Title
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Journal
Journal of Water Resources Planning and Management, 126(1)

ISSN
0733-9496

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Publication Date
2000

DOI
10.1061/(asce)0733-9496(2000)126:1(30)

Peer reviewed

Closed-Form Solution for Coastal Aquifer Management

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Abstract: Ground-water management is posed as the maximization of the expected value of net revenue accruing from the sale of ground water, subject to hydrogeologic, climatic, and environmental conditions. The market price of ground water, the cost of ground-water extraction, and the discount rate are economic factors included in the development of long-term aquifer management policies. Closed-forms solutions were derived for the optimization problem, and the sensitivity of solutions to the market price of ground water, the cost of ground-water extraction, climate conditions (wet or dry), and the initial hydraulic heads was determined. The optimization method was applied to a coastal aquifer and long-term management policies were obtained under a variety of climatic, economic, and hydraulic scenarios.

INTRODUCTION

California's waterscape evolved significantly as a result of the 1987–1992 drought (Loáiciga and Renehan 1997). Water agencies have aggressively looked for alternative, sustainable sources of water. Those sources include water conservation, sewage effluent reclamation, ground water banking, ocean water desalination, and water marketing. With the growth of water markets in California and elsewhere as a background, this work poses the withdrawal and sale of ground water as a long-term revenue maximization problem subject to climatic, hydrogeologic, economic, and environmental constraints. The unique feature of the proposed approach to ground-water management is its reliance on closed-form solutions of the associated optimization problem. The approach presented herein is applicable to relatively homogeneous aquifer systems of limited size, wherein climate and environmental conditions are approximately uniform. The writer's approach is structured to provide flexibility in the assessment of output sensitivity to important factors such as climate change, initial aquifer heads, discount rate, cost of ground-water pumping, and the market price of ground water. The closed-form solutions to the ground-water management problem derived in this work are intended to serve as guidelines for investigating long-term water policy options under a variety of scenarios. In this respect, they differ from operational or real-time aquifer management schemes devised for relatively short time horizons. More complex, numerical approaches to ground-water management are found, e.g., in Willis and Yeh (1987) or Pelmulder et al. (1996).

AQUIFER STORAGE DYNAMICS IN COASTAL AQUIFER

Hydrogeologic Setting

Fig. 1 shows a vertical geologic profile of the main aquifer (called storage unit I) which underlies the city of Santa Barbara, California (from Martin and Berenbrock 1986) and which constitutes the focus of this study. Storage unit I has an estimated ground storage capacity between 9.9×10^6 and 12.3 $\times 10^6$ m^3, and it is delimited by several semipermeable faults. It discharges to the Pacific Ocean along an off-shore fault, which is permeable to ground-water flow. The hydrogeology of storage unit I has been analyzed in Martin and Berenbrock (1986), Freckleton (1989), and McFadden et al. (1991). Storage unit I contains the most productive wells in the study area and is susceptible to seawater intrusion.

Fig. 1 depicts five aquifer layers, here called "zones." Zones 1 and 2 have similar hydraulic properties and constitute the upper (unconfined) aquifer. Zones 3, 4, and 5 are where most of the well screens are located, and they constitute the principal source of ground water in storage unit I. Zones 3, 4, and 5 are lumped into a single production zone for the purpose of analysis in this study. The average hydraulic properties of the unconfined and production zones have been determined from pumping tests in water wells that fully penetrate them (Loáiciga 1997). Ground-water recharge to storage unit I originates from fracture flow in consolidated formations that are part of the mountain range located along the northern aquifer boundary. A smaller amount of recharge stems from seepage in Mission Creek, the main drainage in storage unit I.

