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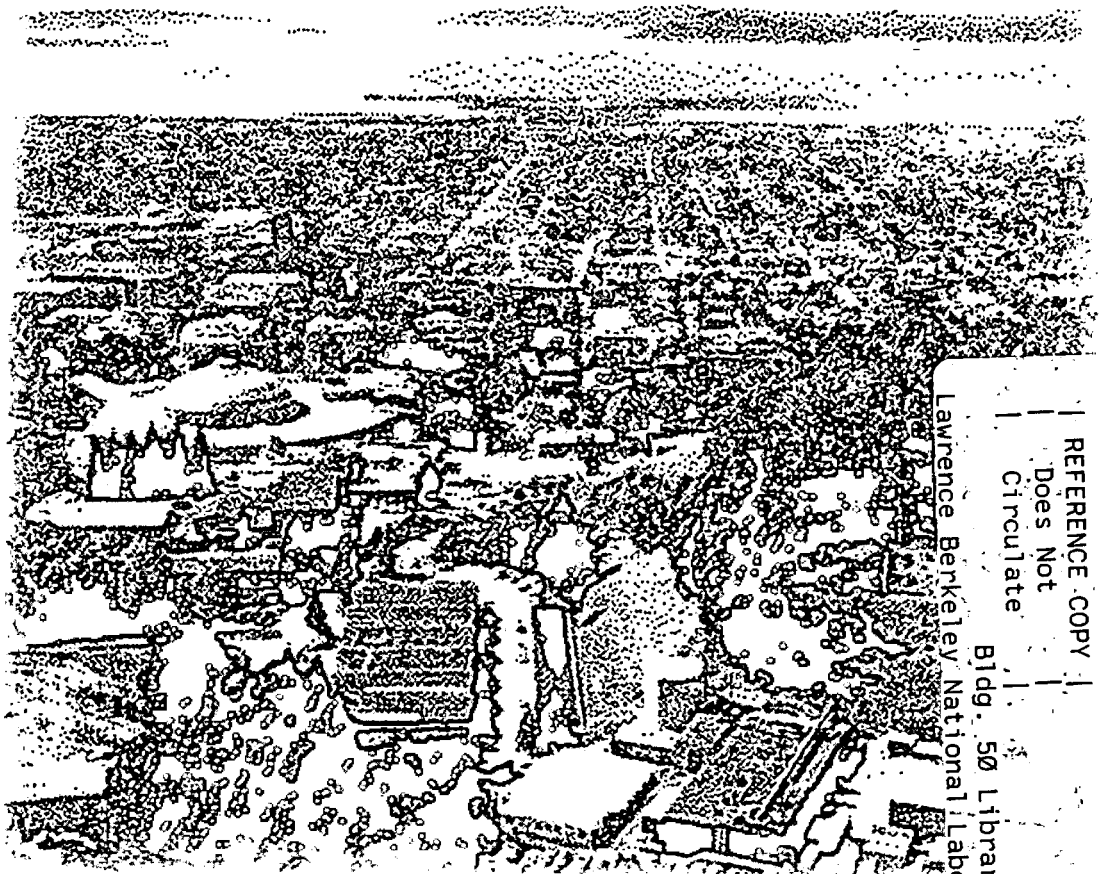
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K.S. Babu and Christopher Kolda

**Physics Division**

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Unification in Higgs Decays**

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# Signatures of Supersymmetry and Yukawa Unification in Higgs Decays

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## Abstract

We show that the branching ratio  $R_{b/\tau} \equiv BR(h^0 \rightarrow \bar{b}b)/BR(h^0 \rightarrow \tau^+\tau^-)$  of the Higgs boson  $h^0$  may usefully differentiate between the Higgs sectors of the Minimal Supersymmetric Standard Model (MSSM) and non-supersymmetric models such as the Standard Model or its two Higgs doublet extensions. Although at tree level  $R_{b/\tau}$  is the same in all these models, only in the MSSM can it receive a large radiative correction, for moderate to large values of the parameter  $\tan\beta$ . Such large corrections are motivated in supersymmetric unified schemes wherein the Yukawa couplings of the  $b$ -quark and the  $\tau$ -lepton are equal at the unification scale; otherwise the  $b$ -quark mass prediction is too large by 15–30% for most of parameter space. Thus accurate measurements of the Higgs branching ratios can probe physics at the unification scale. The branching ratio of  $h^0$  into charm quarks, as well as of the other Higgs bosons ( $H^0, A^0$ ) into  $\bar{b}b, \tau^+\tau^-, \bar{c}c$  can provide additional information about the supersymmetric nature of the Higgs sector.

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# 1 Introduction

The symmetry breaking sector of the Standard Model (SM) is still being vigorously pursued. The Minimal Supersymmetric Standard Model (MSSM) is perhaps the most widely anticipated, for it can explain naturally why the Higgs boson remains light; it is also compatible with the unification of the three gauge couplings.

The Higgs sector of the MSSM is a special case of the more general two-Higgs doublet Standard Model (2HDSM) with three characteristic features [1]. First, as in the generic 2HDSM, the mass of the lightest CP-even Higgs is controlled by dimensionless couplings; however, the dimensionless couplings which appear in the MSSM Higgs potential are simply the  $SU(2) \times U(1)$  gauge couplings. Thus one derives the remarkable (tree-level) result  $m_h < m_Z$ . Second, the Higgs potential of the MSSM can be made real by appropriate field redefinitions and gauge transformations so that, again at tree level, there is no CP violation arising from the Higgs sector itself.

