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Essays in Macroeconomics: Monetary Policy in a Recessionary Environment.

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Essays in Macroeconomics: Monetary Policy in a Recessionary Environment.

By

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Abstract

Essays in Macroeconomics: Monetary Policy in a Recessionary Environment.

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In this dissertation I study recessionary environments in general, placing a special emphasis on the steps that can be taken by policy makers to alleviate the negative consequences of recessions. After the Great Recession, most advanced economies had a prolonged period of low growth and high unemployment that is slowly subsiding. Several economists have proposed the possibility that the economy has entered a regime of secular stagnation where demand factors and the Zero Lower Bound lead to persistently low growth. Given the current worldwide decline in interest rates, secular stagnation is a looming risk that should be taken seriously, as it would have a significant impact on welfare. All chapters of this dissertation relate to these issues. The first chapter provides a general motivation of the topic and discusses the relationships between the remaining chapters. In the second chapter, Going Negative at the Zero Lower Bound, I investigate whether negative rates can provide an antidote against the ZLB, granting monetary authorities more room to perform expansionary policy and combat demand-induced low growth. In the third chapter, The Cyclical Sensitivity in Estimates of Potential Output, I study whether potential output really decreases permanently after prolonged recessions, or if, in contrast, this is a byproduct of a faulty computation of potential output. I also study whether some structural VAR methodologies can avoid these shortcomings. In the fourth chapter, Neo-Keynesian Trade, I build a dynamic model that unifies a detailed trade pattern across regions and sectors with monetary non-neutrality. The nominal friction, a downwardly rigid wage, can lead to unemployment in the face of a negative shock if the monetary authority is unwilling or unable to inflate the economy, and this effect can vary substantially across regions depending on their industrial composition. Overall, the evidence in this dissertation indicates that policy makers should do more to combat slow growth, and that they have tools to do so.

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Chapter 1

General Framework and Introduction

After the Great Recession, most advanced economies had a prolonged period of low growth and high unemployment that is slowly subsiding. Several economists have proposed the possibility that the economy has entered a regime of secular stagnation where demand factors and the Zero Lower Bound lead to persistently low growth. Some indicators that support the possibility of secular stagnation are: the worldwide decline in interest rates, the slow growth of wages in the presence of low unemployment, the remaining weakness in labor force participation, and the underperfomance of inflation relative to inflation expectations. Furthermore, the fact that in the United States (the economy that has performed the best in the last five years), even prolonged periods of fiscal expansion (i.e. the 2018 tax cut) don't lead to sustained inflation, suggests that even after substantial stimulus, the economy remains below its true productive capacity. Secular stagnation is a looming risk that should be taken seriously, as it would likely have a significant impact on welfare.

The discussion in the previous paragraph leads to several important questions. What can central banks do to fight prolonged contractions where the policy rate is limited by the Zero Lower Bound? Do deep recessions permanently decrease the productive capacity of the economy? How do open-economy considerations interact with secular stagnation? The following chapters of this dissertation try to address each of these questions in turn. In the second chapter I investigate whether negative rates can provide an antidote against the ZLB, granting monetary authorities more room to perform expansionary policy and combat demand-induced low growth. In the third chapter I study whether potential output really decreases permanently after prolonged recessions, or if, in contrast, this is a byproduct of a faulty computation of potential output. In the fourth chapter I examine whether trade policies or disruptions can have especially detrimental effects at the global ZLB.

Even though the following chapters are mostly self contained, the three questions posed above are definitely related. The most obvious relationship is that if secular stagnation turns out to be true then the three questions become very relevant, but there are many other interactions. If the output gap remaining in the economy is bigger than what policymakers currently believe it to be, there is more room for monetary and fiscal expansion, and one way this expansion can be achieved is by setting negative nominal interest rates. If international trade linkages propagate low interest rates throughout advanced economies then perhaps there is no need for negative nominal interest rates if an individual country is suffering a recession, but this becomes necessary if the whole world is in the Global Zero Lower Bound. If many countries currently have significant output gaps the world is more likely to be in a Global Zero Lower Bound than in output gaps are mostly closed. This means that all three questions posed above are deeply related, and answers to all are crucial for conducting proper monetary policy in a post-Great-Recession world.

In the aftermath of the Great Recession several central banks started setting negative nominal interest rates in an expansionary attempt, but the effectiveness of this measure remains unclear. In the second chapter of this dissertation, titled: Going Negative at the Zero Lower Bound I study the effectiveness of negative nominal interest rates. Negative rates can stimulate the economy by lowering the interest rates that commercial banks charge on loans, but they can also hurt bank profitability by squeezing deposit spreads. I study the effects of negative rates in a new DSGE model where banks intermediate the transmission of monetary policy. In this context, setting negative rates to fight a recession can undermine bank profitability, and therefore the benefits of lowering rates might be smaller than usual. I use bank-level data to provide evidence for the mechanisms in the model and to estimate its main parameters. I find that monetary policy in negative territory is between 60% and 90% as effective (in welfare terms) as in positive territory, depending on the importance of bank equity for lending.

Since the relative efficiency of rate cuts in negative territory obtained in my paper is high, negative rates may be a useful tool for fighting protracted recessions. However, unboundedly negative rates would not continue being as effective, both because commercial banks would eventually dis-intermediate their deposit franchise and because they could eventually substitute their reserves for cash. For both of these reasons my estimates for the relative efficiency of a cut in the policy rate in negative territory are best thought of as applicable in situations where the policy rate is between 0.5% and -1.5%.

The fact that declines in output since the Great Recession have translated into equivalent declines in measures of potential output is commonly interpreted as implying that output will not return to previous trends. In the third chapter of this dissertation, titled: The Cyclical Sensitivity in Estimates of Potential Output, which is joint work with Olivier Coibion and Yuriy Gorodnichenko, we show that real-time estimates of potential output for the U.S. and other countries respond gradually and similarly to both transitory and permanent shocks to output. Observing revisions in measures of potential output therefore tells us little about whether changes in actual output will be permanent or not. Some structural VAR methodologies can avoid these shortcomings; these approaches suggest a much more limited decline in potential output following the Great Recession. If the amount of slack remaining in the United States and other advanced economies is higher than what policy makers currently believe, this has clear implications for fiscal and monetary policy (e.g., that interest rates should rise at a slower pace that they currently are).

Models in international trade usually incorporate richness across regions and sectors, but they have little room for dynamics or nominal rigidities. Models in open economy macroeconomics are typically the opposite. This divide has prevented economists from studying important topics that require both richness in the trade structure and nominal rigidities. In the fourth chapter of this dissertation, titled: Neo-Keynesian Trade, which is joint work with Andrés Rodriguez-Clare, we build a dynamic model that unifies a detailed trade pattern across regions and sectors with monetary non-neutrality. The nominal friction, a downwardly rigid wage, can lead to unemployment in the face of a negative shock if the monetary authority is unwilling or unable to inflate the economy sufficiently, and this effect can vary substantially across regions depending on their industrial composition. We develop an application of our framework related to the unemployment implications of the China shock between 2000 and 2007, but the model can also be used to study trade or currency wars at the ZLB. The framework that we introduce can be applied to a wide range of salient topics.

Overall, the results discussed in this dissertation indicate that policy makers should do more to combat slow growth. This deficient growth is manifested not only in low GDP growth numbers, but also in low inflation, low labor force participation, and low wage growth. The results presented also indicate that monetary authorities have some tools to combat slow growth, for example negative nominal interest rates, fiscal stimulus, and exchange rate policies.

Chapter 2

Going Negative at the Zero Lower Bound

2.1 Introduction

A long tradition in macroeconomics has proposed the existence of a Zero Lower Bound (ZLB) on nominal interest rates. Intuitively, as cash offers a nominal return of zero percent, agents should not be willing to pay others to keep their money. However, recent experience from the aftermath of the Great Recession has shown that negative nominal interest rates (NNIR) are possible: the central banks of several advanced economies have used them as a policy tool.[1] The Euro Area, Switzerland, Sweden, Denmark, and Japan all implemented NNIR at some point between 2014 and 2018 (Figure 2.1). Even if one abstracts from the Great Recession, the global, secular decline in interest rates increases the likelihood of recessionary episodes where nominal rates hit zero.[2] In this environment, understanding whether negative rates can stimulate the economy is of great importance to academics and policy makers.

Two empirical regularities have been observed across countries setting NNIR: retail deposit rates have remained at zero (failing to follow the policy rate into negative territory), and lending rates have mostly declined. Given these facts, it appears that negative rates can partially stimulate the economy through the transmission mechanisms associated with the lending rate. However, commercial bank profitability could be eroded by a decline in the

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[1] Commercial banks hold substantial reserves that would be costly to keep in cash, and so they are willing to pay the central bank to store their money. However, there is a limit to how much they are willing to pay; this has been termed the physical lower bound (PLB; see, Cœuré, 2016). This paper will not have anything to say about the level of the PLB, and focuses instead on the effectiveness of setting rates below zero but above the PLB.

[2] See Kiley and Roberts (2017) and the references therein.

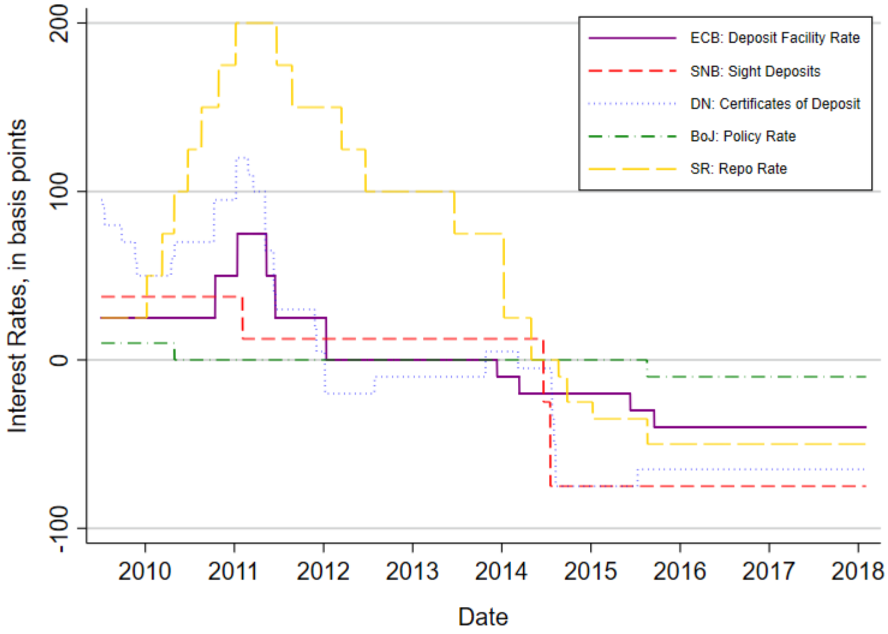

Figure 2.1: Negative rates experience

Notes: This figure shows the rates paid by the Central Bank of Denmark (DN), the ECB, the Central Bank of Sweden (SR), the Swiss National Bank (SNB), and the Bank of Japan (BoJ), in basis points, between 2010 and 2018. The concept of interest rate used for each region is described in the legend. The data was gathered directly from the central banks.

spread between lending and deposit rates. Bank profitability has therefore emerged as one of the most pressing concerns when adopting NNIR.[3] For example, Benoit Cœuré, who serves on the Executive Board of the ECB, said in 2016: "*A reduction in interest rates could harm interest margins, and this could be even more pronounced when rates enter negative territory, due to a potential Zero Lower Bound for retail deposit rates.*" This concern has been echoed in the business press. The Economist wrote in 2016: "*If interest rates go deeper into negative territory, profit margins will be squeezed harder. And if banks are not profitable, they are less able to add to the capital buffers that let them operate safely.*"

In this paper, I study the effects of NNIR on the economy through the lens of a new DSGE model with New Keynesian features where banks intermediate the transmission of monetary policy. In the model, when the central bank sets negative nominal policy rates, deposit rates remain at zero. The lending rate is then affected by two forces. On the one hand, the policy rate decline exerts downward pressure on the loan rate. This is the bank lending channel of monetary policy, which tends to stimulate the economy. On the other hand, the erosion

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[3] Both central banks that implemented negative rates and those that did not have cited bank profitability as a concern; see, Bank of Japan (2016), Danmarks Nationalbank (2015), Bean (2013), and Jackson (2015).

of bank profitability brought about by the decline in the deposit spread will, over time, be transmitted to a decline in bank equity. This leads to upward pressure on the lending rate. I will refer to this as the net-worth channel of monetary policy, which has a contractionary effect. The equilibrium behavior of the lending rate depends on the relative importance of the two channels.

The model features three main frictions affecting the banking sector. First, banks have some monopoly power in lending and managing deposits. As a result, the deposit rate that financial intermediaries pay households is different from the policy rate that the central bank pays on reserves. The policy rate also differs from the rate that borrowers pay commercial banks for loans. This friction is essential for the bank lending channel, since banks are only able to lower their lending rate (despite the fact that their funding costs are constant, i.e., stuck at the ZLB) because of the existence of a profit margin. The second friction is that, after a shock, banks cannot immediately regain their optimal level of equity. Instead, they accumulate capital slowly, through retained earnings. The third friction is that bank equity matters for lending. In particular, banks care about their level of leverage, and they are reluctant to lend when this variable is too high. Frictions two and three lead to the existence of a relevant bank net-worth channel. The combination of the stimulative bank lending channel and the contractionary bank net-worth channel implies that setting NNIR has both beneficial and detrimental effects, the relative importance of which determines the overall usefulness of setting a negative policy rate.

I start by developing a static model of the banking sector that contains only the first friction (bank monopolistic competition). In this model there is a continuum of commercial banks. Each individual bank receives an exogenous level of equity and obtains deposits from consumers. With the resources available after combining their equity and deposits, commercial banks can either provide loans to firms or keep reserves at the central bank. Banks face an upward-sloping deposit supply curve and a downward-sloping loan demand curve. Deposit supply and loan demand for each individual bank arise from the fact that depositors and borrowers have CES preferences across banks. The aggregate amounts of deposits supplied and loans demanded are taken as given for now, as this is a partial equilibrium exercise. Additionally, the model assumes that if a bank sets a negative deposit rate then it obtains no deposits, as consumers could simply save in cash.

In this context, there exists a positive, but small, threshold for the policy rate, denoted by $\tilde{\imath}$, at which the behavior of banks changes. I will refer to the case where the policy rate is above $\tilde{\imath}$ as "Regime 1." In this regime, because of the monopolistic competition setup, each bank sets its loan rate as a mark-up on the policy rate and its deposit rate as a mark-down on it. Consequently, changes in the policy rate are fully passed through to the loan and deposit rates. It will be useful to define the loan spread as the difference between the loan and the policy rate, and the deposit spread as the difference between the policy and the deposit rate. Bank return on equity (ROE) can then be expressed as the sum of three terms:

the policy rate, the loan spread times the loan-to-equity ratio, and the deposit spread times the deposit-to-equity ratio. In Regime 1 the spreads do not change with the policy rate, and so ROE moves one-for-one with the policy rate.

When the policy rate is below $\tilde{\imath}$, denoted "Regime 2," banks would like to set a negative deposit rate to earn their usual deposit spread. However, if they do so they lose all deposits, and so they set a zero deposit rate instead.[4] The loan rate is still set as a mark-up on the policy rate, since holding reserves is the marginal use of bank funds. Therefore, a decline in the policy rate is still fully transmitted into the lending rate, giving rise to the stimulative bank lending channel of NNIR mentioned above. In this regime, the loan spread remains constant, but the deposit spread falls with the policy rate. Consequently, ROE falls more than one-for-one with the policy rate. In this static model, the steep decline in ROE that occurs in Regime 2 after a cut in the policy rate has no perverse effects on the lending rate, due to the lack of dynamics and the absence of additional frictions; the contractionary bank net-worth channel is not operational yet.

The static model has four testable predictions. First, in Regime 2 the deposit rate stops reacting to the policy rate. Second, the lending rate continues to fall with the policy rate even in Regime 2. Third, bank ROE is affected by a cut in the policy rate more in Regime 2 than in Regime 1. Fourth, the higher sensitivity of bank return on equity to the policy rate in Regime 2 is more pronounced for banks that rely heavily on retail deposits for funding. I use bank-level data from more than five thousand banks in 10 advanced regions (i.e., the five advanced regions that have set negative rates and five other comparable advanced regions, including the United States, that have set very low rates) to test these predictions. The first step is to estimate the threshold level $\tilde{\imath}$. A variety of tests confirm the existence of a change in the slope of the response of both the deposit rate and ROE to the policy rate when the policy rate is around 50 basis points. Consequently, I set $\tilde{\imath} = 0.5\%$. I then test the four predictions and find strong support for them in the data. The prediction that the loan rate continues to fall with the policy rate in Regime 2 is especially useful for differentiating between my model and alternative ones that propose that negative rates cannot be expansionary.

I then extend the static bank model to a dynamic setup, introduce frictions two and three (i.e. slow-moving bank capital and the importance of bank equity for lending), and embed this in a DSGE model. In this context, I can study both the beneficial effects of negative rates (expressed through the bank lending channel), as well as the detrimental ones (expressed through the bank net-worth channel). The bank net-worth channel works as follows. First, negative policy rates and the Zero Lower Bound on deposit rates generate a decline in the deposit spread. Second, the decline in the deposit spread translates to a decline in bank ROE that is significantly bigger than the one that would occur after a cut in the policy rate

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[4] There exists a second threshold $\underline{i} < 0$ below which it becomes too costly for banks to accept deposits that earn a negative spread; in that region some banks stop receiving deposits. I postpone this discussion to Section 2.2.

above $\tilde{\imath}$. Third, over time the decline in ROE accumulates to a decline in bank equity, since banks cannot replenish their equity frictionlessly. Finally, the decline in bank equity leads to upward pressure on the loan rate, as banks with less equity require a higher loan rate to be willing to lend.

I calibrate the full model to obtain estimates of the relative efficiency, in welfare terms, of cutting the policy rate below $\tilde{\imath}$ compared to doing so above $\tilde{\imath}$. In the banking sector, the elasticity of loan demand and the importance of bank equity for lending (modeled as the cost of deviating from a target level of leverage) are the most important parameters. I use information on the cross-section of banks in each region to structurally estimate these parameters, leveraging the distribution of loan rates and loan amounts across banks with different levels of equity. The calibrated model indicates that the relative efficiency of a cut in the policy rate below $\tilde{\imath}$ (compared to one above $\tilde{\imath}$) is between 60% and 90%. This estimated relative efficiency is fairly high, and indicates that the harmful implications of negative rates on bank profitability seem to be less serious than previously thought. There are two reasons why the relative efficiency is high despite the existence of the contractionary bank net-worth channel. First, the estimates of the importance of bank equity for lending are small. This is consistent with the fact that in the data, after controlling for bank fixed effects, a decline in the equity of a particular bank does not have a big effect on that bank's lending amount or its loan rate. Second, in the full model, when the policy rate and the loan rate fall, aggregate loan demand increases and banks can switch reserves for loans, decreasing the impact of negative rates on their ROE (this mechanism is not operational in the static model).

There are few academic papers dealing with the topic of NNIR from a theoretical perspective. Rognlie (2015) focuses on money demand while sidestepping the issue of bank profitability. Brunnermeier and Koby (2017) study the "reversal rate," i.e., the level of the interest rate where decreasing the policy rate further becomes contractionary for lending. However, they do so in a partial equilibrium framework without nominal rigidities, and so they cannot analyze whether setting NNIR is optimal. Amador, Bianchi, Bocola, and Perri (2017) investigate how setting negative rates can help a small economy that is experiencing capital inflows, but they do not discuss concerns related to bank profitability. Similarly to my paper, Eggertsson, Juelsrud, and Wold (2017) study NNIR in a monetary DSGE model with banks, but both their assumptions and their conclusions are very different from mine. Their model does not incorporate bank monopoly power and, as a result, NNIR policies are *never* expansionary, regardless of parameter values.[5] By contrast, in my model NNIR can be expansionary or contractionary, as well as welfare-improving or welfare-reducing, depending on parameter values. The Eggertsson et al. (2017) model implies that declines in the policy rate in negative territory are not transmitted to the lending rate, while my model predicts

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[5] The intuition for why interest rate cuts are not expansionary in their model is that if the deposit rate is stuck at zero then banks' funding costs (via deposits) are no longer responsive to the policy rate, and therefore banks (since they do not have monopoly power) are not able to decrease the lending rate.

that they are.[6] I provide evidence that the behavior of the lending rate is consistent with the predictions of my model.

The theoretical framework that I implement is related to papers that study the relation between households and banks, like Kiyotaki and Moore (2012) and Curdia and Woodford (2015). More specifically, it relates to papers that stress the agency problem between households and banks, like Gertler and Karadi (2011, henceforth GK), Gertler and Kiyotaki (2010), and Gerali, Neri, Sessa, and Signoretti (2010, henceforth GNSS). Relative to this literature, the contribution of this paper is to provide a model that combines all the frictions in the financial sector required to allow the study of NNIR with both beneficial and detrimental aspects. In my model, deposits and loans have the same duration, a feature that sidesteps maturity transformation as an aspect of banking. This setup is used for tractability, but it is also motivated by Drechsler, Savov, and Schnabl (2018).

There is a growing empirical literature that studies the effects of NNIR on commercial banks. Ampudia and Van den Heuvel (2017) study the effect of negative rates on banks' stock prices. They try to get at causal identification by using high-frequency techniques, and find that an unexpected decrease in the policy rate has particularly negative effects on banks' stock prices during the negative rate period. Borio, Gambacorta, and Hofmann (2017) discuss the influence of monetary policy on bank profitability, in the context of very low (but not yet negative) rates. They find that low rates and an unusually flat term structure erode bank profitability. Claessens, Coleman, and Donnelly (2017) find that a one percentage point interest rate decline implies an 8 basis points lower net interest margin in normal times, but this effect increases to 20 basis points at low rates. More recent papers, like Basten and Mariathasan (2018), Demiralp, Eisenschmidt, and Vlassopoulos (2017), Eisenschmidt and Smets (2018), and Lopez, Rose, and Spiegel (2018), study the effects of NNIR in Europe and Japan. They generally find that lending volumes have increased, lending rates have fallen, and banks have modified their behavior to reduce the impact of negative rates on their profitability. In contrast to my paper, this literature is atheoretical, and hence cannot interpret these findings in the context of a model that allows for the quantification of the effects of NNIR on the broader economy.

The rest of the paper is organized as follows. Section 2.2 introduces the static banking model and discusses the interest rate spreads that emerge and how banks are hurt disproportionately when the policy rate falls below $\tilde{\imath}$. Section 2.3 uses bank-level data to test the four predictions of the static model. Section 2.4 extends the static model to a fully-fledged DSGE model. Section 2.5 outlines how I use the data to inform the calibration of the full model. Section 2.6 discusses the response of the model economy to a large recessionary shock. Section 2.7 studies the relative efficiency, in welfare terms, of a cut in the policy rate in Regime 2

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[6] While in my model the bank net-worth channel can lead to upward pressure on the lending rate, this only occurs over time, since the decline in bank profitability is only transmitted to a decline in bank equity after some periods.

compared to Regime 1. Section 2.8 concludes.

2.2 The Static Banking Model

This section contains a static and partial equilibrium model of the banking sector that illustrates how a decline in the policy rate, even in negative territory, can be transmitted to a decline in the lending rate. The model also illustrates how negative rates can undermine bank profitability in the presence of a lower bound on the deposit rate. The objects of interest in this section are the (exogenous) policy rate, as well as the (endogenous) deposit and lending rates, and the return on bank equity. The amount lent and the amount of deposits received by each bank are endogenous, but the aggregate amounts are exogenous, due to the partial equilibrium nature of the exercise.

There is a continuum of banks, indexed by j, between zero and one. Each bank is given a certain level of equity as an endowment at the beginning of the period. On the liability side a bank combines equity, denoted by F_j, and deposits, denoted by D_j. Meanwhile, on the asset side, it issues loans L_j and holds reserves H_j. The objective of banks is to maximize their resources at the end of the period, when loans and deposits are repaid. Each bank has some monopoly power that will be modeled using a CES framework. Specifically, each bank faces a downward-sloping loan demand and an upward-sloping deposit supply (even though aggregate loan demand and deposit supply are constant). In this simple setup, due to the presence of curvature in loan demand and deposit supply, there is no need to impose a leverage constraint on banks.

Banks choose the interest rate they charge on loans i^l_j, the amount they lend, the interest rate they pay on deposits i^d_j, the amount of deposits they take, and the amount of reserves they hold in the central bank, which earns the policy rate i, subject to several constraints. The maximization problem that the individual bank j faces is therefore the following:

$$\max_{i^l_j, L_j, i^d_j, D_j, H_j} \quad (1+i^l_j)L_j + (1+i)H_j - (1+i^d_j)D_j$$

$$\text{s.t.}$$

$$L_j = \left(\frac{1+i^l_j}{1+i^l}\right)^{-\varepsilon^l} \mathbf{L} \tag{2.1}$$

$$D_j = \begin{cases} \left(\frac{1+i^d_j}{1+i^d}\right)^{-\varepsilon^d} \mathbf{D} & \text{if } i^d_j \geq 0 \\ 0 & \text{if } i^d_j < 0 \end{cases} \tag{2.2}$$

$$L_j + H_j = F_j + D_j \tag{2.3}$$

$$H_j \geq 0. \tag{2.4}$$

The functional forms of loan demand (equation 2.1), and deposit supply (equation 2.2), are microfounded in Appendix A.1.[7] Equation (2.2) indicates that a bank obtains no deposits if it sets negative nominal deposit rates, since in that case households could save simply by using cash. The aggregate amounts of loans demanded by firms and deposits supplied by households are $\mathbf{L}$ and $\mathbf{D}$ respectively. As mentioned above, these aggregate quantities are not affected by any rates. Equation (2.3) is the balance sheet constraint, which indicates that total assets (loans plus reserves) have to be equal to liabilities plus equity. Equation (2.4) states that reserves at the central bank must be nonnegative.[8]

I assume that $\varepsilon^l > 1$ and $\varepsilon^d < -1$, that all banks have the same amount of initial equity $F_j = \mathbf{F}$, and that $\mathbf{D} > \mathbf{L} > \mathbf{F}$.[9] The formal solution to the bank problem is described in Proposition 1, which is given in Appendix A.1.4 together with its proof. Here I describe the results intuitively. The solution consists of regimes that apply depending on the level of the policy rate. Regime 1 applies when $i \geq \tilde{\imath}$, Regime 2 does when $\underline{i} \leq i < \tilde{\imath}$, and Regime 3 does when $i < \underline{i}$. The thresholds are described below.

In Regime 1, when the policy rate is in "normal" territory (i.e., above $\tilde{\imath}$), all banks set the same (gross) loan and deposit rates, which are given as a mark-up and a mark-down on the gross policy rate:

$$1 + i_j^l = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i), \qquad\qquad 1 + i_j^d = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i).$$

This is reminiscent of the solution to the pricing problem of a monopolistically competitive goods producer.[10] The loan spread is given by $i_j^l - i = (1+i)/(\varepsilon^l - 1)$ and the deposit spread by $i - i_j^d = (1+i)/(1-\varepsilon^d)$, both of these are positive. Even though the spreads technically vary with the policy rate, their slopes with respect to the policy rate (given by $(\varepsilon^l - 1)^{-1}$ and $(1-\varepsilon^d)^{-1}$) are very small.[11] This justifies the claim in the Introduction that in Regime 1 the spreads are (approximately) invariant to the policy rate. In this regime, all banks obtain an amount of deposits equal to the aggregate supply of deposits ($\mathbf{D}$), give an amount of loans equal to the aggregate demand of loans ($\mathbf{L}$), and hold a positive amount of reserves at the central bank ($H_j = \mathbf{F} + \mathbf{D} - \mathbf{L}$). Banks hold reserves not because they are forced to do so

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[7] Specifically, Appendices A.1.1-A.1.2 describe how to obtain equations (2.1) and (2.2) using the CES framework. Appendix A.1.3 shows that the CES formulation can be microfounded through a heterogeneous setup where each agent interacts with a single bank but has stochastic utility across banks (perhaps because of proximity or switching costs).

[8] Banks can borrow from the central bank using the discount window, but this usually carries a high cost and the stigma of being in financial trouble. Hence, I ignore the possibility of borrowing from the central bank.

[9] Since $\varepsilon^l > 1$, a higher loan rate decreases loan demand. Deposits work differently, as costumers are looking for high rates (bank customers supply deposits instead of demanding them). $\varepsilon^d < -1$ indicates that banks that pay a higher rate obtain more deposits.

[10] As an illustrative example consider $i = 3\%$, $\varepsilon^l = 34$, and $\varepsilon^d = -199$; in this case $i^l \approx 6\%$ and $i^d \approx 2.5\%$, for a loan spread of 3% and a deposit spread of 50 basis points. This is similar to the levels observed in the data for advanced countries if one takes long-run averages.

[11] This follows from the fact that the absolute values of ε^l and ε^d are likely to be very high, see footnote 10.

(there is no reserve requirement), but because it is optimal for them to restrict the amount of loans that they provide when they are facing a downward sloping loan demand curve. Consequently, they keep their "unused" funds as reserves in the central bank.

The prescription that $1 + i^d_j = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i)$ implies that i^d_j would become negative when the policy rate falls below the threshold $\tilde{\iota} \equiv -\frac{1}{\varepsilon^d} > 0$. Once the policy rate crosses $\tilde{\iota}$, commercial banks would like to set negative nominal deposit rates in order to obtain their usual spread on deposits; however, if they did so, they would end up losing all their deposits, and so they set a zero deposit rate instead. This is Regime 2, where all banks set $i^d_j = 0$, receive an amount of deposits $\mathbf{D}$, give an amount of loans $\mathbf{L}$, and still hold a positive amount of reserves at the central bank. In this regime the loan rate setting behavior of banks is the same as in Regime 1, since the marginal use of commercial banks' resources is still as reserves at the central bank, and the loan rate is set as a mark-up on that opportunity cost (i.e., $1 + i^l_j = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i)$). This is the sense in which the bank lending channel remains operational below $\tilde{\iota}$; declines in the policy rate are still transmitted to the loan rate as they are above $\tilde{\iota}$.

Notice that when the deposit rate reaches zero banks cannot start turning away the marginal depositor. They either maintain a zero deposit rate and accept all the money that households wish to deposit, or they set a negative deposit rate and lose all deposits. Intuitively, this means that Regime 2 exists because there is a range of low and negative policy rates where banks prefer to receive deposits even if they make a low or negative spread on them, because it allows them to maintain their leverage and earn more on their loan franchise. Regime 2 stops applying when the policy rate crosses the threshold $\underline{i} < 0$, where offering deposits at a zero rate is so costly that at least one commercial bank has incentives to deviate.

Regime 3, which applies when $i < \underline{i}$, is no longer a symmetric equilibrium, since a fraction of the banks still obtains deposits, while the remaining fraction stops doing so. This regime is described in detail in Appendix A.1.4. For the purposes of this section, the important feature of Regime 3 is that the aggregate loan rate i^l is weakly *decreasing* in i. Intuitively, a decline in i creates a disincentive to receive deposits, since some reserves would have to be kept at the central bank, earning a negative i. This (weakly) decreases the fraction of banks that takes deposits, allowing all banks to (weakly) *increase* their loan rate. This effect is reminiscent of the "reversal rate" of Brunnermeier and Koby (2017). Eventually, as the policy rate keeps decreasing, the fraction of banks that does not take deposits becomes independent of the policy rate, and so do all other bank variables, since every bank stops keeping reserves at the central bank.

In order to clarify the channels through which banks earn money, denote end of period equity by F'_j. Using equation (2.3) this can be expressed as

$$F'_j \;=\; (1 + i)F_j + (i^l_j - i)L_j + (i - i^d_j)D_j. \tag{2.5}$$

This expression highlights the fact that banks generate profits via three distinct channels:

1. They can keep their equity as reserves in the central bank, obtaining a gross return of $(1+i)$.
2. They obtain a loan spread of $i^l_j - i$ on each dollar lent.
3. They also obtain a deposit spread of $i - i^d_j$ on each dollar of deposits received. This is the term that gets "squeezed" when the policy rate is too low, i.e., when $i < \tilde{\imath}$.

The "additional" profits mentioned in items 2 and 3 are due to the existence of monopoly power in the banking sector, which is well documented empirically.[12] Bank (gross) return on equity (ROE) is given by

$$\frac{F'_j}{F_j} = \begin{cases} (1+i)\left(1 + \frac{1}{\varepsilon^l - 1}\frac{\mathbf{L}}{\mathbf{F}} + \frac{1}{1-\varepsilon^d}\frac{\mathbf{D}}{\mathbf{F}}\right) & \text{if } \tilde{\imath} \leq i \\ 1 + \frac{1}{\varepsilon^l - 1}\frac{\mathbf{L}}{\mathbf{F}} + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{\mathbf{L}}{\mathbf{F}} + \frac{\mathbf{D}}{\mathbf{F}}\right) & \text{if } \underline{i} \leq i < \tilde{\imath} \\ \left[(\mathbf{L}/\mathbf{F})^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu(i)\right]^{\frac{1}{\varepsilon^l - 1}} (1-\mu(i))^{\frac{1}{1-\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i) & \text{if } i < \underline{i}. \end{cases}$$

This is a continuous function, depicted in Figure 2.2.[13] If one takes $\frac{1}{\varepsilon^l - 1}$ and $\frac{1}{1-\varepsilon^d}$ to be small (which will be the case in my calibration), then the derivative of ROE with respect to the policy rate is roughly one when $i > \tilde{\imath}$, but approximately $(1 + \mathbf{D}/\mathbf{F})$ when $\underline{i} \leq i < \tilde{\imath}$ (i.e., in Regime 2). Since the deposit-to-equity ratio is high in the data (generally between 4 and 9), the effect of the policy rate on ROE will be substantially higher in Regime 2. The intuition for ROE increasing with the policy rate in normal territory (i.e., above $\tilde{\imath}$) is that bank funds can be loaned out or kept as reserves in the central bank; if the rate on reserves increases, then it pushes up the outside option of each individual bank and leads to an increase in the loan rate and ROE. Figure 2.2 is important because the relationship between bank return on equity and the policy rate is a crucial mechanism in the full model I develop in Section 2.4. Recall that in this static model the aggregate amounts of deposits and loans ($\mathbf{L}$ and $\mathbf{D}$) are taken as given, and are independent of the policy rate. Even though later, in the full model, the aggregate amounts of loans and deposits will be affected by the policy rate, the intuition behind Figure 2.2 will remain valid.

The interest rate $\tilde{\imath}$ represents the threshold where further cuts in the policy rate would turn deposit rates negative in the absence of the deposit ZLB. However, since deposit rates are constrained by zero, $\tilde{\imath}$ instead represents the point where further lowering the policy rate starts affecting banks disproportionately, because they cannot charge their usual spread on deposits. Even though cuts in the policy rate above $\tilde{\imath}$ have a negative effect on bank ROE, they have a much more negative effect below $\tilde{\imath}$. The switch between Regimes 1 and 2 occurs before policy rates hit zero (since $\tilde{\imath} > 0$).[14]

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[12] See Berger, Demirgu-Kunt, Levine, and Haubrich (2004), Degryse and Ongena (2008), Drechsler, Savov, and Schnabl (2017), and Drechsler et al. (2018).

[13] The expression for $\mu(i)$ is given in Appendix A.1.4.

[14] How far above zero this happens is governed by ε^d in the static model, but will depend on additional

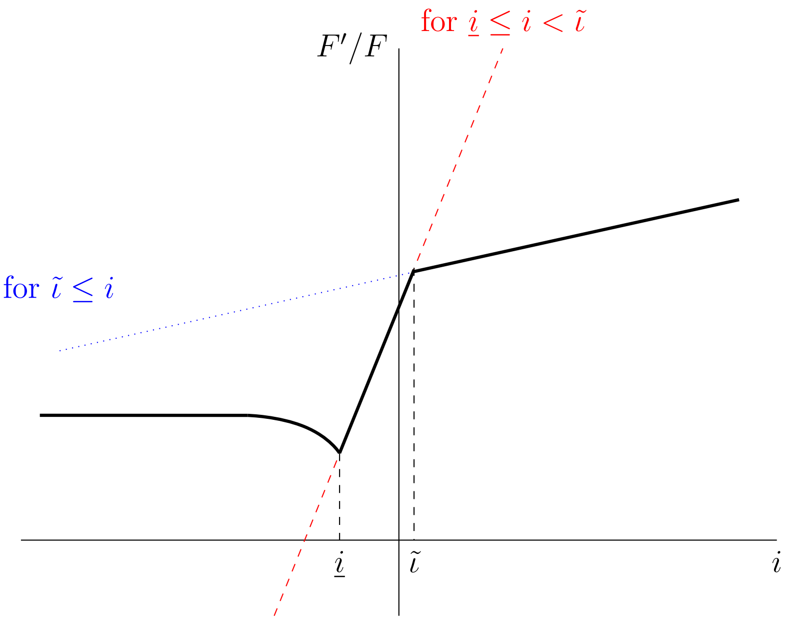

Figure 2.2: Model implied relationship between ROE and i

Notes: This figures describes the model-implied relationship between bank (gross) return on equity (F'/F, denoted ROE), on the y axis, and the policy rate (i), on the x axis. The levels $\tilde{\imath}$ and $\underline{i}$ represent thresholds where commercial banks start reacting differently to the policy rate; their expressions are given in the text.

The threshold $\underline{i}$ represents the point at which fears of "disintermediation" start becoming relevant, since at this point some banks prefer to stop offering certain services (like taking deposits) because they are too unprofitable. The expression for $\underline{i}$ contains the elasticity ε^l (not ε^d), which indicates that this threshold is related to monopoly power in the lending market rather than to monopoly power in the deposit market. Intuitively, even if banks are making low or negative profits while receiving deposits, they can use these funds to make loans and earn the spread between the policy rate and the lending rate, which is governed by ε^l. Notice that the threshold for disintermediation is strictly smaller than zero (i.e., $\underline{i} < 0$), and hence there is some room for policy rates to become negative without raising fears of disintermediation.

Below $\underline{i}$ there is an interval where ROE *increases* as the policy rate decreases. As mentioned above, in that range a decline in i creates a disincentive to receive deposits, since some reserves would have to be kept at the central bank, where they would earn a negative i. Therefore, the fraction of banks that takes deposits decreases, lowering the "threat" that the abundant funds of these banks represents for the aggregate supply of loans. This gives all banks more room to exercise their monopoly power, allowing them to increase their loan rate and consequently their ROE. As the policy rate continues to decline, this perverse effect on

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parameters in the full model described in Section 2.4 and Appendix A.1.5.

ROE disappears, since all banks stop holding reserves at the central bank. For the baseline calibration I describe in Section 2.5, $\underline{i}$ will be around -2.2%.[15] Because this threshold is so low, in the following sections I will ignore the region below $\underline{i}$. Indeed, the lowest level of rates ever set in any country was -75 basis points, far above my estimates of $\underline{i}$. In the full model presented in Section 2.4 I also ignore this region, and confine the analysis to situations where the policy rate is not too far below -2%.

In summary, the model in this section illustrates that there is a range of low and negative rates, between $\tilde{\imath}$ and $\underline{i}$ (roughly between 50 and -200 basis points), where declines in the policy rate are still transmitted to the loan rate due to the presence of bank monopoly power. This implies that rate cuts in this range can be stimulative through the bank lending channel, even in the presence of a ZLB for deposit rates. A decline in the policy rate in the $[\underline{i}, \tilde{\imath}]$ range also leads to a decline in bank ROE which is bigger than the one that would occur if the rate cut happened above $\tilde{\imath}$. In this static model such a decline in ROE does not affect the loan rate (implying that negative rates would only have beneficial effects), but in the full model the decline in ROE can lead to upward pressure in the loan rate, making cuts in the policy rate less effective at stimulating the economy.

2.3 Empirical Analysis

2.3.1 Data Description and Summary Statistics

To test the predictions of the static model, and to later identify some of the parameters of the full model, I compiled a sample of yearly data for individual commercial banks obtained from Fitch Solutions (the same underlying data behind the phased-out Bankscope dataset). The sample spans 28 years (1990–2017) and 19 countries in 10 advanced regions (i.e., the five regions that have set negative nominal rates: the Euro area, Sweden, Switzerland, Denmark, and Japan, and five comparable regions that have set very low rates: U.S., U.K., Canada, Norway and Australia). The data on the policy rates in these countries was obtained directly from their respective central banks.

To reduce the adverse effects of outliers, I excluded banks that: have less than 50 million dollars in total assets, have less than 5 yearly observations, or have extreme values in the quantities of interest.[16] The selected sample includes 5,405 banks.[17] The total number of

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[15] Since the Physical Lower Bound (PLB) mentioned in footnote 1 is probably between -100 and -200 basis points, the PLB may be above $\underline{i}$ or vice versa. Regardless of which one is closer to zero, it would probably be a stretch for central banks to set rates on reserves below -200 basis points.

[16] I exclude banks that in any year have higher than 15% deposit rate, loan rate, net interest margin, or ROAA, higher than 150% ROAE, or a ratio of a specific asset category to total assets that is greater than one. This removes less than 9% of the observations and has a very small effect on the results, but helps with precision.

[17] The breakdown across countries is: 65 in Canada, 83 in Australia, 132 in Norway, 131 in the U.K., 1,235

Table 2.1: Summary statistics for banking variables between 1990 and 2017

Variable	Mean	Std. Dev.	Min.	Max.	N
Rate on Av. Earning Assets	4.57	1.99	0.60	10.50	80086
Deposit Rate	1.02	1.18	0.00	6.62	31615
Net Interest Margin	2.46	0.99	0.01	6.12	80441
ROAA	0.48	0.66	-2.76	3.50	80545
ROAE	5.78	7.91	-43.60	33.07	80202
Log of Net Loans	6.60	1.78	2.84	13.09	84721
Log of Total Customer Deposits	6.71	1.74	2.13	13.14	83532
Log of Equity	4.48	1.76	1.04	10.88	85240
Log of Total Assets	7.13	1.75	4.17	13.91	85311
Customer Deposits to Assets ratio	0.72	0.18	0.01	0.96	83599
Net Loans to Assets ratio	0.62	0.17	0.03	0.97	84823

Notes: This table contains summary statistics for banking variables between the years 1990 and 2017. ROAA stands for return on average assets and ROAE for return on average equity. "N" denotes to the total number of observations across all countries and years.

observations is approximately 85,000. Variables are winsorized at the 0.5% level on each side to further minimize the adverse effects of outliers. Table 2.1 contains some summary statistics of the variables of interest, like the rate paid on average earning assets, which will be used as a measure of the loan rate, the rate paid on customer deposits, the net interest margin, the return on average assets (ROAA), and the return on average equity (ROAE). It also contains other important quantities.

2.3.2 Threshold Effects

The model presented in Section 2.2 has stark predictions regarding the behavior of the variables of interest around the threshold $\tilde{\imath}$ that are summarized in Figure 2.3. The figure shows that above $\tilde{\imath}$ both the deposit rate and the loan rate increase with the policy rate, and that ROE increases as well, but at a relatively slow pace. Below $\tilde{\imath}$ the deposit rate is already at zero and stops responding to declines in the policy rate, the loan rate continues

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in the U.S., 75 in Sweden, 64 in Denmark, 306 in Switzerland, 605 in Japan, and 2,709 in the Euro Area. Appendix Figures A.6 and A.7 contain graphs of the policy rate in the 10 regions in the sample across years.

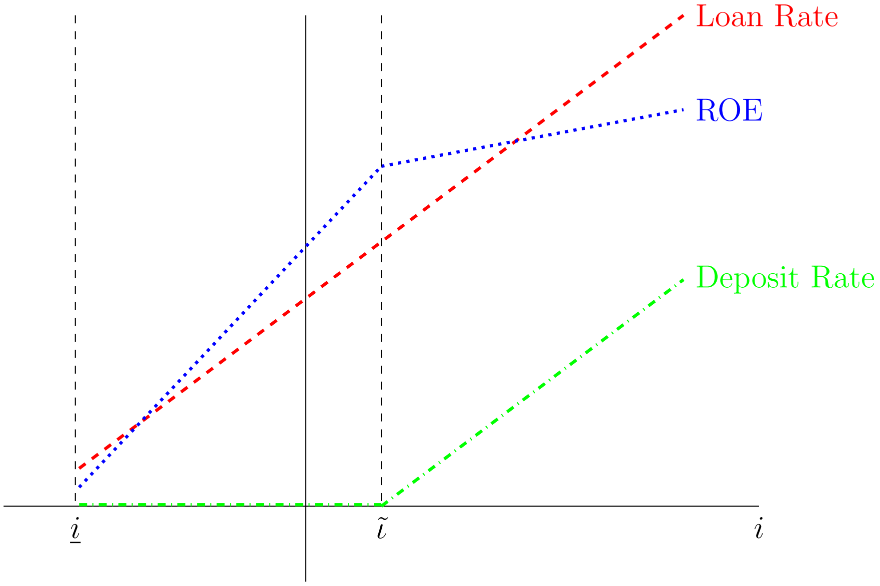

Figure 2.3: Relationship between important variables and i

Notes: This figure describes the relationship, implied by the static model, between the loan rate, the deposit rate, and return on equity (ROE), on one hand, and the policy rate in the other hand. The thresholds $\tilde{\imath}$ and $\underline{i}$ are described in Section 2.2.

to decrease, and return on equity reacts strongly to the policy rate.[18,19]

There are several ways to allow for potentially nonlinear effects.[20] One option is to run a locally weighted regression of the dependent variable at the bank level on the policy rate after residualizing out bank fixed effects and time fixed effects. This is done in Appendix A.2.3, and it supports the predictions of the model, but it does not allow for identification of the threshold value $\tilde{\imath}$. An alternative option is to run regressions of the following type:

$$y_{b,t} = \alpha_b + \delta_t + \beta_1 i_{c(b),t} + \beta_2 (i_{c(b),t} - \tilde{\imath}) D_{c(b),t} + \varepsilon_{b,t}, \tag{2.6}$$

where $y_{b,t}$ is some outcome variable (the deposit rate, the loan rate, or ROAE) for bank b, in country $c(b)$ and year t, and $i_{c(b),t}$ is the policy rate in that country and year. The regressions include a bank fixed effect (α_b) and a year fixed effect (δ_t). The dummy $D_{c(b),t} \equiv \mathbb{1}(i_{c(b),t} > \tilde{\imath})$ is an indicator of whether the policy rate is above $\tilde{\imath}$; the effect of the policy rate on the dependent variable is allowed to have a different magnitude above and below $\tilde{\imath}$. These

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[18] The range of the policy rate presented in Figure 2.3 is entirely above the threshold $\underline{i}$. I ignore the region below $\underline{i}$ because I do not expect to learn anything about this region from the available data, since the negative rates set in advanced countries have never gone below negative 75 basis points while $\underline{i}$ is $\approx -2\%$ in my calibration.

[19] The *levels* of the rates and ROE in Figure 2.3 do not have any particular significance, the important concept being highlighted is their *reaction* to the policy rate.

[20] For comparison, the linear results are reported in Appendix A.2.2.

Table 2.2: Regressions for main variables of interest

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.578^{***}	-0.035	5.004^{***}
	(0.145)	(0.139)	(1.063)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.155	0.479^{***}	-4.194^{***}
	(0.141)	(0.142)	(1.024)
$\beta_1 + \beta_2$	0.423^{***}	0.444^{***}	0.810^{***}
s.e.$(\beta_1 + \beta_2)$	0.026	0.035	0.203
N	80078	31554	80199
R squared	0.93	0.85	0.41
Mean dep. var.	4.58	1.01	5.78

Notes: This table contains the results of regressing the three variables of interest (loan rate, deposit rate, and return on average equity), on bank fixed effects, time fixed effects, the policy rate, and an interaction between the policy rate and an indicator of whether the policy rate is above the threshold $\tilde{\imath}$ (taken to be 50 basis points in this table). S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

regressions require knowing the level of the threshold; I start by setting $\tilde{\imath} = 0.5\%$ and then justify this choice in Section 2.3.3.

Table 2.2 contains the results of the regressions in equation (2.6). The coefficient on the policy rate, denoted β_1, measures the slope below the threshold. The coefficient on $(i_{c(b),t} - \tilde{\imath})D_{c(b),t}$, denoted β_2, measures the difference in slope between the portion below the threshold and the portion above the threshold. Therefore, the sum of the two coefficients $(\beta_1 + \beta_2)$ measures the slope above the threshold. The results conform well to the predictions of the model. The loan rate reacts strongly and significantly to the policy rate below the threshold, and it reacts similarly above the threshold (the significance level increases). The deposit rate does not react to the policy rate below the threshold, but does react strongly and significantly above it. ROAE reacts very strongly to the policy rate below the threshold and mildly above it. Appendix A.2.4 documents the robustness of these results to a number of modifications of the baseline specification, such as including a lag of the dependent variable, including the threshold level as an independent variable in the regression, controlling for the time-varying bank-specific level of equity and assets, or controlling for different indicators of financial or banking crises.

Notice that I do not have exogenous variation in policy rates, and hence these results are simply correlations that hold in the data and have the interpretation of general equilibrium relationships that would hold in a model. Nevertheless, they can be informative of which

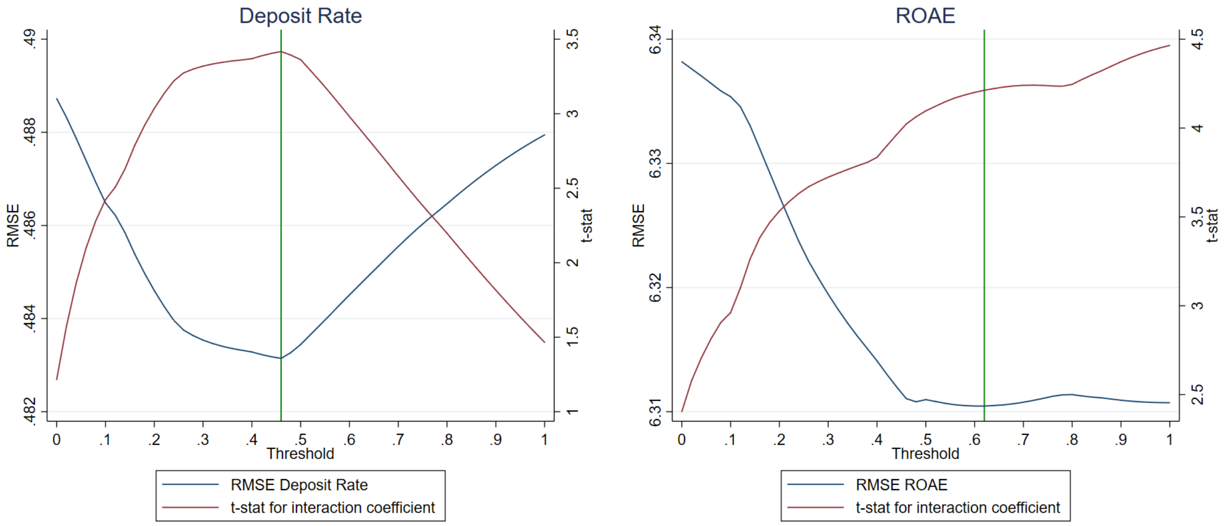

Figure 2.4: RMSE for threshold tests

Notes: This figure plots the root mean squared error (RMSE) and the t-stat on the interaction coefficient (β_2) for the regression in (2.6) across different values of $\tilde{\imath}$. The dependent variable is the deposit rate in the left panel, and return on average equity (ROAE) in the right panel.

mechanisms are operational in the real world. For example, the fact that the loan rate declines with the policy rate below $\tilde{\imath}$ can be used to distinguish between my paper and Eggertsson et al. (2017). In their model the loan rate stops declining once the deposit rate is at zero, while in my model the loan rate continues to decrease due to the presence of bank monopoly power.[21] The evidence in Table 2.2 is consistent with my model.

2.3.3 Identifying the Threshold

The previous regressions require knowledge of the threshold level $\tilde{\imath}$. The full model presented in Section 2.4 implies that $\tilde{\imath}$ is very well approximated by the steady state difference between the policy rate and the deposit rate. In the data, the average difference between the policy rate and the deposit rate is around 50 basis points, both measured through the IMF IFS and in my sample (once I collapse it to the country-year level and then take a simple mean), thus motivating my initial choice of the threshold.[22] It is nevertheless desirable to check whether atheoretical empirical tests on the level of the threshold support this choice.

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[21] In theory the results in Eggertsson et al. (2017) apply for $\tilde{\imath} = 0$ and not for some other level, like 50 basis points. The results of the regressions described in equation (2.6) in the case where $\tilde{\imath} = 0$ are given in appendix Table A.10. The results are similar to the ones reported above, although the significance of β_1 is diminished due to the presence of less observations to identify it.

[22] This 50 basis points difference is obtained both when taking a simple average for all the banks within a country and when using the model implied appropriate CES weighting (using an annual ε^d of -199).

One simple way to identify $\tilde{\imath}$ is to estimate the regressions in equation (2.6) for different possible threshold levels and then choose the one that minimizes the root mean squared error. This quantity (the RMSE) is shown in Figure 2.4 for different possible break levels between a 0% and a 1% annual level of the policy rate for the deposit rate and ROAE. The root mean squared error is minimized between 45 and 65 basis points for both variables. At those levels the t-stat for the interaction coefficient is greater than 2, and hence the null hypothesis of equal slope coefficients above and below the threshold is be rejected. Notice that even though the estimated thresholds for the deposit rate and ROAE are not exactly the same (the threshold is identified at 46 basis points for the deposit rate and 62 basis points for ROAE), they are very close.[23,24]

As pointed out by Hansen (1999), inference in the presence of an unknown threshold is complicated by the presence of a nuisance parameter, because the break point is not present under the null hypothesis. In Appendix A.2.5 I perform a test to identify the threshold based on Hansen's methodology. I also apply a test developed by Chay and Munshi (2015) that uses more information present on the deposit rate data in order to identify the threshold. Both methodologies find a threshold level remarkably close to 50 basis points. Since the model (together with the aggregate data) and the empirical tests all point to a value of $\tilde{\imath}$ that is close to 50 basis points, I will use that as my preferred estimate for $\tilde{\imath}$.

2.3.4 Deposit Channel Evidence

According to the model in Section 2.2, the reason banks are hurt more by a decline in the policy rate below $\tilde{\imath}$ is that they cannot pass it through to their depositors. In the static model all banks have the same amount of customer deposits (since all banks are identical), but in the data banks differ significantly along this dimension. Some banks finance themselves more through equity, bank deposits, or derivatives than through customer deposits. Hence, in the data, banks have different customer-deposits-to-assets (CDA) ratios, and it is possible to analyze how banks are affected by policy rates above and below $\tilde{\imath}$ according to their CDA ratio. My model predicts that banks with a high CDA ratio will be affected more by a decline in the policy rate below $\tilde{\imath}$ than banks with a low CDA ratio, but that both types of banks will be affected similarly above the threshold. To test this I split banks into quintiles

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[23] Appendix Figure A.3 shows the equivalent to Appendix Figure 2.4 once a lag of the dependent variable is included. In this case the threshold is found at exactly the same level of 48 basis points for both the deposit rate and ROAE. While this coincidence is predicted by the model, it is reassuring to find that it holds in the data.

[24] Even though this procedure is predicated on minimizing the RMSE, it will not always find a break point once it is combined with the analysis of the t-statistic for the interaction coefficient. One way to illustrate this is by running the same procedure for the loan rate, which the model predicts should not have a break around the threshold $\tilde{\imath}$. Appendix Figure A.4 displays the results of this test. The RMSE is minimized at 1, but throughout all possible threshold candidates the t-stat for the interaction coefficient is always below 2.

Table 2.3: Regressions to test the deposit channel

	(1) Baseline	(2) Q by Q
Policy Rate	5.004^{***}	
Quintile=1 × Policy Rate		2.653^{**}
Quintile=2 × Policy Rate		3.555^{***}
Quintile=3 × Policy Rate		4.013^{**}
Quintile=4 × Policy Rate		5.816^{***}
Quintile=5 × Policy Rate		8.026^{***}
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-4.194^{***}	
Quintile=1 × $(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$		-2.539^{**}
Quintile=2 × $(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$		-2.605^{**}
Quintile=3 × $(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$		-3.068^{**}
Quintile=4 × $(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$		-4.968^{***}
Quintile=5 × $(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$		-7.131^{***}
$\beta_1 + \beta_2$	0.810^{***}	
Quintile=1 × $(\beta_1 + \beta_2)$		0.113
Quintile=2 × $(\beta_1 + \beta_2)$		0.950^{***}
Quintile=3 × $(\beta_1 + \beta_2)$		0.945^{***}
Quintile=4 × $(\beta_1 + \beta_2)$		0.848^{***}
Quintile=5 × $(\beta_1 + \beta_2)$		0.895^{***}
N	80199	78710
R^2	0.407	0.417
Other FE	Year	Y-Q

Notes: Column (1) reports the results of the regression in equation (2.6) where the dependent variable is return on average equity (ROAE) and column (2) reports the results of the regression in equation (2.7). Quintile 1 includes banks with the lowest CDA ratio, while quintile 5 includes those with the highest. Clustering is done at the country-year level. Bank fixed effects are included. Stars: * for p<.10, ** for p<.05, and *** for p<.01.

according to their CDA ratio the first time I observe them in the panel.[25] Then I run the following regression:

$$ROAE_{b,t} \quad = \quad \alpha_b + \sum_{j=1}^{5} \left(\delta_t^j + \beta_1^j i_{c(b),t} + \beta_2^j (i_{c(b),t} - \tilde{\imath}) D_{c(b),t} \right) I_b^j + \varepsilon_{b,t}, \tag{2.7}$$

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[25] Since the panel is not balanced, simply taking the CDA ratio in 1990 would include very few banks in the sample. One alternative is to obtain a balanced panel between 1995 or 2000 and 2016 and take the CDA ratio in the first year of that panel. Another alternative is to take the average CDA ratio of each bank across years. Both of these options yield similar results. A description of the CDA ratio variable is given in Appendix Table A.11.

where $I_b^j \equiv \mathbb{1}(CDA_b \in Q_j)$ is an indicator that takes the value of one when bank b belongs to quintile j and zero otherwise. In this notation Q_j is the interval that includes all values of the CDA ratio that belong in quintile j. Notice that in this regression the value of the year fixed effect is allowed to vary across quintiles of the CDA ratio (so that there are five sets of time fixed effects). This is equivalent to running the regression in equation (2.6) quintile by quintile.

The results of this regression are given in column 2 of Table 2.3 (for comparison, column 1 replicates column 3 of Table 2.2). The coefficient β_1 increases monotonically across quintiles from 2.65 for the first quintile (banks with the lowest CDA ratio) to 8.02 for the last quintile (banks with the highest CDA ratio). This implies that the aggregate regression masks important heterogeneity across quintiles of the CDA ratio. By contrast, the coefficient for the policy rate above $\tilde{\imath}$, which is given by $\beta_1 + \beta_2$, is very similar for quintiles 2 through 5, at a level of between 0.85 and 0.95. These results conform well to the predictions of the model, and support the notion that having a high CDA ratio leads to a higher impact of the policy rate on ROAE below the threshold $\tilde{\imath}$, but a similar impact above the threshold.

In this section I estimated the parameter $\tilde{\imath}$ to be around 50 basis points, and successfully tested four predictions of the model in Section 2.2. First, once the policy rate falls below $\tilde{\imath}$, the deposit rate stops reacting to it. Second, the lending rate continues to decline with the policy rate even below $\tilde{\imath}$. Third, bank return on equity is more affected by a cut in the policy rate below $\tilde{\imath}$ than above it. Finally, the higher sensitivity of bank return on equity to the policy rate below $\tilde{\imath}$ is more pronounced for banks that rely heavily on customer deposits for funding.

2.4 The Extended Model

The model in Section 2.2 serves to convey useful intuitions but, due to its partial equilibrium nature, cannot speak to the overall effectiveness of setting NNIR. In this section, I develop a richer, general equilibrium, dynamic model where bank equity matters. This will provide a useful laboratory to study the effects of NNIR on the economy, taking into consideration its effects on bank profitability.

There are five types of agents in the model: households, intermediate goods producers, capital producers, retailers, and banks. In addition, there is a government, and a central bank that conducts monetary policy. The model for the capital producers, intermediate good producers, and retailers builds on features from GK, while the model for the banks is a more complicated version of the one described in Section 2.2.

Figure 2.5 illustrates the relationships in the model. Banks are depicted in red, households in green, and firms related to goods (capital good producers, intermediate good producers, and retailers) in blue. Relationships are depicted with directional arrows. Financial relationships

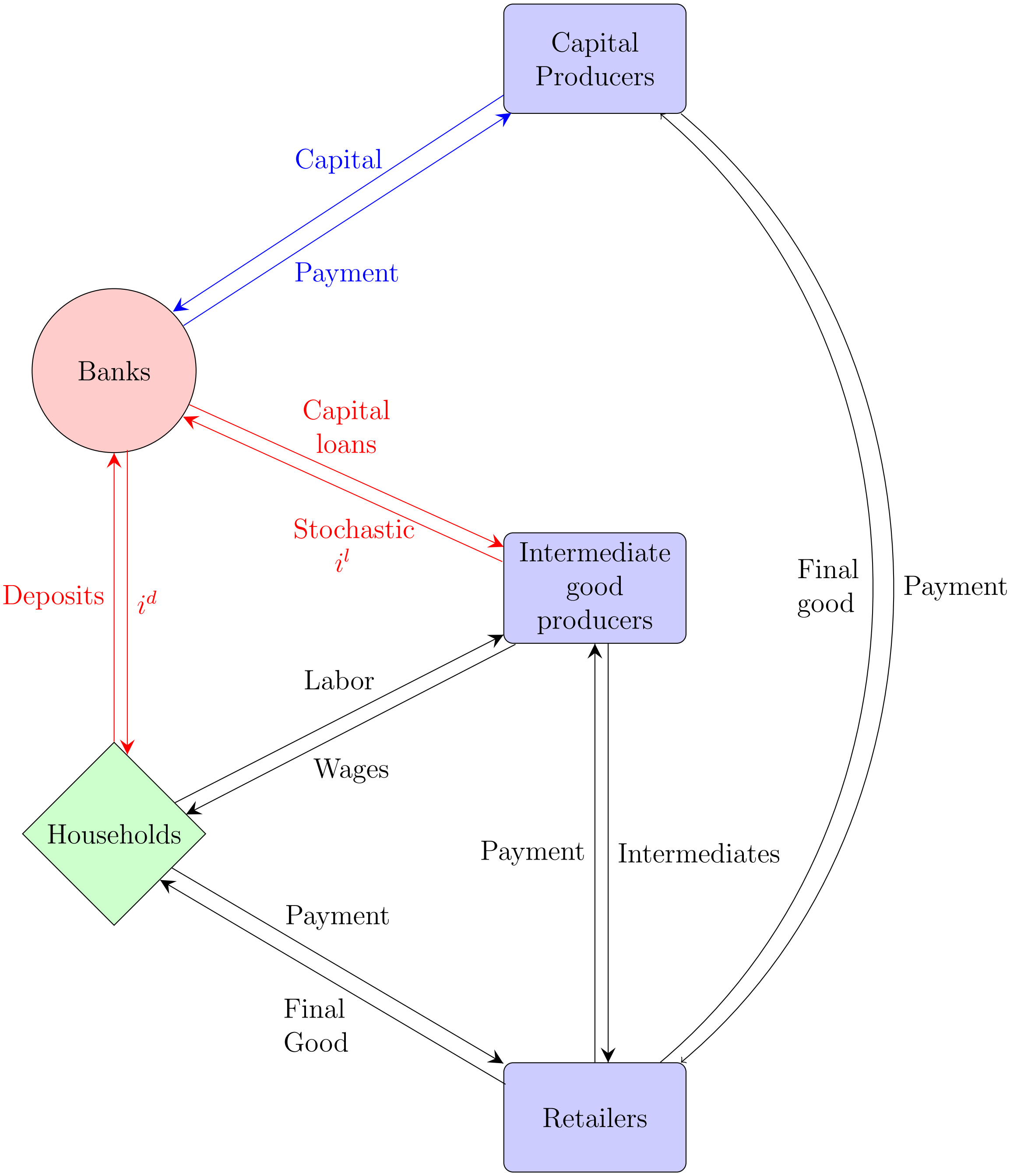

Figure 2.5: Relationships in the full model

Notes: This figure provides a diagrammatic depiction of the relationships in the extended model. Red lines denote financial flows. Blue lines denote capital flows. Black lines depict flows of goods or labor. The flow of profits is not included in the figure.

are in red, capital flows in blue, and other relationships in black. Households deposit money in banks, in return they obtain a deterministic rate i^d. Banks use their equity and deposits from households to buy capital from capital good producers. They lend this capital to intermediate good producers, who in return pay them a stochastic dividend. Intermediate good producers obtain labor from households and pay them a wage in return, they also sell their output to retailers, who differentiate it and set rigid prices. Retailers sell the final good to households and capital producers. The profits of capital good producers and retailers go directly to households. The destination of the profits of intermediate good producers and banks will be described in detail below. The government sector is not included in the diagram for simplicity, but will be explained below.

