UCRL-10600

University of California

Ernest O. Lawrence Radiation Laboratory

HIGH-ENERGY CROSS SECTIONS AND THE CHEW-FRAUTSCHI PARTICLE

Berkeley, California

DISCLAIMER

To be published in Phys. Rev. Letters

UCRL-10600

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

HIGH-ENERGY CROSS SECTIONS AND THE CHEW-FRAUTSCHI PARTICLE

Alberto Pignotti

December 17, 1962

 UCRL-10600

High-Energy Cross Sections and the Chew-Frautschi Particle*

Alberto Pignotti†

Lawrence Radiation Laboratory
University of California
Berkeley, California

December 17, 1962

Two experimental groups have recently reported on the discovery of a $\pi\pi$ resonance at about 1250 MeV that is probably an $I = 0$, $J = 2$ state.[1,2] It was conjectured that this resonance, called f^0, was the "particle" predicted by Chew and Frautschi on the basis of the Regge-poles scheme.[3] In this letter it is shown that this cannot be the case if some reasonable hypotheses on Regge-poles are verified, and if the high-energy experimental data on differential cross sections are considered.[4] Information is obtained from these data by using some bounds that are derived for boson Regge-pole trajectories. Finally, it is suggested that the C. F. particle is likely to have an energy less than or near the ρ-meson energy, and experimental work directed towards testing this possibility is encouraged. The f^0 resonance still can fit into the Regge-poles scheme as possibly belonging to the trajectory proposed by Igi.[5]

In order to derive some bounds for boson Regge trajectories it will be useful to prove the following lemma: Let $\xi(t)$ satisfy the dispersion relation

$$\xi(t) = \frac{1}{\pi} \int_a^\infty \frac{\operatorname{Im} \xi(t')\, dt'}{t' - t} \tag{1}$$

with $\operatorname{Im} \xi(t) > 0$ for $a < t < \infty$. It follows that

$$\xi(t_3) \gtrless \frac{\xi(t_1)\, \xi(t_2)\, (t_2 - t_1)}{(t_2 - t_3)\, \xi(t_2) - (t_3 - t_1)\, \xi(t_1)} \tag{2}$$

for t_1, t_2, and $t_3 \leq a$. If we take $t_1 < t_2$, the $\geq$ sign holds for $t_3 \leq t_1$ or $t_3 \geq t_2$ and the $\leq$ sign for $t_1 \leq t_3 \leq t_2$. Proof: From Eq. (1) we have

$$\operatorname{Im} \xi(t) > 0 \qquad \text{for } \operatorname{Im} t > 0$$

and

$$\left.\begin{array}{l} \operatorname{Re} \xi(t) > 0 \\ \operatorname{Im} \xi(t) = 0 \end{array}\right\} \qquad \text{for } t \text{ real and } -\infty < t < a.$$

Therefore the function

$$\eta(t) = -1/\xi(t) \tag{3}$$

can be represented as[6]

$$\eta(t) = C + A\,t + \int_a^\infty \frac{1+t't}{t'-t}\,\phi(t')\,dt' \tag{4}$$

with A, $\phi(t)$, and C real, and $\phi(t) \geq 0$. Here $\phi(t)$ may include δ functions with positive coefficients.

From Eqs. (3) and (4) it follows for t real and $-\infty < t < a$ that

$$\eta''(t) = [1/\xi^4(t)]\,\{\xi''(t)\,\xi^2(t) - 2\,[\xi'(t)]^2\,\xi(t)\} \geq 0. \tag{5}$$

Therefore, because $\eta(t)$ is a concave function for $t < a$, we have

$$\eta(t_3) \gtrless \eta(t_2)\,\frac{(t_3 - t_1)}{(t_2 - t_1)} + \eta(t_1)\,\frac{(t_3 - t_2)}{(t_1 - t_2)} \tag{6}$$

for $t_1, t_2, t_3 \leq a$. Again, if we take $t_1 < t_2$ the $\geq$ sign holds for $t_3 \leq t_1$ or $t_3 \geq t_2$, and the $\leq$ sign for $t_1 \leq t_3 \leq t_2$. Equation (2) follows trivially from Eqs. (6) and (3). This completes the lemma.

