covariates will no longer correlate with the error terms in the focal model. Additionally, this inclusion allows for the consistent estimation and identification of the effect of social media posts on engagement (Barnow, Cain and Goldberger 1981, Cameron and Trivedi 2005 p. 37). Next, we present our modeling framework to relate multi-platform brand activity with consumer engagement in a dynamic setting.

Empirical model

>>> INSERT FIGURE 2 HERE<<<

The modeling framework for consumer engagement on social media is driven by three important considerations for each brand. First, when a brand creates a social media post, it has the potential to draw responses as consumers with affinity for the brand are likely to directly express their reactions using different engagement measures. Second, the consumer activity generated due to a brand's actions on its page could have a carryover effect to the next period, implying that past activity is sustained over time. Third, we include the spillover effect of a post in one social media platform on another type of social media platform (e.g., effect of Facebook post on consumer activity on Twitter). Finally, because a large number of brand followers might also engage more with the brand across platforms (Colicev et al. 2018) we control for the number of the brand's fan following (*BFF*) on its own social media page on each day across every platform. Colicev et al. (2018) also show that a brand's fan following impacts consumer metrics such as awareness, purchase intent and satisfaction, thus stressing the importance of its inclusion[1]. We specify our model (referred to as *SMM*) as follows:

$$\ln(FBE_{it}) = \lambda_{FB}\ln\left(FBE_{i(t-1)}\right) + \gamma_{FB}\ln(FBP_{it}) + \kappa_{FB,TW}\ln(TWP_{it}) + \kappa_{FB,IN}\ln(INP_{it}) + \eta_1\widehat{FBP_{resid_{it}}} + \eta_2\widehat{TWP_{resid_{it}}} + \eta_3\widehat{INP_{resid_{it}}} + \alpha_1\ln\left(BFF_{it,FB}\right) + \sum_{\substack{j=2 \\ I_j=1, if\ j=i \\ I_j=0\ otherwise}}^{20} \delta_{1j}I_j + \omega_{1it} \tag{4}$$

$$\ln(TWE_{it}) = \lambda_{TW}\ln\left(TWE_{i(t-1)}\right) + \gamma_{TW}\ln(TWP_{it}) + \kappa_{TW,FB}\ln(FBP_{it}) + \kappa_{TW,IN}\ln(INP_{it}) + \eta_4\widehat{FBP_{resid_{it}}} + \eta_5\widehat{TWP_{resid_{it}}} + \eta_6\widehat{INP_{resid_{it}}} + \alpha_2\ln\left(BFF_{it,TW}\right) + \sum_{\substack{j=2 \\ I_j=1, if\ j=i \\ I_j=0\ otherwise}}^{20} \delta_{2j}I_j + \omega_{2it} \tag{5}$$

$$\ln(INE_{it}) = \lambda_{IN}\ln\left(INE_{i(t-1)}\right) + \gamma_{IN}\ln(INP_{it}) + \kappa_{IN,FB}\ln(FBP_{it}) + \kappa_{IN,TW}\ln(TWP_{it}) + \eta_7\widehat{FBP_{resid_{it}}} + \eta_8\widehat{TWP_{resid_{it}}} + \eta_9\widehat{INP_{resid_{it}}} + \alpha_3\ln\left(BFF_{it,IN}\right) + \sum_{\substack{j=2 \\ I_j=1, if\ j=i \\ I_j=0\ otherwise}}^{20} \delta_{3j}I_j + \omega_{3it}, \tag{6}$$

where $\begin{bmatrix}\omega_{1t} \\ \omega_{2t} \\ \omega_{3t}\end{bmatrix} \sim MVN\ (0_{3x1}, \boldsymbol{Q}_{3x3})$. $\boldsymbol{Q}$ is the full variance-covariance matrix that captures both, the unobserved direct effects and indirect spillovers within and across media. The parameters

[1] We thank an anonymous reviewer for this suggestion.

$\lambda_*, \gamma_*, \kappa_{*,*\prime}, \eta_*, \alpha_*$, and δ_{*j} measure the platform specific (*) carryover rates of past engagement within the platform, direct effects of brand posts within the platform, spillover effects of brand posts on other platforms ($*'$), impact of the residuals (generated using the endogeneity correction procedures), the effect of brand fan following within that platform, and the intrinsic effect that the brand (j) has on engagement with its owned social media page on the platform. We control for the brand effect using I_j, which takes a value of one if it is the focal brand (i.e., $j = i$) and is zero otherwise. Figure (2) presents this modeling framework and illustrates how social media posts' direct and spillover effects as well as dynamics due to differential carryovers can drive consumer engagement.

Mathematically, we represent Equations (4)–(6) using a dynamic state space model that captures the intertemporal dynamics in consumer activity within a social media, the within-media brand and consumer activity as well as across media spillovers due to brand and consumer activity on other social media platforms. Specifically, we use the Kalman Filter (Kalman 1960; Naik et al. 1998; Bruce et al. 2012) which proposes a closed form filtering solution to linear Gaussian models to estimate this specification. Due to certain characteristics of the data, the Kalman Filter provides some advantages in our analysis over the existing VAR approach. We briefly describe these advantages in the accompanying Web Appendix.

Due to differing scales and the presence of outliers, we log-transform all the variables (except brand dummies) in the model (Sridhar et al. 2011; Lee et al. 2018; de Vries et al. 2017; Colicev et al. 2018). Therefore, the resulting parameter estimates also function as the social media platform elasticity. The Kalman Filter has been used extensively in marketing, and further details for the estimation procedure are furnished in the review chapter by Naik (2014). We employ the MARSS package in R (Holmes et al. 2020) to simultaneously estimate the equations in (4)-(6) using the Kalman Filter. In the next section, we estimate the model on the data and obtain the results presented in the next section.

Results

Model fit and accuracy

Prior to discussing the results from our focal model, we briefly compare the model fit and predictions to other estimation procedures and specifications. First, maintaining the same model formulation we compare the Kalman Filter (SMM) estimation to the Seemingly Unrelated Regression (SUR, with contemporaneous correlations) and Ordinary Least Squares (OLS, independent equations) procedures. Next, we modify the model to include (1) only direct effects, (2) only direct and spillover effects, and (3) only direct and carryover effects. In addition to the aforementioned models, we also test the focal SMM model against one that also includes both the carryover and the spillover effects between engagement on the social media platforms. Specifically, we modify the SMM model in Equations (4), (5), and (6) to include the potential for spillovers not only due to firm posts, but also due to consumer engagement on alternate social media platforms. We do so by allowing for not only within platform carryovers, but also spillovers between the engagement on other social media platforms used by the firm. We label this model SMM II. To demonstrate the fit and accuracy of the model, we compute the forecasts of consumer engagement across the different platforms using our model as well as the competing models and then compute the mean absolute percentage error (MAPE) of the forecasts. MAPE is computed as the absolute difference between observed engagement and predicted engagement, divided by observed engagement and expressed in percentages. We find that the MAPEs are 11.96% (FB), 5.88% (TW) and 5.39% (IN) for the SMM model, which are also the lowest amongst all tested models. Table 3 presents these results as well as the resultant AIC value. As evident from the table, the SMM model also has the lowest AIC value, further supporting the model. Next, we discuss the estimation results from the retained SMM model.

