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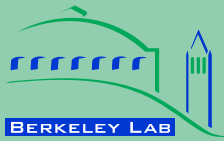
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Parametric System Curves: Correlations Between Fan Pressure Rise and Flow for Large Commercial Buildings

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Environmental Energy

Technologies Division

May 19, 2010

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Parametric System Curves: Correlations between Fan Pressure Rise and Flow for Large Commercial Buildings

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ABSTRACT

A substantial fraction of HVAC energy use in large commercial buildings is due to fan operation. Fan energy use depends in part on the relationship between system pressure drop and flow through the fan, which is commonly called a “system curve”. As a step toward enabling better selections of air-handling system components and analyses of common energy efficiency measures such as duct static pressure reset and duct leakage sealing, this paper shows that a simple four-parameter physical model can be used to define system curves.

Our model depends on the square of the fan flow, as is commonly considered. It also includes terms that account for linear-like flow resistances such as filters and coils, and for supply duct leakage when damper positions are *fixed or are changed independently of static pressure or fan flow*. Only two parameters are needed for systems with *variable-position* supply dampers (e.g., VAV box dampers modulating to control flow). For these systems, reducing or eliminating supply duct leakage does not change the system curve.

The parametric system curve may be most useful when applied to field data. Non-linear techniques could be used to fit the curve to fan pressure rise and flow measurements over a range of operating conditions. During design, when measurements are unavailable, one could use duct design calculation tools instead to determine the coefficients.


Keywords: buildings, HVAC, air distribution, duct design, system curve, fans, leakage

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INTRODUCTION

Typically in North American large commercial buildings, central HVAC systems supply heated or cooled air to conditioned spaces through a complex network of ducts. By adding static and kinetic energy to the airstream, fans generate the large pressure rises needed to circulate air through the typically long duct runs. A substantial fraction of HVAC energy use is due to fan operation.

Fan *electric* power depends on fan air power (product of the airflow through and pressure rise across the fan), fan efficiency, and motor and drive efficiencies. None of these parameters is constant for systems with variable flows or pressures and all are interrelated. For example, although not obvious from manufacturer's data, fan efficiency strongly depends on fan airflow *and* pressure rise.

The pressure rise across the fan must be sufficient to overcome the total pressure drop of the air-handling system. Depending on system configuration, the pressure drop is a function of the duct static pressure set point, duct leakage, and the pressure drops across duct and duct-like elements (e.g., dampers, fittings), coils, and filters that are connected to the fan. It is generally recognized that duct and duct-like pressure drops increase approximately as the square of the flow through them (ASHRAE 2009). However, contrary to common belief, pressure drops across coils and filters behave differently: the pressure drop versus flow relationship is less parabolic and more linear like in some cases such as with wet coils or high-efficiency filters (Rivers and Murphy 1996, Trane 1999, Liu et al. 2003).

For each combination of duct-static-pressure set point, leakage, and element pressure drops, it is possible to plot the associated fan static pressure rise over a range of fan airflows for a given system operating condition. The overall relation between pressure drop and flow defines what is commonly called a "system curve". When duct system characteristics change, such as when the duct static pressure set point is varied, a family of system curves can result.

The system-fan curve intersections that result when a system curve is plotted along with fan performance curves (e.g., power or speed as a function of pressure rise and flow) on a pressure versus flow map define a locus of unique fan operating points. Each of these points has an associated fan efficiency, power, and speed. Knowing the system curve is essential to selecting correctly sized fan and drive components that will be as efficient as possible throughout the fan operating range (ASHRAE 2008, Murphy 2010).

ASHRAE Standard 90.1 (2010), California's Title 24 Nonresidential Alternative Calculation Method Approval Manual (CEC 2008), and many building energy performance simulation programs represent combined fan, motor, and drive system performance using the locus of operating points model. Different generic sets of coefficients are used to represent the various types of fan airflow control for VAV systems (e.g., discharge dampers, inlet vanes, and variable speed control). There are substantial differences between the generic curves, but the literature does not explain the differences. In particular, for all of these generic curves, there is no explicit accounting for fan, belt, motor, variable-frequency-drive (VFD), or distribution system characteristics and the embodied assumptions are undocumented. Consequently, it is unclear what specific air-handling systems they represent and whether the curves include efficiency variations with flow and pressure rise.

Because of their simplified fan models and lack of duct system models, mainstream simulation tools have been unable to simulate the effects of duct static pressure reset or other fan and duct system component improvements. As a result, they cannot be used to demonstrate the energy-saving benefits associated with efficient fan and duct systems. To help overcome the lack of suitable models for fan pressure rise in simulation tools, the purpose of this paper is to develop a simplified physical model of a prototypical air-handling system in a large commercial building that can be used to define *system curves* that account for system leakage and the control of duct static pressure and conditioned space pressure.

CORE MODEL DEVELOPMENT

We seek a simple, yet generic, model for the prototypical variable-air-volume (VAV) air-handling system shown in Figure 1 with the purpose of defining its system curves.