We now assume that a boson Regge trajectory $\alpha(t)$ satisfies the dispersion relation[7]

$$\alpha(t) = \alpha(\infty) + \frac{1}{\pi}\int_a^\infty \frac{\operatorname{Im} \alpha(t')\,dt'}{(t'-t)}\,dt' \tag{H1}$$

with $a > 0$ and[7]

$$\operatorname{Im} \alpha(t) \geq 0, \quad \text{for } a \leq t < \infty. \tag{H2}$$

Therefore we can apply the above lemma to the function $\alpha(t)-\alpha(\infty)$. In particular, if we let $t_1 \to t_2 = 0$ and $t_3 = t$ in Eq. (2), we obtain for $-\infty < t < 0$

$$\alpha(t) > \alpha(\infty) + \frac{[\alpha(0) - \alpha(\infty)]^2}{\alpha(0) - \alpha(\infty) + \alpha'(0)|t|} = L(t), \tag{7}$$

or

$$\alpha'(0) > \frac{1}{|t|}\left\{\frac{[\alpha(0) - \alpha(\infty)]^2}{\alpha(t) - \alpha(\infty)} - \alpha(0) + \alpha(\infty)\right\}. \tag{8}$$

Hence, calling t_0 the value of t at which α vanishes, we have

$$\alpha'(0) > \frac{\alpha(0)[\alpha(0) - \alpha(\infty)]}{-\alpha(\infty)\ |t_0|}. \tag{9}$$

We apply the above bounds to the Pomeranchuk trajectory, which controls the high-energy behavior of differential and total cross sections, and therefore satisfies the condition[3, 8]

$$\alpha(0) = 1. \tag{H3}$$

We also take

$$\alpha(\infty) = -1, \tag{H4}$$

as is the case for the first trajectory for a Yukawa potential.[9] In doing so, we also consider the proof given by Gribov and Pomeranchuck that there is an infinite number of poles satisfying this limit in the relativistic case.[10, 11] Therefore, using Eqs. (7) and (8) and the values obtained for $\alpha(t)$ in the high-energy experiments, we can set a lower bound Ω, for $\alpha'(0)$. On this basis we assume

$$\alpha'(0) > \Omega. \tag{H5}$$

We now examine the possibility that the Pomeranchuk trajectory gives rise to a resonance at an energy-squared value equal to t_R. For this to be so, it is necessary to have

$$\mathrm{Re}\,\alpha(t_R) = 2. \tag{H6}$$

It is well known that this condition is not sufficient. In addition, our experience with the nonrelativistic case shows that at the resonance energy $\mathrm{Re}\,\alpha(t)$ must still be increasing steeply as a function of t, and $\mathrm{Im}\,\alpha(t)$ must still be small. The width of the resonance in the energy variable is related to these two properties and is given approximately by the function[7]

$$\Gamma(t_R) = \frac{1}{\sqrt{t}} \left.\frac{\mathrm{Im}\,\alpha(t)}{\frac{d}{dt}\mathrm{Re}\,\alpha(t)}\right]_{t=t_R} \tag{10}$$

We require that the Pomeranchuk trajectory present these two characteristics at $t = t_R$ by assuming

$$\mathrm{Re}\,\alpha(t) \text{ has at most one inflection point for } t < t_R, \tag{H7}$$

and

$$\mathrm{Im}\,\alpha(t) \leq \mathrm{Im}\,\alpha(t_R)\,\frac{t-4}{t_R-4} \qquad \text{for } a = 4 < t < t_R.\text{[12]} \tag{H8}$$

Although it is not essential, to simplify the following proof we assume further

$$\mathrm{Im}\,\alpha(t) \geq \mathrm{Im}\,\alpha(2t_R - t) \qquad \text{for } t_R < t < 2t_R - 4. \tag{H9}$$

This is still consistent with the properties discussed above.