Retailers are included in order to introduce price rigidity into the model in a tractable way, and they are kept separate from intermediate good firms to avoid complications related to firm specific capital described in Woodford (2005) and Sveen and Weinke (2005).[26] Capital good producers are introduced to be able to have a price of capital that is not fixed at unity, by giving them the capital adjustment costs without encumbering the intermediate good producer's problem with these costs. Having several sectors and adding realistic features, like habit formation and investment adjustment costs, allows the model to capture business cycles in a realistic way, in the tradition of papers like Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007).[27] Since one of the objectives of this paper is a quantitative analysis of the welfare impact of setting negative nominal interest rates, it is important to have a model that is rich enough to match quantitatively the behavior of real-world economies.

2.4.1 Households

There is a continuum of households of measure one. Each household consumes, saves and supplies labor. They save by depositing their money in a continuum of banks, or by holding cash. Household's preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \varphi_t \left(\frac{(C_t - hC_{t-1})^{1-\sigma} - 1}{1-\sigma} - \chi \frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right), \tag{2.8}$$

with $0 < \beta < 1$, $0 < h < 1$, and σ, χ, $\eta > 0$. β is the discount factor, σ is the inverse of the intertemporal elasticity of substitution, χ governs the importance of labor in the utility function, η is the Frisch elasticity of labor supply, and φ_t is a shock to the discount factor.

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[26] Having a rental market for capital instead of firm specific capital would also suffice, but since the market for capital is crucially interrelated to banks, it is simpler to keep retailers separate from intermediate good firms.

[27] A version of the model that also includes variable capital utilization and a different price for new and refurbished capital is available from the author upon request. The results are similar to the baseline model.

C_t is consumption and N_t is labor supply. I allow for habit formation in the consumption behavior of households, captured by the parameter h. Household's deposits in banks are one period nominal contracts that pay the gross nominal interest $(1+i^d_{t-1})$ from $t-1$ to t. Let D_t be the total quantity of deposits that the household lends to banks from period t to period $t+1$, M_t be the amount of cash that households have in period t, W_t be the nominal wage, Π_t be the net nominal payouts to the household from ownership of both nonfinancial and financial firms, and T_t be nominal lump sum taxes. Then the household's budget constraint is given by

$$P_t C_t + D_t + M_t = W_t N_t + \Pi_t - T_t + (1 + i^d_{t-1}) D_{t-1} + M_{t-1}. \tag{2.9}$$

The household's optimality conditions are standard and are given in Appendix A.3.1.

2.4.2 Intermediate Goods Firms

On the production side of the economy nonfinancial firms make intermediate inputs using capital and labor. At the end of period $t-1$, an intermediate goods firm borrows an amount of capital K_t from the banks for use in production during period t. After using capital to produce intermediate goods during t, the firm returns the capital to the bank. There are no capital adjustment costs at the intermediate good producer level, since they simply rent capital.

Let Y^m_t be the amount produced of intermediate goods, K_t be capital, A_t denote total factor productivity and ξ_t denote the quality of capital. The production function is given by

$$Y^m_t = A_t(\xi_t K_t)^\alpha N_t^{1-\alpha}. \tag{2.10}$$

Let P^m_t be the price of intermediate goods output. Then at time t, the firm chooses labor to maximize nominal profits, which are given by

$$\Pi^m_t = P^m_t Y^m_t - W_t N_t - Z_t K_t.$$

These are nominal profits at time t because the firm produces Y^m_t and obtains a price P^m_t for each of those units. It pays W_t to each worker and borrows capital from financial intermediaries. In particular, it borrowed K_t units of effective capital (in the previous period) and pays a dividend of Z_t to each of those units. The optimality condition with respect to labor is

$$(1-\alpha)\frac{P^m_t}{P_t}\frac{Y^m_t}{N_t} \quad = \quad \frac{W_t}{P_t}. \tag{2.11}$$

The dividend Z_t ensures that the profits of intermediate firms are equal to zero:

$$Z_t \quad = \quad P^m_t \alpha \frac{Y^m_t}{K_t}.$$

Consequently, the stochastic, nominal, gross return for banks of having a unit of effective capital is

$$1 + i^l_{t+1} \quad = \quad \frac{Q_{t+1}\xi_{t+1}(1-\delta) + P^m_{t+1}\alpha\frac{Y^m_{t+1}}{K_{t+1}}}{Q_t}. \tag{2.12}$$

Notice that there are no financial frictions between intermediate good firms and banks, that is why intermediate good producers transfer all their residual stochastic returns to banks. This setup, where banks are the residual claimants of intermediate good firms, is used in GK. Additionally, this approach is motivated by two considerations. First, this is meant to capture the Great Recession, where the originating shock, a fall in housing prices, had an important negative effect on bank equity. In my model, the shock originating the recession will be a fall in capital efficiency (ξ_t); if banks just loaned money to intermediate good firms at a deterministic rate this shock would have no major effect on bank equity, and this is not consistent with the experience during the Great Recession. Second, even if banks lend money to firms at a "deterministic" loan rate, the fact that firms might default means that banks end up absorbing some of the risk of intermediate good firms. This could be introduced through a probability of default for intermediate good firms, but that would unnecessarily complicate the analysis.

2.4.3 Capital Producers

The process of producing new capital is subject to flow adjustment costs. The value of newly produced capital is Q_t. Let I_t be investment, then capital evolves according to

$$K_{t+1} \quad = \quad (1-\delta)\xi_t K_t + I_t.$$

Discounted real profits for a capital-producing firm are

$$\max \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t}\Lambda_{t,\tau}\left\{\left(\frac{Q_\tau}{P_\tau} - 1\right) I_\tau - f\left(\frac{I_\tau}{I_{\tau-1}}\right) I_\tau\right\},$$

where $\Lambda_{t,\tau}$ is the household's stochastic discount factor between periods t and τ (excluding the discount factor β). Following Christiano et al. (2005) or GK, f is a function that represents the costs of adjusting the level of investment and that satisfies $f(1) = f'(1) = 0$ and $f''(1) > 0$.[28] The first order condition for investment, which determines Q_t/P_t, is given in Appendix A.3.1.

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[28] I use $f(x) = \zeta\frac{(x-1)^2}{2(1+\beta)}$. The derivative of the price of capital w.r.t. investment evaluated at the S.S. is ζ.

2.4.4 Retail Firms

Each retail firm uses intermediate inputs and costlessly transforms them into a differentiated variety of a retail good. These varieties are aggregated to a final good via a CES aggregator:

$$Y_t = \left(\int_0^1 Y_t(s)^{\frac{\theta-1}{\theta}} \, ds \right)^{\frac{\theta}{\theta-1}}.$$

Demand for a particular differentiated good and the price index are given by

$$Y_t(s) = \left(\frac{P_t(s)}{P_t} \right)^{-\theta} Y_t, \qquad\qquad P_t = \left(\int_0^1 P_t(s)^{1-\theta} \, ds \right)^{\frac{1}{1-\theta}}.$$

As in the traditional Calvo setup, a firm is able to freely adjust its price with probability $1-\gamma$. Thus, the pricing problem of retail firm s is to choose the optimal reset price $P_t^*(s)$ to solve:

$$\max \;\; \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \Lambda_{t,t+r} \frac{P_t}{P_{t+r}} \left[P_t^*(s) - P_{t+r}^m \right] Y_{t+r}(s).$$

The optimality conditions describing the behavior of retail firms are given in Appendix A.3.1.

2.4.5 Banks Redux

The behavior of banks is similar to the one described in Section 2.2, but I introduce four modifications to make the framework richer, more realistic, and easier to match with the other elements of the extended model. I describe each of these changes sequentially.

First, banks are subject to a cost of deviating from a target level of loan-to-equity ratio. The bank pays a quadratic cost (parameterized by a coefficient κ and proportional to outstanding bank equity) whenever the loan-to-equity ratio, $L_t(j)/F_t(j)$, deviates from the target value ν. The part of the quadratic cost to the right of ν is motivated by the fact that regulators will increasingly discourage high levels of leverage. The part to the left of ν is motivated by the fact that investors will punish banks if they have too little leverage, and that bank managers are sometimes rewarded by the gross amount of money they manage. The quadratic cost is a modeling shortcut to capture the fact that bank capital is important in a tractable way, a common choice that has been adopted in several papers. e.g. Gerali, Neri, Sessa, and Signoretti (2010), Campbell (1987), and Drechsler et al. (2017).[29]

Second, I allow banks to face exogenous costs of issuing loans, given by μ_t^l, and benefits of

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[29] The fact that bank equity matters can also be introduced as in Gertler and Karadi (2011), but the quadratic cost is more tractable and easier to implement, specially once the banks have monopoly power and endogenous reserves.

issuing deposits, given by μ_t^d. These are expressed per dollar of loan or deposit issued. The cost of issuing loans is positive (the bank has to monitor the borrowers, pay loan originators, etc), while the cost of issuing deposits could be negative, because it could be seen as a benefit that the bank receives for having a large deposit base, for example attracting more customers or obtaining more publicity (that is why I will depict them as a benefit in my notation).[30]

Third, I allow for the fact that banks can receive a stochastic return from firms, and this can affect their ROE. Banks will not set a deterministic loan rate, but will instead charge each firm a fraction of its total return on capital. In equilibrium, since all banks are symmetric, this fraction will be one, but banks will still face a well defined demand for "loans". The loan return of banks between periods t and $t+1$ will be determined in period $t+1$ and it will contain expectations. This stochastic loan setup is described in Appendix A.1.6. As mentioned in Section 2.4.2, the reason to introduce stochastic loan returns for banks is so that the fall in capital efficiency (which will give rise to the recession I will analyze) has a significant impact on bank equity. This is meant to mimic the Great Recession, where the originating shock affected bank balance sheets substantially.

Finally, I also assume that each period a fraction ς of nominal bank net worth is used up operating the managerial side of the bank. This, together with the fact that banks cannot frictionlessly obtain the optimal amount of equity from households, implies that bank equity is relevant and can take a long time to replenish. With all these assumptions the nominal resources that bank j will have next period (denoted $S_{t+1}(j)$) are given by

$$\begin{aligned} S_{t+1}(j) &= (1+i_{t+1}^l(j)-\mu_t^l)L_t(j)+(1+i_t)H_t(j)-(1+i_t^d(j)-\mu_t^d)D_t(j) \\ &- \varsigma F_t(j)-\frac{\kappa}{2}\left(\frac{L_t(j)}{F_t(j)}-\nu\right)^2 F_t(j). \end{aligned}$$

The bank balance sheet constraint can be used to rewrite this as

$$\begin{aligned} S_{t+1}(j) &= (1+i_t-\varsigma)F_t(j)+(i_{t+1}^l(j)-\mu_t^l-i_t)L_t(j) \\ &+ (i_t+\mu_t^d-i_t^d(j))D_t(j)-\frac{\kappa}{2}\left(\frac{L_t(j)}{F_t(j)}-\nu\right)^2 F_t(j), \end{aligned}$$

which is an extension of equation (2.5), and encapsulates the same 3 ways of making profits, plus the cost of deviating from the target level of loan-to-equity ratio, the exogenous costs of issuing loans and deposits, and the managerial cost of operating the bank. In Appendices A.1.5 and A.1.6 I show that, with the changes mentioned above for the banking sector, the loan rate is given by:

$$\mathbb{E}_t(1+i_{t+1}^l) = \frac{\varepsilon^l}{\varepsilon^l-1}(1+i_t+\mu_t^l)+\kappa\frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{L_t}{F_t}-\nu\right). \tag{2.13}$$

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[30] The reasons to introduce μ_t^l and μ_t^d are to be able to decouple $\tilde{\imath}$ from ε^d, to give the model more flexibility looking forward to the calibration, and to potentially have exogenous variation in intermediation costs.

This is similar to the expression in Section 2.2, with three changes:

1. The loan rate is set as a mark-up over the gross policy rate plus the cost of issuing loans.
2. The amount of bank equity is now relevant; if the loan-to-equity ratio is higher than its target then the expected loan return required by the bank is higher. This occurs because the bank wants to disincentivize lending, in order to lower its leverage.
3. The expression contains expectations due to the stochastic nature of the loan return.

The expression for the deposit rate is given by

$$1 + i_t^d = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i_t + \mu_t^d),$$

when $\tilde{\imath} < i$. This is the same expression in Section 2.2, except for the appearance of the benefit of issuing deposits (μ_t^d). Once the policy rate falls below $\tilde{\imath}$ banks either set a zero deposit rate, or set a negative deposit rate and receive no deposits. Appendix A.1.5 defines the thresholds $\tilde{\imath}$ and $\underline{i}$ for this extended model.

Next, I describe the way bank capital evolves over time. Denote nominal resources net of management costs, previous period equity, and an adjustment for inflation, by X_{t+1}:

$$\begin{aligned} X_{t+1}(j) &\equiv i_t F_t(j) + (i_{t+1}^l(j) - \mu_t^l - i_t)L_t(j) + (i_t + \mu_t^d - i_t^d(j))D_t(j) \\ &- \frac{\kappa}{2}\left(\frac{L_t(j)}{F_t(j)} - \nu\right)^2 F_t(j) - F_t(j)(1-\varsigma)\pi_{t+1}. \end{aligned} \tag{2.14}$$

Notice that this is not a traditional accounting concept, it is simply an intermediate object that will be useful to describe the sluggish evolution of bank capital under the specific framework chosen in this paper. Later on it will become clear why I chose to include the adjustment for inflation in equation (2.14). With this definition of X_{t+1}, next period resources of a bank can be described as

$$S_{t+1}(j) = (1-\varsigma)F_t(j)(1+\pi_{t+1}) + X_{t+1}(j),$$

I assume that a fraction ω of $X_{t+1}(j)$ is kept in the bank and the remaining fraction $1-\omega$ is distributed to the households as dividends. Nominal bank net worth next period is then given by

$$F_{t+1}(j) = (1-\varsigma)F_t(j)(1+\pi_{t+1}) + \omega X_{t+1}(j), \tag{2.15}$$

while the bank pays dividends: $DIV_{t+1}^B(j) = (1-\omega)X_{t+1}(j)$; hence, the sum of all the resources in the bank is used either for accumulating net worth or distributing dividends:

$S_{t+1}(j) = F_{t+1}(j) + DIV^B_{t+1}(j)$, and real bank resources in period $t+1$ are given by

$$\frac{F_{t+1}(j)}{P_{t+1}} = (1-\varsigma)\frac{F_t(j)}{P_t} + \omega\frac{X_{t+1}(j)}{P_{t+1}}. \tag{2.16}$$

It should be emphasized that this particular specification for the evolution of bank capital is not crucial for the implications of the model, the important feature is that it captures the idea of "slow moving" capital, in the sense that banks cannot simply obtain their ideal level of capital frictionlessly.[31] The specific form I have chosen emphasizes the idea that ω governs the effect that "profits" (i.e., X_{t+1}) have on a bank's real resources. If $\omega = \varsigma = 0$, then a bank's real resources are constant, in the sense that they are not affected by any shocks; this is why I chose to incorporate the adjustment for inflation in equation (2.14). The higher ω, the higher the fraction of fluctuations in bank's profits that have to be absorbed by banks themselves.

2.4.6 Resource Constraint, Policy and Shocks

Output is divided between consumption, investment, government expenditure, G_t, and adjustment costs. The economy-wide resource constraint is thus given by

$$Y_t = C_t + I_t + G_t + f\left(\frac{I_t}{I_{t-1}}\right)I_t + \mu^l_t\frac{L_{t-1}}{P_t} - \mu^d_t\frac{D_{t-1}}{P_t} + \varsigma\frac{F_{t-1}}{P_t} + \frac{\kappa}{2}\left(\frac{L_{t-1}}{F_{t-1}} - \nu\right)^2\frac{F_{t-1}}{P_t}, \tag{2.17}$$

additionally, total loans by banks have to equal the total value of capital:

$$L_t = Q_t K_{t+1}. \tag{2.18}$$

I assume monetary policy is characterized by a Taylor rule with interest-rate smoothing. Let i_t be the net nominal interest rate and $\bar{\imath}$ the steady state nominal rate, then

$$i_t = (1-\rho_i)\left(\bar{\imath} + \psi_\pi(\pi_t - \overline{\pi})\right) + \rho_i i_{t-1} + \epsilon^i_t, \tag{2.19}$$

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[31] ς is chosen so that there is a well defined level of bank equity in steady state (where $L/F = \nu$):

$$\varsigma = \omega\left(i + (i^l - \mu^l - i)\nu + (i + \mu^d - i^d)\frac{D}{F}\right).$$

This equation can be interpreted as determining ς for a given level of ω (as well as i, i^l, i^d, ν, D/F, μ^l and μ^d), or as determining ω for a given level of ς. It is based on the requirement than in steady state there is a constant level of bank equity consistent with the law of motion for bank equity given in the text.

where $\rho_i \in [0,1]$, and where ϵ_t^i is an exogenous shock to monetary policy.[32] The processes for the shocks are described in Appendix A.3.1. Technology, discount factor, and government shocks are standard in dynamic New Keynesian models, but will not be emphasized in this paper. The capital efficiency shock will be used to generate the recession which is the object of study. Finally, shocks to reserves are introduced to capture the fact that during the Great Recession most central banks increased their balance sheet by an order of magnitude, and this led to a big increase in the amount of reserves held by commercial banks. This is important because these reserves were later subjected to NNIR. In the main exercise I will keep the level of reserves fixed after the recession, but I will explore different scenarios in extensions. In the baseline model, the exogenous costs and benefits of issuing loans and deposits will be kept constant, i.e., $\mu_t^l = \mu^l$ and $\mu_t^d = \mu^d$.

The equilibrium is characterized by the relevant equations for each of the types of agents in the model, collected in Appendix A.3.1. Appendix A.3.2 describes the steady state of the model.

2.5 Calibration

Given that the objective of this paper is to have a quantitative framework to study the effects of NNIR on the economy, calibrating the values of the parameters in the model is very important. Since the contribution of this paper is concentrated in the banking sector, the parameters in the financial block of the model are the ones that require more discussion, as well as the ones where less inference can be drawn from the literature. I first focus on estimating the value of κ, and then turn to the remaining parameters.

2.5.1 Importance of Bank Equity for Lending

Recall that κ measures the impact of deviating from the target level of the loan-to-equity ratio on the objective function of the bank, and hence also on its lending rate and amount of loans extended. Therefore, a way to learn about κ is by using the cross-section, and studying how banks with different levels of equity differ in their lending behavior.

In the model so far all banks have been homogeneous, which makes it hard to understand the effects of equity on lending. I will now develop a simple model with bank heterogeneity. The current specification for the cost of deviating from the target level of loan-to-equity ratio, $-\kappa/2\left(L_j/F_j - \nu\right)^2$, does not allow for a closed form solution to the heterogeneous

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[32] Lump sum transfers from the government to consumers include the proceeds from seignorage (both base money and reserves at the central bank) and subtract government expenditure in goods:

$$T_t = M_t - M_{t-1} + H_t - (1 + i_{t-1})H_{t-1} - P_t G_t.$$

bank problem. This motivates a change to the following logarithmic specification:

$$-\kappa\nu\frac{L_j}{F_j}\left(\ln\left(\frac{L_j}{F_j}\right)-\ln\nu-1\right)-\kappa\nu^2,$$

which is convenient because it allows the heterogeneous bank model to be solved in closed form. In Appendix A.1.7 I prove that the quadratic specification used so far is the second-order approximation to the logarithmic one around the steady state, which implies that both specifications deliver virtually the same solution to the model with homogeneous banks.[33] I also show that, with the logarithmic specification for the cost of deviating from the target level of leverage, the solution for bank-level log loan amount and loan rate in terms of log bank equity can be written as

$$\begin{aligned} i^l_j &= \alpha+\beta i-\frac{\kappa\nu}{1+\kappa\nu\varepsilon^l}\ln(F_j) \\ \ln(L_j) &= \alpha'+\beta' i+\frac{\kappa\nu\varepsilon^l}{1+\kappa\nu\varepsilon^l}\ln(F_j), \end{aligned}$$

where the expressions for α, α', β, and β' are given in Appendix A.1.7. Thus, a regression of i^l_j on a constant, the policy rate, and $\ln(F_j)$, yields a coefficient on log bank equity of $-\frac{\kappa\nu}{1+\kappa\nu\varepsilon^l}$; and a regression of $\ln(L_j)$ on a constant, the policy rate, and $\ln(F_j)$, yields a coefficient on log bank equity of $\frac{\kappa\nu\varepsilon^l}{1+\kappa\nu\varepsilon^l}$. From these two regressions it is possible to back-out two coefficients, $\kappa\nu$ and ε^l. Denote the coefficient on log bank equity on the loan rate regression by γ_{lr}, and the one on the log loan amount regression by γ_{la}, then

$$-\frac{\gamma_{la}}{\gamma_{lr}}=\varepsilon^l=-\frac{\operatorname{Cov}(\ln(L_j),\ln(F_j))}{\operatorname{Cov}(i^l_j,\ln(F_j))}, \qquad \frac{\gamma_{lr}}{\gamma_{la}-1}=\kappa\nu=\frac{\operatorname{Cov}(i^l_j,\ln(F_j))}{\operatorname{Cov}(\ln(L_j/F_j),\ln(F_j))}.$$

Hence, these two regressions can be used to obtain estimates of $\kappa\nu$ and ε^l jointly. When actually estimating these regressions in the data, it is important to include lags of the dependent variable, since there appears to be sluggishness in loan rates and loan amounts (since the data I have is on total loans outstanding and not on newly issued loans). It is also important to have bank fixed effects that control for time-invariant bank level characteristics (other than equity) that lead to differences in loan rate or log loan amount. Consequently, I run the following regressions:

$$y_{b,t} = \alpha_b+\beta i_{c(b),t}+\eta_1 y_{b,t-1}+\eta_2 y_{b,t-2}+\gamma\ln(F_{b,t-1})+\varepsilon_{b,t}, \tag{2.20}$$

where the dependent variable ($y_{b,t}$) is either the log loan amount or the loan rate of an individual bank. The two parameters of interest are the two γ's (γ_{lr} when $y_b=i^l_b$ and γ_{la} when $y_b=\ln(L_b)$), which are the coefficients on the log level of lagged equity. In the

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[33] The quadratic specification has been used up to this point to convey intuitions easily and to facilitate comparison with other papers, but it would make no difference if the logarithmic specification had been used all along.

Table 2.4: Structural estimation of κ and ε^l, part 1

	(1) USD	(2) JPY	(3) EUR	(4) CHF	(5) GBP
γ_{la}	0.5357	0.3757	0.3833	0.7360	0.4208
γ_{lr}	-0.0203	-0.0094	-0.0102	-0.0072	-0.0107
κ	0.0049	0.0017	0.0018	0.0030	0.0021
ε^l	26.4049	40.0239	37.5096	102.5313	39.1907
$\mathrm{i}^l - i$	0.0386	0.0253	0.0270	0.0098	0.0258

Notes: This table contains the results of the country-level structural estimation of κ and ε^l described in equation (2.20). It contains the 5 largest regions in terms of amount of banks present in the sample: USA (USD), Japan (JPY), the Euro Area (EUR), Switzerland (CHF), and UK (GBP). Table A.13 in the appendix contains the remaining 5 regions.

theoretical framework the log level of equity was dated t, as were the log loan amount and loan rate. It is important to keep in mind that in that framework equity in period t was predetermined, while the loan amount and the loan rate were endogenous. In my data this cannot be guaranteed, so I lag equity by one period.[34] Since the regression includes lags of the dependent variable, I can lag equity more than one period, or instrument it with additional lags of itself, to avoid endogeneity concerns. Given the inclusion of lags of the dependent variable, the relevant coefficient for calculating $\kappa\nu$ and ε^l is not γ, but $\frac{\gamma}{1-\eta_1-\eta_2}$ instead.

Appendix A.2.7 contains the results of the regressions in (2.20) for the full sample, and its implications for the values of ε^l and κ, for different specifications of the regression equation. The baseline specification contains two lags of the dependent variable, as equation (2.20), but instruments $F_{b,t-1}$ with $F_{b,t-3}$ to avoid endogeneity concerns. The regressions yield an estimate of ε^l of 45, which implies an annual spread between the loan rate and the policy rate of about 2.3%, and an estimate of κ of 20 basis points. The estimates for ε^l are realistic, since they imply loan spreads that are close to those observed in the data (i.e. between 2% and 4%).

The heterogeneous bank model allows me to obtain region-specific estimates for κ and ε^l, this will help answer the question of how efficient NNIR are in each region. The results for the biggest regions, using the baseline specification described above, are given in Table 2.4, while the ones for smaller countries are given in appendix Table A.13. The estimates of κ range between 17 basis points for Japan and 50 basis points for the U.S., while the estimates of ε^l are between 26 for the U.S. and 100 for Switzerland. I use these parameter estimates

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[34] Since data is annual, it is hard to argue that equity in a particular year is independent from the loan rate and the loan amount in that same year.

to inform my calibration.

2.5.2 Additional Parameter Values

Table 2.5 describes the calibrated parameter values. Most of the parameters in the blocks pertaining the households, the intermediate good firms, and the capital producing firms are taken directly from GK. The values in the retail block are standard.

The average value for the annual level of the loan rate, the policy rate, and the deposit rate in my database are 6%, 3% and 2.5% respectively. The quarterly value of 0.9937 for β delivers a value of i^d in steady state of 2.5% at the yearly level (0.62% at a quarterly level). $\varepsilon^d = -268$ and $\mu^d = 0.25\%$ then imply a value of i of 3% at a yearly level (0.75% at the quarterly level). Finally $\varepsilon^l = 203$ and $\mu^l = 0.25\%$ then imply a value of i^l of 6% at a yearly level (1.5% at the quarterly level). For the baseline calibration I assume that the μ's and ε's are constant. In this calibration the spread between the policy rate and the deposit rate $(i - i^d)$ is 0.5% annually, the spread between the lending rate and the policy rate $(i^l - i)$ is 3% annually, and the value for $\tilde{\imath}$ is 0.5%, consistent with the evidence presented in Section 2.3.3.

With the gross rate specification for loan demand and deposit supply, reproduced here:

$$L(j) = \left(\frac{1 + i^l(j)}{1 + i^l}\right)^{-\varepsilon^l} L, \qquad D(j) = \left(\frac{1 + i^d(j)}{1 + i^d}\right)^{-\varepsilon^d} D,$$

or the stochastic version of loan demand in the extended model (described in Appendix A.1.6), the elasticities of substitution, ε^d and ε^l, depend on the time horizon. Specifically, the annual elasticities are a fourth of the quarterly elasticities. Hence, the annual levels of these elasticities in the baseline calibration are $\varepsilon^d = -268/4 = -67$ and $\varepsilon^l = 203/4 \approx 50$. The value of ε^l used at the annual level (≈ 50) is very close to the one estimated in Section 2.5.1 (≈ 45).

Given my specification for the evolution of real bank equity described in equations (2.14) and (2.16), the effect of a change in the return on capital (i^l_{t+1}) on the percentage change on bank equity is $\omega\nu$. I choose ω so that the total effect $(\omega\nu)$ is equal to one, since ν is calibrated to 9, this implies that $\omega = 1/9 \approx 0.1111$. This is a normalization, and changing it does not affect the quantitative predictions of the model.[35] The managerial cost of operating the bank, ς, is chosen to be consistent with steady state, this gives a quarterly cost of operating the bank of 1% of equity. ν is chosen to be 9, which is the mean loan-to-equity ratio in my dataset. Likewise, $\overline{H}/\overline{F}$ is chosen to match the 20% reserve-to-asset ratio in my database.

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[35] Provided that the shock originating the recession is also modified to keep the total effect of the shock constant. A greater ω implies that a given shock to ξ (capital productivity) has a greater impact on banks and hence also on the overall economy.

Table 2.5: Calibrated parameter values

Parameter	Value	Description	Target or source
Households			
β	0.9937	Discount rate	Ann. dep. rate of 2.5%
h	0.8150	Habit formation	GK
χ	3.4090	Importance of leisure	GK
η	1.0000	Frisch elasticity of labor supply	Chetty et al. (2011)
σ	1.0000	Inverse of the I.E.S.	Balanced Growth
Intermediate good firms			
α	0.3333	Capital share	GK
δ	0.0250	Depreciation rate	GK
Capital producing firms			
ζ	1.7280	Elasticity of Q to investment	GK
Retail firms			
θ	6.0000	Elasticity of subs. among goods	S.S. mark-up of 20%
γ	0.7500	Prob. of keeping prices fixed	1 year average price spell
Financial intermediaries			
ω	0.1111	Fraction of resources staying in bank	Normalization
ς	0.0100	Bank managerial cost	Consistent with S.S.
ν	9.0000	Loan-to-equity ratio target	Average in my dataset
κ	0.0012	Cost of deviating from leverage target	Estimation
ε^d	-268	Elasticity of substitution for deposits	Ann. policy rate of 3%
ε^l	203	Elasticity of substitution for loans	Ann. lending rate of 6%
μ^d	0.25%	Benefits of issuing deposits	Ann. policy rate of 3%
μ^l	0.25%	Cost of issuing loans	Ann. lending rate of 6%
$\overline{H}/\overline{F}$	2.0000	Reserves over Equity in S.S.	Average in my dataset
Government			
ψ_π	3.5000	Inflation coefficient, Taylor rule	Suggestive
ρ_i	0.8000	Smoothing parameter, Taylor rule	Standard parameter
g	0.2000	Steady state G/Y	GK

Notes: This table contains the parameter values used in the calibration, together with their description and the source where they are taken from or the objective they target.

The most important parameter of the model is κ. The reason this parameter is so important is that, when deciding whether to set NNIR, the central bank has to weigh the fact that it will affect bank profits more than usual. How much bank's profits matter is governed by the importance of deviating from the target level of loan-to-equity ratio, namely κ. This is the parameter that was estimated in Section 2.5.1, there I obtained the value of 50 basis points for the United States at the annual frequency, but this value has to be divided by 4 to convert it into the quarterly frequency. That is why 12.5 basis points was chosen as the baseline value for κ. The full sample delivered a value of $\kappa = 20$ basis points at the annual frequency, which translates to 5 basis points at the quarterly frequency. Since this parameter is so important, I will illustrate the effectiveness of monetary policy for different values of κ in Section 2.7, and I will relate this to the estimates of κ obtained in Section 2.5.1 for different countries.

Now I explore how economically important the cost of deviating from the target level of leverage might be. Consider a change in leverage from 9 (the level in steady state) to 9.9, i.e., an increase of 10%. This decreases available resources next period, via the cost of deviating from target leverage, by a magnitude of:

$$\frac{\kappa}{2}\left(\frac{L}{F}-\nu\right)^2 F = \frac{\kappa}{2}\cdot 0.9^2 \cdot F.$$

By dividing this by F I obtain the change in return on equity from one period to the next. But this is given at a quarterly frequency, so I multiply it by 400 to turn it into annual percentage terms. This means the change in ROE due to the change in leverage is given by $162 \cdot \kappa$, so an increase in leverage of 10% decreases bank return on equity by around 8 basis points annually when κ is 5 basis points, 20 basis points when κ is 12.5 basis points, and 40 basis points when κ is 25 basis points. In GNSS an equivalent exercise would change annual percentage return on bank equity between 5 and 25 basis points, which is close to the range I consider.[36]

Another way to judge the significance of κ in my model is to analyze its effects on the loan rate (instead of on ROE). In the absence of uncertainty and μ^l, the loan rate is given by $1 + i^l = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i + \kappa(L/F - \nu))$, and so an increase in leverage of 10% would lead to an increase in the loan rate of 50 basis points at the annual frequency, this is essentially the moment that was used to identify κ in the regressions done in Section 2.5.1.

I set the response of the policy rate to inflation in the Taylor rule (ψ_π), to 3.5, which is higher than the traditional value of 1.5. I do this because having a higher response to inflation can help the stability properties of the model when the economy hits the ZLB. Changing this value does not have big quantitative implications for the model, provided the size of the shock originating the recession is adjusted to keep the effect on output constant. The value

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[36] The actual specification in GNSS is different, in my notation it would be $\frac{\kappa^G}{2}\left(\frac{F}{L}-\nu^G\right)^2 F$, with calibrated values of $\kappa^G \approx 12$ and $\nu^G \approx 0.09$, this means leverage in steady state is around 11, so an increase of 10% corresponds to a change in F/L from 0.09 to 0.082 and in the return on equity in annualized percentage terms of $\frac{\kappa^G}{2}(0.0818 - 0.09)^2 \cdot 400 \approx 0.16\%$ (16 basis points). Their 2.5% to 97.5% range for κ^G is 4 to 18, which leads to the range of 5 to 25 basis points for the costs of the 10% change in leverage reported above.

of $\rho_i = 0.8$ is standard.

2.6 Recession Under a Taylor Rule

I now analyze how the model economy behaves under three different scenarios. The scenario which has been emphasized so far in this paper is the one where the policy rate can be negative but the deposit rate is constrained to being nonnegative, this is denoted the "Modified ZLB" scenario. I also analyze two scenarios that are more traditional in the literature, the "No ZLB" scenario, where the policy rate and the deposit rate are both unconstrained, and the "Traditional ZLB" scenario, where both the policy and deposit rates are constrained to being nonnegative.

In the No ZLB scenario I log-linearize the model and solve it using traditional methods. In the case of the Modified ZLB scenario I solve the model using the methodology described in Guerrieri and Iacoviello (2015), since the ZLB on the deposit rate represents an occasionally binding constraint. This methodology log-linearizes the model in a piece-wise fashion (one piece when the constraint binds and the other piece when it does not), and then uses perturbation methods to find the period where the economy transitions from one regime to the other. In the case of the Traditional ZLB scenario the same methodology is used, but now there are two occasionally binding constraints, the deposit rate ZLB and the policy rate ZLB.

I study the response of the model economy after a shock to capital productivity; ξ_t falls by 2.5% and this shock is relatively long lived ($\rho_\xi = 0.9$). In the No ZLB scenario this shock will generate a fall in output of roughly 3.5%, a fall in the policy rate, which remains negative for roughly 6 quarters, a fall in the deposit rate, which remains negative for roughly 8 quarters, and a fall in the net worth of financial intermediaries.

Figures 2.6 and 2.7 display the impulse response functions of some the most important variables in the model to the capital productivity shock under the three scenarios mentioned above. The No ZLB scenario corresponds to the dotted blue line, the Traditional ZLB to the solid purple line and the Modified ZLB to the dashed red line. IRFs are expressed as percent deviations from steady state for all variables except for the deposit rate (i_t^d), the policy rate (i_t), the spread between the expected loan rate and the policy rate ($\mathbb{E}_t(i_{t+1}^l - i_t)$), and inflation, whose values are plotted in annualized levels in percentage points. Figures 2.6 and 2.7 display responses for $\kappa = 12.5$ basis points, which is the baseline value (recall that κ governs the importance of deviations from the target loan-to-equity ratio). Appendix Figure A.8 shows the IRFs of additional variables.

Figure 2.6 demonstrates that, on the onset of the recession, both the policy rate and the deposit rate are negative in the No ZLB scenario, the policy rate is negative but the deposit rate is stuck at zero in the Modified ZLB scenario, and both the policy rate and the deposit

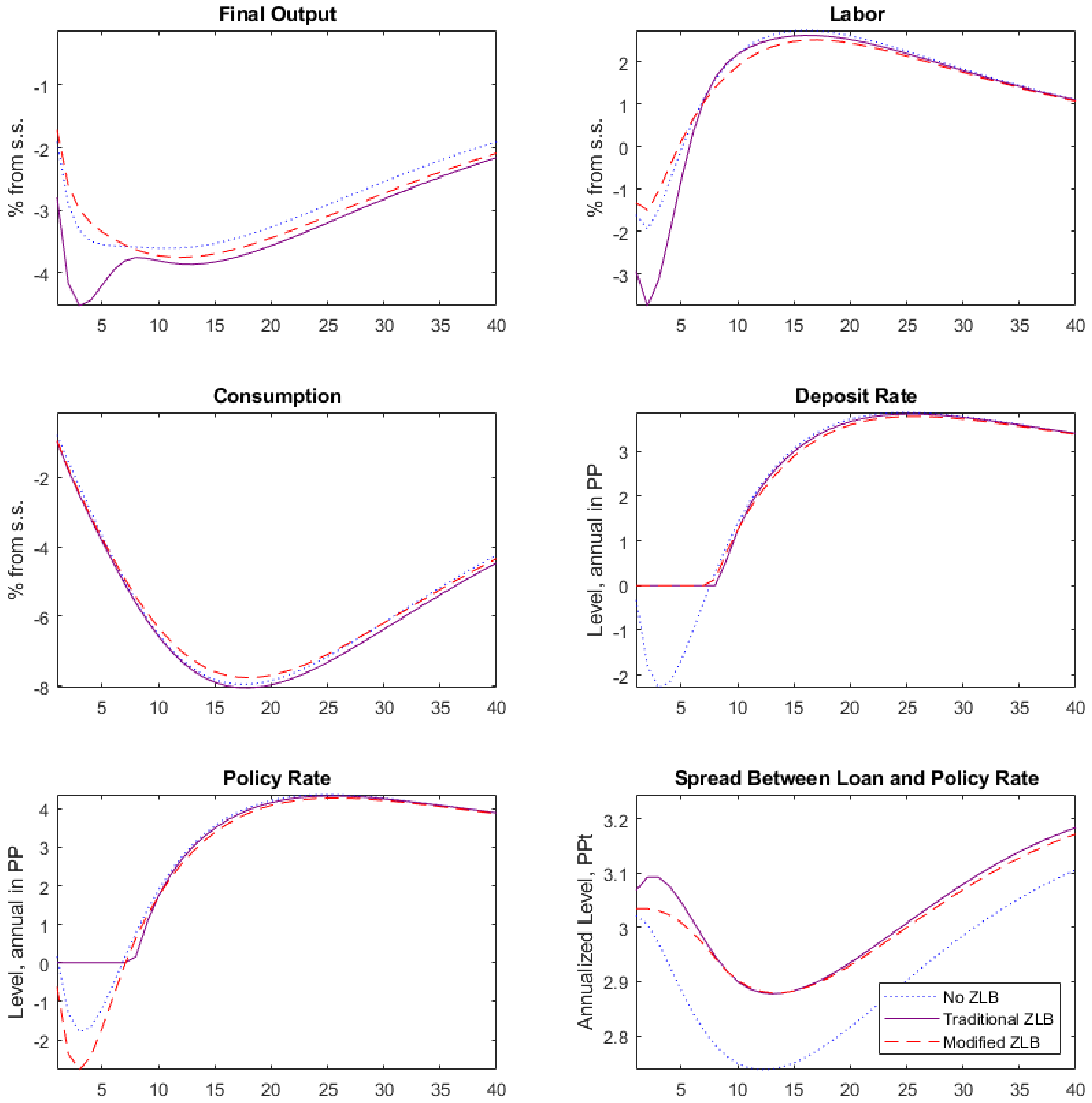

Figure 2.6: IRF's to a capital productivity shock

Notes: This figure depicts the IRF's of some of the main variables in the full model to a capital productivity shock under the "No ZLB" (blue dotted line), "Traditional ZLB" (purple solid line), and "Modified ZLB" (red dashed line) scenarios when $\kappa = 12.5$ basis points. The x axis is given in quarters and the y axis is given in percent deviation from steady state for output, labor and consumption, and in annualized percentage points for the three rates (deposit rate, policy rate, and the spread between loan return and the policy rate).

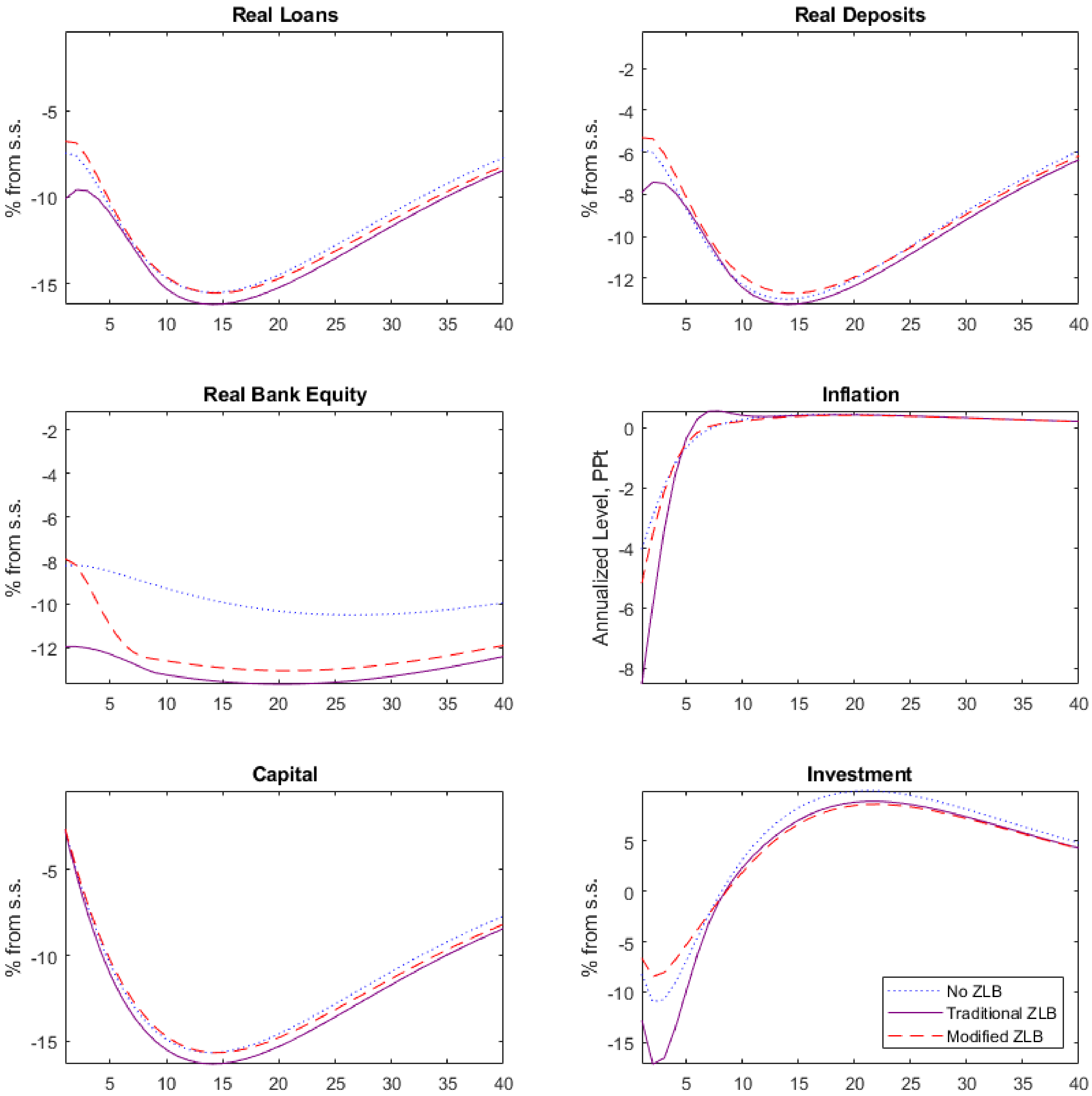

Figure 2.7: More IRF's to a capital productivity shock

Notes: This figure depicts the IRF's of some of the main variables in the full model to a capital productivity shock under the "No ZLB" (blue dotted line), "Traditional ZLB" (purple solid line), and "Modified ZLB" (red dashed line) scenarios when $\kappa = 12.5$ basis points. The x axis is given in quarters and the y axis is given in percent deviation from steady state for everything but inflation (which is given in annualized percentage points).

rate are stuck at zero in the Traditional ZLB scenario. Recall that, in this section, the monetary authority is following a given Taylor Rule, where it reacts to inflation. When the economy is in the Modified ZLB scenario and the deposit rate gets stuck at zero, the instrument of the monetary authority (i.e., the nominal policy rate) has less power compared to the No ZLB scenario, and hence the central bank lowers the policy rate by more to achieve a comparable effect. This explains why the policy rate becomes more negative in this scenario than in the No ZLB one.

The lower policy rate under the Modified ZLB scenario is the reason why output falls by slightly less in that case compared to the No ZLB scenario in the first few periods. Very quickly however, output in the No ZLB scenario overtakes output in the modified ZLB scenario and remains higher than in both other scenarios for most of the relevant quarters of study. It is also possible to observe that initially output falls by more in the Traditional ZLB scenario compared to the Modified ZLB, since in the latter the central bank can still stimulate the economy using the policy rate. But after some time, roughly around quarter 8, output under the Traditional ZLB nearly catches up to output under the Modified ZLB and stays just slightly below it for the following quarters. As far as consumption is concerned the three scenarios are not that different, but consumption under the Traditional ZLB remains lower throughout.

Importantly, bank equity starts off at a similar level in the No ZLB scenario and the Modified ZLB. However, after the periods when the policy rate is negative and the deposit rate is stuck at zero, bank equity in the Modified ZLB scenario falls by almost 4%, and subsequently stays much closer to bank equity under the Traditional ZLB scenario. The loan spread starts at a similar level for the No ZLB and Modified ZLB scenario. However, after the deterioration of bank equity brought about by NNIR, the spread in the Modified ZLB increases relative to the one under the No ZLB scenario, and stays close to the one under the Traditional ZLB scenario.

When I compute the change in welfare from the recession (relative to a situation without the shock) I obtain that the welfare cost of the recession is 98 basis points (of lifetime welfare) under the No ZLB scenario, 101 basis points under the Modified ZLB scenario, and 104 basis points under the Traditional ZLB scenario. Hence, setting negative nominal interest rates in the Modified ZLB is helpful (in the sense that welfare falls less than in the Traditional ZLB), but is not equivalent to having no constraint at all (in the sense that welfare falls more than under the No ZLB scenario). The fact that the differences in welfare between the scenarios are small should not be of concern, this has to do with the fact that I am calculating lifetime welfare (using a low discount rate), whereas the effects of the recession are concentrated in a few quarters after the shock. The fact that the recession has significant welfare effects under the No ZLB scenario is mainly due to the fact that the shock affects capital productivity and is fairly long lived, which implies that it would have serious effects on welfare even under a perfectly efficient economy with no pricing frictions.[37]

In this section I have analyzed the response of the model economy to a recession under a given Taylor rule for the three different scenarios described above, and a given value of κ. This exercise illustrates the differences between the Traditional and the Modified ZLB. However, given that the Modified ZLB seems to be the relevant empirical case, a more interesting exercise is to analyze the response of the model economy to the recession under different stances of monetary policy and different levels of κ, which is what I proceed to do next.

2.7 Effects Under Different Monetary Policy Responses

In this section I keep the size of the recessionary shock the same as in the previous section (ξ_t falls by 2.5% with persistence $\rho_\xi = 0.9$), but focus on the effects of the recession just under the Modified ZLB scenario for different levels of κ and different responses of monetary policy. To analyze different monetary policy stances I look at the level of the policy rate in the first quarter after the recession hits. The central bank can decide to be accommodative by setting very low (including negative) rates, or more restrictive, by setting higher rates. The central bank has this choice in all periods, but to keep the analysis simple I focus on the first period and assume that from period 2 onwards, the central bank simply follows the Taylor rule. Since the Taylor rule has smoothing ($\rho_i = 0.8$) an accommodative stance is translated to the following periods anyway.

First I provide an example of the setup, by illustrating how Figure 2.2 works in the context of the full model. Figure 2.8 shows the policy rate in the first period after the shock, in percentage annualized terms, on the x axis, and bank ROE in percentage annualized terms, on the y axis. The different lines represent different values of κ, from 3 basis points to 150 basis points. Notice that this figure looks similar to Figure 2.2, even though there the setup for the banks was simpler and the amounts of deposits and loans were assumed to be independent of the policy rate. In other words, the mechanisms described in Section 2.2 survive in the richer general equilibrium setup described in Section 2.4. The kink in the figure occurs at the annualized value of 0.5%, which is precisely the value of $\tilde{\imath}$ given the parameters in the baseline calibration.[38]

Figure 2.8 illustrates the fact that under the Modified ZLB the central bank has to worry about hurting bank's profits more than usual when setting negative rates (actually rates smaller than 0.5%). This means that negative rates can be helpful or harmful for the economy as a whole, depending on how important bank equity is. This is studied in Figure 2.9, where

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[37] The fact that the recession has significant welfare effects under the No ZLB scenario is also due to the fact that the central bank does not know the natural rate of interest, divine coincidence does not hold, and the model has several new features compared to the traditional NK model.

[38] The section where $i < \underline{i}$ does not appear in Figure 2.8 because in the baseline calibration the value for $\underline{i}$ is around -2.2% at the annual level.

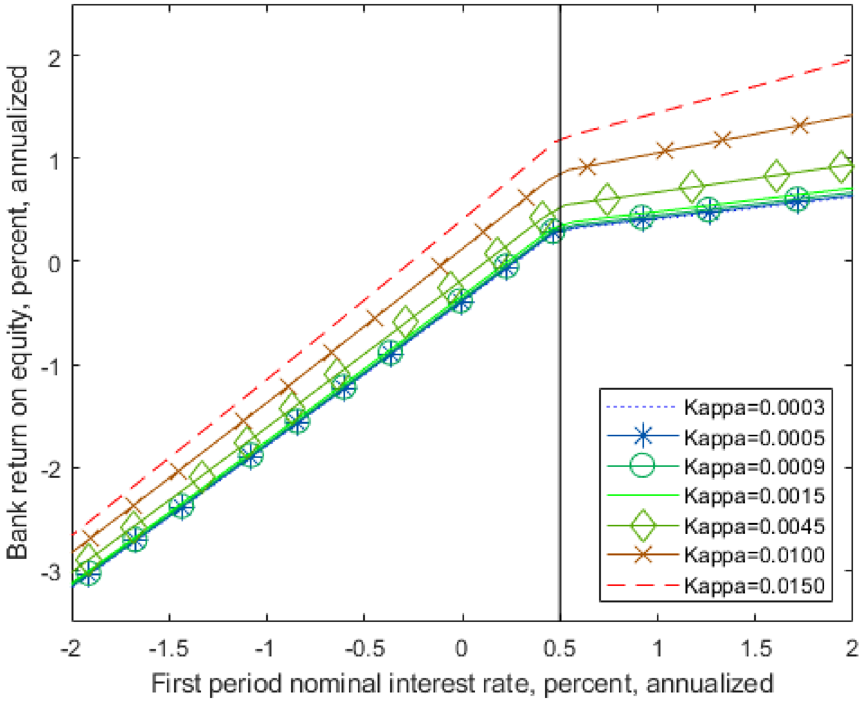

Figure 2.8: Bank ROE vs the policy rate

Notes: This figure plots bank return on equity as a function of the policy rate in the first period after the recessionary capital efficiency shock, for different values of the parameter κ, which parametrizes the cost of deviating from the target level of leverage.

the x axis is the same as the one in Figure 2.8, but the y axis represents the change in welfare from its steady state value in percentage terms. The levels of the welfare measure are similar to the ones mentioned in Section 2.6 (i.e., a fall of between 98 and 104 basis points from the recession). However, in Figure 2.9 the values of the welfare measure have been renormalized so that the welfare change is zero at $\tilde{\imath}$ (which is illustrated by the vertical black line), and so that a one percentage point (annualized) fall in the policy rate from 1.5% to 0.5% increases welfare in one unit. These two normalizations imply that the value of the y axis when the policy rate equals -0.5% measures the *relative efficiency* (in welfare terms) of a cut in the policy rate from 0.5% to -0.5% compared to one from 1.5% to 0.5%.

Figure 2.9 shows that for very low values of κ (like 3 basis points), for which bank equity is almost irrelevant, the efficiency of monetary policy is basically the same under positive and negative rates. For very high values of κ (like 150 basis points), for which bank equity is very important, the efficacy of monetary policy below $\tilde{\imath}$ is roughly half the one above $\tilde{\imath}$, since eroding bank profits is costly. For the baseline value of $\kappa = 12.5$ basis points, the relative efficiency of monetary policy below $\tilde{\imath}$ is roughly 70% of the one above $\tilde{\imath}$.

It is important to point out that the values in Figure 2.9 depend on the value of ρ_i, while the

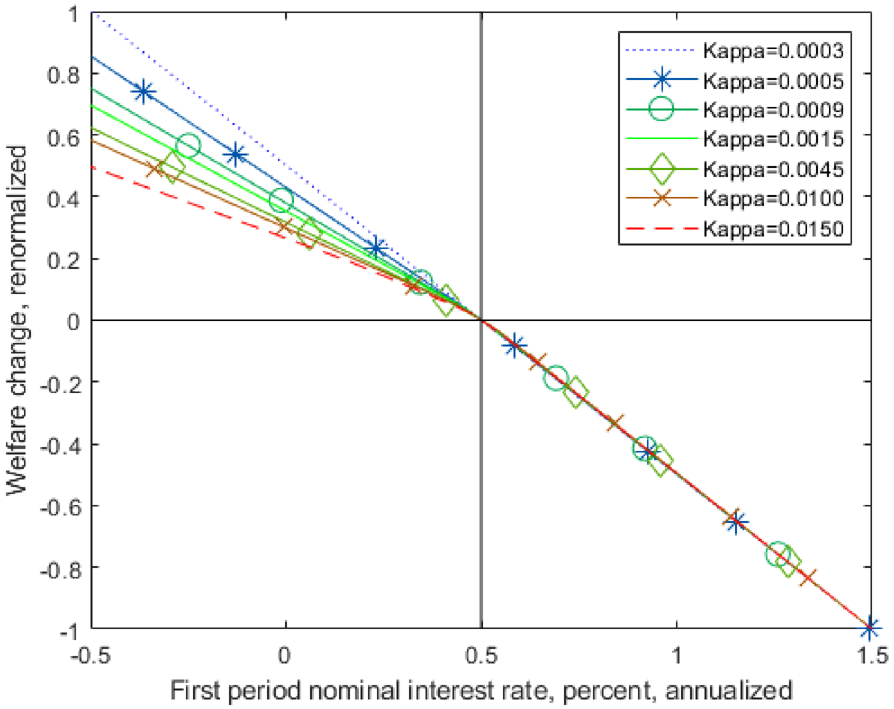

Figure 2.9: Welfare vs the policy rate

Notes: This figure plots a renormalized measured of the change in lifetime welfare as a function of the policy rate in the first period after the recessionary capital efficiency shock, for different values of the parameter κ. The welfare renormalization in the y axis is such that the change in welfare is zero at $i = \tilde{\imath}$ (i.e., 50 basis points) and minus one at $i = 1 + \tilde{\imath}$ (i.e., 150 basis points). The baseline value of $\rho_i = 0.8$ is used for the reaction of the policy rate to inflation in this figure.

other parameters do not have a big impact on the configuration of this figure. To illustrate the impact of ρ_i on the results, Figure 2.10 reproduces Figure 2.9 but for $\rho_i = 0.4$ instead of 0.8. The lower the ρ_i, the faster the relative efficiency of monetary policy in negative territory falls with κ. With $\rho_i = 0.4$ setting negative rates is basically a wash in terms of welfare when $\kappa = 1.5\%$. For κ's even higher than 1.5%, setting negative rates can be detrimental for welfare.

Why does a lower ρ_i lead to lower relative efficiency of a cut in the policy rate below $\tilde{\imath}$ for all but the smallest κ's? Notice that the detrimental effects of NNIR on bank profitability are concentrated at the onset of the recession, when the policy rate is negative but the deposit rate is stuck at zero. Those periods are also when banks are specially vulnerable after having suffered a fall in their equity that originates from the decline in capital efficiency. Consequently, the negative effects of the contractionary bank net worth channel are concentrated in the few periods after the recession. With a low ρ_i the beneficial effects of the expansionary bank lending channel are also concentrated in just a few periods; it is not as useful to hurt

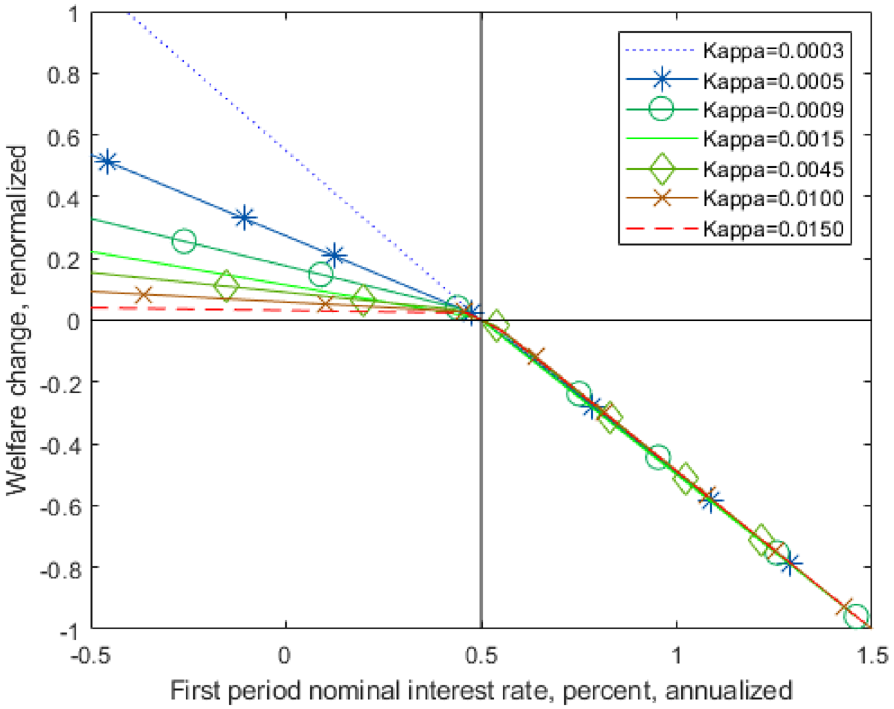

Figure 2.10: Welfare vs the policy rate, low ρ_i

Notes: This figure reproduces Figure 2.9, but uses a smaller value of $\rho_i = 0.4$ for the reaction of the policy rate to inflation (instead of the 0.8 used in Figure 2.9).

banks when they are the most vulnerable for just a few quarters of lower lending rates. By contrast, when ρ_i is high, the beneficial effects of NNIR (expressed through the bank lending channel) extend for more periods, and this increases the relative efficiency of cuts in the policy rate below $\tilde{\imath}$. The takeaway is that hurting banks via NNIR is more useful if the low rate environment engendered by negative rates persists even after banks are starting to rebuild their equity.

After understanding the effects of κ and ρ_i on the efficiency of monetary policy in negative territory relative to that in positive territory, this can be related to the findings in Section 2.5.1 about the differences in κ's across countries. Table 2.6 presents the relative efficiency for the κ's estimated in Section 2.5.1, for different ρ_i's between 0.4 and 0.8. In that table countries are arranged from those with the higher κ (United States, with $\kappa = 49$ basis points at the annual level or 12.25 basis points at the quarterly level) to those with the lowest κ (Japan, with $\kappa = 17$ basis points annually or 4.25 basis points quarterly). The table shows that countries with a lower κ suffer less from hurting their banks through NNIR and hence they end up having a higher relative efficiency of monetary policy below $\tilde{\imath}$. Additionally, within any country, the higher the ρ_i, the higher the relative efficiency of monetary policy below $\tilde{\imath}$. Since traditional estimates of ρ_i tend to be above 0.7, this justifies my range of

Table 2.6: Relative efficiency of monetary policy below $\tilde{\imath}$

ρ_i / Country	0.4	0.5	0.6	0.7	0.8
United States	30.29%	40.23%	50.35%	60.70%	74.70%
Switzerland	42.52%	48.39%	60.62%	68.24%	80.43%
United Kingdom	56.36%	60.63%	71.81%	77.37%	86.40%
Europe	65.21%	68.77%	78.32%	82.66%	89.60%
Japan	69.20%	72.42%	81.08%	84.88%	90.89%

Notes: This table provides the relative efficiency of monetary policy below $\tilde{\imath}$ (described in detail in the text) for different countries and values of ρ_i.

values for the relative efficiency of negative rates between 60% and 90%.

The previous table indicates that negative rates are relatively effective in regions like Japan or Europe and less so in countries like the United States. Notice that the model in this paper does not incorporate any open economy considerations like the ones discussed in Amador et al. (2017) which could be especially relevant for small open economies like Switzerland, and could move its relative position in the previous table. It is natural to assume that different countries, or a particular economy at different points in time, will have different values of κ. Therefore, the usefulness of setting negative rates to fight a deep recession will be specific to a particular context, and each country will have to estimate how useful NNIR would be in its particular context.

There are two reasons why the relative efficiency of a cut in the policy rate below $\tilde{\imath}$ is high despite the existence of the contractionary bank net-worth channel. First, the estimates of the importance of bank equity for lending are relatively small. This is informed by the fact that, after controlling for bank fixed effects, a decline in the equity of a particular bank does not have a big effect on that bank's lending amount or its loan rate. Second, in the full model, when the policy rate and the loan rate fall, aggregate loan demand increases and banks can switch reserves for loans, decreasing the impact of negative rates on their ROE (this mechanism is not operational in the static model of Section 2.2). This result has also been emphasized in an empirical context by Lopez et al. (2018) and Demiralp et al. (2017).

2.8 Conclusion

This paper argues that the ability to set negative policy rates while deposit rates are constrained to being nonnegative is different from not being subject to the ZLB altogether. The former scenario has implications for bank profitability, as it leads to a decline in banks' net worth, which can hinder investment and output growth. Central banks around the world must then be careful when setting negative policy rates and they must take steps to minimize their negative impact on banks' profits. However, the estimates in this paper for the relative efficiency of monetary policy in negative territory are relatively high, and indicate that the effect on commercial bank equity could be less detrimental than previously thought.

The main contribution of this paper is to provide a fully specified DSGE model where the question of negative interest rates and their effects on the economy, and bank profitability, can be studied. Relative to the few previous theoretical papers on NNIR, like Rognlie (2015) and Eggertsson et al. (2017), my paper can capture both beneficial and detrimental effects of negative rates in a monetary general equilibrium model with bank profitability concerns, and determine the relative efficiency of monetary policy in negative territory compared to that in positive territory.

The main finding of this paper is that lowering interest rates below zero can be less effective than lowering them in positive territory, since deposit rates remain stuck at zero and hence bank profits are squeezed. The efficiency of negative nominal rates is then very tightly linked to the importance of bank equity in the economy. For reasonable estimates of this parameter I conclude that the efficiency of monetary policy when the interest rate is below 50 basis points is between 60% and 90% of its value above 50 basis points. The importance of bank equity for lending, and for the overall economy, differs across countries due to different institutional settings, therefore the usefulness of monetary policy in negative territory also differs between countries. For Japan or the Euro Area setting NNIR seems to be relatively efficient, while the US has a lower efficiency.

While this paper strives to provide a comprehensive quantitative model to assess the effects of negative rates on the economy, it makes some simplifications in the interest of parsimony. In what follows I describe several extensions that could improve the realism of the model, but that are beyond the scope of this paper and are therefore left for future research. First, negative rates flatten the yield curve, so they might have effects on bank profitability that cannot be captured in the current framework, where all assets and liabilities have a duration of one period. Allowing for differences in the duration of financial instruments can lead to revaluation effects, as described in Brunnermeier and Koby (2017). Second, negative rates and the associated decline in loan rates can have an effect on the default probability of borrowers. As mentioned in Cœuré (2016), a fall in rates, even in negative territory, can decrease default probabilities, and this would increase the efficiency of monetary policy. Third, the fall in bank profitability can lead to a search for yield, and an increase in risk taking by banks, which can have a negative impact on financial stability and decrease the beneficial effects of negative rates. Fourth, the impact of NNIR on bank profitability depends on the exact fraction of a bank's reserves that are subject to the negative rate. This fraction varies if central banks set an exemption threshold for reserves below which commercial banks earn a zero interest rate. This is something that most central banks setting NNIR already do, by implementing a tiered structure of reserve remuneration.

Understanding the effects of negative rates is a critical task for economists and policy makers in the current environment of persistently low global interest rates. Having a realistic framework that can incorporate both the beneficial and detrimental aspects of negative rates is a good start. Extending that framework to allow for even more realistic aspects of negative rates is an important next step.