>>>INSERT TABLE 3 HERE<<<

>>>INSERT TABLE 4 HERE<<<

Estimation results

Table 4 presents the parameter estimates (at 95% significance level) from the Kalman Filter estimation procedure for the reference brand. The table also provides estimates of the SUR and the OLS estimations. Similar to Naik, Shi and Tsai (2007), we also observe that the SUR and OLS procedures, when compared to Kalman Filter estimations (SMM), determine higher levels (overestimate) of effectiveness of posts at generating engagement and lower levels (underestimate) of platform specific carryover rates. Due to the log-log specification, the estimates also represent elasticities between brand-generated content and consumer engagement. We further investigate the results as they pertain to our propositions, and then assess the impact of the control variables.

Direct effects (P1) The analysis reveals that current posts have a very strong influence on current consumer engagement within the platform. For example, direct effect of *FBP* on *FBE* is

1.71. Instagram exhibits the next highest effect at 0.36, followed by Twitter at 0.2122. The results provide support for P1a and P1b: brand posts do increase engagement within a social media platform, and these increases can vary by platform. Thus, not all platforms yield the same level of engagement, all else being equal. The results also corroborate practitioners' knowledge that Facebook tends to significantly outperform other platforms in terms of the reach and engagement (Comscore 2016).

Spillover effects (P2) Similar to prior research that showed that media in one channel affects the efficacy of media in other channels (Naik and Peters 2009), we too find that brand posts in one social platform impacts engagement on other platforms in the firm's portfolio, and these effects vary by social media platform—confirming P2a and P2b. This result shows that not accounting for spillovers leads to an underestimation of the effects of firm posts and their ability to generate cross-platform engagement. At steady state, the long-term average spillover effects for posts on each medium are as follows: 2.6022 for Facebook, 0.3991 for Twitter and 0.9098 for Instagram. The long-term total spillover effects at steady state for each medium are 5.2044 for Facebook, 0.7982 for Twitter and 1.8197 for Instagram. Facebook exhibits the highest spillover effect compared to the other social media platforms. This indicates that brand posts on Facebook drive significant engagement on the brand's other social media pages. In the case of Instagram and Twitter, the results suggest that the total spillover effects even outweigh the direct effects, indicating that only accounting for direct effects would significantly underweight their importance in the social media platform portfolio. In other words, though the direct effect of a focal social media platform might be small in magnitude, the posts on this medium may play an important role in influencing engagement with the brand's message on other social media due to the focal medium's stronger spillover effects.

Carryover effects (P3) The carryover effects measure the effect of past consumer engagement on current engagement. As predicted by P3a and P3b, posts on social media platforms endure over time (i.e., the effect of past engagement on current engagement is significant), and these effects do vary significantly by platform type. While posts on Facebook have strong current direct and spillover effects, posts on Instagram (0.74) and Twitter (0.66) endure (i.e., carryover rates) for longer than posts on Facebook (0.35). Following Naik (1999), the duration (time required to depreciate to 90% of the initial engagement level) of posts on the social media platforms are 3.58 (FB), 6.82 (TW) and 8.99 (IN) days. Thus, engagement on all social media platforms exhibits some amount of longevity, even in the absence of new posts—with some platforms sustaining engagement for longer periods than others do. This information is managerially relevant because some social media can allow brands to reap their efforts for longer into the future than others. Managerially, this means that brands could allocate resources differentially, depending on the posts longevity in a medium.

Control variables We next examine the control variables in the model. We include these variables to control for the heterogeneity between brands that could explain some of the observed differences in engagement across brands and platforms. Similar to prior literature (Colicev et al. 2018), we find that *Brand Fan Following* has positive and significant impacts on engagement across platforms—thus performing as expected. Specifically, fan following has the largest impact on Instagram engagement (0.11) versus 0.09 for Facebook and 0.04 for Twitter. This result also implies that engagement on these platforms might be driven via different mechanisms and is a useful avenue to investigate for future research.

Finally, we also control for brand effects, and find that brands do generate differential levels of engagement. This variation manifests not only across brands, but also within brands—for example, some brands perform better at driving engagement on Facebook (e.g., Brand 7), while others perform better on Twitter (e.g., Brand 20), or Instagram (e.g., Brand 16). This result shows that some brands might have greater affinity to one type of platform over another—while we do not address this question directly, relating the characteristics of a medium to characteristics of the brand would be a fruitful area for future research.

Allocation

How do these results support managerial decision making? To recommend managerial actions, we compute the *effort* allocation across the multiple social media for a manager using values from the analysis. We assume that effort is proportional to the number of posts. Because we do not have data on actual budgets, effort acts as a proxy that correlates with the actual resource allocation the brand makes. We also assume that the value of an engagement remains equal across social media platforms. Given that the brand's communication efforts aim to maximize engagement, we use elasticity based allocation rules (similar to Dorfman and Steiner 1954) to distribute effort optimally across the different social media platforms. We restrict our allocation rules to be dependent only on the significant parameters from our estimation results. The procedure is as follows. First, we identify the *long-term* direct and spillover platform effects by dividing the direct and spillover estimates by the decay $(1 - carryover)$ of engagement within a media. Next, we determine the cumulative effect (sum of long-term direct and spillover effects) of each platform on generating engagement. Finally, we normalize across media to get the proportional effect of each media as compared to the other.

Table 5 shows the allocation model of communication efforts. Overall, we find allocation of effort as 63% to Facebook, 11% to Twitter, and 26% to Instagram. Thus, given its strong direct and spillover effects, Facebook receives the most amount of effort allocation, Instagram receives the next highest, followed by Twitter. We also determine allocations under the different scenarios shown in Table 3. We find that allocations can vary dramatically depending on whether we account for carryover and spillover effects or not. For example, as shown in the table, if spillovers are ignored (column 5 and 7 in Table 5), firms would significantly underinvest in Facebook and overinvest in Instagram. On the other hand, while carryover effects are significant, they do not seem to have a big impact to how firms would allocate their efforts, implying that spillover effects can have an outsized impact on managerial decision making.