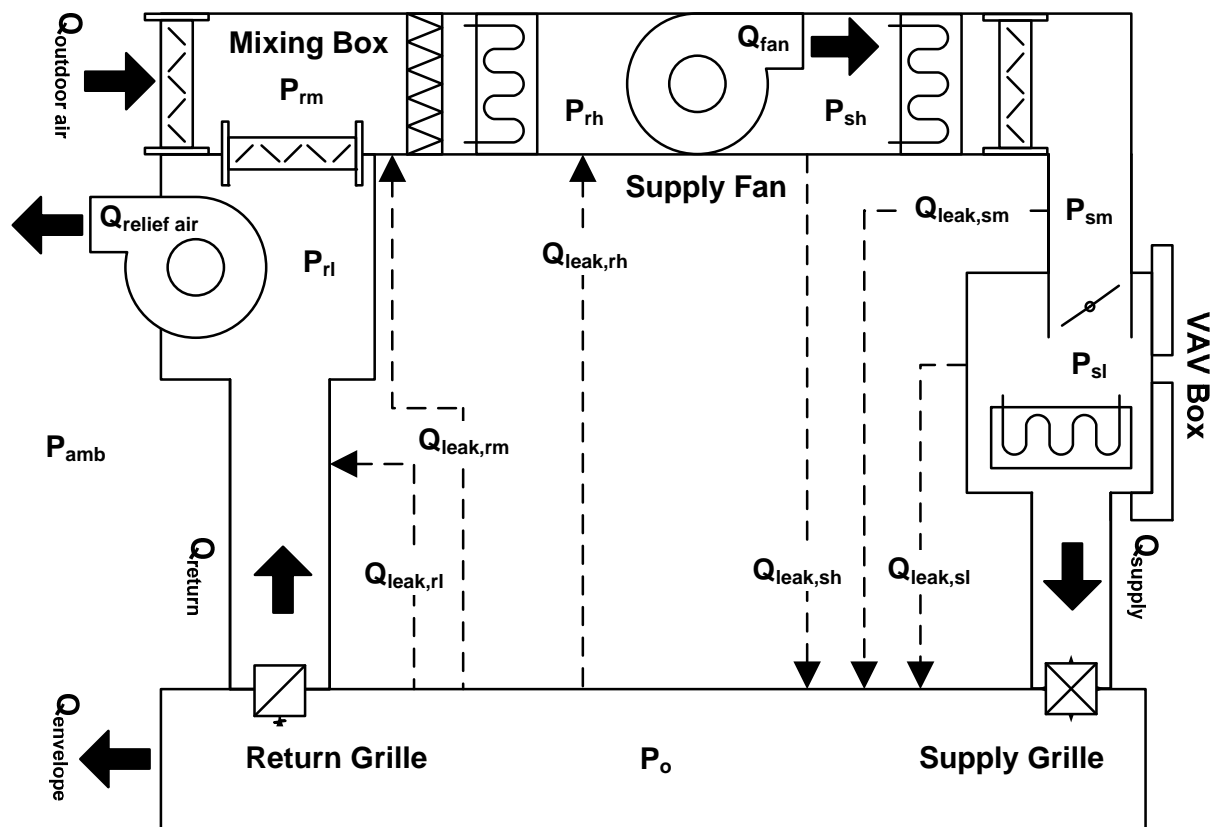


Figure 1. Prototypical Air-Handling System Topography

Our air-handling system is comprised of six sections: a low pressure return, a medium pressure return, a high pressure return, a high pressure supply, a medium pressure supply, and a low pressure supply. We assume that each of these sections and the conditioned space served by the system have uniform (but not necessarily the same) total pressure. We also assume that there are system components that cause pressure changes between each section, between the system and the conditioned space, and between the conditioned space and outdoors, as follows:

- Outdoors (at P_{amb})

- Conditioned space envelope leak with pressure difference ($P_o - P_{amb}$)
- Conditioned space (at P_o)
- Return components with pressure difference ($P_o - P_{rl}$)
- Low pressure return section (at P_{rl})
- Return components with pressure difference ($P_{rm} - P_{rl}$)
- Medium pressure return section (at P_{rm})
- Return components with pressure difference ($P_{rh} - P_{rm}$)
- High pressure return section (at P_{rh})
- Fan with pressure rise ($P_{sh} - P_{rh}$)
- High pressure supply section (at P_{sh})
- Supply components with pressure difference ($P_{sh} - P_{sm}$)
- Medium pressure supply section (at P_{sm})
- Supply components with pressure difference ($P_{sm} - P_{sl}$)
- Low pressure supply section (at P_{sl})
- Supply components with pressure difference ($P_{sl} - P_o$)

For an airtight system (no duct leaks and no outdoor or relief airflows), the fan pressure rise that is needed to move air from the conditioned space through the system and back to the conditioned space is equal to the sum of the total pressure differences across components upstream and downstream of the fan:

$$1 \quad \Delta P_{fan} = [(P_o - P_{rl}) + (P_{rl} - P_{rm}) + (P_{rm} - P_{rh})] + [(P_{sh} - P_{sm}) + (P_{sm} - P_{sl}) + (P_{sl} - P_o)]$$

where P_o is the static pressure in the conditioned space and the spaces surrounding the air-handling system. All pressures are gage pressure. Using P_o as a reference, supply section pressures are assumed to be positive; return section pressures are assumed to be negative.

For each of the six pressure differences in Equation 1, the following presents a simplified physical approximation by equating the pressure difference across each component to the volumetric flow through the component. All flows are referenced to a standard temperature and pressure condition (e.g., 20°C and 101,325 Pa).

The first term in Equation 1 is the pressure difference between the conditioned space and the low-pressure return section (e.g., return ducts and relief air plenum). The pressure change is due to losses through components such as return grilles and duct fittings as well as frictional losses. We assume that all of these components are orifice-like such that the pressure difference is proportional to the square of the return flow going through these components (Sherman 1992, Walker et al. 1997):

$$2 \quad (P_o - P_{rl}) = a_{rl} Q_{return}^2$$

The second term in Equation 1 is the pressure difference between the relief air plenum and the medium-pressure return section (i.e., air-handler mixing box). The pressure change is due to losses through components such as return air dampers and duct fittings as well as frictional losses. Similar to Equation 2, we assume that all of these components are orifice-like:

$$3 \quad (P_{rl} - P_{rm}) = a_{rm} Q_{return}^2$$

The third term in Equation 1 is the pressure difference between the air-handler mixing box and the high pressure return section (i.e., fan inlet). The pressure change is due to components immediately upstream of the fan such as filters, coils, bypass dampers, and fittings as well as frictional losses. We assume that some components are orifice-like and some behave more like pipes dominated by viscous effects such that the pressure difference is proportional to the square of the return flow going through these components plus a term that is linear in flow (Sherman 1992, Walker et al. 1997):