Chapter 3

The Cyclical Sensitivity in Estimates of Potential Output

3.1 Introduction

The Great Recession was characterized not just by large declines in economic activity in most advanced economies, but also ones that have persisted for a decade with no sign of these affected economies "catching up" to previously expected trend levels.[1] If anything, it is the trends that are now being revised down in light of the continuing inability of these economies to close the output gaps first generated in 2008. As illustrated in Figure 3.1a for the U.S., estimates of potential output have been systematically revised downward since the Great Recession, such that all of the current deviations of output from past estimates of potential are now being reinterpreted as permanent declines in the productive capacity of the economy. These large downward revisions imply that the output gap appears closed, and this absence of any remaining slack in the economy is a primary motivation for the Federal Reserve's progressive tightening of monetary policy.

However, before we take these dynamics in the estimates of potential output at face value, we should understand their properties and what determines revisions in these estimates. In this paper, we focus on how real-time estimates of potential output respond to different economic shocks in the U.S. as well as across a wide range of countries. Using a variety of institutional sources for estimates of potential gross domestic product (GDP), we find that real-time estimates of this variable respond to cyclical shocks that have no long-run effects on the economy and under-respond to shocks that do. In all cases, adjustments in real-time estimates of potential GDP are extremely gradual, much like a moving average of

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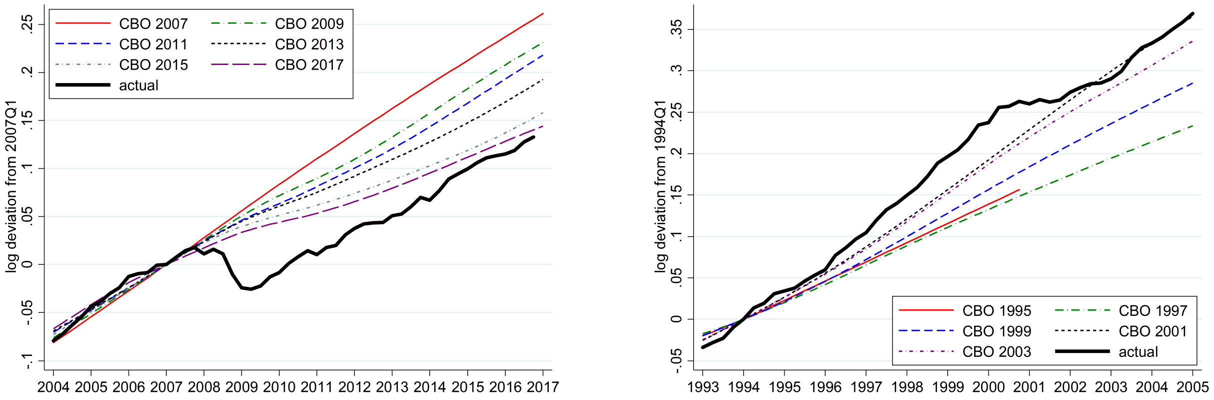
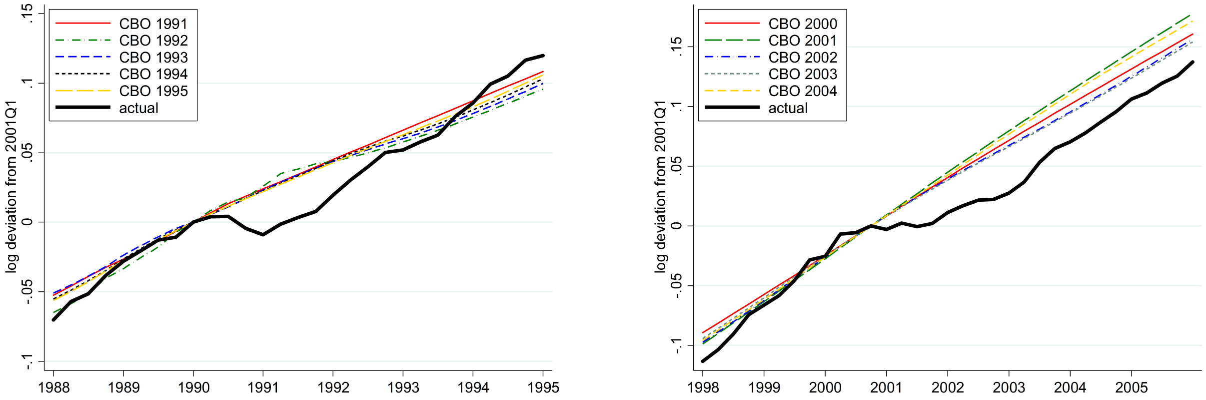
[1] This is joint work with Olivier Coibion and Yuriy Gorodnichenko, their permission to reprint this material as a chapter of the present dissertation has been obtained.



(a) The Great Recession **(b)** The 1990's Productivity Boom



(c) The 1990 Recession **(d)** The 2001 Recession

Figure 3.1: Historical Revisions in CBO Estimates of U.S. Potential Output

Notes: The figure plots estimates of U.S. potential output from the CBO made at different time periods (beginning of the corresponding year). The solid black line represents real GDP in the U.S. In each panel, each series is normalized to zero in 2007 (Panel a), 1994 (Panel b), 1990 (Panel c), and 2000 (Panel d).

past output changes. In fact, given their gradual pace of adjustment to shocks and the fact that these real-time estimates fail to differentiate between shocks that do and do not affect the productive capacity of the economy, there seems to be little value added in estimates of potential GDP relative to simple measures of statistical trends. At a minimum, the fact that estimates of potential GDP are revised, either upward or downward, should not be taken as a sign that future changes in GDP will in fact be more or less persistent than usual but rather indicates little more than that the prior changes in GDP have been persistent.

Because estimates of potential GDP are not necessarily created in the same fashion across institutions, we consider estimates from the Federal Reserve Board (Fed) and from the Congressional Budget Office (CBO) for the U.S. as well as estimates from the International Monetary Fund (IMF) and the Organization for Economic Cooperation and Development (OECD) for a broader cross-section of countries. We complement this with long-term forecasts of output growth from professional forecasters (Consensus Economics). Most public or international organizations follow production function approaches, in which estimates of the potential productive capacity of an economy reflect estimates of the capital stock, potential labor force sizes combined with estimates of human capital, as well as measures of total factor productivity. Hence, estimates of potential output should change when the technological capacity of the economy improves but not in response to purely cyclical variations in employment such as those arising from monetary policies.

To test these propositions, we bring to bear not just a wide range of estimates of potential output but also a range of shock measures. Somewhat surprisingly given the short samples, we find several clear patterns in the data that should give one pause before interpreting changes in estimates of potential output as indicators of permanent changes in output. First, and perhaps most strikingly, while we reproduce the common and well-documented finding that monetary shocks have only transitory effects on GDP, we then document the startling feature that these shocks are followed by a gradual change in estimates of potential GDP. This finding occurs not just in the U.S. but across countries as well and is true for a range of sources of estimates of potential GDP.

We find a similar set of results when we focus on government spending shocks. Regardless of the identification strategy, increases in government spending have transitory effects on GDP, but estimates of potential GDP again display a delayed response to these shocks, ultimately responding to the shock in the same direction as the short-run response of GDP. As with the effects of monetary shocks, the fact that estimates of potential GDP respond so unambiguously to these shocks strongly suggests that real-time estimates of potential GDP are failing to adequately distinguish between permanent and transitory shocks. In this respect, estimates of potential GDP are sensitive to cyclical fluctuations in GDP originating from demand shocks. Turning to supply shocks that should affect potential GDP, the results are more mixed. With productivity shocks, which have immediate and persistent effects on GDP, we find that estimates of potential GDP again respond only very gradually but, after several years, fully incorporate the effects of new productivity levels. With tax shocks, we similarly observe that, after a long delay, estimates of potential GDP eventually catch up to actual changes in GDP. Hence, these two supply shocks provide evidence that real-time estimates of potential output ultimately embody some changes in potential GDP. However, the very slow rate at which information about these shocks is incorporated into estimates of potential GDP points to an insufficient sensitivity of these estimates in response to supply shocks. With oil price shocks, however, an even more severe problem arises. We observe persistent declines in GDP after these shocks, but estimates of potential GDP actually go in the opposite direction. As with demand shocks, this specific type of supply shock therefore also presents a challenge to the view that estimates of potential GDP are actually capturing what they are meant to.

Furthermore, we can consistently reproduce the way in which estimates of potential GDP respond to shocks by applying a one-sided Hodrick-Prescott (HP) filter to real-time GDP data. In the U.S. as well as in the cross-country data, this approach generates impulse responses to shocks that are nearly indistinguishable from those found using the actual estimates of potential GDP from all organizations, including the counter-cyclical behavior of measured potential GDP after oil supply shocks. The HP filter is effectively just a weighted moving-average of recent GDP changes and by construction does not differentiate between the underlying sources of changes in GDP, be they monetary, technological, etc. Thus, a reliance on simple statistical filters like HP by official agencies could readily rationalize why one might observe a gradual response by real-time measures of potential output to any economic shock, even those that have only transitory effects on GDP and that should presumably be stripped out of estimates of potential GDP.

Fortunately, other approaches to identifying potential output can do better. For example, the Blanchard and Quah (1989) approach to identify supply and demand shocks can successfully generate real-time estimates of potential output that are consistent with theoretical predictions. Indeed, when the Blanchard and Quah (1989) approach is applied to real-time data to recover potential output measured as the historical contribution of shocks with permanent effects on output, the resulting real-time estimate of potential output reacts strongly to identified supply shocks (TFP, tax, and oil price shocks) and it does not respond significantly to identified demand shocks (monetary policy and government spending shocks). Hence, it does not suffer from the problems associated with most other measures of potential output. Furthermore, this approach yields a starkly different interpretation for changes in U.S. potential output following the Great Recession. Our estimates imply that the gap between potential and actual output in the U.S. has increased by approximately 5 log percentage points between 2007Q1 (when the gap was likely close to zero) and 2017Q1, leaving ample room for policymakers to close this gap through demand-side policies if they so chose to.

We find similar evidence of a large output gap using other methods to calculate measures of potential output, such as the ones proposed by Gali (1999) (which uses information from labor productivity and hours) or Cochrane (1994) (which brings in additional information from consumption). Using information from inflation to make inferences about potential output through a Phillips curve also points toward significant slack. or one based on an estimated Phillips curve. All these methodologies give similar results, pointing to an increase in the gap of 5-10 percentage points between 2007Q1 and 2017Q1. This assures us that this result is not an artifact of the Blanchard-Quah approach and instead is a feature that is robust to different identification schemes. The idea that significant slack remained in the U.S.

economy through 2017 is also consistent with the low levels of capacity utilization, contained wage growth, and the evolution of labor force participation since the Great Recession.

This paper touches on several literatures. It is most directly tied to recent work since the Great Recession focusing on the possibility of hysteresis: cases where demand shocks lead to permanent effects on the level of economic activity. While there are many mechanisms that can generate such effects (e.g. less R&D during periods of low investment as in Anzoategui, Comin, Gertler, and Martinez (2016), Benigno and Fornaro (2018), and Moran and Queralto (2018)), empirical evidence on hysteresis remains scant, as emphasized in Blanchard (2017), with most estimates of monetary and government spending shocks being consistent with the null that these shocks have no permanent effects on GDP (see Nakamura and Steinsson (2017) and Ramey (2016) for reviews of the literature on monetary and government spending shocks). Recent research has focused on the degree to which the sustained declines in output since the Great Recession have ultimately been interpreted as reflecting declines in potential GDP and therefore expected to be long-lasting. Ball (2014) documents that for most advanced economies, much of the declines in output after the Great Recession have been matched with declines in estimates of potential output. Fatas and Summers (2018) focus on the degree to which fiscal consolidations map first into output changes and then into changes in estimates of potential GDP, with the latter being an indicator that GDP changes will be permanent. Our results suggest that one should draw little inference from the evolution of estimates of potential GDP about the persistence of GDP changes: these estimates fail to exclusively identify supply shocks that should drive potential GDP and instead also respond to transitory demand shocks. The fact that most of the output declines observed since the Great Recession are now attributed to declines in potential GDP implies little other than that these declines have been persistent since estimates of potential GDP fail to adequately distinguish between the underlying sources of changes in GDP.

Our paper also relates to work on news shocks and beliefs about long-run productivity. A strand of literature studies how news about future productivity can have contemporaneous effects on economic activity long before the productivity changes actually occur (e.g. Beaudry and Portier (2006), Barsky and Sims (2011)). In that spirit, Blanchard, Lorenzoni, and L'Huillier (2017) show that revisions in estimates of future potential output are correlated with contemporaneous changes in consumption and investment. If estimates of future potential output were invariant to transitory shocks, then one could entertain a causal interpretation of these correlations as reflecting the effect of news about the future on current economic decisions. But our results call for caution with this type of interpretation: estimates of potential GDP display sensitivity to demand shocks, and this sensitivity calls into question the basis for causal inference of the type made in Blanchard et al. (2017).

A third literature that we build on focuses on the implications of real-time measurement of the output gap for monetary policy. Orphanides and van Norden (2002), for example, illustrate how real-time estimates of potential GDP can, in short samples, be sensitive to the method used to measure either the trend or deviations from trend. Orphanides (2001, 2003, 2004) argues that the Federal Reserve's mismeasurement of the output gap in the 1970s was one of the primary reasons why inflation was allowed to rise so sharply in the 1970s. We are similarly interested in the difficulties with measuring potential output and the output gap, but rather than studying how sensitive estimates of potential output can be to the different statistical techniques used to identify it, we instead characterize whether the historical estimates of potential output from public and international organizations respond to the "correct" shocks. Our estimates imply that just as the Federal Reserve likely over-stimulated the economy in the 1970s because of mismeasurement of potential output, it is now at risk of under-stimulating the economy by underestimating the productive capacity of the economy.

Finally, by comparing actual responses of output after economic shocks to the predictions of agents about these variables, our paper is closely related to recent work studying the expectations formation process of economic agents. Coibion and Gorodnichenko (2012) study the forecast errors of agents to economic shocks and find that these errors are persistent after economic shocks, consistent with models where agents are not fully informed about the state. By comparing the long-run response of GDP to estimates of potential GDP, this paper similarly provides some insight about how these potential GDP estimates are formed.

The paper is organized as follows. Section 3.2 presents information about the estimates of potential output used in the paper. Section 3.3 presents our baseline estimates, using U.S. data, of how estimates of potential GDP respond to economic shocks. Section 3.4 extends these results to a broader range of countries. Section 3.5 presents some examples of how estimates of potential output can be improved. Section 3.6 concludes.

3.2 How Estimates of Potential Output Are Created (and Used)

A seminal description of potential output is in Okun's (1962) presidential address. While the notion of potential or natural levels of output had been discussed as far back as Wicksell (1898) or Keynes (1936), Okun (1962) provided a sharper definition than had been previously utilized as well as guidance about how to estimate potential output (Hauptmeier et al. 2009). Okun emphasized that potential output is a "supply concept, a measure of productive capacity." But it is not designed to represent the maximum amount that an economy could produce. Instead, Okun defines it as the amount that could be produced without generating inflationary pressure. Hence, while potential GDP is related to the non-accelerating inflation rate of unemployment (NAIRU), potential output provides a more comprehensive assessment of how much an economy can produce without triggering above-normal inflation. This interpretation of potential output advocated by Okun serves as the foundation of most approaches to estimating potential output.

Although Okun proposed to estimate potential output through a combination of knowing the NAIRU and applying what subsequently became known as Okun's Law, few organizations follow the specific approach suggested by Okun. As classified in Mishkin (2007), there are three broad classes of methods to construct a measure of potential output: statistical, production function, and structural (DSGE-based). We first review these methods and then discuss how various agencies measure potential output.

Statistical methods typically impose little theoretical structure on the properties of potential output and interpret low-frequency variation in output series as potential output. One example of this approach is to use univariate time series methods, such as autoregressive (AR) models or different types of filters, on actual output to extract a statistical trend component which is then identified with potential output. Another example is given by methods using several variables, such as output, unemployment and inflation, to obtain potential output via an unobserved components model and a Phillips curve (e.g., Kuttner (1994), Staiger, Stock, and Watson (1997)).

In the production function approach, independent estimates of the different inputs that go into the aggregate production function (e.g., labor, capital, multifactor productivity) are plugged into the production function to obtain potential output. Since the objective is to obtain potential output and not actual output, the estimates of the different inputs must correspond to the concept of the maximum (or "normal") amount of each variable that could be used for production without leading to an acceleration of inflation (e.g., the labor force participation rate and a level of natural unemployment should be used instead of the cyclical level of employment). In the latter sense, this approach to estimating potential output remains in the spirit suggested by Okun. This approach is also related to growth accounting, since after log-differentiation of a Cobb-Douglas production function, the growth of potential output can be expressed as the weighted average of the growth rates of the different inputs (see Fernald, Hall, Stock, and Watson (2017) for an application of this approach to the dynamics of output in the post-Great Recession period). Finally, structural approaches use dynamic stochastic general equilibrium (DSGE) models, typically with a New Keynesian structure, to back out potential output. This requires calibrating or estimating the parameters of the model to the relevant economy so that the different shocks hitting the economy can be identified. Once this stage is completed, potential output can be obtained from the solution of the model when certain shocks and frictions are turned off (e.g. Andres, Lopez-Salido, and Nelson (2005)). This methodology is particularly model-dependent and relies heavily on the estimation of a sophisticated model, which given limited variation in macroeconomic data may be a challenge for identification of structural parameters and shocks. Furthermore, because estimated DSGE models have only been used in recent years, there is no historical real-time data available to assess their properties.

The implicit assumptions about the nature of potential output are not identical across methods. The production function approach for example explicitly tries to strip out cyclical factors from estimates of potential output. Statistical filters similarly try to separate cyclical fluctuations in output from changes in the trend, with the latter being equivalent to potential. In contrast, with a New Keynesian DSGE where the potential level of output reflects counterfactual outcomes under flexible prices, transitory "demand" shocks like temporary changes in government spending can affect the level of potential output for some time whereas they would be excluded from estimates of potential under the other two approaches (see Blanchard (2017)). Since our empirical strategy involves studying the response of real-time estimates of potential output to supply (long-lived) vs demand (transitory) shocks, we are adopting an interpretation of potential output which hews most closely to the production function and statistical filtering approaches, in part because this is precisely the conceptual framework that is most often used by statistical and other agencies when they construct estimates of potential.

3.2.1 Congressional Budget office (CBO)

The CBO uses the production function approach for estimating potential output. As described in CBO (2001, 2014), this institution estimates potential output with different methods for five sectors in the economy. The main one is the nonfarm business (NFB) sector, which represents approximately 75 percent of the U.S. economy. The remaining four smaller sectors are agriculture and forestry, households, nonprofit organizations serving households, and government.

In each of these sectors the CBO projects the growth of each input by estimating a trend growth rate for it during the previous and current business cycles (as dated by the National Bureau of Economic Research) and extending that trend into the future. This implies that the trend growth for inputs depends on recent history and on business cycle dating, with possibly large changes in trends when a new business cycle begins. The CBO tries to remove the cyclical component of the growth rate of different variables by estimating the relationships between those variables and a measure of the unemployment rate gap, the difference between the actual unemployment rate and the natural rate of unemployment.

For the nonfarm business sector the CBO uses a production function with three inputs: potential labor, services from the stock of capital and the sector's potential TFP. For the sectors of agriculture and forestry, and nonprofits serving households, potential output is estimated using trends in labor productivity for those sectors. For the household sector, potential output is obtained as a flow of services from the owner-occupied housing stock. Finally, for the government sector, potential output is estimated using trends in labor productivity and depreciation of government capital. Real-time CBO estimates of potential output are available since 1991 at the annual frequency and since 1999 at the semiannual frequency.

Estimates of potential output by the CBO play an important role in fiscal policy discussions in the U.S. When new tax or spending policies are under review by the U.S. Congress, their implications for future tax revenues, government expenditures, and deficits are assessed under assumptions about the long-run future path of the economy, as captured by estimates of potential GDP (although some policies require the CBO to make inferences about how these policies themselves may change potential output over time, e.g. via "dynamic scoring"). How these estimates are formed and how well they separate cyclical from permanent shocks therefore matters for how well these policy measures are scored.

These estimates of potential output are sometimes subject to very large revisions. Prior to the revisions over the course of the Great Recession for example, the CBO had similarly made a sequence of large upward revisions to the projected path of potential output over the course of the 1990s, as illustrated in Figure 3.1b. These upward revisions were tied to the higher than expected productivity growth in the U.S. over this period.[2] Other episodes reveal less dramatic sequences of revisions. For example, Figures 3.1b and 3.1d illustrate the CBO revisions during the two previous U.S. recessions. In both cases, the CBO first started reducing its predicted path of potential output during the recession then ultimately raised them back up again. In the case of the 1990 recession, GDP ultimately overtook estimates of potential output whereas, over the same time horizon of three years after the start of the recession, the CBO continued to estimate a large output gap after the 2001 recession. But in neither case do we observe a systematic pattern of downward revisions toward the path of actual GDP like what was observed after the Great Recession.

3.2.2 Federal Reserve

While preparing macroeconomic projections (historically known as Greenbook forecasts) for meetings of the Federal Open Market Committee (FOMC), the staff of the Federal Reserve Board constructs a measure of the output gap (that is, the difference between actual and potential output) to assist the FOMC's members in their decision making. As pointed out by Edge and Rudd (2016), from the Board of Governors of the Federal Reserve System, the estimate of the output gap from the Greenbook: "... is judgmental in the sense that it is not explicitly derived from a single model of the economy. In particular, the staff's estimates of potential GDP pool and judgmentally weight the results from a number of estimation techniques, including statistical filters and more structural model-based procedures."

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[2] While it is true that some of these revisions were not related to productivity changes, such as the ones coming from the shift to chained GDP, the addition of software, or revisions to NIPA, CBO (2001, p.2) summarized one of the larger revisions as follows, "CBO also altered its method to address changing economic circumstances. In particular, labor productivity has been growing much faster since 1995 than its post-1973 trend. Because that acceleration has coincided with explosive growth in many areas of information technology (IT)... many observers have speculated that the U.S. economy has entered a new era, characterized by more-rapid productivity growth... After analyzing the data and the relevant empirical literature, CBO has concluded that elements of the so-called IT revolution... explain much of the acceleration in the growth of labor productivity during the late 1990s. CBO has incorporated many of those elements into its economic projections."

While describing the evolution of measuring potential output by the Fed, Orphanides (2004) mentions that in the Greenbook estimates: "... the underlying model for potential output was a segmented/time-varying trend. The specific construction methods and assumptions varied over time. During the 1960s and until 1976, the starting point was Okun's (1962) analysis. From 1977 onward, the starting point was Clark's (1979) analysis and later, the related methods explained in Clark (1982) and Braun (1990). Throughout, these estimates of potential output were meant to correspond to a concept of noninflationary "full employment". However, judgmental considerations played an important role in defining and updating of potential output estimates throughout this period, so the evolution of these estimates cannot be easily compared to that of estimates based on a fixed statistical methodology."

More recently, Fleischman and Roberts (2011) describe a methodology to compute potential output using a multivariate unobserved components model that is taken into account by the Federal Reserve Board when producing their judgmental estimates of potential output. Their procedure embeds some parts of many of the methodologies described above: it uses multivariate statistical methods, trend estimation, growth accounting (as in the production function approach) and the relationship between cyclical fluctuations in output and unemployment (as in Okun's law). The authors use data on 9 macroeconomic series: real GDP, real gross domestic income, the unemployment rate, the labor-force participation rate, aggregate hours for the nonfarm business sector, a measure of NFB sector employment, two measures of NFB sector output (measured on the product side and on the income side) and inflation as measured by the CPI excluding food and energy. The common cyclical component of the economy is constrained to follow an AR(2) process and trends in the series are related to each other via structural equations (e.g. Okun's law, production function) to obtain a final measure of the trend of output which is associated with potential output.

Real-time estimates of potential output can be computed from the estimates of actual output and the output gap reported in Greenbooks since 1987.[3] Real-time estimates for the same variables in the 1969-1987 period are provided in Orphanides (2004). For this earlier period, the quality of the estimates is likely to be worse since the estimates sometimes had to be obtained from a variety of sources (e.g., the Council of Economic Advisors) other than the Federal Reserve. As a result, we take the 1987-2011 series as the benchmark and explore the longer time series in robustness checks. Because the Greenbooks only forecast potential output growth for up to a few years, we cannot reproduce Figure 3.1a (the evolution of real-time forecasts of potential GDP during the Great Recession) for Greenbook forecasts.

Estimates of potential output play an immediate role in decision-making by the Federal Reserve. One of the objectives of the FOMC is to stabilize output around potential and whether output is below or above potential is commonly interpreted as having implications for inflation, the other objective targeted by the Federal Reserve. Potential mismeasurement

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[3] This series is available from the Real-Time Data Research Center at the Federal Reserve Bank of Philadelphia. There is a five-year delay period for the release of Greenbook projections.

of the output gap (the difference between actual output and potential) is mentioned (e.g. Orphanides (2001)) as a reason why the Federal Reserve allowed inflation to rise during the 1970s, and Greenspan's perception that potential output was growing unusually rapidly in the 1990s explains why monetary policymakers over this period were less concerned about inflation than they normally would have been given the low unemployment rates (Gorodnichenko and Shapiro (2007)).

3.2.3 International Monetary Fund (IMF)

The IMF provides estimates of potential output for a wide range of countries. There is considerable methodological variation across countries in how the IMF generates estimates of potential output. As summarized in deResende (2014), a study conducted by the IMF's Independent Evaluation Office, "Interviews with staff showed that the use of the macro framework is country-specific and varies greatly in detail and sophistication, ranging from the use of "satellite" models to simply entering numbers based on judgment." In this respect, the IMF approach to measuring potential output is methodologically similar to measures reported in Greenbooks, in the sense that they use a combination of different methods to compute potential output and aggregate them using a great deal of judgement. At the same time, the IMF staff often uses the Hodrick-Prescott filter and/or multivariate methods such as the ones described in Blagrave, Garcia-Saltos, Laxton, and Zhang (2015) to construct measures of potential output. The IMF provides potential output estimates for 27 countries (see Appendix Table B.1 for the list of countries). Nowcasts and one-year-ahead forecasts are available for 2003-2016. Since 2009, the IMF also provides up to five-year-ahead forecasts for potential output.

Estimates of potential output can play an important role in IMF policy decisions. To assess the sustainability of countries' fiscal policies, tax and spending levels are commonly evaluated at the level of potential GDP to control for the cyclical changes in revenues and expenditures that are expected to be transitory, thereby helping to gauge any "structural" fiscal imbalances. These structural imbalances are then the primary focus of policy reforms associated with countries receiving funds from the IMF during times of crisis.

3.2.4 Organization for Economic Cooperation and Development (OECD)

OECD estimates of potential output are based on a production function approach. In particular, the OECD uses a Cobb-Douglas production function with constant returns to scale that combines physical capital, human capital, labor, and labor-augmenting technological progress. Each of these inputs is projected using a trend, and total factor productivity is assumed to converge to a certain degree among different countries in the medium run. As pointed out in Organization for Economic Cooperation and Development (2012): "The degree of convergence in total factor productivity depends on the starting point, with countries farther away from the technology frontier converging faster, but it also depends on the country's own structural conditions and policies." Note that when forecasting potential output in the medium term, the OECD assumes that output gaps close over a period of 4 to 5 years, depending on their initial size. Therefore, one should expect to see above average future growth for countries with large output gaps. Relative to the IMF, the OECD covers more countries and has longer time series (see Appendix Table B.1). For many countries, nowcasts and one-year-ahead forecasts are available since 1989. Since 2005, the OECD also reports five-year-ahead forecasts for potential output. As with the IMF, estimates of potential output in the OECD are commonly used to assess cyclically adjusted fiscal balances and to characterize the need for structural reforms.

3.2.5 Consensus Economics

Consensus Economics, a survey of professional forecasters, does not provide estimates of potential output but they report forecasts for the growth rate of actual output from 1 to 10 years into the future. Since estimates made for several years into the future (for example, years 6 through 10) are likely to be independent of business cycle conditions we use these long-run estimates as an approximation of the growth rate of potential output at the same horizon. These data are available for 12 countries and the starting date varies across countries from 1989 to 1998. Given the wide range of forecasters included in Consensus Economics forecasts, one cannot readily summarize how these forecasts are made. Private forecasts, however, are widely used in both public and international organizations for comparison purposes with in-house forecasts.

3.2.6 Comparison of Potential Output Measures

Table 3.1 documents some basic moments for estimates of the potential output growth rate (nowcasts) produced by the IMF and OECD as well as the forecasted long-term output growth rate from Consensus Economics. We work with growth rates of potential output rather than levels because the definition of output varies across time (base year) and agencies. The growth rate series are highly correlated and generally have similar moments across sources. This is especially true for the IMF and OECD forecasts, which conceptually are measuring the same objects (nowcasts of potential GDP). Consensus forecasts, in contrast, are at a different horizon and are for actual GDP rather than potential GDP. These strong correlations are not driven by outliers. Indeed, there are few large differences across sources and these tend to be concentrated in a handful of countries and periods (Appendix Figure B.1).

Figure 3.2 illustrates that this strong correlation across series is not restricted to differences in growth rates across countries. Time series for the growth rate of U.S. potential output across the different institutions that produce estimates (Greenbook, CBO, IMF, OECD, Consensus Economics long-term forecasts of actual output) track each other closely as well.

Table 3.1: Comparison of IMF, OECD and Consensus Economics

	Institution and output measure		
	IMF, potential output growth rate (nowcast)	OECD, potential output growth rate (nowcast)	CE, 6-10 year ahead forecast for actual output growth rates
Observations	607	1358	581
Mean	1.64	2.30	2.22
St. Deviation	1.10	1.25	0.54
Correlation			
IMF	1.00		
OECD	0.87	1.00	
CE	0.72	0.78	1.00

Notes: The table reports moments of measures of potential output from the IMF and OECD across countries described in Appendix Table B.1, as well as moments of forecasted growth rates of GDP 6-10 years ahead from Consensus Economics. See Section 3.2.6 for details.

There are nonetheless occasional differences across estimates. After the 1990-91 recession, for example, the CBO reduced its estimate of potential GDP growth significantly more than the staff of the Federal Reserve Board, whereas private forecasters hardly changed their long-term forecasts of growth at all. After the Great Recession, the IMF and OECD both lowered their estimates of potential GDP growth far more than the Greenbooks or the CBO, but then revised them back up while the CBO continued to progressively revise its estimates of potential GDP growth down.

Figure 3.3 plots a longer-time series of estimates of potential GDP available from the Greenbooks, as extended backward by Orphanides (2004). In addition, we plot several statistical approaches to estimating potential GDP, including a one-sided 5-year moving average of real-time GDP and a one-sided HP-filter ($\lambda = 500,000$) of real-time GDP. The HP-filter tracks the Greenbook estimate of potential output quite closely, especially since the mid-1980s while the moving-average approach tends to display larger fluctuations. All series co-move relatively closely with a moving-average of capacity-adjusted TFP changes as measured in Fernald (2012).

The persistence in revisions of potential GDP visible in Figures 3.2 and 3.3 suggests some of these revisions might be predictable from recent changes. We evaluate this formally by regressing revisions of potential GDP on lags of itself:

$$(\Delta \log Y^*_{t|t} - \Delta \log Y^*_{t|t-1}) = \alpha + \beta(\Delta \log Y^*_{t-1|t-1} - \Delta \log Y^*_{t-1|t-2}) + error_t \tag{3.1}$$

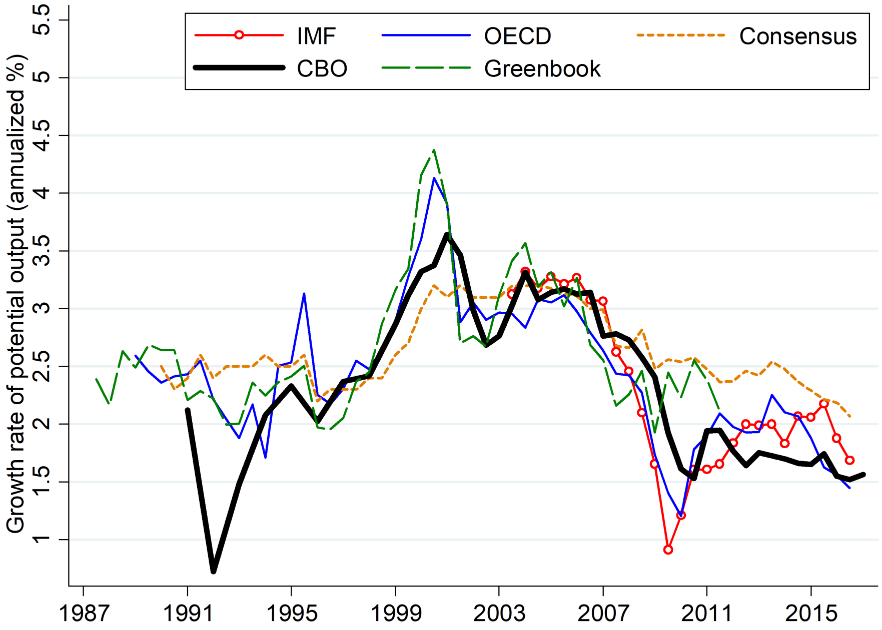

Figure 3.2: U.S. estimates of potential output growth rate

Notes: All series in the figure are real time data at the semi-annual frequency. The potential output for IMF, OECD, and CBO is reported for the current calendar year. Potential output for Greenbooks is the semiannual average of quarterly growth rates of potential output for the quarters in a given semester. Series for Consensus Economics show the 6-10-year-ahead forecast for actual output growth rate (per year).

where $\Delta \log Y^*_{t|s}$ is the growth rate of potential output in time t according to a projection made at time s. We find (Table 3.2) a mild amount of predictability in Greenbook revisions of potential GDP. With CBO, the coefficient on lagged revisions is similar but not significantly different from zero. The results are different for international data, with coefficients on past OECD revisions being not different from zero while those on past IMF and Consensus Economics revisions exhibiting negative predictability.

3.3 How Estimates of U.S. Potential Output Are Adjusted after Economic Shocks

While a limited unconditional predictability is a desirable attribute of estimates of potential GDP, it does not imply that there is no predictability in estimates of potential output conditional on different economic shocks. To assess how estimates of potential output respond to economic shocks, we will combine the estimates described in the previous section with identified measures of economic or policy shocks.

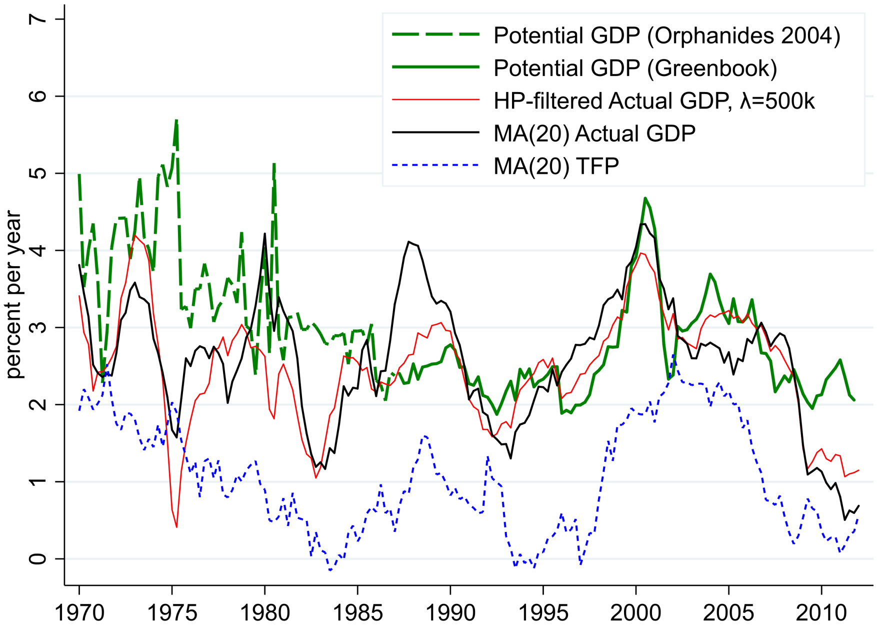

Figure 3.3: U.S. growth rates of potential and actual output

Notes: All series are real-time at the quarterly frequency. Potential output for the pre-1987 period is from Orphanides (2004). Potential output for 1987-2011 is from the Federal Reserve Bank of Philadelphia. Potential is measured as the growth rate of potential output between a given quarter and the next 3 quarters. HP-filtered actual output is calculated as the value of the one-sided HP-filter trend for the quarter given the first vintage of GDP data that covers the given quarter, with HP filter smoothing parameter of 500,000. MA(20) actual output is calculated as the 20-quarter moving average over the current and preceding 19 quarters reported in the first vintage of GDP data that covers the given quarter. MA(20) TFP for a given quarter is the 20-quarter moving average running on the current quarter and the preceding 19 quarters.

3.3.1 Measures of economic shocks

There is a long literature on identifying shocks that potentially drive business-cycle and longer-term fluctuations, particularly for the U.S. (see Ramey (2016) for a survey). Following this literature, we employ several measures of both "demand" and "supply" shocks for the U.S. Our use of the terms "supply" and "demand" reflects certain abuse of terminology. All of the shocks we consider have both supply and demand effects in modern business cycle models. Our classification instead primarily relies on whether these shocks appear to have permanent or transitory effects on GDP. We define demand shocks as those whose real effects appear to be transitory and therefore should not affect estimates of potential output.[4]

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[4] Because the units of these shocks vary, we normalize all shocks to be mean zero and have unit variance.

Table 3.2: Predictability of revisions in estimates of potential

Dependent variable: $(\log Y^*_{t\|t} - \log Y^*_{t\|t-1})$	Source				
	CBO	Greenbook	OECD	IMF	CE
	(1)	(2)	(3)	(4)	(5)
$(\log Y^*_{t-1\|t-1} - \log Y^*_{t-1\|t-2})$	0.204	0.294***	-0.066	-0.154***	-0.355***
	(0.132)	(0.086)	(0.040)	(0.044)	(0.045)
Observations	42	96	1282	548	566
R-squared	0.065	0.085	0.163	0.351	0.288
Number of Countries			31	27	12

Notes: The table presents regressions of the revision in estimates of potential GDP on the previous revision in estimate of potential GDP (equation 1). Newey-West standard errors are in parentheses. "Source" indicates where estimates of potential output come from: Congressional Budget Office (CBO), Greenbooks of the Federal Reserve Board (FED), the Organization for Economic Cooperation and Development (OECD), the International Monetary Fund (IMF) or Consensus Economics (CE). For the latter, revisions are for growth rate of GDP at horizons of 6-10 years. Columns (3)-(5) are across countries and include time and country fixed effects. Within R2 is reported for columns (3)-(5).

For supply shocks, we consider changes in total factor productivity (TFP), oil price shocks and tax shocks. The former are measured as in Fernald (2012), which adjusts Solow residuals for time-varying utilization of inputs. Although these data are somewhat sensitive to vintage (see Kurmann and Sims (2017)), we rely on the final vintage of the data because the data by vintage are available for relatively recent times. For oil price shocks, we use oil supply shocks as identified in Kilian (2009).[5] For tax shocks, we use Romer and Romer (2010)'s narrative measure of exogenous tax changes. To be clear, tax shocks have both demand and supply effects. We denote them here as "supply" shocks because Romer and Romer (2010) document that they have permanent effects on output, and therefore should be captured by estimates of potential GDP.

We consider three identified demand shocks, all related to policy. The first are monetary policy shocks. For the U.S., our baseline measure of these shocks follows the quasi-narrative approach of Romer and Romer (2004). They use the narrative record to construct a consistent measure of policy changes at FOMC meetings since 1969, then orthogonalize these policy decisions to the information available to policymakers at each FOMC meeting, as captured by the Greenbook forecasts prepared by the staff of the Federal Reserve Board before each FOMC meeting. The unexplained policy changes are then defined as the monetary shocks. We use the updated version of these shocks from Coibion, Gorodnichenko, Kueng,

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[5] We also tried using the oil shocks identified by Baumeister and Hamilton (2017) in place of the ones identified by Kilian (2009). The results were very similar and are available from the authors upon request.

and Silvia (2017) and set values after the onset of the zero-bound equal to zero.[6]

The second type of demand shock we consider are the military spending news shocks of Ramey (2016). Using real-time measures of the expected future path of defense spending in the U.S., Ramey constructs a measure of the present discounted value of future defense expenditures each quarter. Changes in these measures from one quarter to the next thus reflect changes in either current or future defense spending.

Finally, we consider a broader measure of government spending shocks, namely differences between ex-post government spending and ex-ante forecasts of that spending following Auerbach and Gorodnichenko (2012). Unlike the Ramey news measure, this measure captures unanticipated short-run changes in government spending but is broader in that it includes more than just military spending.

All three types of demand shocks have repeatedly been found to have only transitory effects on GDP (see Nakamura and Steinsson (2017) and Ramey (2016)), so there is little evidence supporting the hysteresis hypothesis that transitory shocks have long-lived effects on output (and therefore potential) through endogenous productivity or tax responses. As emphasized in Blanchard (2017), these transitory shocks could still affect potential GDP in a transitory fashion in the presence of physical or human capital. As a result, we will study not just the response of nowcasts of potential GDP to these shocks but also of long-run forecasts of potential from the CBO as well as long-run forecasts of GDP growth from private forecasters. The latter two should unambiguously not respond to these transitory shocks. Finally, even if the real-world were characterized by hysteresis, monetary policy-makers explicitly rule out this channel and emphasize that, in their view, monetary policy has only transitory effects on GDP.[7] Their estimates of potential GDP should therefore be invariant to monetary shocks.

3.3.2 Effects of Shocks on Actual Output and Estimates of Potential Output in the U.S.

To provide a benchmark for how we might expect estimates of potential output to respond to economic shocks, we first characterize the response of actual output to these shocks. Specifically, we regress ex-post changes in output on current and past values of a shock as

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[6] We also experimented with monetary policy shocks identified via recursive ordering of VAR residuals as in Bernanke and Blinder (1992) and we found similar results, as documented in Appendix Figure B.5.

[7] For example, in a speech on March 3, 2017, Janet Yellen stated: "Monetary policy cannot, for instance, generate technological breakthroughs or affect demographic factors that would boost real GDP growth over the longer run or address the root causes of income inequality. And monetary policy cannot improve the productivity of American workers. Fiscal and regulatory policies–which are of course the responsibility of the Administration and the Congress–are best suited to address such adverse structural trends."

follows:

$$\Delta \log Y_t = \alpha + \sum_{k=0}^{K} \phi_k \epsilon_{t-k} + error \tag{3.2}$$

where t indexes time (quarters), $\Delta \log Y_t$ is the growth rate of real GDP, ϵ is an identified shock, and *error* is the residual. A key advantage of this moving-average specification is that it allows us to handle data with mixed frequencies and gaps in the time series as well as correlations of the error term. For consistency, we run these regressions at the same time frequency as what is available for estimates of potential output, namely quarterly when comparing to Greenbook forecasts, and semi-annually otherwise. Since Greenbook forecasts of potential output begin in 1987, we run the regression for output over the same time sample. Given the limited number of observations available, we include only one shock at a time (the shocks are roughly uncorrelated). Because the error term is not necessarily white noise, we use Newey-West standard errors everywhere.[8] Impulse responses come directly from the estimates of ϕ. To recover responses of the level of output, we cumulate ϕ_k up to a given horizon. For example, the level responses are ϕ_0 for $h = 0$, $\phi_0 + \phi_1$ for $h = 1$, $\phi_0 + \phi_1 + \phi_2$ for $h = 2$, etc.[9]

For each impulse response, we include 66% confidence intervals and the legend of each associated graph reports the p-values for two types of tests. In parentheses we report the p-value for a test of whether the response of actual output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the response of actual output is different from zero over the entire horizon of the impulse response. These p-values are also included in Panel A of Appendix Table B.2, together with more information that we describe later.

We plot the responses of actual output to each type of shock in Figure 3.4. Panel A focuses on the three supply shocks. In response to a TFP shock, output immediately rises about 0.5% points and remains persistently higher by about that magnitude. Hence, these TFP shocks appear to have permanent effects on output. Tax increases have a (negative) contemporaneous effect on output that is similarly sustained over the entire impulse response horizon. In contrast, negative oil supply shocks have a more delayed effect on output, but are associated with a long-lived decline in GDP. In short, all three supply shocks have the expected long-lived effects on GDP. As a result, we would expect them to be captured by high-quality measures of potential GDP.

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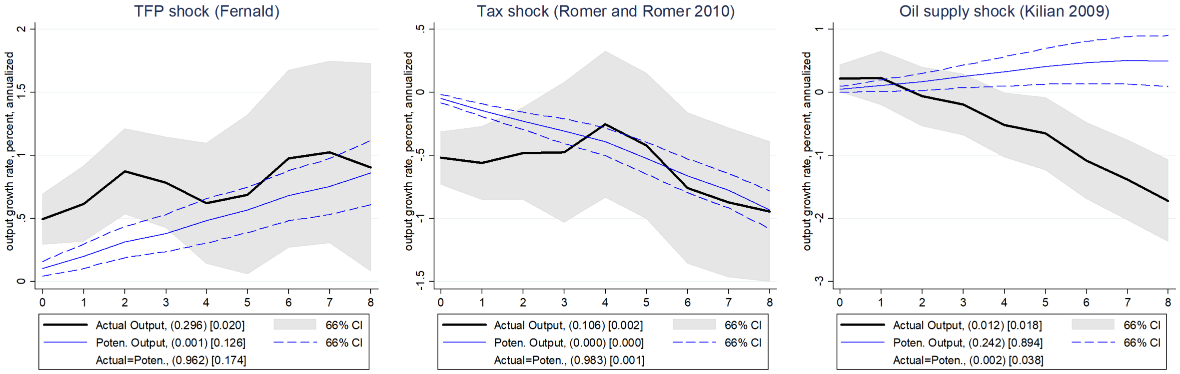
[8] Since the null hypothesis we are testing is that of zero response of output and potential output, the fact that shocks are estimated does not constitute an issue for standard errors and tests of the null hypothesis, as in Pagan (1984).

[9] For monetary policy shocks, we constrain $\phi_0 = 0$ to capture the minimum delay restriction.

Panel A: Supply Shocks



Panel B: Demand Shocks

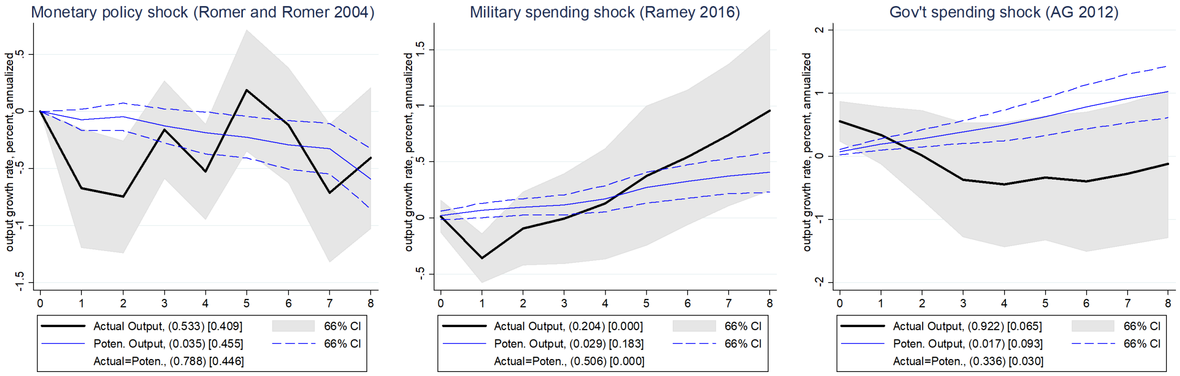


Figure 3.4: Responses of output and Greenbook potential in U.S. to shocks

Notes: The figure reports impulse response functions (IRFs) estimated using equations 3.2 and 3.3. The estimation sample covers the longest possible period with non-missing observations for shocks and potential output (output gap) available at the Federal Reserve Bank of Philadelphia. In parentheses we report the p-value for a test of whether the response of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the response of actual (potential) output is different from zero over the entire duration of the IRF. The last row of the legend reports p-values for a test of equality of responses of actual and potential output at the max horizon (parentheses) and a test of equality of the paths of the responses for actual and potential output are equal across horizons.

Turning to demand-side shocks (Panel B), we again find the expected responses of output. Contractionary monetary policy shocks push output down. The point estimates are much less precise than in Romer and Romer (2004), reflecting the shorter time sample, the fact that monetary shocks are smaller over this limited sample, and the different approach to estimating impulse responses. Increases in expected military expenditures have a delayed positive effect on GDP (which reflects the fact that the expenditures themselves are also generally delayed).[10] Immediate spending shocks as in Auerbach and Gorodnichenko (2012) have transitory short-run effects on GDP and no long-run effects. Demand-side shocks therefore generally deliver cyclical variation in output but no long-run effects on GDP. As a result, we would expect high-quality measures of potential GDP to be insensitive to these shocks.

To characterize the effects of these economic shocks on estimates of potential output, we run equivalent specifications:

$$\Delta \log Y^*_{t|t} = \alpha + \sum_{k=0}^{K} \phi_k \epsilon_{t-k} + error \tag{3.3}$$

where $\Delta \log Y^*_{t|t}$ is the (nowcast) estimated growth in potential in quarter t given information in quarter t at an annualized rate. We first consider Greenbook estimates of potential output and extend our results to alternative estimates of potential in subsequent sections. Responses of the implied level of potential output are constructed in the same way as before. For comparison, we plot the responses of potential output in the same graphs as the responses of actual output, we also include 66% confidence intervals and the p-values for the same tests mentioned above (now for the responses of potential output instead of actual output). Finally, we also include the p-values for a test of whether paths of the responses for actual and potential output are equal over the entire duration of the impulse response (in square brackets) and the p-values of a test of whether the responses are equal at the maximum horizon (in parenthesis). The p-values are also included in Panel A of Appendix Table B.2.

Looking first at TFP shocks, we find that estimates of potential GDP respond very gradually but in the same direction as actual GDP. The shock has little immediate impact on estimates of potential, but after two years, the responses are overlapping and estimates of potential GDP have caught up to actual GDP. Very similar results obtain with tax shocks: estimates of potential GDP are unchanged immediately after the shock, but gradually converge to the path of actual GDP. Hence, with both TFP and tax shocks, one would ultimately attribute the decline in output to a decline in potential output, but only with some delay. One possible reason for delayed responses of forecasts is information rigidity, as suggested in Coibion and Gorodnichenko (2012, 2015b). However, the fact that estimates of potential GDP evolve very gradually after tax shocks (which occur only for large legislative tax changes that staff members at the Board would be well aware of) suggests that other mechanisms must be at play to explain the inertia in real-time estimates of potential output.

Turning to the response to oil price shocks, we find a starkly different response: estimates of potential GDP increase over time while actual GDP falls. In contrast to TFP and tax shocks, in which the long-run response of output is ultimately matched by the response of potential,

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[10] While our horizon of impulse responses is too short to illustrate this, Ramey (2016) shows that news about future military spending has only transitory effects on GDP.

contractionary oil price shocks are associated with sharply falling measured output gaps (Y_t/Y_t^*) in the long run, as estimates of potential are progressively increased while output itself is falling. Policymakers facing a tradeoff between stabilizing inflation (which rises after a negative oil supply shock thereby calling for higher interest rates) and closing the output gap (which is falling and calling for lower interest rates) are therefore perceiving an even starker tradeoff since the rise in the estimate of potential output makes the output gap seem even more negative.[11] This result is not driven by the specific measure of oil supply shocks (we find a similar result with the Kilian (2008) measure of OPEC supply shocks) or by the sample period (we find similar results for alternative periods).

There are several potential explanations for this finding. One is that policymakers are confounding oil supply and demand shocks: if they observe a supply-driven increase in oil prices which they incorrectly attribute to stronger global demand for oil from e.g. improved technology, then this might lead them to revise their estimates of potential GDP upward even as actual GDP is falling. An alternative explanation is that higher oil prices might be perceived as inducing greater investment in new energy sources and alternative energy technologies, which could then raise potential GDP in the long-run even as short-run GDP falls, though there is little evidence that GDP ultimately responds in a positive manner. The available data unfortunately do not enable us to identify the underlying explanation. If nothing else, this result provides a surprising example of how estimates of potential GDP can move in the direction opposite to that of actual GDP.

Turning to demand shocks, we again observe important deviations from what one would expect of estimates of potential GDP. With monetary and both types of fiscal shocks, estimates of potential respond little on impact to these shocks but progressively respond in the same manner as the short-run response of GDP. The transitory decline in GDP after a contractionary monetary shock is followed by a persistent decline in the real-time estimates of potential GDP, while the transitory increase in output after an increase in government spending is followed by a persistent rise in estimates of potential GDP. Hence, these cyclical fluctuations in output lead to the perception among forecasters that they are permanently affecting output, as if they were TFP or tax shocks, despite the fact their effects on income are actually short-lived.

Our results are not limited to these specific examples of identified shocks. For example, we can identify supply and demand shocks jointly as in Blanchard and Quah (1989) by running a VAR with output growth and unemployment and restricting demand shocks to have no long-run effects on output. When we use these supply and demand shocks to characterize the response of real-time estimates of potential output over the same period, we again find that real-time estimates respond very gradually to both shocks, moving in the direction

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[11] The pronounced decline in the perceived output gap after oil supply shocks is consistent with the view that monetary policymakers were too willing to accommodate these shocks with lower interest rates and that this accommodation may have contributed to the Great Inflation of the 1970s.

of the change in output (Appendix Figure B.2). Importantly, because this identification explicitly imposes that only supply shocks have permanent effects on GDP, it addresses the possibility that some demand shocks might have hysteresis effects and therefore should be incorporated in estimates of potential GDP. In short, across identification schemes, we find an over-response of real-time estimates of potential GDP to demand shocks and an under-response to supply shocks.

3.3.3 Robustness of Baseline Results for the U.S.

Because of the relatively short samples involved, we want to verify that our results are robust to a range of reasonable variations. Our first check is on the empirical method used to estimate impulse responses. As an alternative to equations (3.2) and (3.3), we reproduce impulse responses of actual output and nowcasts of potential GDP to each of the shocks using auto-distributed lag specifications to estimate IRFs as in Romer and Romer (2004), namely:

$$\Delta \log Y_t = \alpha + \sum_{j=1}^{J} \delta_j \Delta \log Y_{t-j} + \sum_{k=0}^{K} \phi_k \epsilon_{t-k} + error \tag{3.4}$$

using $J = 4$ and $K = 8$. Results are presented in Appendix Figure B.3. By and large, the results are very similar. With productivity and tax shocks, we continue to find persistent but delayed effects on estimates of potential GDP that are ultimately converging to the responses of actual GDP. Similarly, with all three demand shocks, we find the same qualitative patterns as with the previous empirical specification. The only difference lies in the response to oil supply shocks, where we no longer observe a pronounced rise in estimates of potential GDP. Instead, our estimates instead point toward no response of the nowcasts of potential, suggesting some sensitivity in this result.

One potential source for this empirical sensitivity is the limited time sample. As a result, we replicate our baseline results over an extended time period, where for each shock we now use the maximum time sample available across both the shocks and the Greenbook estimates of potential GDP (1969-2011). The results, presented in Figure 3.5, confirm our baseline findings: there is a delayed but persistent response of the estimates of potential GDP to all shocks. In every case but oil supply shocks, the nowcasts evolve in the direction of the short-run changes in GDP. With oil supply shocks, the estimates of potential GDP rise in an even more pronounced fashion while actual output falls.[12] Hence, the baseline results are not specific to the period since 1987.

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[12] When we apply the ADL specification to oil supply shocks over the whole sample, we find the same result.

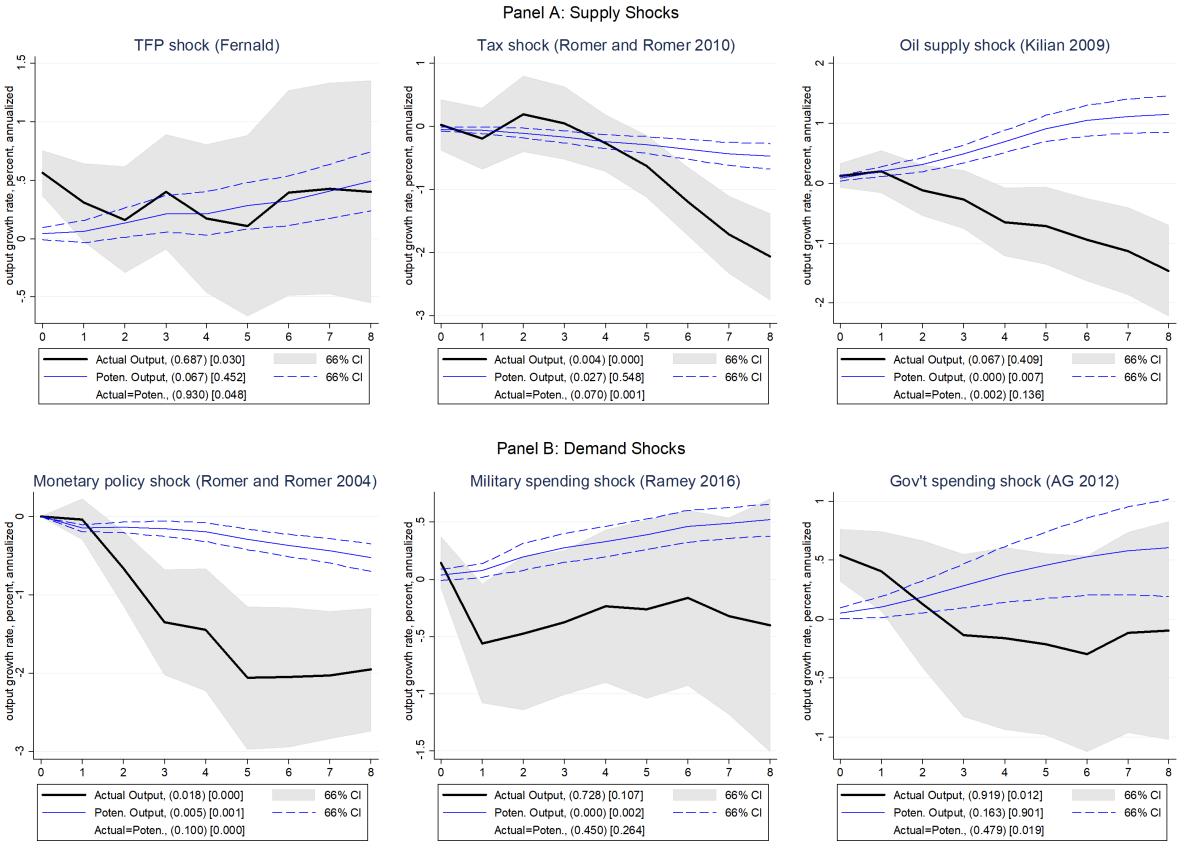

Figure 3.5: Responses of output and Gre. potential (extended sample)

Notes: The figure reports impulse response functions (IRFs) estimated using equation 3.2 and 3.3. The estimation sample covers the longest possible period with non-missing observations for shocks and potential output (output gap) using output gap data starting in 1970. In parentheses we report the p-value for a test of whether the response of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the response of actual (potential) output is different from zero over the entire duration of the IRF. The last row of the legend reports p-values for a test of equality of responses of actual and potential output at the max horizon (parentheses) and a test of equality of the paths of the responses for actual and potential output are equal across horizons.

We also consider whether our results are sensitive to relying on nowcasts of potential GDP growth. Because Greenbooks also include forecasts and backcasts of potential GDP growth (2 years in each direction), we can characterize how the perceived path of potential GDP evolves after each shock. We find very little difference relative to nowcasts, implying that Federal Reserve staff raise or lower the entire path of projected and past potential GDP growth in response to shocks (Appendix Figure B.4).

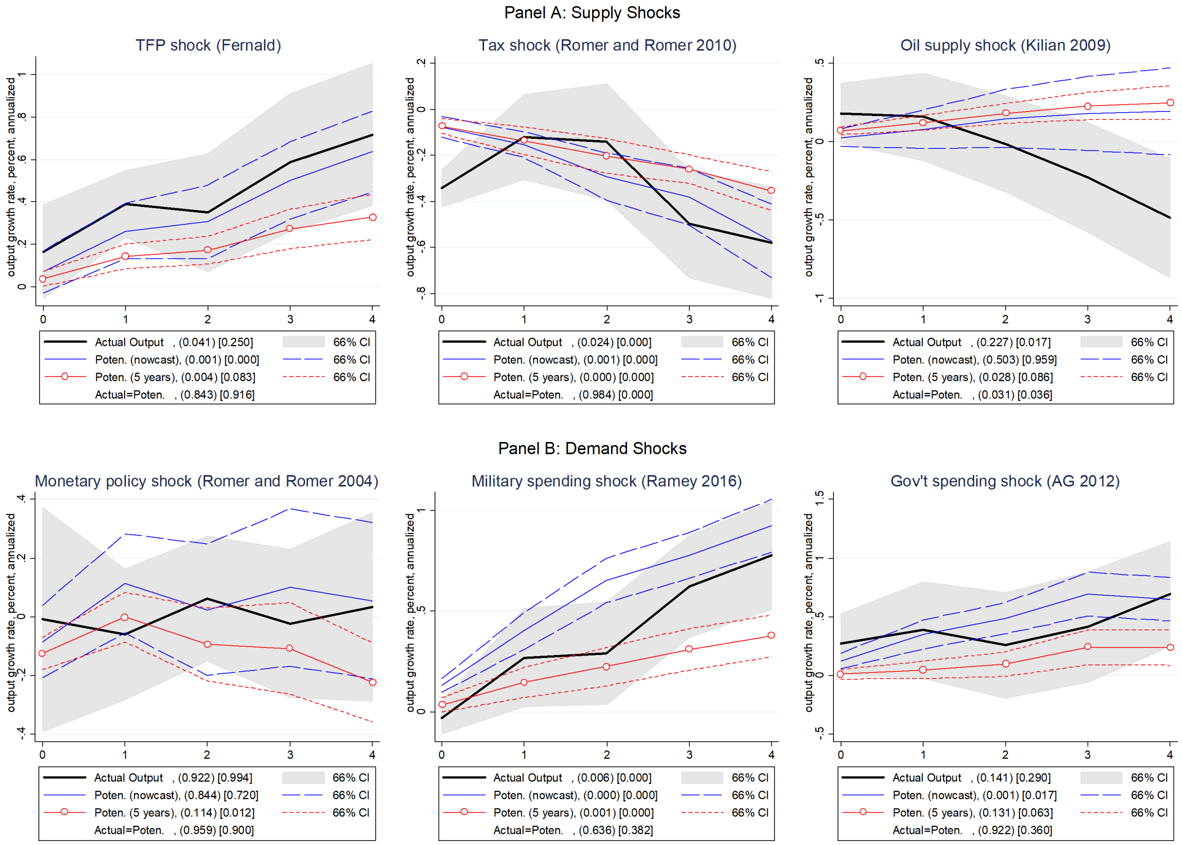

Figure 3.6: Responses of output and CBO potential in U.S. to shocks

Notes: The figure reports impulse response functions (IRFs) estimated using equations 3.2 and 3.3. The estimation sample covers the longest possible period with non-missing observations for shocks and potential output (output gap) available from the Congressional Budget Office. In parentheses we report the p-value for a test of whether the response of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the response of actual (potential) output is different from zero over the entire duration of the IRF. The last row of the legend reports p-values for a test of equality of responses of actual and potential output (nowcast) at the max horizon (parentheses) and for a test of equality of the paths of the responses for actual and potential (nowcast) output are equal across horizons.

Another potential issue with these results is our reliance on estimates of potential GDP from a single source, the staff of the Federal Reserve Board. In Figure 3.6, we reproduce our results using estimates of potential GDP from the Congressional Budget Office. One advantage of CBO estimates is they are available at longer horizons. As a result, we consider both "nowcasts" of potential GDP (equivalent to Greenbook estimates) as well as 5-year ahead forecasts (that is, the growth rate of potential output in five years from the date when a forecast is made). A disadvantage of CBO estimates, as discussed in Section 3.2.1, is that the sample for these is more limited and the time frequency at which forecasts are available is reduced. Not surprisingly, the effects of each shock on GDP are therefore considerably less precisely estimated. However, the responses of the estimates of potential GDP are still quite precise. Qualitatively, we find that CBO estimates of current potential GDP respond much like those from the Greenbooks: gradually but persistently to all shocks. Long-run forecasts of potential GDP generally respond by less than those of current potential GDP. However, they still ultimately respond to demand shocks, implying that the CBO implicitly interprets cyclical shocks as having permanent effects on GDP.