Tank Model for Ground Flow in Storage Unit I

Fig. 2 presents a simplified "tank" model of the ground-water flow system in storage unit I. The production zone (formed by zones 3, 4, and 5 in Fig. 1) is a semiconfined aquifer with a hydraulic head h_2, which is overlain by an unconfined aquifer (zones 1 and 2 in Fig. 1) whose water table is at an elevation h_3. The storage coefficients of the unconfined and confined aquifers are S_3 and S_2, respectively. The hydraulic connection between the unconfined and confined aquifers is modeled by an aquitard of thickness b_2 and hydraulic conductivity K_2. The unconfined and confined aquifers are recharged by a bedrock aquifer that has a hydraulic head h_1 and a storage coefficient S_1. A vertical aquitard of thickness b_1 and hydraulic conductivity K_1 separates the bedrock aquifer from the unconfined and semiconfined aquifers. The situation depicted in Fig. 2 is a simplified parameterization and representation of the hydraulic interaction and ground-water recharge observed in the study area. Fractured rocks serve as a conduit for deep ground-water recharge to semiconfined and unconfined aquifers, where the latter, in addition, receive recharge from channel seepage (represented by R_3 in Fig. 2). The bedrock aquifer is recharged by percolating precipitation in the highlands (R_1 in Fig. 2). The semiconfined aquifer, due to its better water quality, is the source of the water supply. The pumping rate of the confined aquifer is represented by Q in Fig 2. The relative magnitudes of the hydraulic heads in the unconfined and semiconfined aquifers determine the direction of water exchange among them. Discharge is typically from the coastal aquifers to the ocean (of water density $\rho_S = 1.025$ g/cm^3, while the density of freshwater $\rho = 1.0$ g/cm^3), which has a mean sea level elevation h_S, although the oceanward direction of flow can be reversed by low hydraulic heads in the aquifers. The Darcian flows between the coastal aquifers and the ocean are regulated by the intervening hydraulic heads and the thick-

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Note. Discussion open until July 1, 2000. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this technical note was submitted for review and possible publication on December 2, 1997. This technical note is part of the ***Journal of Water Resources Planning and Management***, Vol. 126, No. 1, January/February, 2000. ©ASCE, ISSN 0733-9496/00/0001-0030–0035/$8.00 + $.50 per page. Technical Note No. 17131.

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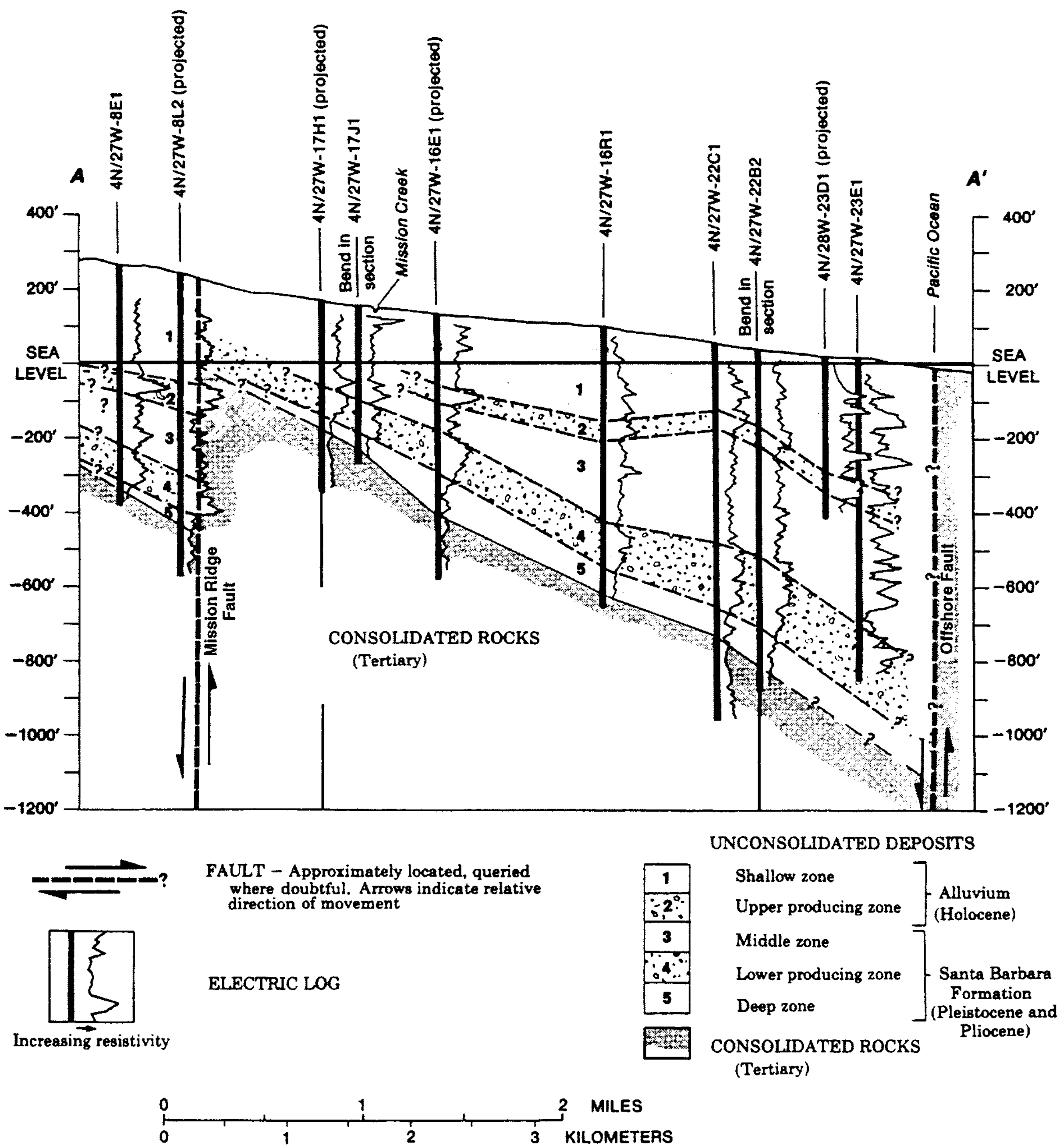