Finally, the MSSM is a so-called “type II” model wherein one Higgs doublet (denoted  $H_u$ ) couples to the  $u$ -quarks, while the second Higgs doublet ( $H_d$ ) couples to the  $d$ -quarks and charged leptons. Such a division of the fields is a requirement in supersymmetric (SUSY) models imposed by holomorphy and anomaly cancellation. But there are other incidental benefits. For example, the large flavor-changing neutral currents endemic to the general 2HDSM are avoided. There is one other side-effect as well: the ratios of branching ratios for a Higgs boson decaying into quarks and leptons in the same class should match the ratio calculated in the SM. This last result implies, for example, that  $R_{b/\tau} \equiv BR(h^0 \rightarrow \bar{b}b)/BR(h^0 \rightarrow \tau^+\tau^-)$  is the same in MSSM as in the SM, roughly  $3m_b^2/m_\tau^2$ . Likewise for the ratio of  $t$ -quarks to  $c$ -quarks, or  $b$ -quarks to  $s$ -quarks, and so on. The invariance of these double ratios has long been known to be a distinguishing feature of any type II 2HDSM, and the MSSM in particular.

All three of the above results can receive significant alterations due to radiative corrections. The mass of the lightest Higgs receives substantial corrections from the heavy  $t$ -quark, increasing its upper bound to roughly 130 GeV [2]. CP violation can also enter the Higgs couplings through spontaneous CP violation in the one-loop effective potential (this turns out to be typically very small [3]) or finite corrections to the Higgs-matter couplings [4]. In this paper we will show that such finite corrections may also significantly shift the ratios of branching ratios, such as  $R_{b/\tau}$ , in interesting regions of SUSY parameter space. Such shifts will *not* occur in the SM or in the non-SUSY 2HDSM and so can serve as an indicator of SUSY. The double ratio  $R_{b/c}$ , of the Higgs branching ratios into  $\bar{b}b$  versus  $\bar{c}c$ , can provide additional information, as can the branching ratios of the other two neutral Higgs bosons. We will also explain how such shifts may provide a new experimental handle on models of grand unification.

It is quite conceivable that a light Higgs boson will be discovered before any supersymmetric particles. If the branching ratio  $R_{b/\tau}$  of the Higgs is measured to be significantly different from the SM prediction, our results suggest, it would be a strong indication of the supersymmetric nature of the Higgs sector.

## 2 The MSSM Higgs sector at tree level

The coupling of the Higgs multiplets to the usual SM fermions is described in a SUSY model via the superpotential:

$$W = y_u Q u^c H_u + y_d Q d^c H_d + y_e L e^c H_d + \mu H_u H_d \quad (1)$$

where the  $y_i$  are the Yukawa couplings ( $3 \times 3$  matrices in generation space), and  $\mu$  is a SUSY-invariant mass parameter which mixes the two Higgs doublets. After electroweak symmetry breaking, the fermions get masses at tree-level of, for example:

$$m_t = y_t v \sin \beta, \quad m_{b,\tau} = y_{b,\tau} v \cos \beta \quad (2)$$

where  $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$  and  $v^2 = \langle H_u \rangle^2 + \langle H_d \rangle^2 \simeq (174 \text{ GeV})^2$ . Perturbativity usually constrains  $\tan \beta$  to lie in the range  $1.4 \lesssim \tan \beta \lesssim 60$ .

Because  $SU(2) \times U(1)$  is broken, the interaction eigenstates  $H_u$  and  $H_d$  also mix. The spectrum of the Higgs sector is then described by 3 Goldstone bosons eaten by  $W^\pm$  and  $Z^0$ , a pair of charged Higgs  $H^\pm$ , a neutral pseudoscalar  $A^0$ , and two neutral scalars  $h^0$  and  $H^0$ , the latter defined so that  $m_h < m_H$ . The mass eigenstates for the two neutral scalars are defined via:

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \text{Re } H_u^0 \\ \text{Re } H_d^0 \end{pmatrix}. \quad (3)$$

The pseudoscalar Higgs is the combination  $A^0 = \sqrt{2}(\cos \beta \text{Im } H_u^0 + \sin \beta \text{Im } H_d^0)$ . Given the two mixing angles  $\alpha$  and  $\beta$ , the couplings of the quarks and leptons are completely determined in terms of their SM values. One finds that, for the lighter scalar, the ratio of the MSSM couplings to those of the SM are simply:

$$\frac{h^0 \bar{t} t |_{\text{MSSM}}}{h^0 \bar{t} t |_{\text{SM}}} = \frac{\cos \alpha}{\sin \beta}, \quad \frac{h^0 \bar{b} b |_{\text{MSSM}}}{h^0 \bar{b} b |_{\text{SM}}} = \frac{h^0 \tau^+ \tau^- |_{\text{MSSM}}}{h^0 \tau^+ \tau^- |_{\text{SM}}} = \frac{-\sin \alpha}{\cos \beta}, \quad (4)$$

where the SM couplings are  $g_2 m_f / 2m_W$  for fermion  $f$ . Note that since the  $b$ -quark and  $\tau$ -lepton both couple to the same Higgs interaction eigenstate, their couplings to the physical Higgs bosons are both shifted by the same amount. Therefore the ratio of branching ratios  $R_{b/\tau} \equiv BR(h^0 \rightarrow \bar{b}b) / BR(h^0 \rightarrow \tau^+ \tau^-)$  is the same in the SM and MSSM, namely  $3m_b^2/m_\tau^2$  up to kinematic factors and Standard Model QCD corrections (we ignore the small QED and electroweak corrections) [5]:

$$R_{b/\tau} = 3 \frac{m_b^2}{m_\tau^2} \left( \frac{m_h^2 - 4m_b^2(m_b)}{m_h^2 - 4m_\tau^2(m_\tau)} \right)^{1/2} \left[ 1 + 5.67 \frac{\alpha_s(m_h)}{\pi} + 29.14 \frac{\alpha_s^2(m_h)}{\pi^2} \right]. \quad (5)$$

(For this letter,  $m_b$  and  $m_\tau$  are to be evaluated in the  $\overline{MS}$  scheme and at the scale  $Q = m_h$  unless otherwise specified. In our numerical calculations we take  $Q = m_Z$ ; we are then missing only a very small residual correction proportional to  $\log(m_h/m_Z)$ .) Defining  $\omega$  by  $R_{b/\tau} = 3(m_b^2/m_\tau^2)(1 + \omega)$  one finds, for  $\alpha_s(m_Z^2) = 0.119$ , that  $R_{b/\tau}$  receives a QCD/phase space enhancement over its tree-level value of  $(1 + \omega) \simeq 1.25$ .

