Title
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Journal
Advances in Physics X, 1(4)

ISSN
2374-6149

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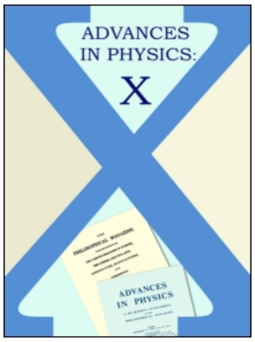
Publication Date
2016-07-03

DOI
10.1080/23746149.2016.1240017

Peer reviewed



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Advances in Physics: X

ISSN: (Print) 2374-6149 (Online) Journal homepage: http://www.tandfonline.com/loi/tapx20

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To cite this article: R. L. Stenzel (2016) Whistler waves with angular momentum in space and laboratory plasmas and their counterparts in free space, Advances in Physics: X, 1:4, 687-710, DOI: 10.1080/23746149.2016.1240017

To link to this article: http://dx.doi.org/10.1080/23746149.2016.1240017

Published online: 04 Nov 2016.

ADVANCES IN PHYSICS: X, 2016
VOL. 1, NO. 4, 687–710
http://dx.doi.org/10.1080/23746149.2016.1240017

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REVIEW ARTICLE

OPEN ACCESS

Whistler waves with angular momentum in space and laboratory plasmas and their counterparts in free space

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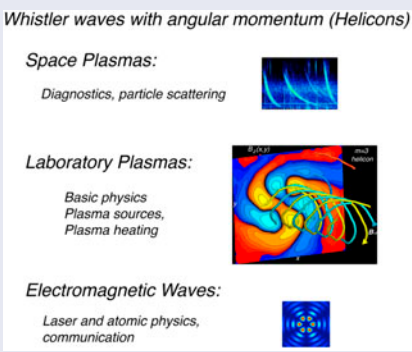
ABSTRACT

Electromagnetic waves with helical phase surfaces arise in different fields of physics such as space plasmas, laboratory plasmas, solid-state physics, atomic, molecular and optical sciences. Their common features are the wave orbital angular momentum associated with the circular wave propagation around the axis of wave propagation. In plasmas these waves are called helicons. When particles or waves change the field momentum they experience a pressure and a torque which can lead to useful applications. In plasmas electrons can damp or excite rotating whistlers, depending on the electron distribution function in velocity space. A magnetized plasma is an anisotropic medium in which electromagnetic waves propagate differently than in space. Phase and group velocities are different such that wave focusing and wave reflections are different from those in free space. Electrons experience Doppler shifts and cyclotron resonance which creates wave damping and growth. All media exhibit nonlinear effects which do not occur in free space. Common and different features of vortex waves in different fields will be reviewed. However, a comprehensive review of this vast field is not possible and further readings are referred to the cited literature.



ARTICLE HISTORY
Received 20 July 2016
Accepted 19 September 2016

KEYWORDS
Whistler waves; helicon modes; wave angular momentum; laboratory experiments; active space experiments; electromagnetic vortex waves

PACS
94.30.Tz Whistler waves; 94.05.Pt wave/wave and wave/particle interactions; 94.30.Tz electromagnetic wave propagation; 52.50.Qt plasma heating by radio-frequency fields; ICR, ICP, helicons

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1. Introduction

This paper deals mainly with a special class of electromagnetic waves, whistler waves, which are very important in plasma physics. These waves have been first discovered in space plasmas, then investigated in solid-state plasmas, laboratory plasmas, industrial plasmas and fusion plasmas. One of the many interesting properties of whistlers is that the wave can propagates like a corkscrew, called helicon. Such vortex waves have been studied first on electromagnetic waves in free space and found many useful applications in laser and atomic physics, communication and astronomy. Although each field is too rich to review thoroughly this paper intends to point out the connection of similar physics in different fields which is sometimes lost by specialization. The paper will address major observations and theories, the accomplishments and frontiers in different fields, the common physical principles, differences in different media, important applications, open topics and possible future developments. Whistlers have been studied for about 100 years and published in thousands of papers which cannot be referenced all, hence review papers are preferred references and the rest are the authors choice.

2. Brief historical background

The name 'whistlers' dates back 100 years to unexplained whistling sounds on telephone lines [1,2]. They were subsequently explained to arise from by lightning pulses, whose waves coupled into the ionosphere where they dispersed upon propagating through an anisotropic plasma. A theory of electromagnetic plane waves in cold plasmas was developed which could explain many of the phenomena [3–5]. In the space age the ground observations were enriched by observations from rockets and satellites [6,7].

Some 50 years ago waves with similar properties were observed in magnetized solids [8]. Whistler modes were also excited in discharge plasmas produced in the laboratory [9]. The waves in bounded plasma columns were termed 'helicons' [10] and a new specialty of low temperature plasma applications evolved [11]. Many laboratory experiments were dedicated to understand the properties of whistler modes in space plasmas [12,13].

The large effort of thermonuclear fusion research requires to heat current-carrying plasmas in tokamak devices [14,15]. Among different waves the whistler modes is now considered useful for inducing plasma currents and heating electrons [16].

In parallel with plasma waves the field of electromagnetic waves in free space has evolved tremendously in the last century with uncountable applications. Since the development of lasers the property of waves with angular momentum has received much attention and many applications. These are waves which propagate along one axis while also rotating around the axis. Such 'helicoid' waves carry linear field momentum and angular field momentum which can be transferred to particles on atomic scales. Helicon waves in plasmas can have the same properties but have just begun to realize its potential usefulness [17]. Thus similar phenomena can exist in different fields of physics ranging from the atomic scale to the astrophysical scale and the purpose of this paper is to point out the common connections.

2.1. Whistler modes vs free space electromagnetic waves

Electromagnetic waves in free space are propagating electric and magnetic fields obeying Maxwell's equations. Faraday's law relates the electric field to a time-varying magnetic field. Ampere's law states that the magnetic field is produced by displacement currents created by the time-varying electric field. The coupled differential equations result in a wave equation predicting that the field propagates at the speed of light. The solution of the wave equation yields plane waves, considered as eigenmodes in a Cartesian coordinate system.

