Lawrence Berkeley National Laboratory

Recent Work

Title

Modulation of Pulsar Signals by Deformation of the Star

Permalink

<https://escholarship.org/uc/item/2t02s379>

Author

Glendenning, N.K.

Publication Date

1989-12-01

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

To be submitted for publication

Modulation of Pulsar Signals by Deformation of the Star

N. K. Glendenning

December 1989

For Reference

Not to be taken from this room

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Modulation of Pulsar Signals by Deformation of the Star[†]

Norman K. Glendenning

*Nuclear Science Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, California 94720*

December 12, 1989

[†]This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Modulation of Pulsar Signals by Deformation of the Star[†]

Norman K. Glendenning

*Nuclear Science Division
Lawrence Berkeley Laboratory
1 Cyclotron Road
Berkeley, California 94720*

December 12, 1989

Abstract

The motion of a rigid axi-symmetric deformed pulsar model in which the body symmetry axis is inclined to the angular momentum axis will modulate the emission from a pulsar introducing such features into individual pulse structure as quasi-periodic drifting, nulling and mode switching, much as is seen in certain pulsars. For very small eccentricity, the period of the modulation can be very long (many orders of magnitude) compared to the pulsar period. In certain situations the modulation due to precession can simulate orbital motion of a pulsar with a companion. It can also produce long null intervals, or otherwise modulate both the intensity and the frequency of the pulsar signals.

PACS 97.60 Gb

[†]This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

Contents

1	Introduction	1
2	Mechanics of the Model	2
3	Qualitative Description of the Pulse Structure	4
4	Compatibility with Observed Frequency Damping	6
5	Sinusoidal Frequency Modulation	7
6	Summary	8

Modulation of Pulsar Signals by Deformation of the Star

Norman K. Glendenning

December 12, 1989

1 Introduction

The well known gross characteristics of pulsar signals are that the pulse profile, averaged over many pulses, and its frequency is very stable. Otherwise the individual pulses generally are different one from the other [1, 2]: (1) Some pulsars produce signals within the average profile with apparently random structure compared to each other. (2) Still others produce successive pulse timings that vary systematically around the mean, the pulses drifting across the average profile and after a few pulses beginning at the other side and repeating a similar sequence of pulses. (3) The drift can occur in either direction. (4) Successive bunches of pulses constituting a cycle in the drift are not precisely the same in number or structure. (5) In some pulsar signals there are nulls, meaning the absence of pulses when they are otherwise expected from the period. (6) In others the average pulse profile switches between several patterns.

We accept the standard view that pulsars are rotating magnetized neutron stars and that the pulse is the manifestation of beamed radiation along the magnetic axis which is fixed in the star. In this paper we discuss the effects of a small deformation of a pulsar, a deformation whose symmetry axis is not aligned with the rotation axis [3]. For simplicity we assume rigid-body motion. The two additional motions introduced by deformation, the precession of the symmetry axis about the angular momentum axis and the rotation of the body about its symmetry axis, will modulate the *intrinsic* radiation pattern produced by the emission mechanism that a fixed observer would see. As we shall show, the modulation can have a remarkable variety of forms, depending, among other factors discussed below, on the relative position of the observer, and the angles between magnetic axis and symmetry axis and between symmetry axis and angular momentum axis, and on the eccentricity. For the purpose of illustrating the modulation of the signal by precession, we shall look at its effect on a constant emission from the surface of the pulsar with a prescribed angular width along the direction of the magnetic field and from both poles. We have nothing to say about the emission mechanism itself, which may be quite complex, and introduce its own structure into the signal [4, 5].

We certainly do not insist on precisely rigid-body motion. This is simply a solvable idealization which has many interesting features that may survive in more flex-

ible motions under the stresses of gravitation, rotation, strong magnetic fields, the short-range tensor interaction which spatially aligns a succession of neutron-proton pairs with their spins, and the magnetic moments of the nucleons, which couples the preceding two.