$$4 \quad (P_{rm} - P_{rh}) = a_{rh} Q_{return}^2 + b_{rh} Q_{return}$$

The fourth term in Equation 1 is the pressure difference between the high pressure supply section (i.e., fan outlet) and medium pressure supply section (e.g., main supply duct). The pressure change is due to components immediately downstream of the fan such as coils, dampers, and duct fittings as well as frictional losses. Similar to Equation 4, we assume that the pressure difference is proportional to the square of the supply flow going through these components plus a term that is linear in flow:

$$5 \quad (P_{sh} - P_{sm}) = a_{sh} Q_{supply}^2 + b_{sh} Q_{supply}$$

The fifth term in Equation 1 is the pressure difference between the main supply duct and the low pressure supply section (i.e., inside the VAV boxes just downstream of their control dampers). The pressure change is primarily due to the control dampers in the VAV boxes, but also includes balancing dampers and duct fittings as well as frictional losses. Similar to Equation 2, we assume that all of these components are orifice-like:

$$6 \quad (P_{sm} - P_{sl}) = a_{sm} Q_{supply}^2$$

The sixth term in Equation 1 is the pressure difference between just downstream of the VAV box damper and the conditioned space. The pressure change is due to coils, duct fittings, and supply grilles, as well as frictional losses (especially for compressed flexible ducts near the grilles). Similar to Equation 4, we assume that the pressure difference is proportional to the square of the supply flow going through these components plus a term that is linear in flow:

$$7 \quad (P_{sl} - P_o) = a_{sl} Q_{supply}^2 + b_{sl} Q_{supply}$$

Substituting Equations 2 through 7 into Equation 1 results in the following expression for an airtight system:

$$8 \quad \Delta P_{fan} = a_r Q_{return}^2 + b_r Q_{return} + a_s Q_{supply}^2 + b_s Q_{supply}$$

where

$$9 \quad a_r = (a_{rl} + a_{rm} + a_{rh}); b_r = b_{rh}; a_s = (a_{sl} + a_{sm} + a_{sh}); b_s = (b_{sl} + b_{sh})$$

Duct Leakage and Ventilation/Pressurization Airflows

If the air-handling system has no duct leaks and no outdoor and relief airflows, then the pressure rise across the fan (Q_{fan}) is described by Equation 8. In general, however, we cannot necessarily assume that there are no leaks (Wray et al. 2005) or that there are no outdoor and relief airflows. To account for the effects of these airflows, we begin by defining the six return and supply section leakage flows plus the envelope leakage flow shown in Figure 1:

$$10 \quad Q_{leak,rl} = f_{rl} Q_{return}; Q_{leak,rm} = f_{rm} [(1 + f_{rl}) Q_{return} + \Delta Q_{oa}]; Q_{leak,rh} = \left[\frac{f_{rh}}{(1 + f_{rh})} \right] Q_{fan}$$

$$11 \quad Q_{leak,sl} = \left[\frac{f_{sl}}{(1 - f_{sl})} \right] Q_{supply}; Q_{leak,sm} = c_{sm} \sqrt{P_{sm} - P_o}; Q_{leak,sh} = f_{sh} Q_{fan}$$

$$12 \quad \Delta Q_{oa} = Q_{outdoor} - Q_{relief} = Q_{envelope} = c_{env} (P_o - P_{amb})^n$$

The terms f_{rl} , f_{rm} , and f_{rh} in Equation 10 and terms f_{sl} and f_{sh} in Equation 11 represent leakage that is proportional to the sum of system flows entering the section containing the leak. The square root term in Equation 11 represents leakage from the medium pressure supply section. It is important to recognize that when $(P_{sm} - P_o)$ is held constant, leakage from this section is not a constant fraction of the flow entering the section. The terms c_{env} and n in Equation 12 describe the flow coefficient and pressure exponent, respectively, for the conditioned space envelope leakage.

Equations 3 through 6 must be modified to account for duct leakage and outdoor and relief airflows. The pressure difference between the relief air plenum and the mixing box as described by Equation 3 becomes:

$$13 \quad (P_{rl} - P_{rm}) = a_{rm} (Q_{return} + Q_{leak,rl} - Q_{relief})^2$$

The pressure difference between the air-handler mixing box and the fan inlet as described by Equation 4 (assuming equal outdoor and relief airflows) becomes:

$$14 \quad (P_{rm} - P_{rh}) = a_{rh} (Q_{return} + Q_{leak,rl} + Q_{leak,rm} + \Delta Q_{oa})^2 + b_{rh} (Q_{return} + Q_{leak,rl} + Q_{leak,rm} + \Delta Q_{oa})$$

The pressure difference between the fan outlet and main supply duct as described by Equation 5 becomes:

$$15 \quad (P_{sh} - P_{sm}) = a_{sh} (Q_{supply} + Q_{leak,sl} + Q_{leak,sm})^2 + b_{sh} (Q_{supply} + Q_{leak,sl} + Q_{leak,sm})$$

The pressure difference between the main supply duct and inside the VAV boxes just downstream of their control dampers as described by Equation 6 becomes:

$$16 \quad (P_{sm} - P_{sl}) = a_{sm} (Q_{supply} + Q_{leak,sl})^2$$

The pressure differences described by Equations 2 and 7 remain the same.

Substituting Equations 2, 7 and 13 through 16 into Equation 1 with the definitions for leakage flows in Equations 10 and 11 results in the following expression for a leaky system with outdoor and relief airflows:

$$17 \quad \begin{aligned} \Delta P_{fan} = & A_r Q_{return}^2 + B_r Q_{return} + A_s Q_{supply}^2 + B_s Q_{supply} \\ & + a_{rm} (Q_{outdoor} - \Delta Q_{oa})^2 + a_{rh} (1 + f_{rm})^2 \Delta Q_{oa}^2 + b_{rh} (1 + f_{rm}) \Delta Q_{oa} \\ & + a_{sh} c_{sm}^2 (P_{sm} - P_o) + b_{sh} c_{sm} \sqrt{P_{sm} - P_o} \end{aligned}$$

where

$$18 \quad A_r = a_{rl} + a_{rm} (1 + f_{rl})^2 + a_{rh} (1 + f_{rl})^2 (1 + f_{rm})^2$$

$$19 \quad B_r = [b_{rh} + 2a_{rh} (1 + f_{rm}) \Delta Q_{oa}] (1 + f_{rl}) (1 + f_{rm}) - 2a_{rm} (1 + f_{rl}) (Q_{outdoor} - \Delta Q_{oa})$$

$$20 \quad A_s = \frac{(a_{sh} + a_{sm})}{(1 - f_{sl})^2} + a_{sl}$$

$$21 \quad B_s = \frac{(b_{sh} + 2a_{sh} c_{sm} \sqrt{P_{sm} - P_o})}{(1 - f_{sl})} + b_{sl}$$