Using assumptions (H1) through (H9) for $\alpha(t)$, we can restrict the parameters of the C. F. particle by setting a lower bound for $\Gamma(t_R)$. Because Ω turns out to be fairly large, we are interested in the region

$$t_R > 1/\Omega \tag{11}$$

For these values of t_R, from assumptions (H3) and (H5) through (H7) it follows that

$$\left.\frac{d}{dt}\,\mathrm{Re}\,\alpha(t)\right]_{t=t_R} < 1/t_R. \tag{12}$$

We now want to put a lower bound on $\operatorname{Im}\alpha(t_R)$ such that, combined with Eqs. (10) and (12), it will provide the desired restriction on the value of $\Gamma(t_R)$. It is clear that such a bound for $\operatorname{Im}\alpha(t_R)$ exists, because if we had $\operatorname{Im}\alpha(t_R) = 0$, from assumptions (H1) and (H8) all the derivatives of α would be positive up to $t = t_R$, and therefore we would have $\operatorname{Re}\alpha(t_R) > 2$, which contradicts assumption (H6). This suggests splitting α into two parts, β and γ, one of which has a zero imaginary part for $t < t_R$. Therefore, we define

$$\left.\begin{aligned} \operatorname{Im}\beta(t) &= \operatorname{Im}\alpha(t) && \text{for } 4<t<t_R \\ &= \operatorname{Im}\alpha(2t_R-t) && \text{for } t_R<t<2t_R-4 \end{aligned}\right\} \qquad (13a)$$

$$\beta(t) = \frac{1}{\pi}\int_4^{2t_R-4} \frac{\operatorname{Im}\beta(t')}{(t'-t)}\,dt' , \qquad (13b)$$

and

$$\gamma(t) = \alpha(t) - \beta(t). \qquad (14)$$

From the symmetry in $\operatorname{Im}\beta$ we have $\operatorname{Re}\alpha(t_R) = \gamma(t_R) = 2$. Using relations (H1), (H2), (H9), (13) and (14), we can apply our lemma to $\gamma(t) - \gamma(\infty)$. We let $t_1 \to t_2 = 0$, and $t_3 \to a = t_R$ in Eq. (2) and obtain

$$\gamma(t_R)-\gamma(\infty) > \frac{[(\gamma(0)-\gamma(\infty)]^2}{\gamma(0)-\gamma(\infty)-t_R\gamma'(0)} > \frac{[1-\alpha(\infty)]^2-2[1-\alpha(\infty)][1-\gamma(0)]}{1-\alpha(\infty)-t_R\,\gamma'(0)} . \qquad (15)$$

Now, from relations (H3), (H5), (H8), (13), and (14), we have

$$\left.\begin{aligned} &\gamma(0) > 1 - \operatorname{Im}\alpha(t_R)\, B_0(t_R) \\ \text{and} \\ &\gamma'(0) > \Omega - \operatorname{Im}\alpha(t_R)\, B_1(t_R), \end{aligned}\right\} \qquad (16)$$

where

$$B_0(t_R) = \frac{1}{\pi(t_R-4)}\left\{-4\ln\left[\frac{2t_R-4}{4}\right] + 2t_R\ln\left[\frac{2t_R-4}{t_R}\right]\right\}$$

and

$$B_1(t_R) = \frac{1}{\pi(t_R - 4)} \ln \left[\frac{t_R^2}{4(2t_R - 4)}\right].$$