2 Mechanics of the Model

In a torque free environment the model pulsar has two constants of motion, the kinetic energy and angular momentum, K and $\mathbf{L}$. The mechanical motion is determined fully by the two scalar constants K and L , all directions being simply referred to the direction of $\mathbf{L}$. Let the symmetry axis, $\mathbf{S}$ make an angle α with the instantaneous angular velocity vector $\boldsymbol{\omega}$ and an angle β with the vector $\mathbf{L}$, respectively. In a body with axial symmetry (say $I_1 = I_2 \neq I_3$), the magnitude ω is fixed and the vectors $\boldsymbol{\omega}$ and $\mathbf{S}$ lie in a plane that rotates about $\mathbf{L}$ with constant precession angular velocity Ω . From the Poinsot construction of mechanics, the above quantities are related by [6],

$$\frac{I_3}{I_1} \tan \beta = \tan \alpha, \quad \Omega \sin \beta = \omega \sin \alpha. \quad (1)$$

There is a third rotation, the body about its own symmetry axis, with angular velocity

$$n = \left(\frac{I_3}{I_1} - 1 \right) \omega \cos \alpha \quad (2)$$

For chosen I_3, I_1 the unknowns are $\alpha, \beta, \omega, \Omega, n$. The two constants of the motion K and L together with the above three equations determine them. For our purpose it is more convenient to choose two of the above five and regard the other three and K and L as dependent. The two most relevant parameters of the problem are Ω and n as will become evident. Define

$$e \equiv \frac{I_3}{I_1} - 1, \quad r \equiv \frac{\Omega}{n} \quad (3)$$

From the first three equations we can also write,

$$n = \frac{e}{1+e} \Omega \cos \beta \quad (4)$$

We note that for $e < 0$ (prolate), the angular velocity, n about the symmetry axis is in the opposite sense to Ω and ω , while for $e > 0$ (oblate) it is in the same sense. Hence $re > 0$. Therefore without loss of generality we can take $0 < \beta < \pi/2$ and consequently

$$1 > \frac{1+e}{re} > 0 \quad (5)$$

For given eccentricity, e , this is a restriction on the ratio of frequencies, r !

$$|r| \equiv \left| \frac{\Omega}{n} \right| > \left| \frac{1+e}{e} \right| \quad (6)$$

For small eccentricities, which are likely in stars, this ratio of frequencies must be greater than one, indeed much greater.

Now we introduce the magnetic axis, $\mathbf{B}$, and let it be fixed in the body making an angle γ with the symmetry axis. It rotates about this axis with angular velocity n . Define a reference system fixed in space with the z-axis defined by the constant direction of $\mathbf{L}$. Let the origin lie at the center of mass of the star. At various instants $\mathbf{B}$, $\mathbf{S}$ and the z-axis are coplanar. At one such instant when $\mathbf{B}$ lies below the $\mathbf{S}$ axis as measured from the z-axis, call the time $t = 0$. After some trigonometry we find that the time-dependent polar angles of $\mathbf{B}$ are,

$$\Theta(t) = \arctan\left(\frac{\sqrt{(\sin\beta + \cos\beta \tan\gamma \cos nt)^2 + \tan^2\gamma \sin^2 nt}}{\cos\beta - \sin\beta \tan\gamma \cos nt}\right) \quad (7)$$

$$\Phi(t) = \Omega(t + t_0) + \arctan\left(\frac{\tan\gamma \sin nt}{\sin\beta + \cos\beta \tan\gamma \cos nt}\right) \quad (8)$$

We explain the choice of t_0 below.

We can think of Φ and Θ in the following way: The pulsar magnetic axis, and therefore the pulsar beam, sweeps by a given azimuth (say the one containing the observer) according to $\Phi(t)$. Its *frequency* is the time derivative, which is the precession frequency, Ω , of the symmetry axis about the fixed angular momentum axis, *modulated* by the derivative of the second term in eq.(8), and is *not* the frequency, ω , of the body itself. The beam oscillates in polar angle according to eq.(7). This introduces a modulation in the *intensity* of the observed signal. Both frequency and intensity of the signal seen by an observer fixed in space from a pulsar whose symmetry axis is inclined to the fixed direction of the angular momentum axis, will be modulated therefore.

Define a variable $\tau = t/T$ with $T = 2\pi/\Omega$. As a consequence of eq.(4) the polar angles of $\mathbf{B}$ are completely defined in terms of τ, β, γ and r satisfying eq.(5), for any chosen eccentricity, e .