Equation 17 would be more convenient if the flows were only in terms of Q_{fan} . To achieve this translation, we can relate Q_{fan} to the supply and return flows using the following two continuity equations:

$$22 \quad Q_{fan} = Q_{return} + Q_{leak,rl} + Q_{leak,rm} + Q_{leak,rh} + \Delta Q_{oa}$$

$$23 \quad Q_{fan} = Q_{supply} + Q_{leak,sl} + Q_{leak,sm} + Q_{leak,sh}$$

Substituting the leakage flow definitions from Equations 10 and 11 into Equations 22 and 23, respectively, and solving for Q_{return} and Q_{supply} :

$$24 \quad Q_{return} = f_r Q_{fan} - \frac{\Delta Q_{oa}}{(1 + f_{rl})}$$

where

$$25 \quad f_r = \left[\frac{1}{(1 + f_{rl})(1 + f_{rm})(1 + f_{rh})} \right]$$

$$26 \quad Q_{supply} = (1 - f_{sl}) \left[Q_{fan} (1 - f_{sh}) - c_{sm} \sqrt{P_{sm} - P_o} \right]$$

Squaring Equations 24 and 26:

$$27 \quad Q_{return}^2 = f_r^2 Q_{fan}^2 - 2f_r Q_{fan} \left(\frac{\Delta Q_{oa}}{(1+f_{rl})} \right) + \left(\frac{\Delta Q_{oa}^2}{(1+f_{rl})^2} \right)$$

$$28 \quad Q_{supply}^2 = (1-f_{sl})^2 \left[Q_{fan}^2 (1-f_{sh})^2 - 2Q_{fan} (1-f_{sh}) c_{sm} \sqrt{P_{sm} - P_o} + c_{sm}^2 (P_{sm} - P_o) \right]$$

We can also express the outdoor airflow entering the system as:

$$29 \quad Q_{outdoor} = f_{oa} Q_{fan} \text{ where } f_{oa} \text{ is the fraction of } Q_{fan} \text{ that is outdoor air.}$$

Combining Equations 12, 17, 24, and 26 through 29, we obtain a general expression for system curves:

$$30 \quad \Delta P_{fan} = Q_{fan}^2 \left[A'_{r1} + a_{rm} \left(\left(\frac{1}{(1+f_{rm})(1+f_{rh})} \right) - f_{oa} \right)^2 + A'_{s1} \right] \\ + Q_{fan} \left[B'_r + B'_{s1} + A'_{s2} c_{sm} \sqrt{P_{sm} - P_o} + A'_{r2} c_{env} (P_o - P_{amb})^n \right] \\ + A'_{r3} c_{env}^2 (P_o - P_{amb})^{2n} + B'_{s2} c_{sm} \sqrt{P_{sm} - P_o} + A'_{s3} c_{sm}^2 (P_{sm} - P_o)$$

where

$$31 \quad A'_{r1} = \left[\frac{a_{rl}}{(1+f_{rl})^2 (1+f_{rm})^2 (1+f_{rh})^2} \right] + \left[\frac{a_{rh}}{(1+f_{rh})^2} \right]$$

$$32 \quad A'_{r2} = 2 \left[\left(\frac{a_{rh} (1+f_{rm})}{(1+f_{rh})} \right) - A'_{r1} (1+f_{rm})(1+f_{rh}) \right]$$

$$33 \quad A'_{r3} = \frac{a_{rl}}{(1+f_{rl})^2}$$

$$34 \quad B'_r = \frac{b_{rh}}{(1+f_{rh})}$$

$$35 \quad A'_{s1} = (a_{sh} + A'_{s3})(1-f_{sh})^2$$

$$36 \quad A'_{s2} = -2A'_{s3}(1-f_{sh})$$

$$37 \quad A'_{s3} = a_{sm} + a_{sl}(1-f_{sl})^2$$

$$38 \quad B'_{s1} = (1-f_{sh})(b_{sh} - B'_{s2})$$

$$39 \quad B'_{s2} = -b_{sl}(1-f_{sl})$$

If we determined all of the coefficients in Equation 30 from a detailed air-handling system design analysis, the model would be complete. In reality, however, we rarely know most of these coefficients and instead need to make some assumptions or diagnostic measurements.

Note that the terms with $(P_o - P_{amb})$ and $(P_{sm} - P_o)$ in Equation 30 depend implicitly on Q_{fan} and ΔQ_{oa} . If Q_{fan} and ΔQ_{oa} are zero, then these terms are also zero. Otherwise, if these pressure differences are maintained at fixed non-zero values (e.g., building and duct static pressure control), Q_{fan} and ΔQ_{oa} need to be greater than or equal to some corresponding minimum values, respectively.

TYPICAL SYSTEM IN A LARGE COMMERCIAL BUILDING

Fixed-Position Supply Dampers

We can simplify Equation 30 by making several assumptions appropriate to typical air-handling systems in large commercial buildings.