From relations (15) and (16) we obtain the lower bound for $\operatorname{Im} \alpha(t_R)$. This result combined with relation (10) and (12) gives the final bound

$$\Gamma(t_R) > \frac{\sqrt{t_R}\,\{[\operatorname{Re}\alpha(t_R) - \alpha(\infty)]\, t_R \Omega + [\alpha(0) - \alpha(\infty)][\alpha(0) - \operatorname{Re}\alpha(t_R)]\}}{2[\alpha(0) - \alpha(\infty)]\, B_0(t_R) + [\operatorname{Re}\alpha(t_R) - \alpha(\infty)]\, t_R B_1(t_R)}$$

$$= \frac{\sqrt{t_R}(3t_R\Omega - 2)}{4B_0(t_R) + 3t_R B_1(t_R)} = \Phi(t_R) \qquad \text{for } t_R > 1/\Omega. \tag{17}$$

In Fig. 1 the function $L(t)$ from relations (7), (H3), and(H4) is plotted for $\alpha'(0) = 1/80$, $1/50$ and $1/35$. The experimental data from the Brookhaven and Cornell groups are also indicated.[13,14] If we apply the inequality (8) to the three data for lower $|t|$, we obtain $\Omega > 1/29$ with 90% confidence. Eventually one may expect this result to be lowered by the presence of other than statistical errors in these data. However, the values of $\Phi(t_R)$ obtained from Eq. (17) and plotted in Fig. 2 show that unless $\alpha'(0)$ is less than 1/80, then $\Gamma(80)$ has to be much larger than the experimental width of the f^0 resonance. Therefore, within the scheme presented here, and provided the high-energy data do not increase far beyond the errors quoted, this resonance cannot be interpreted as the C. F. particle. Nevertheless, this particle could be found at a value of t smaller than or approximately equal to 30.[15,16]

I am indebted to Drs. G. Chew, G. Takeda, and C. Zemach for encouragement and many helpful discussions. I also want to thank Dr. David L. Judd for his kind hospitality at the Lawrence Radiation Laboratory.

FOOTNOTES AND REFERENCES

*Work done under the auspices of the U. S. Atomic Energy Commission, during a fellowship sponsored by the Consejo Nacional de Investigaciones Científicas y Técnicas of Argentina. This does not imply that this institution either approves or assumes any liabilities for the information contained in this report.

†On leave from Facultad de Ciencias Exactas y Naturales, Buenos Aires, Argentina.

1. W. Selove, V. Hagopian, H. Brody, A. Baker, and E. Leboy, Phys. Rev. Letters 9, 272 (1962).
2. J. J. Veillet, J. Hennessy, H. Bingham, M. Block, D. Dryard, A. Lagarrigue, P. Mittner, and A. Rousset, Ecole Polytechnique, Paris and G. Bellini, M. di Corato, E. Fiorini, and P. Nagri, Istituto Nazionale di Fisica Nucleare, Milano, preprint (n. d.).
3. G. F. Chew and S. Frautschi, Phys. Rev. Letters 7, 394 (1961). We will call this particle the C. F. particle.
4. The considerations leading to this conclusion were suggested by an analogous result obtained by A. Ahmadzadeh and I. Sakmar, (Lawrence Radiation Laboratory Report UCRL-10584, December 1962), in which a parameter-dependent form is assumed for Im $\alpha(t)$, and the parameters are determined with the help of the usual properties of the Pomeranchuk Regge pole. I am indebted to these authors for many fruitful discussions and day-to-day communication of their results.
5. K. Igi, Phys. Rev. Letters 9, 76 (1962).
6. J. A. Shohat and J. D. Tamarkin, The Problem of Moments, (American Mathematical Society, New York, N. Y., 1943) p. 23.
7. This and other properties are discussed by G. F. Chew in "The Self-consistent S-Matrix with the Regge Asymptotic Behaviour", CERN preprint TH/292, August 16, 1962, to be published in the Physical Review; further references are given here.

8. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 8, 41 (1962).

9. A. Ahmadzadeh, P. C. Burke, and C. Tate, Lawrence Radiation Laboratory Reports UCRL-10140, March 27, 1962 and UCRL-10216, May 24, 1962.