In a plasma the magnetic field is predominantly produced by moving charges, which for whistler modes are the electrons. In a plasma with a static magnetic field the electron motion differs along and across the static field which creates an anisotropy in the wave propagation. Further complexity arises from two types of currents, the electron Hall current ($\propto \mathbf{E} \times \mathbf{B}$) and the electron displacement current ($\propto \partial E/\partial t$). The result is that whistler modes exhibit dispersion, resonances, different phase and group velocities, and for low frequencies two oblique modes at the same frequency.

In space plasmas it is common to describe whistlers by plane waves. For axial propagation in waveguides or plasma columns cylindrical coordinates are more appropriate. These fields are described as paraxial eigenmodes, i.e. waves propagating along and around the guide axis while forming non-propagating (standing) waves radially. The physics of wall reflections is complicated because in an anisotropic medium the phase and group velocities reflect differently.

Eigenmodes are a mathematical solution to a wave equation subject to boundary conditions. The eigenmode concept does not describe how such modes are produced. In fact, ideal plane waves or Bessel beams are not physically realizable concepts since the source would have to be of infinite size and deliver infinite energy. Finite-size sources such as antennas produce fields which are usually described by a superposition of eigenmodes, i.e. Fourier analysis. In bounded and anisotropic plasmas the antenna-wave coupling becomes a difficult problem.

3. Whistler modes in space plasmas

Aside from the historical discovery whistler wave research in space plasmas has produced a wealth of information of these waves and the plasma environment. First the waves were used to study density and magnetic field of the ionosphere from transit times of wave packets. Then the wave–particle interactions were discovered, such as wave amplification and modulation. The scattering of radiation belt electrons by whistler modes has been discovered and used to explain their confinement time and equilibrium density [18,19]. A nuclear explosion in space creates an electromagnetic impulse and energetic particles which has disastrous effects on satellites [20]. Presently much effort is made to scatter such energetic electrons in velocity space by Doppler-shifted cyclotron resonance [21]. Active wave injections from ground transmitters produced a wealth of nonlinear phenomena, among them involving whistler modes [22–24]. Active wave injections from spacecraft are still in its beginnings [25]. Few electric dipole and magnetic loop antennas have been deployed successfully including long tethers [26]. The nearly collisionless space plasmas are an ideal environment for studying nonlinear whistler wave phenomena. But many of the familiar concepts such as whistler wave propagation, ray tracing to explain refraction, reflection and ducting are theoretical models which have never been tested experimentally since spatially resolved wave measurements cannot be done from a single spacecraft.

3.1. Excitation, detection, and propagation

One process which creates whistler waves in space are lightning strikes from clouds down to Earth or up into the ionosphere [27,28]. In space terminology these waves are ‘whistler waves’ while other excitation sources create ‘whistler modes’. The latter arise from non-Maxwellian electron distribution functions such as electron beams, excess perpendicular electron energy, loss-cone distributions, etc, but also from antennas on spacecraft [29]. The typical properties of whistlers are that they are small amplitude waves, weakly damped, propagating and refracting as WKB modes in weakly nonuniform density and ambient magnetic field. They are assumed to be plane waves which cannot be verified since this would require multipoint measurements. Wave propagation is assumed to be in the direction of $\mathbf{E} \times \mathbf{B}$. The energy flow is obtained numerically by ray tracing. The topology and propagation of wave packets cannot be resolved from single point measurements. Lack of spatially resolved wave measurements is a main reason for performing dedicated laboratory wave experiments.

Transmission experiments have been done with two rockets or satellites [29]. Small electric and magnetic dipole antennas do not radiate like in free space. They do not excite plane waves or paraxial beams. Their radiation patterns are distorted by the spacecraft motion across the ambient magnetic field. Any current-carrying or magnetized object in motion creates a Cherenkov-like wing of whistler or Alfvén waves, depending on the source size compared to the wavelength [30,31]. In order to excite large amplitude whistlers with electric dipoles the sheath nonlinearity creates unknown effects [32,33]. Likewise magnetic loops can distort the near-zone magnetic field which creates unknown radiation patterns. These effects have partially been studied in laboratory experiments which will be described further below.

3.2. Wave–particle, wave–wave interactions

Wave–particle interactions are due to three effects, two of which are well known. The first is Landau resonance, where electrons move along the ambient magnetic field at the phase velocity of the wave. The electrons experiences a dc electric field, provided it is along $\mathbf{B}_0$. A parallel $\mathbf{E}$-field is small for low-frequency whistlers but not negligible in oblique whistlers. The electrons gain energy when the wave is slightly faster than the electrons which causes wave damping. Vice versa if the electrons are faster than the wave the wave grows, resulting in an instability [34]. VLF hiss may be created this way.

The second interaction is due to the Doppler-shifted cyclotron resonance [35]. When electrons move in the direction opposite to the wave propagation they experience an upshifted frequency which can produce cyclotron resonance, $\omega - \mathbf{k} \cdot \mathbf{v}_{\|} = \omega_c$. The electrons gain energy in their motion perpendicular to the magnetic field which damps the wave. Vice versa an instability can arise when the electron distribution has excess perpendicular energy.

The third interaction is possible when a rotating electron interacts with a rotating wave field of a helicon wave. The *transverse* Doppler shift $\omega - \mathbf{k} \cdot \mathbf{v}_{\perp} = \omega_c$ brings the electron into cyclotron resonance when the wave and electron rotate in opposite directions. Some of the orbital angular momentum of the rotating field can be transferred to a rotating electron beam which causes wave damping or, vs, wave growth. When the particle rotates in the direction of the rotating phase the frequency is down shifted and a *perpendicular* Landau resonance can arise, $\omega - \mathbf{k} \cdot \mathbf{v}_{\perp} = 0$. This is a much stronger interaction than the parallel Landau resonance because the wave electric field of whistlers is predominantly perpendicular to $\mathbf{B}_0$. The *transverse* Doppler shift of helicon waves has not yet been explored in space and laboratory plasmas.