The fact that CBO forecasts of long-run potential respond similarly to nowcasts of potential GDP addresses one possible issue raised in Blanchard (2017), namely that demand shocks might have transitory effects on potential output. This can occur even in standard models through a number of channels, such as lower levels of physical capital following periods of disinvestment or lower levels of human capital after extended unemployment stretches. But in these models, demand shocks would still have only transitory effects on potential output, so forecasts of long-run potential output should remain unchanged after demand shocks even if contemporaneous levels of potential were responding to these shocks. The fact that both nowcasts and long-run forecasts of potential respond to demand shocks suggests that the mechanism emphasized in Blanchard (2017) is not driving these results.

In short, we document a systematic response of estimates of potential GDP to shocks that have only cyclical effects on GDP. Furthermore, even some supply shocks have contradictory effects on estimates of potential GDP, in the sense that changes in the latter after oil supply shocks speak little to actual long-run changes in output. Thus, seeing ex-post that declines in GDP seem to be accounted for by changes in potential GDP, as has been the case in the U.S. since the Great Recession, says little about whether the decline in output is likely to persist or can be reversed by standard countercyclical policies.

3.3.4 Explaining Patterns in Impulse Responses

Why are estimates of potential GDP responding to shocks that only have cyclical effects, such as monetary policy and government spending shocks? One possibility is that policy institutions and statistical agencies perceive these shocks as affecting current levels of potential output (e.g., if they affect current capital stocks) but not long-run levels of potential output (as would be implied by e.g. monetary neutrality). This is unlikely to be the case, however, since the long-horizon CBO forecasts of potential GDP respond approximately as much as their nowcasts of potential GDP.

An alternative possibility is that these estimates are relying to a large extent on simple statistical methods to measure trend (potential) levels from actual GDP. As illustrated in

Figure 3.3, one can come close to replicating the real-time Greenbook estimates of potential GDP growth by using a one-sided HP-filter on real-time GDP data available each quarter or by taking a simple one-sided moving-average of recent GDP outcomes. Since these types of methods fail to identify the different potential sources of changes in economic activity, they would naturally lead to slow-moving dynamic responses to all economic shocks that move actual output.

To assess this possibility, we replicate our baseline impulse responses using the same two statistical approaches to estimating potential GDP as in Figure 3.3. In the first case, we apply a one-sided HP-filter with smoothing parameter $\lambda = 500,000$ to real-time data on GDP. In the second, we take a 5-year moving average of real GDP using real-time data. We present the results, along with the responses of potential GDP as measured by the Greenbooks in Figure 3.7 (and the p-values are included in Panel C of Appendix Table B.2). When using the HP-filtered series, we can very closely replicate the response of estimated potential GDP after every shock.[13] With the moving average, the fit is not as strong. The very close fit of the impulse responses using the HP filter, as well as how closely one can reproduce the unconditional time series of historical estimates of potential GDP in Figure 3.3 with an HP-filtered series, suggests that Greenbook estimates of potential GDP incorporate little additional information relative to this purely statistical approach to estimating potential GDP.[14] It is then quite natural for these series to respond to all shocks that affect GDP, even if these movements are transitory in nature. But this endogenous response to cyclical shocks should then not be interpreted as reflecting permanent effects of these shocks on output but rather as a mechanical reaction based on how estimates of potential GDP are constructed. Equivalently, observing a downward revision in real-time estimates of potential GDP is not informative about whether the associated declines in actual GDP are likely to be sustained or not.

Another way to see how closely the HP-filter can mimic real-time estimates of potential GDP, as well as the potential dangers of doing so, is illustrated in Figure 3.8. In Panel A, we plot the time path of potential GDP that would have been estimated in real-time using the HP-filter during the Great Recession period. Specifically, for each quarter, we apply an

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[13] The fact that we can match the increase in estimated potential output after an oil supply shock with the HP-filter points toward a possible identification issue with these shocks. They are identified from a 3-variable VAR of oil production, global economic activity (measured using an index of shipping prices) and oil prices. If oil prices are disproportionately sensitive to U.S. output (rather than global output) or shipping prices are an otherwise imperfect measure of global activity, then one might observe identified oil supply shocks disproportionately happening after sustained U.S. economic expansions (since oil prices and production are endogenous). This could lead an HP-filter of real GDP to rise after an oil supply shock.

[14] The best match of HP-filtered series comes with high values of λ (we use $\lambda = 500,000$). This high value is consistent with a low pass filter that allows only low frequencies with periods of about 10 years and higher. Lower values do not replicate Greenbook measures of potential GDP as closely, as can be seen in Appendix Figure B.6. Similarly, with moving average measures, we can better replicate the dynamic response of Greenbook estimates of potential when averaging over long periods (10-20 years) than over shorter horizons (3-5 years) as illustrated in Appendix Figure B.5.

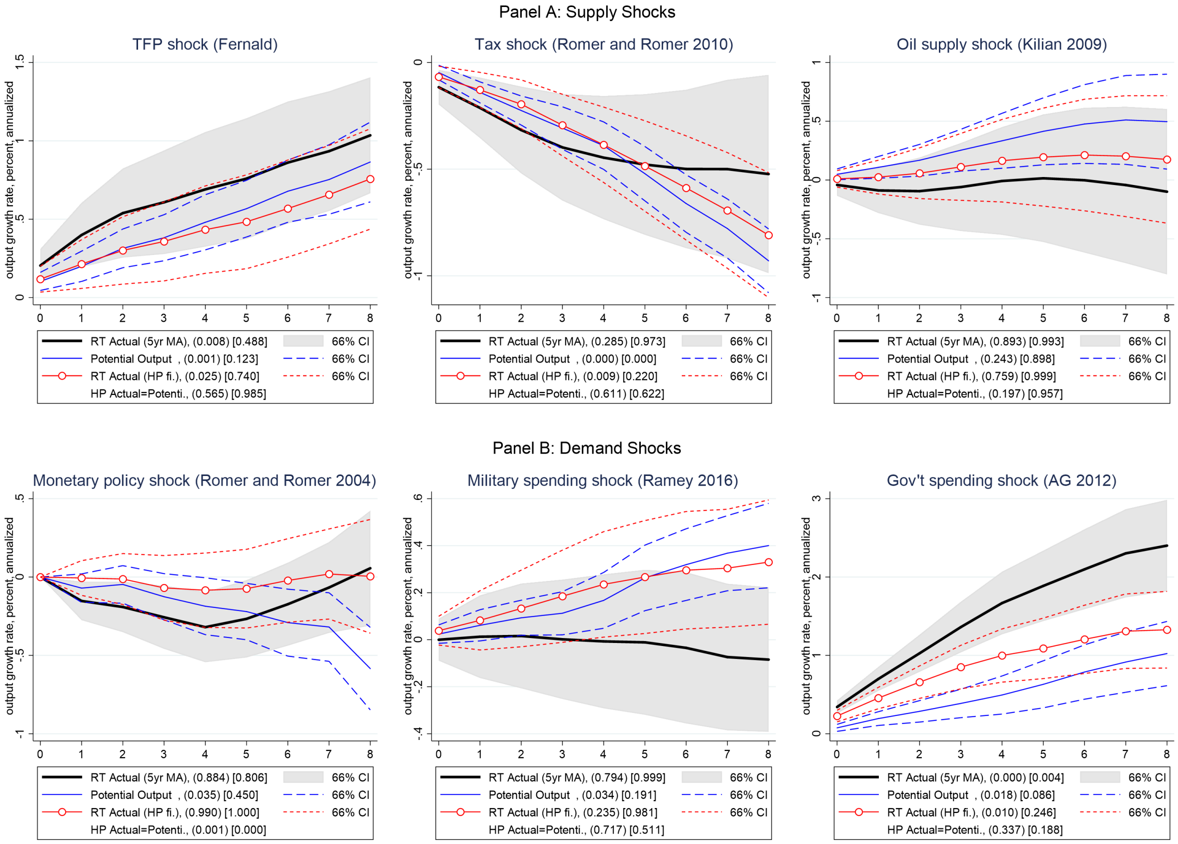

Figure 3.7: Responses of Gre. potential and HP filtered output to shocks

Notes: IRFs estimated using equations 3.2 and 3.3. The sample covers the longest possible period with non-missing observations for shocks and potential output available at Philadelphia FRB. HP-filtered actual is calculated as the value of the HP-filter trend for the quarter given the first vintage of GDP data that covers the given quarter. The smoothing parameter for the HP filter is set at 500,000. 5-year MA actual is calculated as the 20-quarter moving average running on the current quarter and the preceding 19 quarters reported in the first vintage of GDP data that covers the quarter. In parentheses we report the p-value for a test of whether the response of actual (potential) output is different from zero at the max horizon, while in brackets the p-value for whether the path of the response of actual (potential) output is different from zero for all horizons. The last row of the legend reports p-values for a test of equality of responses of actual and potential output at the max horizon (parentheses) and for a test of equality of the paths of the responses for actual and potential.

HP-filter to available data and extract the trend level for that period. We then plot the sequence of these estimates over time, thereby showing the evolution of the implied real-time trend level of GDP during this historical episode for different values of the smoothing parameter. Regardless of the smoothing parameter, estimates of real-time trend output from an HP-filter exhibit a significant downward revision (the magnitude of the revision declines in λ), much like the real-time estimates of official organizations in the U.S., providing another illustration of how closely one can reproduce historical real-time estimates of potential output using a simple statistical filter. The danger of doing so is illustrated in Panel B of Figure 3.8, which replicates this exercise for the Great Depression using data from Ramey and Zubairy (2018). The use of an HP-filter to estimate potential GDP in real-time over the course of the Great Depression would have implied that the output gap was closed sometime between 1934 and 1936, depending on the smoothing parameter. But as illustrated in Figure 3.8, GDP surged thereafter and real-time estimates of potential GDP begin to climb back up. Unless one is prepared to entertain the idea that the Great Depression reflected negative supply shocks that were offset by positive supply shocks in the mid to late 1930s, we interpret this experience as illustrating the potential pitfalls of relying on simple statistical filters to make inferences about potential output during long-lived downturns.[15]

3.4 Cross-Country Evidence on the Incorporation of Shocks into Potential

The Great Recession was of course not limited to the U.S. and the persistence of output declines in most major advanced economies has also been associated with declines in their potential output, as documented in Ball (2014). Indeed, despite widespread lackluster growth by historical standards since the Great Recession, the World Bank recently estimated that advanced economies have an output gap of zero on average, indicating that the large downward revisions to potential output estimated by the CBO for the U.S. since 2007 extend to other advanced economies World Bank (2018). To what extent do the cyclical patterns documented above in estimates of potential GDP generalize to other countries? In this section, we turn to cross-country estimates of potential GDP, both from international organizations as well as from professional forecasters. Using international data gives us many more observations and thus more statistical precision and power.

3.4.1 IMF and OECD Estimates of Potential GDP

We consider first estimates of potential GDP from two international organizations, the IMF and the OECD. Both provide estimates of the level of potential GDP for a wide range of countries.[16]

We follow the same strategy as with the U.S. and compare impulse responses of actual GDP

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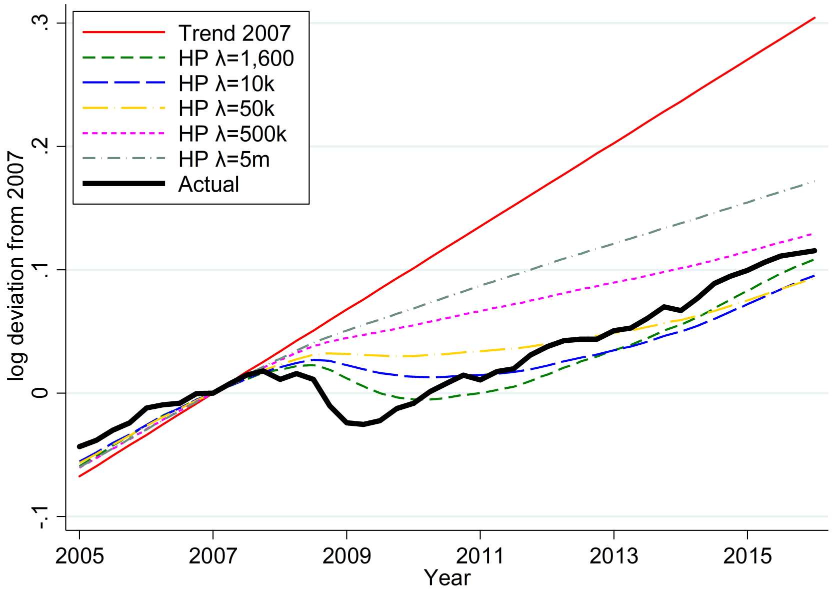
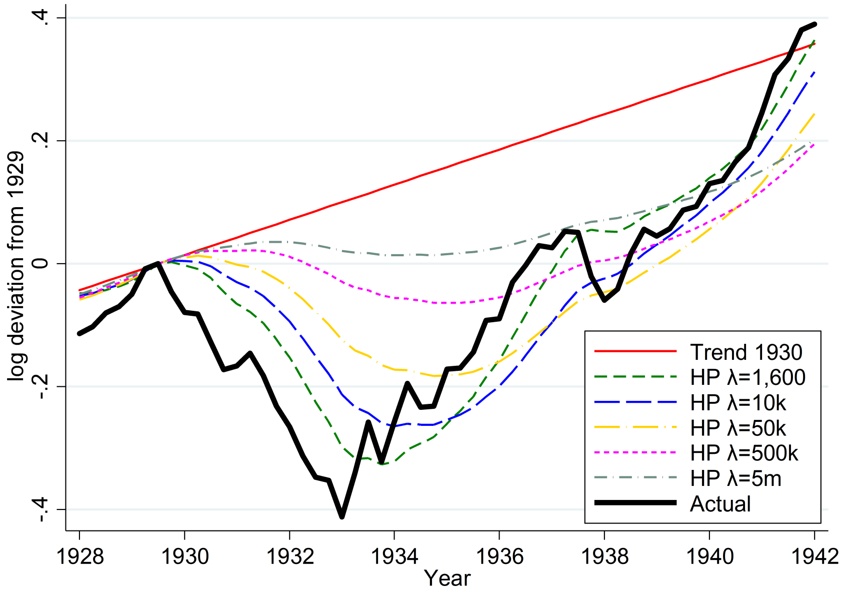
[15] Papell and Prodan (2012) analyze large recessions in the U.S. and other countries using long samples. Consistent with our analysis of the Great Depression, they find that actual output eventually catches up with pre-recession projections of potential output. Gordon and Krenn (2010) document that using a bandpass filter to estimate potential GDP during the Great Depression would similarly imply implausible declines in potential between 1929 and the mid-1930s.

[16] We exclude Norway from our analysis because this country relies heavily on energy exports.



(a) The Great Recession



(b) The Great Depression

Figure 3.8: HP Potential during Great Recession and Great Depression

Notes: The figure report estimates of trend (potential output) generated by the one-sided Hodrick-Prescott filter for various values of the smoothing parameter λ. The filter is recursively applied to the final vintage of the data. For example, an estimate for 2008Q1 uses data only up to 2008Q1, an estimate for 2008Q2 uses data only up to 2008Q2, etc. Data in Panel B are from Ramey and Zubairy (2018).

and estimates of potential GDP from each of these two organizations to different economic shocks. However, because time samples are much shorter for most countries, we pool data across all countries in our sample. In short, for each identified shock ϵ, we estimate the following specifications:

$$\Delta \log Y_{j,t|t} = \alpha_j + \gamma_t + \sum_{k=0}^{K} \phi_k \epsilon_{j,t-k} + error_{j,t} \tag{3.5}$$

$$\Delta \log Y^*_{j,t|t} = \delta_j + \kappa_t + \sum_{k=0}^{K} \psi_k \epsilon_{j,t-k} + error_{j,t} \tag{3.6}$$

where j indicates the country and α_j, δ_j and γ_t, κ_t denote country and time fixed effects respectively. The time frequency is semi-annual, as determined by the frequency of real-time estimates of potential GDP by both the IMF and OECD.

Because of more limited data availability across countries, we cannot identify as many shocks and in the same way as done for the U.S. For productivity, we use innovations in labor productivity, after conditioning on past changes in labor productivity as well as country and time fixed effects.[17] For oil shocks, we continue to use the Kilian measure of oil supply shocks but interact it with a country-specific measure of oil sufficiency (from the International Energy Agency's (International Energy Agency, 2017) World Energy Statistics and Balances, available via the OECD) to distinguish it from the time fixed effects.[18] For monetary policy shocks, we run a VAR for each country on GDP growth, unemployment, inflation and the interest rate and apply a Choleski decomposition on this ordering to recover country-specific interest rate shocks. The VAR has four lags using quarterly data from 1980Q1 until 2016Q4 or as available.[19] Finally, fiscal shocks are differences between ex-post government spending and ex-ante forecasts of government spending from the OECD, following Auerbach and Gorodnichenko (2012).

Turning first to the OECD sample of countries and estimates of potential GDP, Figure 3.9

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[17] Specifically, we use a measure of labor productivity at the semiannual frequency taken from the OECD and then regress it on lags of itself in a panel regression with country and time fixed effects, allowing coefficients on the lags of labor productivity to vary over countries, as well as a dummy for Ireland in 2015 due to its very big outliers in terms of productivity changes. It is important to notice that this OECD measure of labor productivity is highly correlated with other measures of productivity, such as multifactor productivity from the OECD or productivity from EU-KLEMS data.

[18] Oil sufficiency measures what percentage of total oil usage can be satisfied from each country's supply. Hence it ranges from 0 (if the country has no oil supply at all, for example Belgium), passing through 1 (if the country can exactly satisfy its oil demand, for example Australia) up to high numbers like 20 (if the country is a net exporter of oil).

[19] A group of countries is in the eurozone after 1999. For these countries, we construct monetary policy shocks as follows. For the pre-euro period, we run a country-specific VAR and obtain monetary policy as described in the text. For the euro-period, we run a VAR with variables measured at the level of the eurozone. From this VAR, we obtain monetary policy shocks which we append to the shocks identified in the pre-euro period.

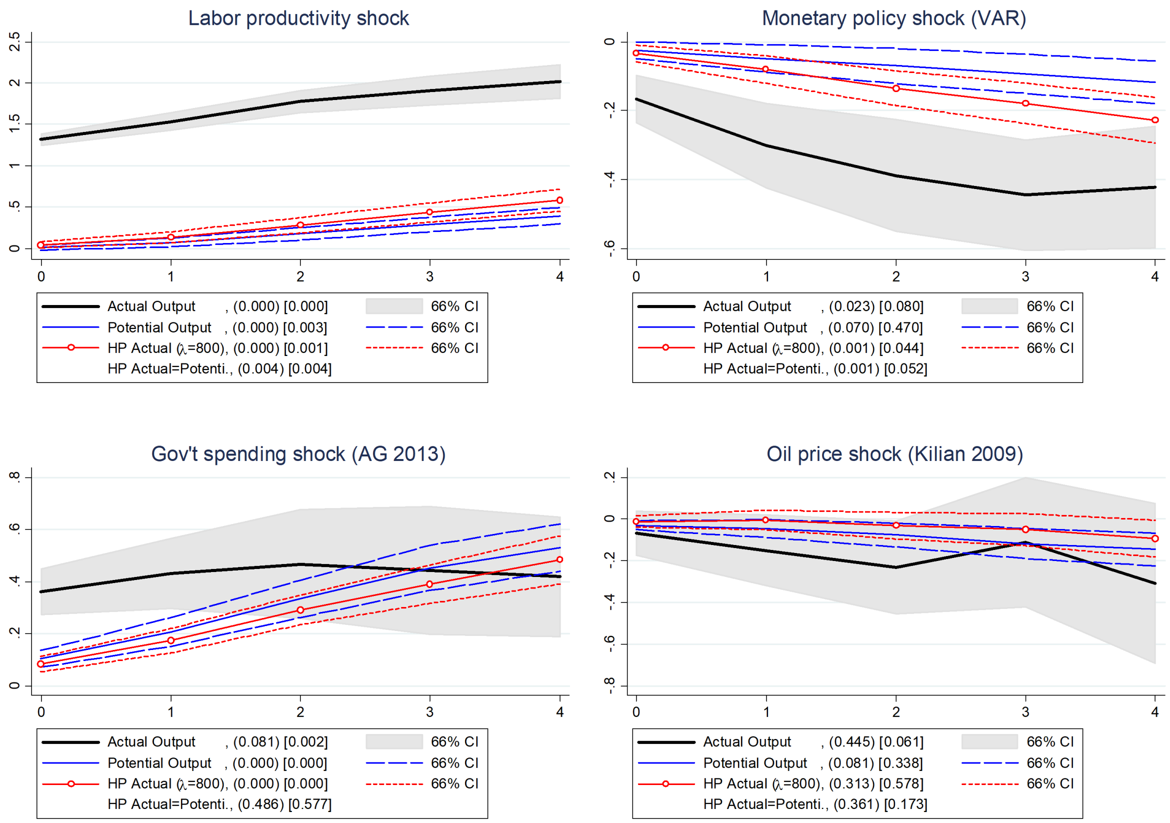

Figure 3.9: Response of the growth rate for actual and OECD's potential

Notes: The figure shows impulse response functions (IRFs) for growth rates of actual and potential output (nowcast). IRFs are estimated using equations (3.5) and (3.6). The horizontal axis measures time in semesters (6 months). The vertical axis measures growth rate of output per year. In parentheses we report the p-value for a test of whether the response of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the response of actual (potential) output is different from zero across all horizons of the IRF. The last row of the legend reports p-values for a test of equality of responses of actual and potential output at the max horizon (parentheses) and for a test of equality of the paths of the responses for actual and potential output are equal across horizons.

presents responses of both GDP and potential to each of the four shocks (the p-values for the same tests discussed in Section 3.3 are included in the figure and summarized in Appendix Table B.3). All four shocks yield the expected changes in GDP. Productivity shocks have an immediate and permanent effect on output while oil supply shocks have a negative albeit delayed persistent effect on output. Both demand shocks have transitory effects on GDP

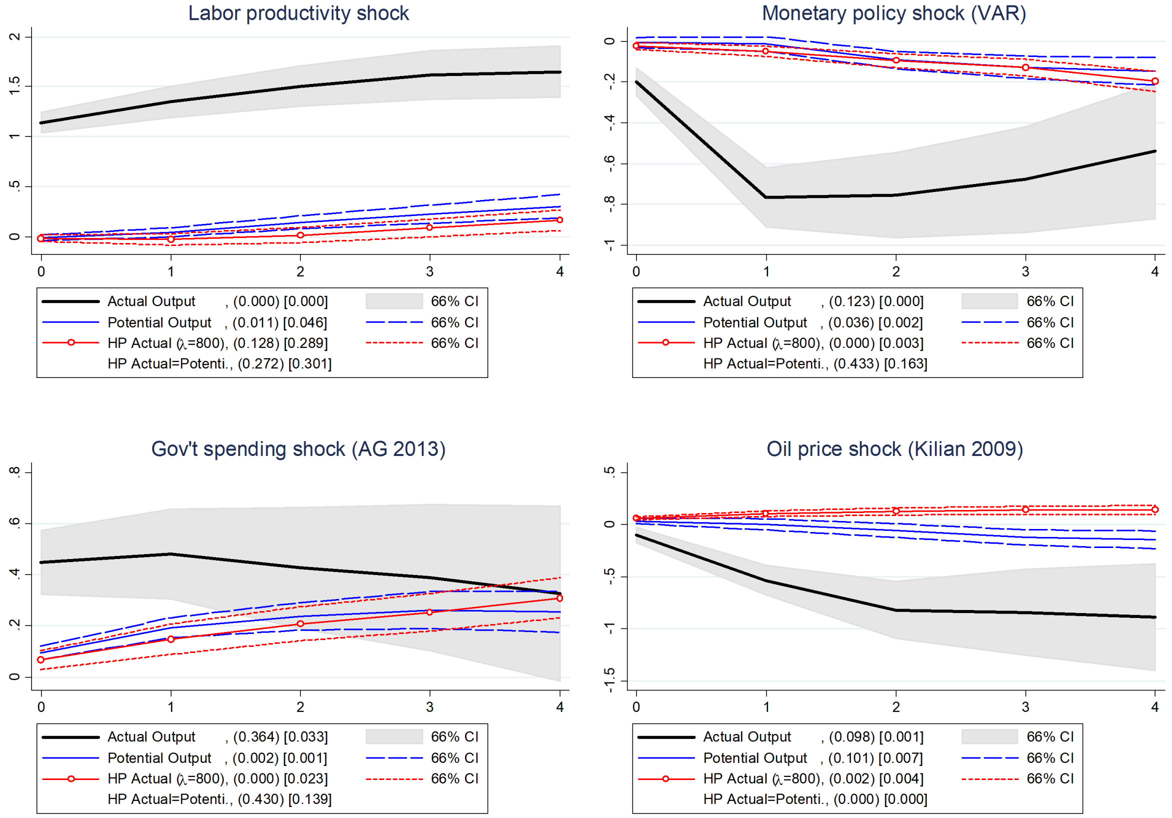

Figure 3.10: Response of the growth rate for actual and IMF's potential

Notes: The figure shows impulse response functions (IRFs) for growth rates of actual and potential output (nowcast). IRFs are estimated using equations (3.5) and (3.6). The horizontal axis measures time in semesters (6 months). The vertical axis measures growth rate of output per year. In parentheses we report the p-value for a test of whether the response of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the response of actual (potential) output is different from zero across all horizons of the IRF. The last row of the legend reports p-values for a test of equality of responses of actual and potential output at the max horizon (parentheses) and for a test of equality of the paths of the responses for actual and potential output are equal across horizons.

which start dissipating around one or one and a half years and are mostly gone after three years (we only show IRF's up to 4 semesters in the figure).

The effects of these shocks on potential GDP are consistent with those obtained for the U.S. In response to productivity shocks, estimates of potential GDP evolve gradually in the direction of actual changes in output. After oil supply shocks, estimates of potential GDP decrease slightly, but this response is very weak. After both demand shocks, estimates of potential GDP gradually and persistently evolve in the same direction as the short-run changes in GDP even though these changes in GDP are transitory. Thus, we observe both the undercyclicality after productivity shocks and over-cyclicality after demand shocks documented in the U.S.

Furthermore, we include in the figure the impulse response of HP-filtered real GDP (constructed for each country using real-time data and a one-sided filter) to each shock. As was the case with the U.S., we find that HP-filtered GDP responds almost identically to each shock as the OECD's estimates of potential GDP. As was the case with the Greenbook estimates of potential GDP, OECD estimates do not appear to capture much more information than what is embodied in a simple univariate filter of real-time actual GDP growth rates, which can account for why their estimates of potential GDP growth rates therefore respond to shocks that have only cyclical effects on GDP.

In Figure 3.10, we produce equivalent results for the IMF sample of countries and IMF estimates of potential GDP. Despite the different countries in the sample, the estimated effects of the shocks on actual GDP are very similar as those found in the OECD sample. The responses of the IMF's estimated levels of potential GDP respond similarly as those from the OECD: they rise inertially after productivity shocks, and respond inertially as well after monetary and fiscal shocks, in the same direction as the short-run response of GDP. Their response after oil supply shocks is equally weak. We also again include for comparison responses of real-time HP-filtered output and find, as with the OECD, that these very closely track the IMF estimates of potential output after shocks, with the only exception again being oil supply shocks.

Overall, the evidence from these two international organizations closely aligns with previous evidence from the U.S.: their estimates of potential GDP are well-approximated by an HP-filter applied to real-time data and therefore seem to respond mechanically to short-run changes in GDP, regardless of the underlying source of economic variation. This suggests that observing revisions in one of these organizations' estimates of potential GDP in a country tells us little about how persistent the concurrent changes in GDP are likely to be.

3.4.2 Private Long-Horizon Forecasts of GDP growth rate

In addition to forecasts from international policy organizations, we consider how private forecasters adjust their beliefs about the long-run GDP growth rate in response to shocks. While forecasts of potential GDP are not readily available, Consensus Economics provides forecasts of GDP at long-horizons on a semi-annual basis. To the extent that cyclical fluctuations in GDP should be complete within 5 or so years, these long-horizon forecasts should be equivalent to forecasts of potential GDP growth at the same horizon.

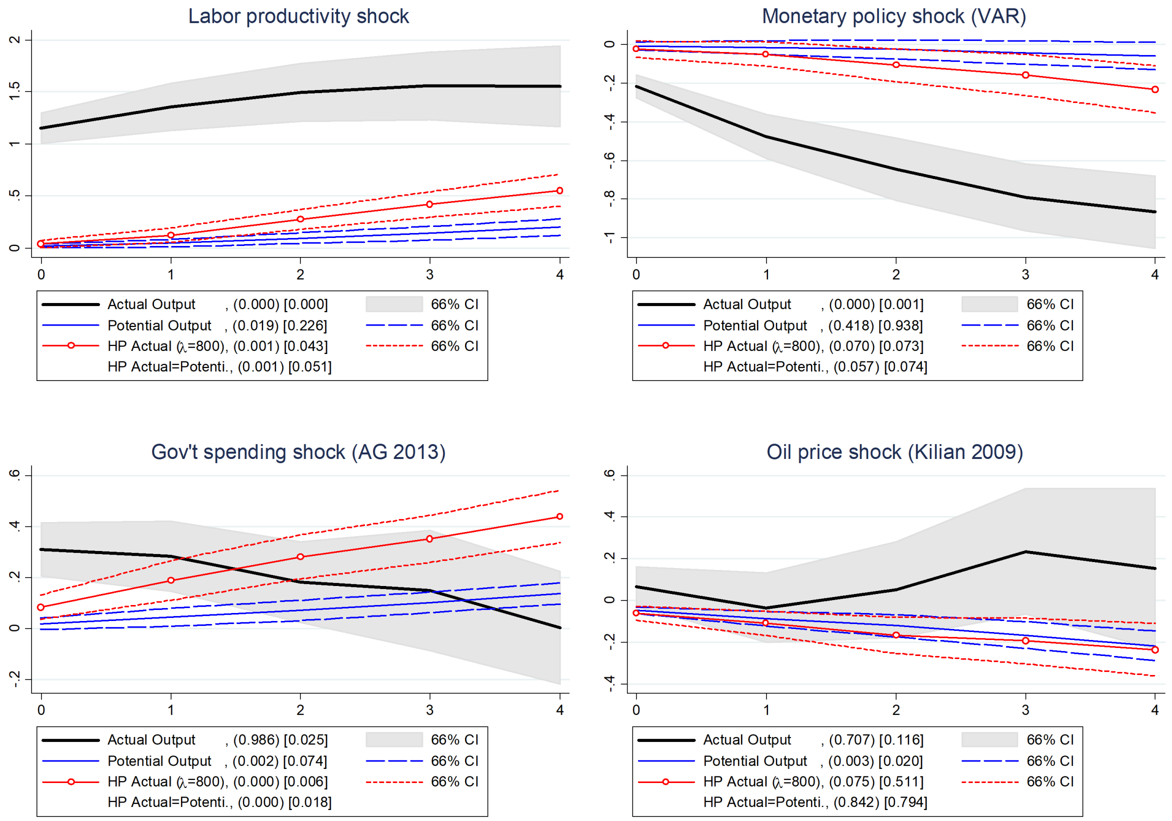

Figure 3.11: Response of the growth rate for actual and CE's forecast

Notes: The figure shows impulse response functions (IRFs) for growth rates of actual output and 6-10-year ahead forecast for actual output growth rate (Consensus Economics). IRFs are estimated using (3.5) and (3.6). The horizontal axis measures time in semesters (6 months). The vertical axis measures growth rate of output per year. In parentheses we report the p-value for a test of whether the IRF of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the IRF of actual (potential) output is different from zero across all horizons of the IRF. The last row of the legend reports p-values for a test of equality of responses of actual and potential output at the max horizon (parentheses) and for a test of equality of the paths of the responses for actual and potential output are equal across horizons.

Using the same shocks as those used with OECD and IMF samples, we replicate our previous results using private forecasts of long-run GDP for the 12 countries for which we have these forecasts. With the different sample of countries and time periods, the impulse responses of actual GDP are broadly similar (Figure 3.11), although the output responses to monetary shocks are more persistent while the response to oil supply shocks is much less precise.

After productivity shocks, private forecasts gradually evolve in the same direction as actual output, therefore replicating the pattern observed with forecasts from public and international organizations. After the two demand shocks, the private sector forecasts also gradually evolve in the direction of the short-run movements in GDP, although the response after monetary shocks is not significant at standard levels. With respect to oil supply shocks, private forecasts of long-run GDP decline gradually.

For comparison, we also plot the implied response of HP-filtered levels of output to the same shocks and countries. For all shocks HP-filtered forecasts evolve in the same direction as private forecasts but more rapidly. This is in contrast to what was found with estimates of potential from public and international organizations when the estimates of potential GDP were almost identical in the impulse responses to those of an HP-filtered level of output. The more inertial response of private forecasters could reflect less rapid information updating or a difference in forecasting horizon (private forecasts are for long-run levels of GDP rather than current estimates of potential GDP).

3.5 Alternative Approaches to Estimating Potential

The apparent inability of available estimates of potential output to differentiate between shocks that have permanent effects and those with only transitory effects raises the question of whether alternative approaches might do better. Obviously, this is a challenging task and developing a single satisfactory method is beyond the scope of the paper. However, we can utilize available tools to get a glimpse of what may constitute a basis for a satisfactory method to estimate potential output. Specifically, we first use the Blanchard and Quah (1989) approach, designed specifically to separately identify supply and demand shocks, to show that long-run restrictions may provide a practical solution to some of the issues we have identified above. We show that this approach implies significantly different estimates of potential output during the Great Recession, and that alternative approaches yield similar conclusions.

3.5.1 Blanchard and Quah Approach to Estimating Potential

In this simple, proof-of-concept exercise, we follow Blanchard and Quah (1989, BQ henceforth) and estimate a bivariate VAR(8) where the variables are output growth and the unemployment rate. The identifying restriction of this model is as follows: supply-side shocks are the structural shocks that have permanent effects on the level of output and demand-side shocks are restricted to have zero effect on the level of output in the long run. We then interpret predicted movements in output driven by supply-side shocks as capturing potential output. The restriction that only supply-side shocks have permanent effects on output is broadly consistent with the responses of demand observed in Figure 3.4 and other results in the literature, namely that monetary and government spending shocks do not seem to have permanent effects on output (e.g. Romer and Romer (2004), Ramey (2016)).

Because BQ and others emphasize the importance of structural breaks, we use a rolling window of 120 quarters.[20] When applying the BQ approach, we use real-time data to ensure that our results are not driven by information not available to the econometrician. In a particular quarter (say 1995Q1) we use the vintages of real output growth and unemployment rate that were available at that point in time (obtained from the FRB of Philadelphia's real time database for macroeconomists), estimate the SVAR with long run restriction using these series and then perform the historical decomposition on this data to recover the component of the growth rate of actual output due to supply-side shocks for the given quarter. That is, we keep only the data point that corresponds to the last quarter in a rolling-window sample. The next quarter's (1995Q2) historical decomposition data point is going to use vintages that were not available yet in 1995Q1, and the previous quarter's (1994Q4) historical decomposition data point used vintages that contained less information and stopped in 1994Q4. This approach therefore uses no more information than what was available to agents in real-time, making our estimates comparable to real-time estimates of US potential GDP growth.

After we recover the time series of the growth rate of output due to supply shocks (that is, our estimate of potential output), we estimate regressions (3.2) and (3.3) on actual output and our estimate of potential output. Figure 3.12 shows the resulting impulse responses. We find that, in contrast to the conventional estimates of potential output, our estimate strongly reacts to supply shocks and exhibits no significant sensitivity to demand shocks. Interestingly, the reaction of our estimate for potential output to a TFP shock is stronger at short horizons than the reaction of actual output. This pattern is consistent with theoretical responses in New Keynesian models where frictions prevent actual output from an immediate adjustment to a productivity shock so that a productivity shock creates a negative output gap in the short run. Despite its simplicity, the BQ approach can therefore make progress toward resolving puzzles in the reaction of conventional estimates of potential output to identified shocks.

The fact that real-time estimates of potential output coming from BQ do not suffer from the same issues as those found from official estimates of potential output is notable. One interpretation of how the latter respond to shocks is that they represent the optimal outcome in the presence of noisy information: if agents cannot differentiate between supply and demand shocks in real-time, then their estimates of potential should slowly respond to each kind of shock. But the fact that the BQ methodology can, in real-time, successfully distinguish between the two kinds of shocks suggests that this is not a binding constraint on real-time analysis but rather reflects the specific methodologies used by each organization to create measures of potential output.[21]

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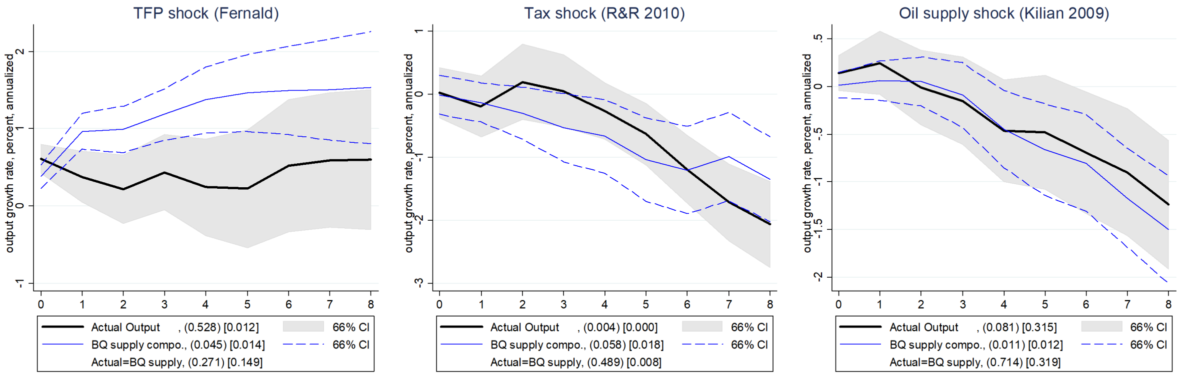
[20] We would like the rolling window to be big for the long-run identifying restriction to work well, but at the same time we would like it to be small to minimize exposure to structural breaks. We compromise by using a rolling window of 120 quarters, but results are similar when we use alternative rolling windows such as 80, 100, 140 or 160 quarters.

[21] Another piece of evidence consistent with this interpretation is that even final (2017) estimates of potential output respond to historical supply and demand shocks in the same qualitative manner as in Figure 3.6

Panel A: Supply Shocks



Panel B: Demand Shocks

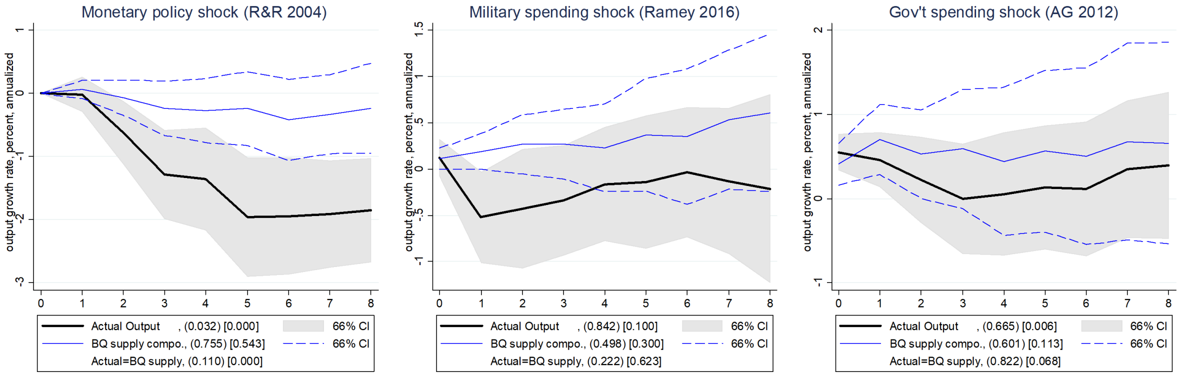


Figure 3.12: Response of the growth rate for actual and SVAR potential

Notes: The figure reports impulse response functions (IRFs) estimated using equation (3.2) and (3.3). The "BQ Supply compo." is the historical contribution of supply-side shocks (identified as in Blanchard and Quah 1989) to output growth rate. The estimation sample covers the longest possible period with non-missing observations for shocks and potential output (output gap) using output gap data starting in 1970. In parentheses we report the p-value for a test of whether the response of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the response of actual (potential) output is different from zero across all horizons of the IRF. The last row of the legend reports p-values for a test of equality of responses of actual and potential output at the max horizon (parentheses) and for a test of equality of the paths of the responses for actual and potential output are equal across horizons.

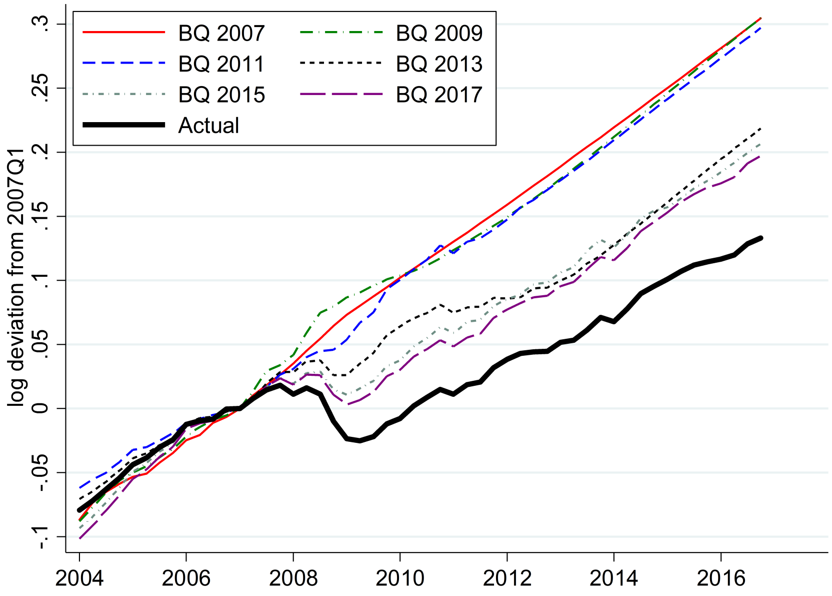
(Appendix Figure B.8). Despite a long delay, revised estimates of potential GDP from official agencies do not successfully distinguish between transitory and permanent shocks, suggesting that this reflects a feature of how these estimates are constructed, not an inability to distinguish between these shocks in real-time.

We can also use the BQ decomposition to revisit how potential output may have changed over the course of the Great Recession. In generating real-time estimates and forecasts of potential output from the BQ methodology, it is important to note that one must take a stand on the long-run growth rate of the economy. Heuristically, we can decompose the growth rate of output growth as $\Delta \log Y_t = g + \Delta \log Y_t^p + \Delta \log Y_t^c$ where g is the long-run growth rate of output, $\Delta \log Y_t^p$ is the growth rate of output due to "supply" shocks with permanent effects on the level of output, and $\Delta \log Y_t^c$ is the growth rate of output due to transitory "demand" shocks. We define the growth rate of potential output as $\Delta \log Y_t^* \equiv g + \Delta \log Y_t^p$. By iterating VAR coefficients from BQ forward, we construct forecasts for $\Delta \log Y_{t+h|t}^* = g + \Delta \log Y_{t+h|t}^p$ given the history of supply shocks up to period t. Then we cumulate $\Delta \log Y_{t+h|t}^*$ over $0, ..., H$ to compute the response of the level of potential output to a shock. Note that in this calculation we follow BQ and assume that shocks do not influence g, the growth rate of output in the long-run. While this assumption is consistent with the fact that the growth rate of output per capita in the U.S. has been remarkably stable at 2 percent per year over the last 150 years (Jones, 2016), it is nonetheless an important assumption. In the context of using BQ for the Great Recession, we apply the long-run growth rate of GDP from the 1977-2007 period (3.1%) and assume that it remains invariant to the Great Recession.

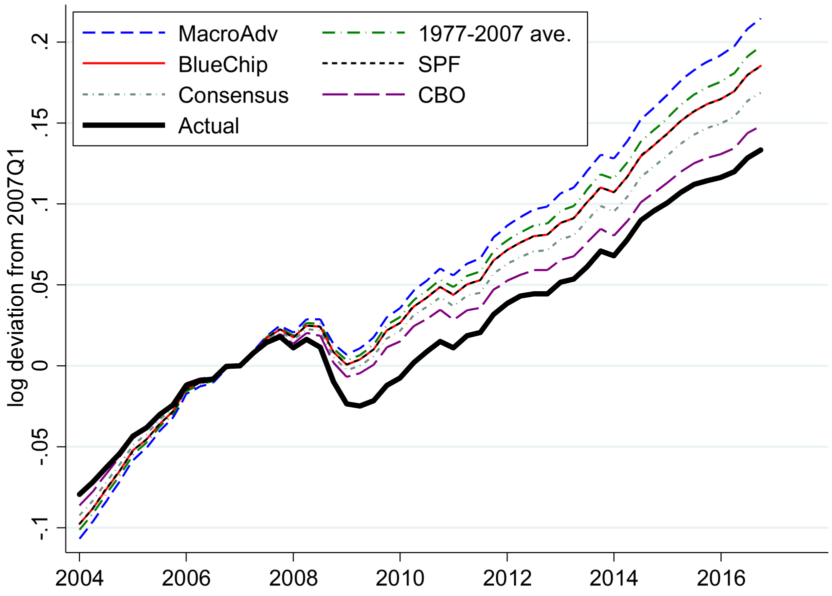
The resulting real-time revisions in potential output from the BQ methodology during the Great Recession are plotted in Panel A of Figure 3.13. Like official estimates, we find that there are declines in potential output during the Great Recession that take some time to uncover: the first significant downward revisions for 2009 potential output occur using the 2013 estimates. But there is little predictability in subsequent revisions: they all closely track the 2013 estimates of the path of output. And unlike the official estimates, the BQ approach points to a large and continuing gap between actual output and potential. By 2016, we estimate U.S. potential output to have grown by approximately 5 log percentage points more than actual output since 2007, a difference which could potentially be closed through the use of demand side policies.

Furthermore, it is likely that BQ estimates represent an overestimate of the decline in potential output. This is because, since the onset of the zero-bound on interest rates, even transitory demand shocks should be expected to have more persistent effects than they normally would given the absence of offsetting monetary policy actions. Since the BQ approach is estimated over a long period, more persistent demand shocks during the ZLB are likely to be in part attributed to "supply shocks" in the BQ decomposition. Some of the estimated decline in potential output since the Great Recession attributed to supply side factors is therefore likely to be transitory in nature, making the output gap even larger than our estimates suggest.

Because of the possible sensitivity of BQ estimates of potential GDP to assumptions about the long-run growth rate, we consider a number of other values for the long-run growth



(a) Assuming Long-Run Growth Rate Equal to Average 1977-2007 Value (3.1%)



(b) Using Alternative Long-Run Growth Rates, BQ 2017 vintage

Figure 3.13: Revisions in BQ Potential during the Great Recession

Notes: Panel A plots the real-time estimates and forecasts of potential GDP following Blanchard and Quah (1989) for different rolling windows. YYYY in "BQ YYYY" shows the last year of the rolling window. See Section 3.5.1 for details. Panel B plots BQ 2017 for different values of g which taken from the sources indicated in the legend.

rate of output that were suggested prior to the Great Recession. We view it as important to restrict our attention to pre-Great Recession estimates because these already include predictable deterministic changes in growth after 2007 (such as from the retirement of the Baby Boomers) but are not contaminated by the persistent changes in output since the Great Recession. Indeed, as we documented using long-run projections of professional/official forecasters in Section 3.4.2, real-time estimates of long-run growth respond to shocks that have only transitory effects, so we should expect these estimates to have been significantly reduced since the Great Recession (as most have in fact been), but this is not informative about whether these changes should be expected to persist.[22]

Given the difficulty inherent in making forecasts about future productivity growth, the main driver of long-run GDP growth, there was significant uncertainty about the long-run future growth rates of U.S. GDP prior to the Great Recession. For example, Macroeconomic Advisers, a prominent economic forecaster, was predicting a relatively high long-run growth rate of 3.3%. Many other professional forecasters were similarly optimistic, with forecasters in both the Blue Chip Economic Forecasts and the Survey of Professional Forecasters predicting long-run growth rates of 3.0%, just under the post-war average of 3.1%. Other forecasters were somewhat more pessimistic. For example, forecasters in Consensus Economics were predicting an average long-run growth rate of 2.8% (there was large disagreement across forecasters: standard deviation is 0.6%). The CBO was even more pessimistic, predicting an average growth rate of just 2.6% in the long-run. We show the implications of each of these assumptions for BQ decompositions since the Great Recession in Panel B of Figure 3.13. Depending on the source of long-term projections, the output gap has fallen anywhere between 15% (Macro Advisers) to 2% (CBO) since the Great Recession.

3.5.2 Alternative Estimates of Potential Output after the Great Recession

While these different estimates from the BQ methodology all imply significant remaining slack, they also point to the difficulty of pinning down the output gap using a single procedure. In this section, we consider several alternative theory-based approaches to investigate the robustness of this finding.

One approach closely related to BQ is from Gali (1999). He proposes to identify technology shocks in a VAR through long-run restrictions by assuming that these shocks change labor productivity in the long-run while other shocks do not. We apply the same 2-variable VAR as used in Gali (1999) on real-time data and define the real-time level of potential output as the level of output coming only from the identified technology shocks. As illustrated in Figure 3.14, this approach points to even smaller changes in potential output over the course of the Great Recession, perhaps due to the narrower interpretation of the types of shocks that affect potential output than in BQ. The 2017 level of potential output is only 5 log

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[22] We find similar results when we adjust output by the size of civilian population (Appendix Figure B.9).

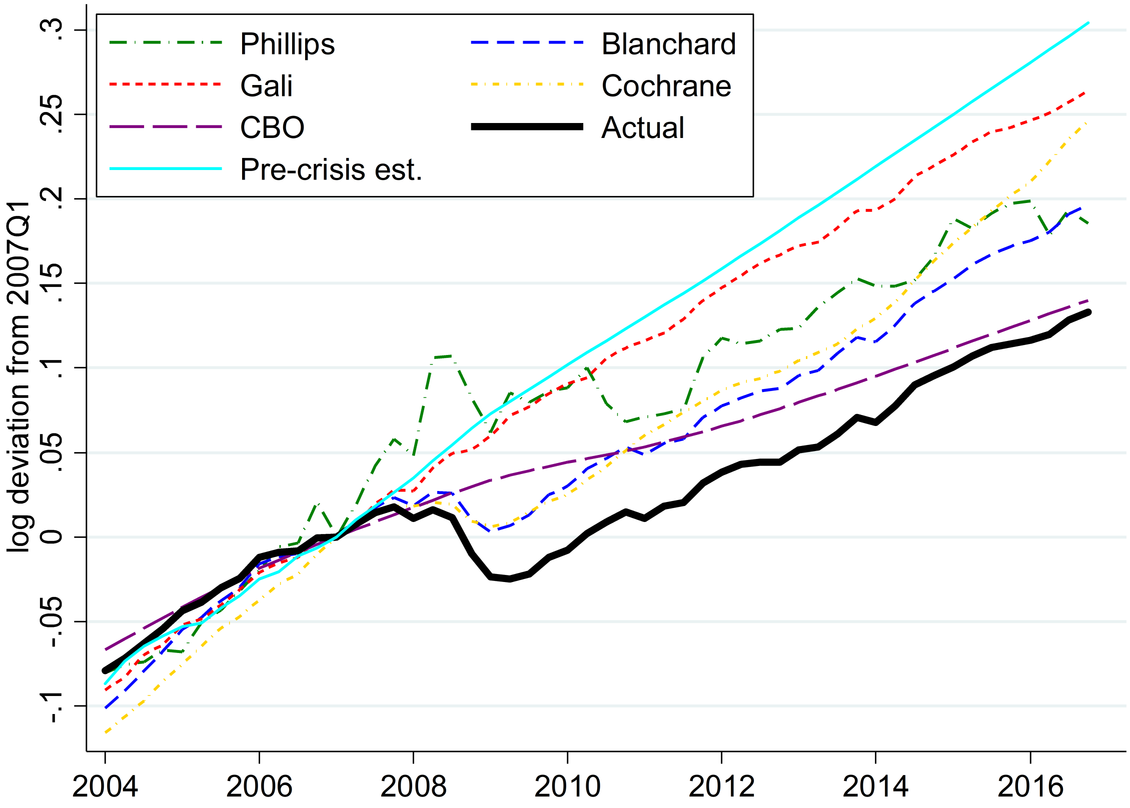

Figure 3.14: Alternative approaches to estimating potential

Notes: The figure plots the 2017 estimates of the path of potential GDP from Gali and Cochrane as well as the Blanchard and Quah (1989, "Blanchard") approach, the Phillips curve ("Phillips"), the CBO estimates of 2017 ("CBO") and 2007 ("Pre-crisis est."). "Actual" denotes the path of Real GDP. See Section 3.5.2 for details.

percentage points lower when estimated using 2017 data than forecasted from 2006 data, yielding a growth in the output gap by 2017 of well over 10 log percentage points relative to 2007.[23]

Cochrane (1994) proposes an alternative approach to identifying permanent changes in GDP by exploiting the consumption/output ratio. Under the Friedman (1957) Permanent Income Hypothesis, consumption changes reflect permanent changes in income so adding information about consumption can help decompose transitory from permanent changes in income. Applying his methodology to real-time data on consumption and GDP and identifying potential GDP as those changes associated with changes in consumption yields a surprisingly

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[23] One could also follow King, Plosser, Stock, and Watson (1991), Gonzalo and Ng (2001) and others to consider VARs that include more than two variables or use other permanent-transitory decompositions.

similar path of revisions in potential output over the Great Recession as the BQ approach, as illustrated in Figure 3.14. As with the Gali (1999) approach, the implied output gap in 2017 is therefore more than 10 log percentage points bigger than in 2007 when applying the same long-run growth rate as in BQ estimates (3.1%).[24]

Importantly, the Cochrane approach is immune to concerns about hysteresis, since it does not try to distinguish between supply and demand shocks based on their long-run effects. If hysteresis is present, then even transitory shocks should have effects on consumption due to their long-lived effects on income. As a result, they would be incorporated into the resulting estimates of potential output. Furthermore, this approach is also likely to overstate the decline in potential output over this time period. If some households are credit-constrained ("hand-to-mouth") and adjust their consumption to transitory income changes, then we will measure declines in potential GDP even from some transitory shocks, thereby overstating the change in potential GDP since the Great Recession and understating the current amount of economic slack.

Closer in spirit to Okun's (1962) approach is to infer information about potential output from the inflation rate. In New Keynesian models, nominal rigidities generate an expectations-augmented Phillips curve which relates inflation to expected inflation and the output gap (or the deviation of unemployment from the natural rate of unemployment). Conditional on observing inflation, expected inflation, and real GDP, one can then use the Phillips curve to infer the potential level of GDP (under the assumption of no markup shocks). Following Coibion and Gorodnichenko (2015b), we estimate an expectations-augmented Phillips curve during the pre-Great Recession period using inflation expectations from the Michigan Survey of Consumers. As shown in Coibion and Gorodnichenko (2015b), conditioning on household forecasts of inflation yields a stable Phillips curve since the 1960s and eliminates the puzzle of the "missing disinflation" during the early years of the Great Recession. We then apply this Phillips curve to the period since the Great Recession to infer what path of potential output is implied to account for inflation dynamics during this time period.

A key advantage of this approach is that it does not rely on long-run restrictions which may be sensitive to structural breaks (Fernald, 2007). We plot a smoothed version of 2017 estimates of potential GDP over the period of the Great Recession in Figure 3.14, along with the 2017 estimates from other approaches for comparison.[25] The implied potential GDP from the Phillips Curve does not decline much until 2011, significantly later than other approaches. However, by 2017, the resulting estimate of potential GDP is close to that of the BQ approach, pointing to an output gap of about 5 log percentage points.

In short, bringing additional information to bear on the identification of potential output, be

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[24] We report results for different vintages for the Gali and Cochrane approaches in Appendix Figure B.10.

[25] We plot a smoothed version because sampling uncertainty in inflation expectations measured by the Michigan Survey of Consumers (500 household participate in the survey in a typical month) generates high-frequency noise in estimates of potential GDP.

it from labor productivity, consumption or inflation, combined with theoretical predictions regarding how these variables relate to potential GDP, largely confirms the findings of the BQ approach. Each approach points to non-trivial revisions in potential output following the Great Recession, but not nearly as large as those coming from the official organizations. This implies that current U.S. output likely remains significantly below potential output, and therefore that further stabilization policies could be warranted.

3.5.3 Can the Output Gap Be Large When Unemployment is Low?

Our view that a significant output gap likely remains in the U.S., a decade after the start of the Great Recession, may seem at odds with the conclusion one might reach from looking at recent U.S. unemployment rates. For example, an output gap of 5% would, using Okun's Law, require a negative unemployment rate gap of approximately 1.5%.[26] With the U.S. unemployment rate having fallen below 4% in April 2018, this would imply a natural rate of unemployment of around 2.5%. In contrast, typical estimates of the NAIRU point toward much higher values (the 2018 CBO estimate is 4.6%). Is it possible to reconcile recent labor market dynamics with our estimates of potential output? In this section, we argue that the answer is unambiguously yes and that it is the alternative view, namely that labor markets are currently very tight, that seems at odds with other economic dynamics.

First, the evidence from a number of other macroeconomic variables is consistent with the view that there remains a lot of economic slack. Consumption dynamics, for example, suggest that permanent declines in income have been quite limited since the recession, as shown in Section 3.5.2. That section also documents that the behavior of inflation relative to inflation expectations is consistent with significant economic slack remaining. Other variables point toward a very similar conclusion. For example, capacity utilization is a commonly used measure of the state of the business cycle. By the end of 2017, utilization was at 77%, well below its average value of 81% over the 1977-2007 period, with only 14% of quarters over that time period having utilization rates of less than 77%. Such low utilization rates by historical standards are hard to reconcile with output being at or above its normal productive capacity. Wages also paint a picture of a labor market that remains slack: annual nominal and real wage growth in the last quarter of 2017 were at the 21st and 6th percentiles respectively of the distribution of their historical values from 1977 to 2007. It is difficult to reconcile tight labor markets with such low growth rates in wages by historical standards.

Second, any statement about the natural rate of unemployment must be tentative at best given the conceptual and measurement issues involved. Indeed, many of the same challenges as those associated with estimating the potential level of GDP are also present in estimating

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[26] For all Okun's Law calculations, we use a coefficient of 3 such that each percentage point change in unemployment gap is associated with a three-percentage point change in output gap (see Knotek (2007) for a range of estimates of Okun's Law).

the natural rate of unemployment so there is little reason to expect one to be significantly better measured than the other. Consistent with this, we observe similar patterns of systematic revisions in estimates of the natural rate of unemployment as we do in estimates of potential GDP. For example, these revisions tend to be in the direction of actual changes in unemployment, much as we observed with potential GDP. Panel B of Figure 3.15 plots projected unemployment rates of professional forecasters at different moments during the recovery, and their estimates of the natural rate of unemployment over time are given in Figure 3.16. When unemployment first began to decline after its peak in the Great Recession, professional forecasters expected a gradual decline in unemployment toward a natural rate that was estimated to be nearly 6%. But as unemployment rates fell over time, professionals continuously revised their estimates of the natural rate downward as well, with their current estimates being just above 4%. Importantly, professional forecasters have been consistently too pessimistic in their unemployment projections since 2011. CBO estimates of the natural rate of unemployment have followed an identical pattern, albeit with smaller changes (Figure 3.16). Panel A of Figure 3.15 shows that FOMC members have similarly adjusted downward the levels toward which they project unemployment rates will converge, though they do not publicly provide explicit forecasts of the natural rate of unemployment.

Third, predictions about nominal variables based on perceptions of a tightening labor market have been significantly off-target in recent years. As described in Section 3.5.2, an expectations-augmented Phillips curve requires a significant output gap to account for inflation dynamics since the Great Recession. But even without imposing an expectations-augmented Phillips curve, forecasts based on tight labor markets have failed to adequately predict inflation. For example, Panel D of Figure 3.15 plots inflation forecasts from the Survey of Professional Forecasters over the course of the Great Recession: these have repeatedly over-predicted inflation since 2013, consistent with professionals over-estimating the tightness in labor markets. A similar pattern is visible using inflation forecasts from the FOMC members over the same period (Panel C of Figure 3.15). The degree of over-estimation of inflation is more limited in FOMC forecasts, but this likely reflects the institutional nature of these forecasts: policy-makers have to present forecasts of inflation that converge to the 2% target or risk casting doubt about their credibility (Tarullo, 2017).

The issues with measuring tightness in labor markets extend beyond the difficulties associated with estimating the natural rate and extend to the challenge of using the unemployment rate as a measure of slack. In an environment where labor force participation exhibits clear business cycle variation, the unemployment rate may not be a sufficient metric of business cycle conditions. The issue is not new: over the course of the late 1990s, for example, Fed Chairman Greenspan allowed unemployment to fall significantly below the then-estimated natural levels of unemployment (the “Greenspan gamble”). Instead of generating a rise in inflation, the result was an increase in labor force participation (from 66.5% in January 1996 to 67.3% in April 2000) which led the CBO to later revise downward its estimate of the 1999 natural rate of unemployment from 5.6% to 4.8%. This endogeneity of the

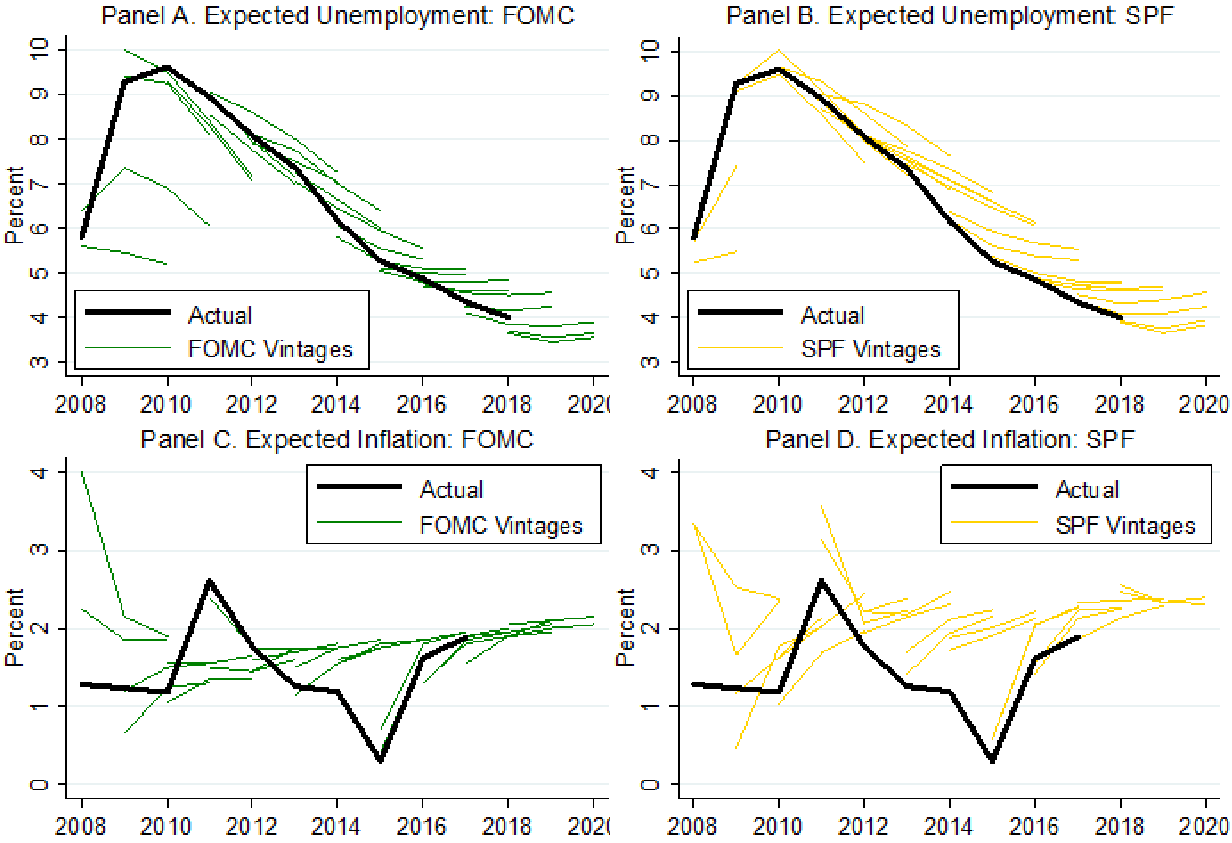

Figure 3.15: Unempl. and inflation forecasts since the Great Recession

Notes: The figures plot realization of unemployment rate (panels A and B) and inflation rate (panels C and D) as well as projections reported in the Survey of Professional Forecasters (SPF; panels B and D) and survey of members of the Federal Open Market Committee (FOMC; panels A and C).

labor force participation rate appears to have become increasingly pronounced since the Great Recession. It is well-known that labor force participation in the U.S. has declined significantly since the start of the Great Recession relative to 2007 projections (Figure 3.17). How much of this decline is likely to reflect an endogenous decision by some to abandon the labor force because of limited job prospects? One way to gauge this is to compare the labor force participation in the U.S. to that of Canada, a country with similar demographic structure and trends and so a frequent benchmark for comparison with the U.S. (see e.g. Card and Freeman (1993)) but also a country which did not experience a serious financial crisis or a recession anywhere near the size of what was experienced in the U.S. As illustrated in Figure 3.17, labor force participation in Canada also declined since 2007, but by far less than in the U.S.: 1.7% vs 3.2%. In fact, the decline in labor force participation in Canada

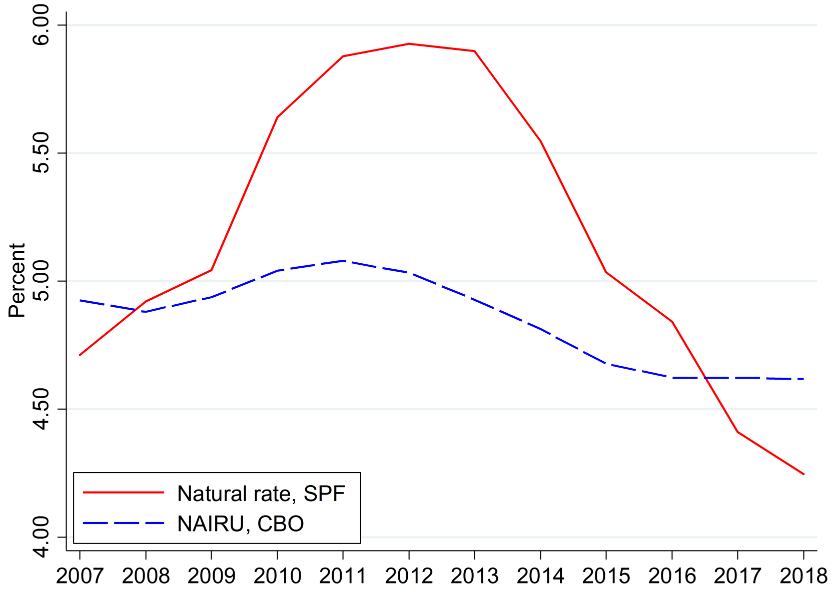

Figure 3.16: Estimates of natural rate of unemployment

Notes: The figure shows time series of Non-Accelerating Inflation Rate of Unemployment (NAIRU) estimated by the Congress Budget Office (CBO) and the consensus real-time estimate of the equilibrium rate of unemployment in the Survey of Professional Forecasters (SPF).

since 2007 corresponds almost exactly to the decline in labor force participation (2.0%) that was predicted to happen in the U.S. in 2007 by the CBO, prior to the start of the Great Recession. Were we to measure the 2017 U.S. unemployment rate relative to a labor force size consistent with a declining participation rate of 2.0% instead of 3.2%, we would have an estimated unemployment rate in 2017 of 5.3% (instead of 4.4%) and an output gap of 5% would imply, via Okun's Law, a natural rate of unemployment of 3.7%.