FIG. 1. Vertical Cross-Section of Aquifer System and Main Hydrostratigraphic Units (After Martin and Berenbrock 1986)

ness (b_S) and hydraulic conductivity (K_S) of the ocean bottom sediments. The dimensions of the aquifers are denoted according to the notation of Fig. 2, where it is assumed that the width of aquifers perpendicular to the plane of the figure is W.

The following system of differential equations describes the time evolution of the hydraulic heads (h) in the various aquifers of Fig. 2 (obtained by applying the equation of conservation of mass in each of the aquifers, i.e., the change of storage equals the difference between inputs and outputs to each aquifer, in conjunction with Darcy's law and the definition of the storage coefficient as the volume of water released per unit area of aquifer per unit of hydraulic head drawdown):

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & k & m \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} + \begin{bmatrix} n \\ p \\ q \end{bmatrix} \tag{1}$$

in which $\dot{h}_i = dh_i/dt$ denotes the rate of time change of the ith hydraulic head (i = 1, 2, 3) and the coefficients a, b, c, d, e, f, g, k, m, n, p, and q are given the following:

$$a = -K_1L_{V2}/(S_1L_1b_1) - K_1L_{V3}/(S_1L_1b_1)$$

$$b = K_1L_{V2}/(S_1L_1b_1)$$

$$c = K_1L_{V3}/(S_1L_1b_1)$$

$$d = K_1L_{V2}/(S_2L_2b_1)$$

$$e = -K_2/(S_2b_2) - K_1L_{V2}/(S_2L_2b_1) - K_SL_{V2}/(S_2L_2b_S)$$

$$f = K_2/(S_2b_2)$$

$$g = K_1L_{V3}/(S_3L_2b_1)$$

$$k = K_2/(S_3b_2)$$

$$m = -K_2/(S_3b_2) - K_SL_S/(S_3L_2b_S) - K_1L_{V3}/(S_3L_2b_1)$$

$$n = R_1/S_1$$

$$p = K_SL_{V2}\rho_Sh_S/(S_2L_2b_S) - Q/(S_2L_2W)$$

$$q = K_SL_S\rho_Sh_S/(S_3L_2b_S) + R_3/S_3$$

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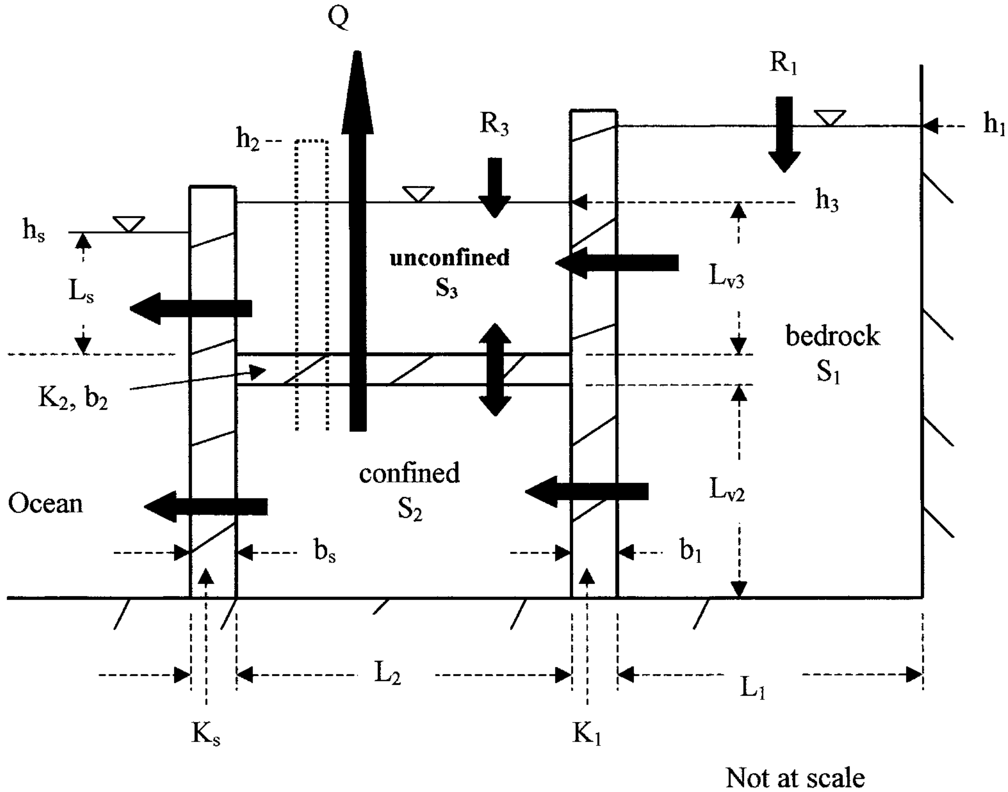

FIG. 2. Conceptual "Tank" Model of Aquifer System