>>>INSERT Table 5 HERE<<<

Figure 3 summarizes the key estimation and allocation results.

>>> INSERT FIGURE 3 HERE<<<

Extensions

Additional social media In this study, we restricted our analysis to three social media: Facebook, Twitter and Instagram. Over time, new social media platforms are likely to emerge (e.g., TikTok) while others could possibly wane (e.g., Tumblr). The proposed model accommodates these changing market conditions due to the flexibility with which the state space modeling approach allows the inclusion of new platforms into the formulation. In this section, we extend the SMM model by incorporating a new social media platform to the existing model. Specifically, we collect data for brand posts and consumer engagement on YouTube. This yields three brands that have a simultaneous presence on Facebook, Twitter, Instagram, and YouTube. We collect YouTube consumer engagement data in the form of likes, comments, dislikes, and views. Following the PCA and endogeneity correction procedures discussed earlier, we determine the consumer engagement (*YTE*) with a brand's YouTube activity (*YTP*). Next, we relate this engagement to brand posts on this medium as well as the other social media used by the firm, to yield Equation (7), which augments the system of equations shown in (4)–(6).

$$\ln(YTE_{it}) = \lambda_{YT}\ln\left(YTE_{i(t-1)}\right) + \gamma_{YT}\ln(YTP_{it}) + \kappa_{YT,FB}\ln(FBP_{it}) + \kappa_{YT,TW}\ln(TWP_{it}) + \kappa_{YT,IN}\ln(INP_{it}) + \eta_{10}\widehat{FBP_{resid_{it}}} + \eta_{11}\widehat{TWP_{resid_{it}}} + \eta_{12}\widehat{INP_{resid_{it}}} + \eta_{13}\widehat{YTP_{resid_{it}}} + \alpha_3\ln(BFF_{it,IN}) + \sum^{3}_{\substack{j=2 \\ I_j=1, if\ j=i \\ I_j=0\ otherwise}} \delta_{4j}I_j + \omega_{4it}\ . \tag{7}$$

For the sake of brevity, we do not display the modified Equations (4)–(6), which will now include the additional social medium. The results of the analysis are presented in Table 6 and they comport with the outcomes found earlier. We find that even when new media are added, the direct, spillover, and differential carryovers of social media exist and in many cases are significant. YouTube posts drive YouTube engagement, as well as engagement on Instagram. Additionally, apart from the significant carryover rates of YouTube engagement, both Facebook and Twitter posts have significant positive effects at driving YouTube engagement. This could occur due to the firm or consumers cross-posting YouTube videos on these platforms. In sum, P1, P2, and P3 are also supported in this analysis when we extend the model to incorporate new social media.

>>>INSERT Table 6 HERE<<<

Alternate outcome measures To explore the carryover, direct, and spillover effects of each medium independently, we need measures that allowed for the estimation of these effects separately. Similar to prior literature, we accomplish this using intermediate measures of engagement in each platform (prior literature use measures like click through rates, views, etc.). However, it would also be useful for firms to determine how these firm actions could affect "hard" metrics like firm financial values or sales. For example, Colicev et al. (2018) and Fossen and Schweidel (2019) show that firm actions on social media have downstream effects on firm valuations (e.g., abnormal returns) and on online activity (online website traffic, online sales). In this section, we relate our findings to similar hard metrics. Specifically, we investigate the effect of multi-platform firm actions and consumer engagement on daily abnormal returns. Using daily data from CRSP database (only 15 brands were publicly traded) and computing abnormal returns to estimate the idiosyncratic risk (similar to Colicev, 2018), we augment the SMM model using Equation (8) and then estimate the model using the Kalman Filter procedure.

$$AR_{it} = \lambda_{AR} AR_{\mathrm{i(t-1)}} + \gamma_{FBE} \ln\left(FBE_{it-1}\right) + \gamma_{TWE} \ln\left(TWE_{it-1}\right) + \gamma_{INE} \ln\left(INE_{it-1}\right) +$$

$$\kappa_{AR,FB} \ln(FBP_{it}) + \kappa_{AR,TW} \ln(TWP_{it}) + \kappa_{AR,IN} \ln(INP_{it}) +$$

$$\eta_{14} \widehat{FBP_{resid_{it}}} + \eta_{15} \widehat{TWP_{resid_{it}}} + \eta_{16} \widehat{INP_{resid_{it}}} + \sum^{15}_{\substack{j=2 \\ I_j=1, if\ j=i \\ I_j=0\ otherwise}} \delta_{5j} I_j + \omega_{5it} \quad (8)$$

Table 7 provides the results of the analysis. We find that social media activity does not influence daily abnormal returns in our dataset. This result could emerge because we track social media data and engagement for brands on the Interbrand survey—namely large well-known brands. Recent research by Du and Osmonbekov (2020) has shown that "the direct effect of advertising on firm value will be stronger for firms not covered by financial analysts than for those that are covered because investors may rely on information flow from financial analysts for covered firms and rely on information flow from advertising in the absence of analyst coverage." The non-significance of social media activity is thus an outcome of the brands in this study being so well known, and widely covered by financial analysts. In sum, the model proposed in Equations (4)–(6) can be extended to provide insights on how social media activity influences more downstream metrics like sales or firm value.

>>>INSERT TABLE 7 HERE<<<

Profit maximizing allocations In Table 5 we derive the effort allocation rules for social media managers to optimize engagement with a post. However, these results only consider managerial actions as they apply to upstream metrics such as consumer engagement. Managers might also be interested in determining how these allocations impact downstream metrics like sales and profits.

Consumer engagement on social media is generally assumed to have positive relationships with measures of firm performance, such as sales (Brodie et al. 2011; Dessart et al. 2015). For example, Kumar et al. (2017) find that firm-generated content on social media positively impacts customer spending. However, some studies note that engagement might not have any impact on sales and in some instances this impact could be negative (Cheung et al 2015; John et al. 2017). More recently, Santini et al. (2020) find, via a meta-analysis, that consumer engagement on social media does indeed directly and indirectly impact firm performance metrics such as sales. They note that such effects arise due to conative activities such as behavioral purchase and patronage intention. In other words, upstream metrics in marketing funnel, such as engagement on social media can have direct impacts on downstream outcomes like sales.