Wave–wave interactions arise from nonlinear coupling between waves which can result in parametric instabilities. Wave energy and momentum have to be conserved. A strong ‘pump’ wave (0) can decay into two lower frequency modes (1,2) and the interaction must obey $\omega_0 = \omega_1 + \omega_2$ and $\mathbf{k}_0 = \mathbf{k}_1 + \mathbf{k}_2$ for plane wave vectors. Examples are the decay of a whistler pump into whistlers [36], Alfvén waves and sound waves. Broadband turbulence can arise from nonlinear wave–wave interactions. The cascading of solar wind Alfvénic turbulence extends into the low-frequency regime of whistler modes where dissipation wave becomes important [37,38]. Density nonuniformities can also couple whistler modes with lower hybrid modes via mode conversion [39,40].

3.3. Importance and future research

Research is a human activity which often begins with a rapidly growing phase of excitement, a saturation phase when most effects are tentatively understood, a continuation phase when new instruments lead to new discoveries, and finally a search of useful applications. Whistler mode research has experienced all of these phases but without experiencing a phase of decline.

One of the recent advances is the development of multipoint measurements by a cluster of satellites. So far only a small number of spatial points can be measured which limits the spatial resolution since at least two data points per wavelength are required by the Nyquist criterion. A multitude of disposable sub satellites may be the next approach. Simultaneous multipoint measurements are needed since the waves are not the same for recurring satellites orbits.

Besides electron whistler modes ion whistlers have also been studied extensively in space plasmas [41,42]. They are useful to remotely diagnose ion species, densities and temperatures. Scattering of ions out of the loss cone has become a goal for future active wave injection experiments. As mentioned above the primary goal is to remediate MeV electrons from nuclear explosions in space [43]. Methods to inject strong whistler modes need to be developed. Ground-based radiation sources require too much power since the coupling of free-space electromagnetic waves into whistler modes is not very efficient. Active whistler wave injections from satellites using efficient antennas [44] have not yet been performed.

Magnetic reconnection has been studied for half a century since it is a fundamental problem of plasma physics, trying to explain particle energization, field topology changes, particle transport, and wave turbulence. Magnetohydrodynamical (MHD) theories have been broadened to include the Electron MHD (EMHD) physics near the null region [45,46]. Whistler mediated fast reconnection signatures such as quadrupole magnetic fields [46,47] and whistler wave emissions have been observed not only in laboratory plasmas [48] but also in space plasmas [49,50]. The propagation of whistler modes in highly nonuniform magnetic fields, densities, and particle distributions is a difficult problem [51]. Without spatial resolution it is difficult to obtain a complete picture of localized field structures such as whistlers near the lunar surface magnetic field anomaly [51,52].

The topic of momentum transport in space and astrophysical plasmas is of great importance [53]. However it is mainly focused on the fluid momentum while the aspect of the field momentum of whistler waves with orbital angular momentum has not yet been considered seriously. Their existence of such waves has been demonstrated in large uniform laboratory plasmas where boundaries and gradients do not play a role [54]. Their investigation could reveal new wave–particle interactions. Active experiments are needed but initial results could be obtained from the wave exhaust from helicon thrusters developed for use on spacecraft.

Another research topic deals with the prediction of earthquakes from space [55,56]. Earthquakes have been observed to modify the properties of the electromagnetic fields in space, and this includes emissions of VLF waves detected in the ionosphere. If reproducible results can be confirmed this research will have an enormous impact. Thus, whistler mode research not only deals with fundamental problems but also begins to address important applications.

4. Whistler modes in laboratory plasmas

Each plasma environment has its own characteristics. Compared to space plasmas laboratory plasmas offered spatial resolution, reproducibility and control over the wave excitation. But early plasma devices were limited by boundary effects and collisions which are nearly absent in space. Solid-state plasmas also relied on external wave excitation and detection. But over time large diameter discharge plasmas were developed [57–60] which nearly eliminated the boundary effects. Nevertheless, plane waves cannot be excited with simple dipole antennas so that a comparison with prevailing theories was not rigorously justified. However, recent antenna studies suggest how to solve this last hurdle with large antenna arrays [61].

Besides fundamental research on whistler modes several important applications inspired researchers in this field. It turned out that helicon modes can produce dense rf plasmas [62]. Without internal electrodes these rf discharges can produce plasmas in reactive gases for etching of semiconductors [11,63]. When the plasma expands along diverging field lines a double layer is formed which can be used for small thrusters on spacecraft [64]. High-frequency whistlers near the cyclotron frequency have been used for a long time to heat electrons [65] and produce plasmas [66].

In magnetic fusion devices the injection of powerful whistler modes is used to heat the electrons and to drive electron currents [16,67]. A rotating rf magnetic field can drive toroidal currents in spheromak devices operating in the Hall MHD regime [68,69].

Laboratory experiments were often designed to model phenomena of interest to space plasmas. Although size, density and magnetic field have to be scaled the physics of Hall reconnection has been clearly demonstrated in laboratory experiments [70,71]. The generation of VLF hiss by electron beams [72] has been shown [34]. Whistler chorus has been generated with energetic electron beams and the different spectra are interpreted by different wave–particle resonances [73]. Whistler wave ducting in narrow density troughs and crests are laboratory observations of relevance to active space experiments [74]. The scattering of electrons by whistler modes has recently been demonstrated in the laboratory [43,75]. Whistler instabilities by anisotropic distributions have also been observed [76]. Many nonlinear phenomena have been studied in the laboratory which are not found in space [13] Comparison of electric dipole antennas and magnetic loops show that the latter have a much higher radiation efficiency for low-frequency whistlers than dipoles [44].