TABLE 1. Values of Hydrogeologic Parameters (See Fig. 2)

Parameter (1)	Value (2)	Dimensions (3)
K_1	182.5	m/yr
K_2	36.5	m/yr
K_s	36.5	m/yr
S_1	0.125	—
S_2	0.05	—
S_3	0.05	—
ρ_s/ρ_w	1.025	—

TABLE 2. Aquifer Geometry Data (See Fig. 2)

Parameter (1)	Value (2)	Dimensions (3)
b_1	10	m
b_2	5	m
b_s	5	m
h_s	75	m
L_1	10^3	m
L_2	2.5×10^3	m
L_s	20	m
L_{v2}	50	m
L_{v3}	variable	m
W	5×10^3	m

Table 1 shows the value of all the hydrogeologic parameters used in this work. Table 2 contains aquifer geometry data. All other data are presented in the following sections.

Closed-Form Solution for Ground-Water Flow

The system of differential equations in (1) can be written in compact form in vector/matrix notation as follows:

$$\dot{h} = \mathbf{A}\mathbf{h} + \mathbf{B} \tag{2}$$

with the initial condition $\mathbf{h}(t = 0) = h_0$, where $\mathbf{h}$ is a 3×1 vector of hydraulic heads: $\mathbf{A}$ is a (3×3) matrix of coefficients (i.e., aquifer parameters and geometry); and the (3×1) vector $\mathbf{B}$ is, in addition to hydrogeologic parameters and geometry parameters, a function of aquifer stresses such as pumping and ground-water recharge.

Letting λ_i, $\boldsymbol{\mu}_i$, and $\boldsymbol{\nu}_i$ denote an eigenvalue and the (3×1) right and left normalized column eigenvectors of the matrix $\mathbf{A}$, respectively, for $i = 1, 2, 3$, it is known that the solution of (2) is given by the following expression:

$$h(t) = \left\{\sum_1^3 e^{\lambda_i t}\boldsymbol{\mu}_i\boldsymbol{\nu}_i^T\right\} h_0 + \left\{\sum_1^3 \frac{e^{\lambda_i t} - 1}{\lambda_i}\right\} \boldsymbol{\mu}_i\boldsymbol{\nu}_i^T B \tag{3}$$

in which T denotes the transpose of a (column) vector. Eq. (3) gives a closed-form solution for aquifer hydraulic heads as a function of time, initial conditions, aquifer recharge, and pumping rate.