Supply Sections

- In general, pressure losses in a typical supply system are dominated by losses through supply control dampers and duct fittings. Therefore, we assume that the effect of “linear” resistance components (e.g., coils and filters, if any) downstream of the fan is small such that b_{sl} and $b_{sh}=0$.
- Most high pressure supply sections (e.g., fan outlet) are quite short compared to the rest of the supply system (Fisk et al. 2000) and are likely tighter per unit surface area compared to the rest of the system (on average, supply ducts upstream of the VAV box damper are about 3 times tighter compared to the downstream ducts, Wray et al. 2005). Also, the ratio of leakage pressure differences between this section and the rest of the system may be only about 2 (medium pressure section) to 10 (low pressure section). Accordingly, we ignore the leakage out of this section and set $f_{sh}=0$.
- For a typical system, we can set $f_{sl}=0$ and add the effects of this low pressure duct leakage flow to the supply grille pressure coefficient a_{sl} because, to the fan, supply leakage of this type “looks” like a supply grille. If one attempts to measure total supply flow by adding all the known grille flows, it might be important to consider f_{sl} separately, but for now we do not.

Similar to the leakage from the medium pressure supply ducts, the low pressure supply duct leakage can be represented by:

$$40 \quad Q_{leak,sl} = c_{sl} \sqrt{P_{sl} - P_o}$$

where c_{sl} is the flow coefficient for the low pressure supply duct leakage.

Therefore, the combined supply grille flow and leakage flow is:

$$41 \quad Q_{supply} + Q_{leak,sl} = \frac{1}{\sqrt{a'_{sl}}} \sqrt{P_{sl} - P_o}$$

where

$$42 \quad a'_{sl} = \frac{1}{\left(\sqrt{\frac{1}{a_{sl}}} + c_{sl}\right)^2}$$

Return Sections

- Most medium and high pressure return sections (e.g., mixing box and fan inlet) are short compared to the rest of the return system. Thus, like the high pressure supply section, we ignore leakage into these sections and set $f_{rm}=0$ and $f_{rl}=0$.
- For a typical system, we can set $f_{rl}=0$ and add the effects of this low pressure duct leakage flow to the return grille pressure coefficient a_{rl} because, to the fan, return leakage of this type “looks” like a return grille. If one attempts to measure total return flow by adding all the known grille flows, it might be important to consider f_{rl} separately, but for now we do not.

Similar to the leakage from the low pressure supply ducts, the low pressure return duct leakage can be represented by:

$$43 \quad Q_{leak,rl} = c_{rl} \sqrt{P_o - P_{rl}}$$

where c_{rl} is the flow coefficient for the low pressure supply duct leakage.

Therefore, the combined return grille flow and leakage flow is:

$$44 \quad Q_{return} + Q_{leak,rl} = \frac{1}{\sqrt{a'_{rl}}} \sqrt{P_o - P_{rl}}$$

where

$$45 \quad a'_{rl} = \frac{1}{\left(\sqrt{\frac{1}{a_{rl}}} + c_{rl}\right)^2}$$

- We assume that the envelope leakage is small enough that the difference between the outdoor and relief airflows needed to maintain a small envelope pressure difference (e.g., 5 to 10 Pa) is a small fraction of the fan flow. Consequently, we set the terms with $\Delta Q_{oa} = c_{env} (P_{amb} - P_o)^n = 0$. For buildings with leaky envelopes, this term might be important.

Applying these assumptions to Equations 30 through 39, our model can be applied to systems with *fixed-position* supply dampers, such as a constant-air-volume system:

$$46 \quad \Delta P_{fan} = Q_{fan}^2 \left[a'_r + a_{rm} (1 - f_{oa})^2 + a'_s \right] + b_r Q_{fan} - 2Q_{fan} (a_{sm} + a'_{sl}) c_{sm} \sqrt{P_{sm} - P_o} + (a_{sm} + a'_{sl}) c_{sm}^2 (P_{sm} - P_o)$$

where, in this case, $a'_s = (a_{sh} + a_{sm} + a'_{sl})$ and $a'_r = (a_{rh} + a'_{rl})$

There are four terms in Equation 46. The first one looks like the industry “standard” system curve in which the fan pressure is proportional to the square of the fan flow, but we have given it a functional form that includes physical pressure difference parameters and an outdoor air fraction. The second term accounts for pressure drop components (specifically in the return) with “linear” behavior such as coils and filters. The third and fourth terms account for supply duct leakage. They are both a function of the duct static pressure difference that is often used as a control pressure.

In principle, all of the coefficients can be found by knowing the details of the entire system and then calculating the individual pressure drop coefficients. In practice, however, the details of real buildings will not be known sufficiently.

Practically speaking, Equation 46 may be more useful when fitting field data. One can make measurements at different combinations of fan flows, outside air fractions, and static pressures and then use non-linear techniques to fit the data and determine the coefficients. If we can measure fully at different outside air settings, damper positions, and fan speeds, then we can in fact regress to find all of the parameters in this equation.

One may not have the full ability to vary all the variables independently. For example, consider the case in which one cannot significantly vary the outside air fraction (or there is no outside air). Then, Equation 46 becomes:

$$47 \quad \Delta P_{fan} = Q_{fan}^2 [a''_r + a'_s] + b_r Q_{fan} - 2Q_{fan} (a_{sm} + a'_{sl}) c_{sm} \sqrt{P_{sm} - P_o} + (a_{sm} + a'_{sl}) c_{sm}^2 (P_{sm} - P_o)$$

where $a''_r = a'_r + a_{rm} (1 - f_{oa})^2$ with fixed f_{oa}

Note that simply fitting Equation 47 to measured flows and pressure rises would not provide sufficient information to uniquely determine all of the parameters in Equation 47. For example, we can determine the sum of return and supply pressure loss coefficients, but not each individually. This does not impact our ability to use the equation to model the air-handling system, but it does limit our ability to make physical interpretations. Consequently, we would instead fit the measured flows and pressure rises to the following equation:

$$48 \quad \Delta P_{fan} = \alpha Q_{fan}^2 + \beta Q_{fan} + \gamma Q_{fan} \sqrt{P_{sm} - P_o} + \delta (P_{sm} - P_o)$$

where

$$49 \quad \alpha = a'_r + a_{rm} (1 - f_{oa})^2 + a'_s \text{ with fixed } f_{oa}$$

The coefficient α might dominate the pressure drop and thus be the only significant one.

$$50 \quad \beta = b_r$$

The coefficient β will be zero if there are no significant linear resistance elements.

$$51 \quad \gamma = -2(a_{sm} + a'_{sl}) c_{sm}$$

The coefficient γ will be zero if there is no duct leakage and will be negative otherwise.

$$52 \quad \delta = (a_{sm} + a'_{sl})c_{sm}^2$$

The dimensionless coefficient δ will be zero if there is no duct leakage and will be positive otherwise.

