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QUENCH PROTECTION FOR SUPERCONDUCTING SOLENOIDS WITH A CONDUCTING BORE TUBE*

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ABSTRACT

A method to check the design of the quench protection of a superconducting solenoid with a conducting bore tube is described and justified. These checks assume the most pessimistic circumstances. Instructions on how to revise the magnet parameters if the tests fail are given. The dependence on the current, the diameter and the length of the solenoid are discussed.

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1. Purpose

In this paper, we set requirements for a quench protection system to insure that any superconducting magnet is safe against destruction during a quench. The requirements apply to the case of a superconducting D.C. solenoid with a bore tube made of electrically conducting material.[1-3] Some of the conclusions can be extended to other kinds of magnets in the limit where the volume of the bore tube is equal to zero. The computations that have to be carried out necessitate only the use of graphs and of a hand calculator.

It is difficult to describe quenches accurately because of the many parameters involved.[2-4] All we know is that a quench is a phenomenom that turns part or all the coil normal. To derive our rules, we have developed a pessimistic quench theory, i.e. a theory where our simplifying approximations always make things worse than real life. For example, though heat conductivity helps both in reducing the temperature of the coil's hottest point and in decreasing the coil current by turning more of the coil normal, these aspects of heat conductivity are ignored. The requirements determined are more stringent, but the equations are simpler. It follows that a magnet may very well survive a quench if its quench circuit violates our requirements, but we are sure it is safe if its quench circuit abides with them.

The theory relies on phenomena derived from basic principles only, such as energy conservation and Maxwell equations. Later on, we may acquire more confidence in some helpful phenomena that we are yet unable to describe accurately. Also, we may use more sophisticated mathematical solutions. At that time, we intend to update our theory and derive less stringent rules based on less pessimistic assumptions. For the time being, in this paper, we take an extreme position in favor of security.

The maximum temperature which can be reached in the superconductor is a function of the integral current density squared with time. The maximum value for this integral which results in safe magnet operation is about 10^{17} $A^2m^{-4}s$ for copper based superconductor and about $4x10^{16}$ $A^2m^{-4}s$ for aluminum based superconductor. The role of the bore tube is to reduce the current density in the conductor very quickly without inducing large transient voltages. The coupled differential equations which describe the coil-bore tube circuit are given in Section 4. A solution to these equations is given in Section 5. This solution, which is simplified, shows that the bore tube does reduce the current in the coil very quickly.

Sections 6, 7, 8, and 9 present the pessimistic approximation of the hot spot temperature in the superconducting coil. An external resistor is assumed. If the quench is detected quickly and the power supply is disconnected, the pessimistic theory shows that high current density magnets with closely coupled bore tubes can be made to quench safely. Section 10 takes into account the change of resistance of the bore tube with temperature. Sections 11, 12 and 13 show that for the same peak voltage across the coil, a varistor, which is a resistor with non-linear voltage versus current characteristics, causes the current in the magnet to drop even faster than a resistor with linear voltage versus current characteristics. The varistor will then result in lower hot spot temperature.

Section 14 summarizes the rules for calculating the hot spot temperature using the pessimistic theory. Section 15 shows how the first-trial design parameters can be modified to ensure a safe magnet. Section 16 shows, by example, the design of a quench protection system that will result in safe quenching of the LBL A and B Magnets and the final TPC Detector Magnet.

For solenoids with an iron yoke, Sections 17 and 18 show the scaling laws for magnet safety. These laws act as quide lines for designing large safe magnets based on previous safe designs. In general, the following can be said for magnet safety when scaling a magnet from one size to another: (1) If the magnet quenches safely at its maximum current, it will quench safely at all currents lower than its maximum current. (2) For a given stored energy, fewer number of turns will result in lower transient voltages. (3) For a given hot spot temperature, bore tube temperature and transient voltage conditions, the current density in the coil must drop as the square root of the magnet stored energy.

2. Problems associated with a quench.

During a quench, one of the dangers is burn out. A small piece of wire ends up collecting too much of the magnet energy and gets damaged by overheating. It takes a finite time to heat a piece of material to a given temperature with a given current density. A sure way to keep this phenomenon under control is to make sure the current i in the coil is turned off quickly after the beginning of the quench. In section 7 and in ref. 5, it is shown how an upper limit T_{lim} for the temperature anywhere in the magnet coil can be derived, knowing the initial current i_o in the magnet, the wire characteristics and the effective time τ_J

$$\tau_J = \int \left(\frac{i}{i_o}\right)^2 dt \qquad (2.1)$$

The smaller τ_J is, the smaller T_{lim} will be. If one makes sure that the coil current i drops quickly enough as a function of the time t, the quantity τ_J computed by equation (2.1) is small and the corresponding T_{lim} will be an acceptable value for the temperature. Therefore, anywhere in the coil, the temperature will be less than T_{lim} and there will be no burn out.

Another danger during the quenches comes from the voltages that are developed in the coil. They may exceed the value allowed by the insulation. The voltages developed between any two points of the coil are related to the change of flux ϕ. The total inductive voltage v_ϕ across the whole coil is

$$v_\phi = -n_c \frac{d\phi}{dt} \qquad (2.2)$$

where n_c is the number of turns of the coil and ϕ the flux in one turn of the magnet coil. If one makes sure that v_ϕ is less than the value which the insulation can withstand, we insure the coil against damage from excessive voltage. For this purpose, the variations of the flux ϕ have to be kept slow.

In a magnet where the only contribution to the flux ϕ comes from the current i, security against burnout requires short time constants but protection against excessive voltages requires long time constants. It may be impossible to find a compromise between the two.

If the magnet is a solenoid with an electrically conducting bore tube,[1-3] then the current i can be turned off quickly to avoid burnout while the flux ϕ decays slowly because of the induced current i' in the bore tube.[3,4] Thus the coil voltage can be kept small. There is no danger of excessive voltage in the bore tube either, because it is a coil with only one turn. Most of the energy will end up in the bore tube, but there is no danger of overheating in the bore tube if its volume can absorb the total magnet energy, because a bore tube made of homogenous material will heat up evenly.

3. The electric circuit.

The diagram of fig. 1 respresents the electric circuit corresponding to the magnet coil, its bore tube, its current supply and a quench protection circuit. During normal operation, the switch S_o is closed and the current $i = i_o$ flows through the magnet coil. There is no current in the bore tube because it is made of conducting, not superconducting material.

When a quench develops in the magnet coil, we assume that it is detected soon after it has started and that an automatic device then opens the switch S_o disconnecting the power supply. The coil current i is first switched into the coil shunt shown on fig. 1, that, for the time being, we assume to be a pure resistance R_{ext}. The resistor in the coil circuit provokes the decay of the current i, thus some decay of the flux ϕ. The change in ϕ induces a current i' in the bore tube circuit that limits $\frac{d\phi}{dt}$, which in turn limits the value of the maximum voltage v_ϕ. After a short while (see section 5, eq. (5.12) and fig. 2), the current i finds a new equilibrium and decays slowly with i'. Then, the ratio of the two currents is given by

$$\frac{i}{i'} \cong \frac{n_c R'}{R} \tag{3.1}$$

where R is the resistance of the coil circuit and R' that of the bore tube. If R' is small enough, the current i is small and the quantity τ_J of equation (2.1) can be made acceptable.

4. Basic equations after switch opening.

After the quench has been detected and the switch S_0 shown on Fig. 1 is open, the relevant equations are the standard equations that described two systems magnetically coupled to one another.

$$\begin{cases} L\frac{di}{dt} + Ri + M\frac{di'}{dt} = 0 \\ L'\frac{di'}{dt} + R'i' + M\frac{di}{dt} = 0 \end{cases} \tag{4.1}$$

We define the quantity ψ (measured in amps) equal to the flux through the bore tube divided by the mutual inductance M. M is the scale factor that makes ψ comparable to the current i in the coil. The product $n_c \cdot \psi$ is the total number of ampere turns from all sources producing flux in the bore tube. The low resistance of the bore tube will oblige ψ to always vary slowly.

$$\begin{cases} \psi = \frac{L'}{M} i' + i \\ i' = \frac{M}{L'} (\psi - i) \end{cases} \quad (4.2)$$

$\tau = \frac{L}{R}$ = Time constant of coil circuit if the bore tube were not conducting. (4.3)

$\tau' = \frac{L'}{R'}$ = Time constant of bore tube. (4.4)

$r_\tau = \frac{\tau}{\tau'}$ = Ratio of time constants (4.5)

$$\varepsilon = 1 - \frac{M^2}{LL'} \quad (4.6)$$

ε is assumed to be small, less than a few %.

$$\varepsilon \frac{di}{dt} + \frac{i}{\tau} + (1 - \varepsilon)\frac{d\psi}{dt} = 0 \quad (4.7)$$

$$\begin{cases} \tau' \frac{d\psi}{dt} + \psi - i = 0 & (4.8) \\ \varepsilon\tau' \frac{di}{dt} + (1 + \frac{1}{r_\tau} - \varepsilon) i - (1 - \varepsilon)\psi = 0 & (4.9) \end{cases}$$

Let us define t = 0 as the time at which S_o opens. The initial conditions at t = 0 are almost the same conditions as before quench. The fore, at t = 0 :

$$i = i_o \quad (4.10)$$

$$i' = 0 \quad (4.11)$$

$$\psi = i_o \quad (4.12)$$

5. Time evolution of the current just after switch opening with a resistor shunt.

After the switch S_o opens, the initial behavior of the current i can be described by the solution of equations (4.8) and 4.9) with τ and τ' constant. Before much heat is deposited in the bore tube, its temperature is low and its resistivity ρ' is essentially constant at low temperatures, wether it is made of copper or aluminum (see fig. 3). Therefore R' is equal to its minimum value R'_{min}. If the quench is detected soon, the coil resistance is still small just after S_o opens and the resistance of the coil circuit is essentially the resistance R_{ext} of the external shunt.

$$\left.\begin{aligned} \tau &= \frac{L}{R} \cong \frac{L}{R_{ext}} = \tau_{ext} \\ \tau' &= \frac{L'}{R'} \cong \frac{L'}{R'_{min}} = \tau'_{max} \end{aligned}\right\} \qquad (5.1)$$