FORMULATION OF GROUND-WATER MANAGEMENT MODEL

The pumping rate Q (in m^3/yr) is the decision or management variable in the writers' formulation of the ground-water extraction optimization problem. Q is the annual pumping rate that maximizes the expected present value of the net revenue stream that is generated from ground-water extraction over an indefinitely long period of time. Q is subject to the aquifer recharge and ground water flow dynamics, dictated by (3), plus prescribed constraints that ensure the long-term viability of the ground-water resource. Aquifer recharge is largely driven by rainfall. The annual rainfall (R, in mm/yr) has been fitted with a two-parameter (α and β) gamma distribution (where Γ denotes the gamma function) as follows:

$$f_R(r) = \frac{\alpha^{\beta+1} r^{\beta} e^{-\alpha r}}{\Gamma(\beta + 1)} \tag{4}$$

with a mean $(\beta + 1)/\alpha$ (=0.46 m/yr) and a variance $(\beta + 1)/\alpha^2$ (=6.25×10^{-2} m^2/yr^2). The ground-water recharge by rainfall percolation in the bedrock aquifer, R_1, is equal to a fraction

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f_R of annual rainfall, i.e., $R_1 = f_R R$, in which $f_R = 0.15$ (Freckleton 1989). Likewise, seepage recharge along Mission Creek channel to the unconfined aquifer, R_3, is a fraction of annual rainfall, i.e., $R_3 = f_S R$, where $f_S = 0.04$ (McFadden et al. 1991). The other source of statistical uncertainty is the (annual) real discount rate S, which, following Arrow and Intriligator (1986), is modeled by a two-parameter (φ and γ) gamma distribution with mean $(\gamma + 1)/\varphi$ (=0.10) and variance $(\gamma + 1)/\varphi^2$ (=2.5 × 10^{-3}). The real discount rate considers the joint effect of the nominal interest rate and the rate of inflation (the latter increases the price of water over time).

The market price of ground water is denoted by P (in \$/m^3 of ground water). The cost of ground-water extraction, C (in \$/m^3 of ground water) is given by $C = d^* - b^* h_2$ in which $d^* = 2$ (\$/m^3) and $b^* = 1.25 \times 10^{-2}$ (\$/m$^3 \cdot$m) are parameters, and h_2 (in m) is the hydraulic head in the production zone (semiconfined aquifer). The head h_2 is the second row of the vector solution (3) and it depends, among other things, on the pumping rate Q. The parameter b^* represents the slope of the cost function C. The net revenue that accrues from the sale of one unit of ground water is $P - [d^* - b^* h_2]$. The number of annual units of ground water pumped is Q (in m^3/yr), so that in any one year the net revenue from ground-water sales is $Q \cdot \{P - [d^* - b^* h_2]\}$. The present value (i.e., in year $t = 0$) of the net revenue accruing in year t is $Q \cdot \{P - [d^* - b^* h_2]\} \cdot e^{-St}$. Therefore, the expected present value of the net revenue stream generated by the pumping rate Q is obtained by integrating $Q \cdot \{P - [d^* - b^* h_2]\} \cdot e^{-St}$ over time t (a long time period, in years) and weighing it by the probability distribution functions of ground-water recharge and discount rate $[f_S(s)]$, the latter two integrated over their respective domains. Specifically, the objective function of the ground-water management problem is given by

$$F = \max_{w \cdot r \cdot tQ} \int_0^\infty \int_0^\infty \int_0^\infty \{P - (d^* - b^* h_2)\} Q e^{-st} f_S(s) f_R(r)\, dr\, ds\, dt \tag{5}$$

The optimization problem embodied by (5) is subject to the expected value of aquifer dynamics and to constraints on hydraulic heads, which are not allowed to fall below the freshwater equivalent of the ocean mean sea level $[=(\rho_S/\rho) h_s$, in which ρ is the density of freshwater = 1.0 g/cm^3]. Specifically, (5) is subject to

$$\bar{\dot{h}} = \mathbf{A}\bar{\mathbf{h}} + \bar{\dot{\mathbf{B}}} \tag{6}$$

and

$$\bar{h} \geq \frac{\rho_S}{\rho} h_S \tag{7}$$

in which the overbar denotes expected value.