4.1. Solid-state plasmas

Electrons in solids can move rapidly compared to positive charges (holes). When a strong magnetic field is applied the EMHD conditions [45] can be found in solids [8]. In order to limit collisional effects low temperatures are required. Such solid-state plasmas support whistler modes as in plasmas. The waves are excited and detected with magnetic loops externally to the solid-state sample. Due to wave reflections at the boundaries of the solid slab or cylinder resonances arise. From the resonance conditions, wavelengths and dispersions can be obtained. These in turn yield the electron density and damping, hence are useful to diagnose the solid-state properties [77]. Bounded whistler modes with angular field rotation were termed 'helicon' waves [10]. The wave propagation was modeled like that of electromagnetic waves in cylindrical waveguides, except for the Ohms law of EMHD: the radial field dependence is described by Bessel functions of the first kind, which describes radial standing waves due to wall reflections. The 'paraxial' propagation ($\parallel \mathbf{B}_0$) produces plane phase fronts for the lowest order helicon mode ($m = 0$) and spiraling phase fronts for higher order helicons. The dispersion relation ω vs $k_\parallel$ for these 'eigenmodes' differs from that of plane waves without boundaries, ω vs k_{total}. The same concept of helicon eigenmodes has been applied to whistler modes in gaseous plasmas which will be described below.

Helicons in solid-state plasmas also exhibit nonlinear wave–wave phenomena. For example, parametric instabilities between helicons and sound waves have been seen [78].

4.2. Gaseous plasmas

A wide range of whistler mode phenomena have been studied in laboratory plasmas. These start from simple experiments to observe linear waves, measure their dispersion and damping. In bounded plasmas the waves cannot be plane waves such that the comparison with theory was only an approximation. The excitation of these waves was done with electric and magnetic antennas whose radiation patterns, radiation resistance and nonlinear perturbations have received much attention [79,80]. The radiation pattern of 'point' antennas whose dimensions are small compared to the wavelength exhibit 'resonance cones' whose opening angle is the group velocity angle for oblique cyclotron resonance [81]. These patterns have not only been observed in the laboratory [82] but also in active experiments in space plasmas [83]. Whistler mode refraction in density and magnetic field nonuniformities have been studied [84]. As the diagnostics improved the complete time–space dependence of whistler modes has been mapped [85]. Fourier transformation into $\omega - \mathbf{k}$ space allowed a meaningful comparison between observation and plane wave theory [86]. From 3D spatial data the current density [87] and magnetic helicity [88] have been obtained unambiguously. Instead of antennas whistlers have also been excited by a modulated small diameter electron beam traveling at the speed of the whistler mode [89]. Whistlers have also been produced by parametric decay of Bernstein waves [90,91]. vs, whistlers can decay by parametric instabilities of whistlers into lower frequency whistlers and ion acoustic waves [92]. Whistlers have also been produced by mode conversion during ionospheric modification experiments [93]. Broadband whistler modes are also excited by thermal fluctuations [94].

Wave–particle interactions have been investigated in various experiments. Large diameter electron beams produce whistler instabilities by Landau resonance [34] with obliquely propagating whistlers, a process also suggested to explain the emission of VLF hiss in the magnetosphere [95]. The interaction of energetic electrons with whistler waves via the Doppler-shifted cyclotron resonance has been verified [96]. Temperature anisotropy has been shown to excite whistler waves [97,98].

4.3. Nonlinear whistlers

Since the dispersion of whistlers depends on density and magnetic field a perturbation of these parameters by the wave produces nonlinear effects. Density perturbations by ponderomotive force or thermal pressure can produce a density trough which guide whistlers [99–101]. Focusing resonance cones produces density holes, fast ions, and low-frequency turbulence [82]. Whistler modes whose wave magnetic field exceeds the ambient field have been studied in high beta plasmas. When the wave field opposes the ambient field the net field topology forms force-free field termed a propagating ‘whistler spheromak’ [97]. In spite of null points in the total field these force-free structures are stable. When the wave packet enhances the ambient field a propagating ‘whistler mirror’ is excited. The propagation speed and damping depend on the field topology. Thus a continuous wave produces alternating spheromak and mirror topologies whose different propagation speeds creates non-sinusoidal waves with harmonic spectra. In the toroidal null line of the whistler spheromak the electrons are strongly accelerated and heated by current-driven ion sound turbulence. The heating creates temperature anisotropies which in turn excite whistler instabilities [76]. Thus a whistler spheromak becomes a moving radiation source of unstable whistlers. The energy for electron heating and wave emissions is supplied by the fields of the whistler spheromak which consequently leads to strong damping. The conversion of stored magnetic energy to electron energy is an example of magnetic reconnection. It occurs in a toroidal null line with an O-type null [102] rather than the traditional X-type null [71]. Field line ‘merging’ in an O-type null results in complete magnetic field line annihilation. The out-of-plane field of a 3D O-point exhibits the characteristic quadrupole shape [103] which is caused by the convection of frozen-in magnetic field lines by the in-plane electron fluid flow [45].

Nonlinear whistler modes can even exist in field-free plasmas. When the wave amplitude is large enough to magnetize the electrons it propagates in its own magnetic field [104]. Without a guide field the wave propagation becomes extremely nonlinear. The wave spread decreases its amplitude, which creates cyclotron resonance, wave absorption, and the end of wave propagation.

In the opposite limit of very small amplitudes the magnetic fluctuations from thermal fluctuations have been studied in the regime of whistler modes. Cross-correlations verified that the magnetic fluctuations are whistlers with an *1/f* spectrum [94]. Unlike the solar wind spectrum it cannot be explained by cascading since waves excited by thermal fluctuations are completely linear.

4.4. Whistler mediated reconnection

Whistler modes have become important in the physics of magnetic reconnection. While early models described the neutral sheet region as a diffusion region the present models distinguish an ideal outer MHD and an inner EMHD region. The fast field propagation of whistler modes enhances the reconnection rate. The whistlers are nonlinear wave packets in a highly nonuniform background field of the neutral sheet. Early experiments demonstrated reconnection in the EMHD regime [70,102,105]. This topic is still of great current interest [48,106]. Whistler modes have been observed in reconnection regions in the laboratory [105,107] and in space [108] but their excitation mechanism and effects remain subjects of research.