In this study, we do not have access data that allows us to link upstream engagement to downstream sales and optimize accordingly. However, to aid managerial decision making, we extend the methodology developed in this paper and the results in Table 5 by proposing a novel analytical approach that firms can utilize to optimize their posts when accounting for both consumer engagement as well as the potential sales generated due to increased engagement. Using the extant literature as a guide, we account for the direct contribution of engagement by linking the engagement on a platform to sales and then determine the optimal (profit-maximizing) number posts a manager should make on the platform at any point in time. First, we generalize the engagement Equations (4), (5), and (6) for any social media i. For simplicity we assume that J social media platforms are available to the manager. Then, the log of engagement (E_i^*) on a platform i due to M_i posts on social media i (the focal medium) and M_j posts on alternate platforms j at time 't' yields the generalized social media engagement model $E_{it}^* = \lambda_i E_{it-1}^* + \gamma_i \ln(M_{it}) + \sum_{j,j\neq i} \kappa_{i,j} \ln(M_{jt}) + \omega_{it}$. For simplicity of exposition, we ignore the other terms here. Following Santini et al. (2020), we directly link the engagement on each social media platform to downstream sales as follows:

$$S_t = \lambda_s S_{t-1} + \sum_{j=1}^{J} \psi_j E_{jt}^* + \epsilon_{St}, \tag{9}$$

where, S_t is the sales at time t, λ_S is the sales carryover term—i.e., the contribution of sales at time $t-1$ to time t, and ψ_j denotes the sales effect of engagement generated on each of the social media platforms j (inclusive of i), and ϵ_{st} is the sales shock at time t. We next define a profit function that accounts for the profits from sales as well as the cost of posting on each platform. The profit function is assumed as follows: $\pi_t = mS_t - \sum_j c_{jt} M_{jt}$. The term '$m$' represents the margin generated from sales, c_{jt} indicates the cost of posting M_{jt} posts (administrative, production costs, etc.) on the social media platform j at time t. Using this profit function, the generalized social media engagement model and the sales model in Equation (9), we define the Hamiltonian at each instant t as shown in the Web Appendix. Analytically solving the Hamiltonian yields the profit maximizing number of posts. Proposition 4 presents the generalized solution to the optimal number of posts M_{it}^{*} on a specific social media platform i at any time t on any platform i.

P4: For any social media platform i, the optimal number of posts M_{it}^{*} at time t is

$$M_{it}^{*} = \frac{m}{c_{it}(\rho+1-\lambda_S)}\left[\frac{\psi_i\gamma_i}{\rho+1-\lambda_i} + \sum_{j,j\neq i}\left(\frac{\psi_j\kappa_{j,i}}{\rho+1-\lambda_j}\right)\right]. \tag{10}$$

The term ρ in Equation (10) denotes the time-discount factor. Equation (10) reveals that the optimal number of posts on any social media platform i depends directly on (1) the effectiveness of the posts in generating engagement within the platform (γ_i), (2) the platform's corresponding sales contribution (ψ_i), (3) the indirect effect ($\kappa_{j,i}$) of posts in social media i on engagement in social media j, and (4) the sales contribution of social media j, ψ_j. It also shares an inverse relationship with the decay rate (given by $1 - carryover$) of overall sales and the engagement on each medium. In sum, the proposition provides the profit maximizing allocation of social media posts on each medium at every point in time, accounting for the direct effect, cross-effects and the sales contributions of each medium.

Brand-by-brand estimations Finally, we also estimate Equations (4)–(6) individually for each brand (without brand controls), and then compute the average parameter effects for parameters of importance to this study. Table 8 provides the parameter estimates and lists the resulting allocations. We find that the recommended allocations are comparable with those recommended by the SMM model in Table 5.

>>>INSERT TABLE 8 HERE<<<

Managerial implications

Our research has several implications for marketing practitioners using social media. First, we utilize the most common intermediate metrics for observing consumer engagement on social media and relate these metrics to brand actions across multiple social media platforms. (Mochon et al. 2017; Phua and Ahn 2016; Schondienst et al. 2012; Lipsman et al. 2012; Metaxas et al. 2015; de Vries, et al. 2012). The use of engagement metrics by platform allows us to measure the effect of a post on each platform separately, and identify carryover effects distinctly.

Second, we develop a generalizable, easy to implement social media modeling framework that relates multi-platform social media marketing actions to consumer responses. We find that the methodology is robust, can be modified easily to include new media, and is easily implementable using popular statistical packages. The methodologies presented here apply to brands regardless of their size or revenues. By collecting simple measures such as social media posts and likes, practitioners can delve deeper using similar frameworks and readily available open source analytical tools.

Third, we study the dynamic nature of social media posts and user engagement. Using a dynamic framework, practitioners can determine the impact of previous posts on current engagement, and how the post endures into future. Because the dynamics vary by media, brand managers can vary actions across media by observing the carryover effects—and timing their actions using the average media carryover rate for their brand. Using the carryover rates, we find that posts on Instagram endure longer than posts on Twitter and Facebook.

Fourth, we evaluate the direct and spillover effects of social media posts within and across platforms. The nature of these cross-platform effects imply that there exist some mutually reinforcing effects across media, and marketers must be cognizant of these effects and post media on social platforms recognizing these effects. Our modeling framework provides an avenue for marketers to quantify these direct and cross-platform effects. The model finds that the indirect effect of social media platforms in some instances can be bigger than direct effects in generating engagement. Thus, practitioners should strive to not under-estimate the effect of a medium simply because it does not generate much engagement within itself. They must also determine each post's cross-platform effects. In sum, each post creates a direct effect within the focal platform, an indirect/spillover effect across platforms and is capable of carrying its effect into the future due to the dynamic nature of social media engagement.

Finally, using the estimates of elasticities from the model, we provide a heuristic for how managers can allocate efforts across social media. Overall, we find that Facebook commands the most resource allocation at 63%, followed by Instagram at 26%, and then Twitter at 11%.

Conclusions and future research

To conclude, the SMM framework and subsequent analyses reveals useful implications for marketing practitioners by identifying the within platform efforts, cross platform synergies and continued impact on future engagement. Using data on social media posts (across platforms) from 20 brands we empirically validate the model. The SMM framework provides a measurement and validation process to help practitioners make efficient effort allocation decisions by identifying the effects correctly. The allocation decisions, dependent on the elasticity of each platform, likely lead to stronger overall consumer engagement. Finally, this study also provides the optimal ROI based allocations by linking upstream metrics like engagement to downstream metrics like sales.