Knowing the direction of $\mathbf{B}$ as a function of the time variable τ , we can calculate the angle between the magnetic axis and an observer as a function of this variable. We have not chosen the x-axis within the constant, $\Phi = \Omega t_0$. Let it now be chosen so that the observer lies in the x,z-plane. This defines the constant t_0 introduced above. Let the angle between the observer and the z-axis (direction of $\mathbf{L}$) be η . Then the time-dependant angle between observer and magnetic axis is

$$\delta(\tau) = \sin\eta \sin\Theta(\tau) \cos\Phi(\tau) + \cos\eta \cos\Theta(\tau) \quad (9)$$

We illustrate the effects of the above motion by considering the signal that a fixed observer would see if a constant beam of radiation is emitted with a gaussian distribution in angle measured from the magnetic axis $\mathbf{B}$. The intensity of the signal seen by an observer will be proportional to

$$I(\tau) = \exp\left(-\left(\frac{\delta(\tau)}{\Delta}\right)^2\right) + (\delta \rightarrow \delta - \pi) \quad (10)$$

The two terms take into account the radiation from both poles. Some authors believe that as well as, or instead of, the above emission pattern, the radiation is emitted in a cone centered at the magnet axis, or both[7]. For the cone case we take the intensity proportional to

$$I(\tau) = \left(\frac{\delta(\tau)}{\Delta}\right)^2 \exp\left(-\left(\frac{\delta(\tau)}{\Delta}\right)^2 + 1\right) + (\delta \rightarrow \delta - \pi) \quad (11)$$

To have a chance of seeing the signal the observer's orientation has to approximately satisfy the following relation,

$$\beta + \gamma + \Delta \geq \eta \geq \beta - \gamma - \Delta \quad (12)$$

where the role of Δ is obvious, and the other parts of the limits follow from eq.(7).

3 Qualitative Description of the Pulse Structure

Let us describe in words the motion that is executed by the magnetic axis. For a star with symmetry axis and rotation axis *coincident* and the magnetic axis inclined at an angle γ with the rotation axis, the magnetic axis traces out a cone. For the deformed star described above with symmetry axis inclined to the fixed angular momentum axis, it traces a surface that is bounded by two cones, at angles $\beta \pm \gamma$ with respect to the z-axis, completing a cycle in time $2\pi/n$. For small eccentricities, appropriate to stars, the period of the rotation of the body about its own axis is much longer than the precession of the body's symmetry axis about the z-axis. Consequently the surface traced by the magnetic axis will spiral between the above two cones, but the time to complete one cycle will be longer than the time for the magnetic axis to precess once around the z-axis by the ratio of frequencies $r = \Omega/n$. This surface will eventually retrace itself only if this ratio of frequencies is a rational number. Essentially that will never be the case for long, because of radiation damping. So in general the surface traced by the magnetic axis in the course of time will not retrace itself (not be periodic). The radiated signal has an angular width about the magnetic axis. Each pulse intensity and timing seen by an observer fixed in space as the symmetry axis of the star rotates with precession frequency Ω will depend on the particular cut of the intrinsic radiation pattern his line of sight makes as the magnet axis sweeps by with the time dependent polar and azimuthal angles given by frequency eq.(7,8). *Therefore any sequence of pulses seen by an observer will never be repeated in intensity, structure and in timing.* However there will occur approximate repetition of sequences as we see below, a quasi-periodicity with period $2\pi/n$.

To gain further insight into the signals that an observer fixed in space would see from such a model pulsar, recall that the motion of the magnetic axis described in space fixed axis by eqs.(7,8), is simply the superposition of the precession of $\mathbf{S}$ about the z-axis with angular velocity Ω and the rotation of $\mathbf{B}$ about $\mathbf{S}$ with angular velocity n , both making constant angles β and γ with their respective rotation axes. Let $\mathbf{S}$ define a z' -axis with x' -axis in the plane defined by the z and z' -axis and with the x' -axis below the xy -plane. Then the polar angle of $\mathbf{B}$ in this (uniformly precessing)