4.5. Transient currents carried by whistlers

Whistler modes are also involved in transient and localized magnetic structures which can be considered whistler wave packets. An example is the propagation of a pulsed electron beam injected into a magnetized dense plasma. The beam current produces a magnetic field which propagates at the group velocity of whistler modes. If the beam electrons propagate faster than the wave, the beam front is current is neutralized by a return current supported by the background electrons. The beam current and its return current propagate as a transient whistler wave packet into the plasma. Transport of currents by whistler modes has been verified in laboratory experiments [109].

A time variation can also arise from a convective derivative of a moving dc current system. An example is the electrodynamic tether in space [110]. The cross-field motion of a long wire perpendicular to the magnetic field induces a voltage which drives a current from end electrodes through the plasma. Since the current source rapidly moves through the stationary plasma the tether induces transient currents whose spectrum falls into the whistler regime. The succession of moving transient currents gives rise to a wing-like current or magnetic structure, termed a ‘whistler wing’, as demonstrated in a laboratory experiment [111]. Whistler wings are also emitted from a moving object with a constant magnetic field, which has been observed in space [112,113].

4.6. Helicons

A class of whistler modes, termed ‘helicons’, describes low-frequency whistler modes in bounded plasma columns [114,115]. Recently it has been pointed out that helicons also exist in uniform plasmas [116]. The interest in bounded helicon waves arose since they produce dense plasmas for plasma processing and for thrusters in space [62]. As regards the wave properties the geometry suggests to describe helicons in cylindrical coordinates. The helicon wave theory has been adopted from waveguide modes for electromagnetic waves, where the displacement current is replaced but by Ohm’s law for an EMHD plasma. The solution to the wave equation assumes radial standing waves, described by Bessel functions, and phase propagation in axial and azimuthal directions, called ‘paraxial’ propagation, such as $B_z \propto J_m(k_r r)\exp[i(m\phi + k_z z - \omega t)]$. Eigenmodes are formed by integer azimuthal wavenumbers m and radially quantized wavenumbers $k_r = m/r$ due to reflecting boundaries. This concept has also been used in solid-state plasmas [8] and space plasmas for wave propagation in narrow density ducts [117]. This wave theory is questionable since in an anisotropic plasma the group and phase velocities differ, which yields different reflections for phase and group velocities. Experimentally, paraxial wave propagation is usually not observed in helicon experiments [118–120]. The eigenmode theory does not address how these modes are excited. Actual helicon modes are highly nonlinear since the wave produces the plasma and the density determines the wave topology. These difficult nonlinear boundary value problems are presently attempted to explain by computer simulations [121].

As in unbounded plasmas helicons have for a fixed frequency two spatial modes with different radial wavelengths, a helicon wave, and a shorter wavelength Trivelpiece-Gould mode (TG) [122–124]. These two modes also exist in unbounded plasmas where the difference lies in the angle of wave propagation. TG modes are highly oblique whistler modes near the angle of oblique cyclotron resonance where they become highly electrostatic and damped. The latter has been thought to explain the anomalous damping of helicon modes and efficient heating and ionization.

A characteristic feature of helicon waves is their linear and angular field momentum. The former creates radiation pressure on interacting objects such as the electrons. The latter exerts a torque on particles or waves which alter the field rotation. Since whistler modes with helical phase surfaces exist in both bounded and unbounded plasmas, a more general definition of helicons should be whistler modes with orbital angular momentum [17]. The angular momentum has two contributions, one from the circular polarization of whistlers and another one from the azimuthal wave propagation, referred to as orbital angular momentum. An exception might be the $m = 0$ mode which has no azimuthal propagation or orbital angular momentum. The angular momentum properties of whistler modes have not yet received attention in helicon physics or space plasmas but

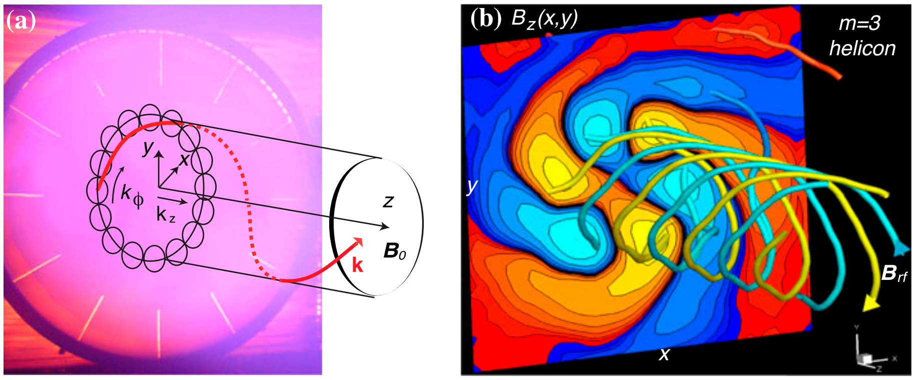

Figure 1. Formation and properties of a helicon mode in a uniform plasma.

Notes: (a) Picture of the discharge plasma produced by a 1 m diam oxide coated cathode. A phased array of identical magnetic loop antennas of 4 cm diam is inserted into the uniform plasma center to excite low-frequency whistler modes with angular and axial propagation. The red line indicates schematically the direction of phase propagation. (b) Snapshot of contours of the axial magnetic field component $B_z(x, y)$ at $z = 15$ cm from the antenna, obtained by measurements and field superpositions. The spiral arms of the contours indicate radial propagation. Field line tracing indicates that this $m = 3$ helicon mode has 3 pairs of opposing and nested spiral field lines. Parameters: $f = 5, f_c = 14, f_p = 3000$ MHz.

its investigation started in basic plasma experiments in laboratory plasmas [54]. Before addressing this topic it is useful to review methods of producing waves with angular momentum.