Variable-Position Supply Dampers

Our equations so far assume that damper positions never change or are changed independently of static pressure or fan flow. In some systems, however, damper positions are not completely independent of the supply flow and supply pressures. For example, in most VAV systems, the VAV box damper positions change continuously to adjust the supply flows to meet space loads. To account for this behavior, we can rewrite Equation 23 to account for the operation of the control dampers (still assuming the supply leakage parameter f_{sh} is zero):

$$53 \quad Q_{fan} = Q_{sd} + Q_{leak,sm} = f_d c_d \sqrt{P_{sm} - P_o} + c_{sm} \sqrt{P_{sm} - P_o}$$

where

$$54 \quad c_d = 1 / \sqrt{a_{sm} + a'_{sl}}$$

The first term in Equation 53 represents the total supply flow through all of the VAV box dampers (Q_{sd}) and the second represents leakage from the medium pressure supply section. c_d represents the combined flow coefficient for the medium- and low-pressure supply duct sections including the VAV box dampers, when all of the dampers are at their coincident widest opening (for a VAV system, during normal operation, most dampers coincidentally are less than fully open). f_d is the net flow fraction through all dampers combined (0=all closed, 1=maximum coincident opening). Note that, when the fan is operating, f_d typically has a minimum value greater than zero (such as 0.3) to maintain ventilation air supply to the conditioned space (and to avoid over pressurizing the supply ducts and stalling the fan).

The pressure difference ($P_{sm} - P_o$) in Equation 53 can be expressed as the following sum:

$$55 \quad (P_{sm} - P_o) = (P_{sm} - P_{sl}) + (P_{sl} - P_o) = a_{sm} Q_{sd}^2 + a'_{sl} Q_{sd}^2$$

Equation 55 can be applied to two damper modulation cases (one without f_d and one with) as follows:

$$56 \quad (P_{sm} - P_o) = a_{sm} Q_{sd}^2 + a'_{sl} Q_{sd}^2$$

$$57 \quad (P_{sm} - P_o) = a_{sm,fd} (f_d Q_{sd})^2 + a'_{sl} (f_d Q_{sd})^2$$

Using Equations 56 and 57, the ratio between the case with f_d to the one without ($f_d = 1$) is:

$$58 \quad \frac{a_{sm,fd}}{a_{sm}} = \frac{(P_{sm} - P_o) - f_d^2 a'_{sl} Q_{sd}^2}{f_d^2 [(P_{sm} - P_o) - a'_{sl} Q_{sd}^2]}$$

Substituting the definition for Q_{sd} from Equation 53 into Equation 58 and simplifying:

$$59 \quad \frac{a_{sm,fd}}{a_{sm}} = \left(\frac{1}{f_d}\right)^2 + \frac{a'_{sl}}{a_{sm}} \left[\left(\frac{1}{f_d}\right)^2 - 1 \right]$$

Equation 59 indicates that the pressure coefficient a_{sm} when all of the dampers are at their coincident widest opening can be modified by a factor involving $(1/f_d)^2$ to account for damper operation.

We can now modify Equation 46 to account for damper operation:

$$60 \quad \Delta P_{fan} = Q_{fan}^2 \left[a'_r + a_{rm} (1 - f_{oa})^2 + a''_s \right] + b_r Q_{fan} \\ - 2Q_{fan} \left[a''_s c_{sm} \sqrt{P_{sm} - P_o} \right] + a'''_s c_{sm}^2 (P_{sm} - P_o)$$

where

$$61 \quad a''_s = a_{sh} + a_{sm} \left\{ \left(\frac{1}{f_d}\right)^2 + \frac{a'_{sl}}{a_{sm}} \left[\left(\frac{1}{f_d}\right)^2 - 1 \right] \right\} + a'_{sl} = a_{sh} + (a_{sm} + a'_{sl}) \left(\frac{1}{f_d}\right)^2$$

$$62 \quad a'''_s = (a_{sm} + a'_{sl}) \left(\frac{1}{f_d}\right)^2$$

It would be useful to translate the net flow fraction (f_d) that is implicitly included in Equation 60 to something more useful, because this fraction cannot be easily measured. To accomplish this translation, we rearrange Equation 53 and use it to define an expression involving the pressure coefficient modifier in Equations 61 and 62:

$$63 \quad \left(\frac{1}{f_d}\right) = \frac{c_d \sqrt{P_{sm} - P_o}}{Q_{fan} - c_{sm} \sqrt{P_{sm} - P_o}} = \left(\frac{c_d \sqrt{P_{sm} - P_o}}{Q_{fan}}\right) \left[1 / \left(1 - \frac{c_{sm} \sqrt{P_{sm} - P_o}}{Q_{fan}}\right) \right]$$

The rightmost term of Equation 63 has the form $1/(1-x)$, which can be represented by a geometric series determined using a Maclaurin series expansion with $|x| < 1$:

$$64 \quad 1/(1-x) = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \dots$$

Assuming that the net flow fraction through all dampers combined is 0.3 or larger and the duct leakage fraction f_{sm} is 0.1 or less when all of the dampers are at their coincident widest opening, we can use the first three terms of the series expansion in Equation 64 to approximate Equation 63:

$$65 \quad \left(\frac{1}{f_d}\right) \approx \left(\frac{c_d \sqrt{P_{sm} - P_o}}{Q_{fan}}\right) \left(1 + \frac{c_{sm} \sqrt{P_{sm} - P_o}}{Q_{fan}} + \frac{c_{sm}^2 (P_{sm} - P_o)}{Q_{fan}^2}\right)$$

Having smaller net flow fractions or leakier ducts might require using more terms of the series expansion to maintain the same accuracy.