Erceg and Levin (2014) provide another way to gauge the cyclical sensitivity of labor force participation during the Great Recession by exploiting the cross-state variation in employment outcomes. They find that states which experienced larger increases in unemployment during the Great Recession also experienced larger declines in labor force participation over subsequent years, a feature we verify over a longer time span in Appendix Figure B.11. They find that each percentage point of higher unemployment is associated with a 0.3% decline in the labor force participation rate. Extrapolating this to the aggregate economy, the increase in the national unemployment rate by 5 percentage points between 2007 and 2009 should therefore be expected to generate an approximately 1.5 percentage point decline in labor force participation. Hence, endogenous labor force participation can account for all of the unexpected decline in the labor force participation rate observed since the Great Recession.[27]

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[27] Erceg and Levin (2014) focus on the labor force participation rate for prime-aged adults. In Appendix

Accounting for this changing labor force participation of the unemployed yields an adjusted unemployment rate of 5.8% for 2017 and, via Okun's Law and an estimated output gap of 5%, a natural rate of unemployment of 4.1%.

This sensitivity of both the measured unemployment rate and the estimated natural rate of unemployment should give one pause when thinking about the cyclical state of the economy based on the labor market. The endogeneity of labor force participation puts typical values of both in question. Because estimates of potential output are not being normalized by an endogenous variable the way unemployment rates are, this provides another reason to focus on measuring output gaps rather than unemployment gaps. However, estimating potential output is no panacea to the measurement problems associated with labor market variables. As Okun (1962) observed, "The quantification of potential output is at best an uncertain estimate and not a firm, precise measure." Indeed, estimating potential output is hard because statistical issues are magnified by sensitivity to economic assumptions. For example, forecasts for actual output are routinely associated with wide confidence bands (e.g., standard errors for the Fed and private one-year-ahead forecasts are often greater than one percentage point). Since potential output is aimed to project long-run dynamics, sampling uncertainty is amplified in these projections. This uncertainty is further exacerbated by using long-run restrictions as in BQ and similar methods in relatively short samples. Structural breaks and low-frequency variation in the data add another layer of complexity.

The sensitivity of potential output estimates to variation in economic assumptions is equally humbling. For example, BQ and similar approaches assume that g, the long-run growth rate of potential output, does not respond to economic shocks but conceivably g may persistently react to these shocks. Because even small differences in growth rates are compounded into large magnitudes over time, a weak sensitivity of g to shocks can translate into significant variation in potential output estimates. Concretely, if we overstate g by 0.1 percent per year, over ten years we can overstate the output gap by 1 percentage point.[28] In fact, our baseline estimates of the long-term growth of potential from the BQ approach, which we also apply to the consumption (Cochrane) approach and the productivity (Gali) approach, have potential output growing at 3.1%. This is above the current CBO estimate of 2.2% and above the SPF (2017) mean estimate of 2.3%. These latter sources justify their low estimates because of projections of declining labor force participation as the population ages and lower rates of population growth, as well as a continuation of the current productivity slowdown.[29] This

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Figure B.11, we present equivalent results using changes in total labor force participation from 2007 to 2017 across states. We find that a one percentage increase in the unemployment rate between 2007 and 2009 is associated with a 0.15 decline in the labor force participation rate through 2017, or half the sensitivity found by Erceg and Levin (2014). Hence, our estimates imply that the aggregate rise in unemployment from 2007 to 2009 can account for three-fourths of the unpredictable component of the decline in labor force participation.

[28] The degree of uncertainty about what value to use for g is large. Gordon (2014), for example, argues that g is likely to be only 1.6% per year between 2014 and 2020, well under the CBO's forecast of 2.2% a year, and far below the historical average of 3.1% (1947-2017 sample).

[29] However, both of these slow-moving demographic factors should have already been incorporated in the

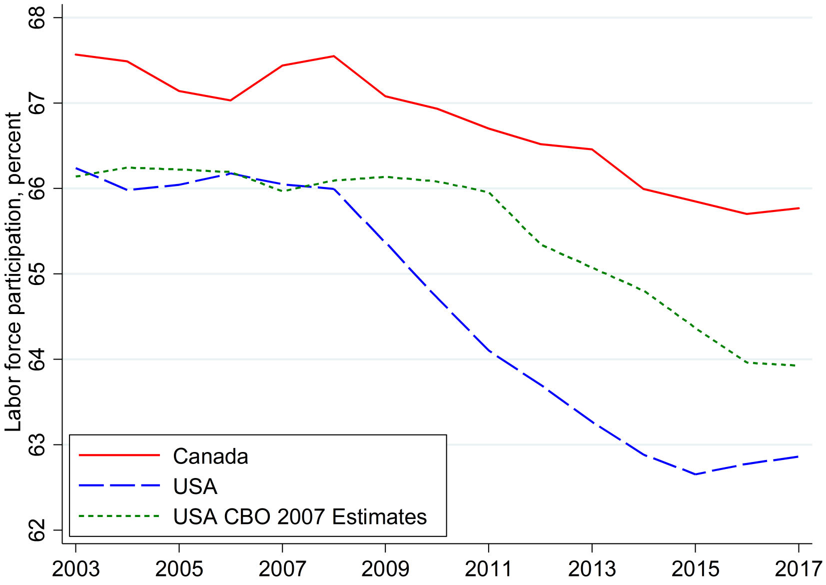

Figure 3.17: Labor force participation in Canada and U.S.

Notes: The figure plots time series of actual and projected labor force participation rates. The U.S. actual series are from the Bureau of Labor Statistics. The Canadian series is from Statistics Canada. The 10-year-ahead projection (as of 2007) for the participation rate in the U.S. is from the Congressional Budget Office (CBO).

difference in growth rates is an essential part of why we obtain large positive gaps. Our more optimistic estimates, based on statistical averages, assume that one or more of those assumptions are incorrect, although it is beyond the scope of this paper for us to examine precisely which is incorrect.[30] As a result, because estimating potential output is inherently so challenging, one should interpret our estimates in this section, and indeed all estimates of the potential level of output, as tentative. This uncertainty surrounding estimates of potential output and the natural rate of unemployment imply that risk management should be a primary consideration in policy-makers' decision-making process.

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projections these institutions made in 2007 for the 2007-2017 period, which we used as our estimates of long term growth in the exercise done in Panel B of Figure 3.13. Those estimates made by CBO and forecasters in the SPF in 2007 for the 2007-2017 period still imply significant output gaps remaining in 2017. The fact that these institutions have revised their expectations of long-run growth so much since the great recession is then more likely due to business cycles factors than to demographic factors, which is why we didn't use them in our baseline specification.

[30] Additionally, low expected growth rates going forward from 2017 don't necessarily imply that potential output growth rate was low from 2007 to 2017, which is the relevant question for our estimates of the output gap.

3.6 Conclusion

Our results speak to two distinct but related questions. The first is how real time estimates of potential output respond to transitory vs. permanent economic shocks and therefore how we should interpret revisions in estimates of potential output observed in the data. The second is how high-quality real time estimates of potential should react to economic shocks.

With respect to the first question, we provide robust evidence that real-time estimates of potential output respond to all identified economic shocks, be they transitory or permanent. Observing a sequence of revisions in estimates of potential output, like those since the start of the Great Recession, therefore tells us little about whether declines in GDP are likely to be permanent or transitory. Instead, approaches like Blanchard and Quah (1989) that explicitly distinguish between temporary and long-lived shocks are much more successful in this respect. Importantly, they suggest that current U.S. GDP is significantly below its longer run potential and therefore that the U.S. economy remains in need of ample stimulus from monetary and fiscal authorities.

In terms of how high-quality estimates of potential should respond to shocks, the answer is sensitive to the concept of potential output one has in mind and the purpose that it is supposed to serve. For an agency like the IMF that is concerned with constructing cyclically adjusted balances and long-run fiscal trends, the relevant measure of potential output is precisely one that strips out cyclical variation in GDP and identifies long-run changes. Our results suggest that the current methods used by these agencies are largely unsuccessful in this respect: their revisions are contaminated by transitory shocks and respond too slowly to long-lived shocks. For example, tax cuts that have immediate and permanent effects on output are not fully reflected in official estimates of potential output for several years, suggesting the effects of tax changes on projected revenues are likely overstated. In this sense, our results are related to Blanchard and Leigh (2013) who argue that the IMF underestimates the fiscal multipliers of austerity measures.

At the same time, it is important to bear in mind the severe constraints that hamper the ability of public and private organizations to estimate potential GDP in real-time. Not only are there profound statistical and economic challenges involved, as described in Section 3.5.3, but tight budgetary restrictions also make the systematic creation and updating of these estimates in real-time a significant challenge for public institutions. The political implications of estimates of potential GDP created by these agencies also present additional constraints on officials' ability to experiment with alternative procedures. The objective of our paper should therefore not be interpreted as criticizing these particular organizations but rather as highlighting the limitations of the methods that are currently being relied upon for both fiscal and monetary policy-making as well as proposing some potential alternatives.

The approaches that we consider here, either because they explicitly distinguish between transitory and permanent shocks like Blanchard and Quah (1989) or incorporate additional information like consumption or inflation, can help address some of the limitations of currently used methods and lead to improved estimates of cyclically-adjusted levels of GDP. It is likely that there remains much room for further improvement in the real-time measurement of potential output. One strategy would be to combine some of the different approaches used in this paper (as well as others), in the hope that combining different sources of information could augment the precision of the resulting estimates. A complementary approach might be to consider the dynamics of potential GDP jointly with the natural rate of unemployment and the natural rate of interest, concepts that are closely related but typically estimated separately. Since theory implies a tight link between these different measures, considering their joint determination might also lead to more precise estimates. But until new research provides more refined and reliable estimates of potential GDP, we should likely heed Okun's (1962) warning that "[m]eanwhile, the measure of potential must be used with care."

Chapter 4

Neo-Keynesian Trade

4.1 Introduction

There is a sharp divide in the field of international economics.[1] In the subfield of international trade models can usually incorporate many sectors, goods, and countries, but there is little room for monetary considerations or dynamic aspects. On the other hand, in the subfield of international macroeconomics the main focus is monetary, and dynamic aspects are well developed, but the models are usually simplified to either two goods, two countries or two sectors. Given the rich trade pattern of international trade models, introducing dynamics and nominal rigidities is fairly complicated. In contrast, due to the elaborate dynamic and nominal structure of models used in open economy macroeconomics, as well as the fact that those models are usually log-linearized, introducing a realistic trade structure is usually challenging.

To a first approximation this divide between the two branches is justified, because the main questions they are trying to answer are fairly different, and best studied with the approach that each branch has taken. Nevertheless, this divide has prevented economists from studying a range of interesting questions that require both richness in the trade structure and nominal rigidities. The first such question regards the gains from trade in the short and medium run, when restrictions in the adjustment of prices and wages prevent a frictionless reallocation of workers across sectors. This question is particularly relevant given the political nature of trade liberalization, since a leader advocating for these policies during the first or second year of their administration would face elections 2 or 3 years after implementing them. A second question relates to the aggregate demand effects of sectoral reallocation shocks brought about by a foreign productivity surge which is biased towards particular sectors. It is important to

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[1] This is joint work with Andrés Rodriguez-Clare, his permission to reprint this material as a chapter of the present dissertation has been obtained. This paper is still work in progress.

understand if such a shock would have asymmetric effects on regions with different sectoral compositions, and if the central monetary authority should react in a certain way.

To study these questions we build a model that breaches the aforementioned divide, combining a rich trade structure that incorporates a large number of goods, sectors, and regions, with a monetary framework that integrates nominal rigidities. The monetary friction, a downwardly rigid nominal wage, can lead to unemployment in the face of a negative trade shock, if the monetary authority is unwilling or unable to inflate the economy sufficiently (perhaps because there are un-modeled costs of inflation that it is not willing to tolerate or because of the presence of a ZLB on nominal interest rates). This last caveat about the willingness to tolerate inflation is related to the nominal anchor in the economy. In open economy macro models this nominal anchor is usually assumed to be a Taylor rule where the nominal interest rate reacts to inflation and perhaps an output gap. For simplicity we will instead assume that world nominal demand in dollars grows at a constant rate. This nominal anchor allows the model to be solved using a simple tatonnement algorithm, and it can be understood as as capturing the presence of a global ZLB.

We develop an application of our framework regarding the effects on the United States of the increase in Chinese productivity during the 2000's, popularized as the "China shock" in Autor, Dorn and Hanson (2013, henceforth referred to as ADH). In this case we will have 50 regions within one country that will represent the 50 states inside the U.S. These 50 regions share a currency (the dollar), while the remaining 37 regions (other countries) will each have an independent currency. One of the remaining countries will be China, and an increase in its manufacturing productivity between 2000 and 2007, calibrated from the data, will generate different amounts of unemployment in the different states of the U.S. This will depend on their initial industrial composition, their "exposure" to China as described in ADH. We calibrate the parameter in the downward nominal wage rigidity to match the effect of Chinese exposure on unemployment found in ADH. We find that the China shock is responsible for around 1.3 percentage points of unemployment in the U.S. in 2005 (this is around a fourth of total unemployment in the U.S. that year, which was 5.1%), but this average masks a great deal of heterogeneity, since some states don't suffer any unemployment from the China shock, while others suffer up to 4.5% unemployment in 2005 due to it.

Our paper is related to several research areas in international economics. First, it follows in the footsteps of a large literature in international trade that analyzes the impacts of trade on different regions. Papers like Autor, Dorn, and Hanson (2013), Caliendo, Dvorkin, and Parro (2015), or Galle, Rodriguez-Clare, and Yi (2017) study the regional implications of trade shocks, with an eye towards the effect of the China shock on the U.S. economy and its welfare implications. In general these papers have limited dynamics, and don't deal with nominal rigidities. A recent strand of the trade literature, including Eaton, Kortum, Neiman, and Romalis (2016), Reyes-Heroles (2017), and Ravikumar, Santacreu, and Sposi (2017), has started incorporating dynamics into more traditional trade models, but still don't incorporate monetary frictions. On the trade side, one of the closest papers to ours is Eaton, Kortum, and Neiman (2013), which tries to study unemployment in a rich trade model and references nominal rigidities, but the dynamic aspects are not well developed and the applicability of their framework is limited.

On the side of open economy macroeconomics, classic contributions like Clarida, Gali, and Gertler (2002) or various papers by Gali and Monacelli (2005, 2008, 2016), have introduced nominal rigidities in models with trade, but they don't deal with detailed, real-world, trade patterns. Schmitt-Grohe and Uribe (2016) uses a downward nominal wage rigidity to study the effects of trade shocks on a small open economy, but it doesn't deal with multiple countries, this paper motivates our use of the downward nominal wage rigidity. Caballero, Farhi, and Gourinchas (2015) studies the international aspects of monetary frictions at the ZLB, but, in the macro tradition, has a very simple trade structure. Choudhri, Faruqee, and Tokarick (2011) is a macro style paper studying the implications of nominal rigidities for the gains from trade, but it is limited to two countries.

A number of papers in closed economy macroeconomics, like Nakamura and Steinsson (2014), Beraja, Hurst, and Ospina (2016), or Chodorow-Reich and Wieland (2017) deal with regional heterogeneity in general equilibrium models, but they still don't incorporate a rich trade structure than can match real-world trading patterns.

Some papers are related to our topic but are harder to classify. Ruhl (2008) tries to reconcile the low trade elasticities (between 0.5 and 3) used in open economy macroeconomics, with the high trade elasticities (between 5 and 10) used in international trade models. In his model the movement of firms into and out of exporting in response to temporary changes in productivity, or permanent changes in tariffs, drives the two very different elasticities measured. Arkolakis, Doxiadis, and Galenianos (2015) studies the adjustment of Greece to the Euro crisis and finds that wages seems to have adjusted roughly as they were expected to do so according to pre-crisis data, but prices haven't. However, their model doesn't have well developed dynamics and it is unclear what aspect of their model delivers the results.

The rest of the paper proceeds as follows: Section 4.2 introduces the general framework that incorporates a rich trade structure with dynamic aspects and nominal rigidities. After introducing the model, this section also discusses the equilibrium, exact hat algebra, and the tatonnement algorithm that can be used to solve the model. Section 4.3 describes the application of this framework to the study of the response of the United States economy to the China shock in the 2000-2007 period. Section 4.4 includes extensions to the general framework and further applications. Finally, Section 4.5 concludes.

4.2 A Quantitative Trade Model with Nominal Rigidities

We present a multi-sector quantitative trade model with an input-output structure as in Caliendo and Parro (2015), but extended to allow for multiple periods and downward nominal wage rigidity. We assume that the United States is composed of multiple regions. Since our intention here is to focus on the role of nominal rigidities in affecting employment, we assume that there is no labor mobility across those regions, an extension to allow for such mobility is left for future work.

4.2.1 Basic Assumptions

Our presentation of the consumption, production, and trade sides of the model will be brief since this is well known. There are M regions in the U.S., plus $I-M$ regions outside of the U.S. (for a total of I regions). There are S sectors in the economy (indexed by s or k). In each region (indexed by i or j) and each period a representative consumer devotes all income to expenditure $P_{j,t}C_{j,t}$, where $C_{j,t}$ and $P_{j,t}$ are aggregate consumption and the price index in region j in period t, respectively. Aggregate consumption is a Cobb-Douglas aggregate of consumption across the S different sectors with expenditure shares $\alpha_{j,s}$. As in a multi-sector Armington trade model, consumption in each sector is a CES aggregate of the consumption of the good of each of the I regions, with elasticity of substitution $\sigma_s > 1$ in sector s.

Each region produces good k with a Cobb-Douglas production function using labor and intermediates with shares $\phi_{j,k}$ and $\phi_{j,sk}$, respectively, with $\phi_{j,k} + \sum_s \phi_{j,sk} = 1$. Under perfect competition, given iceberg trade costs $\tau_{ij,k} \geq 1$, assuming that intermediates are aggregated in the same way as consumption goods, and letting $W_{i,t}$ denote the wage in region i at time t, the price in country j of good k produced by i at time t is $A_{i,k,t}^{-1}\tau_{ij,k}W_{i,t}^{\phi_{i,k}}\prod_s P_{i,s,t}^{\phi_{i,sk}}$, where $P_{i,s,t}$ is the price index of sector s in country i at time t and is given by

$$P_{j,k,t}^{1-\sigma_k} = \sum_i \left(A_{i,k,t}^{-1}\tau_{ij,k}W_{i,t}^{\phi_{i,k}} \prod_s P_{i,s,t}^{\phi_{i,sk}} \right)^{1-\sigma_k}. \tag{4.1}$$

4.2.2 Downward Nominal Wage Rigidity

In the standard trade model, labor market clearing is that the sum of labor used across sectors in a region be equal to the inelastic labor supply, $L_{i,t} \equiv \sum_{s=1}^{S} L_{i,s,t} = \overline{L}_{i,t}$. We depart from the standard model and instead follow Schmitt-Grohe and Uribe (2016) by assuming that there is downward nominal wage rigidity (DNWR), which might lead to an employment level that is strictly below labor supply,

$$L_{i,t} \leq \overline{L}_{i,t}. \tag{4.2}$$

All prices and wages up to now have been expressed in U.S. dollars. In contrast, the downward nominal wage rigidity of a region is in terms of its local currency unit. Letting $W_{i,t}^{LCU}$ denote the wage of region i at time t in local currency units, the DNWR takes the following form:

$$W_{i,t}^{LCU} \geq \delta W_{i,t-1}^{LCU}, \qquad \delta \geq 0.$$

Let $E_{i,t}$ denote the exchange rate between the local currency unit of region i and the local currency unit of region 1 (which is the U.S. dollar) in period t, this is given in dollars per local currency units of region i. This implies that $W_{i,t} = W_{i,t}^{LCU} E_{i,t}$, and hence the DNWR in dollars entails

$$W_{i,t} \ \geq \ \frac{E_{i,t}}{E_{i,t-1}} \delta W_{i,t-1}.$$

Since all regions within the U.S. share the dollar as their local currency unit then $E_{i,t} = 1$ and $W_{i,t}^{LCU} = W_{i,t} \ \forall \ i \leq M$. This means that the DNWR in states of the U.S. takes the familiar form $W_{i,t} \geq \delta W_{i,t-1}$. For the $I - M$ regions outside of the U.S., the LCU is not the dollar and so the behavior of the exchange rate will affect how the DNWR affects the real economy. For our baseline analysis we assume that the exchange rate against the dollar is fully flexible in all countries outside the U.S. This implies that those countries have no DNWR in terms of dollars. The DNWR in dollars can then be simply captured by

$$W_{i,t} \geq \delta_i W_{i,t-1}, \qquad \delta_i \geq 0, \tag{4.3}$$

with

$$\delta_i = 1 \ \forall \ i \leq M \qquad \text{and} \qquad \delta_i = 0 \ \forall \ i > M.$$

Besides equations (4.2) and (4.3), we additionally have the complementary slackness condition:

$$(\overline{L}_{i,t} - L_{i,t})(W_{i,t} - \delta_i W_{i,t-1}) = 0. \tag{4.4}$$

4.2.3 Nominal Anchor

So far we have introduced nominal elements to the model (i.e. the DNWR), but we haven't introduced a nominal anchor that constraints or determines nominal quantities and prevents nominal wages from rising so much in each period as too make the DNWR always non-binding. The idea here would be that each country has a central bank that is not willing to allow inflation to be too high, because inflation is costly. In traditional macro models this is usually implemented via a Taylor rule, where the nominal interest rate reacts to inflation in order to keep price growth in check. Since our model is stylized and doesn't incorporate interest rates, we will use a nominal anchor that tries to capture the same idea in a simple way that lends itself to quantitative implementation and captures important real world elements.

In particular we will assume a nominal anchor which says that world nominal GDP in dollars grows at a constant rate across years, i.e.:

$$\sum_{i=1}^{I} W_{i,t} L_{i,t} = \gamma \sum_{i=1}^{I} W_{i,t-1} L_{i,t-1}. \tag{4.5}$$

This can be interpreted as saying that world aggregate demand in dollars grows at a gross rate of γ. At first sight this might seem like a weird choice for a nominal anchor, since it wouldn't obviously correspond to the monetary policy of any of the countries. Nonetheless it has some desirable properties, for example it can lead to unemployment even in the context of two countries that have a single region each, it can be seen as capturing a fixed level of aggregate demand in the context of a global liquidity trap, it can motivate "currency wars" since countries might want to manipulate their exchange rate to bring aggregate demand to their home country. This rule can also be seen as indicating that one of the countries (for example the U.S. in our specification) has a nominal GDP targeting rule where the target is for world GDP instead of just for U.S. GDP, again this might seem like a strange idea, but it makes sense for the reasons cited above.

In section 4.4 we will explore the consequences of using different nominal anchors that capture different elements of the global economy. For example a U.S. level nominal GDP targeting, or a limit to the growth of the wage in a particular region in dollars terms (this would combine fighting inflation in that country with preventing an appreciation), which could be a plausible nominal anchor for the China shock application.

4.2.4 Equilibrium

Letting $R_{i,s,t}$ denote total revenues in sector s of country i, noting that the demand of industry k of country j of intermediates from sector s is $\phi_{j,sk} R_{j,k,t}$, and allowing for exogenous deficits as in Dekle, Eaton, and Kortum (2007), the equilibrium for sector s in country i can be written as

$$R_{i,s,t} = \sum_{j=1}^{I} \lambda_{ij,st} \left(\alpha_{j,s} \left(W_{j,t} L_{j,t} + D_{j,t} \right) + \sum_{k} \phi_{j,sk} R_{j,k,t} \right), \tag{4.6}$$

where

$$\lambda_{ij,k,t} \equiv \frac{(A_{i,k,t}^{-1}\tau_{ij,k,t}W_{i,t}^{\phi_{i,k}}\prod_s P_{i,s,t}^{\phi_{i,sk}})^{1-\sigma_k}}{\sum_{r=1}^{I}(A_{r,k,t}^{-1}\tau_{rj,k,t}W_{r,t}^{\phi_{r,k}}\prod_s P_{r,s,t}^{\phi_{r,sk}})^{1-\sigma_k}}$$

are sector-k trade shares in period t. Given last-period wages $\{W_{i,t-1}\}$, the period t equilibrium is a set of wages $\{W_{i,t}\}$, sector-country prices indices $\{P_{i,s,t}\}$ and revenues $\{R_{i,s,t}\}$ such that equations (1)-(6) hold.

If δ is low enough or the exchange rate can depreciate (e.g., $\delta_i = 0$) then wages can adjust downwards in the required magnitude without causing unemployment, while if γ is high enough then again there would be no unemployment since no downward adjustment is needed in the wage. However, there are combinations of parameters δ_i and γ that will lead to unemployment after the shock, although there would then be a decline in unemployment towards zero as the DNWR and the nominal anchor allow for adjustment year after year.

We clarify that having multiple regions is not critical for the shock to lead to unemployment given the particular form of our nominal anchor. To see this, imagine that the U.S. was composed of a single region and consider a shock that required a decline in the U.S. wage relative to the rest of the countries, for example, this could be a negative productivity shock in the U.S. If γ was high enough then the adjustment could take place without unemployment in the U.S. since wages in dollars in the rest of the world could increase enough to generate the necessary relative wage adjustment. However, if γ is low and δ is high, this full adjustment would not be possible and there would be (temporary) unemployment in the U.S.

With multiple regions in the U.S. we could also have unemployment with the more natural nominal anchor rule that simply said that

$$\sum_{i=1}^{M} W_{i,t}L_{i,t} = \gamma \sum_{i=1}^{M} W_{i,t-1}L_{i,t-1}.$$

Imagine that there was a shock that affected only one of the regions in the U.S., requiring that the wage in that region fall relative to the wages of the other regions. With a low enough γ and a high enough δ, this cannot take place, and so there would be (temporary) unemployment after the shock. We want the regional heterogeneity to be able to study the differential impact of shocks across areas. Additionally, in a "monetary union" with a common exchange rate, the response heterogeneity is coming only from the differential impact of the shocks, and not from differential responses to it via monetary policy.

4.2.5 Hat Algebra

Our goal is to use a calibrated version of the model above to compute the welfare effects of a trade shock or the closing of a country's trade deficit. We want to do this using actual data for U.S. states as well as outside countries, but without having to calibrate technology levels and iceberg trade costs along the transition and without requiring data on nominal wages or available labor (since this would require taking a stance on what efficiency units we are measuring things in). To do so, we follow the exact hat algebra methodology of Dekle et al. (2007) and the extension of that methodology to dynamic settings proposed in Caliendo et al. (2015). Our counterfactual exercises then, only require data on nominal GDP, $Y_{i,t} \equiv W_{i,t} L_{i,t}$, trade deficits, $D_{i,t}$, and trade shares $\lambda_{ij,s,t}$ at time zero, $t = t_0$, whatever shocks we are interested in, and the model's parameters, namely δ, γ, $\{\alpha_s\}$, $\{\phi_{i,s}\}$, and $\{\phi_{i,sk}\}$.

We will use the variable $\hat{x}_t$ to denote x_t/x_{t-1} for any underlying variable x. To express the equilibrium system in hats and only leave it in terms of observable data in period zero (when we assume the economy was in a steady state where every country had full employment) we follow an iterative process described in Appendix C.1. There we show that the equilibrium system in hats is given by:

$$\begin{aligned}
\hat{R}_{i,s,t} R_{i,s,t-1} &= \sum_{j=1}^{I} \hat{\lambda}_{ij,s,t} \lambda_{ij,s,t-1} \left(\alpha_{j,s} (\hat{W}_{j,t} \hat{L}_{j,t} Y_{j,t-1} + \hat{D}_{j,t} D_{j,t-1}) + \sum_{k=1}^{S} \phi_{j,sk} \hat{R}_{j,k,t} R_{j,k,t-1} \right) \\
\hat{\lambda}_{ij,k,t} &= \frac{\left(\hat{A}_{i,k,t}^{-1} \hat{\tau}_{ij,k,t} \hat{W}_{i,t}^{\phi_{i,s}} \prod_{s=1}^{S} \hat{P}_{i,s,t}^{\phi_{i,sk}} \right)^{1-\sigma_k}}{\sum_{r=1}^{I} \lambda_{rj,k,t-1} \left(\hat{A}_{r,k,t}^{-1} \hat{\tau}_{rj,k,t} \hat{W}_{r,t}^{\phi_{r,k}} \prod_{s=1}^{S} \hat{P}_{r,s,t}^{\phi_{r,sk}} \right)^{1-\sigma_k}} \\
\hat{P}_{j,k,t}^{1-\sigma_s} &= \sum_{i=1}^{I} \lambda_{ij,s,t-1} \left(\hat{A}_{i,k,t}^{-1} \hat{\tau}_{ij,k,t} \hat{W}_{i,t}^{\phi_{i,k}} \prod_{s=1}^{S} \hat{P}_{i,s,t}^{\phi_{i,sk}} \right)^{1-\sigma_k} \\
\hat{L}_{i,t} &\leq \frac{1}{1 - u_{i,t-1}} \\
\hat{W}_{i,t} &\geq \delta_i \\
0 &= \left(\frac{1}{1 - u_{i,t-1}} - \hat{L}_{i,t} \right) (\hat{W}_{i,t} - \delta_i) \\
\sum_{i=1}^{I} \hat{W}_{i,t} \hat{L}_{i,t} Y_{i,t-1} &= \gamma \sum_{i=1}^{I} Y_{i,t-1},
\end{aligned}$$

where $u_{i,t} \equiv \frac{\bar{L}_{i,t} - L_{i,t}}{\bar{L}_{i,t}}$ is the unemployment rate of country i at time t. For each period t this is a system of equations which we can use to solve for the quantities that we care about (the $\hat{W}_{i,t}$ and $\hat{L}_{i,t}$ for all i) given the objects that we already know from the previous period ($Y_{i,t-1}$, $\lambda_{ij,s,t-1}$, $D_{i,t-1}$ and $u_{i,t-1}$ for all i, j, s) and the time t shocks ($\hat{A}_{i,s,t}$, $\hat{D}_{i,t}$ and $\hat{\tau}_{ij,s,t}$ for all i, j, s). Thus, starting at $t = 1$ we can solve this system with information on $Y_{i,0}$, $\lambda_{ij,s,0}$, $D_{i,0}$ and $u_{i,0}$ for all i, j, s and the shocks ($\hat{T}_{i,s,1}$, $\hat{D}_{i,1}$ and $\hat{\tau}_{ij,s,1}$ for all i, j, s) and obtain $\hat{W}_{i,1}$ and $\hat{L}_{i,1}$ for all i, from these we can also obtain $Y_{i,1}$, $\lambda_{ij,s,1}$, $D_{i,1}$ and $u_{i,1}$ for all i, j, s. Then we can move forward to period 2 and solve for $\hat{W}_{i,2}$ and $\hat{L}_{i,2}$ for all i. We can keep doing this process to solve the system forward while requiring only period zero information and the shocks hitting the economy.

4.2.6 Tatonnement

In the case where there are no intermediate inputs, i.e. $\phi_{j,sk} = 0 \ \forall j, s, k$, and when using the world nominal GDP targeting as a nominal anchor, we can develop a very nice tatonnement algorithm to solve the model. We first rewrite the equilibrium system in hats as:

$$
\begin{aligned}
\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} &= \sum_{s=1}^{S} X_{i,s,t} && (4.7)\\
\hat{L}_{i,t} &\leq L^U_{i,t} \equiv \frac{1}{1-u_{i,t-1}} && (4.8)\\
\hat{W}_{i,t} &\geq \delta_{i,t} && (4.9)\\
0 &= \left(L^U_{i,t} - \hat{L}_{i,t}\right)(\hat{W}_{i,t} - \delta_{i,t}) && (4.10)\\
\sum_{i=1}^{I} \hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} &= \gamma \sum_{i=1}^{I} Y_{i,t-1} && (4.11)
\end{aligned}
$$

where:

$$
X_{i,s,t} = \sum_{j=1}^{I} \frac{\lambda_{ij,s,t-1}\hat{T}_{i,s,t}(\hat{\tau}_{ij,s,t}\hat{W}_{i,t})^{1-\sigma_s}}{\sum_{r=1}^{I}\lambda_{rj,s,t-1}\hat{T}_{r,s,t}(\hat{\tau}_{rj,s,t}\hat{W}_{r,t})^{1-\sigma_s}} \alpha_{j,s}\left(\hat{W}_{j,t}\hat{L}_{j,t}Y_{j,t-1} + \hat{D}_{j,t}D_{j,t-1}\right).
$$

We could also write $L^U_{i,t} = \frac{1}{\prod_{q=1}^{t-1}\hat{L}_{i,q}}$. Recall that:

$$
\sum_{i=1}^{I}\sum_{s=1}^{S} X_{i,s,t} = \sum_{i=1}^{I} \hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1}.
$$

We will implement a "tatonnement" algorithm in the spirit of Alvarez and Lucas (2007), but that also accounts for the fact that labor is an endogenous quantity and that we have inequality constraints at each point in time. The algorithm takes a set of pre-determined variables ($Y_{i,t-1}$, $D_{i,t-1}$, $L^U_{i,t}$ and $\lambda_{i,s,t-1}$), a set of shocks ($\hat{T}_{i,s,t}$, $\hat{\tau}_{ij,s,t}$ and $\hat{D}_{i,t}$) and a guess vector for the changes in wages and labor ($\hat{W}_{i,t}$ and $\hat{L}_{i,t}$), and it returns a new guess for the changes in wages and labor ($\hat{W}'_{i,t}$ and $\hat{L}'_{i,t}$).

If there was never any unemployment we could use the equivalent of the Alvarez and Lucas (2007) algorithm modified for our context. Specifically we would set:

$$
\hat{W}'_{i,t} = (1-\nu)\hat{W}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}} \qquad (4.12)
$$

$$\hat{L}'_{i,t} \;=\; 1, \tag{4.13}$$

where ν is a parameter of the algorithm that satisfies $0 < \nu < 1$. The actual algorithm is more complicated, since it has to account for endogenous labor and inequality constraints. We set:

$$\hat{W}'_{i,t} \;=\; \max\left\{\frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{L^U_{i,t}}, \delta_{i,t}\right\} \tag{4.14}$$

$$\hat{L}'_{i,t} \;=\; \min\left\{L^U_{i,t}, \frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{\delta_{i,t}}\right\}. \tag{4.15}$$

In appendix C.2 we prove that if the initial guess of the algorithm $\hat{W}_{i,t}$ and $\hat{L}_{i,t}$ satisfies equations (4.8)-(4.11) then the new guess $\hat{W}'_{i,t}$ and $\hat{L}'_{i,t}$ will also satisfy equations (4.8)-(4.11). Meanwhile the iterations of the algorithm will lead to equation (4.7) also being satisfied (the algorithm will converge), so provided a suitable initial guess, the algorithm will converge to a solution for the equilibrium system in changes (hats).

The intuition for the algorithm is similar to the one in Alvarez and Lucas (2007), and can be described as follows. Start with equation (4.12) but now imagine that instead of just the change in the wage, it provides the rationale for updating the change in nominal GDP, we would obtain:

$$\hat{W}'_{i,t}\hat{L}'_{i,t} \;=\; (1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}. \tag{4.16}$$

This is a sensible updating mechanism, but the problem is that is just provides a new guess for the product of the change in wages and labor while we need to obtain individual guesses. That is where the additional equations (4.8)-(4.10) come in, they will allow us to obtain individual updated guesses.

Equation (4.16) determines a parabola in $(\hat{W}'_{i,t}, \hat{L}'_{i,t})$ space, such as the ones depicted in red, green, or blue in Figure 4.1. Equations (4.8)-(4.10), on the other hand, dictate that the new guesses $\hat{W}'_{i,t}$ and $\hat{L}'_{i,t}$ must be situated in the thick black portions of the L^U or δ lines. If the parabola described in equation (4.16) looked like the red one, then we could find the point we are after as its intersection with the $\hat{W}'_{i,t} = \delta_{i,t}$ line, which is labeled G'' in the figure. But if the parabola described in equation (4.16) looked like the blue one, then the point we are after would be its intersection with the $\hat{L}'_{i,t} = L^U$ line, labeled G in the figure. In general we can find the intersection of the parabola with both lines and then pick the point with the highest $\hat{W}'_{i,t}$ and the lowest $\hat{L}'_{i,t}$. That is precisely what equations (4.14) and (4.15) do.

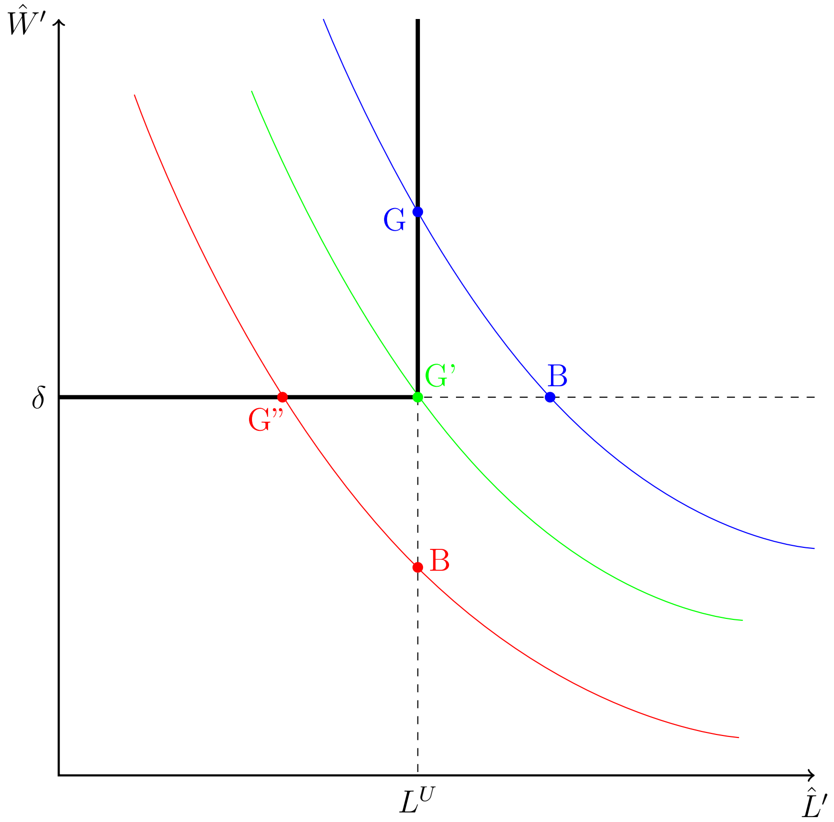

Figure 4.1: Illustration of the tatonnement algorithm

Notes: This figure provides an illustration of the tatonnement algorithm described in Section 4.2.6 and Appendix Section C.2.

4.3 China Shock and Unemployment

Now that we have introduced the general model with rigidities, we will apply it to studying the China shock and the unemployment effects that this shock might have. We first discuss the data we use and how we calibrate the model, then we discuss the effects of the China shock without nominal rigidities and then we discuss the effects under nominal rigidities.

4.3.1 Data and Calibration

We use data on output, deficits and trade shares for 50 U.S. states, 36 additional countries and an aggregate rest of the world region, for a total of 87 regions. The data was provided by Caliendo, Dvorkin, and Parro (2015) and corresponds to their base year of 2000. The international country level data comes from the World Input-Output Database (WIOD) while the data for U.S. states comes from the 2002 Commodity Flow Survey. There are 22 sectors, of which, as described by CDP, 12 are manufacturing sectors and 8 are service sectors, there is also a construction sector and a combined wholesale and retail trade sector.

The $\alpha_{i,s}$ Cobb-Douglas shares are taken directly from the CDP data, the trade elasticity σ_s is assumed to be constant across sectors and it takes the value of 6, consistent with the trade literature. For inter-temporal comparisons, when computing welfare, we use a β of 0.97 (at the annual level). For the results reported next we assume that $\phi_{j,sk} = 0 \ \forall j, s, k$, i.e. that there are no intermediate inputs, but the results are robust to having intermediate inputs, as we show in an extension available upon request. Next we need to calibrate the increase in Chinese manufacturing productivity $\hat{A}_{i,s,t}$ for $i = \text{China}$, $s = 1, ..., 12$ and $t = 2000, ..., 2007$, we only calibrate the productivity increase for the first 12 sectors since these are the ones classified as "manufacturing", the other 10 sectors are roughly non-tradable.

We calibrate Chinese productivity changes to match the predicted changes in import values or shares from China to the U.S., as in Caliendo et al. (2015) or Galle et al. (2017). By "predicted" we mean that we don't use the actual changes in import values or shares from China to the U.S., but the values we would predict for those variables by regressing them on the changes in import values or shares from China to other 8 high income countries. These 8 high income countries are the same ones used in ADH (or the subset of them available in the 2013 version of the WIOD which are Australia, Germany, Denmark, Spain, Finland, and Japan, the countries of New Zealand and Switzerland are included in the "other country" category of ADH but are not included in the WIOD).

We need to calibrate $\hat{A}_{China,s,t}$ for $s = 1, ..., 12$ and $t = 2001, ..., 2007$, these are 84 parameters. We don't want to do this year by year since this would add too much noise to the estimates. So we decompose $\hat{A}_{China,s,t}$ as $\hat{A}_{China,s,t} = \hat{B}_s \hat{F}_t$, this means we have to estimate only 19 parameters instead of 84. We do this in two steps instead of all together. We first pick $\hat{B}_s$ for the 12 manufacturing sectors to match the predicted changes in imports from china by sector, we obtain these predicted values from a regression like:

$$\Delta M_{US,C,s}^{2007-2000} = \alpha + \beta \Delta M_{OC,C,s}^{2007-2000} + \varepsilon_s.$$

This is a regression with 12 data points, but it still has a very high R^2. When we run this regression with a constant it can deliver negative values for the predicted changes in U.S. imports. Values that are more negative than the initial imports from China and that would imply that the U.S. would need to have negative (gross) imports from China, this is impossible and so the model crashes. To fix this we can either set the negative changes to zero or run the regression without a constant. We pick the $\hat{B}_s$ to match these predicted changes in a two period model without wage rigidity (since there are only two periods there is a single change in imports and that is what we match).

Once we have obtained the $\hat{B}_s$ in the way described above we proceed to obtain the $\hat{F}_t$'s. We do so to match the predicted changes in total (across manufacturing sectors) imports across years. These changes are obtained from a regression like:

$$\Delta M_{US,C,t} \quad = \quad a + b\Delta M_{OC,C,t} + \varepsilon_t.$$

This is a regression with just 7 data points but, surprisingly, it still has a high R^2. Now in a model with 8 periods (2000-2007) we choose the $\hat{F}_t$'s such that the productivity changes in China are $\hat{A}_{China,s,t} = \hat{B}_s\hat{F}_t$ and they deliver a change in total (across manufacturing sectors) imports from China given by the ones obtained from the previous regression.

We will use the following measure of exposure to China, defined for a certain state i in the U.S.:

$$\text{Exposure}_i \equiv \sum_{s=1}^{S} \frac{Y_{i,s,2000}}{Y_{i,2000}} \frac{\Delta_{2000}^{2007} M_{C-US,s}}{EXP_{US,s,2000}},$$

where $Y_{i,s,2000}$ is the production of state i in sector s in year 2000 and it is taken directly from the CDP data, $Y_{i,2000}$ is the total production of state i across all 22 sectors in year 2000, also taken from the CDP data, $EXP_{US,s,2000}$ is total U.S. expenditure on sector s in year 2000, also from the CDP data, and $\Delta_{2000}^{2007} M_{C-US,s}$ is the change in imports from China to the U.S. from 2000 to 2007 in sector s, where the value in 2007 comes from the model without rigidities. This measure is very similar to the measure of exposure used in ADH, but it is slightly different since they use employment and we use production (we don't have employment and don't want to take a stance on the efficiency units of things). Since the measures have different means we re-normalize our exposure measure to have the same mean as the measure in ADH for comparability purposes.

4.3.2 Results Without Rigidities

By setting $\delta = 0$ we can study the results of the model without any nominal rigidities. Since in this case our model is fairly standard our results should be similar to other estimates of the China shock that were obtained under similar assumptions and using similar data.

In this case lifetime welfare in the U.S. (the unweighted mean across states) increases in around 23 basis points from the China shock (this amount is almost unchanged if we weigh states by size). This is consistent with the results in Galle et al. (2017). We can also do a scatter plot of the welfare change against exposure to China across all U.S. states, this is shown in Figure 4.2. There, we can observe that all states have a positive welfare change, since they benefit more from the increase in Chinese productivity (and the fall in prices that it brings about) that what they lose by being less competitive. Another interesting feature we can observe in the figure is that the welfare effect of the China shock is positively correlated with exposure if there are no rigidities. The reason this occurs is that regions which are more exposed to China also consume more of their income in goods where the Chinese productivity increase is greater, and so they benefit more from the fall in prices. To provide support for this explanation we can generate a "fake" welfare level, calculated by computing price indexes for all states in the U.S. with the average Cobb-Douglas coefficients for the U.S. instead of the individual Cobb-Douglas coefficients of that state. This is shown in Figure 4.3. There, it can be seen that once prices are computed with a single set of expenditure weights, the expected result, that states that are more exposed to China have smaller welfare increases from the China shock, is obtained.

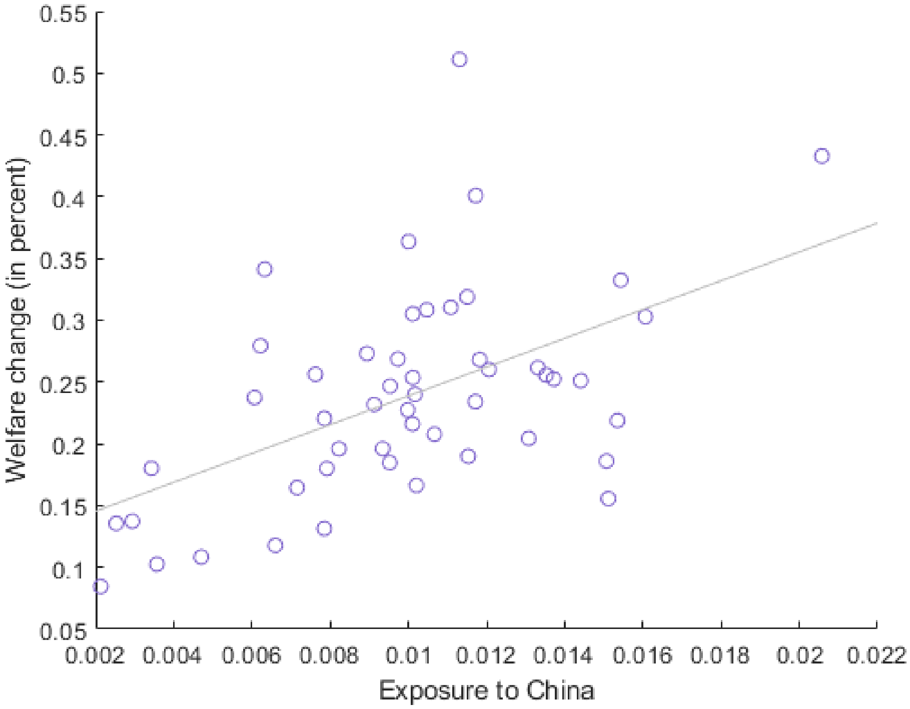

Figure 4.2: Welfare changes vs exposure to China, no rigidities

Notes: Scatter plot of welfare changes across states in the U.S. versus exposure to China for the case of no rigidities.

4.3.3 Results With Rigidities

Once there are rigidities the γ and δ parameters of the nominal anchor and the downward nominal wage rigidity, as well as the exchange rate policies of different countries become relevant. In this section we will assume that all countries outside the U.S. have enough exchange rate flexibility to never hit their downward nominal wage rigidity constraint. We will also assume that the γ parameter is 1, and hence the burden of adjustment is put on δ. We want to choose δ to match the effect of exposure to China on unemployment to the empirical estimates obtained in ADH. These authors find a 0.22 increase in unemployment (2000-2007) for each additional \$1000 of exposure to China. By doing this we obtain $\delta = 0.9886$ which roughly falls in the range of Schmitt-Grohe and Uribe (2016).

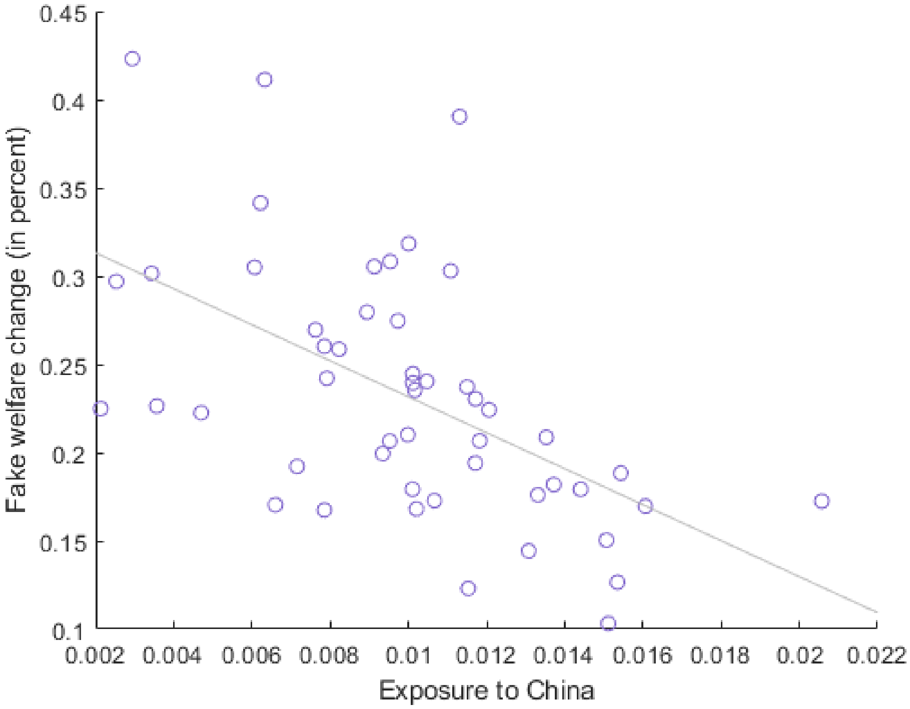

Figure 4.3: "Fake" welfare changes vs exposure, no rigidities

Notes: Scatter plot of "fake" welfare changes (computed with the same price index for all states in the U.S.) across states versus exposure to China for the case of no rigidities.

In Figure 4.4 we can see the coefficients that we would obtain for different choices of δ between 0.985 and 0.99, and we can see that for the calibrated level of 0.9886 the coefficient is approximately 0.22 which is the value obtained in ADH. Schmitt-Grohe and Uribe (2016) obtained a "central" estimate of 0.996 at the quarterly level (once adjusting for inflation and growth, so this corresponds well to the choice we made of keeping γ fixed at one), which would imply a annual δ of 0.984, which is close to our value of 0.9886. But then Schmitt-Grohe and Uribe (2016) say that they will use a "conservative" estimate of δ of 0.96 at the annual level, which is farther away from the value that we obtain. It is true that the values obtained by Schmitt-Grohe and Uribe (2016) are for Argentina and for European countries, so these values might not correspond exactly to values in the U.S., nevertheless we think it is a good sign that our estimate of δ is in the ballpark of the ones discussed in Schmitt-Grohe and Uribe (2016).

We can obtain the reaction of wages, labor, nominal income, real income, prices, etc., for all the 87 regions. For a low delta (lower than 0.985 or so) no region has unemployment while for high deltas (for example 0.99) almost all regions in the U.S. suffer unemployment while the wage is stuck at its lower bound. For values in between, for example our calibrated value

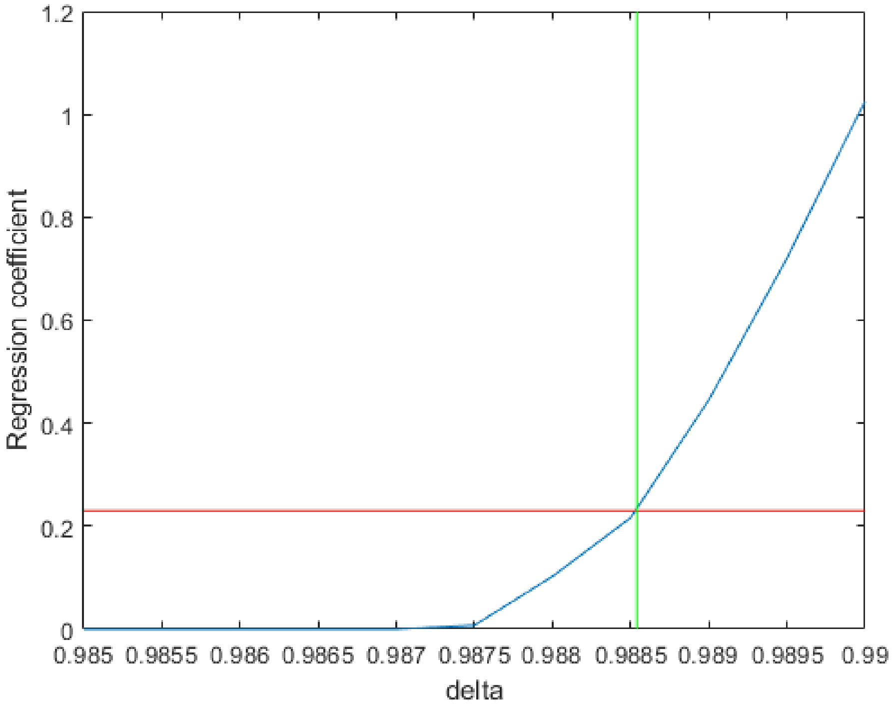

Figure 4.4: Coefficient for exposure on unemployment

Notes: ADH coefficient for the regression of exposure on unemployment obtained for different levels of the DNWR parameter δ.

of 0.9886, only some states suffer unemployment, and those are more affected by the China shock. We can plot a map with unemployment across states for all the years 2000-2007 to see how unemployment appears around 2003-2004 and disappears around 2006-2008. We can also plot a welfare map across states, welfare against exposure to China and aggregate welfare in the U.S. against delta, this is done in figures 4.5, 4.6, and 4.7 respectively.

In Figure 4.7 we can see that as δ increases unemployment becomes more significant and the welfare increases from the China shock start becoming smaller. Eventually, when δ is high enough, the welfare change flips from a gain to a loss, this happens for a δ higher than 0.99. For our calibrated δ of 0.9886 the welfare gain from the China shock is still positive and around 15 basis points, which is around two thirds of the total welfare gain that would come about in the absence of rigidities. This aggregate welfare change across the U.S. is a simple average across states, if we weigh states by size the results are very similar.

As δ increases the effect of exposure on welfare starts becoming smaller and eventually it becomes negative. This happens because the negative effect of having more unemployment if you are more exposed to China, as well as becoming less competitive, starts outweighing

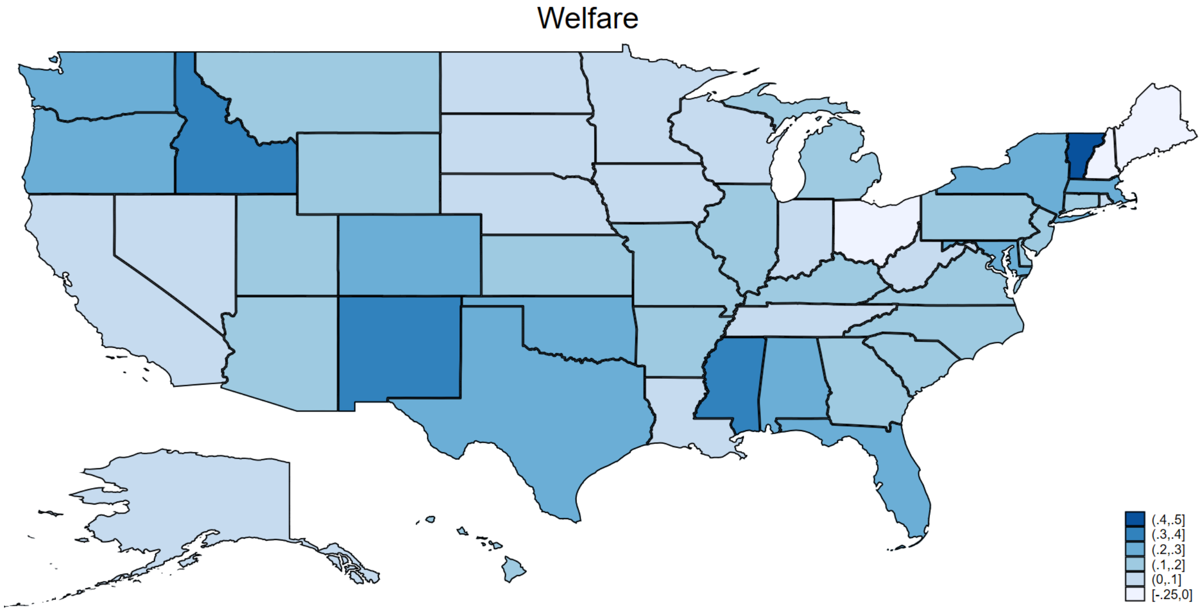

Figure 4.5: Welfare map

Notes: Welfare map across the states of the U.S. for the calibrated DNWR parameter δ of 0.9886.

the positive effect of getting cheaper goods from China. For our calibrated δ of 0.9886 the slope of exposure of actual welfare is already negative, meaning that states that are more exposed to China suffer bigger losses from the China shock, this is even more true is we look at "fake" welfare. For different δ's we can see how the slope changes in Figure 4.8.

For δ's higher than 0.988 the slope of the relationship is positive, for $\delta = 0.988$ the slope is roughly zero and for higher δ's the slope becomes negative. As we mentioned in the section with no rigidities, fake welfare, computed using the same single set of Cobb-Douglas exponents when calculating the price index, is indeed negatively correlated with exposure even when there are no rigidities, which seems to indicate that the "strange" sign does come from differences in sectoral shares that evidence some sort of home bias (in the macro sense, not the trade sense). We can see the effects of exposure on this "fake" welfare in Figure 4.9.

4.4 Extensions

4.4.1 Results With U.S. Nominal GDP Targeting

If we do exactly the same calibration as above but instead use a U.S. level nominal GDP targeting instead of the world nominal GDP targeting we obtain that for the previously calibrated level of δ, 0.9886, no state of the U.S. would suffer unemployment. This would

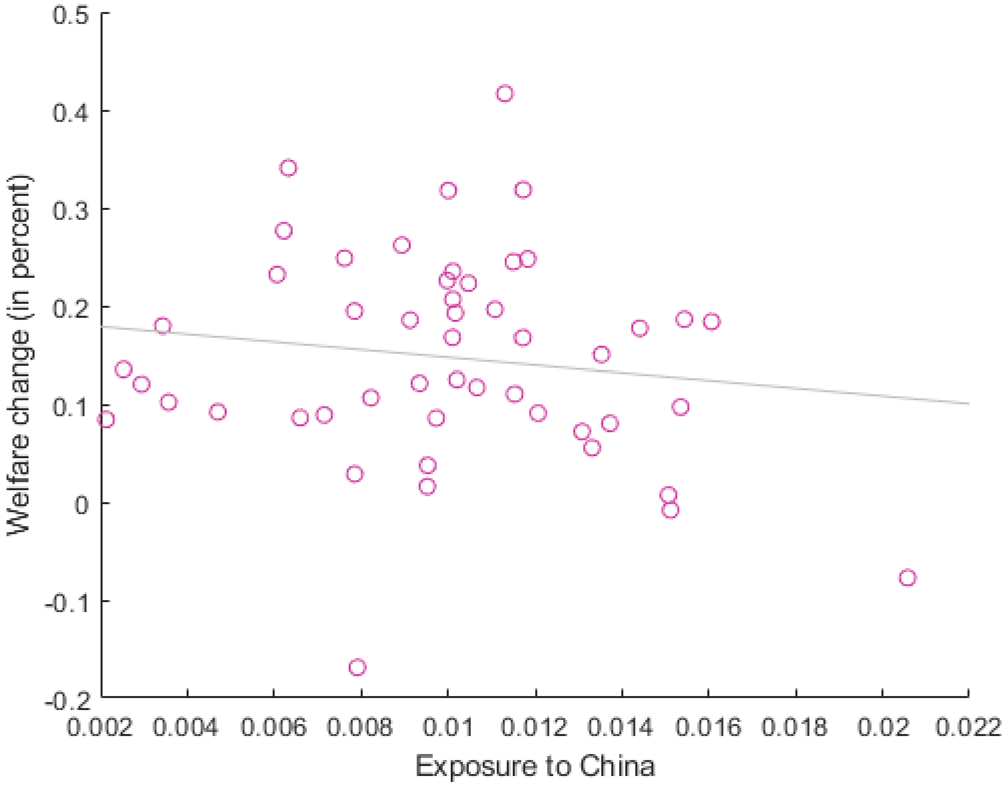

Figure 4.6: Welfare vs Chinese exposure for calibrated δ

Notes: Scatter plot of welfare changes across states in the U.S. versus exposure to China for the case of the rigidities implied by the calibrated δ of 0.9886.

indicate that the China shock between 2000 and 2007 would be too small to generate unemployment in the U.S. if all the other countries had flexible exchange rate policies and didn't mind seeing their currency appreciate (possibly a lot in the case of China).

In order to generate unemployment in some states of the U.S. under this nominal anchor we would need to assume an extremely high δ, for example one higher than 0.999. The δ that matches the ADH coefficient of exposure on unemployment is even higher, at 0.9998. With this last δ we obtain a similar result as the one obtained under the global GDP targeting rule that the welfare change in the U.S. with rigidities is about two thirds of the one without rigidity (which is obviously still the same, 23 basis points).