Whistler modes can be excited by electric or magnetic antennas inside or adjacent to plasmas. In space plasmas electric dipoles are preferred [25] since large magnetic loops have been difficult to deploy [125]. In laboratory plasmas it is generally known that helicon waves are best excited with magnetic antennas rather than electric dipoles. The reason is that the magnetic field energy of whistlers is much larger than the electric field energy, so that it is best to couple to the wave magnetic field. The magnetic field of electric dipoles is limited by small displacement currents while magnetic loops carry large conduction currents and produce large wave magnetic fields. Both electric and magnetic dipoles excite $m = +1$ helicon modes when their dipole moments are aligned across $\mathbf{B}_0$.

The simplest magnetic antenna is a current-carrying loop. When the loop axis is aligned with the magnetic field $\mathbf{B}_0$ there is no azimuthal phase rotation which results in an $m = 0$ mode, whose topology is that of a succession of alternating magnetic vortices [126]. If the loop axis is orthogonal to $\mathbf{B}_0$ the propagating field forms an $m = +1$ helicon mode. Its topology can be described by a dipole field with axis perpendicular to $\mathbf{B}_0$ but rotating azimuthally as the wave propagates axially [127].

In order to produce plane waves the antenna has to have a plane surface with constant amplitude and phase and lateral dimensions large compared to the wavelength. This is difficult to do in space where the wavelengths are of order of kilometers. In large uniform laboratory plasmas it can be approximately

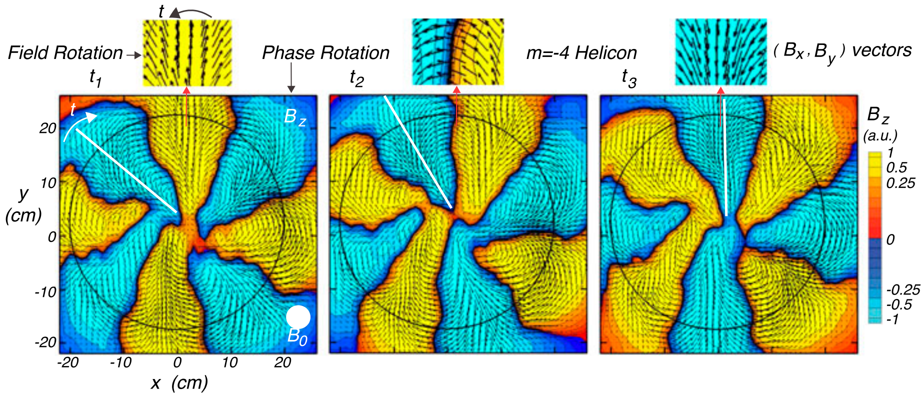

Figure 2. A whistler mode with azimuthal field rotation in a uniform plasma, also called a helicon mode [116].

Notes: The wave is excited by a phased antenna array located on the black circle. The propagating field is displayed at an instant of time by contours of the axial field component and vector fields for the transverse wave field. The phase of the wave (a line of $B_z = \text{const}$ such as a crest indicated by the white line) rotates clockwise with increasing time, indicating here a helicon with mode number $m = -4$. The field vectors rotate counter clockwise as seen in the enlarged pictures on the top. This identifies the wave as a whistler mode whose field polarization is always right-hand circular with respect to the axial field $\mathbf{B}_0$.

achieved with an antenna array [54,128]. Oblique plane waves are emitted when the plane is inclined with respect to $\mathbf{B}_0$ or when a phase delay is introduced between the array elements. Cylindrical waves are best excited by a circular disc-shaped antenna array. In order to produce a paraxial helicon mode the phase is shifted in azimuthal direction and the surface normal is parallel to $\mathbf{B}_0$. Azimuthal eigenmodes are formed when an integer number m of wavelengths are formed for one azimuthal rotation. The sign of the phase shift determines the direction of phase rotation. Helicons with positive or negative m-numbers can propagate equally well, but their field topologies are not identical.

An example of helicon mode excitation is shown in Figure 1. The array antenna is shown schematically in Figure 1(a), superimposed on a picture of a 1 m diam discharge plasma. Although a single ring antenna does not produce a plane wave it excites a helicon mode. The azimuthal phase shift from loop to loop creates rotating antenna field which excites an axially propagating whistler mode with azimuthal field rotation, i.e. a helicon mode. Figure 1(b) shows contours of one field component, $B_z(x, y)$. It has three pairs of positive and negative peaks, which forms an $m = 3$ helicon mode. In time the mode structure rotates in $+\phi$ direction. Since the phase also propagates radially outward and inward the rotation produces spiral-shaped contours. When all three field components are combined the field lines of the wave is obtained. Field line tracing reveals m pairs of nested helices with opposite field line directions. The spirals rotate left-handed in space in the direction of wave propagation. The field lines are approximately tangential to phase surfaces constructed from isosurfaces $B_z(x, y, z) = \text{const}$. The phase surfaces are helical screw surfaces.

It is worth pointing out that the wave rotation is not related to the rotation of the field vectors, which is usually called the field polarization. For electron whistler modes the field polarization is right-hand circular in time with respect to the static magnetic field $\mathbf{B}_0$ [5]. The wave rotation of helicons can be both in $+\phi$ or $-\phi$ direction depending on the phasing of the antenna array. An example of opposite field and vector rotations is shown in Figure 2 which displays contours of $B_z(x,y)$ and vector fields of $(B_x, B_y)(x,y)$ at three consecutive instants of time ($\Delta\omega t \simeq 45°$). The quadrupole contours show that the mode number is four. The rotation of a phase contour, enhanced by a white line, rotates clockwise with increasing time t_1–t_3, confirming that it is an $m = -4$ helicon mode. Each vector rotates counterclockwise, as best seen from the enlarged small section of the vector field. The polarization is right-hand circular.