system is $\phi' = nt$ and $\theta' = \gamma$. As already noted, the sign of n depends on the sign of e . Taking note that $\mathbf{S}$ is precessing in space say in counterclockwise sense, we then can say with respect to a fixed observer in space that if $0 < \phi' < \pi$ then the space-fixed polar angle Φ of the magnetic axis will be advanced compared to the symmetry axis, while if $\pi < \phi' < 2\pi$, it will be retarded. The position of greatest advancement or retardation occurs at $\phi' = \pi/2, 3\pi/2$. The positions when the magnetic axis is neither advanced or retarded is $\phi' = 0, \pi$. For each cycle of the pulsar (Ω), the magnetic axis will rotate by only a small angle, $\Delta\phi' = 2\pi(n/\Omega)$. What the observer sees will depend on his polar angle $\theta_{obs} = \eta$ in the space fixed axis. If $\theta_{obs} \approx \beta \pm \gamma$ he will see a succession of pulses near the angular frequency Ω but progressing from slightly advanced to retarded or vice versa, ie drifting in one sense or the other across the average pulse profile, with possible null pulses between one group of drifting pulses and the next. In Fig. 1 we show a sequence of individual pulses, plotted over the portion of the precession period for which the signal can be seen from the observer's position in space, and stacked one over the other. In this way we display both the sense of intensity variation and the frequency modulation. In contrast, the timing of the signal from a pulsar whose symmetry axis coincides with the rotation axis, would occur at constant frequency, if the source were a constant beam. Whether a sequences of drifting pulses is separated by nulls or by weaker pulses drifting in the opposite sense depends on the the beam width Δ , on γ , and on the observer's orientation. A sequence can also consist of pulses that drift from advanced to retarded positions with a frequency modulation that is approximately sinusoidal, interspersed with many nulls, as in Figs. 2 and 3. The cycles described are generally quasi-periodic, and not exactly so unless the ratio of frequencies $r = \Omega/n$ is a rational number. The sum of pulses and nulls in a cycle is approximately r . The parameters corresponding to the figures can be found in Table 1.

If the observer's orientation has about the same polar angle as the symmetry axis, $\eta \approx \beta$, then he will see the pulsar only when $\phi' \approx \pi/2, 3\pi/2$, if $\Delta \ll \gamma$. As ϕ' approaches either of these positions, the observer will see a sequence of pulses that occur very close in frequency (and near to the precession frequency) followed by a null period followed by another series of pulses that again occur near the precession frequency but which are displaced in timing from the first group, and correspond to the other position of ϕ' . One group is advanced and the other retarded, as in Fig. 4.

Table 1: Parameters that define the pulsar characteristics corresponding to the Figures.

Fig. no.	e	Ω/n	β/π	γ/π	η/π	Δ/π	$ eg(\beta) $
1	-0.05	25	0.225	0.2	0.35	0.1	0.09
2	-0.05	50	0.376	0.1	0.33	0.07	0.17
4	-0.05	50	0.376	0.15	0.33	0.07	0.17
5	-0.05	50	0.376	0.15	0.38	0.1	0.17
6	-0.05	25	0.225	0.05	0.2	0.04	0.09

There follows another null period and then a sequence at timings close to the first group and so on. The number of nulls will be equal if $\eta = \beta$, and otherwise there will be alternating number of nulls. The number of pulses seen near each of these positions of ϕ' will depend on the beam width and on the ratio of frequencies n/Ω , and will be approximately $(\Delta/2\pi)(\Omega/n)$. The average pulse profile of each group will be the reflection symmetric image of the other (but only approximately, because of the quasi-periodicity). This pattern resembles the phenomena exhibited by certain pulsars that is referred to as mode switching. This is another example of the quasi-periodicity.

Under some circumstances pulses from both poles can be seen. One circumstance under which this can happen is when the angle between the symmetry axis and the magnetic axis is sufficiently large. An example of interpulse signals is shown in the intensity-pulse plot of Fig. 5.

As a final example, note that if the pulse emission pattern itself is more complicated than the gaussian pattern of eq.(11), say a cone emission as in eq.(12), then the individual pulses can vary not only in timing, but also in shape, as seen in Fig. 6.