If the external shunt is a resistor, the system of equations (4.8) and (4.9) has two time constants given by

$$\tau_{S,L} = \tau'_{max} \frac{1 + r_{\tau,min}}{2} \left(1 \pm \sqrt{1 - 4\varepsilon \frac{r_{\tau,min}}{(1+r_{\tau,min})^2}}\right) \qquad (5.2)$$

where $$r_{\tau,min} = \frac{\tau_{ext}}{\tau'_{max}} \cong \frac{n_c^2 R'_{min}}{R_{ext}} \qquad (5.3)$$

Our systems involve small ε's, therefore the long time constant τ_L and the short time constants τ_s are given by

$$\tau_L \cong \tau'_{max} + \tau_{ext} \cong \tau'_{max}(1 + r_{\tau,min}) \tag{5.4}$$

$$\tau_S = \frac{\epsilon\tau_{ext}\tau'_{max}}{\tau_L} \cong \epsilon\tau'_{max}\frac{r_{\tau,min}}{1 + r_{\tau,min}} \tag{5.5}$$

As a function of time, the current i and the quantity ψ are described by the sum of two exponentials as shown on Fig. 2.

$$i = i_S\, e^{(-\frac{t}{\tau_S})} + i_L\, e^{(-\frac{t}{\tau_L})} \tag{5.6}$$

$$\psi = \psi_S\, e^{(-\frac{t}{\tau_S})} + \psi_L\, e^{(-\frac{t}{\tau_L})} \tag{5.7}$$

$$i_L = i_o\frac{\tau_{ext}-\tau_S}{\tau_L - \tau_S} \cong i_o\frac{r_{\tau,min}}{1 + r_{\tau,min}} \tag{5.8}$$

$$i_S = i_o\frac{\tau_L-\tau_{ext}}{\tau_L - \tau_S} \cong \frac{i_o}{1 + r_{\tau,min}} \tag{5.9}$$

$$\psi_L = i_o\frac{\tau_L}{\tau_L-\tau_S} \cong i_o \tag{5.10}$$

$$\psi_S = - i_o\frac{\tau_S}{\tau_L-\tau_S} \cong 0 \tag{5.11}$$

These formulae verify the initial conditions (4.10) and 4.12). Once the short time constant exponential has disappeared, i and ψ decay with the same time constant τ_L and they are bound by the following relations:

$$i \cong \frac{r_\tau}{1 + r_\tau}\psi \tag{5.12}$$

$$\tau'\frac{d\psi}{dt} + \frac{\psi}{1 + r_\tau} \cong 0 \tag{5.13}$$

The τ_S exponential is just a transient to readjust i to its equilibrium value given by (5.12), i.e. a response to the sudden opening of the switch S_0. In the limit where $\varepsilon = 0$, τ_S is zero, and the current i drops instantaneously from i_o to the value given by (5.12) at the time t = 0. This behavior of the approximation $\varepsilon = 0$ is also shown on fig. 2. Equation (5.8) shows that during the long time constant, the real current i is slightly inferior to its value with the approximation $\varepsilon = 0$ if $r_{\tau\,min}$ is less than 1.

6. Evolution when R' is not constant.

When an appreciable amount of energy is deposited in the bore tube, the resistance R' (therefore $\frac{1}{\tau'}$) will start to increase. A look at equations (4.8) and (4.9) shows that even when R' is changing, ψ will vary the time scale τ', while i will readjust in a time less than $\varepsilon\tau'$ to a value such that (5.12) and (5.13) are verified. Now r_τ is time dependent

$$r_\tau = \frac{n_c^2 R'}{R} = \frac{n_c^2 R'}{R_c + R_{ext}} \tag{6.1}$$

Since R_c, R and R' are time dependent resistances. With all these considerations in mind, equations (5.12) and (5.13) are still verified to a good approximation when r_τ can no longer be considered as constant. Then, there are convenient variables to consider, E_{em} the electromagnetic energy, the energy E_H dissipated in the coil circuit and the heat $E_{H'}$ deposited in the bore tube.

$$E_{em} \cong \frac{1}{2} L\, \psi^2 \tag{6.2}$$

$$\frac{dE_H}{dt} \cong Ri^2 = \frac{2r_\tau}{(1+r_\tau)^2}\ \frac{E_{em}}{\tau'} \tag{6.3}$$

$$\frac{dE_H'}{dt} = R'i'^2 = \frac{2}{(1+r_\tau)^2}\ \frac{E_{em}}{\tau'} \tag{6.4}$$

$$\frac{dE_{em}}{dt} + \frac{2}{1+r_\tau}\frac{E_{em}}{\tau'} = 0 \qquad (6.5)$$

E_H and $E_{H'}$ are bound by the relation

$$\frac{dE_H}{dE_{H'}} = r_\tau = \frac{n_c^2 R'}{R_c+R_{ext}} \qquad (6.6)$$

These equations are valid in addition to (5.12) and (5.13).

7. Derivation of the burnout safety rule.

During the quench, let's consider the temperature evolution of the hottest point of the wire at a given time. Its temperature T corresponds to an enthalpy $h_{wire}(T)$ per unit of volume and to the resistivity $\rho_{wire}(T)$. The current density is

$$j_{wire} = \frac{i}{A_{wire}} \qquad (7.1)$$

where A_{wire} is the wire cross sectional area. h_{wire} and ρ_{wire} are averages over the wire material composed of superconducting filaments and of a matrix of, let's say, copper. If ρ_{Cu} is the resistivity of copper and r_{SC} the copper to superconductor ratio

$$\rho_{wire} < \rho_{Cu}\ \frac{1+r_{SC}}{r_{SC}} \qquad (7.2)$$

The current produces an amount of heat equal to $\rho_{wire}\ j_{wire}^2$ per unit of volume. Some of this heat is left on the wire element to increase its enthalpy h and some of that heat is conducted away. No heat is conducted inward since the element is the hottest point in the coil. At that point

$$\frac{dh_{wire}}{dT} \frac{dT}{dt} < \rho_{wire} j^2_{wire} < \rho_{Cu} \frac{1+r_{SC}}{r_{SC}} \frac{i^2}{A^2_{wire}} \quad (7.3)$$

Approximating the average enthalpy of the wire per unit of volume by the enthalpy of copper, we have

$$\frac{1}{\rho_{Cu}} \frac{dh_{Cu}}{dT} \frac{dT}{dt} < \frac{1+r_{SC}}{r_{SC}} \frac{1}{A^2_{wire}} i^2 \quad (7.4)$$

Integrating (7.4) over time from the beginning of the quench, we obtain as in ref. 5, an upper limit T_{lim} for the final temperature of the hottest point of the wire. T_{lim} satisfies the relation

$$F_J (T_{lim}) = J \quad (7.5)$$

where

$$J = \frac{i_o^2}{C_J} \tau_J \quad (7.6)$$

$$\tau_J = \int_0^\infty \frac{i^2}{i_o^2} dt \quad (7.7)$$

$$C_J = \frac{r_{SC}}{1+r_{SC}} A^2_{wire} \quad (7.8)$$

$$F_J (T) = \int_{T_{crit}}^{T} \frac{1}{\rho_{Cu}} \frac{dh}{dT} dT \quad (7.9)$$

τ_J depends on the time evolution of the current i after the beginning of the quench, C_J is a characteristic of the superconducting wire and F_J is a function of temperature and of the matrix material only. Different examples of functions $F_J(T)$ are plotted on fig. 4 for coppers of several resistance ratios

$$r_R = \frac{\rho_{Cu}\ (273°K)}{\rho_{Cu}\ (4°K)} \tag{7.10}$$

At any point of the wire, the temperature is less than T_{lim} computed using equations (7.5) to (7.9). If τ_J can be made small enough, T_{lim} is acceptable and the burnout danger is avoided.

Similar calculations can be made when aluminum based superconducting wires are used. The same formulae can be used, but $F_J(T)$ should be read from the curves of fig. 5 instead.

8. Approximations for the computation of τ_J.

The integral τ_J of equation (7.7) results from different contributions

$$\tau_J = t_{So} + \Delta\tau_{J,S} + \Delta\tau_{J,L} \tag{8.1}$$

t_{So} is the time for the quench to be detected and for the switch S_o to open. Until $t = t_{So}$, the current i is essentially the same as the initial value i_o. $\Delta\tau_{J,L}$ is the contribution made after the opening of S_o, neglecting the transient response. It can be computed with the approximation $\varepsilon = 0$ represented by (5.12), and (6.2) to (6.6). $\Delta\tau_{J,S}$ is a correction due to the fact that ε is not zero and i is not given by (5.12) during the short time constant exponential.

The computation of t_{So} involves a study of the quench detection that will be the subject of another report. An exact estimate of $\Delta\tau_{J,L}$ and $\Delta\tau_{J,S}$ requires an estimation of the coil resistance R_c, i.e. a good description of the quench propagation. A pessimistic estimate is obtained by setting $R_c = 0$ at all times. We will first prove that this approximation is pessimistic.

Actually, we intend to show that an underestimate of the resistance R of the coil circuit leads to an overestimate, i.e. a pessimistic estimate of τ_J. For $\Delta\tau_{J,L}$, equations (6.3) and (6.6) are valid. Therefore,

$$\left.\begin{aligned} R' \frac{dE_{H'}}{dE_H} &= \frac{R}{n_c^2} \\ R &= R_c + R_{ext} \end{aligned}\right\} \qquad (8.2)$$

If we know a real solution for a given R for each value of E_H, we can consider the effect of a negative change in R just at one instant. The function $\int R' dE_{H'}$ (and therefore $E_{H'}$) will be smaller after the change of R as can be seen from integrating (8.2). Thus a larger fraction of the energy E_o will end up in the coil circuit. E_H will be larger at the end of the quench. Furthermore,

$$\Delta\tau_{J,L} \cong \int \frac{1}{i_o^2} i^2 \, dt = \int \frac{1}{i_o^2} \frac{1}{R} \frac{dE_H}{dt} dt = \int \frac{1}{i_o^2} \frac{dE_H}{R} \qquad (8.3)$$

will be larger because R is smaller and because the range of integration of E_H will be larger too.

For the transient contribution $\Delta\tau_{J,S}$, a look at equation (4.9) shows that a decrease in R, (i.e. an increase in r_τ) induces a less negative value for $\frac{di}{dt}$, therefore larger values for i while ψ is not effected very much.