The energy flow of low-frequency whistlers is highly field aligned such that large wave amplitudes are mainly found in the flux tube subtended by the antenna, almost irrespective of the direction of the phase propagation. A radially phased full array excites conical waves. When the cone angle equals the Gendrin angle the group velocity is field aligned forming a collimated Gendrin beam. Focusing the phase does not enhance the amplitude [61].

Superposition of oppositely propagating waves produces standing waves. Opposite azimuthal rotation forms azimuthal standing waves, opposite radial propagation forms radial standing waves and paraxial propagation, and finally axial standing waves are also readily produced [54,129]. Unless the opposing waves have the same amplitude no standing wave nulls arise. Radial wall reflections cannot produce pure standing waves since radial wave propagation is orthogonal to the axial group velocity, hence strongly damped. The observed radial propagation of helicon modes should be described by Hankel functions but not Bessel functions which describe standing waves. Oblique reflection are different for group and phase velocities [5]. The wave propagation of whistler modes in narrow plasma columns is still poorly understood, leave alone the nonlinear excitation by antennas [118].

TG modes, defined as helicon modes near the oblique resonance cone angle, $\cos\theta_{res} = \omega/\omega_c$, have also been observed in unbounded plasmas [61]. They arise in high m-number helicons near the axis where the azimuthal wave number $k_\phi = m/r$ increases as $r \to 0$. With $k_\phi >> k_r$ the TG mode propagates predominantly azimuthally which is different from the radial TG mode propagation of low m-number helicons in bounded plasmas. The evidence for TG modes is shown in Figure 3 for an $m = 8$ helicon mode. A snapshot of $B_z(x,y)$ contours shows the 16 alternating poles of the circular antenna array of radius $r = 8$ cm. But the azimuthal phase structure disappears radially inward. Furthermore, the wave amplitude and wave energy (Figure 3(b)) also nearly vanish in the interior of the helicon. For $k_\phi >> k_z$ the resultant k-vector becomes highly oblique to $\mathbf{B}_0$ and the wave is absorbed by oblique cyclotron resonance. The wave absorption may lead to electron heating on axis, i.e. the angular momentum and wave

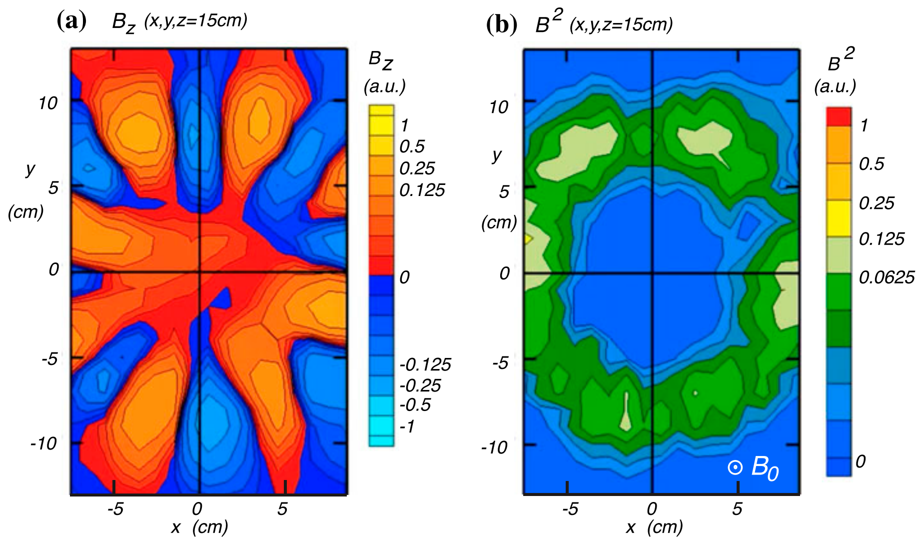

Figure 3. Trivelpiece-Gould modes in a uniform plasma, defined as highly oblique whistler modes in helicons with large mode numbers.

Notes: (a) Contours of the axial field component excited by an antenna array at $z = 0$ consisting of an 8 cm radius ring of 16 loops with alternating polarities. The antenna excites an axially propagating, non-rotating 5 MHz helicon wave. The azimuthal wavelength decreases toward the axis which creates a highly oblique propagation angle near cyclotron resonance. The mode is predominantly electrostatic and strongly damped. (b) Contours of the energy density showing no magnetic energy in the center of the $m = \pm 8$ helicon.

energy are transferred to the electrons. Wave absorption also prevents radial wave reflections and standing waves, i.e. helicon wave theory does not hold for high order helicons.

4.7. Important applications and future research

Helicon research is mainly driven by applications. Helicon plasma sources found many applications in plasma processing [130]. The next application was based on the observation of a potential double layer at the end of a plasma column with flaring magnetic fields [64,131]. It accelerated the ions which can be used for thrusters on spacecraft to extend their lifetime normally limited by drag. The high density achieved in helicon devices also provides a suitable background plasma for laser-plasma accelerators [132].

The significance of the angular momentum has so far only been pointed out but not yet demonstrated or applied. A new effect is a transverse Doppler shift. A rotating ‘observer’ sees a different frequency than the wave frequency. If the ‘observer’ is a rotating electron it may encounter cyclotron resonance when the wave and particle rotate in opposite directions. This is analogous to the familiar Doppler shift for motion along the direction of wave propagation. The transverse wave–particle resonance may energize electrons or excite helicon instabilities if the electrons have excess perpendicular energy. Another interesting effect is that high m-number helicons are field free in the center. The azimuthal wavenumber increases toward the axis and the wave is absorbed by oblique cyclotron resonance. Note that for nearly parallel energy flow the wave can only be observed in the flux tube of the antenna since it is attenuated radially. This implies that a helicon eigenmode or whistler Bessel beam, both obeying $B_z \propto J_m(k_r r)\,\mathrm{expi}\,(m\phi + k_z z - \omega t)$, cannot be excited with a localized antenna. A plane wave can neither be excited by an antenna of finite size.