It should be noted that we have illustrated only six possibilities from a multiple infinity of effects that precession can cause. If any particular pulsar precesses, the modulation of the observed signal can be highly varied, depending on the relative position of the observer and the other parameters that describe the deformation and the motion as detailed above.

4 Compatibility with Observed Frequency Damping

For many pulsars the period and its rate of change are measured. Periods range from milliseconds to seconds with most in the range 2/10 to 2 seconds, and the rate of change of periods lie in the range $10^{-18} < \dot{T} < 10^{-12}$. The damping is believed to be caused by magnetic dipole radiation or gravitational radiation for which $\dot{T} \propto 1/T$ and $1/T^3$ respectively. The gravitational damping will occur only if there is a time changing quadrupole mass transport, and so will occur in our model. The measured damping places an upper bound on the eccentricity, or more precisely on e times a function of the angle between the symmetry axis and the angular momentum axis, β . Gravitational radiation yields a rate of change of the period according to,

$$\begin{aligned} \dot{T} &= (2\pi)^4 \frac{4}{25} \frac{1}{T^3} M R^2 e^2 \sin^2 \beta (16 \sin^2 \beta + \cos^2 \beta) \\ &= 1.6 \times 10^{-12} \left(\frac{s}{T}\right)^3 \left(\frac{R}{10 \text{ km}}\right)^2 \frac{M}{M_\odot} e^2 g^2(\beta) \end{aligned} \quad (13)$$

where $g^2(\beta)$ stands for the function of β in the first line and s stands for the unit second. For most of our examples, $eg(\beta) \sim 0.1$. Hence for a nominal solar mass pulsar with radius 10 km, $\dot{T} \approx 1.6 \times 10^{-14} (s/T)^3$. This defines a curve in the $\dot{T} - T$ plane above which many pulsars lie. Model parameters used in the figures are compatible with those pulsars that lie above this curve.

5 Sinusoidal Frequency Modulation

Under the condition $\tan \gamma \ll \tan \beta$ we have,

$$\begin{aligned}\Phi &\approx \Omega(t + t_0) + \arctan\left(\frac{\tan \gamma}{\sin \beta} \sin nt\right) \\ \dot{\Phi} &\approx \Omega + \left\{1 + \left(\frac{\tan \gamma \sin nt}{\sin \beta}\right)^2\right\}^{-1} n \frac{\tan \gamma}{\sin \beta} \cos nt\end{aligned}\quad (14)$$

We see that the frequency modulation can be sinusoidal, as for a pulsar whose signal is modulated by orbital motion with a companion. However, the frequency of the modulation, n , and its amplitude, $\Delta\Omega$, are not necessarily compatible with a particular observation, since they are both defined dominantly in terms of e [8]. Eq.(4) gives the relation between the precession (pulsar) frequency, Ω , and the modulation frequency, n , and we can read the amplitude of the frequency modulation from eq.(14),

$$\frac{\Delta\Omega}{\Omega} \approx \frac{n \tan \gamma}{\Omega \sin \beta} \ll \left| \frac{e}{1 + e} \right| \quad (15)$$

For example, the eight hour modulation of the 1/2 ms pulses observed for PSR1987A [9] is not compatible with precession [8, 10]. The observed amplitude is too large. However the 2 hr modulation that has been reported in an improved analysis of the data [11] is compatible with precession as we can see very simply. The measurement of the time rate of change of the period, observed as $\dot{T} < 3 \times 10^{-14}$, places a constraint on the eccentricity through eq(13), and tells us that $eg(\beta) < 10^{-7}$. This is satisfied by $e \leq 10^{-6}$. The second constraint is imposed on the ratio of modulation frequency to pulsar frequency, eq.(4,5). The third constraint is on the amplitude of the frequency modulation, given by eq.(15). Identifying the long two hour period with the rotation of the star about its own symmetry axis whose angular velocity we denoted by n , eq.(4) provides the constraint $e \cos \beta = \frac{2}{3}10^{-7}$. Together the first two constraints read,

$$\frac{2}{3} < \frac{e}{10^{-7}} < 10 \quad (16)$$

and are compatible. Thus the deformed pulsar model can account for the period of the pulsar and its two hour modulation period with an eccentricity that satisfies the constraints imposed by the small rate of change of the pulsar period and with the ratio of the millisecond to the two hour period. The third constraint is also satisfied, namely the observed relative amplitude of the two hour modulation is $|\Delta\Omega/\Omega| \sim 1.2 \times 10^{-8}$ which satisfies eq.(15). So the two hour frequency modulation could be caused by precession due to a small non-axisymmetric deformation that is compatible with the measured rate of change of the millisecond period.