SOLUTION TO OPTIMIZATION PROBLEM

The solution, Q^*, to the optimization problem embodied by (5)–(7) was obtained by incorporating the constraint [(6)] into the integral equation [(5)], followed by integration and maximization of the resulting expression. The following solution was obtained (in which $d_1 = 77.4$, $e_1 = -3.95 \times 10^{-7}$, and $f_1 = 0.491$):

$$Q^* = \underset{(\text{if } Q^* > 0)}{\text{smaller of}} \left[\begin{array}{c} \dfrac{\dfrac{\rho_S}{\rho} h_S - \dfrac{\beta + 1}{\alpha} f_1 - d_1}{e_1} \\ -\dfrac{1}{2L_2^*} \left\{ \dfrac{P - d^*}{b^*} + \dfrac{\beta + 1}{\alpha} L_1^* + L_0 \right\} \end{array} \right];$$

$$\text{else } Q^* = 0 \tag{8}$$

Prior to defining L_0, L_1^*, and L_2^* in (8), let us introduce the following terms [in which the constants $k_{11} = -0.00268$; $k_{12} = 0.506$; $k_{13} = -0.500$; $k_1 = -0.229$; $k_{21} = -0.339$; $k_{22} = 0.187$; $k_{23} = 0.191$; $k_2 = -2.89$; $k_{31} = 0.343$; $k_{32} = 0.308$; $k_{33} = 0.309$; and $k_3 = -74.3$ arise from the integration of (5), and h_{01}, h_{02}, and h_{03} represent the initial hydraulic heads in aquifers 1, 2, and 3, respectively]:

$$a_i = \sum_{j=1}^{3} k_{ij} H_{oj} + k_i \quad (i = 1, 2, 3) \tag{9}$$

and [letting λ_i, $i = 1, 2, 3$, represent the eigenvalues of the matrix $\mathbf{A}$ in (2); while φ and γ are parameters in the distribution of the discount rate and equal 40 and 3, respectively]:

$$I_i = \int_0^\infty (u + \varphi|\lambda_i|)^{-1} u^\gamma e^{-u}\, du \quad (i = 1, 2, 3) \tag{10}$$

L_0 is defined by

$$L_0 = d_1 + \frac{\gamma}{\Gamma(\gamma + 1)} \sum_{i=1}^{3} a_i I_i \tag{11}$$

The expressions for L_1^* and L_2^* are as follows (in which the following constants are required: $b_1^* = -2.69 \times 10^{-9}$; $b_2^* = 14.7 \times 10^{-9}$; $b_3^* = 378 \times 10^{-9}$; $c_1 = 0.00134$; $c_2 = 0.0124$; and $c_3 = -0.505$):

$$L_1^* = f_1 + \frac{\gamma}{\Gamma(\gamma + 1)} \sum_{i=1}^{3} c_i I_1 \tag{12}$$

$$L_2^* = e_1 + \frac{\gamma}{\Gamma(\gamma + 1)} \sum_{i=1}^{3} b_i^* I_i \tag{13}$$

Eqs. (8)–(13) show the dependence of the optimal pumping rate Q^* on all economic (P, b^*, d^*, γ, φ) and climatic (α, β) parameters, as well as on initial hydraulic heads (h_{01}, h_{02}, and h_{03}).

RESULTS AND CONCLUSIONS

Data

Hydrogeologic and aquifer geometry data are presented in Tables 1 and 2, respectively. The current market price of ground water, P, is \$1.0/m^3. Two other values are considered in this work, \$0.5/m^3 and \$1.5/m^3, to assess the impact of price on management policies. The cost of ground-water pumping depends primarily on the cost-slope parameter b^*, and this is allowed to vary between slightly below 0.0124 and slightly above 0.0127 to assess its impact on optimal pumping rates. The parameter b^* has a value of 0.0125 in the study area. Two sets of initial hydraulic heads are considered in this study. The first set corresponds to so-called ''high'' initial conditions (h_{01}, h_{02}, h_{03} equal to 150, 135, and 130 m, respectively) characterized by high hydraulic heads in the three aquifers and steep hydraulic gradients that drive ground-water flow among the aquifers. The second set of initial conditions, termed ''low,'' consists of low hydraulic heads ((h_{01}, h_{02}, h_{03}, equal to 125, 120, and 120 m, respectively) and associated hydraulic gradients. The set of initial conditions labeled high is used as a baseline case, since it represents conditions currently occurring in the study area. Solutions are found for a number of combinations of economic, climatic, and initial hydraulic heads.