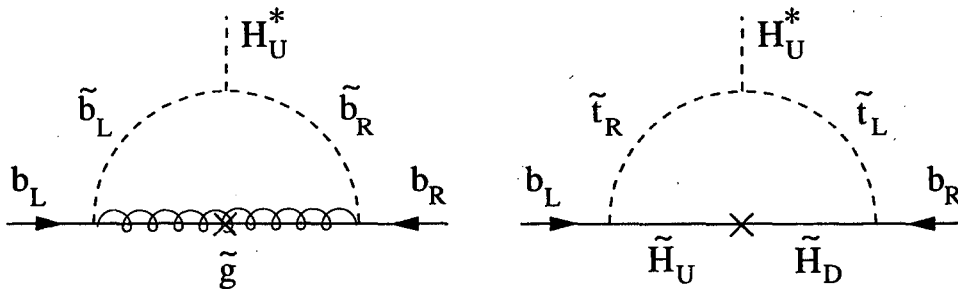


Figure 1: Leading threshold contributions to  $\epsilon_b$ .

### 3 Finite corrections at one-loop

Although the  $b$ -quark does not couple to  $H_u^0$  at tree level, it picks up a small coupling to  $H_u^0$  at one-loop through the diagrams in Fig. 1. Such diagrams were studied earlier in the context of neutron electric dipole moments in Ref. [6] and in the context of radiative fermion mass generation in Ref. [7]. Their existence is due to the interesting fact that  $b$ -squarks *do* couple to  $H_u^0$ , despite the fact that  $b$ -quarks do not, through the interaction  $y_b \mu \tilde{b}_L \tilde{b}_R^* H_u^{0*} + h.c.$  Since the coupling is loop-suppressed it is small. However, if  $v_u \gg v_d$  (*i.e.*,  $\tan \beta$  is large), the contribution of these loops to the  $b$ -mass is enhanced by  $\tan \beta$  and can therefore become quite significant.

The significance of such finite corrections for Yukawa unification schemes in grand unified theories was noticed in Ref. [8]. Specifically, in a class of minimal SO(10) models, one expects unification of all third generation Yukawas at the GUT scale:  $y_t(m_{GUT}) = y_b(m_{GUT}) = y_\tau(m_{GUT})$ . Such a unification demands large  $\tan \beta$  to compensate the large hierarchy in the masses of the  $b$ - and  $t$ -quarks:  $m_t/m_b = (y_t/y_b) \tan \beta$ . In the context of these unified models, it was realized that the one-loop contribution from Fig. 1 could significantly shift the  $b$ -mass, obscuring the simple relation between  $m_b$  and  $y_b$ . This should become clear shortly. The same class of corrections have also been studied in a wide variety of applications by other authors. Our discussion is particularly close in spirit to that of Ref. [9] in which the intimately related question of Higgs production and decay at the Tevatron was studied, including the effects of these corrections. We will confine ourselves to the question of Higgs decays in order to avoid entangling the physics we wish to highlight with production effects. (This is a reasonable approach especially for Higgs production at an electron or muon collider.)

Begin by writing the most general coupling of a  $b$ -quark and  $\tau$ -lepton to the neutral Higgs fields:

$$\mathcal{L} = y_b \bar{b}_L b_R H_d^0 + \epsilon_b y_b \bar{b}_L b_R H_u^{0*} + (b \leftrightarrow \tau) + h.c. \quad (6)$$

where we assume for simplicity that  $y_{b,\tau}$  and  $\epsilon_{b,\tau}$  are all real. For the remainder of this letter we will also assume that the scale of SUSY-breaking is larger than that of electroweak-breaking, consistent with our experimental knowledge of SUSY, so that



our effective Lagrangian is an accurate description of the physics. The parameter  $\epsilon_b$  receives contributions, after electroweak- and SUSY-breaking, from a number of diagrams, the most important being those in Fig. 1, which yield (taking all parameters real):

$$\epsilon_b = \frac{2\alpha_3}{3\pi} \mu M_3 f(M_3^2, m_{b_L}^2, m_{b_R}^2) + \frac{y_t^2}{16\pi^2} \mu A_t f(\mu^2, m_{t_L}^2, m_{t_R}^2) \quad (7)$$

where

$$f(m_1^2, m_2^2, m_3^2) \equiv \frac{1}{m_3^2} \left[ \frac{x \log x}{1-x} - \frac{y \log y}{1-y} \right] \frac{1}{x-y} \quad (8)$$

for  $x = m_1^2/m_3^2$ ,  $y = m_2^2/m_3^2$ . Numerically  $\epsilon_b$  is of order 2%, though the precise value will depend on the supersymmetric spectrum. There are also contributions with internal  $SU(2) \times U(1)$  gauginos, but these are typically suppressed by  $\alpha_{1,2}/\alpha_3$  compared to the gluino diagram<sup>1</sup>. Notice that  $\text{sgn}(\epsilon_b)$  is model-dependent and cannot be predicted without further input. Since there are no QCD-enhanced contributions for the  $\tau$ , nor a light right-handed  $\nu_\tau$ , then to a good approximation  $\epsilon_\tau \simeq 0$ ; we will comment later on the possibility of  $\epsilon_\tau \neq 0$ .

Including the corrections, the  $b$ -quark mass can be written

$$m_b = y_b v_d + y_b \epsilon_b v_u = y_b (1 + \epsilon_b \tan \beta) v \cos \beta. \quad (9)$$

Meanwhile, the coupling of the  $b$ -quark to the light Higgs is given by:

$$\mathcal{L}_{h\bar{b}b} = \frac{1}{\sqrt{2}} (-y_b \sin \alpha + y_b \epsilon_b \cos \alpha) h^0 \bar{b} b. \quad (10)$$

Including the corrections, the ratio of branching ratios,  $R_{b/\tau}$ , can be expressed (including the usual phase space/QCD corrections) as:

$$\begin{aligned} R_{b/\tau} &= 3 \frac{y_b^2}{y_\tau^2} (1 - \epsilon_b / \tan \alpha)^2 (1 + \omega) \\ &= 3 \frac{m_b^2}{m_\tau^2} \frac{(1 - \epsilon_b / \tan \alpha)^2}{(1 + \epsilon_b \tan \beta)^2} (1 + \omega). \end{aligned} \quad (11)$$

(This last formula can also be derived from Eq. (3.11) of Ref. [9].)