There is another interesting possibility. One of our examples above (fig. 2,3) showed a long period modulation in which the signal was very weak over about half of the long period. Now the pulsar in SN1987A was not found two weeks after its first sighting at the time of the next search. Perhaps the subsequent searches

have occurred during the portions of the period when the magnetic axis has been rotated too far out of our line of sight by a very small eccentricity. Identify the modulation frequency with a time of the order of a week. This, according to eq.(6) would require an eccentricity, $e \gtrsim 8 \times 10^{-10}$, even smaller than those quoted above, relegating gravitational radiation to a very small role in the rate of change of the period. We have tentatively proposed the deformed pulsar model, in this case with minuscule eccentricity, as an explanation of the disappearance of the signal [8]. If this explanation is correct, it should reappear and disappear at cyclic intervals with a period anywhere greater than about an hour to weeks, years or centuries. The duration of the appearance need not be the same as the duration of the disappearance. This depends on $\beta, \gamma, \Delta, \eta, e$. The smaller the eccentricity, the longer the period of the complete cycle, according to eq.(4).

In support of the two long-term cycles discussed in the preceding two paragraphs, we cite an earlier suggestion that precession may be the most natural way to understand the 35 day cycle of the 1.24 s pulsar, Hercules X-1 [12, 13], or the 4.8 hour period of the 12.6 ms γ -ray pulses of Cygnus X-3 [14].

6 Summary

The modulation of signals received by an observer from a pulsar that is in simple rotation about its symmetry axis is believed to be the origin of the pulsed signals, not the emission mechanism. Here we suggest that in certain pulsars, in which the symmetry axis and angular momentum axis do not coincide, additional modulations will be superposed on the emitted radiation. There is a rich variety in possible modulations, depending on the degree of eccentricity and the two angles between its axis, and the angular momentum axis and the magnetic axis, as well as the observers orientation. We assumed that the source of radiation was *constant* in intensity and distributed around the magnetic axis with a simple pattern. In real pulsars whatever effects discussed here that are caused by rotation about an axis that is not the symmetry axis, are *superimposed* on the radiation which is produced in the complex and possibly time-varying environment of the magnetosphere. The source itself may vary in intensity and frequency and its distribution about the magnetic axis may be more complex than assumed in our examples. The simple source is chosen so as to most clearly exhibit the effects of precession.

Depending on the observer's orientation with respect to the angular momentum axis, and the parameters defining the pulsars motion, the modulation can manifest itself as drifting of individual pulses either in an advanced or retarded sense, with nulls or weaker pulses connecting the groups of drifting pulses. Under other circumstances the modulation can produce a group of pulses whose periods are nearly the same followed by nulls and then another group at a shifted timing, but again having nearly the same periods. The latter pattern resembles mode switching. Under certain circumstances an observer would see a sequence of pulses whose frequency is approximately sinusoidally modulated, and whose intensity varies, perhaps falling below the detection threshold so that a sequence of visible pulses is followed by nulls. In the

absence of radiation damping, the above cycles would be periodic only if Ω/n is a rational number, say 50.45. Each cycle would consist of a total of about 50 pulses some of which may be nulls. Every 5045 pulses the same sequence of cycles would recur. However, in practice, the above cycles must be quasiperiodic because precise periodicity can occur only if the ratio of precession frequencies is a rational number; because of radiation damping this situation, though occasionally attained, cannot be maintained.