10. V. N. Gribov and I. Ya. Pomeranchuk, in 1962 Annual International Conference on High Energy Physics at CERN (CERN Scientific Information Service, Geneva, 1962).

11. We will present here some other results that follow from relations (2), and (H1) through (H4). In Eq. (2) let $t_1 = t_0$, $t_2 = 0$, and $t_3 = t$; then for $t_0 < t < 0$ we have

$$\alpha(t) < -1 + 2t_0/(t_0 + t). \qquad \text{(F1)}$$

Let $t_1 \to t_2 = t$, $t_3 = 0$; it follows that, for $-\infty < t < 0$,

$$\alpha'(t) < \frac{1 - \alpha^2(t)}{2|t|},$$

and in particular, if we set $t = t_0$ and use Eq. (11),

$$\alpha'(t_0) < 1/(2|t_0|) < \alpha'(0)/4.$$

12. This last hypothesis is generous near threshold, because there we know [see A. O. Barut and D. E. Zwanziger, Phys. Rev. 127, 974 (1962)] $\mathrm{Im}\,\alpha(t) \sim (t-4)^{\mathrm{Re}\,\alpha(4) + 1/2}$ and $\mathrm{Re}\,\alpha(4) > 1$. Moreover, the maximum for $\mathrm{Re}\,\alpha(t)$ is expected to occur at approximately the same energy as the inflection point for $\mathrm{Im}\,\alpha(t)$. Therefore, as $\mathrm{Re}\,\alpha(t)$ has not yet reached its maximum at the resonance energy, $\mathrm{Im}\,\alpha(t_R)$ is presumably still concave, which is stronger than required by assumption (H8).

13. W. F. Baker, E. W. Jenkins, A. L. Read, G. Cocconi, V. T. Cocconi, and J. Orear, Phys. Rev. Letters 9, 221 (1962).

14. The reader is referred also to the data of A. N. Diddens, E. Lillithun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, Phys. Rev. Letters 9, 111 (1962). If one considers all their data, the result $\alpha(-70) < 0$ is definitely established. However, their results for $t > -50$ are too linear and therefore not consistent with Eq. (F1) in footnote 11. If one takes only their data for $(s/2M^2) > 10.5$, their results agree with those of Baker et al.,[13] although with larger errors.