The problem with this specification, and the reason why it is necessary to assume an extremely high level of δ to have unemployment effects from the China shock if one assumes that the nominal anchor is simply a U.S. level nominal GDP targeting is that in this case there is no limit to the appreciation that other countries can endure, so the exchange rates of

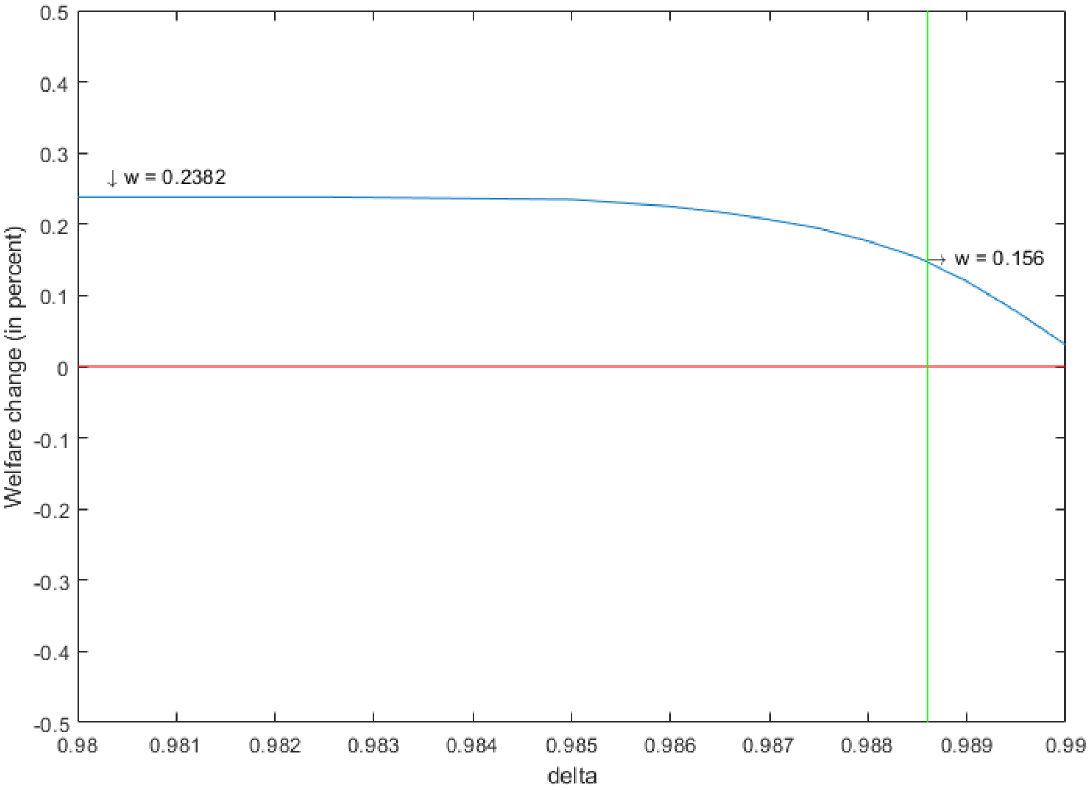

Figure 4.7: Aggregate U.S. welfare against δ

Notes: Scatter plot of "fake" welfare changes (computed with the same price index for all states in the U.S.) across states in the U.S. versus exposure to China for the case of the rigidities implied by the calibrated DNWR parameter δ of 0.9886.

other countries can carry all the burden of the adjustment and the only adjustments within the U.S. come from differential effects of the China shock which are not that heterogeneous. That is why we want to investigate a nominal anchor were we limit the growth of the Chinese wage, we do so next.

4.4.2 Results With Limit to Chinese Wage Growth

Here we limit the increase in the growth of the Chinese wage in dollars to X% per year. Since this limit is to the growth in the Chinese wage in dollars it is like combining a limit to the growth in the Chinese wage in yuan and a limit to the appreciation that the yuan can have against the dollar. The logic here is that the Chinese monetary authority is not willing to tolerate very high levels of (wage) inflation or very high levels of yuan appreciation against the dollar, the first because it would have the traditional detrimental effects of inflation,

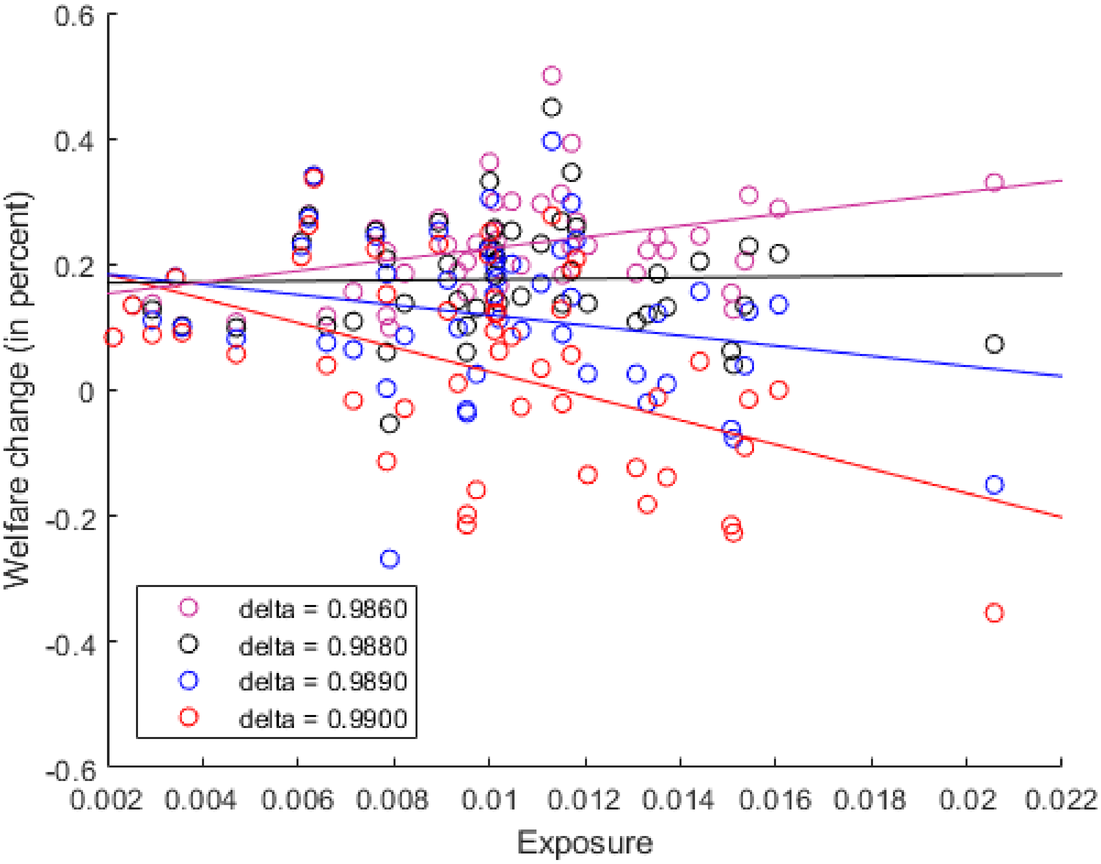

Figure 4.8: Welfare vs exposure for several δ's

Notes: Scatter plot of welfare changes across states in the U.S. versus exposure to China for the case of the rigidities implied by several different levels of δ.

and the second because it would have detrimental effects on competitiveness. Since we are just constraining the Chinese wage in dollars to grow slower than X% a year, this is a very generous bound and should provide a lower limit to the effects of the nominal rigidity on the China shock.

Here the difficulty that we face is that we have to chose X and δ together. One option would be to take X from the data, and that would say that in no year between 2000 and 2008 China had more than 5% inflation or 5% appreciation, so we could decide to use $X = 10$ or $X = 15$ to be generous. Then we would need to calibrate δ, maybe in the same way that we did it in the section with the world nominal GDP targeting. The problem in this case is that there can only be unemployment when China is stuck at its upper limit, which probably would only occur in 2003 and 2004, so by 2005 probably all unemployment will be gone, this means that the unemployment average we use for the ADH regressions of exposure

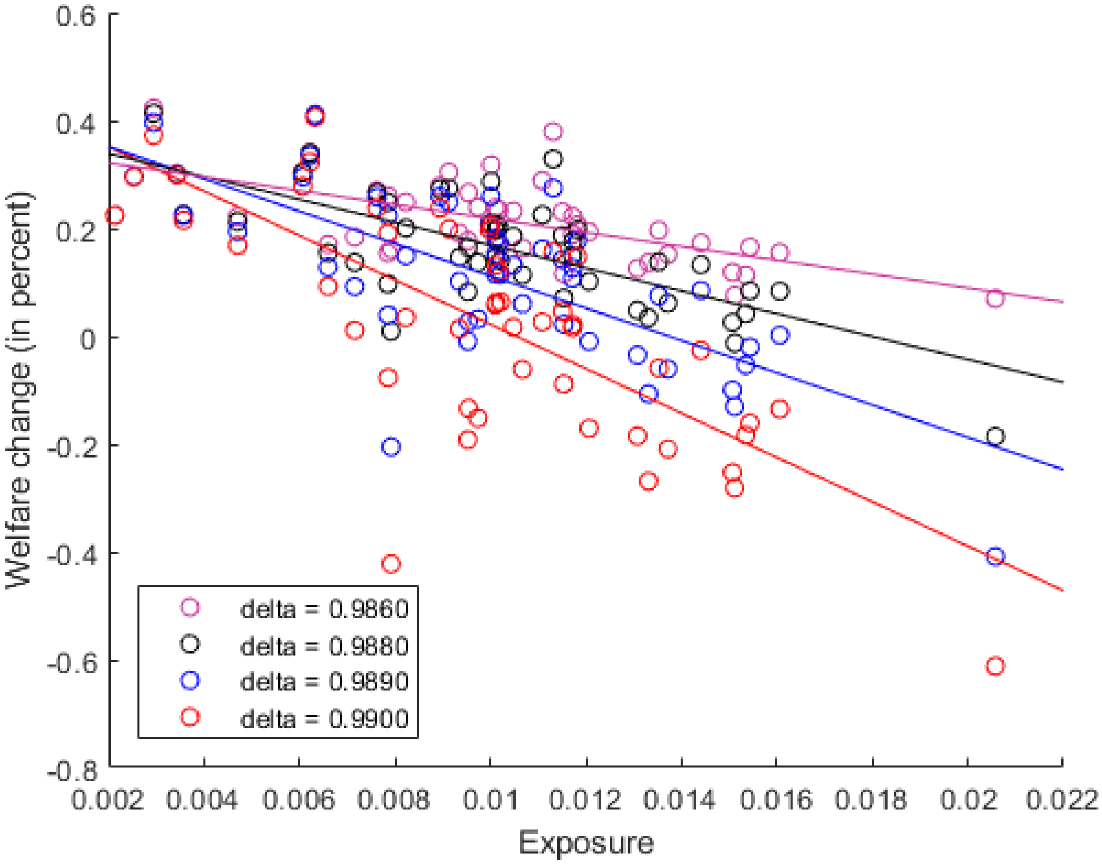

Figure 4.9: “Fake” Welfare vs exposure for several δ’s

Notes: Scatter plot of “fake” welfare (computed with the same price index for all states in the U.S.) changes across states in the U.S. versus exposure to China for the case of the rigidities implied by several different levels of δ.

on unemployment will be several years with 0 unemployment and 1 or 2 years with positive unemployment, this makes it difficult to deliver a high coefficient. With X=20% it is difficult to match the ADH coefficient. With X=15% it is possible to match the ADH coefficient with a δ of around 0.945 (a lot lower than in the case of the world nominal GDP targeting), but only when unemployment is measured as an average over the whole period (2000-2007) and not just as an average over 2006-2008 as in ADH. With such a delta the welfare gain from the China shock in the U.S. is somewhere between 4 and 8 basis points (a lot lower than in the world nominal GDP targeting case), between a third and a fourth of the effect without rigidity. In this specification unemployment is a lot more short lived than with the world nominal GDP targeting.

4.5 Conclusion

In this paper we build a Neo-Keynesian model of trade to capture the fact that unemployment can be possible after trade shocks due to nominal rigidities. Our model combines the richness in the trade structure of international trade models (several regions, countries, and sectors) with the rich dynamic structure and nominal rigidities of open economy macro models. The nominal rigidity, which is a downwardly rigid nominal wage, can generate unemployment if nominal demand is not growing sufficiently fast, captured in the model by having nominal demand grow at a constant pace.

We apply this model to quantify the effects of the China shock across states in the United States, with a realistic calibration, but several simplifying assumptions. We find that the China shock is responsible of up to 1.3 percentage points of U.S. unemployment, but this can go as high as 4.5 percentage points for the states affected the most. Regarding welfare, we find that on aggregate the welfare increase in the U.S. with nominal rigidities, 15 basis points, is about two thirds of the one that would occur without rigidities (22 basis points). Importantly, the effect is still positive (even though it could be negative if nominal rigidities were even higher) and we can disaggregate it across regions to show which states suffered the most from the China shock through high unemployment.

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Appendices

Appendix A

Additional Negative Rates Material

A.1 Bank Related Derivations

A.1.1 Loan Market

Here I solve the problem of an agent (in the model described in Section 2.4 it would represent a firm) that has to decide how much to borrow from each bank subject to a CES constraint. As discussed by GNSS, and in Appendix A.1.3, there are several ways to justify the CES constraint, some of them are: switching costs, asymmetric information, menu costs and regulatory restrictions. Agent s seeks a total amount of real loans equal to $l_t(s)$, he borrows an amount $l_t(s,j)$ from each bank j and faces the following constraint:

$$\left[\int_0^1 l_t(s,j)^{(\varepsilon_t^l-1)/\varepsilon_t^l} dj\right]^{\varepsilon_t^l/(\varepsilon_t^l-1)} \geq l_t(s),$$

which indicates that the loans he gets from individual banks are aggregated via a CES aggregator into the total loans he obtains. ε_t^l is the elasticity of substitution between banks, which for now I allow to vary with time. This elasticity will be assumed to be greater than one, as traditional in the monopolistic competition framework. Each bank charges the agent a net interest rate $i_t^l(j)$. Demand for this agent can be derived from minimizing over $l_t(s,j)$ the total repayment (including principal) due to the continuum of banks j:

$$\int_0^1 (1+i_t^l(j))l_t(s,j)dj,$$

subject to the constraint given above. I assume that loan customers minimize total repayment to banks $(1+i^l)$ instead of net interest payments (i^l). Total payments are more suitable that interest payments, since in the full dynamic model there is a time difference between when loans are taken out and when they are repaid. The gross formulation also leads to expressions that are a lot more suitable in a ZLB context.

The Lagrangian for this problem is:

$$\mathcal{L} = \int_0^1 (1+i_t^l(j))l_t(s,j)dj - \lambda \left(\left[\int_0^1 l_t(s,j)^{(\varepsilon_t^l-1)/\varepsilon_t^l} dj \right]^{\varepsilon_t^l/(\varepsilon_t^l-1)} - l_t(s) \right).$$

The F.O.C. w.r.t. $l_t(s,j)$ yields:

$$l_t(s,j) = \left(\frac{1+i_t^l(j)}{\lambda} \right)^{-\varepsilon_t^l} l_t(s),$$

where:

$$\lambda \;=\; \left(\int_0^1 (1+i_t^l(j))^{1-\varepsilon_t^l} dj \right)^{\frac{1}{1-\varepsilon_t^l}}.$$

So, denoting $1+i_t^l \equiv \lambda$, one can write demand for a particular bank j coming from client s as:

$$l_t(s,j) = \left(\frac{1+i_t^l(j)}{1+i_t^l} \right)^{-\varepsilon_t^l} l_t(s).$$

A.1.2 Deposit Market

Now I analyze the problem of an agent that instead of needing to borrow from banks wants to lend to banks via deposits (this will represent households in the full model). Assume that households want to maximize total repayment from deposits subject to total deposits (as aggregated through a CES aggregator) being smaller or equal to the amount available to deposit. In this case demand by agent s seeking an amount of real deposits equal to $d_t(s)$ can be derived from maximizing over $d_t(s,j)$ the total amount obtained from the continuum of banks j:

$$\int_0^1 (1+i_t^d(j))d_t(s,j)dj,$$

subject to:

$$\left[\int_0^1 d_t(s,j)^{(\varepsilon_t^d-1)/\varepsilon_t^d} dj \right]^{\varepsilon_t^d/(\varepsilon_t^d-1)} \leq d_t(s),$$

where the elasticity ε_t^d is required to be smaller than -1, which means that the exponent of the terms inside the integral is greater than one, implying this function is convex. The

Lagrangian for this problem is:

$$\mathcal{L} = \int_0^1 (1 + i_t^d(j))d_t(s,j)dj - \lambda \left(\left[\int_0^1 d_t(s,j)^{(\varepsilon_t^d - 1)/\varepsilon_t^d} dj \right]^{\varepsilon_t^d/(\varepsilon_t^d - 1)} - d_t(s) \right).$$

This is exactly the same problem as in the loan case, which means that the solution will be the same. In this case the solution can be written as:

$$d_t(s,j) = \left(\frac{1 + i_t^d(j)}{1 + i_t^d} \right)^{-\varepsilon_t^d} d_t(s).$$

But remember that now ε_t^d is negative (in particular smaller than -1), which means that customers put more deposits in a particular bank the higher that bank's deposit rate is.

A.1.3 Microfoundation of Bank CES

Here I provide simple microfoundations for the use of a CES aggregator across individual banks. One possible objection to such a setup could be that it is implausible that all consumers borrow from all banks, since a more accurate representation of reality probably is that each consumer borrows from one (or at most two) banks. Here I show how a model where each consumer chooses to borrow from a single bank and is subject to an stochastic utility of borrowing from each bank can deliver the same demand for loans as the one that emerges from the CES approach. The different stochastic utilities across individuals of borrowing from specific banks can represent proximity, switching costs, tastes, or asymmetric information. The presentation is inspired by Anderson, de Palma, and Thisse (1988) and Anderson, Palma, and Thisse (1989).

Assume that there is an individual consumer that lives for two periods, denoted 0 and 1. This consumer has a total income of $\overline{Y}$ in the second period and he can consume in both periods. To consume in period 1 is easy for this consumer, he can do it directly, but to consume in period 0 he most borrow against his future income $\overline{Y}$ through one of a continuum of banks between zero and one (indexed with j). The decision process of this consumer happens in two stages. In the first stage, the consumer decides which bank he wants to borrow from, and in the second stage he chooses the amount he wants to borrow. Suppose that the outcome of the first stage is that the consumer decides to borrow from bank j. Assume that the direct utility function of the consumer conditional on his choice of bank j is:

$$U(C_{0j}, C_1) = \ln(C_{0j}) + \beta \ln(C_1),$$

Where β is the discount factor between periods. The first period, second period, and aggregate budget constraints of the consumer (again conditional on the choice of bank j) are:

$$C_{0j} = B_j$$

$$\begin{aligned} C_1 &= \overline{Y} - (1+i^l_j)B_j \\ (1+i^l_j)C_{0j} + C_1 &= \overline{Y}, \end{aligned}$$

where $1+i^l_j$ is the interest rate charged between periods 0 and 1 by bank j (which is known by the consumer with certainty). The solution to this problem is:

$$C_{0j} = \frac{\overline{Y}}{1+\beta}\frac{1}{1+i^l_j}, \qquad C_1 = \frac{\beta}{1+\beta}\overline{Y},$$

and the indirect utility function conditional on borrowing from bank j is:

$$v(1+i^l_j) = (1+\beta)(\ln(\overline{Y}) - \ln(1+\beta)) + \beta\ln(\beta) - \ln(1+i^l_j).$$

Then, as in Anderson et al. (1988), assume that the first stage (the bank choice stage), is described by a stochastic utility approach:

$$V_i = v(1+i^l_j) + \mu\varepsilon_j,$$

where μ is a positive constant and ε_j is a random variable with zero mean and unit variance. Assuming that the ε_j random variables are independently and identically distributed with type-1 extreme value distribution (Gumbel), then the probability for a consumer of choosing bank j is given by:

$$Pr(j) = Pr(V_j = \max_r V_r) = \frac{e^{v(1+i^l_j)/\mu}}{\int_0^1 e^{v(1+i^l_r)/\mu}dr} = \frac{(1+i^l_j)^{-\frac{1}{\mu}}}{\int_0^1 (1+i^l_r)^{-\frac{1}{\mu}}dr},$$

as in McFadden (1973). Substituting $1/\mu$ for $\varepsilon^l - 1$ (which is positive since $\varepsilon^l > 1$) the previous expression can be rewritten as:

$$Pr(j) = \frac{(1+i^l_j)^{1-\varepsilon^l}}{\int_0^1 (1+i^l_r)^{1-\varepsilon^l}dr} = \left(\frac{1+i^l_j}{1+i^l}\right)^{1-\varepsilon^l},$$

where i^l is the aggregate loan rate defined in Appendix A.1.1. Multiplying C_{0j} by this probability one obtains:

$$C_{0j}Pr(j) \;=\; \frac{\overline{Y}}{1+\beta}\frac{1}{1+i^l}\left(\frac{1+i^l_j}{1+i^l}\right)^{-\varepsilon^l}.$$

Interpret $\frac{\overline{Y}}{1+\beta}\frac{1}{1+i^l}$ as aggregate borrowing and denote it L. Additionally, interpret $C_{0j}Pr(j)$ as the amount borrowed from bank j once the whole population of consumers are taken into account, and denote this by L_j. Then:

$$L_j = \left(\frac{1+i^l_j}{1+i^l}\right)^{-\varepsilon^l} L,$$

which is the same expression that one obtains directly from the CES aggregator. This shows that a heterogeneous borrower approach with stochastic utility and extreme value shocks works as a microfoundation for the CES aggregator in the case of a homogeneous borrower. A similar process can be followed to microfound deposit supply.

A.1.4 Solution to the Simple Bank Problem

Recall the following maximization problem for an individual bank j (which is one out of a continuum of identical banks between zero and one) described in Section 2.2:

$$\begin{aligned}
\max_{i^l_j, L_j, i^d_j, D_j, H_j} \quad & (1+i^l_j)L_j + (1+i)H_j - (1+i^d_j)D_j \\
L_j &= \left(\frac{1+i^l_j}{1+i^l}\right)^{-\varepsilon^l} L \\
D_j &= \begin{cases} \left(\frac{1+i^d_j}{1+i^d}\right)^{-\varepsilon^d} D & \text{if } i^d_j \geq 0 \\ 0 & \text{if } i^d_j < 0 \end{cases} \\
L_j + H_j &= F_j + D_j \\
H_j &\geq 0.
\end{aligned} \tag{A.1}$$

Proposition 1: *Consider the bank problem described in equation* (A.1)*, additionally assume that* $\varepsilon^l > 1$*,* $\varepsilon^d < -1$*, and* $D > L > F$*. The solution is described by several regimes that apply depending on the level of the policy rate* i*. Regime 1 applies when* $i \geq \tilde{\imath}$*, Regime 2 applies when* $\underline{i} \leq i < \tilde{\imath}$*, Regime 3A applies when* $\underline{\underline{i}} \leq i < \underline{i}$*, and Regime 3B applies when* $i < \underline{\underline{i}}$*. The thresholds are given by:*

$$\begin{aligned}
\tilde{\imath} &= -\frac{1}{\varepsilon^d} > 0 \\
\underline{i} &= \frac{\left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l-1} - \frac{1}{\varepsilon^l-1}\frac{L}{F} - 1}{1 + \frac{1}{\varepsilon^l-1}\frac{L}{F} + \frac{D}{F} - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l-1}} < 0 \\
\underline{\underline{i}} &= \frac{\left(1+\frac{D}{F}\right)^{\frac{1}{\varepsilon^l}} - 1 - \frac{D}{\varepsilon^l F}}{1 + \frac{D}{F} - \left(1+\frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}} < \underline{i},
\end{aligned}$$

Regime 1: *In this regime all banks obtain an amount of deposits* $D_j = D$*, lend out an amount* $L_j = L$*, and hold an amount of reserves* $H_j = F + D - L > 0$*. All banks set the same loan rate and deposit rate, whose expressions are given by:*

$$1 + i^l_j = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i), \qquad\qquad 1 + i^d_j = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i).$$

Bank return on equity is given by:

$$ROE_j \equiv \frac{F'_j}{F_j} - 1 = (1 + i)\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{1}{1 - \varepsilon^d}\frac{D}{F}\right) - 1.$$

Regime 2: *In this regime all banks obtain an amount of deposits* $D_j = D$*, lend out an amount* $L_j = L$*, and hold an amount of reserves* $H_j = F + D - L > 0$*. All banks set the same loan rate and deposit rate, whose expressions are given by:*

$$1 + i^l_j = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i), \qquad\qquad i^d_j = 0.$$

Bank return on equity is given by:

$$ROE_j = \frac{1}{\varepsilon^l - 1}\frac{L}{F} + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right).$$

Regime 3A: *In this regime a fraction of banks* $\mu(i)$ *sets a negative deposit rate, doesn't obtain any deposits and lends out its equity at the loan rate:*

$$1 + i^l_{ND} = \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon}}(1 + i^l),$$

where ND *stands for "No Deposits" and* i^l *is the aggregate loan rate, given by:*

$$1 + i^l = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i)\left(\frac{1 - \mu(i)}{1 - \mu(i)\,(F/L)^{\frac{\varepsilon^l - 1}{\varepsilon^l}}}\right)^{\frac{1}{1 - \varepsilon^l}}.$$

These banks don't keep any reserves at the central bank. The remaining fraction of banks $(1 - \mu(i))$ *sets a zero deposit rate and obtains an amount of deposits* D*, lends out an amount:*

$$L_D = \left(\frac{L^{\frac{\varepsilon - 1}{\varepsilon}} - \mu(i)F^{\frac{\varepsilon - 1}{\varepsilon}}}{1 - \mu(i)}\right)^{\frac{\varepsilon}{\varepsilon - 1}},$$

at a rate: $1 + i^l_D = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i)$*, and keeps an amount of reserves at the central bank given by:* $H_D = F + D - L_D$*. The fraction of banks not taking deposits* $\mu(i)$ *is defined implicitly* *by the following expression:*

$$\frac{\varepsilon^l}{\varepsilon^l-1}(1+i)\frac{L_D}{L}\left(\left(\frac{L_D}{L}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}-1\right) \;=\; \mu\cdot(1+i)\cdot\left(\frac{F}{L}+\frac{D}{L}-\frac{L_D}{L}\right)-\mu\cdot\frac{D}{L}.$$

Regime 3B: *In this regime a fraction of banks μ^* sets a negative deposit rate, doesn't obtain any deposits and just lends out its equity at the loan rate i^l_{ND}. These variables are given by:*

$$\begin{aligned}\mu^* &= \frac{(1+D/F)^{\frac{\varepsilon^l-1}{\varepsilon^l}}-(L/F)^{\frac{\varepsilon^l-1}{\varepsilon^l}}}{(1+D/F)^{\frac{\varepsilon^l-1}{\varepsilon^l}}-1}\\ 1+i^l_{ND} &= \frac{\frac{D}{F}}{\left(1+\frac{D}{F}\right)^{\frac{\varepsilon^l-1}{\varepsilon^l}}-1},\end{aligned}$$

These banks don't keep any reserves at the central bank. The remaining fraction of banks $(1-\mu^*)$ sets a zero deposit rate and obtains an amount of deposits D, lends out an amount $L_D = F + D$ at a rate i^l_D, where:

$$1+i^l_D \;=\; \frac{\frac{1}{F/D+1}}{1-\left(1+\frac{D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}}$$

These banks also don't keep any reserves at the central bank. In this case the aggregate loan rate is given by:

$$1+i^l \;=\; \frac{\frac{D}{F}\left(\frac{F}{L}\right)^{\frac{1}{\varepsilon^l}}}{\left(1+\frac{D}{F}\right)^{\frac{\varepsilon^l-1}{\varepsilon^l}}-1}$$

Proof: First verify the claims about the thresholds. Since $\varepsilon^d < 0$ it is obvious that $\tilde{\imath} > 0$. To prove that $\underline{i}$ is negative I will prove that the denominator is positive and the numerator is negative. First the denominator:

$$1+\frac{1}{\varepsilon^l-1}\frac{L}{F}+\frac{D}{F}-\left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l-1} > \frac{1}{\varepsilon^l-1}\frac{L}{F}\left(1-\left(\frac{L}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right)+\frac{D}{F}-\left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}.$$

The last expression is positive because $D/F > L/F > (L/F)^{1/\varepsilon^l}$ and $L/F > 1$ which implies $\left(\frac{L}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} < 1$. The numerator of $\underline{i}$ is negative, to see this write it as a function of L/F, $\text{numerator}_{\underline{i}} = f(L/F)$, where:

$$f(x) \;=\; x^{1/\varepsilon^l}\frac{\varepsilon^l}{\varepsilon^l-1}-\frac{x}{\varepsilon^l-1}-1$$

Since the second derivative is always negative for $x > 1$, the function is maximized where $f'(x) = 0$, which is at $x = 1$. Additionally $f(1) = 0$, which implies that the function is always negative for values of x greater than 1. Since $L/F > 1$ this means that the numerator of $\underline{i}$ is always negative. In the explanation for Regime 3A I will prove that $\underline{i} > \underline{\underline{i}}$. Now I proceed to prove the claims about the regimes.

Regime 1: In this regime both rates are set according to their unconstrained F.O.C.s, which due to the concavity of the objective function guarantees that banks achieve a maximum. The loan and deposit subproblems are independent, due to the presence of positive reserves. The unconstrained loan subproblem is to maximize $(i^l_j - i)L_j$ subject to loan demand. The solution is given by:

$$1 + i^l_j \; = \; \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i).$$

Similarly, the unconstrained deposit subproblem is to maximize $(i - i^d_j)D_j$ subject to deposit supply. The solution is given by:

$$1 + i^d_j \; = \; \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i).$$

No bank has incentives to deviate since they are all acting optimally. If a bank decided to stop keeping reserves at the central bank it would have to lend more and this requires lowering its loan rate, which is not optimal. If a bank decided to stop taking deposits it would stop earning its deposit spread, which is also suboptimal. The constrain $H > 0$ is satisfied in this regime, since $H = F + D - L > 0$. The deposit ZLB is satisfied as long as the deposit rate paid by banks is greater or equal than zero, which occurs as long as:

$$i \; \geq \; -\frac{1}{\varepsilon^d} \equiv \tilde{\imath}.$$

In this regime:

$$ROE_j \equiv \frac{F'_j}{F_j} - 1 \; = \; (1 + i)\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{1}{1 - \varepsilon^d}\frac{D}{F}\right) - 1.$$

Regime 2: In this regime the constraint that reserves are positive holds for the same reason as in the previous regime: $H_j = F + D - L > 0$. All banks set a zero deposit rate, so the deposit ZLB also holds. Since banks still hold reserves at the central bank they solve the unconstrained loan subproblem, which yields the same solution as it did in Regime 1. In this regime ROE is given by:

$$ROE_j \; = \; \frac{1}{\varepsilon^l - 1}\frac{L}{F} + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right).$$

Banks don't want to start taking deposits by setting a positive deposit rate, since their unconstrained problem would deliver a negative rate. Banks also don't want to deviate to stop having reserves at the central bank, since this would require deviating from their maximizing loan rate. An individual bank might want to deviate to set a negative deposit rate, obtain no deposits and just lend out its full equity at the rate that allows to do so. This deviating bank would need to lend out F, so it could charge a loan rate of:

$$i_j^l \;=\; \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i) - 1.$$

Banks are better by not deviating if:

$$\begin{aligned}
\left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i) - 1 \;&\leq\; \frac{1}{\varepsilon^l - 1}\frac{L}{F} + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right) \\
i \;&\leq\; \frac{\left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l-1} - 1 - \frac{1}{\varepsilon^l-1}\frac{L}{F}}{1 + \frac{1}{\varepsilon^l-1}\frac{L}{F} + \frac{D}{F} - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l-1}} \equiv \underline{i}.
\end{aligned}$$

It is also easy to check than in this case banks that deviate to not taking deposits wouldn't want to keep reserves at the central bank, since the rate that they earn on loans is bigger than the policy rate.

Regime 3A: Since banks not taking deposits lend an amount equal to F, then the amount lent by banks taking deposits (denoted L_D) has to equal:

$$L_D \;=\; \left(\frac{L^{\frac{\varepsilon^l-1}{\varepsilon^l}} - \mu F^{\frac{\varepsilon^l-1}{\varepsilon^l}}}{1-\mu}\right)^{\frac{\varepsilon^l}{\varepsilon^l-1}}.$$

Since banks taking deposits are still keeping reserves at the central bank they must still satisfy their unconstrained loan subproblem F.O.C. which delivers:

$$(1 + i_D^l) = \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i).$$

Banks taking deposits must lend out L_D, which requires:

$$1 + i^l \;=\; \left(\frac{L_D}{L}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i).$$

Additionally, banks that do not take deposits must lend out F, which requires:

$$1 + i_{ND}^l \;=\; \left(\frac{L_D}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i).$$

It is now possible to calculate the ROE for both types of banks:

$$\frac{F'_{ND}}{F_{ND}} - 1 \quad = \quad i^l_{ND},$$

and:

$$\frac{F'_D}{F_D} - 1 \quad = \quad \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i)\frac{L_D}{F} + (1+i)\frac{F + D - L_D}{F} - \frac{D}{F} - 1.$$

So, the profits of the two types of banks are equal when:

$$\begin{aligned} i^l_{ND} &= \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i)\frac{L_D}{F} + (1+i)\frac{F + D - L_D}{F} - \frac{D}{F} - 1 \\ 1 + \frac{i}{1+i}\frac{D}{F} &= \frac{\varepsilon^l}{\varepsilon^l - 1}\frac{L_D}{F}\left[\left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} - \frac{1}{\varepsilon^l}\right]. \end{aligned}$$

This expression of i and μ can be written as $F(i, \mu) = 0$, where:

$$F(i, \mu) \quad = \quad 1 + \frac{i}{1+i}\frac{D}{F} - \frac{\varepsilon^l}{\varepsilon^l - 1} g(\mu)^{\frac{1}{\varepsilon^l}} + \frac{g(\mu)}{\varepsilon^l - 1},$$

and:

$$g(\mu) = \frac{L_D}{F} = \left(\frac{(L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu}{1 - \mu}\right)^{\frac{\varepsilon^l}{\varepsilon^l - 1}}.$$

Since $g(\mu)$ is an increasing function of $\frac{a-\mu}{1-\mu}$, where $a = (L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} > 1$, it is evident that $g'(\mu) > 0$. Also notice that:

$$\frac{\partial F}{\partial \mu} \quad = \quad \frac{g'(\mu)}{\varepsilon^l - 1}\frac{(L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - 1}{(L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu} > 0.$$

Using the implicit function theorem this means that the expression $F(i, \mu) = 0$ implicitly defines μ as a function of i. The derivative of $\mu(i)$ is given by:

$$\frac{\partial \mu}{\partial i} = -\frac{\frac{\partial F}{\partial i}}{\frac{\partial F}{\partial \mu}},$$

which is negative because:

$$\frac{\partial F}{\partial i} = \frac{1+i-i}{(1+i)^2}\frac{D}{F} = \frac{1}{(1+i)^2}\frac{D}{F} > 0.$$

The level $\mu(i) = 0$ is a solution when:

$$1+i+i\frac{D}{F} \;\; = \;\; (1+i)\frac{\varepsilon^l}{\varepsilon^l - 1}\frac{L}{F}\left[\left(\frac{L}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} - \frac{1}{\varepsilon^l}\right]$$

Which occurs when $i = \underline{i}$. The requirement that the amount of reserves held at the central bank by commercial banks taking deposits must be positive can be expressed as:

$$\begin{aligned} 0 \;\; &\leq \;\; F + D - \left(\frac{L^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu F^{\frac{\varepsilon^l - 1}{\varepsilon^l}}}{1-\mu}\right)^{\frac{\varepsilon^l}{\varepsilon^l - 1}} \\ \mu \;\; &\leq \;\; \frac{(1+D/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - (L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}}}{(1+D/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - 1} \equiv \overline{\mu}. \end{aligned}$$

It is easy to see that this limit level $\overline{\mu}$ is greater than zero and smaller than one. When $\mu = \overline{\mu}$ the amount lent by banks taking deposits is $1 + D/F$, this is obvious from the fact that this limit was derived from $H_D = 0$, and that occurs when $1 + \frac{D}{F} = \frac{L_D}{F}$. This can be introduced into the expression for $\mu(i)$ to obtain the level of the policy rate that delivers $\overline{\mu}$:

$$\begin{aligned} 1 + \frac{i}{1+i}\frac{D}{F} \;\; &= \;\; \frac{\varepsilon^l}{\varepsilon^l - 1}\frac{L_D}{F}\left[\left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} - \frac{1}{\varepsilon^l}\right] \\ i \;\; &= \;\; \frac{\left(1+\frac{D}{F}\right)^{\frac{1}{\varepsilon^l}} - 1 - \frac{D}{\varepsilon^l F}}{1 + \frac{D}{F} - \left(1+\frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}} \equiv \underline{\underline{i}}. \end{aligned}$$

This interest rate is smaller than $\underline{i}$, since at $\underline{i}$ it is the case that $\mu = 0$, at $\underline{\underline{i}}$ it is the case that $0 < \mu < 1$, and $\mu(i)$ is a decreasing function of i. Now I want to show that i^l is a decreasing function of i. Recall that:

$$1 + i^l \;\; = \;\; \left(\frac{L_D}{L}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l - 1}(1+i),$$

which means that as i increases, there are two effects on i^l, the increase in i directly increases i^l, but the fall in μ brought about by the increase in i lowers L_D/F and this tends to lower

i^l. To see which effect dominates take derivatives:

$$\frac{\partial i^l}{\partial i} = \frac{1}{\varepsilon^l}\left(\frac{L_D}{L}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} \frac{\partial (L_D/L)}{\partial i}\frac{\varepsilon^l}{\varepsilon^l-1}(1+i)+\left(\frac{L_D}{L}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l-1}.$$

To obtain the derivative $\frac{\partial(L_D/F)}{\partial i}$ apply the implicit function theorem directly to:

$$F(i,L_D/F)=1+\frac{i}{1+i}\frac{D}{F}-\frac{\varepsilon^l}{\varepsilon^l-1}\frac{L_D}{F}\left[\left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}-\frac{1}{\varepsilon^l}\right],$$

to obtain:

$$\frac{\partial(L_D/F)}{\partial i} = -\frac{\frac{\partial F}{\partial i}}{\frac{\partial F}{\partial L_D/F}}=-\frac{\frac{1}{(1+i)^2}\frac{D}{F}}{\frac{1}{\varepsilon^l-1}\left(1-\left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right)}.$$

Introducing this in the expression for $\frac{\partial i^l}{\partial i}$ one gets:

$$\frac{\partial i^l}{\partial i} = \left(\frac{L_D}{L}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l-1}-\left(\frac{L_D}{L}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\frac{\frac{1}{1+i}\frac{D}{L}}{1-\left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}}.$$

This is equal to zero if:

$$\frac{\varepsilon^l}{\varepsilon^l-1}\left(1-\left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right) = \frac{1}{1+i}\frac{D}{L_D}.$$

At $\underline{i}$, where $L_D/F=1+D/F$ the previous expression becomes:

$$\begin{aligned}(1+\underline{i})\frac{\varepsilon^l}{\varepsilon^l-1}\left(1-\left(1+\frac{D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right) &= \frac{D}{L_D}\\ \frac{1}{1+\frac{D}{F}-\left(1+\frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}}\left(1+\frac{D}{F}-\left(1+\frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}\right) &= 1\end{aligned}$$

Which is always true. This proves that at $i = \underline{\underline{i}}$, the derivative $\frac{\partial i^l}{\partial i}$ is equal to zero. It is also possible to show that the second derivative of i^l w.r.t. i is negative:

$$\begin{aligned}
\frac{\partial^2 i^l}{\partial i^2} &= -\frac{1}{\varepsilon^l}\left(\frac{L_D}{L}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1} \frac{\frac{1}{(1+i)^2}\frac{D}{L}}{\frac{1}{\varepsilon^l - 1}\left(1 - \left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right)} + \frac{1}{(1+i)^2} \frac{\frac{D}{L}}{\left(\frac{L_D}{L}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \left(\frac{F}{L}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}}} \\
&- \frac{\frac{1}{1+i}\frac{D}{L}}{\left(\left(\frac{L_D}{L}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \left(\frac{F}{L}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}}\right)^2} \frac{\varepsilon^l - 1}{\varepsilon^l}\left(\frac{L_D}{L}\right)^{-\frac{1}{\varepsilon^l}} \frac{\frac{1}{(1+i)^2}\frac{D}{L}}{\frac{1}{\varepsilon^l - 1}\left(1 - \left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right)}
\end{aligned}$$

The first two terms cancel out and then it is clear than the second derivative of i^l w.r.t. i is negative, which means that throughout the whole Regime 3A the highest i^l is achieved at $i = \underline{\underline{i}}$. In this regime ROE is given by:

$$ROE_j = \frac{F_j'}{F_j} - 1 = i^l_{ND} = \left(\frac{(L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu(i)}{1 - \mu(i)}\right)^{\frac{1}{\varepsilon^l - 1}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i) - 1$$

Regime 3B: Banks not taking deposits lend out F:

$$1 + i^l_{ND} = \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon}}(1 + i^l)$$

Banks taking deposits lend out $F + D$:

$$1 + i^l_D = \left(\frac{F + D}{L}\right)^{-\frac{1}{\varepsilon}}(1 + i^l)$$

ROE for both types of banks must be equalized.

$$\begin{aligned}
(1 + i^l_{ND}) &= (1 + i^l_D)\left(1 + \frac{D}{F}\right) - \frac{D}{F} \\
1 + i^l &= \frac{\frac{D}{F}\left(\frac{F}{L}\right)^{\frac{1}{\varepsilon^l}}}{\left(1 + \frac{D}{F}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - 1}
\end{aligned}$$

Recall that in Regime 3A, when $\mu = \overline{\mu}$, it is the case that:

$$1 + i^l = \left(\frac{L_D}{F}\right)^{\frac{1}{\varepsilon^l}}\left(\frac{F}{L}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + \underline{\underline{i}}) = \frac{\frac{D}{F}\left(\frac{F}{L}\right)^{\frac{1}{\varepsilon^l}}}{\left(1 + \frac{D}{F}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - 1}$$

This means that the highest loan rate charged in Regime 3A is precisely the same rate charged in Regime 3B. It is easy to see that both types of banks satisfy all constraints and have no profitable deviations. ROE for all banks in this regime is given by:

$$ROE_j = \frac{F_j'}{F_j} - 1 = i^l_{ND} = \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon}} (1+i^l) - 1 = \frac{\left(1+\frac{D}{F}\right)^{\frac{1}{\varepsilon^l}} - 1}{1 - \left(1+\frac{D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}}$$

A.1.5 Problem of the Bank with a Target Leverage Ratio

In this section I describe a more general version of the bank model described in Section 2.2. Here, as in Section 2.4, banks are going to be subject to a cost of deviating from a target level of loan-to-equity ratio, the particular form of this cost is similar to the one in GNSS. The bank pays a quadratic cost (parameterized by a coefficient κ and proportional to outstanding bank net worth) whenever the loan-to-equity ratio $L(j)/F(j)$ deviates from the target value ν.

Banks also face costs of issuing loans given by μ^l and benefits of issuing deposits given by μ^d, these are per dollar of loan or deposit issued, and they could be positive or negative. The cost of issuing loans is positive (the bank has to monitor the borrowers, pay loan originators, etc), while the cost of issuing deposits could plausibly be negative, as it could be seen as a benefit that the bank receives for having a large deposit base, for example attracting more customers or obtaining more publicity (that is why I will depict them as a benefit in my notation).

I will make the assumption that $D + F > L > F$, which indicates that the total amount of loans being demanded is possible to cover with the total amount of deposits being supplied plus total bank equity, but it is not possible to cover simply with bank equity. The problem of the bank is:

$$\begin{aligned}
\max_{i^l(j),L(j),i^d(j),D(j),H(j)} \quad & (1+i^l(j)-\mu^l)L(j) + (1+i)H(j) - (1+i^d(j)-\mu^d)D(j) \\
& -\frac{\kappa}{2}\left(\frac{L(j)}{F(j)} - \nu\right)^2 F(j) \\
\text{s.t.} \quad & \\
L(j) \quad & \leq \left(\frac{1+i^l(j)}{1+i^l}\right)^{-\varepsilon^l} L \\
D(j) \quad & = \begin{cases} \left(\frac{1+i^d(j)}{1+i^d}\right)^{-\varepsilon^d} D & \text{if } i^d(j) \geq 0 \\ 0 & \text{if } i^d(j) < 0 \end{cases} \\
L(j) + H(j) \quad & = F(j) + D(j)
\end{aligned}$$

$$H(j) \geq 0$$

In Regime 1 all banks charge the same loan rate, which is set according to the unconstrained loan subproblem F.O.C.:

$$1 + i^l(j) = \varepsilon^l(i^l(j) - i - \mu^l) - \kappa\varepsilon^l \left(\left(\frac{1 + i^l(j)}{1 + i^l} \right)^{-\varepsilon^l} \frac{L}{F(j)} - \nu \right).$$

In equilibrium, since all the banks are symmetric, this becomes:

$$1 + i^l = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i + \mu^l) + \kappa \frac{\varepsilon^l}{\varepsilon^l - 1} \left(\frac{L}{F} - \nu \right).$$

So, it is clear that in this regime the loan interest rate increases not only with the policy rate and the cost of issuing loans, but also with the amount of loans being made (L). So this is a type of loan supply curve. In this regime the F.O.C. for the deposit rate is:

$$0 = -\left(\frac{1 + i^d(j)}{1 + i^d} \right)^{-\varepsilon^d} D - \varepsilon^d(i + \mu^d - i^d(j)) \left(\frac{1 + i^d(j)}{1 + i^d} \right)^{-\varepsilon^d - 1} \frac{D}{1 + i^d},$$

hence the solution for the deposit rate is:

$$1 + i^d(j) = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i + \mu^d).$$

Notice that this net deposit rate is only greater than zero if:

$$-\frac{1}{\varepsilon^d} - \mu^d \leq i.$$

Defining $\tilde{\imath} \equiv -\frac{1}{\varepsilon^d} - \mu^d$, I can say that as long as $i \geq \tilde{\imath}$ commercial banks want to set nonnegative deposit rates. In this regime bank resources at the end of the period are given by:

$$\begin{aligned} F' &= (1 + i) \left(F + \frac{1}{\varepsilon^l - 1} L + \frac{1}{1 - \varepsilon^d} D \right) + \frac{\mu^l}{\varepsilon^l - 1} L + \frac{\mu^d}{1 - \varepsilon^d} D \\ &+ \kappa \left(\frac{L}{F} - \nu \right) \left(\frac{\varepsilon^l}{\varepsilon^l - 1} L - \frac{L}{2} + \frac{\nu F}{2} \right). \end{aligned}$$

I assume that $\left(\frac{L}{F} - \nu\right)$ is small enough and $\frac{\varepsilon^l}{\varepsilon^l - 1} - 1$ is also small enough that I can approximate $\left(\frac{L}{F} - \nu\right) \frac{\varepsilon^l}{\varepsilon^l - 1}$ with $\left(\frac{L}{F} - \nu\right)$, then the previous expression can be written as:

$$\frac{F'}{F} = \left(1 + \frac{1 + \mu^l}{\varepsilon^l - 1} \frac{L}{F} + \frac{1 + \mu^d}{1 - \varepsilon^d} \frac{D}{F} \right) + i \left(1 + \frac{1}{\varepsilon^l - 1} \frac{L}{F} + \frac{1}{1 - \varepsilon^d} \frac{D}{F} \right) + \frac{\kappa}{2} \left(\frac{L}{F} - \nu \right) \left(\frac{L}{F} + \nu \right).$$

Once $i < \tilde{\imath}$ commercial banks start setting a deposit rate of exactly zero and continue to obtain deposits. In this case bank resources at the end of the period are given by:

$$F' = (1+i)\left(F + \frac{1}{\varepsilon^l - 1}L\right) + \frac{\mu^l}{\varepsilon^l - 1}L + (i + \mu^d)D + \kappa\left(\frac{L}{F} - \nu\right)\left(\frac{\varepsilon^l}{\varepsilon^l - 1}L - \frac{L}{2} + \frac{\nu F}{2}\right),$$

with the same approximation used above this becomes:

$$\frac{F'}{F} = \left(1 + \frac{1+\mu^l}{\varepsilon^l - 1}\frac{L}{F} + \mu^d\frac{D}{F}\right) + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right) + \frac{\kappa}{2}\left(\frac{L}{F} - \nu\right)\left(\frac{L}{F} + \nu\right).$$

All banks setting a zero deposit rate and obtaining deposits continues to be an equilibrium until any single bank would have an incentive to deviate. This deviating bank would set its loan rate to lend out its full equity:

$$1 + i^l_{DEV} = \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon^l}}(1 + i^l)$$

Equity at the end of the period for the deviating bank would be given by:

$$F'_{DEV} = (1 + i^l_{DEV} - \mu^l)F_{DEV} - \delta_i\frac{\kappa}{2}(1-\nu)^2 F_{DEV},$$

where δ_i is an indicator that can be set equal to 1 if one wishes to still include the cost of deviating from target leverage when banks are not taking deposits and equal to zero if one wishes not to include it. Since the "bank" is no longer taking household deposits it could be argued that it is no longer a "bank" and should not be subject to regulatory oversight. In order to discover the level of the policy rate at which it would start paying off to deviate I compare the ROE from the two scenarios:

$$\begin{aligned} &\left(1 + \frac{1+\mu^l}{\varepsilon^l - 1}\frac{L}{F} + \mu^d\frac{D}{F}\right) + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right) + \frac{\kappa}{2}\left(\frac{L}{F} - \nu\right)\left(\frac{L}{F} + \nu\right) \\ = &\left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}\left(\frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i + \mu^l) + \kappa\frac{\varepsilon^l}{\varepsilon^l - 1}\left(\frac{L}{F} - \nu\right)\right) - \mu^l - \delta_i\frac{\kappa}{2}(1-\nu)^2, \end{aligned}$$

Defining:

$$\underline{i} = -\frac{1 + \frac{1+\mu^l}{\varepsilon^l-1}\frac{L}{F} + \mu^d\frac{D}{F} + \frac{\kappa}{2}\left(\frac{L}{F} - \nu\right)\left(\frac{L}{F} + \nu\right) - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l-1}\left(1 + \mu^l + \kappa\left(\frac{L}{F} - \nu\right)\right) + \mu^l + \delta_i\frac{\kappa}{2}(1-\nu)^2}{1 + \frac{1}{\varepsilon^l-1}\frac{L}{F} + \frac{D}{F} - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l-1}},$$

then it is possible to say that Regime 2 is active when $\tilde{\imath} > i \geq \underline{i}$. For my baseline calibration I obtain $\tilde{\imath} = 0.5\%$ and $\underline{i} \approx -2\%$ at the annual level with $\delta_i = 0$ (with $\delta_i = 1$ $\underline{i}$ is much more negative). Notice that when $\mu^d = \mu^l = \kappa = 0$ the expressions just given simplify to the ones given in Section 2.2.

A.1.6 Loan Market Under Uncertainty

Here I will modify the problem of the borrowing firm from the one described in Appendix A.1.1 so that it is consistent with the stochastic nature of loan returns described in the banking framework of Section 2.4. The reasons for this change are sketched in Section 2.4.5.

There will be a firm seeking to obtain a known amount of funding from a continuum of banks. Each bank j will charge a multiple $\mu(j)$ of the return of the project (so they would earn $\mu(j)$ times the return of the project if they were allocated the whole project) and they will be allocated a fraction $\gamma(j)$ of the total project. Hence bank j will have to be paid an amount $\mu(j)\gamma(j)$ of the total return of the project. The firm will want to minimize this amount subject to the CES aggregation of the $\gamma(j)$ being equal to one. That is, they want to minimize:

$$\int_0^1 \mu(j)\gamma(j)dj,$$

subject to:

$$\left[\int_0^1 \gamma(j)^{(\varepsilon_t^b-1)/\varepsilon_t^b}dj\right]^{\varepsilon_t^b/(\varepsilon_t^b-1)} \geq 1.$$

The F.O.C. implies that:

$$\gamma(j) = \left(\frac{\mu(j)}{\lambda}\right)^{-\varepsilon_t^b},$$

where:

$$\lambda = \left[\int_0^1 \mu(j)^{1-\varepsilon_t^b}dj\right]^{\frac{1}{1-\varepsilon_t^b}},$$

so, denoting $\mu \equiv \lambda$, I can write demand for a particular bank j coming from client i as:

$$\gamma(j) = \left(\frac{\mu(j)}{\mu}\right)^{-\varepsilon_t^b}.$$

Since all the banks will be symmetric, they will all charge $\mu(j) = \mu$ and they will all be allocated a fraction of 1 of the total project (remember there is a measure 1 of banks so this is consistent with the total fraction allocated being 1).

With this demand schedule and this setup the loan subproblem of the bank is:

$$\max_{\gamma_t(j),\mu_t(j)} \quad \left[\mathbb{E}_t(1+i_{t+1}^l)\right]\mu_t(j)\gamma_t(j)L_t - (1+i_t+\mu^l)\gamma_t(j)L_t - \frac{\kappa}{2}\left(\frac{\gamma_t(j)L_t}{F_t(j)} - \nu\right)^2 F_t(j)$$

s.t.

$$\gamma_t(j) = \left(\frac{\mu_t(j)}{\mu_t}\right)^{-\varepsilon^l}.$$

Differentiating with respect to $\mu_t(j)$ I obtain the F.O.C.:

$$0 = \frac{\varepsilon^l - 1}{\varepsilon^l}\left[\mathbb{E}_t(1+i^l_{t+1})\right]\mu_t(j) - (1 + i_t + \mu^l) - \kappa\left(\left(\frac{\mu_t(j)}{\mu_t}\right)^{-\varepsilon^l}\frac{L_t}{F_t(j)} - \nu\right).$$

In equilibrium, since all the banks are symmetric this becomes:

$$0 = \frac{\varepsilon^l - 1}{\varepsilon^l}\left[\mathbb{E}_t(1+i^l_{t+1})\right]\mu_t - (1 + i_t + \mu^l) - \kappa\left(\frac{L_t}{F_t} - \nu\right).$$

Also, since all the banks are symmetric $\mu_t(j) = \mu_t$, and since in equilibrium μ_t must always equal one (total returns on each unit of capital must always be $(1 + i^l_{t+1})$), then I set $\mu_t = 1$ and obtain:

$$\mathbb{E}_t(1+i^l_{t+1}) = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i_t + \mu^l) + \kappa\frac{\varepsilon^l}{\varepsilon^l - 1}\left(\frac{L_t}{F_t} - \nu\right).$$

A.1.7 Heterogeneous Bank Model

The new logarithmic specification described in section 2.5.1 is:

$$-\kappa\nu\frac{L_j}{F_j}\left(\ln\left(\frac{L_j}{F_j}\right) - \ln\nu - 1\right) - \kappa\nu^2.$$

Denote this function by $f(x)$, where $x = L/F$, then:

$$\begin{aligned} f(x) &= -\kappa\nu x\left(\ln(x) - \ln\nu - 1\right) - \kappa\nu^2 \\ f'(x) &= -\kappa\nu\left(\ln(x) - \ln\nu\right) \\ f''(x) &= -\frac{\kappa\nu}{x}. \end{aligned}$$

Hence the second order approximation to the new functional form (around $x = \nu$ which is the steady state value of L/F) is the following:

$$\begin{aligned} f(x) &\approx^2 f(\nu) + f'(\nu)(x-\nu) + \frac{f''(\nu)}{2}(x-\nu)^2 \\ &= -\frac{\kappa}{2}(x-\nu)^2. \end{aligned}$$

This means that the quadratic specification that I have used so far is the second order approximation to the logarithmic specification around the steady state. The reason that the logarithmic specification is convenient is because it allows to solve the heterogeneous bank model in a simple way (after using an approximation). When using the specification in logs, the bank problem (abstracting from the managerial cost, the exogenous costs and benefits of issuing loans and deposits, the stochastic nature of the loan return, and the constraints that reserves are nonnegative and deposits rates are nonnegative) is:

$$\max_{H_j,L_j,D_j,i_j^l,i_j^d} \quad (1+i)H_j + (1+i_j^l)L_j - (1+i_j^d)D_j - \left[\kappa\nu\frac{L_j}{F_j}\left(\ln\left(\frac{L_j}{F_j}\right) - \ln(\nu) - 1\right) + \kappa\nu^2\right]F_j$$

$$\text{s.t.} \quad L_j = \left(\frac{1+i_j^l}{1+i^l}\right)^{-\varepsilon^l} L, \quad D_j = \left(\frac{1+i_j^d}{1+i^d}\right)^{-\varepsilon^d} D, \quad L_j + H_j = F_j + D_j.$$

The F.O.C. with respect to the deposit rate is unchanged, and it implies that all banks set the same deposit rate. The F.O.C. w.r.t. the loan rate is now:

$$1+i_j^l = \frac{\varepsilon^l}{\varepsilon^l-1}\left(1+i+\kappa\nu\left(\ln\left(\frac{L_j}{F_j}\right) - \ln(\nu)\right)\right).$$

If I impose symmetry so that $L_j = L$ for all j, and $F_j = F$, I obtain:

$$1+i^l = \frac{\varepsilon^l}{\varepsilon^l-1}\left(1+i+\kappa\nu\left(\ln\left(\frac{L}{F}\right) - \ln(\nu)\right)\right),$$

which is very similar to what I had in the homogeneous bank model used before, except that $\nu(\ln(L/F) - \ln(\nu))$ has replaced $(L/F - \nu)$, but the second one is the first order approximation to the first. Taking natural logs in the expression before imposing symmetry one gets:

$$\ln(1+i_j^l) = \ln\left(\frac{\varepsilon^l}{\varepsilon^l-1}\right) + \ln\left(1+i+\kappa\nu\left(\ln\left(\frac{L_j}{F_j}\right) - \ln(\nu)\right)\right).$$

I can approximate this as:[1]

$$i_j^l = \ln\left(\frac{\varepsilon^l}{\varepsilon^l-1}\right) + i + \kappa\nu\ln(L_j) - \kappa\nu\ln(F_j) - \kappa\nu\ln(\nu).$$

This is linear in the net rates and the logs of quantities, which is convenient because demand is also linear in those things (after a similar approximation):

$$\ln(L_j) \approx -\varepsilon^l(i_j^l - i^l) + \ln(L).$$

---

[1] This assumes that interest rates are small and that $\kappa\nu(\ln(\frac{L_j}{F_j}) - \ln(\nu))$ is also small, these are very plausible things to assume for all the parameter values and shock sizes that I will use. In numerical simulations I confirmed that the approximation works very well

Introduce this in the previous equation for the loan rate and simplify to obtain:

$$i_j^l \;=\; \frac{1}{1+\kappa\nu\varepsilon^l}\left(\ln\left(\frac{\varepsilon^l}{\varepsilon^l-1}\right)+i+\kappa\nu\varepsilon^l i^l+\kappa\nu\ln(L)-\kappa\nu\ln(F_j)-\kappa\nu\ln(\nu)\right).$$

Now introduce i_j^l in the expression for $\ln(L_j)$ to obtain:

$$\ln(L_j) \;=\; -\frac{\varepsilon^l}{1+\kappa\nu\varepsilon^l}\left(\ln\left(\frac{\varepsilon^l}{\varepsilon^l-1}\right)+i-\kappa\nu\ln(F_j)-\kappa\nu\ln(\nu)-i^l-\frac{\ln(L)}{\varepsilon^l}\right).$$

It can also be shown that the aggregate loan rate is given by

$$i^l \;=\; \ln\left(\frac{\varepsilon^l}{\varepsilon^l-1}\right)+i+\kappa\nu\left(\ln\left(\frac{L}{\tilde{F}}-\nu\right)\right),$$

where:

$$\tilde{F} \;=\; \left(\int_0^1 F_j^{\frac{\varepsilon^l-1}{\varepsilon^l+1/(\kappa\nu)}}dj\right)^{\frac{\varepsilon^l+1/(\kappa\nu)}{\varepsilon^l-1}}.$$

Hence, the equations for bank-level loan rate and loan amount can be expressed as:

$$\begin{aligned}
i_j^l &= \ln\left(\frac{\varepsilon^l}{\varepsilon^l-1}\right)+i+\kappa\nu\left(\ln\left(\frac{L}{\tilde{F}}-\nu\right)\right)-\frac{\kappa\nu}{1+\kappa\nu\varepsilon^l}\ln\left(\frac{F_j}{\tilde{F}}\right)\\
\ln(L_j) &= \ln(L)+\frac{\kappa\nu\varepsilon^l}{1+\kappa\nu\varepsilon^l}\ln\left(\frac{F_j}{\tilde{F}}\right).
\end{aligned}$$

These two equations can be rewritten as

$$\begin{aligned}
i_j^l &= \alpha+\beta i-\frac{\kappa\nu}{1+\kappa\nu\varepsilon^l}\ln(F_j)\\
\ln(L_j) &= \alpha'+\beta' i+\frac{\kappa\nu\varepsilon^l}{1+\kappa\nu\varepsilon^l}\ln(F_j),
\end{aligned}$$

where:

$$\begin{aligned}
\alpha &= \ln\left(\frac{\varepsilon^l}{\varepsilon^l-1}\right)+\kappa\nu\left(\ln\left(\frac{L}{\tilde{F}}-\nu\right)\right)+\frac{\kappa\nu}{1+\kappa\nu\varepsilon^l}\ln(\tilde{F})\\
\alpha' &= \ln(L)-\frac{\kappa\nu\varepsilon^l}{1+\kappa\nu\varepsilon^l}\ln(\tilde{F})\\
\beta &= 1\\
\beta' &= 0.
\end{aligned}$$

A.2 Additional Empirical Results

A.2.1 More Summary Statistics

Table A.1: Summary Statistics for Banking Variables 1990-2017

	All Countries		NR Countries		Other Countries	
	Mean	N	Mean	N	Mean	N
Rate on Av. Earning Assets	4.57	80086	4.17	56385	5.53	23701
Deposit Rate	1.02	31615	0.89	19884	1.24	11731
Net Interest Margin	2.46	80441	2.20	56408	3.07	24033
ROAA	0.48	80545	0.32	56481	0.84	24064
ROAE	5.78	80202	4.41	56455	9.03	23747
Log of Net Loans	6.60	84721	6.47	60239	6.91	24482
Log of Total Customer Deposits	6.71	83532	6.58	59388	7.04	24144
Log of Equity	4.48	85240	4.27	60568	5.00	24672
Log of Total Assets	7.13	85311	7.03	60605	7.39	24706
Customer Deposits to Assets ratio	0.72	83599	0.71	59446	0.75	24153
Net Loans to Assets ratio	0.62	84823	0.61	60291	0.66	24532

Notes: This table contains more summary statistics for banking variables in the period 1990-2017 split between negative rate and non-negative rates regions.

A.2.2 Linear Results

As a starting point to study the relationships between the policy rate and the variables of interest one could run linear regressions of the following type:

$$y_{b,t} = \alpha_b + \delta_t + \beta i_{c(b),t} + \varepsilon_{b,t}, \tag{A.2}$$

where $y_{b,t}$ is some outcome variable for bank b, in country $c(b)$ and year t, and $i_{c(b),t}$ is the policy rate in that country and year. The regressions include a bank fixed effect (α_b) and a year fixed effect (δ_t). The results of these regressions without a lag of the dependent variable are given in Table A.2, while the ones that include a lag are given in Table A.3. In all these regressions the coefficient on the policy rate is positive and significant, which means that the loan rate, deposit rate and ROAE all move together with the policy rate and in the same direction. These results are well known in the literature. The first lag of the dependent variable is also positive and significant, while if a second lag is included (results not shown) it is generally nonsignificant. This indicates that roughly two-thirds of the effects of the policy rate on the loan rate and the deposit rate happens in the first year and the remaining one-third happens during the second year. With return on equity the first year effect is roughly 85% of the total.

The fact that movements in the policy rate do not fully translate to the deposit rate is qualitatively and quantitatively consistent with the idea of the "deposit channel of monetary policy" developed in Drechsler et al. (2017). In my results the spread between the policy rate and the deposit rate increases by 40 bps with a 100 bps increase in the policy rate, while in their results the increase in the policy rate is 54 bps. The numbers are very similar, and can easily be accounted for by the difference in time periods and countries being analyzed.