The contribution of the transient to τ_J will be larger if R is decreased during the transient period.

In conclusion, the smaller R, the larger τ_J. Solving equations with $R_C = 0$ and $R = R_{ext}$ provides a pessimistic estimation of the burnout problem. Any underestimation of R_{ext} will also lead to a pessimistic evaluation of the burnout problem, a property that will be used in section 11.

When the external shunt is a resistor, there is an absolute relation between the energy $E_{ext,f}$ dumped in the external shunt and the quantity $\Delta\tau_J$'s after S_o is open

$$E_{ext,f} = R_{ext} \int i^2 \, dt = R_{ext} \, i_o^2 (\Delta\tau_{JS} + \Delta\tau_{J,L}) \qquad (8.4)$$

$$\frac{E_{ext,f}}{E_o} = \frac{2R_{ext}}{L} (\Delta\tau_{J,S} + \Delta\tau_{J,L}) \qquad (8.5)$$

Since τ_J is overestimated in our estimation, $E_{ext,f}$ is overestimated too.

9. Estimation of $\Delta\tau_{J,S}$ and $\Delta\tau_{J,L}$ when R' = constant

When the energy E_o stored in the magnet is too small to heat the bore tube to the point when R' is temperature dependent, then we can assume R' to be a constant at the same time as we assume $R_C = 0$. Then, $\Delta\tau_{J,S}$ and $\Delta\tau_{J,L}$ can be computed by integrating $(\frac{i}{i_o})^2$ with i given by expression (5.6), (5.8) and (5.9). The contribution of the long time constant is

$$\Delta\tau_{J,L} = \left(\frac{i_L}{i_o}\right)^2 \frac{\tau_L}{2} = \left(\frac{\tau-\tau_S}{\tau_L-\tau_S}\right)^2 \frac{\tau_L}{2} \cong \frac{\tau_{ext}}{2} \frac{r_{\tau,min}}{1+r_{\tau,min}} \qquad (9.1)$$

$$\tau_{ext} = \frac{L}{R_{ext}} \qquad (9.2)$$

$$r_{\tau,min} = \frac{n_c^2 R'_{min}}{R_{ext}} = \frac{L}{R_{ext}\,\tau'_{max}} \tag{9.3}$$

The correction due to the short time constant exponential amounts to

$$\Delta\tau_{J,S} \cong \int \left[\left(\frac{i_S}{i_o}\right)^2 e^{\left(-\frac{2t}{\tau_S}\right)} + s\,\frac{i_S\,i_L}{i_o^2}\, e^{\left(-\frac{t}{\tau_S}\right)} \right] dt$$

$$= \frac{\varepsilon\tau_{ext}}{2}\;\frac{1+4r_{\tau,min}}{(1+r_{\tau,min})^3} \tag{9.4}$$

$\Delta\tau_{JS}$ is important with respect to $\Delta\tau_{J,L}$ of (9.1) only if r_τ is small of the same order as ε. Then

$$\Delta\tau_{J,S} \cong \frac{\varepsilon\tau_{ext}}{2} \tag{9.5}$$

When the energy E_o in the magnet is large enough to heat the bore tube to a degree where R' is no longer a constant, the variation of R' is important only after the transient is over. Therefore expressions (9.4) or (9.5) can be kept for the contribution of the short time constant.

10. Computation of $\Delta\tau_{J,L}$ when R' is not a constant

The effect of the changing R' is felt only after the transient is over. We can use the approximation $\varepsilon = 0$, therefore equation (8.2), in addition to the approximation $R_c = 0$. Since the total energy $E_{ext,f}$ dumped in the shunt is the same as the total energy $E_{H,f}$ dumped in the coil circuit, we can write a relation between $E_{ext,f}$ and the total energy $E_{H',f}$ dumped in the bore tube.

$$E_{ext,f} = E_{H,f} = \int dE_H = \frac{n_c^{\,2}}{R_{ext}} \int_0^{E_{H',f}} R' \, dE_{H'} =$$

$$= \frac{(\pi \; D \; n_c)^2}{V' \; R_{ext}} \int_0^{E_{H',f}} \rho' \, dE_{H'} = \frac{(\pi \; D \; n_c)^2}{R_{ext}} \int_0^{h'_f} \rho' \, dh' \qquad (10.1)$$

where D is the diameter and V' the volume of the bore tube, ρ' its resistivity and h' its enthalpy per unit of volume

$$h' = \frac{E_{H'}}{V'} \qquad (10.2)$$

We define the function G'(h') of the bore tube material.

$$G'(h') = \int \rho' \, dh' \qquad (10.3)$$

G'(h') can be plotted on a graph such as fig. 6. Equation (10.1) shows that there is a point on the curve G'(h') with coordinates h'_f and G'_f and such that

$$G'_f = G'(h'_f) \qquad (10.4)$$

$$G'_f = \frac{E_{ext,f} \; R_{ext}}{(\pi \; D \; n_c)^2} \qquad (10.5)$$

$$h'_f = \frac{E_{H'f}}{V'} \qquad (10.6)$$

Since the energies dumped in the shunt $E_{ext,f}$ and in the bore tube $E_{H',f}$ add up to E_o at the end of the quench, E_o being the total system energy.

$$E_o = V'h_f' + \frac{(\pi\ D\ n_c)^2}{R_{ext}}\ G_f' \tag{10.7}$$

$$\frac{h_f'}{h'_{max}} + \frac{G_f'}{G'_{max}} = 1 \tag{10.8}$$

where $$h'_{max} = \frac{E_o}{V'} \tag{10.9}$$

$$G'_{max} = \frac{E_o\ R_{ext}}{(\pi\ D\ n_c)^2} = \rho'_{ext}\ h'_{max}$$

$$\rho'_{ext} = \frac{R_{ext}\ V'}{(\pi\ D\ n_c)^2} \tag{10.9}$$

The point of abscissa h_f' and ordinate G_f' lies on the straight line drawn between the point of abscissa h'_{max} on the h' axis and the point of ordinate G'_{max} on the G'axis on fig. 6. The intersection of that straight line with the curve G'(h') corresponding to the actual bore tube material has the ordinate G_f'. This is a procedure to find G_f'. Then, using (8.5) and (10.5), we get the relation between $\Delta\tau_{J,L}$ and the estimate of $E_{ext,f}$ with the assumption $\epsilon = 0$, i.e. with the assumption $\Delta\tau_{J,S} = 0$

$$\Delta\tau_{J,L} = \frac{L}{2R_{ext}E_o}\ \frac{(\pi D n_c)^2}{R_{ext}}\ G_f' = \frac{\tau_{ext}}{2}\ \frac{G_f'}{G'_{max}} = \frac{\tau_{ext}}{2}\ \frac{h'_{max}-h_f'}{h'_{max}} \tag{10.11}$$

Examples of functions G'(h') are plotted on fig. 7 for aluminum bore tubes of several resistance ratios

$$r_R' = \frac{\rho'(273°K)}{\rho'(4°K)} = \frac{\rho'\ (273°K)}{\rho'_{min}} \tag{10.12}$$

For magnets without a conducting bore tube, we set $V' = 0$. Therefore

$$\Delta\tau_J = \frac{\tau_{ext}}{2} \tag{10.13}$$

11. Use of a "varistor" in the shunt.

After the switch S_o is open, the maximum of the current i in the coil occurs in the beginning, when $i = i_o$. That is also the maximum value of the current in the shunt. Let $\mathcal{V}(i)$ be the voltage characteristics of that shunt, the voltage v_c accross the coil is at all times

$$v_c = \mathcal{V}(i) \tag{11.1}$$

The maximum voltage accross the coil is the voltage at the beginning

$$v_{c,max} = \mathcal{V}(i_o) \tag{11.2}$$

If the shunt is a resistor, as we have considered so far

$$\mathcal{V}(i) = R_{ext}\, i \tag{11.3}$$

$$v_{c,max} = R_{ext}\, i_o \tag{11.4}$$

R_{ext} cannot be chosen as large as one would like because the voltage $v_{c,max}$ given by (11.4) cannot exceed the maximum value allowed by the insulation. There is a maximum value for R_{ext} from voltage limitation. On the other hand, the prevention of burnout may require a high value of R_{ext}. A suitable compromise may be hard to find.

It is interesting to notice that the largest voltage occurs just after S_o opens, during the transient period. Onthe other hand, the largest contribution to τ_J (most likely $\Delta\tau_{J,L}$) occurs after the transient is over and when the current is already substantially reduced. It makes sense to look for shunts that would decrease the voltage and increase τ_J during the transient period when the voltage is the problem. This shunt should do the opposite later on when τ_J is the problem. These shunts would have non linear characteristics, with a large resistance at low current and a small resistance at higher current, i.e. during the transient period. Such shunts can be made out of thyrites. They are sometimes called varistors. They have a more or less constant voltage for a rather large range of current.

The voltage characteristics of a varistor made out of thyrites is shown on fig. 8. It can be approximated by the function

$$\mathcal{V}(i) \cong a\, i^b \tag{11.5}$$

with

$$b = 0.2 \text{ to } 0.25 \tag{11.6}$$

An exact solution for i is difficult to obtain. We will make a pessimistic approximation, reducing the resistance R of the coil circuit as we did in Section 8. The characteristics of the varistor will be first modified into a straight line crossing the origin and a point of i* and of ordinate v* on the real varistor characteristics (fig. 8). For $i < i^*$ we will take

$$\mathcal{V}(i) = \frac{v^*}{i^*}\; i \tag{11.7}$$

The value i* is an arbitrary value less than i_o, but i* must be larger than any value of i after the transient is over (see Section 13 for a method to check the validity of that statement). Under this condition, the approximation (11.7) of the characteristics leads to a pessimistic estimation of $\Delta\tau_{J,L}$. The procedure of Section 10 and formula (10.11) to estimate $\Delta\tau_{J,L}$ can be applied pessimistically, making the identification

$$R_{ext} = \frac{v^*}{i^*} \tag{11.8}$$

The choice of i* may have to be revised later in view of the consequences, as it is explained in Section 15. For a first trial, one may try $i^* = \frac{1}{3}$ or $\frac{1}{2}\, i_o$.