Rearranging Equation 65 and substituting into Equations 61 and 62 to translate the damper operation variable:

$$66 \quad a_s'' = a_{sh} + (a_{sm} + a'_{sl}) \left(\frac{c_d \sqrt{P_{sm} - P_o}}{Q_{fan}} + \frac{c_d c_{sm} (P_{sm} - P_o)}{Q_{fan}^2} + \frac{c_d c_{sm}^2 (P_{sm} - P_o)^{1.5}}{Q_{fan}^3} \right)^2$$

$$67 \quad a_s''' = (a_{sm} + a'_{sl}) \left(\frac{c_d \sqrt{P_{sm} - P_o}}{Q_{fan}} + \frac{c_d c_{sm} (P_{sm} - P_o)}{Q_{fan}^2} + \frac{c_d c_{sm}^2 (P_{sm} - P_o)^{1.5}}{Q_{fan}^3} \right)^2$$

Substituting Equations 66 and 67 into Equation 60 and grouping like terms:

$$68 \quad \Delta P_{fan} = Q_{fan}^2 \left[a'_r + a_{rm} (1 - f_{oa})^2 + a_{sh} \right] + b_r Q_{fan} + (a_{sm} + a'_{sl}) c_d^2 (P_{sm} - P_o) + \frac{1}{Q_{fan}^3} \left[-2(a_{sm} + a'_{sl}) c_d^2 c_{sm}^3 (P_{sm} - P_o)^{2.5} \right] + \frac{1}{Q_{fan}^6} \left[(a_{sm} + a'_{sl}) c_d^2 c_{sm}^6 (P_{sm} - P_o)^4 \right]$$

Assuming that the last two terms are very small compared to ΔP_{fan} , Equation 68 becomes:

$$69 \quad \Delta P_{fan} = Q_{fan}^2 \left[a'_r + a_{rm} (1 - f_{oa})^2 + a_{sh} \right] + b_r Q_{fan} + (a_{sm} + a'_{sl}) c_d^2 (P_{sm} - P_o)$$

Equation 69 represents a system curve that depends in part on VAV box damper operation. The field advantage to this form is that it can be used in the typical case where dampers are being controlled to meet a varying space conditioning load.

Thus, without having to assume fixed positions for supply dampers, the form of the equation to fit measured data remains:

$$70 \quad \Delta P_{fan} = \alpha Q_{fan}^2 + \beta Q_{fan} + \gamma Q_{fan} \sqrt{P_{sm} - P_o} + \delta (P_{sm} - P_o)$$

where

$$71 \quad \alpha = a'_r + a_{rm} (1 - f_{oa})^2 + a_{sh}$$

The coefficient α still might be the dominant one, but will be smaller than in the fixed-position damper case, because it does not include the pressure coefficients for the low and medium pressure supply sections.

$$72 \quad \beta = b_r$$

The coefficient β is the same as in the fixed-position damper case and will be zero when there are no significant linear pressure drop components.

$$73 \quad \gamma = 0$$

The coefficient γ is zero (different compared to the fixed-position damper case).

$$74 \quad \delta = (a_{sm} + a'_{sl})c_d^2 = (a_{sm} + a'_{sl}) \left[\frac{1}{(a_{sm} + a'_{sl})} \right] = 1$$

The coefficient δ is one (also different compared to the fixed-position damper case), which means that changes to the pressure difference ($P_{sm} - P_o$) directly affect the fan pressure rise.

DISCUSSION

Our development has shown that a simplified physical model of a typical commercial duct system with a fixed outdoor air fraction results in a system curve of the following form:

$$75 \quad \Delta P_{fan} = \alpha Q_{fan}^2 + \beta Q_{fan} + \gamma Q_{fan} \sqrt{P_{duct}} + \delta P_{duct}$$

where $P_{duct} \equiv (P_{sm} - P_o)$

The following discusses the meaning of the four terms in Equation 75. The first term, which depends on the square of the fan flow, is the principal component of most system curves. Very often it is the only one considered, but that would only be correct with fixed-position dampers, no duct leakage, and no linear resistance components. Our physical derivation indicates that the coefficient for this term depends in part on the outdoor air fraction. The influence of this fraction on the total pressure rise depends on its magnitude relative to other resistances in the system.

The second term accounts for significant flow resistances in the system where the pressure difference is linearly proportional to the flow. Some filters and coils in the return may need this term to be adequately described. If there are no linear components or if the fan flow is maintained in a narrow range, this term could be zero or small compared to the other terms.

The third term, which depends on the fan flow and the square root of the supply duct pressure P_{sm} , accounts in part for duct leakage in the supply system when damper positions are *fixed or are changed independently of static pressure or fan flow*. In this case, reducing or eliminating supply duct leakage results in a different system curve. This, however, might be only a minor “correction” to the simple system curves generally used. Note that the parameter P_{sm} in Equation 75 can be thought of as either total or static pressure. In the latter case, the dynamic pressure component of P_{sm} is simply included as part of the first term.

Interestingly, the third term is zero when the VAV box dampers are modulated to control flow. Consequently, with *variable-position* supply dampers, reducing or eliminating supply duct leakage does not change the system curve. This behavior, however, does not mean that supply duct leakage has no effect. For example, at a given space conditioning load, reducing supply duct leakage will tend to increase supply duct static pressures (P_{sm} and P_{sl}) for the same fan flow. To maintain P_{sm} at its set point, the flow through the fan will decrease. Because the pressure difference ($P_{sm} - P_{sl}$) has decreased, the VAV box dampers will open somewhat to maintain the same flow (Q_{supply}) to the conditioned space. The net result is a lower operating point on the same system curve, which means that the fan pressure rise and fan air power (product of fan pressure rise and flow) will also decrease.

The last term also accounts in part for duct leakage in the supply system when damper positions are *fixed or are changed independently of static pressure or fan flow*. This term indicates

that the same fan pressure rise can be achieved by raising the duct pressure and closing dampers. The only change in the system in such a case is that the duct leakage flow may increase. The coefficient for this term is equal to one when the VAV box dampers are modulated to control flow. In both cases, this term may be the most important “correction” to the simple system curves generally used, especially at low flows.