Our study focused on social media activity of top-ranking brands, and is subject to some limitations. For future research, we recommend that researchers expand the set of brands studied and obtain data that includes qualitative information (comments, text of tweets, emoticons) which allows marketers to not only optimize the quantity of engagement, but also the quality of engagement. Research can delve into types of engagement that can arise from individuals acting as brand ambassadors and sharing brand content with their network, and how this behavior links to downstream outcomes like satisfaction, purchases or firm value. Due to data constraints, we cannot control for content type in our analysis. Future research could also account for content types and their role in promoting cross-platform engagement, their effects on endogeneity correction and on the allocation of posts. Finally, future research can also expand on this study by using other outcome variables such as social media specific visit or purchase logs that allow for separate identification of the effect of each platform on outcomes such as website/store visits and sales.

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Figure 1: Example of spillover of engagement between social media platforms

Figure 2: Model Structure: Social Media Engagement with Direct and Spillover Effects of Posts

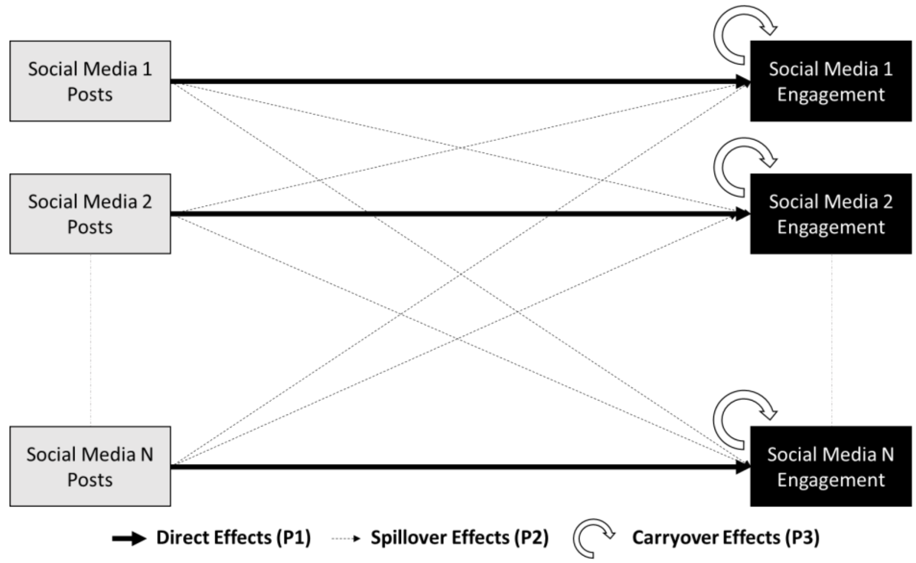

Figure 3 (a): Model Estimates for Carryover, Direct and Spillover Effects

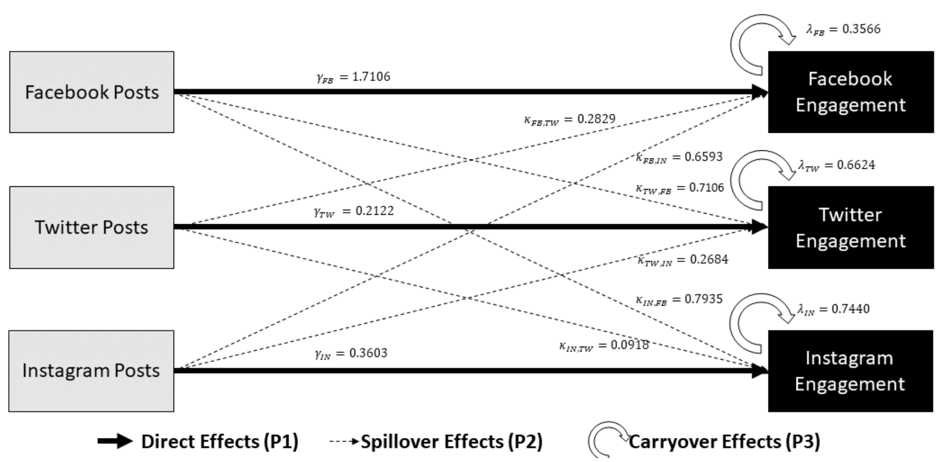

Figure 3 (b): Resource Allocation Across Social Media Platforms

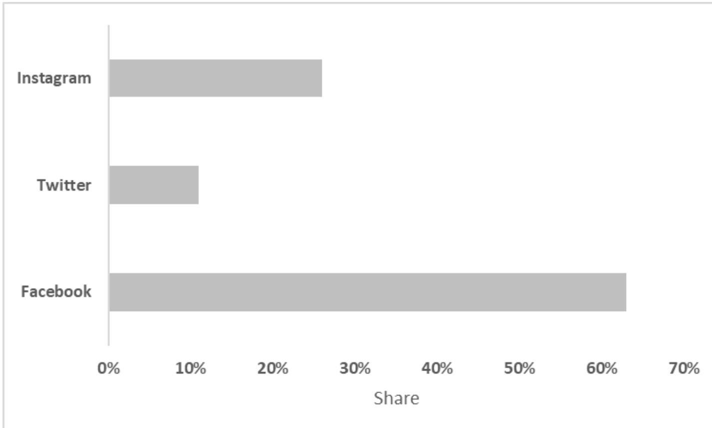

Table 1. Comparison with Relevant Research in Marketing

Study	Social Media	Effects	Brand Fans	Allocation Recommendation	New Insights
Stephen & Galak (2012)	Blogs, Online communities	Direct Effects	Yes	No	Social earned media affect sales and have higher elasticity than traditional earned media's
Goh, Hing & Lin (2013)	Facebook	Direct Effects	No	No	Earned media show greater impact than owned in driving consumer purchases
Kumar, Rishika, Janakiraman & Kannan (2015)	Facebook	Direct Effects	No	No	Owned media with strong receptivity in social media has positive impact on customer behavior such as spend and cross buy.
Hewett, Rand, Rust & Van Heerde (2016)	Twitter	Direct Effects	No	No	Companies benefit from using social media for personalized customer responses, although there is still a role for traditional brand communications
Lovett & Staelin (2016)	Twitter	Direct Effects	No	No	Earned and paid media play a central role in developing and maintaining entertainment brands
De Vries, Gensler & Leeflang (2017)	Facebook, Twitter	Direct Effects	No	No	Social media activities of the firm along with traditional advertising can enhance brand building and customer acquisition.
Zhan & Kim (2017)	Facebook	Direct Effects	No	No	Social CRM capability relates to firms' performance and its customer engagement.
Lee, Hosanagar, & Nair (2018)	Facebook	Direct Effects	No	No	Facebook messages reveal that brand characteristics and promotions influence customer path to purchase.
Colicev, Malshe, Pauwels, & O'Connor (2018)	Facebook, Twitter, YouTube	Direct and Carryover Effects	Yes	No	Brand fan following improves customer mindset metrics-brand awareness, purchase intent and customer satisfaction.
Ilhan, Kübler & Pauwels (2018)	Facebook	Direct and Carryover Effects	Yes	No	Fan posts induce broader social-media brand engagement as they substantially increase and prolong the effects of managerial actions such as communication campaigns and new-product introductions.
Kupfer, Vor Der Holte, Kübler, & Hennig-Thorau(2018)	Facebook	Direct Effects (of Partner Brands)	Yes	No	A partner brand's social media power potential, power exertion, and their interaction lead to higher composite product sales
Fossen & Schweidel (2019)	Twitter	Direct Effects	Yes	No	Ads that air in programs with more social TV activity see increased ad