Note that, since $i^* < i_o$, we have

$$v_{c,max} = \mathcal{V}(i_o) < R_{ext}\, i_o \tag{11.9}$$

Therefore, there is less voltage on the coil if we use the varistor than if we use a resistor of resistance R_{ext} in the shunt, i.e. for the same value of $\Delta t_{J,L}$. That happens because the varistor maintains the voltage across the coil near maximum value, i.e. maximum efficiency at all times, not only at the beginning of the quench.

12. $\Delta\tau_{J,S}$ with a varistor

The transient is different from (5.6) though. It may correspond to a larger value of $\Delta\tau_{J,S}$ than the expression (9.4) since, now, the resistance $\mathcal{V}/i$ is smaller than R_{ext} in the beginning of the transient. We define, as in (5.8),

$$i_L = i_o \frac{r_{\tau,min}}{1+r_{\tau,min}} = \frac{i_o}{1+\frac{R_{ext}\tau'}{L}} \qquad (12.1)$$

using (9.3) and the value of R_{ext} given by (11.8). The value i_L of (12.1) is about the value reached after the transient period once the approximation

$$\mathcal{V}(i) = R_{ext}\, i \qquad (12.2)$$

is assumed. During the transient period, the current i and the voltages change from the values $(i_o, \mathcal{V}(i_o))$ to $(i_L, R_{ext} i_L)$ following the varistor characteristics. We will now make a second assumption, even more pessimistic, that the characteristics will follow a straight line between the point $(i_o, \mathcal{V}(i_o))$ and the point $(i_L, R_{ext}\, i_L)$ during the transient period (fig. 8).

$$\left.\begin{aligned} v_c - R_{ext}\, i_L &= R_{transient}\,(i_o - i_L) \\ R = \frac{v_c}{i} &= \frac{R_{ext}\, i_L}{i} + \frac{R_{transient}\,(i-i_L)}{i} \end{aligned}\right\} \qquad (12.3)$$

where

$$R_{transient} = \frac{\mathcal{V}(i_o) - R_{ext}\, i_L}{i_o - i_L} \qquad (12.4)$$

Then, we have to replace $\frac{1}{r_\tau}$ in equation (4.9) by

$$\frac{1}{r_\tau} = \frac{\tau'}{\tau} = \frac{R\tau'}{L} = \frac{v_c \tau'}{L\ i} = \frac{R_{ext}\tau'\ i_L}{L\ i} + \frac{R_{trans}\tau'(i-i_L)}{L\ i} \quad (12.5)$$

ψ stays around its initial value i_o, therefore

$$\varepsilon\tau' \frac{di}{dt} + i + \frac{R_{ext}\ \tau'\ i_L}{L} + \frac{R_{transient}\ \tau'}{L}(i-i_L) \cong i_o \quad (12.6)$$

$$\varepsilon\tau' \frac{d(i-i_L)}{dt} + \left(1 + \frac{R_{transient}\ \tau'}{L}\right)(i-i_L) =$$

$$= i_o - i_L - \frac{R_{ext}\ i_L\ \tau'}{L} = i_o - i_L\left(1+\frac{R_{ext}\ \tau'}{L}\right) = 0 \quad (12.7)$$

Using (12.1) as an expression for i_L

The transient can be approximated by

$$i = i_L + \frac{i_o}{1+r_{\tau,min}}\, e^{\left(-\frac{t}{\tau_S}\right)} \quad (12.8)$$

where $$\tau_S = \frac{\varepsilon\tau'_{max}}{1+\frac{\tau'_{max}}{\tau_{transient}}} \quad (12.9)$$

$$\tau_{transient} = \frac{L}{R_{transient}} \quad (12.10)$$

$$\Delta\tau_{J,S} = \int\left[\left(\frac{i}{i_o}\right)^2 - \left(\frac{i_L}{i_o}\right)^2\right] dt =$$

$$= \frac{\tau_S}{2}\ \frac{1+4\ r_{\tau,min}}{(1+r_{\tau,min})^2} \quad (12.11)$$

$\Delta\tau_{J,S}$ contains the factor ϵ through the term τ_S. It is important in comparison with $\Delta\tau_{J,L}$ only if $\Delta\tau_{J,L}$ is small, therefore if $r_{\tau,min}$ is small. Therefore, when it is important,

$$\Delta\tau_{J,S} \cong \frac{\tau_S}{2} = \frac{\epsilon\tau'_{max}}{2\left(1+\frac{\tau'_{max}}{\tau_{transient}}\right)} \tag{12.12}$$

13. Maximum current after transient in the varistor case.

We have to make sure that the approximation of $\mathcal{V}(i)$ being linear as expressed by (11.7) is indeed a pessimistic approximation after the transient. Then we can use formula (10.11) for a pessimistic estimation of $\Delta\tau_{J,L}$ as it was assumed to be in section 11. Suppose that, for the approximated solution using (11.7), the current never reaches the value i* after transient. Then the approximated resistance R_{ext} of (11.8) is always less than the real ratio $\mathcal{V}(i)/i$ occurring in real life with the varistor. By the same argument as the one used in section 8, one shows that the real current i is smaller than the approximated one, that it never reaches the value i* either, and that the real $\Delta\tau_{J,L}$ is less than its approximation (10.11). Now we have to find out if the approximated current i ever reaches the value i* once the transient is over.

After the transient is over, equations (5.12), (5.13) and (6.4) are valid.

$$\psi = \frac{1+r_\tau}{r_\tau}\, i \tag{13.1}$$

$$\frac{d\psi}{dt} = \frac{-\psi}{\tau'(1+r_\tau)} = -\frac{i}{r_\tau\tau'} = \frac{-i}{\tau_{ext}} = -\frac{R_{ext}\, i}{L} \tag{13.2}$$

$$\frac{dE_{H'}}{dt} = \frac{L\psi^2}{\tau'(1+r_\tau)^2} = \frac{\tau_{ext}\, R_{ext}\, i^2}{\tau'\, r_\tau^{\,2}} = \frac{R_{ext}\, i^2}{r_\tau} \tag{13.3}$$

Differentiating (13.1) with respect to time, we get the following relations at the maximum of i,

$$\frac{di}{dt} = 0 \qquad (13.4)$$

$$\frac{d\psi}{dt} = -\frac{i}{r_\tau^2}\frac{dr_\tau}{dt} = -\frac{i}{r_\tau^2}\frac{dr_\tau}{dE_{H'}}\frac{R_{ext}\, i^2}{r_\tau} = -\frac{i^3 R_{ext}}{r_\tau^3}\frac{dr_\tau}{dE_{H'}} \qquad (13.5)$$

Comparing (13.2) to (13.5), we find that the maximum of i occurs when

$$\frac{r_\tau^3}{\frac{dr_\tau}{dE_{H'}}} = Li^2 \qquad (13.6)$$

In order to find out if i exceeds i* at its maximum, we compute the value h'* of h' that would correspond to a maximum of i equal to i*. From h'*, we can compute the corresponding total energy E* in the system. If E* is found larger than E_o, the hypothesis i = i* leads to an unphysical result and is consequently wrong; i will not exceed i*. With our approximation (12.2), the current i will exceed i*, however, if E* is smaller than E_o. To find h'* we define

$$\rho'_{ext} = \frac{R_{ext}\, V'}{(\pi D\, n_c)^2} = \frac{G'_{max}}{h'_{max}} \qquad (13.7)$$

$$g'^* = \frac{Li^{*2}}{V'}\, \rho'^2_{ext} \qquad (13.8)$$

and the function g'(h') such that

$$g'(h') = \frac{\rho'^3}{\frac{d\rho'}{dh'}} \qquad (13.9)$$

Examples of functions $g'(h')$ are plotted on fig. 9 for aluminum bore tubes and several resistance ratios r_R'. The maximum of i occurs at i^* if the corresponding value h'^* of h' is such that

$$g'(h'^*) = \frac{\rho'^3}{\frac{d\rho'}{dh'}} = \frac{\rho'^3}{V'\frac{d\rho'}{dE_{H'}}} = \frac{V'^2}{V'(\pi D)^4}\frac{R'^3}{\frac{dR'}{dE_{H'}}} =$$

$$= \frac{V'}{(\pi D)^4}\left(\frac{R_{ext}}{n_c^2}\right)^2 \frac{r_\tau^3}{\frac{dr_\tau}{dE_{H'}}} = \frac{V'R_{ext}^2}{(\pi D n_c)^4} Li^{*2} =$$

$$= \rho_{ext}'^2 \frac{Li^{*2}}{V'} = g'^* \qquad (13.10)$$

Therefore h'^* can be read on the relevant curve on fig. 9 once g'^* is computed. It is the abscissa for the ordinate g'^*. From h'^*, we can determine the corresponding value ρ'^* of the resistivity ρ' on fig. 3 and $G'^* = G'(h'^*)$ using the proper function $G'(h'^*)$ on fig. 6 or 7. The total energy E^* in the system is given by the sum of the energy $E_{H'}$ in the bore tube, E_{ext} in the shunt and E_{em} the electromagnetic energy.

$$E_{H'} = V'h'^* \qquad (13.11)$$

$$E_{em} = \frac{L\psi^2}{2} = \frac{Li^{*2}}{2}\left(\frac{1+r_\tau}{r_\tau}\right)^2 = \frac{Li^{*2}}{2}\left(1 + \frac{\rho'_{ext}}{\rho'^*}\right)^2 \qquad (13.12)$$

$$E_{ext} = R_{ext}\int i^2 dt = \frac{(\pi D n_c)^2}{R_{ext}} G'(h'^*) = \frac{V'}{\rho'_{ext}} G'^* \qquad (13.13)$$

Using (13.3) to replace i^2 in the integral.

The condition of validity for the choice of i* on the varistor characteristic amounts to

$$\frac{E^*}{E_o} = \frac{E_{H'}}{E_o} + \frac{E_{ext}}{E_o} + \frac{E_{em}}{E_o} > 1 \qquad (13.14)$$

$$\frac{h'^*}{h'_{max}} + \frac{G'^*}{G'_{max}} + \left(\frac{i^*}{i_o}\right)^2 \left(1 + \frac{\rho'_{ext}}{\rho'^*}\right)^2 > 1 \qquad (13.15)$$

14. Summary of the rules and calculations.

All symbols are in the MKS system of units.

a) Magnet parameters.