Wave–wave interactions may be as important as wave–particle interactions. Parametric instabilities conserve energy and momentum. Little is known about the parametric decay of rotating fields, leave alone the combined axial and rotating **k**-vectors of helicons and other decay modes.

The refraction of helicon waves presents an interesting scientific problem, i.e. the conservation of momentum and energy. In order to change the direction of propagation a force and torque has to be applied which must come from the medium, i.e. the electrons. Ray tracing is a concept of Snell's law and group-phase velocity relations, but does not consider conservation laws.

Whistler modes are just one branch of electromagnetic waves in plasmas. Vortex waves may also be produced with the lower frequency Alfvén waves or higher frequency ordinary and extraordinary waves [133]. These topics have not yet been explored.

Antenna arrays for receiving helicon waves have neither been discussed. If each loop signal of a circular array is recorded the field rotation can be obtained by a phase shifts, i.e. which yields a scan over m-numbers. Plane waves propagating through regions of fluid vorticity become helicons hence the field analysis provides a remote sensing capability for plasma properties.

5. Electromagnetic waves with angular momentum

It has been long known that electromagnetic waves in free space can carry angular momentum [134]. In 1936 it was experimentally verified that a circularly polarized light beam can exert a torque on matter [135]. When the wave propagates both axially and azimuthally, it possesses an axial field momentum and an azimuthal orbital angular field momentum, $\hbar k_z + \hbar m/r$. The latter can also exert a torque on matter [136]. In analogy to a rotating and spinning particle the field polarization produces a 'spin' angular momentum and the field rotation an 'orbital' angular momentum [137]. Spin angular momentum can be converted to angular orbital momentum [138,139]. Vortices with $m > 0$ have a phase singularity and an intensity minimum on axis. The angular momentum properties also apply to electromagnetic waves in media such as whistler modes in plasmas.

'Paraxial' waves have field dependencies $A(r)\exp\,[i(m\phi + k_z z - \omega t)]$ where $A(r)$ is the radial amplitude dependence and $m = k_\phi r$ the azimuthal eigenmode number and k_z is the axial wave number. The phase surface is helical and the field is called a 'vortex field' [140] or 'helicoid' [137]. These field properties exist at any frequency and have been studied over a wide spectrum from radio waves[141] to X-rays [142].

The interest in waves with angular momentum is driven by many exciting applications [143]. With a powerful laser beam objects at the atomic and molecular scale can be manipulated, calling it an 'optical tweezer' [144]. Lasers have also been used to manipulate heavy charged grains in dusty plasmas but the force was due to thermophoretic effects instead of the field pressure or torque [145]. Strong helicon waves may trap light electrons but this has not yet been observed.

In order to produce waves with angular momentum the wave passes through a medium whose refractive index varies in azimuthal direction. This sentence became partially scrambled concept can be used throughout the spectrum of waves leading to diverse applications. For example microwaves at a single frequency can be transmitted with different m numbers eigenmodes which can be modulated independently, thereby increasing the information rate m-fold [141,146]. Light reflected from meta-material surfaces can produce vortex fields useful for integrated optical devices in nano photonics [147].

In astronomy radio and optical waves can be converted with phase plates into vortex waves. High m-number beams have the property of forming hollow light beams. This is useful for observing weak sources in the vicinity of bright sources, which are suppressed by the vortex null [148]. In plasma helicons the central region becomes nearly field-free due to oblique cyclotron resonance. This arises when the azimuthal scale length $k_\phi = m/r$ becomes shorter than the electron inertial length, c/ω_p.

Bessel beams are paraxial waves without radial propagation [149]. They are exact solutions of the wave equation in cylindrical coordinates, predicting a field dependence $J_m(k_r r)\exp[i(m\phi + kz - \omega t)]$, where J_m is the Bessel function of order m and k_r is a radial wavenumber. Such beams have no spread and are self-healing around scattering objects, i.e. diffraction-free [150]. The lowest order Bessel beam has an intensity maximum on axis with half width less than a wavelength. Unfortunately, like plane waves, ideal Bessel beams cannot be generated since the source would have to be infinitely wide in radial direction and provide an infinite wave energy. Finite size beams exhibit radial propagation and beam spread.

Note that helicon eigenmodes have the same field dependence as Bessel beams but, as in waveguides, the radial field dependence is terminated at nodes due to boundary reflections. Without reflections the wave propagation is not paraxial. But a magnetized plasma is an anisotropic medium which exhibits different directions for phase and group velocities. The radial field dependence cannot be described by a Bessel function if there is no radial energy flow. However, whistler beams can be collimated without reflecting boundaries. Parallel energy flow can arise for two angles of wave propagation, $\theta = 0$ and $\theta_{\text{Gendrin}} = \arccos(2\omega/\omega_c)$. However, a circular antenna array has radial boundaries where the beam spreads.

6. Conclusions

Electromagnetic waves with vortex phase fronts exist in various fields of physics. In plasmas they form helicon waves which have been studied in small plasma columns and in large uniform plasmas. The latter result implies that helicon modes also exist in space plasmas where whistler waves are ubiquitous. The property of orbital angular momentum is common to all spiraling waves. When such waves interact with matter many interesting applications can arise. Some of them are well understood, such as radiation pressure and torque, others still need to be explained, such as the strong absorption of helicon waves. The conservation of momentum and energy during refraction in a nonuniform density or magnetic field is a virtually untouched topic. Surprisingly, rather little research on vortex fields has been done in space plasma physics since it would require spatially resolved field measurements and large antenna arrays. Applications differ in different fields such as space plasmas or photonics, but the basic physics of the waves is very similar. Thus, the topic of vortex fields is interdisciplinary. It is also active, growing and useful.

Acknowledgements

The author gratefully acknowledges valuable research contributions, in particular the digital data acquisition and processing, by Dr. J. Manuel Urrutia. The author also thanks Prof Shinohara for inviting me to write this review article.

Disclosure statement

No potential conflict of interest was reported by the author.

Funding

This work was supported by the National Science Foundation, currently under [grant number NSF-PHY 1414411].

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