A couple of comments are now in order. First, perturbativity requires that  $\epsilon_b \ll 1$ , though not necessarily  $\epsilon_b \tan \beta \ll 1$ . Thus, the  $b$ -quark mass can receive a significant correction from Fig. 1. Second, the values of  $\alpha$  and  $\beta$  are correlated in the MSSM via:

$$\sin 2\alpha = -\frac{m_A^2 + m_Z^2}{m_H^2 - m_h^2} \sin 2\beta \simeq -\frac{m_A^2 + m_Z^2}{|m_A^2 - m_Z^2|} \sin 2\beta \quad (12)$$

where the final approximation holds in the large  $\tan \beta$  limit. (We use the exact relation between  $\alpha$  and  $\beta$  in our numerical results.) Thus in the so-called ‘‘Higgs

<sup>1</sup>If there is a significant contribution to  $\epsilon_b$  coming from diagrams other than those of Fig. 1, then these can be simply absorbed into  $\epsilon_b$  and the rest of our discussion is unchanged.

decoupling limit" of the MSSM, in which  $m_A \rightarrow \infty$ , one finds  $\alpha \rightarrow \beta - \pi/2$  and so  $R_{b/\tau}$  approaches its SM value, given by Eq. (5). Thus we expect to see the largest deviations of  $R_{b/\tau}$  from its SM value for relatively light  $A^0$ . We can expand  $R_{b/\tau}$  in the limit of  $m_A \gg m_Z$ :

$$R_{b/\tau} \simeq 3 \frac{m_b^2}{m_\tau^2} (1 + \omega) \left\{ 1 - 4 \frac{m_Z^2}{m_A^2} \frac{\epsilon_b \tan \beta}{1 + \epsilon_b \tan \beta} \right\}. \quad (13)$$

In this form, the shift in  $R_{b/\tau}$  away from its SM value,  $\delta R_{b/\tau}$ , can be written as a function only of the shift in the  $b$ -mass coming at one-loop,  $\delta m_b/m_b = \epsilon_b \tan \beta$  (our  $\delta m_b/m_b$  appears as  $\Delta(m_b)$  in Ref. [9]):

$$\frac{\delta R_{b/\tau}}{R_{b/\tau}} \simeq -4 \frac{m_Z^2}{m_A^2} \frac{\delta m_b/m_b}{1 + \delta m_b/m_b}. \quad (14)$$

(If there is a significant contribution to  $\epsilon_\tau$ , then so long as  $\epsilon_\tau \tan \beta \ll 1$ , one can simply replace  $\epsilon_b$  with  $(\epsilon_b - \epsilon_\tau)$  in Eq. (13) and corresponding formulae, and replace  $\delta m_b/m_b$  with  $(\delta m_b/m_b - \delta m_\tau/m_\tau)$  in Eq. (14) and corresponding formulae. If  $\epsilon_\tau \tan \beta \sim 1$ , similarly simple forms occur.)

In Fig. 2 we have shown contours of  $\delta R_{b/\tau}/R_{b/\tau}$  in the  $m_A - \delta m_b/m_b$  plane; the plot uses the full result of Eq. (11) for  $\tan \beta = 30$ , though the dependence of the plot on  $\tan \beta$  is unobservable for any  $\tan \beta \gtrsim 5$ . Contours are shown for  $\delta R_{b/\tau}/R_{b/\tau}$  of +2.5, 5, 10, 15, 20, 25% (solid lines in lower half plane), and -2.5, -5, -10, -15, -20, -25% (dotted lines in upper half plane). Even though the plot has no apparent dependence on  $\tan \beta$  it is unlikely that any large effect would be observed at small values of  $\tan \beta$ . That is to say, although the  $\tan \beta$  dependence can be absorbed into  $\delta m_b/m_b$ , it would be very difficult to generate appreciable shifts in the  $b$ -quark mass without the enhancement provided by large  $\tan \beta$ . Typically, one would require  $\tan \beta \gtrsim 10$  or so, for  $\delta m_b/m_b \sim 20\%$ .

It is well-known that radiative corrections in the MSSM coming from heavy top squark loops can significantly alter the scalar Higgs mass matrix, lifting the lighter scalar above the  $Z$  mass. These same corrections alter the relation between  $\alpha$  and  $\beta$ , slowing the decoupling that occurs as  $m_A \rightarrow \infty$ . Each of the elements of the scalar Higgs mass matrix is shifted by corrections, but for the sake of simplicity, we will only keep the leading term which shifts the diagonal  $H_u^0$  piece by an amount [2]:

$$\delta m^2 = \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \log \left( \frac{m_t^2}{m_t^2} \right). \quad (15)$$