Symbol	Definition	Section	Units
A_{wire}	= cross sectional area of wire	(7.1)	meters^2
C_J	= a constant that is characteristic of a particular type wire.	See (7.8)	meters^4
D	= diameter of bore tube	(10.1)	meters
E_{em}	= electro-magnetic energy	(6.2)	MJ
$E_{ext,f}$	= final energy dumped in external shunt	(8.4)	MJ
E_H	= heat deposited in coil circuit	(6.3)	MJ
$E_{H,f}$	= final energy dumped in coil circuit	(10.1)	MJ
$E_{H'}$	= heat deposited in bore tube	(6.4)	MJ
$E_{H',f}$	= final heat deposited in bore tube	(10.1)	MJ
E_o	= total system energy	(10.7)	MJ
E*	= total energy in system supposing the current i is maximum when i = i*	(13.14)	MJ
$F_J(T)$	= enthalpy integral of conductivity for matrix material	(7.9)	$A^2 sec/m^4$
$F_J(T_{lim})$	= value of $F_J(T)$ at $T = T_{lim}$	(7.5)	$A^2 sec/m^4$
G'(h')	= enthalpy integral of resistivity for bore tube material	(10.3)	$v^2 sec/m^2$
G'_f	= value of G'(h') for $h' = h'_f$	(10.4)	$v^2 sec/m^2$
G'_{max}	= maximum possible value of G', had all energy ended in external resistor	(10.9)	$v^2 sec/m^2$
g'*	= quantity defined by	(13.8)	
h_{Cu}	= enthalpy per unit volume of copper only	(7.4)	MJ/m^3
h_{wire}	= enthalpy per unit volume of the wire	(7.3)	MJ/m^3
h'	= enthalpy per unit volume of bore tube material	(10.2)	MJ/m^3
h'_f	= final enthalpy of bore tube	(10.6)	MJ/m^3
h'_{max}	= maximum bore tube enthalpy possible if all E_o ended in bore tube	(10.9)	MJ/m^3

h'^*	= enthalpy of bore tube if and when current maximum is i^*	(13.10)	MJ/m^3
i	= coil current at any instant	(2.1)	A
i_L	= initial value of long time constant contribution	(5.6)	A
i_o	= current in coil at time of opening switch	(2.1)	A
i_S	= initial value of short time constant component of i	(5.6)	A
i^*	= a selected current to help approximate varistor characteristic. i^* is arbitrary within limits (see section 13)	(11.7)	A
i'	= current in bore tube as a function of time	(3.1)	A
J	= time integral of the square of the current density	(7.6)	
j_{wire}	= current density in wire	(7.1)	A/m^2
L	= inductance of coil circuit	(4.1)	H
L'	= inductance of bore tube circuit	(4.1)	H
M	= mutual inductance between coil and bore tube	(4.1)	H
n_c	= number of turns in coil	(2.2)	
R	= resistance of coil circuit $\{R = R_c + R_{ext}$ (8.2)$\}$	(3.1)	Ω
R_c	= resistance of coil alone - a function of time	(6.1)	Ω
R_{ext}	= external resistance of the shunt	(5.1)	Ω
$R_{transient}$	= slope $d\mathcal{V}/di$ of varistor characteristics in the approximation used for the transient	(12.3)	Ω
R'	= resistance of bore tube	(3.1)	Ω
R'_{min}	= minimum resistance of bore tube	(5.1)	Ω
r_R	= resistivity ratio of matrix material at 273°K/4°K	(7.10)	
r_{SC}	= ratio of copper to superconductor (by volume average)	(7.2)	
r_τ	= τ/τ'	(4.5)	
$r_{\tau,min}$	= ratio of τ_{ext}/τ'_{max}	(5.3)	
r'_R	= resistivity ratio $\frac{\rho' 273°K}{\rho'_{min}}$ of bore tube material	(10.12)	
S_o	= switch name	(fig.1)	
T	= temperature of hot spot of wire	(7.1)	°K

T_{lim}	= highest possible quench caused temperature for hottest spot in the coil	(2.1)	°K
t	= independent variable time	(2.1)	sec
t_{So}	= time for quench to be detected and S_o to open	(8.1)	sec
V'	= volume of bore tube	(10.1)	m^3
v	= varistor voltage at any instant	(12.1)	V
v_c	= coil voltage at any instant	(11.1)	V
$v_{c,max}$	= maximum coil voltage	(11.2)	V
v_ϕ	= inductive voltage across the coil	(2.2)	V
v^*	= varistor voltage corresponding to current i^*	(11.7)	V
$\mathcal{V}(i)$	= real volt-amp characteristics of varistor shunt	(11.1)	V
$\Delta\tau_J$	= effective time (after S_o opens) of current flow	(10.13)	sec
$\Delta\tau_{J,L}$	= effective time after S_o opens (neglecting short transient)	(8.1)	sec
$\Delta\tau_{J,S}$	= effective time correction due to short transient	(8.1)	sec
ϵ	$= 1 - \frac{M^2}{LL'}$ = ratio of leakage inductance to L	(4.6)	
ρ_{Cu}	= resistivity of copper	(7.2)	Ω-m
ρ_{wire}	= resistivity of wire (average for superconducting and matrix)	(7.2)	Ω-m
ρ'	= resistivity of bore tube material	(5.1)	Ω/m^3
ρ'_{ext}	= a fictitious bore tube resistivity representing the external resistor in the approximation $\epsilon = 0$	(10.9)	Ω-m
ρ'_{min}	= minimum resistivity of bore tube = resistivity at 4°K	(10.12)	Ω-m
τ	= L/R time constant of coil circuit if bore tube were not conducting	(4.3)	sec
τ_{ext}	= L/R_{ext} time constant of coil circuit if coil resistance $R_c = 0$	(5.1)	sec
τ_J	= effective time of current flow in coil after quench (depends on time evolution of current)	(2.1)	sec
τ_L	= long time constant	(5.2)	sec
τ_S	= short time constant	(5.2)	sec
$\tau_{transient}$	= time constant produced by element $R_{transient}$	(12.10)	sec

τ'	= L'/R' time constant of bore tube	(4.4)	sec
τ'_{max}	= L'/R'_{min}	(5.1)	sec
ϕ	= magnetic flux through one turn of magnet coil	(2.2)	
ψ	= flux in bore tube divided by M	(4.2)	A
ψ_L	= initial value of long time constant component of ψ	(5.10)	A
ψ_S	= initial value of small time constant component of ψ	(5.11)	A

b) Computed constants

$$C_J = \frac{r_{SC}}{1+r_{SC}} A^2_{wire} \qquad (14.1)$$

$$\ell_{wire} = \pi D n_c \, , \qquad (14.2)$$

$$\tau'_{max} = \frac{L V'}{\ell^2_{wire} \rho'_{min}} \qquad (14.3)$$

$$\tau_{ext} = \frac{L}{R_{ext}} \qquad (14.4)$$

$$\rho'_{ext} = \frac{R_{ext} V'}{\ell^2_{wire}} \qquad (14.5)$$

$$r_{\tau,min} = \frac{\rho'_{min}}{\rho'_{ext}} \qquad (14.6)$$

$v_{c,max} = R_{ext} i_o$ if resistor is used (14.7)

$= \mathcal{V}(i_o)$ = voltage given by the varistor characteristics if varistor is used

$$i_L = i_o \frac{r_{\tau,min}}{1+r_{\tau,min}} \qquad (14.8)$$

$$R_{transient} = \frac{v_{c,max} - R_{ext} i_L}{i_o - i_L} \qquad (14.9)$$

$\tau_{transient} = \frac{L}{R_{transient}}$ if varistor is used (14.10)

$$E_o = \frac{1}{2} L i_o^2 \qquad (14.11)$$

$$h'_{max} = \frac{E_o}{V'} \qquad (14.12)$$

$$G'_{max} = \rho'_{ext} h'_{max} = \frac{E_o R_{ext}}{(\pi D n_c)^2} \qquad (14.13)$$

$$g'^* = \frac{L i^{*2}}{V'} \rho'^2_{ext} \qquad (14.14)$$

h'^* = Read from fig. 9

c) Functions to be plotted or to select on figs. 4, 5, 7, 9 or 3

$$F_J(T) = \int \frac{1}{\rho} \frac{dh}{dt} dt \qquad (14.15)$$

$$G'(h') = \int \rho' \, dh' \qquad (14.16)$$

$$g'(h') = \frac{\rho'^2}{\frac{d\rho'}{dh'}} \qquad (14.17)$$

$$\rho'(h') \qquad (14.18)$$

d) Checks and rules

1) Is $v_{c,max}$ acceptable?

2) If varistor is used, check that $i_L < i^*$.

3) Determine h'^* such that

$$g'(h'^*) = g'^* \quad \text{from fig. 9} \qquad (14.19)$$

Then $\rho'^* = \rho'(h'^*)$ from fig. 3 (14.20)

$$G'^* = G'(h'^*) \quad \text{from fig. 6} \qquad (14.21)$$

$$\frac{E^*}{E_o} = \frac{h'^*}{h'_{max}} + \frac{G'^*}{G'_{max}} + \left(\frac{i^*}{i_o}\right)^2 \left(1 + \frac{\rho'_{ext}}{\rho'^*}\right)^2 \qquad (14.22)$$

Is $\frac{E^*}{E_o}$ larger than 1?