Table A.2: Linear regressions for main variables of interest, no lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.428^{***}	0.424^{***}	0.928^{***}
	(0.027)	(0.039)	(0.236)
N	80078	31554	80199
R squared	0.93	0.85	0.40
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.58	1.01	5.78

Notes: This table contains the results of the linear regressions described in equation (A.2) for the main variables of interest. SE in parenthesis, clustering is done at the Country-Year Level. Stars: * for p<.10, ** for p<.05, and *** for p<.01.

Table A.3: Linear regressions for main variables of interest, including a lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.322^{***}	0.324^{***}	0.729^{***}
	(0.017)	(0.032)	(0.227)
L.Rate on Av. Earning Assets	0.480^{***}		
	(0.023)		
L.Deposit Rate		0.500^{***}	
		(0.046)	
L.ROAE			0.333^{***}
			(0.033)
N	74096	28209	74209
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.49	0.93	5.66

Notes: Linear regressions for main variables of interest, including a lag. Clustering is done at the Country-Year Level. Stars: * for p<.10, ** for p<.05, and *** for p<.01.

A.2.3 Locally-Weighted Regressions

The results of the locally weighted regressions described in section 2.3.2, with and without a lag of the dependent variable respectively, are shown in Figures A.1 and A.2. The results are consistent with the predictions of the model. The loan rate decreases with the policy rate both for high and low levels of the policy rate, and at a similar rate in both cases. The deposit rate does not react much to the policy rate at low levels of the policy rate but does react for high levels. Return on equity reacts strongly to the policy rate at low rates but reacts less at high rates.

While somewhat informative, this approach does not allow for the identification of the break point, because the residualized measure has no direct connection with the underlying policy rate. In the figures I draw a vertical line at certain level of the residualized policy, but this level varies depending on whether or not the regressions include a lag, and (even though it is informed by the average level at which the residualized measure would hit the 50 basis point threshold) it is chosen somewhat arbitrarily. That is why in the text I focus on the regression threshold framework which allows for a very clear analysis of the threshold.

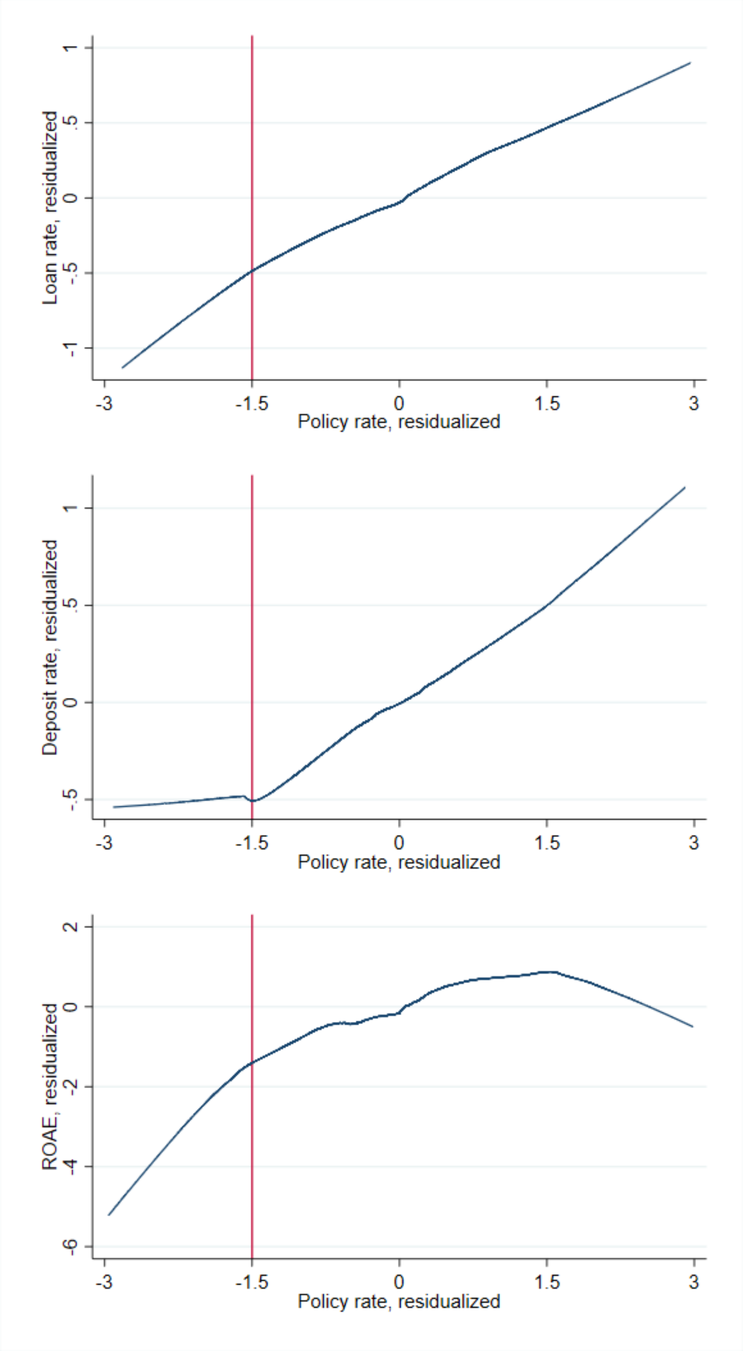

Figure A.1: Locally-weighted regressions, including a lag

Notes: This figure contains the behavior of the loan rate, deposit rate and return on average equity (ROAE) with respect to the policy rate in the selected sample of banks. All quantities have been residualized using bank fixed effects, year fixed effects and one lag of the dependent variable, and clustered at the country level. The graphs show the line from a locally weighted regression using tricube weighting with 0.5 bandwidth.

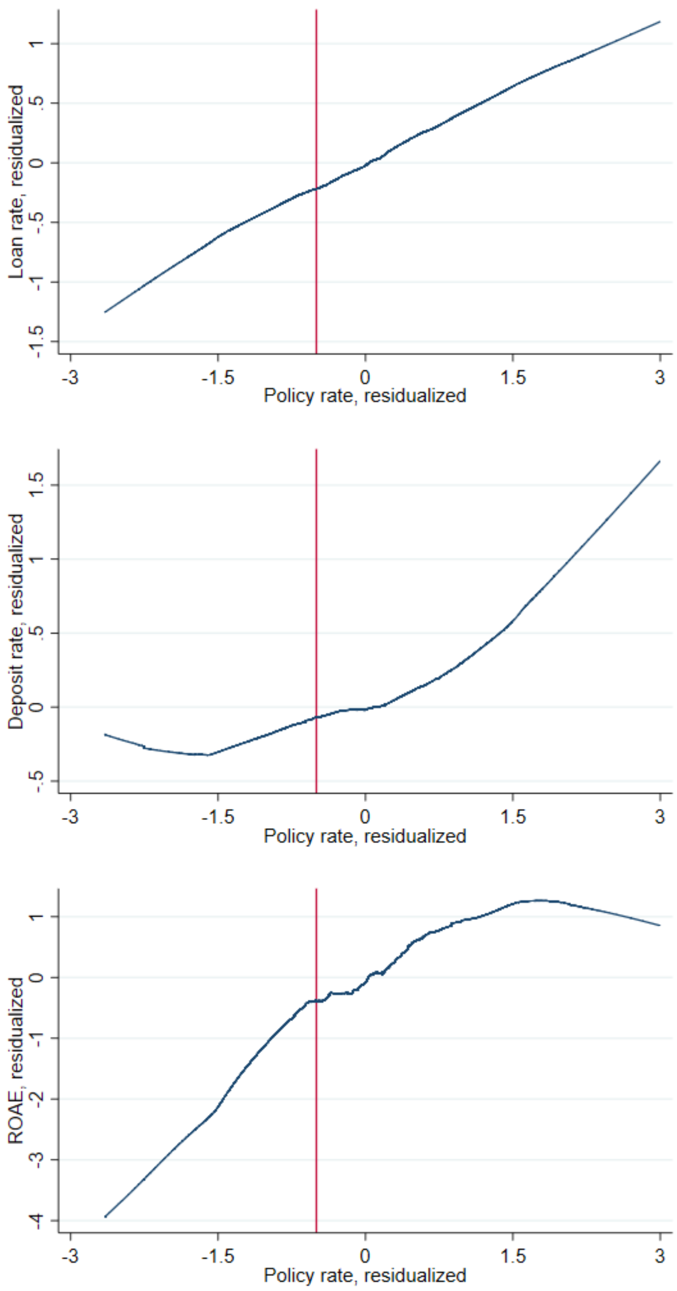

Figure A.2: Locally-weighted regressions, no lag

Notes: This figure contains the behavior of the loan rate, deposit rate and return on average equity (ROAE) with respect to the policy rate in the selected sample of banks. All quantities have been residualized using bank fixed effects and year fixed effects, and clustered at the country level. The graphs show the line from a locally weighted regression using tricube weighting with 0.5 bandwidth.

A.2.4 Robustness of Threshold Effects

In this appendix I document the robustness of the results presented in section 2.3.2 to different modifications of the baseline specification. Table A.4 contains the results of the regressions that include a lag of the dependent variable. Tables A.5 and A.6 present the results of regressions that allow for a break in level at the threshold $\tilde{\imath}$. Table A.7 contains the results of regressions that include bank-level time-varying characteristics, like the amount of bank equity or bank assets (either contemporaneous or lagged one period). Finally, Tables A.8 and A.9 present the results of regressions that control for indicators of banking or financial crises. The tables present the results for the Reinhart and Rogoff (2014) indicator for all crises, but the results are similar if one uses their systemic indicator, the indicators in Laeven and Valencia (2013) or the Romer and Romer (2017) indicator. In all of these cases the results are qualitatively similar to the ones from the baseline specification.

Table A.4: Main regressions, lag of dependent variable

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.389^{***}	0.130^{*}	3.499^{***}
	(0.089)	(0.077)	(0.857)
$(i-\tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.068	0.204^{**}	-2.853^{***}
	(0.087)	(0.092)	(0.815)
L.Rate on Av. Earning Assets	0.480^{***}		
	(0.023)		
L.Deposit Rate		0.492^{***}	
		(0.046)	
L.ROAE			0.326^{***}
			(0.032)
$\beta_1 + \beta_2$	0.320^{***}	0.334^{***}	0.646^{***}
s.e.$(\beta_1 + \beta_2)$	0.017	0.032	0.208
N	74096	28209	74209
R squared	0.95	0.92	0.48
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.49	0.93	5.66

Notes: SE in parenthesis, clustering is done at the Country-Year Level. Stars: * for p<.10, ** for p<.05, and *** for p<.01.

Table A.5: Main regressions, change in level at $\tilde{\imath}$, no lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.397^{***}	-0.197	4.809^{***}
	(0.132)	(0.156)	(1.193)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	0.001	0.593^{***}	-4.027^{***}
	(0.127)	(0.150)	(1.115)
$\mathbb{1}(i \geq \tilde{\imath})$	0.336^{***}	0.277^{**}	0.361
	(0.098)	(0.121)	(0.867)
$\beta_1 + \beta_2$	0.397^{***}	0.396^{***}	0.782^{***}
s.e.$(\beta_1 + \beta_2)$	0.026	0.044	0.224
N	80078	31554	80199
R squared	0.93	0.85	0.41
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.58	1.01	5.78

SE in parenthesis, clustering is done at the Country-Year Level.

Stars: * for p<.10, ** for p<.05, and *** for p<.01.

Table A.6: Main regressions, change in level at $\tilde{\imath}$, lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.365^{***}	0.143	3.494^{***}
	(0.106)	(0.099)	(1.005)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.048	0.195^{*}	-2.848^{***}
	(0.100)	(0.102)	(0.927)
$\mathbb{1}(i \geq \tilde{\imath})$	0.046	-0.023	0.010
	(0.076)	(0.083)	(0.844)
L.Rate on Av. Earning Assets	0.477^{***}		
	(0.023)		
L.Deposit Rate		0.494^{***}	
		(0.047)	
L.ROAE			0.326^{***}
			(0.032)
$\beta_1 + \beta_2$	0.317^{***}	0.338^{***}	0.645^{***}
s.e.$(\beta_1 + \beta_2)$	0.019	0.035	0.223
N	74096	28209	74209
R squared	0.95	0.92	0.48
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.49	0.93	5.66

SE in parenthesis, clustering is done at the Country-Year Level.

Stars: * for p<.10, ** for p<.05, and *** for p<.01.

Table A.7: Main regressions, controlling for bank equity and assets

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.380^{***}	0.103	3.953^{***}
	(0.088)	(0.073)	(0.930)
$(i-\tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.059	0.236^{***}	-3.307^{***}
	(0.086)	(0.087)	(0.882)
Log of Equity	-0.054	-0.040	3.466^{***}
	(0.038)	(0.037)	(0.787)
Log of Total Assets	0.072^{*}	0.228^{***}	-2.793^{***}
	(0.038)	(0.048)	(0.661)
L.Rate on Av. Earning Assets	0.479^{***}		
	(0.023)		
L.Deposit Rate		0.485^{***}	
		(0.046)	
L.ROAE			0.310^{***}
			(0.032)
$\beta_1 + \beta_2$	0.321^{***}	0.338^{***}	0.646^{***}
s.e.$(\beta_1 + \beta_2)$	0.017	0.031	0.199
N	74037	28185	74182
R squared	0.95	0.92	0.48
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.49	0.93	5.66

SE in parenthesis, clustering is done at the Country-Year Level.

Stars: * for p<.10, ** for p<.05, and *** for p<.01.

Table A.8: Main regressions, banking crisis indicators, no lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.704^{***}	-0.244	9.110^{***}
	(0.204)	(0.217)	(1.765)
$(i-\tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.301	0.671^{***}	-8.489^{***}
	(0.205)	(0.210)	(1.745)
Reinhart and Rogoff BCI	0.047	0.302^{***}	-1.689^{***}
	(0.075)	(0.087)	(0.633)
$\beta_1 + \beta_2$	0.403^{***}	0.427^{***}	0.621^{***}
s.e.$(\beta_1 + \beta_2)$	0.028	0.038	0.210
N	68960	24716	69049
R squared	0.93	0.86	0.42
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.85	1.15	5.87

SE in parenthesis, clustering is done at the Country-Year Level.

Stars: * for p<.10, ** for p<.05, and *** for p<.01.

Notes: Main regressions, controlling for Reinhart and Rogoff's banking crisis indicators, no lag of the dependent variable

Table A.9: Main regressions, banking crisis indicators, lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.369^{**}	-0.154	6.950^{***}
	(0.154)	(0.159)	(1.540)
$(i-\tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.044	0.486^{***}	-6.384^{***}
	(0.152)	(0.165)	(1.521)
Reinhart and Rogoff BCI	0.039	0.146^{**}	-1.435^{**}
	(0.070)	(0.066)	(0.675)
L.Rate on Av. Earning Assets	0.446^{***}		
	(0.025)		
L.Deposit Rate		0.442^{***}	
		(0.050)	
L.ROAE			0.301^{***}
			(0.034)
$\beta_1 + \beta_2$	0.324^{***}	0.331^{***}	0.565^{***}
s.e.$(\beta_1 + \beta_2)$	0.018	0.033	0.217
N	63001	21418	63074
R squared	0.95	0.92	0.48
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.78	1.06	5.74

SE in parenthesis, clustering is done at the Country-Year Level.

Stars: * for p<.10, ** for p<.05, and *** for p<.01.

Notes: Main regressions, controlling for Reinhart and Rogoff's banking crisis indicators, no lag of the dependent variable

Table A.10: Main regressions, $\tilde{\imath} = 0$

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.463	0.032	3.586^{***}
	(0.310)	(0.324)	(1.168)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.035	0.402	-2.675^{**}
	(0.304)	(0.331)	(1.113)
$\beta_1 + \beta_2$	0.428^{***}	0.434^{***}	0.911^{***}
s.e.$(\beta_1 + \beta_2)$	0.027	0.039	0.232
N	80078	31554	80199
R squared	0.93	0.85	0.40
Entity FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Clustering	C-Y	C-Y	C-Y
Mean dep. var.	4.58	1.01	5.78

SE in parenthesis, clustering is done at the Country-Year Level.

Stars: * for p<.10, ** for p<.05, and *** for p<.01.

A.2.5 Additional Threshold Tests

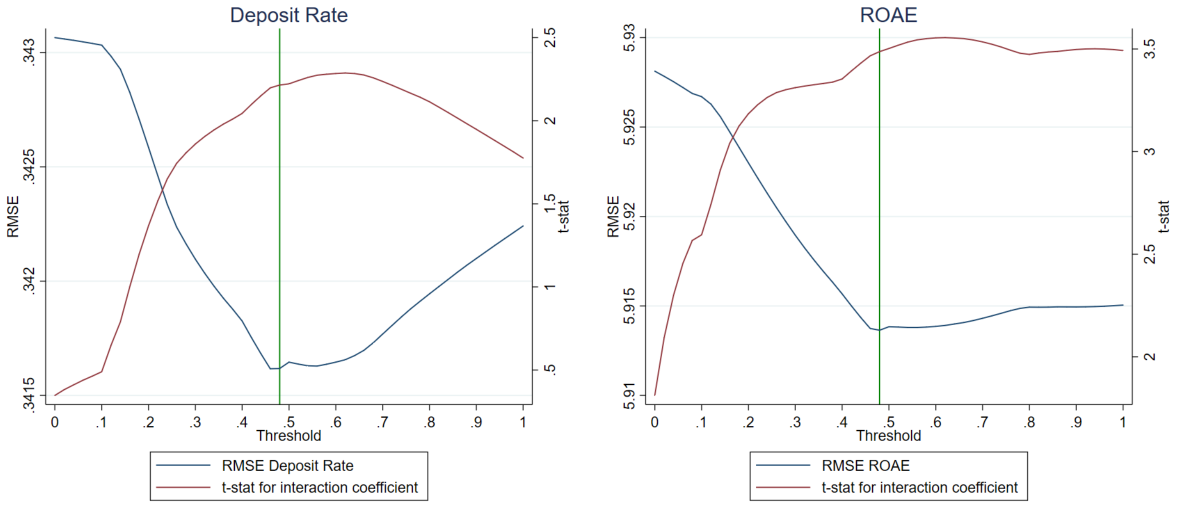

Figure A.3: RMSE for threshold tests, lag

Notes: These figures contain the RMSE and t-stat on the interaction coefficient β_2 for Regression (2.6) with a lag of the dependent variable for the deposit rate and ROAE for different values of the threshold level $\tilde{\iota}$.

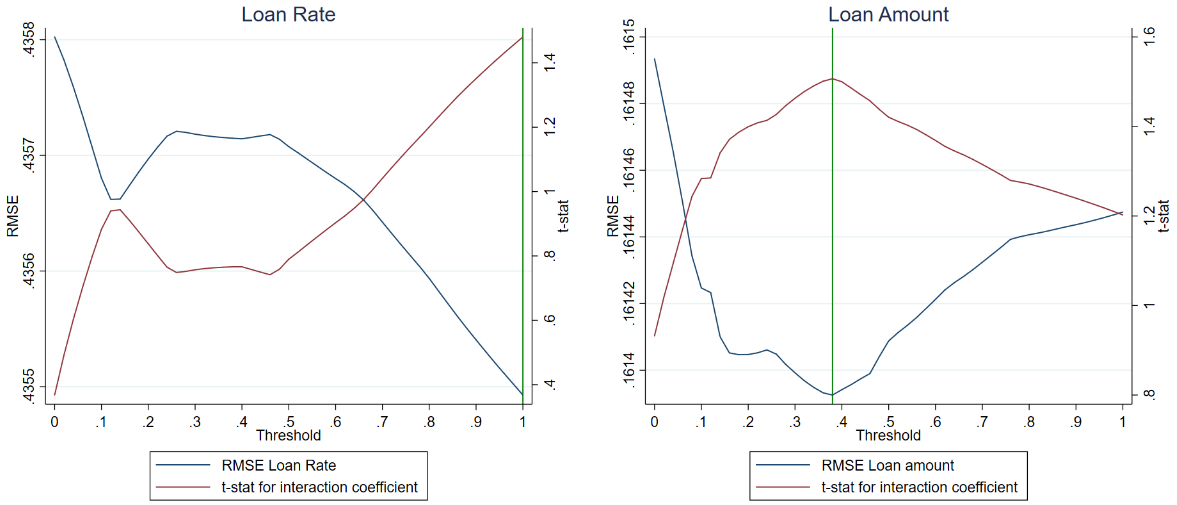

Figure A.4: RMSE for threshold tests, other variables

Notes: These figures contain the RMSE and t-stat on the interaction coefficient β_2 for Regression (2.6) with a lag of the dependent variable for the loan rate and the loan amount for different values of the threshold level $\tilde{\iota}$.

As pointed out by Andrews (1993) and Hansen (1999), inference in the presence of an unknown threshold is complicated by the presence of a nuisance parameter, this comes from the fact that the break point is not present under the null-hypothesis. Andrews (1993) proposes the use of supremum statistics to solve this issue and Hansen (1999) proposes a bootstrap-based method. The Hansen methodology is only theoretically applicable for nondynamic panels and it requires a balanced panel, so to apply it in my context I turn my dataset into a balanced panel and do not include the lag of the dependent variable. Running the Hansen (1999) procedure on data for the deposit rate (a balanced panel between 2009 and 2016 with 1986 banks per year) identifies a break point at 62 basis points and rejects the null-hypothesis of no break point at the 1% level. Running it on data for ROAE (a balanced panel between 2006 and 2016 with 3401 banks per year) identifies a break point at 47 basis points and also rejects the null-hypothesis of no break point at the 1% level.[2]

Another possible test for the threshold level using equation (2.6) for the deposit rate, that utilizes information on both β_1 and β_2, is based on Chay and Munshi (2015). In my setup, as in theirs, if the true threshold is picked, β_1 should be close to zero and β_2 should be positive. If a threshold candidate below the true threshold is picked, β_1 should still be estimated as zero but β_2 should be estimated as a smaller quantity. If a threshold candidate above the true threshold is picked, β_1 should now be estimated as positive but β_2 should be estimated as a higher quantity. The authors develop a test for the joint hypothesis that $\beta_1 = 0$ and $\beta_2 > 0$ based on the test statistic:

$$\Upsilon^{CM} = \frac{\left[\phi\left(\frac{\hat{\beta}_1}{h}\right)\right]^2}{\left[\phi\left(\epsilon\right)\right]^2} \frac{\hat{\beta}_2^2}{\hat{V}_{\beta_2}}, \tag{A.3}$$

where ϕ is a symmetric and continuous function that reaches its maximum value at zero (the authors use the normal p.d.f.), h is a scale parameter, ϵ is the value below which the normalized baseline slope coefficient, $\frac{\hat{\beta}_1}{h}$, is treated as "zero", and $\hat{V}_{\beta_2}$ is the estimated variance of β_2. The authors use the normalization by h because $\hat{\beta}_1$ will be further away from zero when the outcome variable has a larger mean or variance, the authors set h to be the standard deviation of the outcome under consideration, in my case the standard deviation of the deposit rate. I will set $\epsilon = 0$, as the authors do, to be conservative. The statistic is distributed as a chi-square with one degree of freedom. The statistic is presented in Figure A.5 for the case that does not include a lag of the dependent variable, the statistic is maximized at $\tilde{\imath} = 0.46$ (46 basis points). The p-value of the statistic is given in the secondary axis, and the green horizontal line indicates the 5% threshold for rejection of the null. This means that for thresholds between 10 and 80 basis points there is evidence that $\beta_1 = 0$ and $\beta_2 > 0$ which is what the model predicts. Like the Hansen (1999) procedure, the Chay and Munshi procedure was in theory developed for a nondynamic panel, that is why I

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[2] I choose the starting year of the panel to maximize the total number of observations remaining in the balanced panel. This year turns out to be 2009 for the case of the deposit rate and 2006 for ROAE. Running the test for the deposit rate starting the balanced panel in 2006 produces similar results.

do not include the lag of the dependent variable, but including it yields similar results.

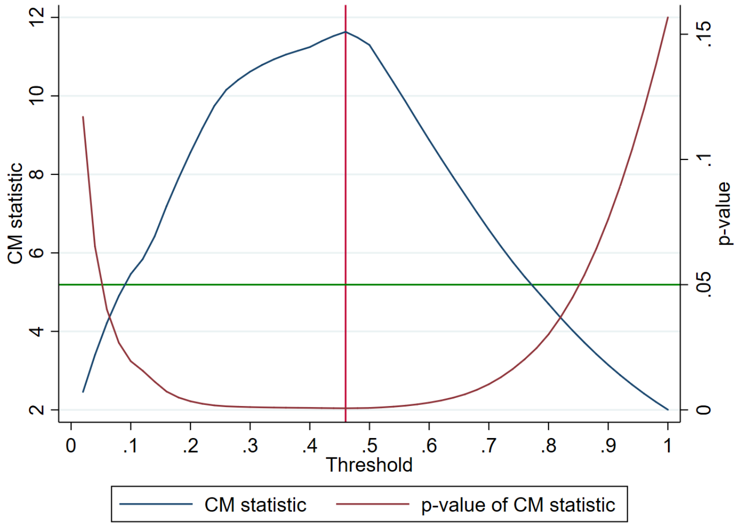


Figure A.5: CM statistic for the deposit rate

Notes: This figure contains the Chay and Munshi (CM) statistic for the deposit rate based on equation (2.6) and its p-value, for different values of the threshold level $\tilde{\iota}$.

A.2.6 Quintile Description

Table A.11: CDA for different quintiles

Quintile	Mean	Std. Dev.	Min.	Max.	N
First	0.43	0.18	0.01	0.63	16726
Second	0.70	0.03	0.63	0.75	16717
Third	0.78	0.02	0.75	0.82	16721
Fourth	0.85	0.02	0.82	0.88	16738
Fifth	0.91	0.02	0.88	0.96	16697

Notes: Description of the CDA ratio variable for different quintiles

A.2.7 Additional Structural Estimation Results

The following table summarizes the results of the regressions in equation (2.20), when assuming that $\nu = 9$ (which is the mean loan-to-equity ratio in my dataset). Column 1 presents the results when the regression includes two lags of the dependent variable and $F_{b,t-1}$ is instrumented with $F_{b,t-3}$, to avoid potential endogeneity with the two lags of the dependent variable. Column 2 presents the results when the regression includes just one lag of the dependent variable and $F_{b,t-1}$ is instrumented with $F_{b,t-2}$. Column 3 presents the results when the regression does not include any lags of the dependent variable and $F_{b,t-1}$ is not instrumented.

Table A.12: Aggregate structural estimation of κ

	(1)	(2)	(3)
γ_{la}	0.4551	0.3586	0.6625
γ_{lr}	-0.0100	-0.0086	-0.0104
κ	0.0020	0.0015	0.0034
ε^l	45.4540	41.4951	63.7593
$\mathrm{i}^l - i$	0.0222	0.0244	0.0158

Notes: This table contains the results of the aggregate structural estimation of κ and ε^l described in equation (2.20).

The results are similar for the three models and deliver an estimate of κ between 15 and 35 basis points, together with an estimate of ε^l between 40 and 60 (at the annual level). These estimates of ε^l would deliver a steady state wedge between the loan rate and the policy rate of between 1.5% and 2.5% which is reasonable. The baseline specification will be the first one, since it is necessary to include the lags of the dependent variable to control for the sluggishness in rate setting and lending behavior.

The following table contains the region-level results of the regressions for Denmark, Sweden, Canada, Australia and Norway.

Table A.13: Structural estimation of κ and ε^l, part 2

	(1) DKK	(2) SEK	(3) CAD	(4) AUD	(5) NOK
β_{la}	0.7841	0.6053	1.2854	0.1653	-0.5330
β_{lr}	-0.0047	-0.0030	0.0588	-0.0081	0.0099
κ	0.0024	0.0008	0.0229	0.0011	-0.0007
ε^l	167.9998	202.4073	-21.8763	20.3058	54.0099
$\mathrm{i}^l - i$	0.0060	0.0050	-0.0447	0.0505	0.0187

Notes: This table contains the results of the country level structural estimation of κ and ε^l described in equation (2.20). This table contains the 5 smallest regions (in terms of amount of banks present in the sample from that region) Denmark (DKK), Sweden (SEK), Canada (CAD), Australia (AUD) and Norway (NOK).

A.3 Model Solution

A.3.1 Equilibrium Equations

The equilibrium is characterized by the relevant equations for each of the types of agents in the model. Households have an intratemporal condition for labor supply, an Euler equation, the definition of the marginal utility of consumption and the definition of the stochastic discount factor:

$$\begin{aligned}
\chi N_t^{\frac{1}{\eta}} &= \phi_t \frac{W_t}{P_t} \\
1 &= \mathbb{E}_t \left(\beta \Lambda_{t,t+1} (1 + i_t^d) \frac{P_t}{P_{t+1}} \right) \\
\phi_t &= (C_t - hC_{t-1})^{-\sigma} - \beta h \frac{\varphi_{t+1}}{\varphi_t} \mathbb{E}_t (C_{t+1} - hC_t)^{-\sigma} \\
\Lambda_{t,t+1} &= \frac{\phi_{t+1}}{\phi_t} \frac{\varphi_{t+1}}{\varphi_t}.
\end{aligned}$$

Intermediate goods firms have their production function, a labor demand equation and the definition of the return on capital:

$$\begin{aligned}
Y_t^m &= A_t (\xi_t K_t)^\alpha N_t^{1-\alpha} \\
(1-\alpha) \frac{P_t^m}{P_t} \frac{Y_t^m}{N_t} &= \frac{W_t}{P_t} \\
1 + i_t^l &= \frac{\frac{Q_t}{P_t} \xi_t (1-\delta) + \frac{P_t^m}{P_t} \alpha \frac{Y_t^m}{K_t}}{\frac{Q_{t-1}}{P_{t-1}}} \frac{P_t}{P_{t-1}}.
\end{aligned}$$

Capital producing firms have the evolution of capital and the F.O.C. for the price of capital:

$$\begin{aligned}
K_{t+1} &= (1-\delta)\xi_t K_t + I_t \\
\frac{Q_t}{P_t} &= 1 + f\left(\frac{I_t}{I_{t-1}}\right) + f'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} - \mathbb{E}_t \beta \Lambda_{t,t+1} f'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2.
\end{aligned}$$

Retail firms have equations for price setting, the evolution of prices, the dispersion of prices and the relationship between final output and intermediate output:

$$\begin{aligned}
1 &= (1-\gamma) \left(\frac{P_t^*}{P_t}\right)^{1-\theta} + \gamma \left(\frac{P_{t-1}}{P_t}\right)^{1-\theta} \\
\theta \Gamma_t^1 &= (\theta - 1) \Gamma_t^2 \\
\Gamma_t^1 &= \phi_t \varphi_t \frac{P_t^m}{P_t} Y_t + \gamma \beta \mathbb{E}_t \left(\frac{P_t}{P_{t+1}}\right)^{-\theta} \Gamma_{t+1}^1
\end{aligned}$$

$$
\begin{aligned}
\Gamma_t^2 &= \phi_t \varphi_t \frac{P_t^*}{P_t} Y_t + \gamma\beta \mathbb{E}_t \frac{P_t^*}{P_{t+1}^*} \left(\frac{P_t}{P_{t+1}}\right)^{-\theta} \Gamma_{t+1}^2 \\
Y_t^m &= Y_t v_t^p \\
v_t^p &= \gamma \left(\frac{P_{t-1}}{P_t}\right)^{-\theta} v_{t-1}^p + (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{-\theta}.
\end{aligned}
$$

Banks have equations for the deposit rate, the loan rate, bank profits, bank equity evolution and the bank balance sheet constraint:

$$
\begin{aligned}
1 + i_t^d &= \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i_t + \mu_t^d) \\
\mathbb{E}_t(1 + i_{t+1}^l) &= \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i_t + \mu_t^l) + \kappa \frac{\varepsilon^l}{\varepsilon^l - 1}\left(\frac{L_t}{F_t} - \nu\right) \\
\frac{P_t}{P_{t-1}} \frac{X_t}{P_t} &= i_{t-1} \frac{F_{t-1}}{P_{t-1}} + (i_t^l - \mu_{t-1}^l - i_{t-1}) \frac{L_{t-1}}{P_{t-1}} + (i_{t-1} + \mu_{t-1}^d - i_{t-1}^d) \frac{D_{t-1}}{P_{t-1}} \\
&- \frac{\kappa}{2}\left(\frac{L_{t-1}}{F_{t-1}} - \nu\right)^2 \frac{F_{t-1}}{P_{t-1}} - \frac{F_{t-1}}{P_{t-1}}(1-\varsigma)\pi_t \\
\frac{F_t}{P_t} &= (1-\varsigma)\frac{F_{t-1}}{P_{t-1}} + \omega \frac{X_t}{P_t} \\
\frac{L_t}{P_t} + \frac{H_t}{P_t} &= \frac{F_t}{P_t} + \frac{D_t}{P_t}.
\end{aligned}
$$

Finally one has the resource constraint, Taylor rule and the condition saying that total loans most equal the value of capital:

$$
\begin{aligned}
Y_t &= C_t + I_t + G_t + f\left(\frac{I_t}{I_{t-1}}\right) I_t + \mu_t^l \frac{P_{t-1}}{P_t}\frac{L_{t-1}}{P_{t-1}} - \mu_t^d \frac{P_{t-1}}{P_t}\frac{D_{t-1}}{P_{t-1}} \\
&+ \varsigma \frac{P_{t-1}}{P_t}\frac{F_{t-1}}{P_{t-1}} + \frac{\kappa}{2}\left(\frac{L_{t-1}}{F_{t-1}} - \nu\right)^2 \frac{P_{t-1}}{P_t}\frac{F_{t-1}}{P_{t-1}} \\
i_t &= (1-\rho_i)\left(\overline{\iota} + \psi_\pi(\pi_t - \overline{\pi}) + \psi_y \frac{Y_t - Y_t^*}{\overline{Y}}\right) + \rho_i i_{t-1} + \epsilon_t^i \\
\frac{L_t}{P_t} &= \frac{Q_t}{P_t} K_{t+1}.
\end{aligned}
$$

This is a system of 23 equations in 23 unknowns $(N, \phi, W/P, \Lambda, i^d, \pi, C,\ Y^m, K, P^m/P,$ $i^l, Q/P,\ I, P^*/P,\ \Gamma^1, \Gamma^2, Y,\ v^p, i, L/P, F/P, X/P, D/P)$ which can be used to solve for the equilibrium. The processes for the shocks are given by:

$$
\begin{aligned}
A_t &= \overline{A}^{1-\rho_a} A_{t-1}^{\rho_a} e^{\epsilon_t^a} \\
G_t &= \overline{G}^{1-\rho_g} G_{t-1}^{\rho_g} e^{\epsilon_t^g} \\
H_t &= \overline{H}^{1-\rho_h} H_{t-1}^{\rho_h} e^{\epsilon_t^h}
\end{aligned}
$$

$$
\begin{aligned}
\xi_t &= \xi_{t-1}^{\rho_\xi} e^{\epsilon_t^\xi} \\
\varphi_t &= \varphi_{t-1}^{\rho_\varphi} e^{\epsilon_t^\varphi}.
\end{aligned}
$$

A_t is the technology of intermediate good producers, G_t is government expenditure in goods, H_t is the total amount of central bank reserves, ξ_t is capital efficiency and φ_t is the shock to the discount factor. Additionally $\mu_t^l = \mu^l$ and $\mu_t^d = \mu^d$. I choose $\overline{A} = 1$ as a normalization, $\overline{G}$ is pinned down by the equation $\frac{\overline{G}}{\overline{Y}} = 0.2$ and $\overline{H}$ is pinned down by the parameter $\overline{H}/\overline{F}$ which as I explain in the calibration section is set to 2. The steady state value of capital efficiency ξ is 1, as is the steady state value of φ, which is just a normalization.

A.3.2 Steady State

In steady state with zero inflation the equations simplify a lot. For the retailers, for example, $P^* = P$, $v^p = 1$, $Y^m = Y$ and $\frac{P^m}{P} = \frac{\theta-1}{\theta}$. I can also get rid of the equations for Λ and Q/P. This way the 6 equations for retailers plus 2 additional equations are eliminated. Then I get rid of superfluous equations, like $i = \bar{\imath}$, the balance sheet of the banks (which just defines D/P), the equation for X/P, the equation for F/P and the equation for L/P. I obtain the following system:[3]

$$
\begin{aligned}
\chi N^{\frac{1}{\eta}} &= \phi \frac{W}{P} \\
1 &= \beta(1+i^d) \\
\phi &= C^{-\sigma}(1-h)^{-\sigma}(1-\beta h) \\
Y &= AK^\alpha N^{1-\alpha} \\
(1-\alpha)\frac{\theta-1}{\theta}\frac{Y}{N} &= \frac{W}{P} \\
\alpha\frac{\theta-1}{\theta}\frac{Y}{K} + (1-\delta) &= (1+i^l) \\
1+i^d &= \frac{\varepsilon^d}{\varepsilon^d-1}(1+i+\mu^d) \\
1+i^l &= \frac{\varepsilon^l}{\varepsilon^l-1}(1+i+\mu^l) \\
Y &= C+I+G+\mu^l\frac{L}{P} - \mu^d\frac{D}{P} + \varsigma\frac{F}{P} \\
I &= \delta K.
\end{aligned}
$$

Then I can get rid of the Euler equation and the bank equations defining interest rates, since $1+i^d = \frac{1}{\beta}$, $1+i = \frac{\varepsilon^d-1}{\varepsilon^d}\frac{1}{\beta} - \mu^d$ and $1+i^l = \frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{\varepsilon^d-1}{\varepsilon^d}\frac{1}{\beta} - \mu^d + \mu^l\right)$. I can also get rid of investment (it is just δK), ρ, the real wage and the definition of output (I assume $G = gY$).

---

[3] Recall that $\frac{L/P}{F/P} = \nu$ and $L/P = K$, hence I know that $F/P = K/\nu$, also from the balance sheet of the banks I know $\frac{L}{F} + \frac{H}{F} = \frac{D}{F} + 1$ so $\frac{D/P}{F/P} = \nu + \frac{H}{F} - 1$, which implies $D/P = \left(\nu + \frac{H}{F} - 1\right)\frac{K}{\nu}$

I obtain the 3 equation system:

$$\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-h)^{-\sigma}(1-\beta h)(1-\alpha)\frac{\theta-1}{\theta}AK^{\alpha}N^{-\alpha} \\
\alpha\frac{\theta-1}{\theta}AK^{\alpha-1}N^{1-\alpha}+(1-\delta) &= \frac{\varepsilon^{l}}{\varepsilon^{l}-1}\left(\frac{\varepsilon^{d}-1}{\varepsilon^{d}}\frac{1}{\beta}-\mu^{d}+\mu^{l}\right) \\
(1-g)AK^{\alpha}N^{1-\alpha} &= C+\left(\delta+\mu^{l}-\mu^{d}\left(1+\frac{H/F-1}{\nu}\right)+\frac{\varsigma}{\nu}\right)K.
\end{aligned}$$

The capital labor ratio can be obtained from the second equation:

$$\begin{aligned}
\alpha\frac{\theta-1}{\theta}AK^{\alpha-1}N^{1-\alpha}+(1-\delta) &= \frac{\varepsilon^{l}}{\varepsilon^{l}-1}\left(\frac{\varepsilon^{d}-1}{\varepsilon^{d}}\frac{1}{\beta}-\mu^{d}+\mu^{l}\right) \\
Z\equiv\frac{K}{N} &= \left(\frac{\alpha A(\theta-1)}{\theta\left(\frac{\varepsilon^{l}}{\varepsilon^{l}-1}\left(\frac{\varepsilon^{d}-1}{\varepsilon^{d}}\frac{1}{\beta}-\mu^{d}+\mu^{l}\right)+\delta-1\right)}\right)^{\frac{1}{1-\alpha}}.
\end{aligned}$$

I can introduce this into the other two equations and further introduce the first in the third to obtain:

$$N^{\sigma+\frac{1}{\eta}} = \frac{(\theta-1)(1-\alpha)AZ^{\alpha}(1-\beta h)}{\theta\chi(1-h)^{\sigma}\left((1-g)AZ^{\alpha}-\left(\delta+\mu^{l}-\mu^{d}\left(1+\frac{H/F-1}{\nu}\right)+\frac{\varsigma}{\nu}\right)Z\right)^{\sigma}}.$$

And from this variable all the other ones can be backed out. In particular consumption is given by:

$$C = \left(\frac{\chi\theta N^{\frac{1}{\eta}}}{AZ^{\alpha}(1-\alpha)(\theta-1)(1-h)^{-\sigma}(1-\beta h)}\right)^{-\frac{1}{\sigma}}.$$

A.4 Additional Figures

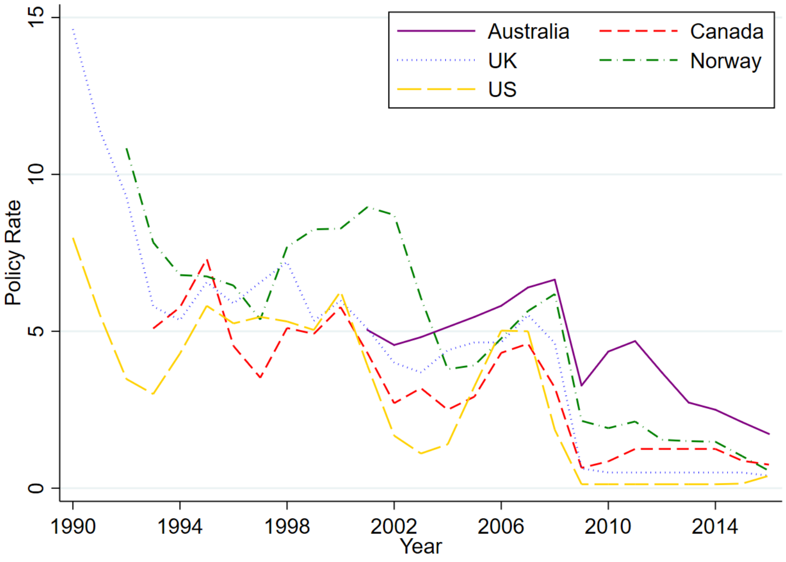


Figure A.6: Policy rate across years, positive rates

Notes: This figure contains the policy rate across years for the regions in the sample not setting negative rates.

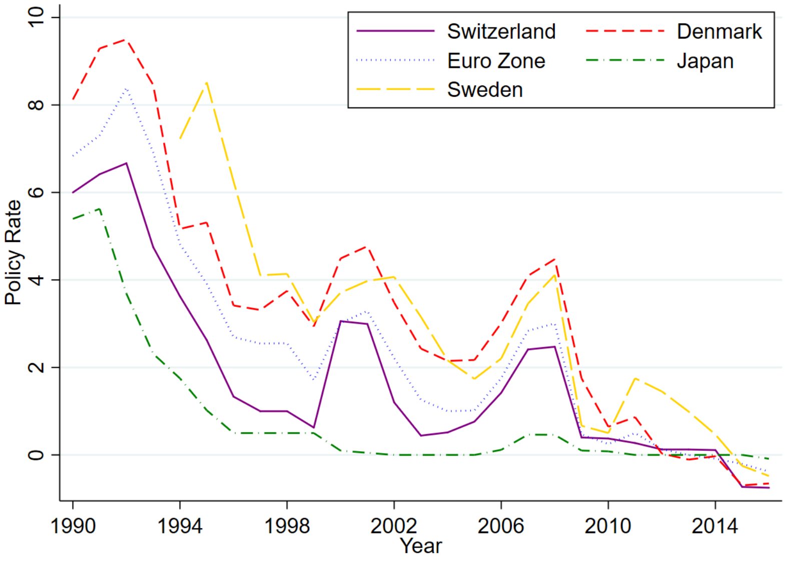


Figure A.7: Policy rate across years, negative rates

Notes: This figure contains the policy rate across years for the regions in the sample setting negative rates.

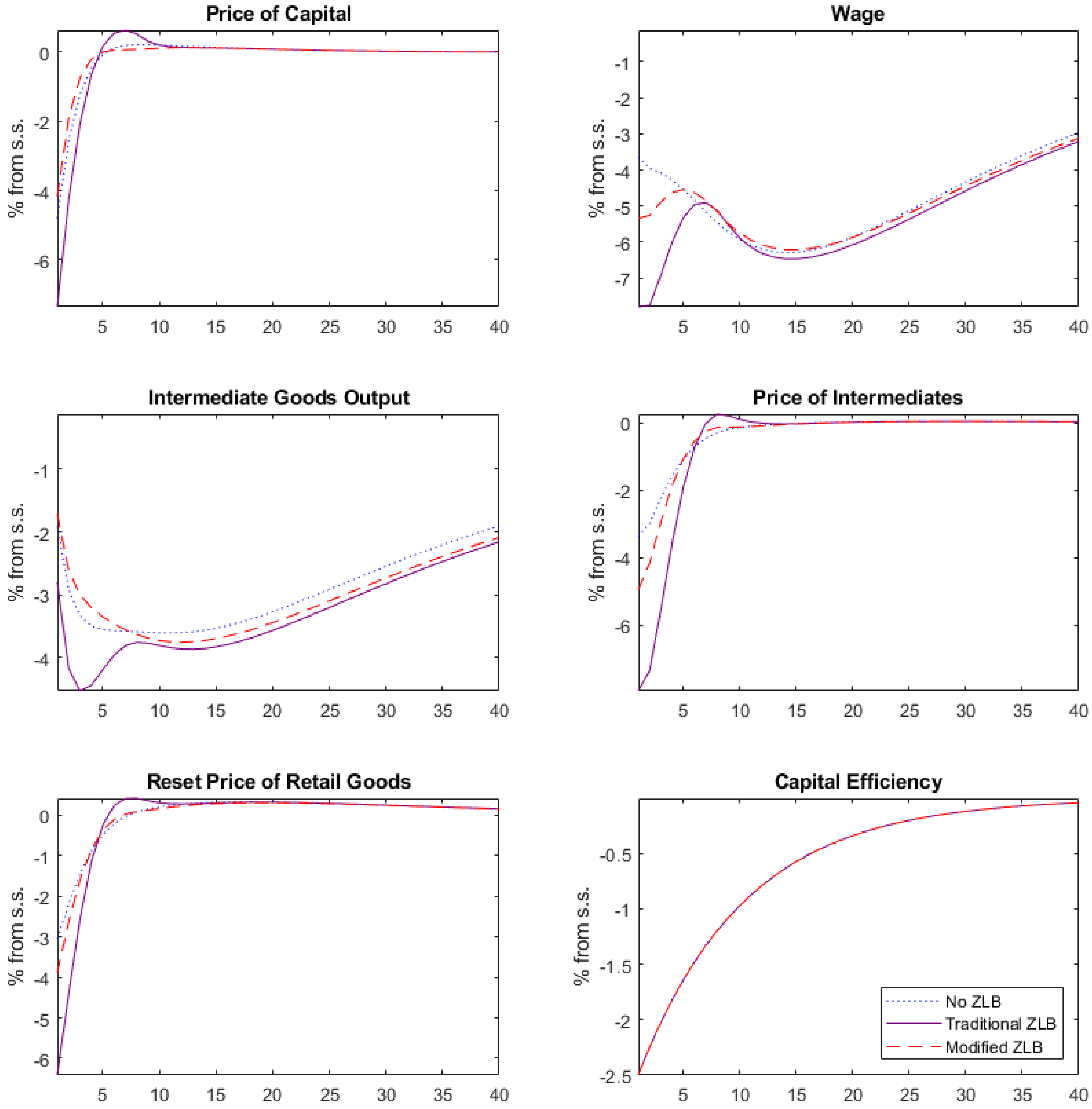


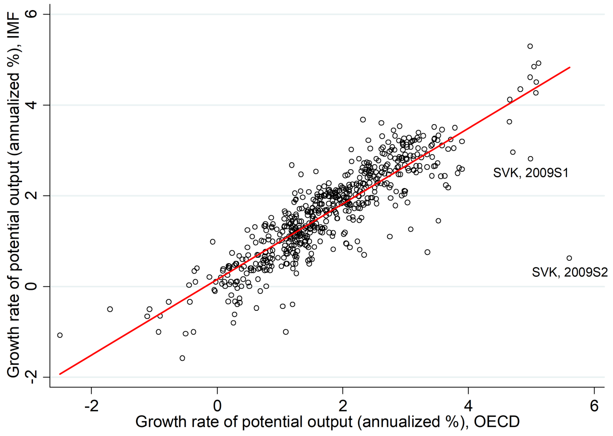
Figure A.8: IRF's to capital productivity shock

Notes: IRF's to capital productivity shock under "no ZLB" (blue dotted line), "traditional ZLB" (purple solid line) and "modified ZLB" (red dashed line) when $\kappa = 12.5$ basis points. The x axis is given in quarters and the y axis is given in percent deviations from steady state.

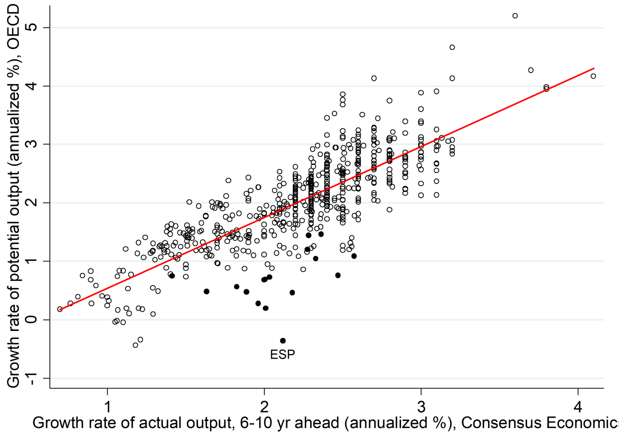
Appendix B

Additional Potential Output Material

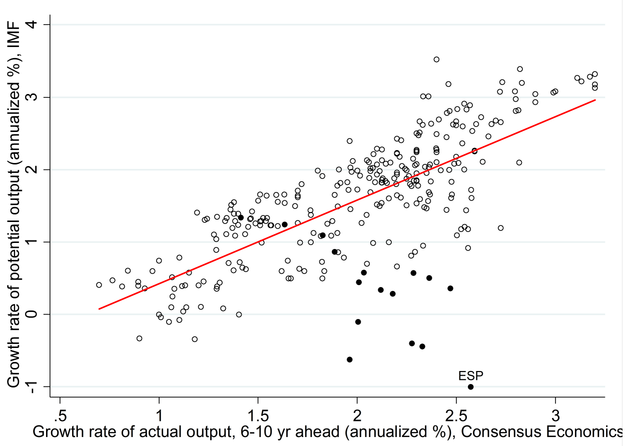
B.1 Figures



(a) IMF vs OECD



(b) OECD vs. Consensus Economics



(c) IMF vs Consensus Economics

Figure B.1: Comparison of IMF, OECD, and CE potential

Notes: Filled markers in (b) and (c) show observations for Spain in the 09-16 period.

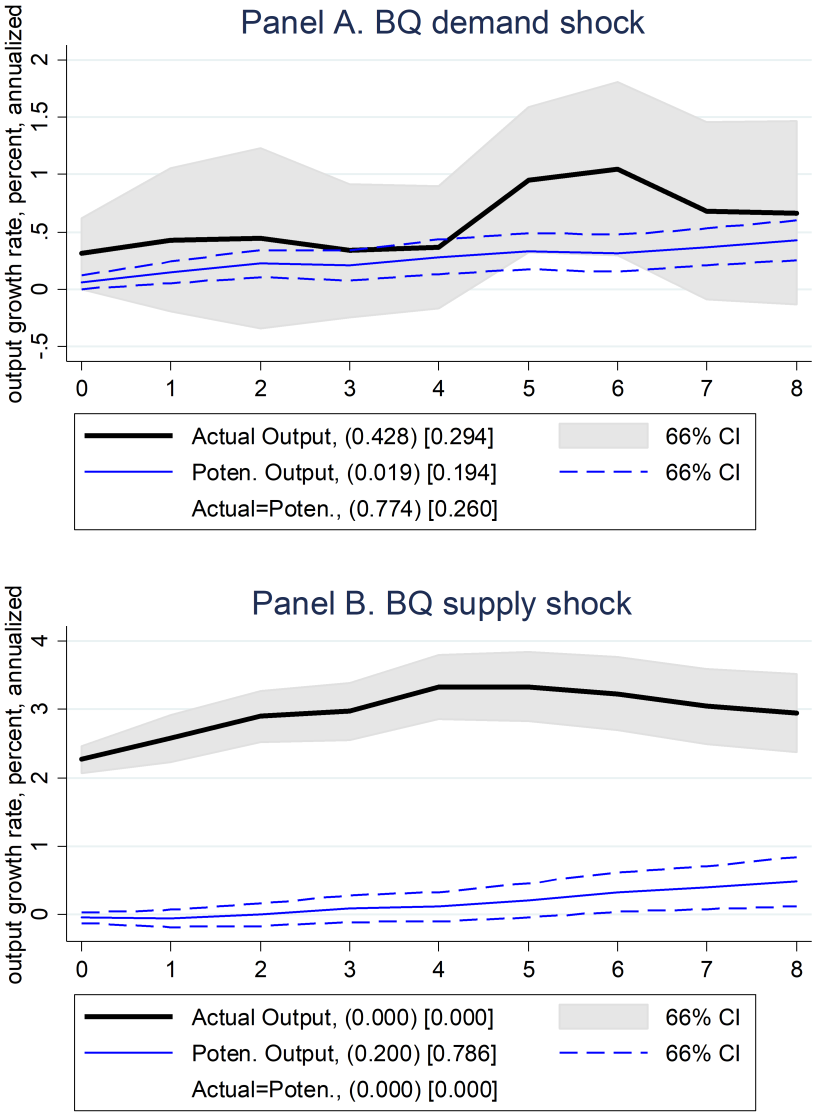
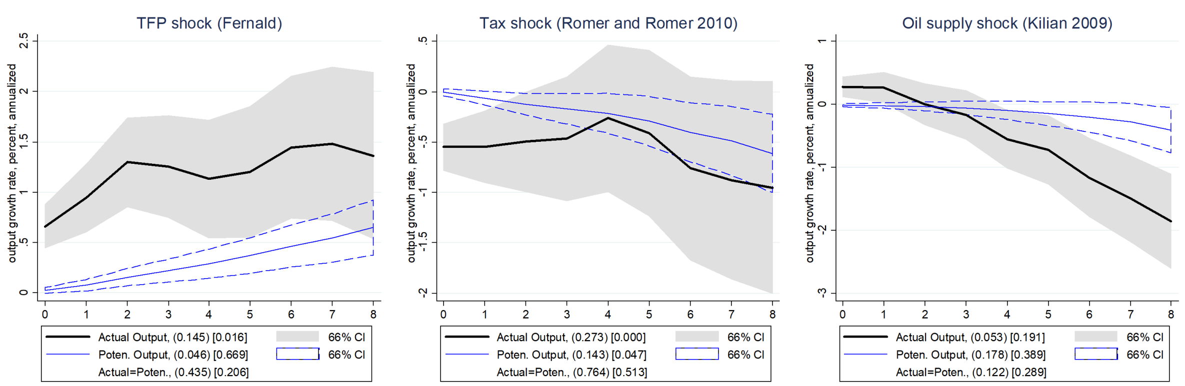
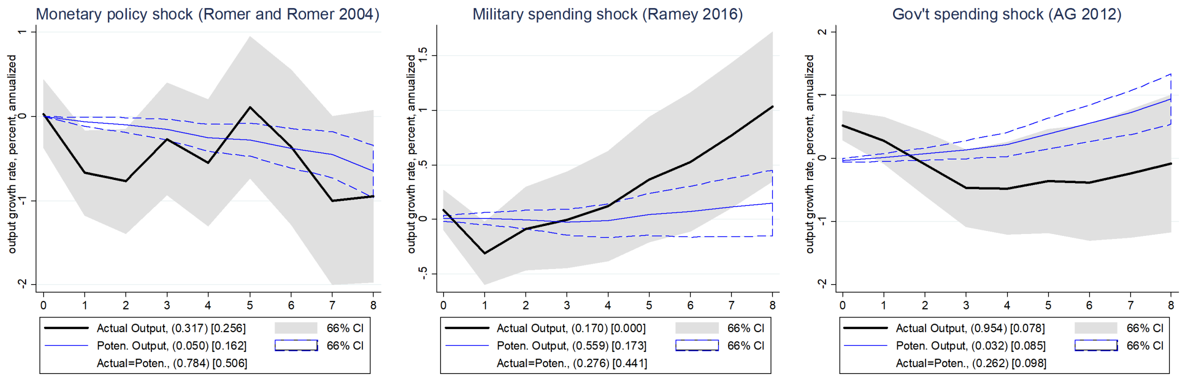


Figure B.2: Responses to BQ Identified Supply and Demand Shocks

Notes: The figure reports impulse response functions (IRFs) estimated using equation (2) and (3). The estimation sample covers the benchmark time period for Greenbook forecasts. "Supply" and "Demand" shocks are identified as in Blanchard and Quah (1989). In parentheses we report the p-value for a test of whether the IRF of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the IRF of actual (potential) output is different from zero over the entire duration of the IRF. The last row of the legend reports p-values for a test of equality of IRFs of actual and potential output at the max horizon (parentheses) and a test of equality of the paths of the responses for actual and potential output are equal across horizons.



Panel A: Supply Shocks



Panel B: Demand Shocks

Figure B.3: Responses of Output and Gre. Potential, ADL specification

Notes: The figure reports impulse response functions (IRFs) estimated using equation (4), which is an auto-distributed lag specification. The estimation sample covers the longest possible period with non-missing observations for shocks and potential output (output gap) available at the Federal Reserve Bank of Philadelphia. In parentheses we report the p-value for a test of whether the IRF of actual (potential) output is different from zero at the max horizon (8 quarters), while in square brackets we show the p-value for a test of whether the path of the IRF of actual (potential) output is different from zero over the entire duration of the IRF. The last row of the legend reports p-values for a test of equality of IRFs of actual and potential output at the max horizon (parentheses) and a test of equality of the paths of the responses for actual and potential output are equal across horizons.

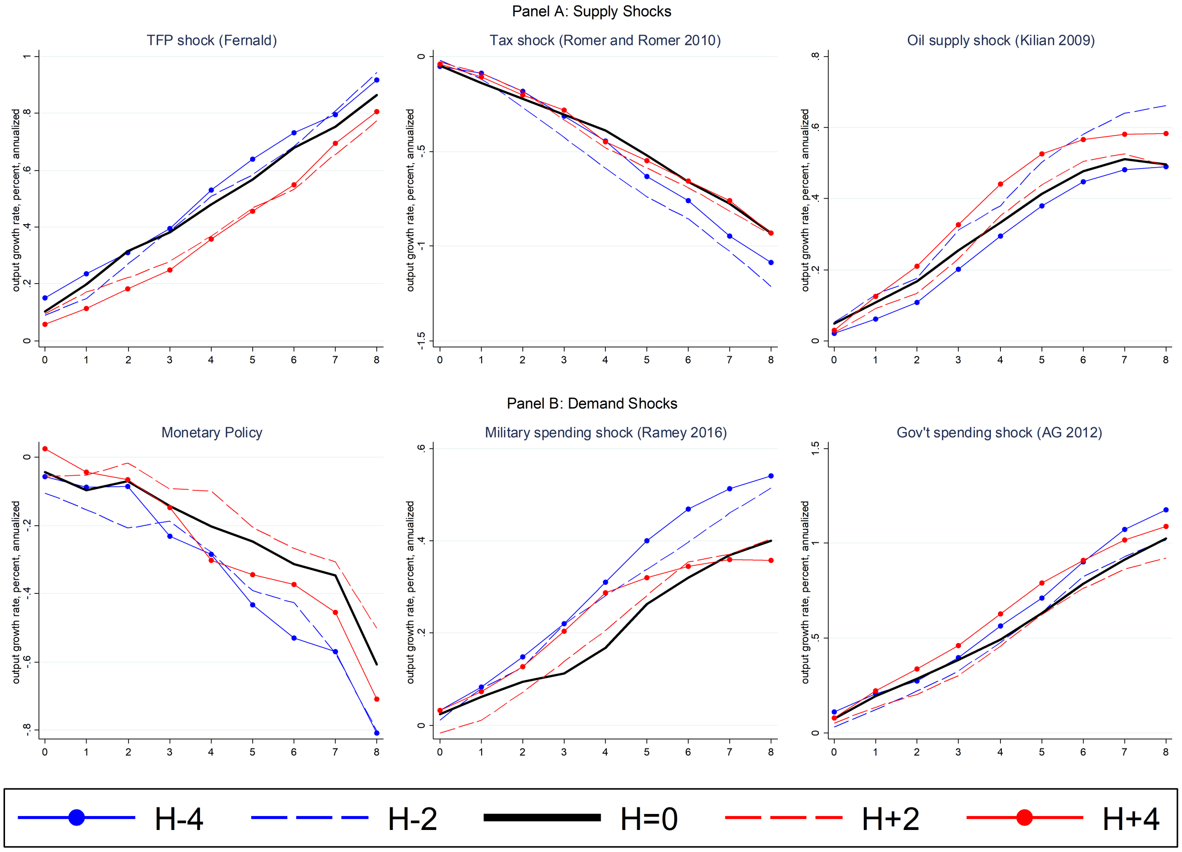
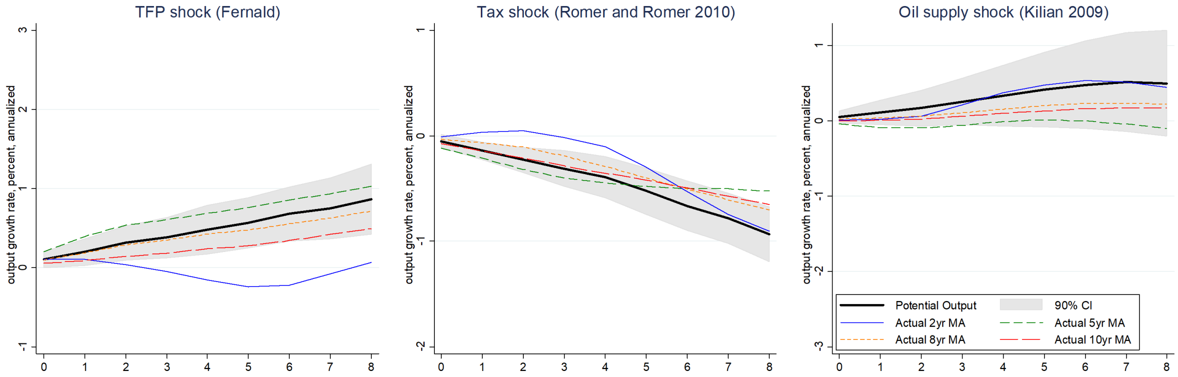


Figure B.4: Responses of Backcasts and Forecasts of Potential Output

Notes: The figure shows impulse responses of horizon $H + k$ growth rate of potential output to structural shocks. $k > 0$ corresponds to forecasts, $k < 0$ correspond to backcasts, $k = 0$ is the nowcast (which corresponds to the results reported in Figure 6). All data are from Greenbooks.

Panel A: Supply Shocks



Panel B: Demand Shocks

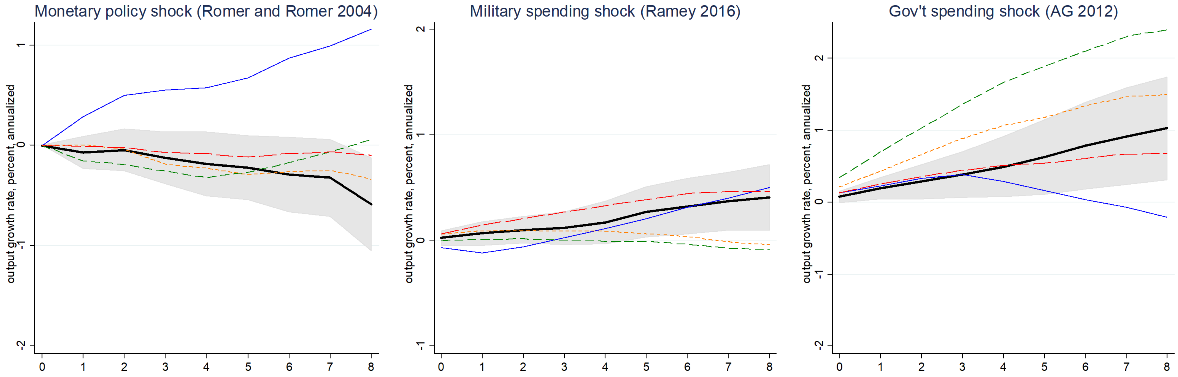
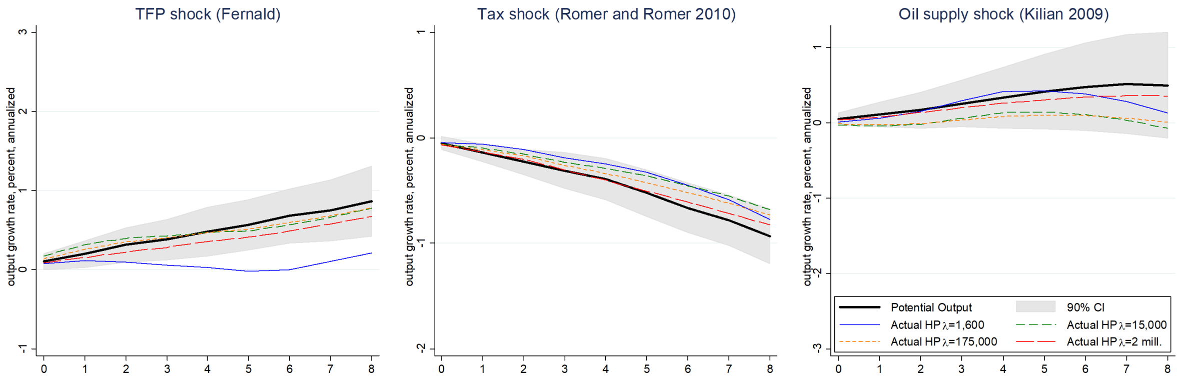


Figure B.5: Responses of Moving-Averages of U.S. Output to Shocks

Notes: The figure reports impulse response functions (IRFs) estimated using equations (2) and (3). The estimation sample covers the longest possible period with non-missing observations for shocks and potential output (output gap) available at the Federal Reserve Bank of Philadelphia.