The ratio of the period of the quasi-periodic cycle to the pulsar period is governed by the eccentricity and the angle between symmetry axis and angular momentum axis in the form $(e \cos \beta)^{-1}$. If this is of the order of tens or hundreds, precessional effects resemble, in generic fashion, features of individual pulse structure as observed in some pulsars such as drifting, nulling and mode switching. This possibility appears not to have been discussed previous to this work. If $(e \cos \beta)^{-1}$ is many orders of magnitude, precessional effects will be observed as long-time cycles of the order of days, weeks or months, where the principle observable effect will be the modulation of the signal intensity, as perhaps seen in Hercules X-1, and suggested earlier. (Tracking the corresponding frequency modulation over such time scales is probably very difficult.)

We have set down the detailed equations governing this general behavior in the idealization of rigid motion in which we took two of the principle moments of inertia to be equal. Freed from these restrictions, the pulse patterns would have even greater complexity.

Acknowledgements: I appreciate helpful discussions with M. G. Redlich and W. Swiatecki. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of Nuclear Physics, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

References

- [1] R. N. Manchester and J. H. Taylor, *Pulsars*, W. H. Freeman and Co., San Francisco, 1977.
- [2] J. M. Rankin, *Astrophys. J.* **274** (1983) 333; loc. cit. **301** (1986) 901.
- [3] N. K. Glendenning, *Neutron Stars, Fast Pulsars, Supernovae and the Equation of State of Dense Matter* in NATO Advanced Study Institute, "The Nuclear Equation of State", ed. by W. Greiner, Peniscola, Spain, May 21-June 3, 1989, To be published by Plenum, London (LBL-27293).
- [4] E. Flowers and M. A. Ruderman, *Astrophys. J.* **215** (1977) 302.
- [5] P. B. Jones, *Mon. Not. R. Astr. Soc.* **209** (1984) 569.
- [6] J. L. Synge and B. A. Griffith *Principles of Mechanics* McGraw-Hill, N.Y. (1949) 425.
- [7] D. C. Backer, *Astrophys. J.* **209** (1976) 895 (B76).

- [8] N. K. Glendenning, Phys. Rev. Let. **63** (1989) 1443.
- [9] J. Kristian, C. R. Pennypacker, J. Middleditch, M. A. Hamuy, J. N. Imamura, W. E. Kunkel, R. Lucinio, D. E. Morris, R. A. Muller, S. Perlmutter, S. J. Rawlings, T. P. Sasseen, I. K. Shelton, T. Y. Steinman-Cameron and I. R. Tuohy, NATURE **385** (1989) 234.
- [10] R. W. Nelson, L. S. Finn and I. Wasserman, Cornell preprint CRSR 914, (1989).
- [11] J. Middleditch et. al. *Further Analysis of the PSR1987A Discovery Data*, preprint, Aug, 1989, to be published in Science.
- [12] K. Brecher, Nature **239** (1972) 325.
- [13] J. Trumper, P. Kahabka, H. Ogelman, W. Pietsch and W. Voges, Astrophys. J. **300** (1986) L63.
- [14] K. J. Orford, Astrophys. and Space Sci. **129** (1987) 181.

Fig. 1 Pulses that would be seen from the model deformed pulsar defined by the parameters of Table 1, for a continuous source of radiation. Pulses are stacked modulo the precession period $T = 2\pi/\Omega$. This example illustrates drifting pulses separated by nulls. If the observer were positioned near $\eta = \beta - \gamma$ instead of $\beta + \gamma$ he would see a drift in the opposite direction, as would be the case if the sign of the eccentricity were reversed.

Fig. 2 Frequency modulation is approximately sinusoidal in this case, with intensity of observed signal varying as in Fig. 3. As in all cases the signal source is constant.

Fig. 3 Intensity of individual observed pulses corresponding to Fig. 2, for one complete cycle of rotation of the pulsar about its own symmetry axis.

Fig. 4 Illustrating pulses that alternate in timing with approximately the same frequency in each bunch. The plot covers two complete cycles.

Fig. 5 Observed intensity of pulses from one end of pulsar occur near the integer units of t/T and weaker pulses seen from the other end occur near $1/2$ integer units. The source at each end is constant and equal in intensity.

Fig. 6 Drifting pulses for the cone shaper emission pattern of eq.11.

LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
1 CYCLOTRON ROAD
BERKELEY, CALIFORNIA 94720