At large  $\tan \beta$ , one measures  $\delta m^2$  simply by discovering the light Higgs, since  $\delta m^2 \simeq m_h^2 - m_Z^2$  to a good approximation. The shifted  $\alpha$  is now given by:

$$\sin 2\alpha = -\frac{(m_A^2 + m_Z^2)}{\Delta} \sin 2\beta \quad (16)$$

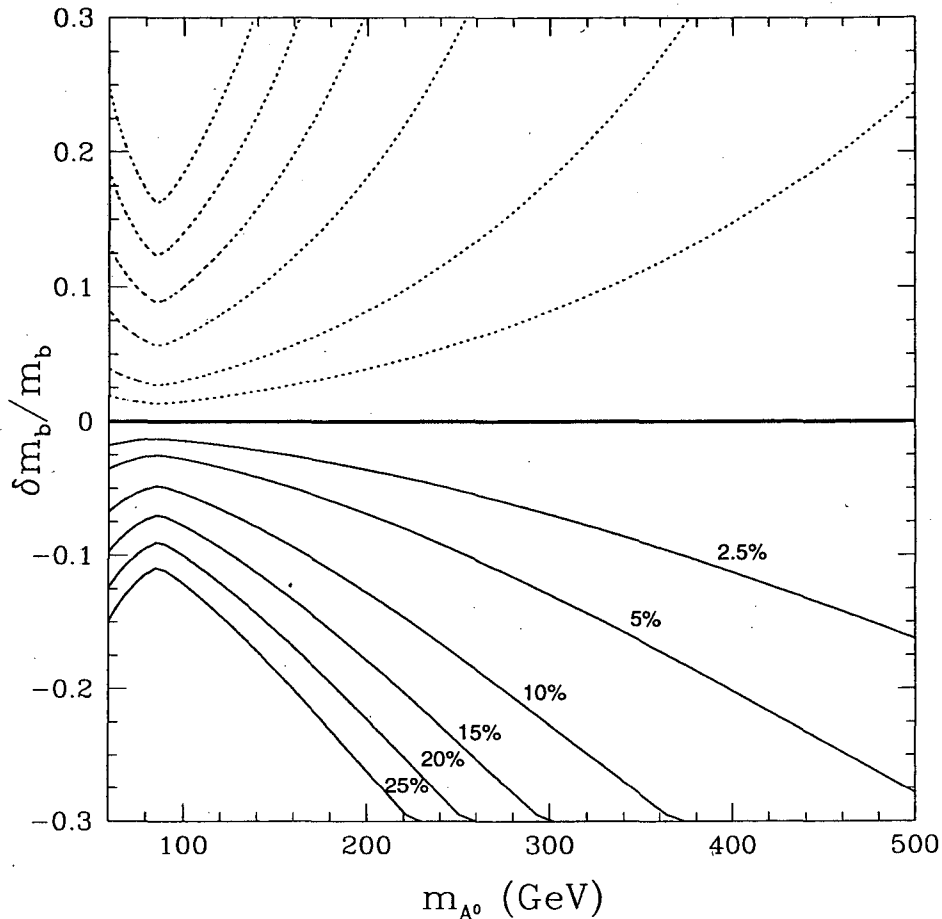


Figure 2: Contours of  $\delta R_{b/\tau}/R_{b/\tau}$  in the  $m_A - \delta m_b/m_b$  plane. The central dark contour is  $\delta R_{b/\tau}/R_{b/\tau} = 0$ ; dotted contours are negative values of the corresponding labelled solid contours. The figure does not include the leading radiative corrections to the Higgs mass matrix, which are discussed in the text. The figure holds for all  $\tan \beta \gtrsim 5$ .

where

$$\Delta = \left[ (m_A^2 + m_Z^2 + \delta m^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta - 4m_A^2 \delta m^2 \sin^2 \beta - 4m_Z^2 \delta m^2 \cos^2 \beta \right]^{1/2}. \quad (17)$$

In the large  $\tan \beta$  limit, Eq. (16) simplifies to

$$\sin 2\alpha \simeq -\frac{m_A^2 + m_Z^2}{|m_A^2 - m_h^2|} \sin 2\beta \simeq -\left(1 + \frac{2m_Z^2 + \delta m^2}{m_A^2}\right) \sin 2\beta \quad (18)$$

where the last equality holds if we also take the large  $m_A$  limit. These show clearly that for  $\delta m^2 > 0$  the radiative corrections slow down the decoupling as  $m_A \rightarrow \infty$ .

Thus Eq. (14) is corrected by replacing  $m_Z^2$  with  $(m_Z^2 + \delta m^2/2)$ . For  $m_h = 120$  GeV, for example, the radiative corrections increase  $|\delta R_{b/\tau}/R_{b/\tau}|$  from the values shown in Fig. 2 by roughly 40%. (That is, the curve labeled 10% would correspond to 14% after radiative correction.) There are additional, though generally smaller, corrections to the other entries in the scalar Higgs mass matrix. Though these corrections have the ability to shift the mixing angle,  $\alpha$ , they are more model-dependent and we do not analyze them here; see however Refs. [10, 9] for a discussion of some possible effects.

We can define another double ratio  $R_{b/c} \equiv BR(h^0 \rightarrow \bar{b}b)/BR(h^0 \rightarrow \bar{c}c)$ . In the MSSM, including the finite radiative corrections, this is given by

$$R_{b/c} = \frac{m_b^2}{m_c^2} (\tan \alpha \tan \beta)^2 \left[ \frac{1 - \epsilon_b/\tan \alpha}{1 + \epsilon_b \tan \beta} \right]^2 \left[ \frac{1 + \epsilon_c/\tan \beta}{1 - \epsilon_c \tan \alpha} \right]^2, \quad (19)$$

where  $\epsilon_c y_c$  is the radiatively generated coefficient to the  $H_d^{0*} \bar{c}c$  vertex. Note that there is no  $\tan \beta$  enhancement associated with  $\epsilon_c$  ( $\epsilon_c \simeq \epsilon_b \sim 2\%$  from the gluino graph), so the rightmost bracket in Eq. (19) goes to 1 and  $\delta R_{b/c}/R_{b/c}$  becomes identical to  $\delta R_{b/\tau}/R_{b/\tau}$ . Thus simultaneous measurement of a shift in  $R_{b/c}$  will provide crucial supporting evidence for supersymmetry.