Results for Baseline (High) Initial Hydraulic Heads and Current Climate

Fig. 3 shows the optimal pumping rate Q^* obtained for a range of values of the cost-slope parameter b^*, and for three levels of ground-water prices (P = \$1/m^3, the current price; P

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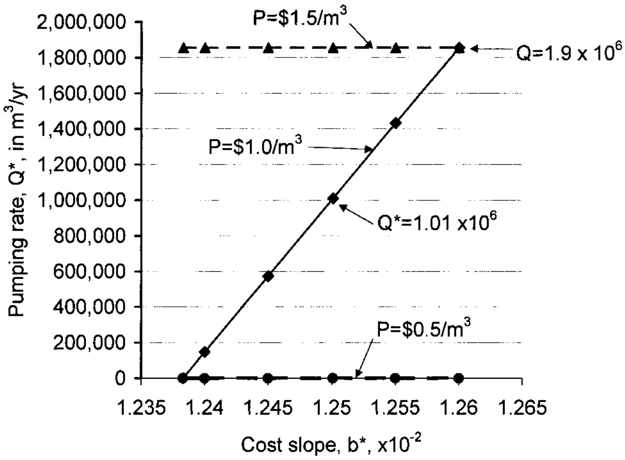

FIG. 3. Pumping Rate Q^* as Function of Cost-Slope and Ground Water Price under Current Climate and High Initial Hydraulic Heads

= \$0.5/m^3; and P = \$1.5/m^3). Hydraulic heads start at a high level, and the climate is characterized by the current mean annual rainfall of 0.46 m/yr with a standard deviation of annual rainfall equal to 0.25 m/yr. The parameter b^* measures the rate of change of pumping cost as a function of the hydraulic head in the productive aquifer, h_2. Notice that as the cost-slope parameter *increases*, the cost of pumping *decreases*. The value of the cost-slope parameter under current conditions is b^* = \$0.0125/m$^3 \cdot$m, for which the optimal pumping rate is $Q^* = 1.01 \times 10^6$ m^3/yr, as shown in Fig. 3. The parameter b^* is allowed to vary between slightly below 0.0124 and 0.0126 in Fig. 3 to illustrate its impact on the optimal pumping rate. When P = \$1/m^3, as b^* approaches its lower limit, the cost of pumping increases to the point where the optimal strategy is not to extract any ground water (i.e., $Q^* = 0$). As b^* increases, the cost of ground extraction diminishes and, thus, Q^* increases to a maximum of $Q^* \cong 1.9 \times 10^6$ m^3/yr, the largest rate at which the aquifer can be pumped without inducing seawater intrusion. Notice how the line corresponding to a price of ground water of \$1/m^3 (labeled P = 1 in Fig. 3) shows a nearly linear relationship between the optimal pumping rate Q^* and the cost-slope parameter b^* in the feasible range of pumping rate (i.e., in the range of 0 to 1.9×10^6 m^3/yr). Fig. 3 shows also the optimal pumping rate as a function of the cost-slope parameter when the market price of ground water P = \$1.5/m^3 or P = \$0.5/m^3. In the former case, the marginal net revenue per unit of extracted ground water is so large that the pumping rate equals its maximum feasible value of 1.9×10^6 m^3/yr for all levels of the cost-slope parameter b^*. In the latter case, the net revenue turns out to be negative for all levels of pumping cost considered. Thus, the optimal pumping strategy is to not extract any ground water, i.e., $Q^* = 0$ when P = \$0.5/m^3.

Impacts of Changes in Model Parameters and Initial Conditions

Table 3 shows the optimal pumping rate Q^* and associated expected net revenue for a range of values of the cost-slope parameter b^* and for three mean annual rainfall conditions: (1) mean rainfall equal to present value (=0.46 m/yr); (2) a wetter rainfall scenario (mean annual rainfall 10% higher than the current mean value); and (3) a dryer rainfall scenario (mean annual rainfall 10% lower than the current mean value). The price of ground water is P = \$1.0/m^3 in the calculations results shown in Table 3. It is seen in Table 3 that the variations in the cost-slope parameter b^* induce changes in the pumping rate Q^* and the expected net revenue F^* that are much larger than those due to the changes in the mean annual rainfall. The pumping rate and expected revenue in a wetter climate increase relative to those achieved in the current climate for all levels of the cost-slope parameter. In a dryer climate, the pumping rate and expected revenue decrease relative to those calculated for the current climate for all levels of the cost-slope parameter. Evidently, the aquifer recharge increases in a wetter climate, which leads to higher pumping rates relative to the present climate, provided that the cost of pumping and the price of ground water remain constant. Diminished aquifer recharge in a dryer climate produces the opposite effect, thus leading to lower pumping rates than those achieved under the current climate when all other factors remain fixed. It can be seen in Table 3 that the present value of net revenue obtained under the present cost (b^* = \$0.0125/m$^3 \cdot$m), water price, and climate is \$63,500.