					responsiveness in terms of subsequent online shopping behavior
Krijestorac, Garg, & Mahajan(2020)	Video Platforms (YouTube, Vimeo etc.)	Direct and Lead-Lag Spillover for Video Ads	No	No	Video introduction on a ‘lag’ platform increases view growth in the lead platform, indicating spillovers due to eWoM between video platforms
Liu, Dzyabura and Mizik (2020)	Flickr, Instagram	Direct Effects	No	No	Brand portrayal in the consumer posts on social media reflects consumers’ brand perceptions
This Study	Facebook, Twitter, Instagram & YouTube (in extension)	Differential Direct, Spillover and Carryover Effects by Social Media	Yes	Elasticity-based allocation	Brand posts on one social medium generates engagement both within and across social media. These effects vary by social medium. The study also exemplifies a resource allocation model using the differential direct, indirect and carryover effects.

Table 2A. Descriptive statistics of Social Media Posts across all 20 brands

Variables	Mean	Std.	Median
FB Posts	1.91	1.36	1.40
FB Likes	8930.17	13297.89	2470.20
FB Comments	388.86	458.10	272.30
FB Shares	830.28	1349.73	264.05
FB Brand Fan Following	24611887.00	25584270.57	20049868.95
TW Posts	57.20	145.08	9.15
TW Retweets	398.04	491.53	181.65
TW Replies	168.88	219.74	67.70
TW Mentions	645.56	611.74	456.20
TW Brand Fan Following	3937738.17	4287824.32	2738956.45
IN Posts	1.53	1.33	1.05
IN Likes	101305.50	188721.02	11156.65
IN Comments	394.66	535.58	101.35
IN Brand Fan Following	3583513.19	5234798.37	971856.75

Table 2B. Average Values of Principal Components Analysis (PCA) Scores and Variance Explained by the First Factor by Social Media (across all 20 brands)

Social Media	**Engagement Type**	**PCA Score**	**Variance Explained**
Facebook	Likes	0.956	97.405
	Shares	0.143	
	Comments	0.078	
Twitter	Retweets	0.445	93.419
	Mentions	0.834	
	Retweets	0.159	
Instagram	Likes	0.999	99.785
	Comments	0.007	

Table 3. Model Validation: Average values of MAPE and AIC by Model

Model	MAPE(FB)	MAPE(TW)	MAPE(IN)	AIC
SMM	**11.96**	**5.88**	**5.39**	**16,222.55**
SMM II (with Cross-Platform Engagement Spillover)	11.97	6.07	5.44	16,226.93
SUR	12.04	6.05	6.08	16,695.57
OLS	12.02	6.01	6.05	16,698.87
Direct Effects only	13.10	10.16	8.93	20,873.24
Direct and Spillover Effects (no Carryover)	12.40	8.19	7.43	18,129.41
Carryovers and Direct Effects (no Spillover)	12.00	6.25	5.50	16,763.54

Table 4: Parameter Estimates (at 95% significance)

Model	**SMM(Kalman Filter)**			**SUR**			**OLS**		
Parameters	**FBE**	**TWE**	**INE**	**FBE**	**TWE**	**INE**	**FBE**	**TWE**	**INE**
Carryover	0.3566	0.6624	0.7440	0.158	0.54	0.45	0.181	0.585	0.485
Facebook Posts	1.7106	0.7106	0.7935	2.125	0.92	1.408	2.08	0.828	1.311
Twitter Posts	0.2829	0.2122	0.0918	0.355	0.281	0.253	0.341	0.26	0.236
Instagram Posts	0.6593	0.2684	0.3603	0.823	0.342	0.535	0.801	0.312	0.502
Brand Fan Following	0.0910	0.0499	0.1154	n.s.	0.073	0.351	n.s.	0.061	0.345
FBP_{resid}	-0.5571	-0.5185	-0.8219	-0.795	-0.715	-1.467	-0.756	-0.631	-1.376
TWP_{resid}	-0.2298	-0.0825	-0.0800	-0.277	-0.129	-0.216	-0.268	-0.117	-0.203
INP_{resid}	-0.4196	-0.1401	n.s.	-0.551	-0.18	n.s.	-0.54	-0.161	n.s.
Brand 2	1.5094	0.9323	1.3109	2.13	1.377	2.993	2.08	1.215	2.797
Brand 3	1.3031	1.0011	1.5730	1.781	1.399	3.312	1.742	1.246	3.095
Brand 4	3.3332	1.4707	1.7532	4.431	2.041	3.932	4.311	1.837	3.668
Brand 5	1.8666	0.3057	0.9361	2.654	0.51	2.336	2.573	0.44	2.178
Brand 6	1.9763	1.0155	0.8114	2.717	1.459	1.945	2.644	1.311	1.81
Brand 7	3.7972	1.2820	1.4200	5.08	1.808	3.393	4.936	1.628	3.17
Brand 8	1.7052	0.4526	0.2680	2.472	0.737	1.088	2.396	0.649	1.019
Brand 9	2.0335	n.s.	1.1771	2.932	n.s.	3.15	2.85	n.s.	2.943
Brand 10	2.6697	n.s.	1.5142	3.708	0.283	3.538	3.606	0.24	3.278
Brand 11	0.8472	0.8281	1.4707	1.253	1.188	3.017	1.241	1.034	2.81
Brand 12	1.8932	0.6302	1.0811	2.751	0.998	3.052	2.662	0.889	2.859
Brand 13	1.7690	0.4638	1.1677	2.459	0.699	2.877	2.39	0.622	2.682
Brand 14	0.6466	0.3628	0.4144	1.01	0.575	1.254	0.976	0.51	1.168
Brand 15	3.0084	1.0725	1.6870	4.094	1.545	3.979	3.976	1.386	3.718
Brand 16	0.3859	0.4259	1.7360	0.678	0.663	4.28	0.654	0.59	4.001
Brand 17	2.2808	0.6559	1.2812	3.121	0.965	3.075	3.033	0.86	2.871
Brand 18	1.0946	0.5432	0.4869	1.601	0.817	1.286	1.556	0.722	1.193
Brand 19	1.0267	0.7508	1.0328	1.496	1.092	2.467	1.456	0.971	2.299
Brand 20	0.3566	0.6624	0.7440	0.158	0.54	0.45	0.181	0.585	0.485

Note: n.s. denotes that the estimate is not significant at 95%.