Panel A: Supply Shocks



Panel B: Demand Shocks

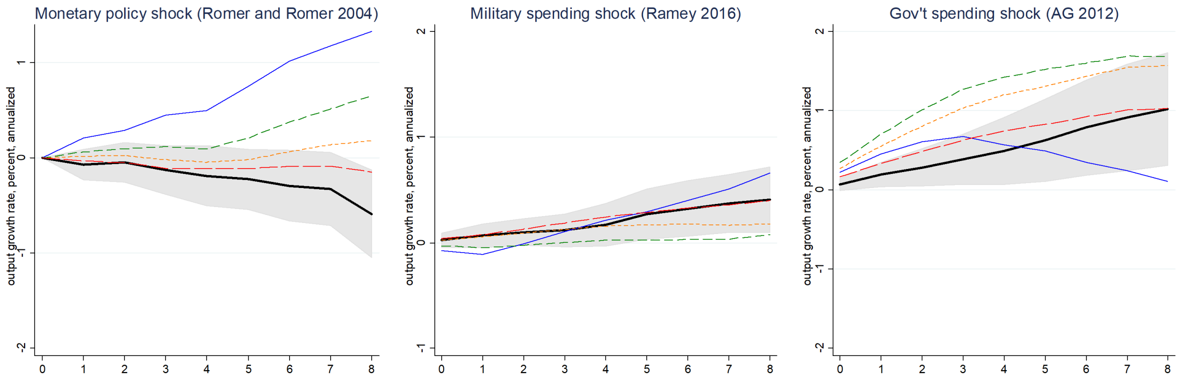
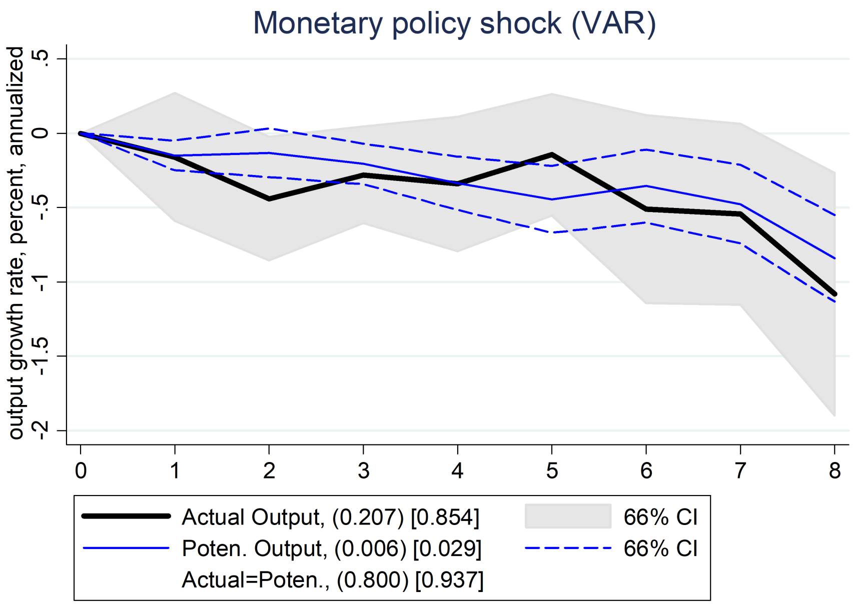
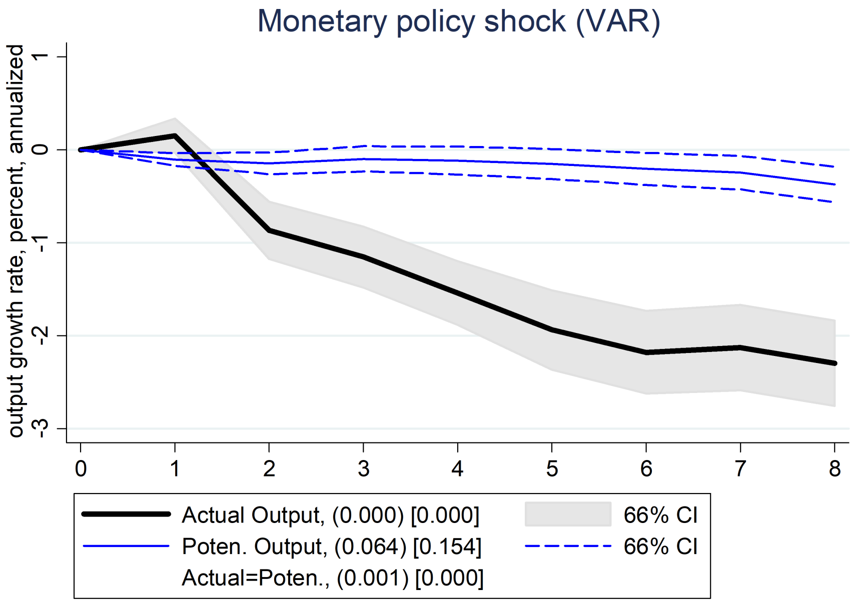


Figure B.6: Responses of HP-filters of Real-Time U.S. Output to Shocks

Notes: The figure reports impulse response functions (IRFs) estimated using equations (2) and (3). The estimation sample covers the longest possible period with non-missing observations for shocks and potential output (output gap) available at the Federal Reserve Bank of Philadelphia.



(a) 1987-2011 sample (current quarter)



(b) 1969-2011 sample (Orphanides; 3-quarters ahead)

Figure B.7: Robustness of Responses to Identification of Monetary Shocks

Notes: The figure reports impulse responses (IRFs) estimated using equations (2) and (3). The estimation sample covers the longest possible period with non-missing observations for shocks and potential output (output gap) available at the FRB of Philadelphia (left panel) and the extended measure of potential from Orphanides in right panel. Monetary shocks are identified from a trivariate VAR(4) using Cholesky.

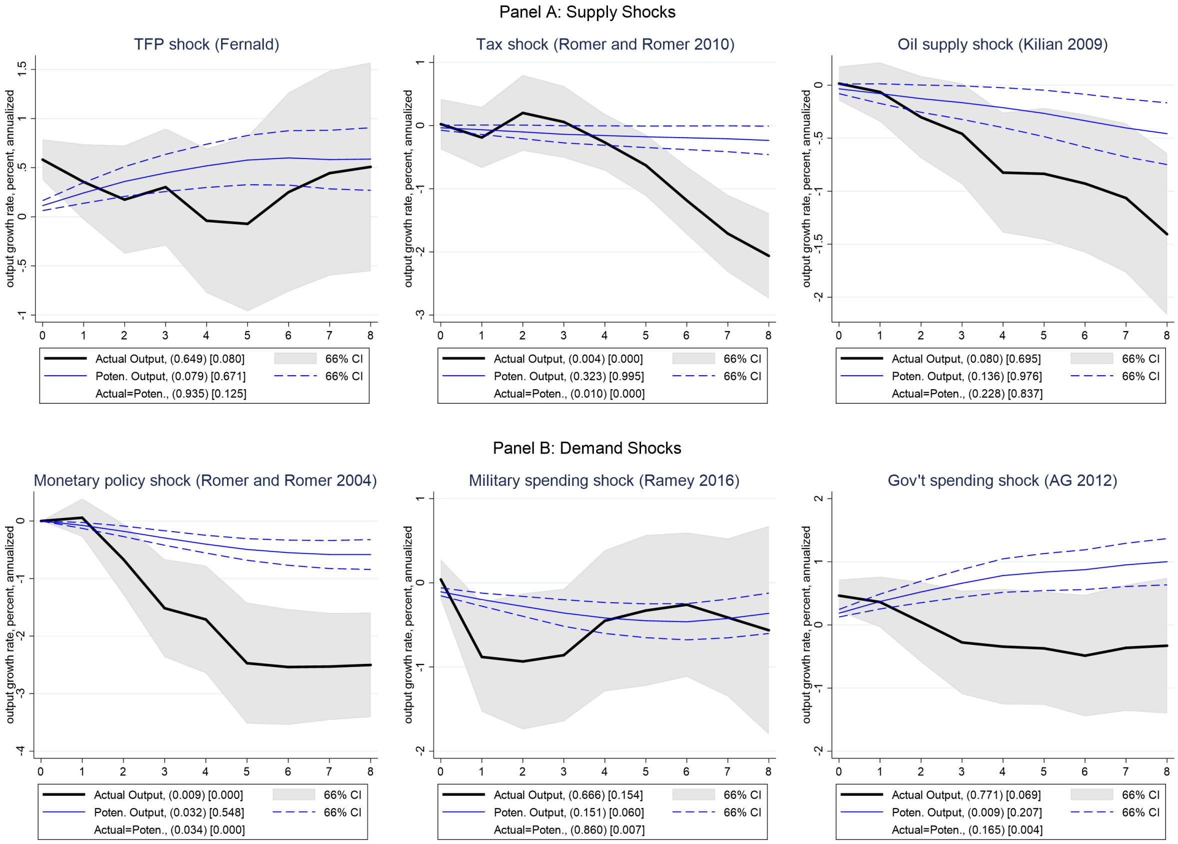


Figure B.8: Responses of Final CBO Estimates of Potential to Shocks

Notes: The figure reports impulse response functions (IRFs) estimated using equations (2) and (3). The estimation is identical to the baseline, except using final (2017) CBO estimates of potential GDP instead of real-time estimates.

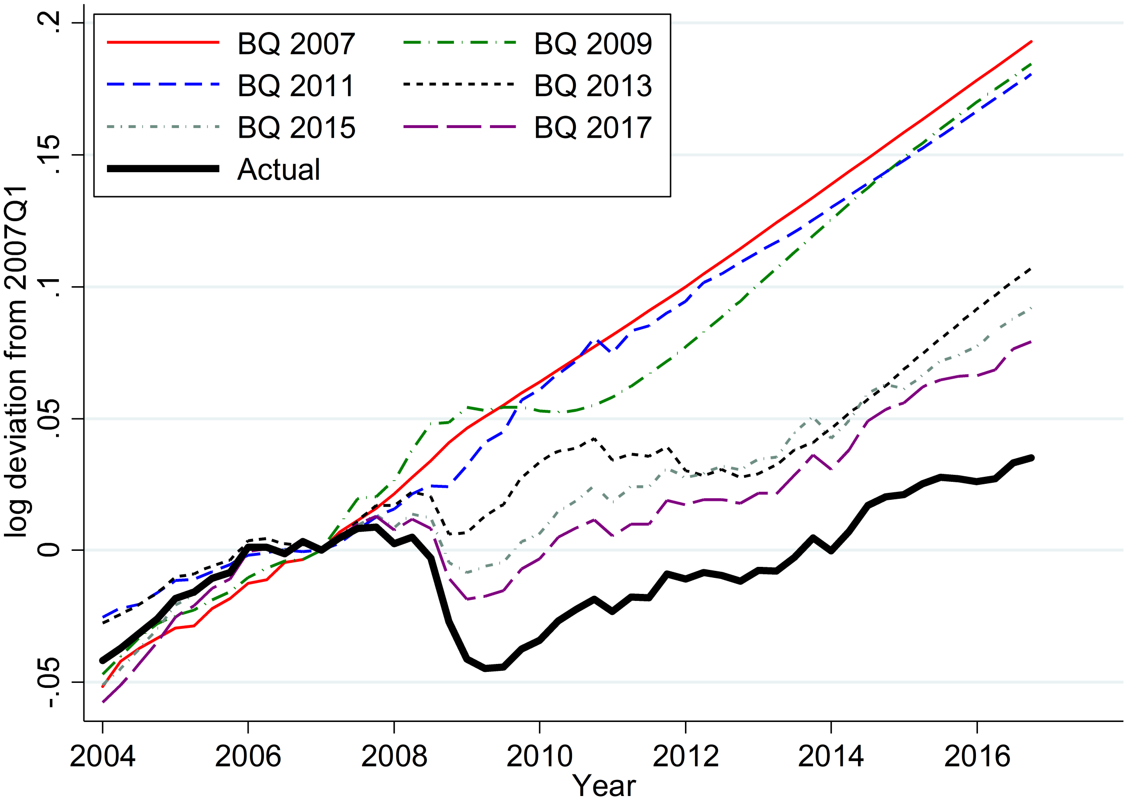
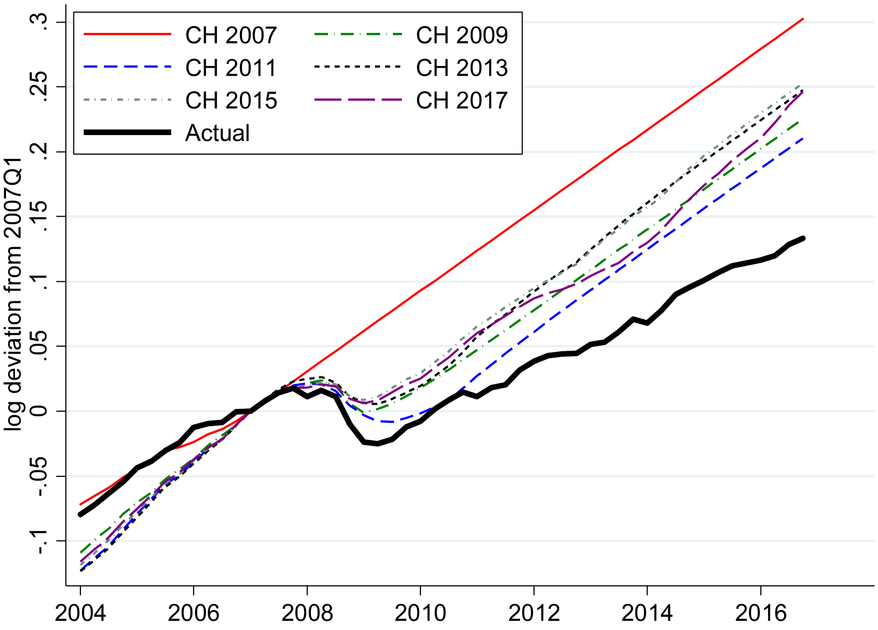
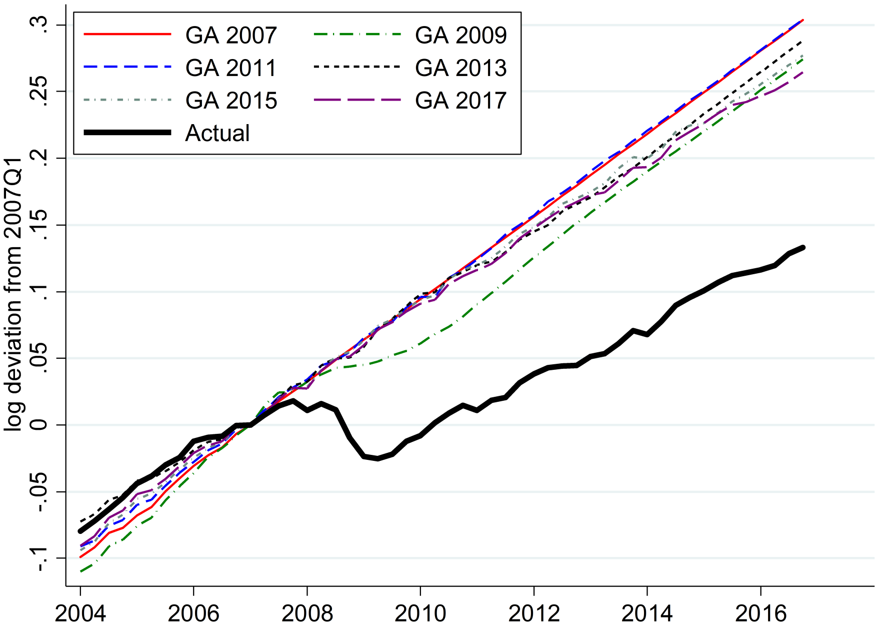


Figure B.9: Robustness of Responses to BQ Estimates of Monetary Shocks

Notes: The figure reports results for the series normalized real GDP by non-institutional civilian population and use the 1947-2007 period to compute trend growth for the normalized variable.



(a) Cochrane (1994) approach



(b) Gali (1999) approach

Figure B.10: Revisions in Potential during the Great Recession

Notes: The figure shows real-time estimates of changes in potential output since 2007 using the method of Cochrane (1994) in panel A and Gali (1999) in Panel B. The solid black line is actual GDP. See Section 5.2 for details.

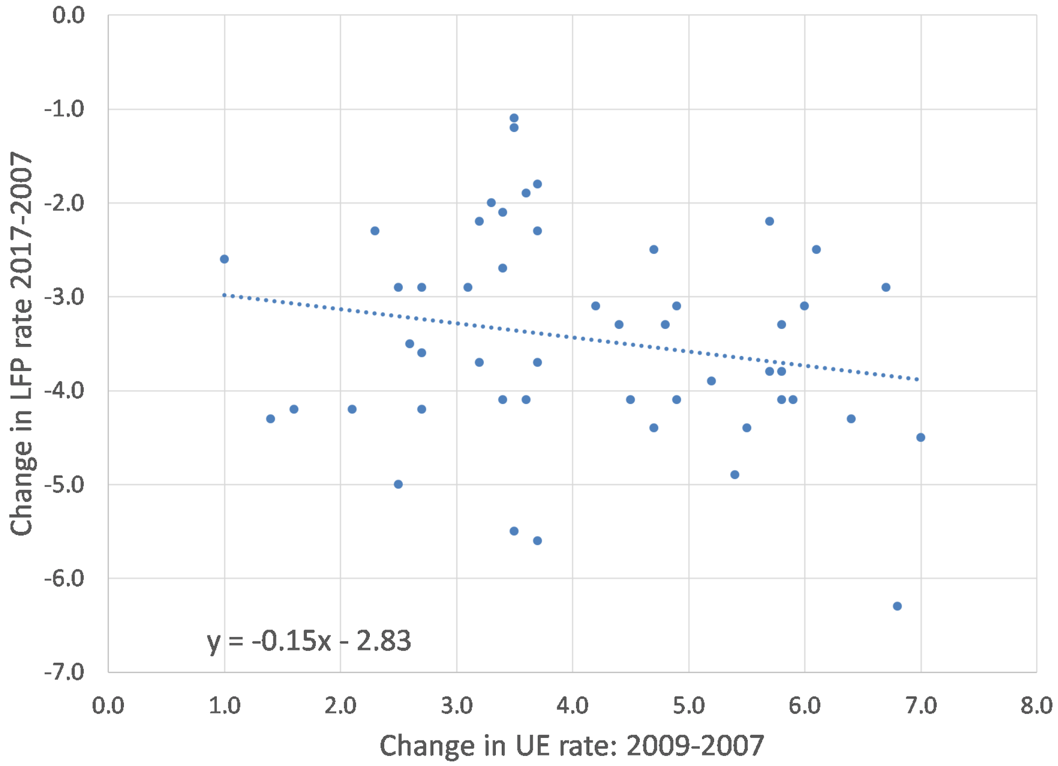


Figure B.11: State Unemployment and Changes in Labor Participation

Notes: The figure shows the evolution of total labor force participation from 2007-2017 for each U.S. state (vertical axis) relative to the change in their unemployment rate from 2007-2009.

B.2 Tables

Table B.1: Data coverage for cross-country analysis

Country	Prod. Shock	Oil Shock	Monetary Shock	Fiscal Shock	Actual IMF	Potential IMF	Actual OECD	Potential OECD
Australia	1981-2018	1980-2016	1983-2016	1998-2014	2003-2016	2003-2016	1986-2016	1989-2016
Austria	No data	1980-2016	1989-2016	1998-2014	2003-2016	2003-2016	1986-2016	1989-2016
Belgium	1981-2018	1980-2016	1984-2016	1998-2013	2003-2016	2003-2016	1986-2016	1989-2016
Canada	1981-2018	1980-2016	1994-2016	1987-2014	2003-2016	2003-2016	1986-2016	1989-2016
Switzerland	No data	1980-2016	1994-2016	1998-2014	No data	No data	1986-2016	1989-2016
Cyprus	No data	1980-2015	2001-2016	No data	2003-2016	2009-2016	No data	No data
Czech Republic	1994-2018	1990-2016	1996-2016	1998-2009	No data	No data	1996-2016	2005-2016
Germany	1992-2018	1980-2016	1994-2016	1987-2014	2003-2016	2003-2016	1986-2016	1989-2016
Denmark	No data	1980-2016	1984-2016	1998-2010	2003-2016	2009-2016	1986-2016	1989-2016
Spain	No data	1980-2016	1987-2016	1998-2012	2003-2016	2003-2016	1986-2016	1989-2016
Estonia	1996-2018	1990-2016	1995-2016	2010-2014	2003-2016	2012-2016	2008-2016	2011-2016
Finland	1981-2018	1980-2016	1989-2016	1998-2014	2003-2016	2003-2016	1986-2016	1989-2016
France	1981-2018	1980-2016	1983-2016	1987-2014	2003-2016	2003-2016	1986-2016	1989-2016
United Kingdom	1981-2018	1980-2016	1990-2016	1987-2014	2003-2016	2003-2016	1986-2016	1989-2016
Greece	No data	1980-2016	No data	1998-2001	2003-2016	2009-2016	1986-2016	1989-2016
Hungary	No data	1980-2016	2002-2016	1998-2003	No data	No data	1996-2016	2005-2016
Ireland	1991-2018	1980-2016	2000-2016	1998-2014	2003-2016	2003-2016	1996-2016	1996-2016
Iceland	1981-2018	1980-2016	1999-2016	1998-2014	No data	No data	1986-2016	2000-2016
Italy	1981-2018	1980-2016	1984-2016	1987-2014	2003-2016	2003-2016	1986-2016	1989-2016
Japan	1981-2018	1980-2016	1994-2016	1987-2014	2003-2016	2003-2016	1986-2016	1989-2016
Korea	1981-2018	1980-2016	1994-2016	1999-2014	2003-2016	2012-2016	1997-2016	2005-2016
Luxembourg	1986-2018	1980-2016	1997-2016	1998-2014	2003-2016	2012-2016	1986-2016	2005-2016
Malta	No data	1980-2015	No data	No data	2003-2016	2009-2016	No data	No data
Netherlands	1981-2018	1980-2016	1984-2016	1998-2014	2003-2016	2003-2016	1986-2016	1989-2016
Norway	1981-2018	1980-2016	1981-2016	1998-2014	2003-2016	2003-2016	1986-2016	1989-2016
New Zealand	1990-2018	1980-2016	1987-2016	1998-2014	2003-2016	2003-2016	1986-2016	1989-2016
Poland	No data	1980-2016	1997-2015	1998-2011	No data	No data	1996-2016	2005-2016
Portugal	1981-2018	1980-2016	1993-2016	1998-2014	2003-2016	2003-2016	1986-2016	1994-2016
Slovak Republic	No data	1980-2016	2001-2016	2008-2009	2003-2016	2009-2016	2000-2016	2005-2016
Slovenia	No data	1992-2016	1997-2016	2014-2014	2003-2016	2009-2016	2008-2016	2010-2016
Sweden	1981-2018	1980-2016	1984-2016	1998-2014	2003-2016	2003-2016	1986-2016	1989-2016
Turkey	No data	1980-2016	2001-2016	1998-2002	No data	No data	1986-2016	2005-2016
United States	1981-2018	1980-2016	1981-2016	1987-2014	2003-2016	2003-2016	1986-2016	1989-2016

Notes: The table describes time periods for which shocks and measures of potential output are available for each country and source of data. See Section 2 for descriptions of measures of potential GDP, and Sections 3 and 4 for details on construction of shocks.

Table B.2: P-values for tests for U.S. data

	Actual		Potential		Equality of actual and potential IRFs	
	IRF is equal to zero pointwise	IRF is zero at the max horizon	IRF is equal to zero pointwise	IRF is zero at the max horizon	pointwise	at the max horizon
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Greenbook, 1987-2011, Measure of actual = actual						
TFP shock	0.02	0.30	0.13	0.00	0.17	0.96
Government spending shock, (AG 2012)	0.07	0.92	0.09	0.02	0.03	0.34
Tax shock (RR 2010)	0.00	0.11	0.00	0.00	0.00	0.98
Military spending shock (Ramey 2016)	0.00	0.20	0.18	0.03	0.00	0.51
Oil price shock (Kilian 2009)	0.02	0.01	0.89	0.24	0.04	0.00
Monetary policy shock (RR 2004)	0.41	0.53	0.46	0.04	0.45	0.79
Panel B. Greenbook, 1969-2011, Measure of actual = actual						
TFP shock	0.03	0.69	0.45	0.07	0.05	0.93
Government spending shock, (AG 2012)	0.01	0.92	0.90	0.16	0.02	0.48
Tax shock (RR 2010)	0.00	0.00	0.55	0.03	0.00	0.07
Military spending shock (Ramey 2016)	0.11	0.73	0.00	0.00	0.26	0.45
Oil price shock (Kilian 2009)	0.41	0.07	0.01	0.00	0.14	0.00
Monetary policy shock (RR 2004)	0.00	0.02	0.00	0.01	0.00	0.10
Panel C1. 1987-2011, Measure of actual = 5yr MA of last vintage of actual						
TFP shock	0.44	0.02	0.13	0.00	0.99	0.94
Government spending shock, (AG 2012)	0.04	0.00	0.09	0.02	0.41	0.07
Tax shock (RR 2010)	0.98	0.87	0.00	0.00	0.10	0.08
Military spending shock (Ramey 2016)	0.96	0.22	0.18	0.03	0.54	0.02
Oil price shock (Kilian 2009)	0.97	0.30	0.89	0.24	0.24	0.00
Monetary policy shock (RR 2004)	0.31	0.46	0.46	0.04	0.00	0.01
Panel C2. 1987-2011, Measure of actual = 5yr MA of real time actual						
TFP shock	0.49	0.01	0.13	0.00	0.98	0.57
Government spending shock, (AG 2012)	0.00	0.00	0.09	0.02	0.08	0.01
Tax shock (RR 2010)	0.97	0.29	0.00	0.00	0.33	0.36
Military spending shock (Ramey 2016)	1.00	0.79	0.18	0.03	0.78	0.12
Oil price shock (Kilian 2009)	0.99	0.89	0.89	0.24	0.95	0.14
Monetary policy shock (RR 2004)	0.81	0.88	0.46	0.04	0.00	0.01
Panel C3. 1987-2011, Measure of actual = HP of real time actual						
TFP shock	0.51	0.01	0.13	0.00	0.95	0.27
Government spending shock, (AG 2012)	0.21	0.01	0.09	0.02	0.20	0.99
Tax shock (RR 2010)	0.09	0.00	0.00	0.00	0.34	0.57
Military spending shock (Ramey 2016)	0.78	0.08	0.18	0.03	0.06	0.96
Oil price shock (Kilian 2009)	1.00	0.42	0.89	0.24	0.91	0.47
Monetary policy shock (RR 2004)	1.00	0.64	0.46	0.04	0.00	0.00
Panel D. CBO, 1991-2011, Measure of actual = actual						
TFP shock	0.25	0.04	0.00	0.00	0.92	0.84
Government spending shock, (AG 2012)	0.29	0.14	0.02	0.00	0.36	0.92
Tax shock (RR 2010)	0.00	0.02	0.00	0.00	0.00	0.98
Military spending shock (Ramey 2016)	0.00	0.01	0.00	0.00	0.38	0.64
Oil price shock (Kilian 2009)	0.02	0.23	0.96	0.50	0.04	0.03
Monetary policy shock (RR 2004)	0.99	0.92	0.72	0.84	0.90	0.96

Notes: P-values for responses of actual GDP (columns 1-2) or estimates of potential GDP (columns 3-4) in response to shocks. Column 1 tests null that actual GDP is always zero in IRFs, column 2 tests null that its response is zero at the max horizon of IRFs. Columns 3 and 4 are equivalent but for potential GDP. Column 5 tests the null that the IRFS of actual GDP and estimated potential are the same at all horizons while column 6 tests the null they are the same at the final horizon. Panels A and C (1, 2 and 3) use the same measure of potential GDP (Greenbook 1987-2001); what changes between these panels is the measure of actual GDP (panel A uses the last vintage of actual output, panel C1 uses a 5 year MA of the last vintage of actual output, panel C2 uses a 5 year MA of actual output in real time and panel C3 uses an actual output in real time filtered with the HP method).

Table B.3: P-values for tests for international data

	Measure of actual output		Potential output		Equality of IRFs for measure of actual and potential output	
	IRF is equal to zero pointwise	IRF is zero at the max horizon	IRF is equal to zero pointwise	IRF is zero at the max horizon	pointwise	at the max horizon
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. IMF, Measure of actual = actual						
TFP shock	0.00	0.00	0.05	0.01	0.00	0.00
Oil price shock (Kilian 2009)	0.00	0.10	0.01	0.10	0.01	0.17
Monetary policy shock (VAR)	0.00	0.12	0.00	0.04	0.00	0.13
Government spending shock, (AG 2012)	0.03	0.36	0.00	0.00	0.09	0.83
Panel B. IMF, Measure of actual = HP of real time actual						
TFP shock	0.29	0.13	0.05	0.01	0.30	0.27
Oil price shock (Kilian 2009)	0.00	0.00	0.01	0.10	0.00	0.00
Monetary policy shock (VAR)	0.00	0.00	0.00	0.04	0.16	0.43
Government spending shock, (AG 2012)	0.02	0.00	0.00	0.00	0.14	0.43
Panel C. OECD, Measure of actual = actual						
TFP shock	0.00	0.00	0.00	0.00	0.00	0.00
Oil price shock (Kilian 2009)	0.06	0.45	0.34	0.08	0.12	0.96
Monetary policy shock (VAR)	0.08	0.02	0.47	0.07	0.29	0.12
Government spending shock, (AG 2012)	0.00	0.08	0.00	0.00	0.00	0.58
Panel D. OECD, Measure of actual = HP of real time actual						
TFP shock	0.00	0.00	0.00	0.00	0.00	0.00
Oil price shock (Kilian 2009)	0.58	0.31	0.34	0.08	0.17	0.36
Monetary policy shock (VAR)	0.04	0.00	0.47	0.07	0.05	0.00
Government spending shock, (AG 2012)	0.00	0.00	0.00	0.00	0.58	0.49
Panel E. Consensus Economics, Measure of actual = actual						
TFP shock	0.00	0.00	0.23	0.02	0.00	0.00
Oil price shock (Kilian 2009)	0.12	0.71	0.02	0.00	0.07	0.37
Monetary policy shock (VAR)	0.00	0.00	0.94	0.42	0.03	0.00
Government spending shock, (AG 2012)	0.03	0.99	0.07	0.00	0.02	0.58
Panel F. Consensus Economics, Measure of actual = HP of real time actual						
TFP shock	0.04	0.00	0.23	0.02	0.05	0.00
Oil price shock (Kilian 2009)	0.51	0.08	0.02	0.00	0.79	0.84
Monetary policy shock (VAR)	0.07	0.07	0.94	0.42	0.07	0.06
Government spending shock, (AG 2012)	0.01	0.00	0.07	0.00	0.02	0.00

Notes: The table reports p-values for different statistics of responses of actual GDP (columns 1-2) or estimates of potential GDP (columns 3-4) in response to shocks listed in the table using different measures of potential GDP. Column 1 tests null that actual GDP is always zero in IRFs while column 2 tests null that its response is zero at the maximum horizon of IRFs. Columns 3 and 4 are equivalent but for responses of the estimates of potential GDP. Column 5 tests the null that the IRFS of actual GDP and estimated potential are the same at all horizons while column 6 tests the null they are the same at the final horizon. See Section 4 for details. Notice also that the measure of potential output is the same in panels A and B, in panels C and D and in panels E and F, what differs between these pairs is that the first uses the last vintage of actual output as a measure of actual output while the second uses real time actual output filtered with an HP filter with $\lambda = 800$.

Appendix C

Additional Neo-Keynesian Trade Material

C.1 Hat Algebra

As described in the text we will use the variable $\hat{x}_t$ to denote x_t/x_{t-1} for any variable x. We will also denote trade shares with λ:

$$\lambda_{ij,s,t} = \frac{T_{i,s,t}(\tau_{ij,s,t}W_{i,t})^{1-\sigma_s}}{\sum_{r=1}^{I} T_{r,s,t}(\tau_{rj,s,t}W_{r,t})^{1-\sigma_s}}$$

Then we know that hat trade shares are:

$$\begin{aligned}
\hat{\lambda}_{ij,s,t} &= \frac{\hat{T}_{i,s,t}(\hat{\tau}_{ij,s,t}\hat{W}_{i,t})^{1-\sigma_s}}{\sum_{r=1}^{I} T_{r,s,t}(\tau_{rj,s,t}W_{r,t})^{1-\sigma_s} / \sum_{q=1}^{I} T_{q,s,t-1}(\tau_{qj,s,t-1}W_{q,t-1})^{1-\sigma_s}} \\
&= \frac{\hat{T}_{i,s,t}(\hat{\tau}_{ij,s,t}\hat{W}_{i,t})^{1-\sigma_s}}{\sum_{r=1}^{I} \hat{T}_{r,s,t}(\hat{\tau}_{rj,s,t}\hat{W}_{r,t})^{1-\sigma_s} T_{r,s,t-1}(\tau_{rj,s,t-1}W_{r,t-1})^{1-\sigma_s} / \sum_{q=1}^{I} T_{q,s,t-1}(\tau_{qj,s,t-1}W_{q,t-1})^{1-\sigma_s}} \\
&= \frac{\hat{T}_{i,s,t}(\hat{\tau}_{ij,s,t}\hat{W}_{i,t})^{1-\sigma_s}}{\sum_{r=1}^{I} \lambda_{rj,s,t-1}\hat{T}_{r,s,t}(\hat{\tau}_{rj,s,t}\hat{W}_{r,t})^{1-\sigma_s}}
\end{aligned}$$

To express the equilibrium system in hats and only leave it in terms of observable data in period zero (where we assume the economy was in a steady state where every country had full employment), we follow an iterative process. We first describe the procedure followed in period 1 (where quantities in period 0 are observed because they are the steady state we want to depart from), and after that we describe the procedure followed in the subsequent periods. The market clearing conditions can be written as:

$$\begin{aligned}
\hat{W}_{i,1}\hat{L}_{i,1}W_{i,0}L_{i,0} &= \sum_{s=1}^{S}\sum_{j=1}^{I}\hat{\lambda}_{ij,s,1}\lambda_{ij,s,0}\alpha_{j,s}\left(\hat{W}_{j,1}\hat{L}_{j,1}W_{j,0}L_{j,0}+\hat{D}_{j,1}D_{j,0}\right)\\
\hat{W}_{i,1}\hat{L}_{i,1}Y_{i,0} &= \sum_{s=1}^{S}\sum_{j=1}^{I}\frac{\lambda_{ij,s,0}\hat{T}_{i,s,1}(\hat{\tau}_{ij,s,1}\hat{W}_{i,1})^{1-\sigma_s}}{\sum_{r=1}^{I}\lambda_{rj,s,0}\hat{T}_{r,s,1}(\hat{\tau}_{rj,s,1}\hat{W}_{r,1})^{1-\sigma_s}}\alpha_{j,s}\left(\hat{W}_{j,1}\hat{L}_{j,1}Y_{j,0}+\hat{D}_{j,1}D_{j,0}\right)
\end{aligned}$$

The upper bound on total labor becomes:

$$\begin{aligned}
L_{i,1} &\leq \overline{L}_{i,1}\\
\hat{L}_{i,1}L_{i,0} &\leq \hat{\overline{L}}_{i,1}\overline{L}_{i,0}\\
\hat{L}_{i,1} &\leq 1
\end{aligned}$$

Where in the last step we are using the fact that since we are leaving an steady state (where there is full employment because the downward nominal wage rigidity doesn't bind then $L_{i,0}=\overline{L}_{i,0}$). We have also used the fact that since total labor doesn't change across time (in the short run) then $\hat{\overline{L}}_{i,1}=1$. The lower bound on the wage translates to hats very easily (part of the reason why we chose it):

$$\hat{W}_{i,1}\geq\delta_{i,1}$$

Then the complementary slackness condition is:

$$0=(1-\hat{L}_{i,1})(\hat{W}_{i,1}-\delta_{i,1})$$

Finally the first nominal anchor (global GDP targeting) becomes:

$$\begin{aligned}
\sum_{i=1}^{I}W_{i,1}L_{i,1} &= \gamma\sum_{i=1}^{I}W_{i,0}L_{i,0}\\
\sum_{i=1}^{I}\hat{W}_{i,1}\hat{L}_{i,1}Y_{i,0} &= \gamma\sum_{i=1}^{I}Y_{i,0},
\end{aligned}$$

The second one (U.S. GDP targeting) becomes:

$$\sum_{i=1}^{M}\hat{W}_{i,1}\hat{L}_{i,1}Y_{i,0} = \gamma\sum_{i=1}^{M}Y_{i,0}$$

And the limit on nominal wage in region h becomes:

$$\hat{W}_{h,1} \leq \gamma_1 \quad \text{and} \quad \left(\sum_{i=1}^{I}(1-\hat{L}_{i,1})\right)(\hat{W}_{h,1}-\gamma_1)=0$$

So, in summary, the equilibrium system is:

$$\begin{aligned}
\hat{W}_{i,1}\hat{L}_{i,1}Y_{i,0} &= \sum_{s=1}^{S}\sum_{j=1}^{I}\frac{\lambda_{ij,s,0}\hat{T}_{i,s,1}(\hat{\tau}_{ij,s,1}\hat{W}_{i,1})^{1-\sigma_s}}{\sum_{r=1}^{I}\lambda_{rj,s,0}\hat{T}_{r,s,1}(\hat{\tau}_{rj,s,1}\hat{W}_{r,1})^{1-\sigma_s}}\alpha_{j,s}\left(\hat{W}_{j,1}\hat{L}_{j,1}Y_{j,0}+\hat{D}_{j,1}D_{j,0}\right)\\
\hat{L}_{i,1} &\leq 1\\
\hat{W}_{i,1} &\geq \delta_{i,1}\\
0 &= (1-\hat{L}_{i,1})(\hat{W}_{i,1}-\delta_{i,1})
\end{aligned}$$

For all regions i, together with one of the following nominal anchors:

$$\begin{aligned}
\sum_{i=1}^{I}\hat{W}_{i,1}\hat{L}_{i,1}Y_{i,0} &= \gamma\sum_{i=1}^{I}Y_{i,0}\\
\sum_{i=1}^{M}\hat{W}_{i,1}\hat{L}_{i,1}Y_{i,0} &= \gamma\sum_{i=1}^{M}Y_{i,0}\\
\hat{W}_{h,1} &\leq \gamma_1 \quad \text{and} \quad \left(\sum_{i=1}^{I}(1-\hat{L}_{i,1})\right)(\hat{W}_{h,1}-\gamma_1)=0
\end{aligned}$$

This is a system of equations which we could use to solve for the quantities that we care about (the $\hat{W}_{i,1}$ and $\hat{L}_{i,1}$ for all i) given data ($Y_{i,0}$, $D_{i,0}$ and $\lambda_{ij,s,0}$) and shocks ($\hat{T}_{i,s,1}$, $\hat{D}_{i,1}$ and $\hat{\tau}_{ij,s,1}$). Once we have the $\hat{W}_{i,1}$ we can obtain the $\hat{\lambda}_{ij,s,1}$, and from them the $\lambda_{ij,s,1}$ (since we know the $\lambda_{ij,s,0}$). These will be useful as they will be needed in the solution for the next period. We can also recover $Y_{i,1}=W_{i,1}L_{i,1}$ as $\hat{W}_{i,1}\hat{L}_{i,1}Y_{i,0}$ and $D_{i,1}$ as $\hat{D}_{i,1}D_{i,0}$.

Now lets continue to period t and do the same process where now we can use as data $Y_{i,t-1}$, the whole past path of labor changes $\hat{L}_{i,t-1}, \hat{L}_{i,t-2}, ..., \hat{L}_{i,1}$, the past trade shares $\lambda_{ij,s,t-1}$ and $D_{i,t-1}$. The only condition that looks a little different is the upper bound on total labor:

$$\begin{aligned}
L_{i,t} &\leq \overline{L}_{i,t}\\
\frac{L_{i,t}}{L_{i,t-1}}\frac{L_{i,t-1}}{L_{i,t-2}}\cdots\frac{L_{i,1}}{L_{i,0}}L_{i,0} &\leq \overline{L}_{i,0}\\
\hat{L}_{i,t}\hat{L}_{i,t-1}\cdots\hat{L}_{i,1} &\leq 1\\
\prod_{q=1}^{t}\hat{L}_{i,q} &\leq 1
\end{aligned}$$

The equilibrium system in hats then is:

$$\begin{aligned}
\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} &= \sum_{s=1}^{S}\sum_{j=1}^{I}\frac{\lambda_{ij,s,t-1}\hat{T}_{i,s,t}(\hat{\tau}_{ij,s,t}\hat{W}_{i,t})^{1-\sigma_s}}{\sum_{r=1}^{I}\lambda_{rj,s,t-1}\hat{T}_{r,s,t}(\hat{\tau}_{rj,s,t}\hat{W}_{r,t})^{1-\sigma_s}}\alpha_{j,s}\left(\hat{W}_{j,t}\hat{L}_{j,t}Y_{j,t-1}+\hat{D}_{j,t}D_{j,t-1}\right)\\
\prod_{q=1}^{t}\hat{L}_{i,q} &\leq 1\\
\hat{W}_{i,t} &\geq \delta_{i,t}\\
0 &= \left(1-\prod_{q=1}^{t}\hat{L}_{i,q}\right)(\hat{W}_{i,t}-\delta_{i,t})
\end{aligned}$$

For all regions i, together with one of the following nominal anchors:

$$\begin{aligned}
\sum_{i=1}^{I}\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} &= \gamma\sum_{i=1}^{I}Y_{i,t-1}\\
\sum_{i=1}^{M}\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} &= \gamma\sum_{i=1}^{M}Y_{i,t-1}\\
\hat{W}_{h,t} &\leq \gamma_t \quad \text{and} \quad \left(\sum_{i=1}^{I}\left(1-\prod_{q=1}^{t}\hat{L}_{i,q}\right)\right)(\hat{W}_{i,t}-\gamma_t)=0
\end{aligned}$$

This is a system of equations which we can use to solve for the quantities that we care about (the $\hat{W}_{i,t}$ and $\hat{L}_{i,t}$ for all i) given the objects that we already know ($Y_{i,t-1}$, $\lambda_{ij,s,t-1}$, $D_{i,t-1}$ and the whole history of previous $\hat{L}_i$) and shocks ($\hat{T}_{i,s,t}$, $\hat{D}_{i,t}$ and $\hat{\tau}_{ij,s,t}$).

C.2 Tatonnement

Recall that the algorithm given in the text is described by the following rules:

$$\begin{aligned}
\hat{W}'_{i,t} &= \max\left\{\frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t}+\nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{L^U_{i,t}},\delta_{i,t}\right\}\\
\hat{L}'_{i,t} &= \min\left\{L^U_{i,t},\frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t}+\nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{\delta_{i,t}}\right\}
\end{aligned}$$

Here we prove that if the initial guess of the algorithm $\hat{W}_{i,t}$ and $\hat{L}_{i,t}$ satisfies equations (4.8)-(4.11) then the new guess $\hat{W}'_{i,t}$ and $\hat{L}'_{i,t}$ will also satisfy them.

Since our algorithm is described by a max and a min, it is immediate that conditions (4.8) and (4.9) are satisfied. Also notice that it cannot happen that:

$$\begin{aligned}\hat{W}'_{i,t} &= \frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{L^U_{i,t}} \\ \hat{L}'_{i,t} &= \frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{\delta_{i,t}}\end{aligned}$$

Since that would require:

$$\begin{aligned}\frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{L^U_{i,t}} &\geq \delta_{i,t} \\ L^U_{i,t} &\geq \frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{\delta_{i,t}}\end{aligned}$$

Putting these two things together we get:

$$\frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{L^U_{i,t}} \geq \delta_{i,t} \geq \frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{L^U_{i,t}}$$

Which is impossible unless both inequalities hold with equality. (in which case all the relevant conditions are satisfied anyway). This means that unless we are in a knife edge case (where everything works fine anyway) we are either going to be in the point:

$$\left(\hat{L}'_{i,t}, \hat{W}'_{i,t}\right) = \left(L^U_{i,t}, \frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{L^U_{i,t}}\right)$$

Or in the point:

$$\left(\hat{L}'_{i,t}, \hat{W}'_{i,t}\right) = \left(\frac{(1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}}}{\delta_{i,t}}, \delta_{i,t}\right)$$

Which means that complementary slackness (condition (4.10)) is satisfied. It is also clear that we always have:

$$\begin{aligned}\hat{W}'_{i,t}\hat{L}'_{i,t} &= (1-\nu)\hat{W}_{i,t}\hat{L}_{i,t} + \nu\frac{\sum_s X_{i,s,t}}{Y_{i,t-1}} \\ \hat{W}'_{i,t}\hat{L}'_{i,t}Y_{i,t-1} &= (1-\nu)\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} + \nu\sum_s X_{i,s,t}\end{aligned}$$

We can sum this expression over all i's to obtain:

$$\begin{aligned}\sum_{i=1}^{I}\hat{W}'_{i,t}\hat{L}'_{i,t}Y_{i,t-1} &= (1-\nu)\sum_{i=1}^{I}\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1}+\nu\sum_{i=1}^{I}\sum_{s=1}^{S}X_{i,s,t}\\ &= (1-\nu)\sum_{i=1}^{I}\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1}+\nu\sum_{i=1}^{I}\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1}\\ &= \sum_{i=1}^{I}\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1}\end{aligned}$$

Which means that if the initial guess of the algorithm (the $\hat{W}_{i,t}$ and $\hat{L}_{i,t}$) satisfies condition (4.11) then the new iteration (the $\hat{W}'_{i,t}$ and $\hat{L}'_{i,t}$) will also satisfy it. We conclude that if the initial guess of our algorithm satisfies conditions (4.8)-(4.11) then so will all subsequent guesses. Additionally the algorithm will converge to a solution that satisfies (4.7) and so it will be a solution to our equilibrium system. We don't yet have a proof that the algorithm will converge on a solution that satisfies (4.7), but this has been the case under the diverse set of conditions and parameters that we have tested so far.

Another important question is where to start the algorithm, since we need an initial guess that satisfies conditions (4.8)-(4.11). There is a straightforward way to make the initial guess if all the $\delta_{i,t}$ satisfy $\gamma \geq \delta_{i,t}$. In that case group all regions that have $L^U_{i,t} > 1$ into a group called G_1 and all regions that have $L^U_{i,t} = 1$ into a group called G_2. Assign initial guess $\hat{L}_{i,t} = 1$ to all regions. Assign initial guess $W_{i,t} = \delta_{i,t}$ if $i \in G_1$ and $W_{i,t} = \hat{W}^G$ if $i \in G_2$, where:

$$\begin{aligned}\sum_{i=1}^{I}\hat{W}_{i,t}\hat{L}_{i,t}Y_{i,t-1} &= \gamma\sum_{i=1}^{I}Y_{i,t-1}\\ \sum_{i=1}^{I}\hat{W}_{i,t}Y_{i,t-1} &= \gamma\sum_{i=1}^{I}Y_{i,t-1}\\ \sum_{i\in G_1}\delta_{i,t}Y_{i,t-1}+\sum_{i\in G_2}\hat{W}^GY_{i,t-1} &= \gamma\sum_{i=1}^{I}Y_{i,t-1}\\ \hat{W}^G &= \frac{\gamma\sum_{i=1}^{I}Y_{i,t-1}-\sum_{i\in G_1}\delta_{i,t}Y_{i,t-1}}{\sum_{i\in G_2}Y_{i,t-1}}\end{aligned}$$

We have to prove that this initial guess satisfies equations (4.8)-(4.11). For $i \in G_1$ equation (4.8) is satisfied because $\hat{L}_{i,t} = 1 \leq L^U_{i,t}$, equation (4.9) is satisfied with equality because $W_{i,t} = \delta_{i,t}$, which also means (4.10) is satisfied. For $i \in G_2$ equation (4.8) is satisfied with equality because $\hat{L}_{i,t} = 1 = L^U_{i,t}$, which also means (4.10) is satisfied, equation (4.9) is satisfied because $W_{i,t} = \hat{W}^G \geq \delta_{i,t}$. We can prove this last statement by showing that $\hat{W}^G \geq \gamma$ and using our assumption that $\gamma \geq \delta_{i,t}$ for all i. To show that $\hat{W}^G \geq \gamma$ notice that this happens iff:

$$\begin{aligned}
\frac{\gamma \sum_{i=1}^{I} Y_{i,t-1} - \sum_{i \in G_1} \delta_{i,t} Y_{i,t-1}}{\sum_{i \in G_2} Y_{i,t-1}} &\geq \gamma \\
\gamma \sum_{i=1}^{I} Y_{i,t-1} - \sum_{i \in G_1} \delta_{i,t} Y_{i,t-1} &\geq \gamma \sum_{i \in G_2} Y_{i,t-1} \\
\gamma \sum_{i \in G_1} Y_{i,t-1} - \sum_{i \in G_1} \delta_{i,t} Y_{i,t-1} &\geq 0 \\
\sum_{i \in G_1} (\gamma - \delta_{i,t}) Y_{i,t-1} &\geq 0
\end{aligned}$$

Which is true since $\gamma \geq \delta_{i,t}$ for all i. Notice that for the first period this initial guess would say that everyone has a labor hat of 1, no one has unemployment, everyone has a wage hat of γ and hence the world nominal GDP target is satisfied. For periods after the first one, regions with unemployment in the previous periods get assigned their DNWR target (as if they still had slack), and regions that didn't have unemployment in the previous period get a higher wage to fulfill the world GDP targeting. The problem with this guess is that it requires having $\gamma \geq \delta_{i,t}$, since $\delta_{i,t} = \kappa_{i,t}\delta = \frac{E_{i,t}}{E_{i,t-1}}\delta$, which is imposing that $\frac{E_{i,t}}{E_{i,t-1}} \leq \frac{\gamma}{\delta}$, since $E_{i,t}$ is given in dollars per local currency units of region i, an upper bound on the exchange rate change in region i is like requiring that this region doesn't want to appreciate too much.

C.3 Labor supply

Lets think about the model without wage rigidity but where there is a labor supply equation, the equations would be:

$$\begin{aligned}
W_{i,t} L_{i,t} &= \sum_{s=1}^{S} \sum_{j=1}^{I} \frac{T_{i,s,t} (\tau_{ij,s,t} W_{i,t})^{1-\sigma_s}}{\sum_{r=1}^{I} T_{r,s,t} (\tau_{rj,s,t} W_{r,t})^{1-\sigma_s}} \alpha_{j,s} (W_{j,t} L_{j,t} + D_{j,t}) \\
\frac{W_{i,t}}{P_{i,t}} &= \psi L_{i,t}^{1/\eta}
\end{aligned}$$

Where the second equation is a standard intra-temporal condition for labor supply derived from GHH preferences (we could also use standard KPR preferences, this wouldn't make much difference in the present context of no savings, although the deficits definitely pose a problem). Now use the definition of the price index:

$$P_{i,t} = \prod_{s=1}^{S} \left(\frac{P_{i,s,t}}{\alpha_{i,s}} \right)^{\alpha_{i,s}}$$

$$
\begin{aligned}
P_{i,s,t} &= \left(\sum_{j=1}^{I} P_{ji,s,t}^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}} \\
P_{ji,s,t} &= \tau_{ji,s,t}\frac{W_{j,t}}{A_{j,s,t}}
\end{aligned}
$$

This can be expressed as:

$$
\begin{aligned}
P_{i,t} &= \prod_{s=1}^{S}\left(\frac{P_{i,s,t}}{\alpha_{i,s}}\right)^{\alpha_{i,s}} \\
P_{i,s,t} &= \left(\sum_{j=1}^{I} T_{j,s,t}(\tau_{ji,s,t}W_{j,t})^{1-\sigma_s}\right)^{\frac{1}{1-\sigma_s}}
\end{aligned}
$$

And as:

$$
P_{i,t} = \prod_{s=1}^{S}\left(\sum_{j=1}^{I} T_{j,s,t}(\tau_{ji,s,t}W_{j,t})^{1-\sigma_s}\right)^{\frac{\alpha_{i,s}}{1-\sigma_s}} \alpha_{i,s}^{-\alpha_{i,s}}
$$

If we make $\psi = \overline{L}^{-\frac{1}{\eta}}$ From the labor supply we would know:

$$
L_{i,t} = \overline{L}\frac{W_{i,t}^{\eta}}{P_{i,t}^{\eta}}
$$

Introducing this in the market clearing condition we get:

$$
\frac{W_{i,t}^{1+\eta}}{P_{i,t}^{\eta}}\overline{L} = \sum_{s=1}^{S}\sum_{j=1}^{I}\frac{T_{i,s,t}(\tau_{ij,s,t}W_{i,t})^{1-\sigma_s}}{\sum_{r=1}^{I} T_{r,s,t}(\tau_{rj,s,t}W_{r,t})^{1-\sigma_s}}\alpha_{j,s}\left(\frac{W_{j,t}^{1+\eta}}{P_{j,t}^{\eta}}\overline{L} + D_{j,t}\right)
$$

Notice that if $\eta = 0$ we would be back to the standard equations. Since given all the W_i's we know all the P_i's then we could in theory easily solve this in Matlab using a solver. It would also be interesting to know if this equation "tatonnes" following the procedure of Alvarez and Lucas.

C.4 Input-Output Loop

C.4.1 Levels

Here we add an input output loop to the previous model because we need such an structure to properly use the data provided by CDP. The way it works is that there are I regions and S sectors and to produce output in each region and sector, firms need to combine labor with all the sectoral aggregates (the version of them available in that country). That is, the production function in region i. sector s, and time t is given by:

$$Y_{i,s,t} = A_{i,s,t} L_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^{S} M_{i,ks,t}^{\phi_{i,ks}},$$

where $M_{i,ks,t}$ is the amount of the aggregate sector k good used for production in sector s in country i at time t, $\phi_{i,s}$ is the labor share in region i, sector s, $\phi_{i,ks}$ is the share of inputs that sector s uses from sector k in country i, and we know that $1 - \phi_{i,s} = \sum_{k=1}^{S} \phi_{i,ks}$. We also know that the aggregate price in region i, sector s is given, according to the Armington assumption, by the CES aggregator:

$$P_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I} P_{ji,s,t}^{1-\sigma_s}$$

Additionally, due to the Cobb-Douglas assumption in production and the fact that firms are in perfect competition, we know that the price of the good in sector s produced in region i and consumed in that same country is:

$$P_{ii,s,t} = \frac{W_{i,t}^{\phi_{i,s}}}{A_{i,s,t}} \prod_{k=1}^{S} P_{i,k,t}^{\phi_{i,ks}}$$

And the prices of the good in sector s produced in region i but consumed in region j are given by:

$$P_{ij,s,t} = \tau_{ij,s,t} P_{ii,s,t}$$

Combining the last three equations we obtain:

$$P_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I} \left(\tau_{ji,s,t} \frac{W_{j,t}^{\phi_{j,s}}}{A_{j,s,t}} \prod_{k=1}^{S} P_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s}$$

Which, for each time period t, is a system of $I \times S$ equations in $I \times S$ unknowns that can be used to solve for the $P_{i,s,t}$'s given the trade costs ($\tau_{ji,s,t}$'s), technologies ($A_{j,s,t}$'s), wages ($W_{j,t}$'s), labor shares ($\phi_{j,s}$'s) and input output coefficients ($\phi_{j,ks}$) (note that we don't allow the labor shares and input output coefficients to vary with time for simplicity). So we are done with production, and consumption also seems to be unchanged, so now I will look at equilibrium. We know that the share of total expenditure in region i, sector s, at time t that is used in goods coming from region j is given by:

$$\lambda_{ji,s,t} = \left(\frac{P_{ji,s,t}}{P_{i,s,t}}\right)^{1-\sigma_s} = \frac{P_{ji,s,t}^{1-\sigma_s}}{\sum_{r=1}^{I} P_{ri,s,t}^{1-\sigma_s}} = \frac{\left(\tau_{ji,s,t}\frac{W_{j,t}^{\phi_{j,s}}}{A_{j,s,t}}\prod_{k=1}^{S} P_{j,k,t}^{\phi_{j,ks}}\right)^{1-\sigma_s}}{\sum_{r=1}^{I}\left(\tau_{ri,s,t}\frac{W_{r,t}^{\phi_{r,s}}}{A_{r,s,t}}\prod_{k=1}^{S} P_{r,k,t}^{\phi_{r,ks}}\right)^{1-\sigma_s}}$$

We know that the amount demanded for the good of region i and sector s is given by:

$$R_{i,s,t} = \sum_{j=1}^{I} \lambda_{ij,s,t} Z_{j,s,t},$$

where $Z_{j,s,t}$ denotes total expenditure in region i in the sector s composite. We know, from Cobb-Douglas preferences and technology that:

$$Z_{j,s,t} = \alpha_{j,s}(Y_{j,t} + D_{j,t}) + \sum_{k=1}^{S} \phi_{j,sk} R_{j,k,t},$$

where $Y_{i,t}$ is income in region i at time t, which is just given by $W_{i,t}L_{i,t}$ (as always $D_{i,t}$ are the deficits). Combining the previous two equations we obtain:

$$R_{i,s,t} = \sum_{j=1}^{I} \lambda_{ij,s,t}\left(\alpha_{j,s}(W_{j,t}L_{j,t} + D_{j,t}) + \sum_{k=1}^{S} \phi_{j,sk} R_{j,k,t}\right)$$

This can be used to solve for the $R_{i,s,t}$ as a function of wages. And in fact this is a linear equation in the R's, so it is relatively easy to solve. Of this demand ($R_{i,s,t}$), which is also total production, we know that a fraction $\phi_{i,s}$ is payed to labor, so we can write:

$$\begin{aligned} W_{i,t}L_{i,s,t} &= \phi_{i,s}R_{i,s,t} \\ W_{i,t}L_{i,t} &= \sum_{s=1}^{S} \phi_{i,s}R_{i,s,t} \end{aligned}$$

C.4.2 Hat Algebra

Now that I understand this in levels proceed to do it in hats. First we need to express the system for prices:

$$P_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I} T_{j,s,t}\left(\tau_{ji,s,t} W_{j,t}^{\phi_{j,s}} \prod_{k=1}^{S} P_{j,k,t}^{\phi_{j,ks}}\right)^{1-\sigma_s}$$

In term of hats. But this is easily done as:

$$\hat{P}_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^{I} \lambda_{ji,s,t-1} \hat{T}_{j,s,t} \left(\hat{\tau}_{ji,s,t} \hat{W}_{j,t}^{\phi_{j,s}} \prod_{k=1}^{S} \hat{P}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s}$$

Then continue with the trade shares:

$$\lambda_{ji,s,t} = \frac{T_{j,s,t} \left(\tau_{ji,s,t} W_{j,t}^{\phi_{j,s}} \prod_{k=1}^{S} P_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s}}{\sum_{r=1}^{I} T_{r,s,t} \left(\tau_{ri,s,t} W_{r,t}^{\phi_{r,s}} \prod_{k=1}^{S} P_{r,k,t}^{\phi_{r,ks}} \right)^{1-\sigma_s}}$$

As usual, we can express these in hats as:

$$\hat{\lambda}_{ji,s,t} = \frac{\hat{T}_{j,s,t} \left(\hat{\tau}_{ji,s,t} \hat{W}_{j,t}^{\phi_{j,s}} \prod_{k=1}^{S} \hat{P}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s}}{\sum_{r=1}^{I} \lambda_{ri,s,t-1} \hat{T}_{r,s,t} \left(\hat{\tau}_{ri,s,t} \hat{W}_{r,t}^{\phi_{r,s}} \prod_{k=1}^{S} \hat{P}_{r,k,t}^{\phi_{r,ks}} \right)^{1-\sigma_s}}$$

Then we need to express the system for revenues:

$$R_{i,s,t} = \sum_{j=1}^{I} \lambda_{ij,s,t} \left(\alpha_{j,s}(W_{j,t} L_{j,t} + D_{j,t}) + \sum_{k=1}^{S} \phi_{j,sk} R_{j,k,t} \right)$$

In hats. There is no nice way to do this, so we just leave it in terms of past revenues:

$$\hat{R}_{i,s,t} R_{i,s,t-1} = \sum_{j=1}^{I} \hat{\lambda}_{ij,s,t} \lambda_{ij,s,t-1} \left(\alpha_{j,s}(\hat{W}_{j,t} \hat{L}_{j,t} Y_{j,t-1} + \hat{D}_{j,t} D_{j,t-1}) + \sum_{k=1}^{S} \phi_{j,sk} \hat{R}_{j,k,t} R_{j,k,t-1} \right)$$

With this we can easily turn the equilibrium conditions into hats:

$$\begin{aligned} W_{i,t} L_{i,t} &= \sum_{s=1}^{S} \phi_{i,s} R_{i,s,t} \\ \hat{W}_{i,t} \hat{L}_{i,t} Y_{i,t-1} &= \sum_{s=1}^{S} \phi_{i,s} \hat{R}_{i,s,t} R_{i,s,t-1} \end{aligned}$$

And that is all we need to solve the system in hats. But we also need to construct data for the initial period as done by CDP. We have in the data that they provided 3 things. Labor shares for all regions and sectors, input-output matrix for all regions (although it is the same one for all of the US) and bilateral trade flows are in the data provided by CDP. We need to construct the alphas and income in each country and revenues.

C.4.3 Fixing the Data for New Alphas

Imagine we had new alphas and we wanted to obtain the data that is compatible with these new alphas. The equilibrium system to obtain the data for the new alphas is the following:

$$\begin{aligned}
\hat{R}_{i,s,t}R_{i,s,t-1} &= \sum_{j=1}^{I}\hat{\lambda}_{ij,s,t}\lambda_{ij,s,t-1}\left(\alpha'_{j,s}(\hat{W}_{j,t}Y_{j,t-1}+D_{j,t-1})+\sum_{k=1}^{S}\phi_{j,sk}\hat{R}_{j,k,t}R_{j,k,t-1}\right)\\
\hat{\lambda}_{ij,k,t} &= \frac{\left(\hat{W}_{i,t}^{\phi_{i,s}}\prod_{s=1}^{S}\hat{P}_{i,s,t}^{\phi_{i,sk}}\right)^{1-\sigma_k}}{\sum_{r=1}^{I}\lambda_{rj,k,t-1}\left(\hat{W}_{r,t}^{\phi_{r,k}}\prod_{s=1}^{S}\hat{P}_{r,s,t}^{\phi_{r,sk}}\right)^{1-\sigma_k}}\\
\hat{P}_{j,k,t}^{1-\sigma_s} &= \sum_{i=1}^{I}\lambda_{ij,s,t-1}\left(\hat{W}_{i,t}^{\phi_{i,k}}\prod_{s=1}^{S}\hat{P}_{i,s,t}^{\phi_{i,sk}}\right)^{1-\sigma_k}\\
\sum_{i=1}^{I}\hat{W}_{i,t}Y_{i,t-1} &= \sum_{i=1}^{I}Y_{i,t-1}
\end{aligned}$$

In this system the R_{t-1}, λ_{t-1}, α', Y_{t-1}, D_{t-1}'s are all data, and the $\hat{W}$, $\hat{P}$, $\hat{R}$ and $\hat{\lambda}$'s are the outcomes.