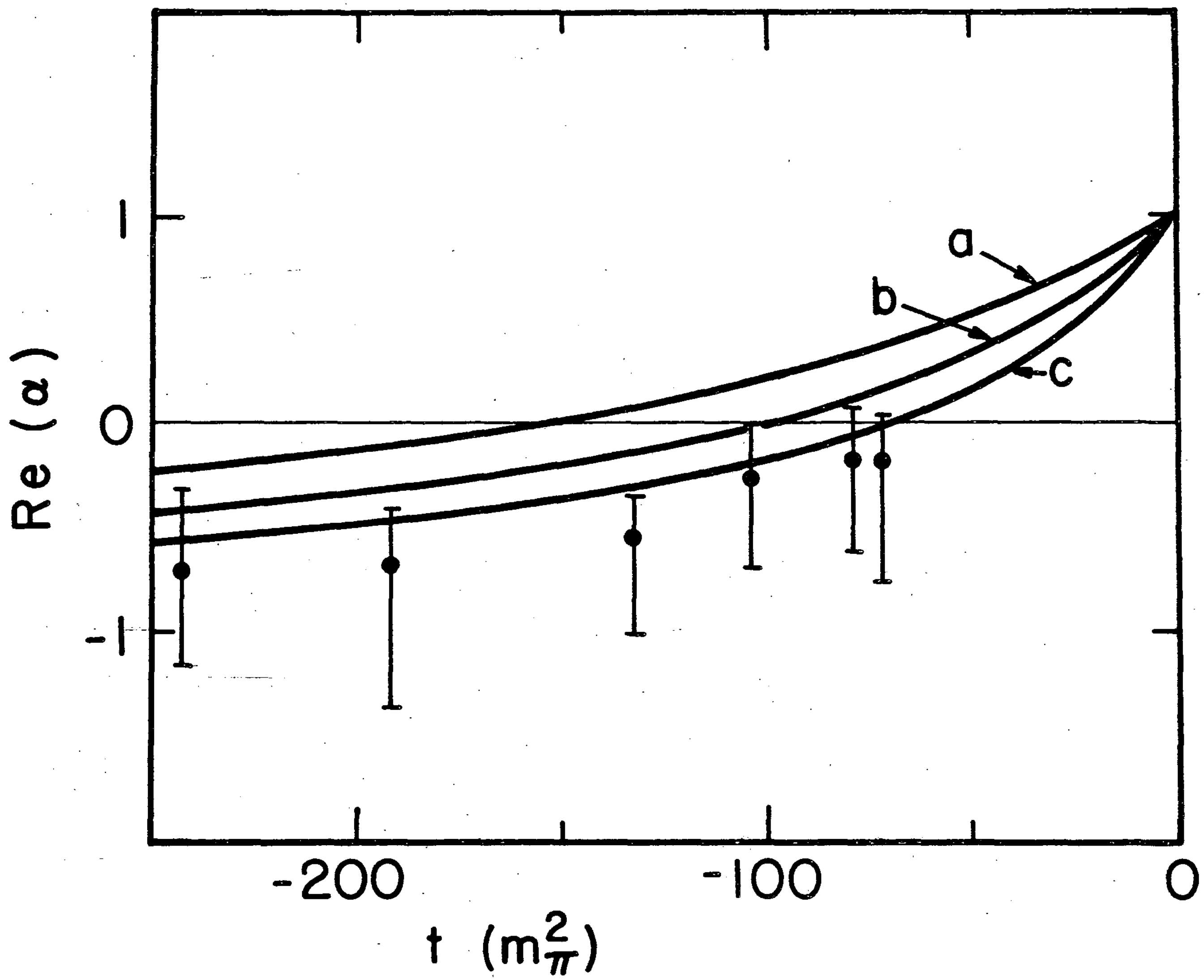
15. Chew has pointed out to me that both groups reporting on the f_0 have observed some anomalies in the angular distribution around the ρ peak that could be attributed to an $I = 0$ state because they were not present in the analogous experiment with $\pi^+\pi^0$ or $\pi^-\pi^0$. On the other hand, I am indebted to Dr. Takeda for indicating that if the C. F. particle has about the same energy as the ρ meson, its width is likely to be less than the experimental resolution, on the basis of the measured cross sections for ρ^+ and ρ^-. A. Ahmadzadeh and I. Sakmar (see Lawrence Radiation Laboratory, Report UCRL-10584, December 1962) have also suggested the possible connection of the C. F. particle with other experimental information for $t < 30$.

16. The possibility of the Pomeranchuk trajectory "bending down" soon and never reaching 2 cannot be excluded. However, this is not likely to occur if the quantum numbers of the f^0 have been assigned correctly, which would imply that another trajectory with the same quantum numbers as the Pomeranchuk, but below it at $t = 0$, is able to reach $\mathrm{Re}\,\alpha = 2$.