The same analysis can be repeated for the heavier scalar Higgs,  $H^0$ , simply by replacing  $-1/\tan \alpha \rightarrow \tan \alpha$  in Eq. (11), and for the pseudoscalar,  $A^0$ , by replacing  $\tan \alpha \rightarrow -\tan \beta$  in the numerator of Eq. (11). Note that for the  $H^0$  and  $A^0$  decoupling does not occur, as expected. At large  $\tan \beta$ , the expressions for  $H^0$  and  $A^0$  simplify to the same form, so that the shift in either ( $\equiv R_{b/\tau}^A$ ) can be written:

$$\delta R_{b/\tau}^A/R_{b/\tau}^A \simeq (1 + \delta m_b/m_b)^{-2} - 1. \quad (20)$$

Because there is no dependence on  $m_A$  (nor in this form on  $\tan \beta$ ), the result is particularly easy to examine. In Fig. 3 we do just that, showing the shift in  $R_{b/\tau}^A$  as a function of the shift in the  $b$ -quark mass. Notice that a 25% shift in the  $b$ -mass translates into a 75% correction to  $R_{b/\tau}^A$ !

## 4 Implications for SUSY searches

It is entirely conceivable, if not likely, that the lightest Higgs will be discovered prior to the discovery of any SUSY partners. It is then a fair question to ask: is this Higgs the SM Higgs, a SUSY Higgs, or some other? If the Higgs is found to have appreciable deviations in  $R_{b/\tau}$  from the SM case, though not 100%, we believe that this will be a fair argument that SUSY exists and will be found at higher energies. In the SM itself, there is no source of large corrections to  $R_{b/\tau}$  besides those already shown in Eq. (5). In non-SUSY extensions of the SM Higgs sector, there is no simple source for *large* corrections to  $R_{b/\tau}$ . For example, in the type II 2HDSM, there is a diagram with a top quark and charged Higgs boson which corresponds closely to the second diagram

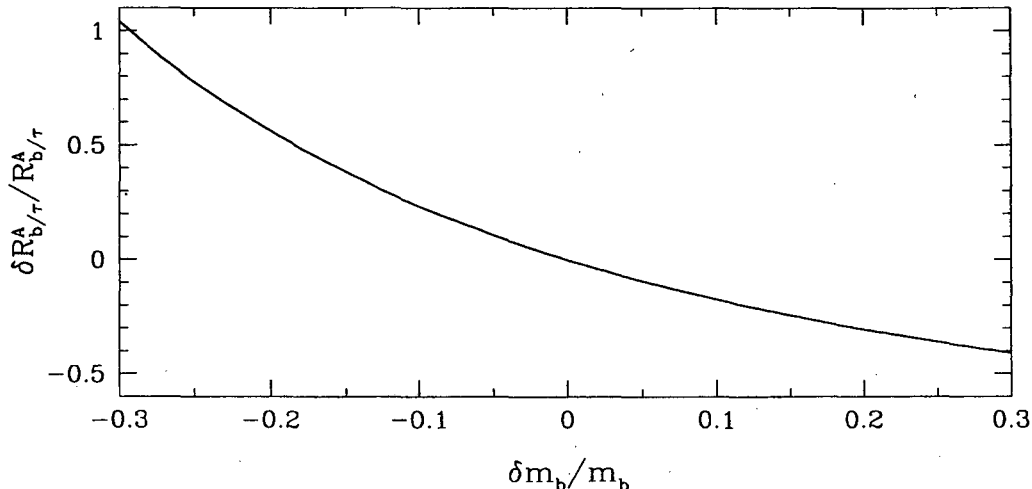


Figure 3: Corrections to  $R_{b/\tau}^A$  as a function of  $\delta m_b/m_b$ . The figure holds for all  $\tan\beta \gtrsim 5$ .

of Fig. 1, it can be shown to lack the  $\tan\beta$  enhancement that the SUSY diagrams share [8], making it very difficult to generate large enough  $\delta m_b/m_b$  to be observable.

We should remark that there are ways to distinguish the MSSM Higgs sector from other (perhaps less motivated) versions of the non-SUSY 2HDSM by studying the decays of  $h^0$  alone. For example, in the type I model, where all the fermions couple to a single Higgs, the predictions for  $R_{b/\tau}$  will be identical to that of the SM. In the type III model, where  $H_u$  couples to  $u$ -quarks and charged leptons while  $H_d$  couples to  $d$ -quarks, already at the tree level  $R_{b/\tau}$  is different from its SM value. This case can be tested by the measurements of three observables:  $R_{b/\tau}$ ,  $R_{b/c}$  and  $\sigma \cdot BR(h^0 \rightarrow \bar{b}b)$ , where  $\sigma$  denotes the production cross section for  $h^0$ . Since there are no large radiative corrections in this model, these three observables depend only on two parameters, *viz.*,  $\alpha$  and  $\beta$  (apart from  $m_h$ ), so consistency of this scenario can be directly tested.

Even if there are early indications of SUSY in the Higgs decays, this would not be equivalent to saying that SUSY is light. On the contrary, the diagrams that contribute to  $\epsilon_b$  are non-decoupling — as the SUSY scale increases, these diagrams approach a non-zero constant, so long as  $m_A$  remains light. Thus corrections to Higgs decays widths may be the only indication of SUSY for quite some time. This could be a particularly interesting probe then of SUSY models in which much of the SUSY spectrum remains quite heavy.

## 5 Implications for grand unification

The simplest and most elegant grand unification theories (GUTs), SU(5) and SO(10), group otherwise different fermions into common representations at the unification scale. For example, in SU(5) the  $b_R$  and  $\tau_L$  are part of a single  $\bar{\mathbf{5}}$  representation,

while the  $b_L$  and  $\tau_R$  are part of a single **10**; in  $\text{SO}(10)$ , all the above are grouped together into a single **16** spinor representation. For minimal GUT Higgs sectors, the grouping imply  $y_b = y_\tau$  for  $\text{SU}(5)$  and  $y_t = y_b = y_\tau$  for  $\text{SO}(10)$ , all evaluated at the unification scale [11]. In most of the simplest extensions of the GUT Higgs sectors, the  $b-\tau$  unification of  $\text{SU}(5)$  and  $\text{SO}(10)$  survives, though not always the full  $t-b-\tau$  unification of  $\text{SO}(10)$ .