Non-Dimensionalizing the System Curve

We can recast the system curve with fixed- or variable-position supply dampers using dimensionless parameters derived from fan laws and hydrodynamics. Non-dimensionality allows us to scale the system curve for different size systems. Many fan efficiency and speed curves also can be expressed in terms of this dimensionless pressure.

The key dimensionless variable is pressure non-dimensionalized by the flow through the fan, air density ρ at the fan inlet, and fan wheel diameter D_{fan} :

$$76 \quad P' \equiv \frac{D_{fan}^4 \Delta P_{fan}}{\rho Q_{fan}^2}$$

We can also define a Reynolds Number as:

$$77 \quad Re \equiv \frac{\rho Q_{fan}}{\mu D_{fan}}$$

where the air density ρ and viscosity μ are determined at standard conditions (as is often the case for fan manufacturers). Non-dimensionalizing Equation 46 using Equations 76 and 77 for the fixed position damper case results in:

$$78 \quad P'_{fan} = \frac{D_{fan}^4}{\rho} \left[a'_r + a_{rm} (1 - f_{oa})^2 + a'_s \right] + \frac{D_{fan}^3}{\mu Re} (b_r) \\ + \frac{D_{fan}^2}{\sqrt{\rho}} \left[-2(a_{sm} + a'_{sl})c_{sm} \right] \sqrt{P'_{duct}} + \left[(a_{sm} + a'_{sl})c_{sm}^2 \right] P'_{duct}$$

Or equivalently from a more parametric view:

$$79 \quad P'_{fan} = \alpha' + \beta' / Re + \gamma' \sqrt{P'_{duct}} + \delta' P'_{duct}$$

In the case of *fixed-position* dampers:

$$80 \quad \alpha' = \frac{D_{fan}^4}{\rho} \left(a'_r + a_{rm} (1 - f_{oa})^2 + a'_s \right)$$

$$81 \quad \beta' = \frac{D_{fan}^3}{\mu} (b_r)$$

$$82 \quad \gamma' = \frac{D_{fan}^2}{\sqrt{\rho}} \left[-2(a_{sm} + a'_{sl})c_{sm} \right]$$

$$83 \quad \delta' = (a_{sm} + a'_{st}) c_{sm}^2$$

In the case of *variable-position* dampers:

$$84 \quad \alpha' = \frac{D_{fan}^4}{\rho} (a'_r + a_{rm} (1 - f_{oa})^2 + a_{sh})$$

$$85 \quad \beta' = \frac{D_{fan}^3}{\mu} (b_r)$$

$$86 \quad \gamma' = 0$$

$$87 \quad \delta' = 1$$

The Reynolds number enters this problem because of linear flow resistance elements and also can account for developing flow. Otherwise, the dimensionless system curve is only a function of dimensionless pressures.

CONCLUSIONS

As a step toward providing a suitable model for fan pressure rise in mainstream building energy simulation tools, this paper has shown that a simplified four-parameter physical model can be used to define system curves for a prototypical air-handling system in a large commercial building. The first term depends on the square of the fan flow. Very often it is the only one considered, but that would only be correct with fixed-position dampers, no duct leakage, and no linear resistance components. Our physical derivation indicates that the coefficient for this term depends in part on the outdoor air fraction. The influence of this fraction on the total pressure rise depends on its magnitude relative to other resistances in the system. The second term accounts for linear-like flow resistances such as filters and coils. The last two terms account for duct leakage in the supply system when damper positions are *fixed or are changed independently of static pressure or fan flow*. In this case, reducing or increasing the amount of duct leakage results in a different system curve. The coefficients for these two terms are zero if there is no duct leakage. The last term may be the most important “correction” to the simple system curves generally used, especially at low flows.

Interestingly, the third term is zero and the coefficient in the last term is one when the VAV box dampers are modulated to control flow. For these systems, damper positions are not completely independent of the supply flow and supply pressures. Consequently, with *variable-position* supply dampers, reducing or eliminating supply duct leakage does not change the system curve. This behavior, however, does not mean that supply duct leakage has no effect. Reduced duct leakage results in a lower operating point on the same system curve, which means that the fan pressure rise and fan air power (product of fan pressure rise and flow) will also decrease.

Practically speaking, the parametric system curve may be most useful when fitting field data. In principle, all of the coefficients can be found by knowing the details of the entire system and then calculating the individual pressure drop coefficients. In practice, however, the details of real buildings will not be known sufficiently. One can make measurements at different combinations of fan flows, outside air fractions, and static pressures and then use

non-linear techniques to fit the data and determine the coefficients. If we can measure fully at different outside air settings, damper positions, and fan speeds, then we can in fact regress to find all of the parameters in the equation.

NOMENCLATURE

a, A	Pressure loss coefficient [Pa-s ² /m ⁶]
b, B	Linear pressure-loss coefficient {Pa-s/m ³ }
c	Duct leakage coefficient [m ³ /(s-Pa ^{1/2})]
D	Fan wheel outer diameter [m]
f	Dimensionless duct leakage coefficient
f_{oa}	Dimensionless outside air fraction
f_d	Dimensionless net flow fraction through supply dampers
P	(Static) gage pressure [Pa]
Q	(Volumetric) airflow [m ³ /s]
Re	Reynolds number
$\alpha, \beta, \gamma, \delta$	System characterization coefficients
Δ	Difference
ρ	Air density [kg/m ³]
μ	Air viscosity [Pa s]

Subscripts:

amb	Outdoors (ambient)
env	Building envelope
fan	Properties of the fan in the system
h, m, l	High, medium, or low pressure side of system respectively
n	Pressure exponent
o	“Outside” the system (e.g., conditioned space)
$outdoor, oa; relief$	Outdoor or relief air respectively
$r, return; s, supply$	Return or supply side of system respectively
sd	Supply damper

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