If yes, the point (i^*v^*) on the varistor characteristics is good for a pessimistic evaluation of $\Delta\tau_{J,L}$. If no, another point with a higher i^*, may have to be chosen (see subsection 15c).

e) Determination of $\Delta\tau_{JL}$ and $\Delta\tau_{JS}$

On the graph showing $G'(h')$ fig. 7 for aluminum bore tube, draw the straight line between the point of abscissa h'_{max} on the h' axis and the point of ordinate G'_{max} on the G' axis. Read the coordinates h'_f and G'_f of the intersection with curve $G'(h')$

Compute

$$\Delta\tau_{J,L} = \frac{\tau_{ext}}{2}\frac{G'_f}{G'_{max}} = \frac{\tau_{ext}}{2}\left(1 - \frac{h'_f}{h'_{max}}\right) \qquad (14.23)$$

If resistor is used compute

$$\Delta\tau_{J,S} = \frac{\epsilon\tau_{ext}}{2} \qquad (14.24)$$

If a varistor is used, compute

$$\Delta\tau_{J,S} = \frac{\epsilon\tau'_{max}}{2\left(1+\frac{\tau'_{max}}{\tau_{transient}}\right)} \qquad (14.25)$$

f) Burnout condition

Knowing t_{So}, $\Delta\tau_{JS}$ and $\Delta\tau_{JL}$,

compute

$$\tau_J = t_{So} + \Delta\tau_{JS} + \Delta\tau_{JL} \qquad (14.26)$$

$$J_f = \frac{i_o^2\,\tau_J}{C_J} \qquad (14.27)$$

Read T_{lim} such that

$$F_J\,(T_{lim}) = J_f \qquad (14.28)$$

T_{lim} is an upper limit for the temperature in the coil. Is it acceptable?

g) Further checks

1) The energy in the shunt will be less than our approximation $E_{ext,f}$, which is such that

$$\frac{E_{ext,f}}{E_o} = \frac{2(\Delta\tau_{JS} + \Delta\tau_{JL})}{\tau_{ext}} \qquad (14.29)$$

Is this amount of energy acceptable for the shunt?

2) Since our approximations overestimate $E_{ext,f}$ (see Section 8 and equation (8.3) for justification), the energy $E_{H'}$ is underestimated by $E_{H',f}$. However,

$$E_{H'} < E_o \qquad (14.30)$$

i.e. $$h' < h'_{max} \qquad (14.31)$$

On fig. 7, there is a temperature scale for h'. From this scale, one can read a maximum temperature T'_{lim} for the bore tube, the temperature corresponding to h'_{max}.

$$h'_{max} = h'(T'_{lim}) \qquad (14.31)$$

Is T'_{lim} acceptable?

15. Revision of design parameters.

a) If the rules expressed in subsections 14d 1), 14d 2), 14d 3), the burnout condition 14f) and the checks of 14g 1) and 14g 2) are satisfied, the magnet is really safe against burnout. Since our theory is pessimistic, the temperatures will be lower than computed here.

b) If condition 14d 1) is violated another shunt with less voltage for the current i_o is needed, a lower resistor or a different varistor.

c) If a varistor is used and checks of subsection 14d 2) or 14d 3) are not O.K., the circuit may still be acceptable, but the choice of the point (i*v*) on the varistor characteristics may have been unfortunate. Another choice with a higher value of i*, but still smaller than i_o, can be made. If all the checks are O.K. with any choice of (i*v*), the magnet is safe against burnout.

d) If a varistor is used and if the burnout condition of 14 f) shows that T_{lim} is not acceptable, things may still be O.K. if another choice of (i*v*) is made on the varistor characteristics. The next choice should correspond to a lower value of i*.

e) If i* cannot be increased because of the result of 14d 2) or 14d 3), or if a resistor is used, a violation of the burnout condition 14f imposes another revision of the circuit parameters. Another shunt with more voltage for the same current may be considered.

f) An increase in shunt voltage is of course possible only until the condition of 14d 1) is violated. Then, to abide with our rules, either the number of turns n_c should be decreased or the volume of the bore tube V' should be increased. Of course, if n_c is decreased, i_o and A_{wire} has to be increased by the same factor to maintain the same field and current density in the magnet. (See section 18.b)

g) If the check of subsection 14g 2) is not O.K., the volume of the bore should be increased.

h) If 14g 1) fails, the shunt capacity to take energy should be increased or, again, n_c could be decreased or V' increased.

k) Of course, if the magnet parameters violate our rules, the magnet may still be O.K. Our estimations are all pessimistic. It may be wise to first try a very simple quench protection circuit and to test the magnet nondestructively using the method of ref. 5. If necessary, the quench circuit can later be made more sophisticated.

16. <u>Application to two examples of magnets protected by varistors.</u>

Magnet	Coils A and B[3,4]	TPC Magnet[6]
D	1.05	2.09
V'	0.020	0.22
$r_{R'}$	14	14
ρ'_{min}	1.82×10^{-9}	1.8×10^{-9}
r_R	73	73
r_{SC}	1.0	1.65
A_{wire}	7.8×10^{-7}	2.4×10^{-6}
n_c	1667	2000
L	1.89	4.92
ε	0.022	0.0105
i_o	670	2115
t_{So}	28 ms	18 ms(44ms)
varistor element	69 W 60100	68 W 60100
n_S=# of elements in series	1	6
$n_{/\!/}$ =# of elements in parallel	8	30
i^*	224	400
v^*	1344	4000
$C_J = A^2_{wire}\, r_{sc}/(1+r_{sc})$	3×10^{-13}	3.6×10^{-12}
$\ell_{wire} = \pi D\, n_c$	5500	13100
$\lambda = V'/\ell^2_{wire}$	6.6×10^{-10}	1.28×10^{-9}
$\tau'_{max} = L\lambda/\rho'_{min}$	0.69	3.5
$E_o = Li_o^2/2$	420KJ	11 MJ
$h'_{max} = E_o/V'$	21 MJ/m^3	50 MJ/m^3
$T'_{lim} = h'^{-1}(h'_{max})$	75°K	110°K
$R_{ext} = v^*/i^*$	6 Ω	10 Ω
$\tau_{ext}/2 = L/2R_{ext}$	.157	.246
$\rho'_{ext} = \lambda\, R_{ext}$	4.0×10^{-9}	1.28×10^{-8}
$r_{\tau,min} = \rho'_{min}/\rho'_{ext}$	0.46	0.142
$i_L = i_o r_{\tau,min}/(r_{\tau,min}+1)$	211	263
$i_L < i^*$?	yes	yes

(continued)

Magnet	Coils A and B	TPC Magnet
$G'_{max} = \rho'_{ext}\ h'_{max}$	.084	0.64
$v_{max} = \mho(i_o)$	1800	5800
$R_{trans} = (v_{max} - R_{ext} i_L)/(\dot{i}_o\ i_L)$	1.16 Ω	1.71 Ω
$r_{\tau,trans} = R_{trans}\ \tau'_{max}/L$	.42	1.21
$g'* = \left(L/V'\right)(i*\rho'_{ext})^2$	7.6×10^{-11}	5.9×10^{-10}
$h'*$	1.8 MJ/m^3	20 MJ/m^3
$\rho'*$	2.15×10^{-9}	4.0×10^{-9}
$G'*$	.005	.057
$h'*/h'_{max}$	.09	0.40
$G'*/G'_{max}$	.06	.09
$i*/i_o$	0.33	0.189
$\rho'_{ext}/\rho'*$	1.86	3.2
$(i*/i_o)(1+\rho'_{ext}/\rho'*)$	0.96	0.79
same thing square	0.91	0.63
$E*/E_o$	1.06	1.12
$E*/E_o > 1$?	yes	yes
h'_f	13 MJ/m^3	38 MJ/m^3
G'_f	.032	.148
$\Delta\tau_{JL} = (\tau_{ext}/2)(G'_f/G'_{max})$	59 ms	57 ms
$\Delta\tau_{JS} = \frac{\xi}{2}\tau'_{max}/(1+r_{\tau,trans})$	5 ms	8 ms
$\tau_J = t_{So}+\Delta\tau_{JL}+\Delta\tau_{JS}$	92 ms	83 ms (109ms)
$r_E = 2(\Delta\tau_{JL}+\Delta\tau_{JS})/\tau_{ext}$	.41	.26
$j_J^2 = i_o^2/C_J$	1.50×10^{18}	1.24×10^{18}
$J = j_J^2 \cdot \tau_J$	1.38×10^{17}	1.03×10^{17} (1.35×10^{17})
$T_{lim} = F^{-1}(J)$	320°K	180°K (300°K)
$E_{ext,f} = E_o\ r_E$	171 kJ	2.9 MJ
$E_{ext,f}/(n_\ell\ n_s)$	21 kJ	16 kJ
Magnet O.K. ?	yes	yes

17. Dependence on i_o.

Consider a magnet with an adequate quench circuit designed according to the rules of section 14. The design value of the current is i_{od} and the magnet is safe for quenches when the current is i_{od}. When the current is being turned on, the magnet may quench before the current reaches the design value i_{od}. Therefore, the quench circuit has to be adequate also when the current has any value $i_o < i_{od}$. This may look as an obvious property of the quench circuit. We are, however, giving a full justification hereafter.

Following the checking procedure of section 14 for $i_o < i_{od}$, we will compute new values for E_o, h'_{max} and G'_{max}. They will be reduced by the factor $(i_o/i_{od})^2$. G'_f, determined on a graph such as fig. 6, will be smaller than for i_{od}, therefore $i_o^2 \Delta\tau_{J,L}$ will be smaller too. $\Delta\tau_{JS}$ will be the same for a resistor in the shunt (equation 14.24) and smaller for a varistor. Our experience with induced quenches[2,4] shows that t_{So} increases when i_o decreases, but $i_o^2 t_{so}$ still decreases. It follows that J_f of eq. (14.27) is smaller for i_o than i_{od}. For a varistor, E* will be the same. Therefore the test 14.d 3) will be O.K. for i_o if it is O.K. for i_{od}. The maximum voltage is smaller whether there is a resistor or a varistor in the shunt. It follows that the quench circuit and the magnet satisfy the rules of section 14 for any $i_o < i_{od}$ if it satisfies them for i_{od}.

Below design value, the protection against destruction is actually easier than at design value. Probably, one can often design a simpler quench protection circuit that would satisfy the requirement of section 14 for a current $i_o = i_{om} < i_{od}$ but that would not for $i_o = i_{od}$. That simpler circuit can safely be used to test the magnet up to i_{om}. Since our requirements of section 14 are pessimistic, the simple quench circuit may happen to be adequate for quench protection also when $i_o = i_{od}$, in spite of the fact that it violates the rules of of section 14 for $i = i_{od}$. To find out if such is the case, one can use the testing procedure of ref. 5, first inducing quenches with currents less than i_{om} and augmenting the current step by step untill the burnout limit of ref. 5 is reached. If that limit is above i_{od}, one ends up with the simpler quench circuit and it is safe. If the burnout limit of ref. 5 is below i_{od}, then one may have to return to the sophisticated original design of the quench circuit.