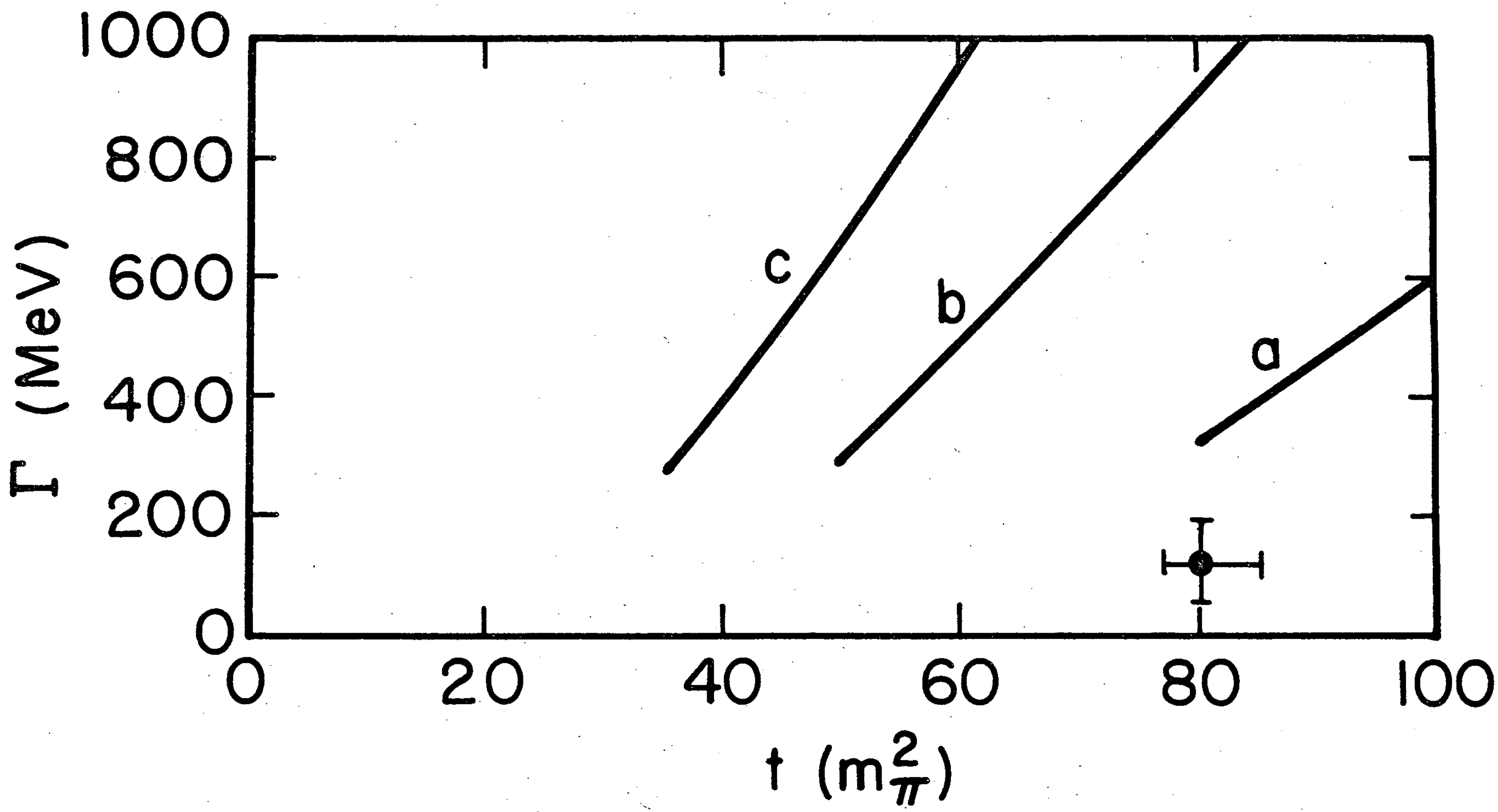
FIGURE LEGENDS

Fig. 1. Curves a, b, and c are lower bounds for $\alpha(t)$ given by Eqs. (7), (H3), and (H4) for $\alpha'(0) \leq 1/80$, 1/50, and 1/35, respectively. The experimental data are from Ref. 13.

Fig. 2. Curves a, b, and c show lower bounds for $\Gamma(t_R)$ given by Eq. (17) for $\alpha'(0) \geq 1/80$, 1/50, and 1/35, respectively. The experimental values for the energy squared and width of the f^0 are also indicated (Refs. 1 and 2).



MU-28927



MU-28928