However, if we extract  $y_b$  simply from Eq. (2), then explicit calculations of Yukawa unification within the context of the MSSM and assuming a “grand desert” between the SUSY and GUT scales find that  $b-\tau$  unification does not occur for generic values of  $\tan\beta$  [12]. In fact, for most  $\tan\beta$ , one finds that the physical  $m_b$  predicted by unification is much larger than measured; alternatively, given the measured  $m_b$ , one find  $y_b < y_\tau$  at the unification scale. Only at special values of  $\tan\beta$  does  $b-\tau$  unification match experiment. These special values correspond either to  $y_t$  pseudo-fixed points ( $m_t^{\text{pole}}/\sin\beta \simeq 205 \text{ GeV}$ ) or  $y_b$  and  $y_\tau$  pseudo-fixed points ( $\tan\beta \simeq 60$ ). For all other values of  $\tan\beta$ , one generally finds:

$$0.75 \lesssim \frac{y_b}{y_\tau} \Big|_{\text{GUT}} \lesssim 0.85 \quad \iff \quad -0.25 \lesssim \frac{m_{b,\text{exp}} - m_{b,\text{pred}}}{m_{b,\text{pred}}} \lesssim -0.15 . \quad (21)$$

It has been traditional to plot the regions allowed/disallowed by Yukawa unification in the parameter space of  $M_t$  and  $\tan\beta$ ; however with the errors on measurements of  $M_t$  now small, we have chosen instead to plot the difference between the  $b$ - and  $\tau$ -Yukawa couplings at the GUT scale as a function of  $\tan\beta$  and  $\alpha_s(m_Z)$ , as seen in Fig. 4. (For the plot, we have used  $m_b(m_b) = 4.2 \text{ GeV}$ .) At very large and very small values of  $\tan\beta$ , it is impossible to talk about unification because the theory is non-perturbative (the hatched region). In the small regions near the edge of perturbativity (the dark regions), one does indeed find rough unification of  $m_b$  and  $m_\tau$  in the MSSM. However, throughout the whole central region of the plot  $b-\tau$  unification fails, usually by 15–25% as shown in the contours.

Thus, if  $b-\tau$  unification is to survive, we must either live at the pseudo-fixed point of some Yukawa coupling, or we must use the one-loop corrections to the  $b$ -quark mass to *reduce*  $m_b$  in order to bring agreement with the low measured value. Thus minimal GUT unification implies  $-25\% \lesssim \delta m_b/m_b \lesssim -15\%$  typically [8, 13].

With this “prediction” in hand, we can go back to Figs. 2 and 3. In Fig. 2, we see that in the region of interest of  $\delta m_b/m_b$ , large deviations in  $R_{b/\tau}$  can be expected. Even for  $m_A = 300 \text{ GeV}$ , we can expect shifts approaching +15%. If  $R_{b/\tau}$  is normalized experimentally by the  $h^0 \rightarrow \bar{b}b$  branching ratio, then shifts in  $R_{b/\tau}$  greater than zero correspond to suppressed  $h^0 \rightarrow \tau^+\tau^-$  branching ratios.

In Fig. 3, we find that GUT-motivated shifts in the  $b$ -mass correspond to shifts in  $R_{b/\tau}^A$  of nearly 80%. Normalizing to the  $A^0, H^0 \rightarrow \bar{b}b$  branching ratios, we now find a suppression of the  $A^0, H^0 \rightarrow \tau^+\tau^-$  branching ratios of nearly half. Thus GUTs would seem to prefer relatively large shifts in  $R_{b/\tau}$  and  $R_{b/\tau}^A$ , with definite signs corresponding to suppressed decays to  $\tau$ 's relative to  $b$ 's.

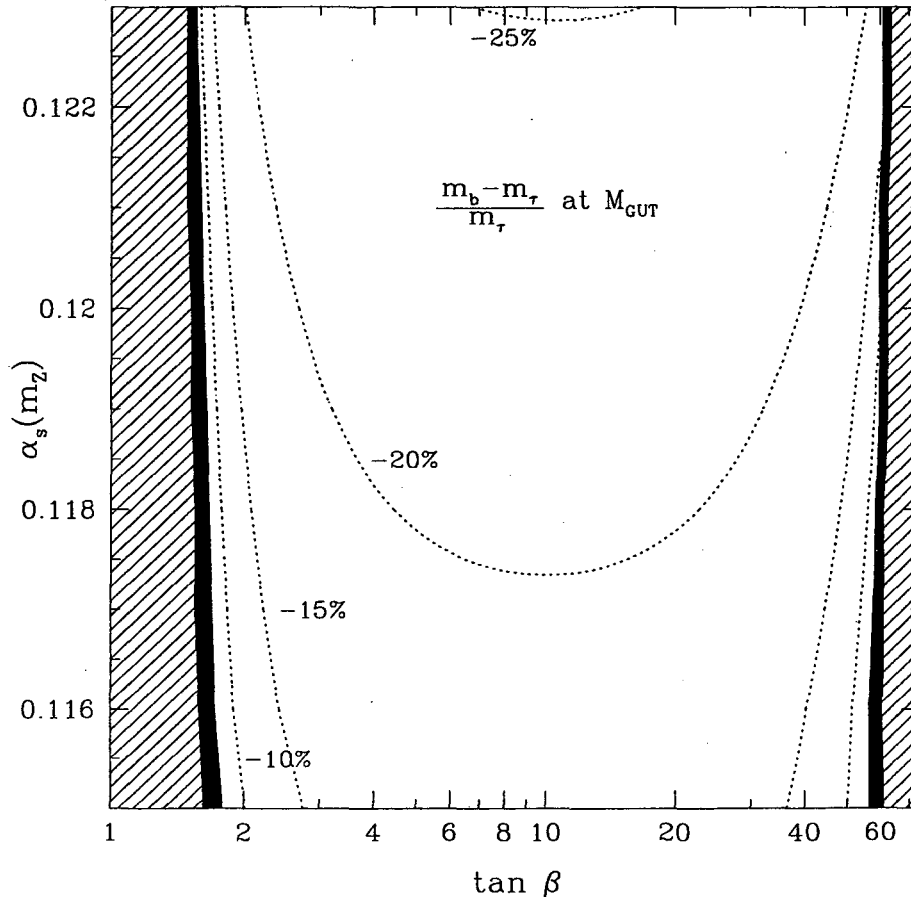


Figure 4: Mismatch between the measured  $b$ - and  $\tau$ -Yukawa couplings at the GUT scale. The hatched regions to the left and right are disallowed by perturbativity constraints. The dark regions exhibit approximate Yukawa unification. In the central regions, the contours label values of  $(y_b - y_\tau)/y_\tau$ , evaluated at the GUT scale.