18. Solenoids with an iron yoke.

If there is an iron yoke, the formulae for infinite solenoids can be applied.

a) Scaling in size.

It is possible to build solenoids with the same field, different diameters D, different lengths ℓ but with the same values of T_{lim}, T'_{lim} and $v_{c,max}$ according to the procedure of section 14. As a function of the two variables D and ℓ, it is sufficient to scale all the parameters according to the following rules.

Thickness of the bore tube =	$e' \sim D$	(18.1)
Number of turns =	$n_c \sim$ constant	(18.2)
Current =	$i_o \sim \ell$	(18.3)
External resistor =	$R_{ext} \sim 1/\ell$	(18.4)
Self inductance =	$L \sim D^2/\ell$	(18.5)
External time constant =	$\tau_{ext} \sim D^2$	(18.6)
Total energy	$E_o \sim D^2\ell$	(18.7)

h'_{max}, G'_{max} and therefore G'_f and h'_f are the same.

$$\Delta\tau_J \sim D^2 \qquad (18.8)$$

We assume that the quench detection is set in such a way that $t_{\ell o}$ scales like $\Delta\tau_J$, i.e like D^2.

The product $i_o^2 \tau_J \sim D^2 \ell^2$ (18.9)

To get the same T_{lim}, J_f should be the same and C_J needs to scale like $D^2\ell^2$ The simplest way is to keep r_{sc} constant and scale A_{wire} like $D\ell$. This way the current density in the superconductor varies but it may not matter. To keep the current density the same, one has to vary A_{wire} and r_{sc} such that

$$C_J = \frac{r_{sc}}{1+r_{sc}} A_{wire}^2 \sim D^2 \ell^2 \qquad (18.10)$$

$$j_{sc} = \frac{i_o (1+r_{sc})}{A_{wire}} \sim \text{constant} \qquad (18.11)$$

Of course, if one has to redesign a magnet and change its diameter or its length, the rules for scaling above may not necessarily lead to the best design, but they provide an easy solution.

b) Change in n_c

For a given D, ℓ, and field B the number of turns n_c is a design parameter that can be varied to adjust $v_{c,max}$ and i_o without changing G'_{max}, h'_{max} and therefore T'_{lim} and T_{lim}. The corresponding scaling has to apply.

$$i_o \sim 1/n_c \qquad (18.9)$$

$$A_{wire} \sim 1/n_c \qquad (18.10)$$

For a resistor

$$R_{ext} \sim n_c^2 \qquad (18.11)$$

For a varistor, the number of elements in parallel $\sim 1/n_c$ (18.12)

in series $\sim n_c$ (18.13)

$$i^* \sim 1/n_c \qquad (18.14)$$

$$v_{c,max} \sim n_c \qquad (18.15)$$

$$C_J \sim 1/n_c^2 \qquad (18.16)$$

$$v_{c,max} \cdot i_o \sim R_{ext}\, i_o^2 \sim \text{constant} \qquad (18.17)$$

$$\tau_{ext} = \frac{L}{R_{ext}} \sim \text{constant} \qquad (18.18)$$

c) Determination of T_{1im}

For a solenoid with an iron yoke, h'_{max}, G'_{max} and τ_{ext} amount to

$$h'_{max} = \frac{B^2}{32\cdot 10^{-7}} \frac{D}{e'} \qquad (18.19)$$

$$G'_{max} = 10^{-7} \frac{R_{ext} i_o^2}{2\ell} \sim \frac{v_{c,max} i_o}{\ell} \qquad (18.20)$$

$$\tau_{ext} = \frac{L}{R_{ext}} = \frac{2E_o}{R_{ext} i_o^2} = \frac{B^2}{16\times 10^{-7}} \frac{\ell D^2}{R_{ext} i_o^2} = \frac{B^2 D^2}{32\times 10^{-7} G'_{max}} \qquad (18.21)$$

From h'_{max}, G'_{max} and a graph like fig. 6, we can determine G'_f and therefore the $\Delta\tau_{J's}$ of (14.23) to (14.25). From them we determine T_{1im} using (14.26) to (14.28).

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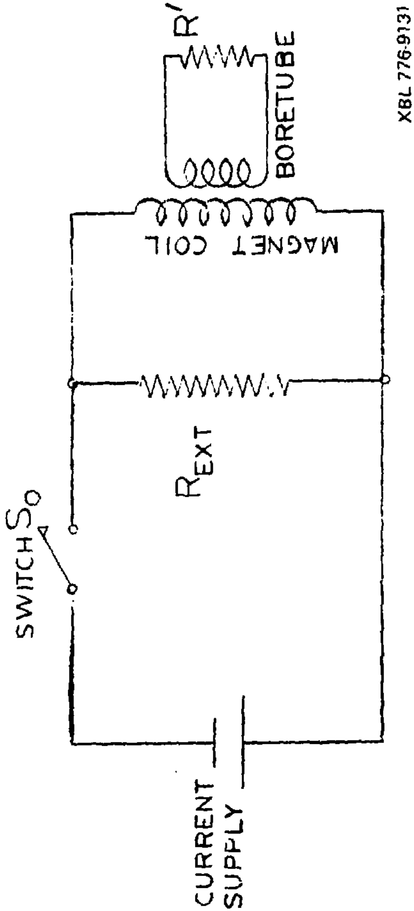
R'
BORETUBE
MAGNET COIL
R_{EXT}
SWITCH S_O
CURRENT SUPPLY
XBL 776-9131

Fig. 1

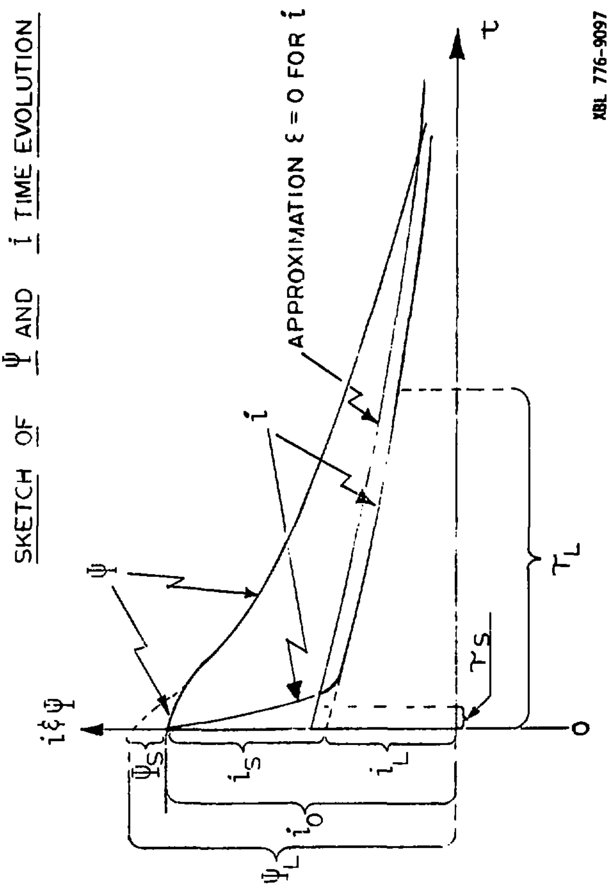
SKETCH OF Ψ AND i TIME EVOLUTION
APPROXIMATION ε = 0 FOR i
i & Ψ
Ψ
i
τ
τ_S
τ_L
Ψ_S
i_S
i_L
i_O
Ψ_L
O
XBL 776-9097

Fig. 2

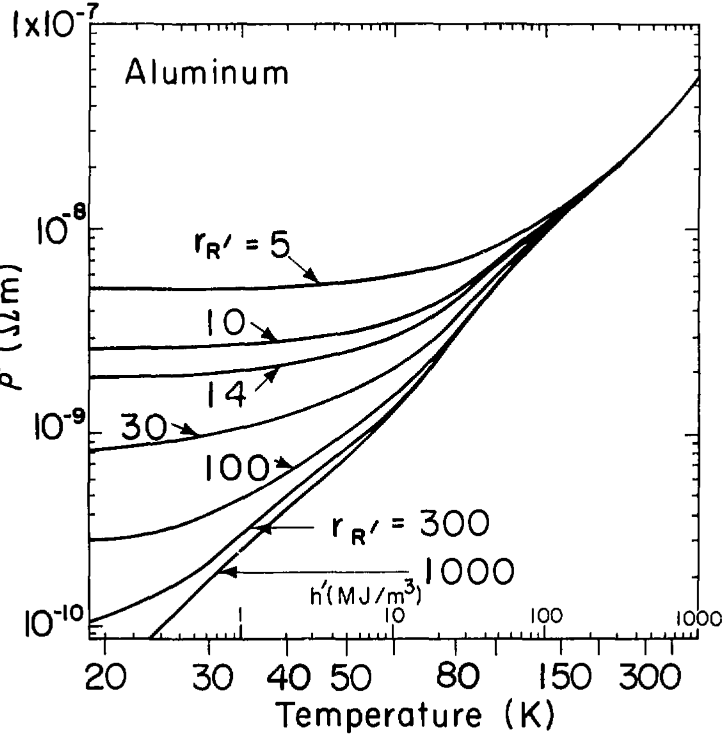

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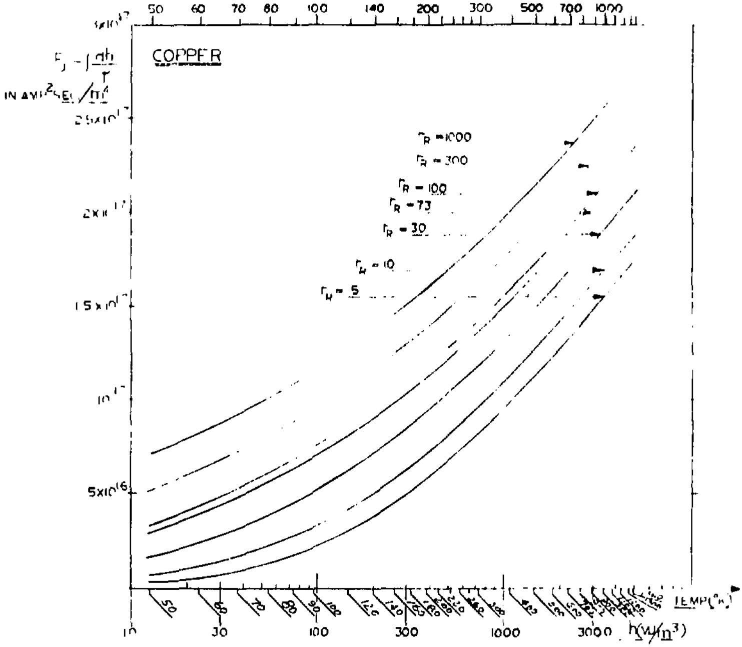
Fig. 3