Table 5: Allocations Across Social Media

Social Media	**SMM**	**SUR**	**OLS**	**Direct and Carryover Effects Only (SMM)**	**Direct and Spillover Effects Only (SMM)**	**Direct Effects Only (SMM)**
Facebook	63%	63%	63%	30%	64%	47%
Twitter	11%	13%	13%	15%	13%	17%
Instagram	26%	24%	24%	55%	23%	36%

Table 6: Parameter Estimates for YouTube

Parameters	**FBE**	**TWE**	**INE**	**YTE**
Carryover Effects	0.482	0.744	0.625	0.958
Facebook Posts	1.97	0.564	0.990	0.176
Twitter Posts	n.s.	0.199	0.168	0.064
Instagram Posts	n.s.	n.s.	0.308	n.s.
YouTube Posts	n.s.	n.s.	0.203	0.080
Brand Fan Following	n.s.	n.s.	0.308	0.010
$\boldsymbol{FBP_{resid}}$	-0.461	n.s.	-0.537	n.s.
$\boldsymbol{TWP_{resid}}$	n.s.	n.s.	n.s.	n.s.
$\boldsymbol{INP_{resid}}$	n.s.	n.s.	n.s.	n.s.
$\boldsymbol{YTP_{resid}}$	n.s.	n.s.	n.s.	n.s.
Brand 2	2.22	0.081	2.05	0.140
Brand 3	3.25	0.255	0.519	0.226

Note: n.s. denotes that the estimate is not significant at 95%.

Table 7: Parameter Estimates for AR model

Parameters	**AR**	**FBE**	**TWE**	**INE**
Carryover	0.3636	0.7411	0.7928	0.9000
Facebook Posts	*-0.0859*	0.9804	0.4913	0.4319
Twitter Posts	*0.1681*	0.2801	0.1710	0.1126
Instagram Posts	*0.5422*	0.7188	0.4593	0.4272
Brand Fan Following		*-0.0095*	0.0505	0.1041
$\boldsymbol{FBP_{resid}}$	*0.5698*	-0.3582	-0.3358	-0.4111
$\boldsymbol{TWP_{resid}}$	*0.0291*	-0.2655	-0.0876	-0.1043
$\boldsymbol{INP_{resid}}$	*-0.6698*	-0.6706	-0.3927	-0.2323
Brand 2	*-0.5392*	-0.4660	0.2138	*-0.0611*
Brand 3	*-0.9340*	*-0.1063*	0.3920	0.2891
Brand 4	*-0.2441*	0.9833	0.7508	0.4557
Brand 5	*0.0473*	*-0.1053*	-0.1034	-0.0986
Brand 6	*0.5979*	0.2084	0.4539	*-0.0194*
Brand 7	*-0.0931*	-0.5733	-0.1827	-0.5548
Brand 8	*0.3775*	-0.8256	*0.0035*	*0.0030*
Brand 9	*0.0247*	-0.6270	-0.1739	-0.3294
Brand 10	*-0.0478*	*0.0185*	*0.0035*	*0.0362*
Brand 11	*0.4266*	-0.5210	*-0.0554*	-0.2737
Brand 12	*0.3814*	-0.7022	-0.1824	-0.3931
Brand 13	*-0.3124*	-0.3142	*0.0721*	-0.2241
Brand 14	*0.2604*	-0.2922	0.2411	*0.0182*
Brand 15	*0.0049*	-0.6776	*-0.0750*	-0.2870

(Italicized values are not significant at 95%)

Table 8: Brand-by-Brand Allocation model*

Parameters	**FBE**	**TWE**	**INE**	**Allocations**
Carryover	0.7654	0.7784	0.9439	
Facebook Posts	1.7092	0.806	0.8439	57%
Twitter Posts	0.624	0.389	0.225	18%
Instagram Posts	0.6593	0.2684	0.411	25%
Brand Fan Following	1.6817	0.9940	1.3228	

*Note: Average of significant values only.

Web Appendix

I. Benefits of the Kalman Filter approach

The Kalman Filter (Kalman 1960; Naik et al. 1998; Bruce et al. 2012) proposes a closed form filtering solution to linear Gaussian models to estimate this specification. We use the Kalman Filter because it forecasts values of all variables and it dynamically updates variable estimates when new information arrives (Naik 2014). Second, the Kalman filter can handle data without a need for balancing autocorrelation and data lag selection to gain stationarity. Thus, it allows for the intrinsic knowledge and direct interpretation of the variables reducing over parametrization of high order lagged data. Third, it can handle irregularly spaced data. Its recursive structure allows real time execution as it keeps only previous state parameters instead of data history. Finally, the Kalman gain also allows for correction in cases where measurement errors could exist (Durbin, J. and Koopman, S. J. 2012). We note that the VAR model is an alternative approach suited to estimating such dynamic models, however due to the advantages outlined above, we proceeded with the Kalman Filter model.

II. Determining the Allocations of Social Media Posts to Optimize ROI

In this section we derive the proof for Proposition 4. The key question considers how managers allocate posts across multiple social media platforms to maximize profit subject to (1) the engagement generated across the platforms by firms post and (2) the sales engendered due to the engagement generates by firm actions. We note that this analysis generalizes to any set of platforms, $j = 1, \ldots, J$, and not just the three media we consider in the empirical analysis. Let M_{it} be the count of posts made by a firm on platform i at time t. The engagement generated by the post on platform i at time t is given by E_{it}, and E_{it}^{*} represents the natural log of E_{it}. This leads us to a generalized engagement equation given by,

$$E_{it}^{*} = \lambda_i E_{it-1}^{*} + \gamma_i \ln(M_{it}) + \sum_{j, j \neq i} \kappa_{i,j} \ln(M_{jt}) \qquad \text{(W1)}$$

where j denotes the other social media platforms, λ_i the carryover of engagement within the platform, γ_i the direct effects on engagement of posts on platform i, and $\kappa_{i,j}$ the effects of posts on platform j in generating engagement on platform i, also known as the spillover effects.