Fig. 4 shows the impact of changes in the initial hydraulic heads on the optimal pumping rate for a range of values of the cost-slope parameter and for a price of ground water P = \$1.0/m^3. Two sets of initial hydraulic heads are considered herein,

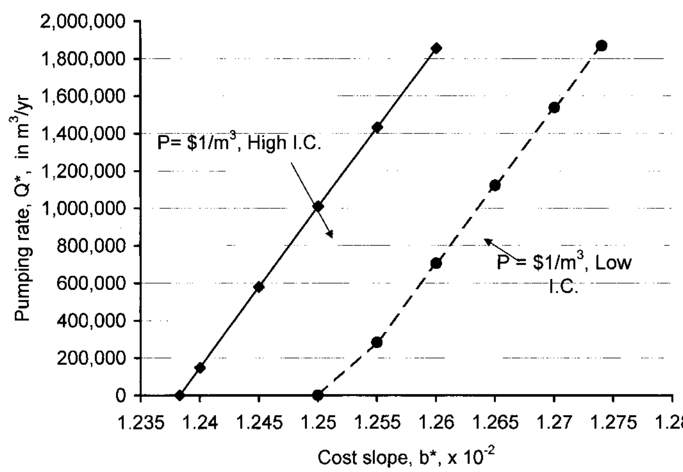

FIG. 4. Pumping Rate as Function of Cost-Slope for Both High and Low Initial Hydraulic Heads under Current Climate and Price of Water = \$1/m^3 (I.C. Initial Hydraulic Head Conditions)

TABLE 3. Pumping Rate (Q^*) and Expected Net Revenue (F^*) as Function of Annual Rainfall: High Initial Hydraulic Heads and Water Price = \$1/m^3

Cost slope ($b^* \times 10^{-2}$) (1)	Mean Rain Q^* (m^3/yr) (2)	Mean Rain F^* (\$/yr) (3)	+10% Rain Q^* (m^3/yr) (4)	+10% Rain F^* (\$/yr) (5)	−10% Rain Q^* (m^3/yr) (6)	−10% Rain F^* (\$/yr) (7)
1.2389	0	0	37,000	86	0	0
1.240	148,000	1,360	176,000	1,920	119,000	880
1.245	580,000	20,900	608,000	23,000	551,000	18,900
1.250	1,010,000	63,500	1,036,000	67,120	979,000	59,900
1.255	1,433,000	128,900	1,461,000	134,000	1,404,000	123,700
1.260	1,855,000	216,800	1,882,000	223,000	1,826,000	210,000

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i.e., ''high'' and ''low,'' whose values and meanings were discussed earlier in the section concerning data specification. The two curves depicted in Fig. 4 are drawn for the range of cost-slope parameter for which the pumping rates meet the feasibility criteria in each case. It is apparent from Fig. 4 that, for any given value of the cost-slope parameter b^*, the pumping rates in the case of high initial hydraulic heads exceed those associated with low initial hydraulic heads. This follows from the larger water fluxes that occur in the aquifer system when initial heads are high compared with those that take place when initial hydraulic heads are low. In addition, Fig. 4 indicates that the curve corresponding to low initial hydraulic heads is shifted right relative to the curve associated with high initial heads. Thus, to achieve the same level of optimal pumping rate, the cost of pumping under low initial conditions must be lower than that which prevails under high initial hydraulic heads (notice that a lower pumping cost is synonymous to a larger cost-slope parameter). In fact, for the current cost of pumping, exemplified by $b^* = 0.0125$, the pumping rate under low initial conditions drops to zero. Therefore, it would not be profitable to extract ground water were the initial hydraulic heads to drop to the low level considered herein.

ACKNOWLEDGMENT

This research was partially funded by grant ATM-9711491 from the National Science Foundation.

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