XBL 776 9130

Fig. 4

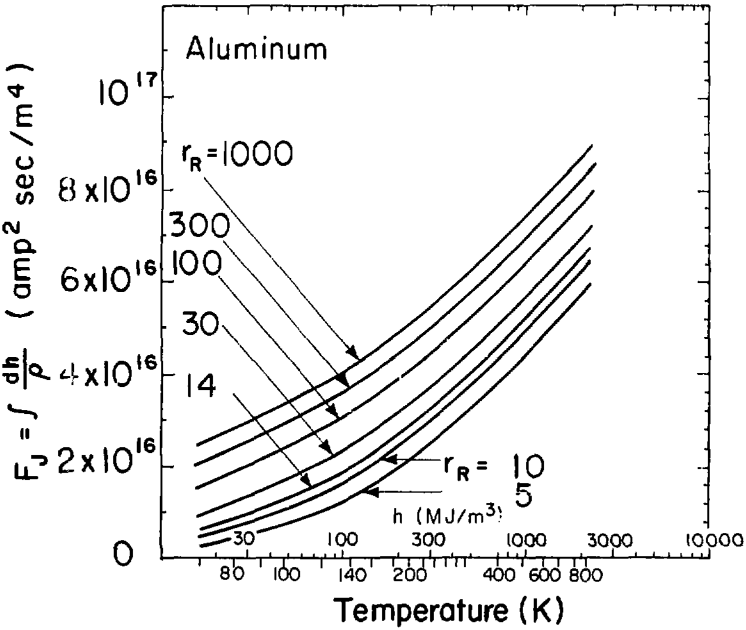


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Fig. 5

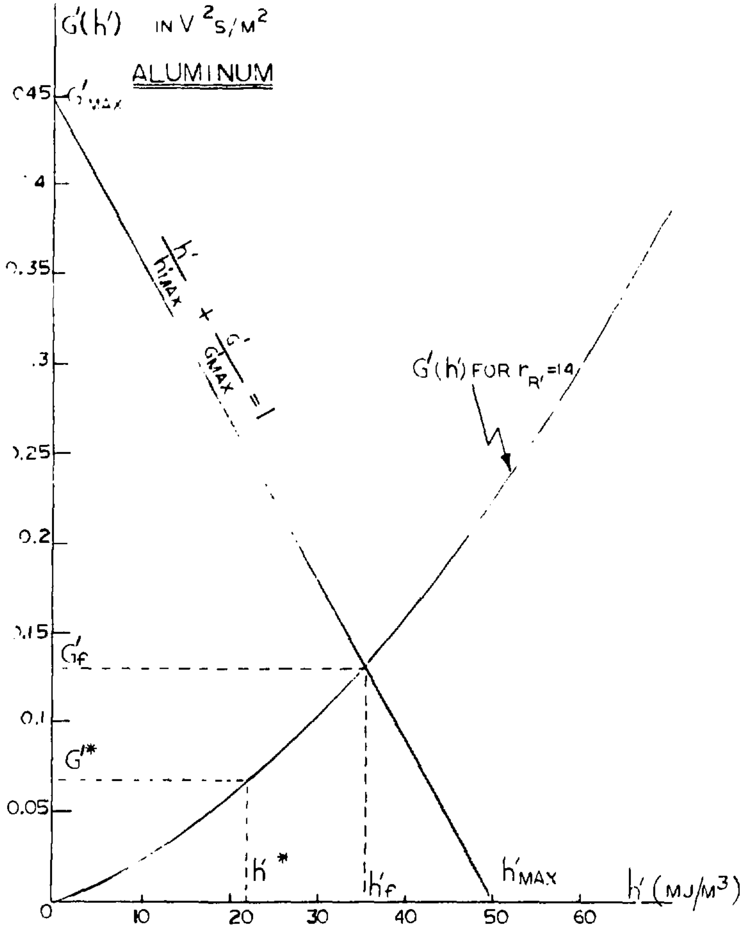

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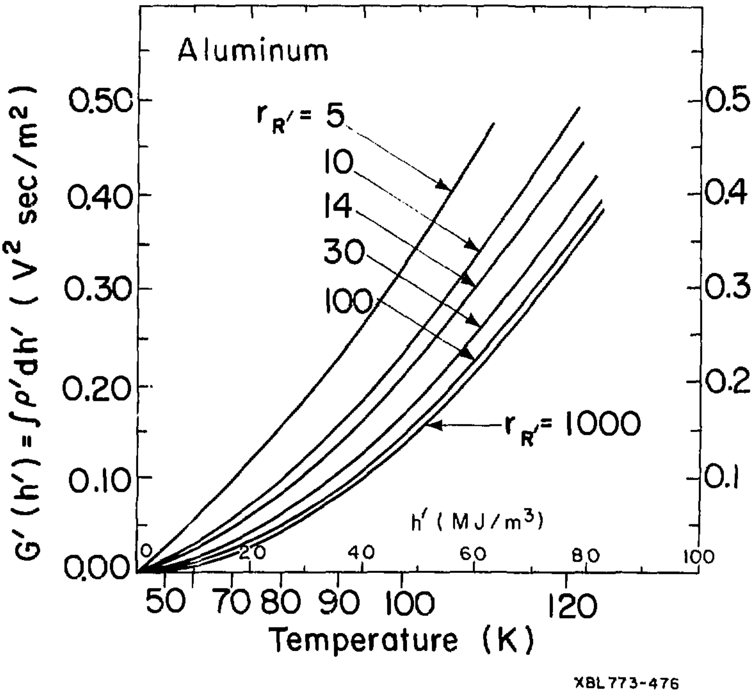
Fig. 6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10
14
30
100
$r_{R'}$ = 1000
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0.40
0.30
0.20
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0.00
0.5
0.4
0.3
0.2
0.1
G′ (h′) = ∫ρ′dh′ (V² sec/m²)
h′ (MJ/m³)
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20
40
60
80
100
50
70
80
90
100
120
Temperature (K)
XBL 773-476

Fig. 7

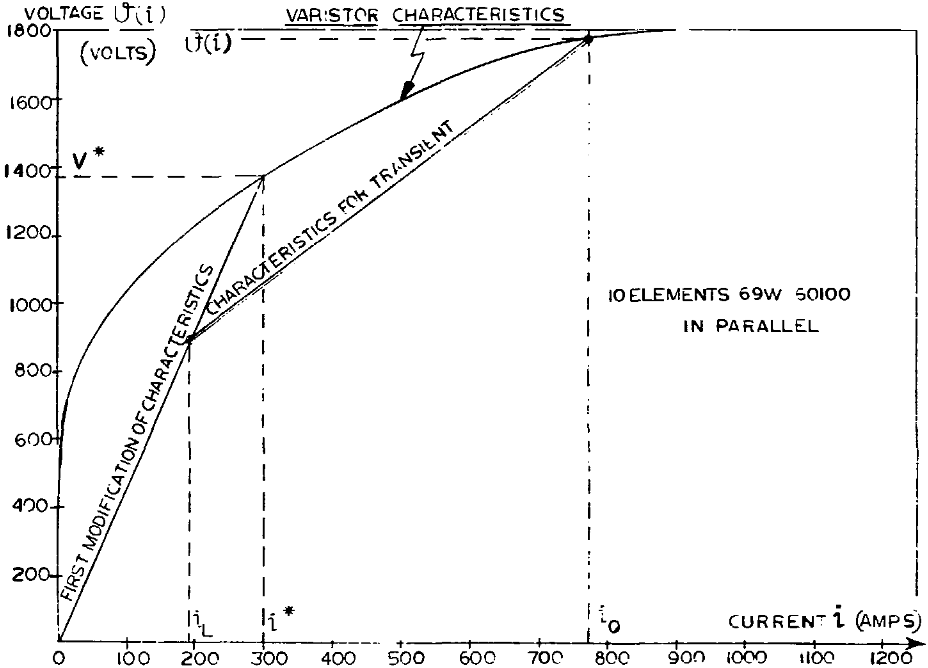

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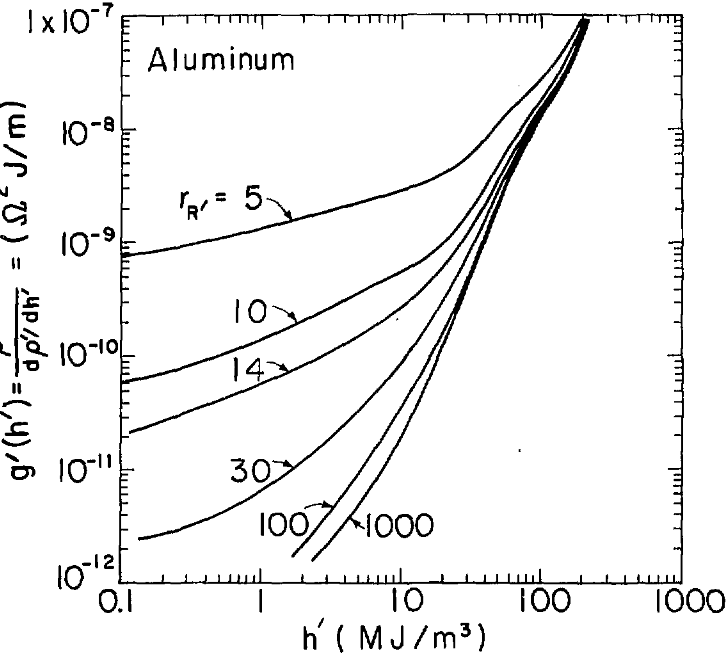
Fig. 8



XBL 773-477

Fig. 9