This represents then a rare opportunity to probe GUT physics more carefully, and in the unexpected regime of Higgs decays. If the Higgs branching ratios are found to shift, and to do so with signs and magnitudes consistent with  $b - \tau$  unification, this would be additional circumstantial evidence in favor of a real unified gauge group. On the other hand, large shifts in the wrong direction would certainly constitute an argument against the simpler classes of unified models.

## 6 Experimental comparison

In order to experimentally detect deviations of  $R_{b/\tau}$  from its SM value, the SM value itself must be well-understood. To do so requires careful measurement of three pa-

rameters:  $m_b$ ,  $m_\tau$  and  $\alpha_s$ . Of these,  $m_\tau$  is already extremely well-measured and requires no further discussion.  $\alpha_s$  has been measured over the last several years with increasing precision. Global fits to data over a wide range of energies gives  $\alpha_s(m_Z) = 0.119 \pm 0.002$  [14]. Since  $\alpha_s$  enters primarily as a 25% radiative correction in  $R_{b/\tau}$ , the error due to the uncertainty in  $\alpha_s$  is small.

It is  $m_b$  itself which is hardest to determine. The most direct, though least precise, experimental determination of  $m_b$  is through three jet heavy quark production at LEP, where the effects of the small  $m_b$  are enhanced by the details of the jet clustering algorithms. An analysis by DELPHI gives  $m_b(m_Z) = 2.67 \pm 0.50$  GeV [15], which translates to  $m_b(m_b) = 3.91 \pm 0.67$ . The  $\Upsilon$  system provides another clean experimental measurement, though one in which theoretical effects are harder to disentangle. Using QCD moment sum rules for inclusive  $b$ -production in  $e^+e^-$  collisions, and assuming that the  $\Upsilon$  saturates the higher moments, a recent value of  $m_b(m_b) = 4.13 \pm 0.06$  GeV [16] has been extracted. The latest lattice extraction yields  $m_b(m_b) = 4.15 \pm 0.20$  GeV [17]. The theoretical errors in these estimate seem to be hard to quantify, a conservative range of  $4.1 \text{ GeV} < m_b(m_b) < 4.4 \text{ GeV}$  has been quoted in Ref. [14], roughly an uncertainty of  $\pm 3.5\%$ .

While the LEP-derived values of  $\alpha_s$  are actually calculated at the  $Z$ -pole (and then run down to  $Q = m_b$  for comparison), the values derived from heavy meson systems need to be run from  $Q = m_b$  up to  $Q = m_Z \simeq m_h$  for use in Eq. (5). At one-loop, the QCD renormalization group equations for  $m_b$  can be solved:

$$m_b(m_Z) = m_b(m_b) \left( \frac{\alpha_s(m_Z)}{\alpha_s(m_b)} \right)^{12/23} \quad (22)$$

Thus, if  $m_b(m_b)$  were known with infinite precision, there would still be a 2% uncertainty in  $R_{b/\tau}$  from the current uncertainty in  $\alpha_s(m_Z)$ . However, that uncertainty is presently overwhelmed by the uncertainties in  $m_b$  itself. Thus an important aim of future experimental and theoretical work should be to get the errors on  $m_b$  down to the 2% level. Given that the present error on  $m_b(m_b)$  is about 3-4%, such an improvement does not appear to be beyond reach, especially with forthcoming experimental efforts at the  $B$ -factories and theoretical efforts in lattice gauge theories. Only when the uncertainties in the SM prediction for  $R_{b/\tau}$  are below the few percent level will an unequivocal measurement of  $\delta m_b/m_b$  be possible in  $h^0$  decays for a large portion of the parameter space. Barring that, we must wait for the discovery of the  $H^0$  and/or  $A^0$  where the effects can be expected to be much larger. Of course, if  $\delta m_b/m_b$  is large while  $m_A$  remains under a few hundred GeV, observation of deviations in  $h^0$  decays will be possible even with the current precision in  $m_b$ .

## 7 Conclusions

Throughout this article, we have tried to keep our approach as model-independent as possible. However such an approach can fail under special circumstances, or can mask



other interesting physics. For example, the spectrum of SUSY sparticles could be too close to the weak scale to allow them to be completely decoupled from our effective Lagrangian, requiring a calculation of the full radiative corrections in the spirit of Ref. [18]; however such a calculation will generically lead to even larger deviations in  $R_{b/\tau}$ . We have also not specified the regions of the full MSSM parameter space which can lead to sizable  $\epsilon_b$ ; in some (but not all) regions of that parameter space, large  $\epsilon_b$  may imply correspondingly large radiative contributions to the Higgs mass matrix and  $b \rightarrow s\gamma$ . In the approach we have taken, model-dependent correlations of this kind cannot be examined.

Yet even without a study of the complete parameter space of the MSSM, we can make the following statement: in the MSSM, unlike non-SUSY 2HDSMs, there can be significant shifts in the ratio  $BR(\phi \rightarrow \bar{b}b)/BR(\phi \rightarrow \tau^+\tau^-)$  for  $\phi = h^0, H^0$  or  $A^0$ . These shifts in the  $h^0$  decays may be our first indication of SUSY, long before SUSY partners, or the additional Higgs bosons, themselves are discovered. There is also a strong correspondence between the shifts in the Higgs branching ratios and Yukawa unification in minimal GUT models. In this way, significant departures of the Higgs branching ratios away from their SM values could teach us simultaneously about SUSY *and* GUT physics.

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