Following Santini et al. (2020), we define a sales function that depends on the lagged sales (via carryover λ_s) generated by the firm as well as the direct contribution (ψ_j) to sales due to engagement generated on a social media platform j.

$$S_t = \lambda_s S_{t-1} + \sum_{j=1}^{J} \psi_j E_{jt}^{*} \qquad \text{(W2)}$$

Using (W2), we next define the profit function for the firm as shown in (W3), where m determines the average margin obtained from sales, and c_{jt} the costs of posting on a platform. These costs are assumed to be general in nature and could be varied and include the costs of ad production, development, posting etc.

$$\pi_t(M_1, M_2, \dots, M_J, S_t) = mS_t - \sum_j c_{jt} M_{jt} \tag{W3}$$

Next we set up the dynamic optimization problem that seeks to maximize $J(M_1, M_2, \dots, M_J) = \sum_t e^{-\rho t} \pi_t(M_1, M_2, \dots, M_J, S_t, c_{1t}, c_{2t}, \dots, c_{Jt})$ for a given discount rate ρ subject to (W1) and (W2). We next rewrite (W2) and W(3) in matrix form for J social media platforms.

$$\underbrace{\begin{bmatrix} S_t \\ E^*_{1t} \\ \vdots \\ E^*_{Jt} \end{bmatrix}}_{(J+1)\text{ x }1} = \underbrace{\begin{bmatrix} \lambda_S & \psi_1 & \cdots & \psi_J \\ 0 & \lambda_1 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \lambda_J \end{bmatrix}}_{(J+1)\text{ x }(J+1)} \underbrace{\begin{bmatrix} S_{t-1} \\ E^*_{1t-1} \\ \vdots \\ E^*_{Jt-1} \end{bmatrix}}_{(J+1)\text{ x }1} + \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0 \\ \gamma_1 & \kappa_{1,2} & \cdots & \kappa_{1,J} \\ \kappa_{2,1} & \gamma_2 & \cdots & \kappa_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{J,1} & \kappa_{J,2} & \cdots & \gamma_J \end{bmatrix}}_{(J+1)\text{ x }J} \underbrace{\begin{bmatrix} M_{1t} \\ M_{2t} \\ \vdots \\ M_{Jt} \end{bmatrix}}_{J\text{ x }1} \tag{W4}$$

We can then re-express (W4) as (W5) by subtracting the state vector given by $\begin{bmatrix} S_{t-1} \\ E^*_{1t-1} \\ \vdots \\ E^*_{Jt-1} \end{bmatrix}$.

$$\begin{bmatrix} \Delta S_t \\ \Delta E^*_{1t} \\ \vdots \\ \Delta E^*_{Jt} \end{bmatrix} = \begin{bmatrix} -(1-\lambda_S) & \psi_1 & \cdots & \psi_J \\ 0 & -(1-\lambda_1) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & -(1-\lambda_J) \end{bmatrix} \begin{bmatrix} S_{t-1} \\ E^*_{1t-1} \\ \vdots \\ E^*_{Jt-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \gamma_1 & \kappa_{1,2} & \cdots & \kappa_{1,J} \\ \kappa_{2,1} & \gamma_2 & \cdots & \kappa_{2,J} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{J,1} & \kappa_{J,2} & \cdots & \gamma_j \end{bmatrix} \begin{bmatrix} M_{1t} \\ M_{2t} \\ \vdots \\ M_{Jt} \end{bmatrix} \tag{W5}$$

To solve this dynamic optimization problem, we apply the discrete time maximum principle to derive the optimal post allocation across the social media platforms. Thus, the Hamiltonian at each instant t is given by

$$\boldsymbol{\mathcal{H}_t} = mS_t - \sum_{j=1,\dots,J} c_{jt} M_{jt} + \mu_{st+1}\left(-(1-\lambda_s)S_t + \sum_{j=1}^{J} \psi_j E^*_{jt}\right) + \sum_{\substack{j=1\dots J \\ Note:\, i \in j}} \mu_{jt+1}\left(-(1-\lambda_i)E^*_{it} + \gamma_i \ln(M_{it}) + \sum_{j,j\neq i} \kappa_{i,j} \ln(M_{jt})\right) \tag{W6}$$

where μ_s and $\mu_1, \dots, \mu_J$ are the co-state variables corresponding to the sales and engagement equations. We set stationary co-states in order to derive the analytical insights. The conditions for optimality are:

(a) $\frac{d\mathcal{H}_t}{dM_i} = 0 = -c_{it} + \frac{\mu_{it}\beta_1}{M_i} + \sum_{\substack{j=1 \\ j\neq i}}^{J} \frac{\mu_{jt}\kappa_{j,i}}{M_i}$ that yields $M_i = \frac{1}{c_{it}}\left(\mu_{it}\beta_1 + \sum_{\substack{j=1 \\ j\neq i}}^{J} \mu_{jt}\kappa_{j,i}\right)$ (W7)

(b) $\Delta\mu_{st} = 0 = \rho\mu_{st} - \frac{d\mathcal{H}_t}{dS_t} = \rho\mu_{st} - m - \mu_{st}(\lambda_s - 1)$ that yields $\mu_{st} = \frac{m}{\rho+1-\lambda_s}$. (W8)

(c) $\Delta\mu_{it} = 0 = \rho\mu_{it} - \frac{d\mathcal{H}_t}{dE^*_{i,t}} = \rho\mu_{it} - \mu_s\psi_i - \mu_{it}(\lambda_i - 1)$ that yields $\mu_{it} = \frac{m\psi_i}{(\rho+1-\lambda_s)(\rho+1-\lambda_i)}$ (W9)

Solving equations (W7), (W8) and (W9) and substituting appropriately, yields the following optimal allocation solution.

$$M^*_{it} = \frac{m}{c_{it}(\rho+1-\lambda_s)}\left[\frac{\psi_i\gamma_i}{\rho+1-\lambda_i} + \sum_{j,j\neq i}\left(\frac{\psi_j\kappa_{j,i}}{\rho+1-\lambda_j}\right)\right]. \quad \text{(W10)}$$

Equation (W10) proves Proposition 4 and furnishes the profit maximizing allocation of posts across multiple social media platforms. As evident from the result, the optimal number of posts M_{it} at time t are a function of the (1) the effectiveness of the posts in generating engagement within the platform (γ_i), (2) the platform's corresponding sales contribution (ψ_i), (3) the indirect effect ($\kappa_{j,i}$) of posts in social media i on engagement in social media j, and (4) the sales contribution of social media j, ψ_j.

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