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"I NO LONGER SKIP THE NUMBERS." AN ANALYSIS OF STUDENTS' USE OF STATISTICAL LITERACY PRACTICES

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Abstract

"I No Longer Skip the Numbers." An Analysis of Students' Use of Statistical Literacy Practices

Julianne Foxworthy Gonzalez

This dissertation documented how undergraduate students made sense of data in news media. The participants were 30 undergraduate students enrolled in a course called "Numbers and Social Justice." The study used argument analysis (Toulmin, 2003) and ethnographic methodology to examine students' written work in a naturalistic setting. Course lessons and assignments used open-ended tasks focused on socio-political issues using real data from newspapers, media outlets, or social media (Engledowl & Weiland, 2021; Weiland, 2019). Recordings of class meetings (on zoom) and students' written work were used to explore the following questions: 1) How did students respond in writing to a news article with statistical claims, data, and visual representations of data? 2) What arguments did students write? When students made arguments, what types of arguments did they make? 3) What statistical literacy practices did students use? 3a) Did these change over time and with the opportunity to revise for four focal students? Analysis of students' essays provided evidence that asset-based instruction centered on statistical practices can support students from non-STEM majors in learning to critically analyze data-based claims. Both the course design and analysis used an asset perspective of students, a perspective that is important for documenting the strengths of learners from non-dominant communities. Analysis documented students from non-STEM majors using data in arguments,

participating in statistical literacy practices, and generating and refining their written arguments about data in a news article. The study shows that students who previously felt marginalized from mathematics learned to use statistical practices through instruction that used social justice topics, focused on goals, and provided opportunities to write and revise. Analysis uncovered the need for students to notice goals, highlighted how goals are central for revising arguments, and organized lists of statistics skills into clusters based on inferred goals for students' statistical literacy practices. The study has implications for supporting students in learning to construct stronger arguments about data. In particular, the study shows that writing and revising can provide opportunities for students to slow down, help teachers to notice students' strengths and offer feedback focused on goals, and may support students in improving their arguments.

Acknowledgments & Dedication

When I was an undergrad, I read an essay that I've never forgotten. The author wrote, "The [person] who makes a vow makes an appointment with [them]self at some distant time and place. The danger of it is that [them]self should not keep the appointment," (Chesterton, 2015, p.16). There are many people who have helped me keep this rash vow of completing a PhD, and to them I owe much gratitude.

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Chapter 1: Statement of the Problem

Contemporary life is marked by an abundance of statistical information. The COVID-19 pandemic and the increasing occurrence of climate catastrophes reminds us that people must critically examine statistical information to assess risk and make decisions that impact them, their communities, and the planet. News media, advertising, and social media companies make use of data in increasingly sophisticated – and sometimes unscrupulous – ways to persuade their audiences (Engledowl & Weiland, 2021; Shaughnessy, 2007). Moreover, government policies are debated using data, with both sides presenting evidence to support their position. It is therefore a democratic imperative for people to understand and interrogate these data, how they are represented, and the conclusions that are inferred from them. Indeed, Cobb (1999) cautioned that a lack of conceptual understanding in statistics may result in "de facto disenfranchisement" (p. 38). One needs at least some familiarity with the rules of a system to participate in it, and numerical evidence can be intimidating and mystifying to the uninitiated.

It is perhaps unsurprising that many students accept statistical claims presented in text without critical analysis (Watson & Moritz, 1997). The absence of norms for critiquing statistical claims and, perhaps, the knowledge to do so has unpropitious implications for society. This has critical significance for marginalized communities and their allies for whom statistical literacy is one tool to combat and correct systemic injustice. All this creates a pressing need for students to develop

statistical literacy, not just use statistical formulas and vocabulary, but understand the concepts and larger social context that give statistics meaning.

Even before the start of the coronavirus pandemic, before people across the globe were inundated with daily updates to data and graphs, the interpretation of which seemed to have life-or-death consequences (Watson & Callingham, 2020), statistics educators and researchers were predicting a future in which every person would need statistical literacy (Gal, 2003; Wallman, 1993). Historically, statistics curricula have focused on descriptive statistics and the precise execution of procedures, and that has contributed to a lack of conceptual understanding and inferential reasoning amongst students (Cobb, 2007; Harradine, Batanero, & Rossman, 2011). Despite increased attention to statistical literacy in the research literature and in various national curricula (e.g., The Common Core State Standards and the California Mathematics Framework in the US, Ethiopia's move to incorporate statistics in K–12 classrooms, and Australia's 2013 national curriculum) there continues to be great variation in how statistical literacy is taught in schools (ProCivicStat Partners, 2018; Zieffler, Garfield, & Fry, 2018).

Research literature provides descriptions of statistical literacy with lists of knowledge, skills, and dispositions that students need to make sense of and critique data in our world (e.g., Gal, 2002; Ridgway et al., 2018). While it is useful to identify these concepts and skills, there is a danger that these lists may be used in classrooms in disconnected ways that leave students with the impression that statistics is merely a set of procedures they don't really understand (Bakker & Derry, 2011). There is also

a danger that researchers and educators reduce statistical literacy to procedural fluency or basic skills (Gal, 2018). There is a need to describe what statistical literacy looks like in action. In this dissertation, I propose mathematical practices[1] as a sociocultural framework for conceptualizing statistical literacy. This framework considers not only students' use of strategies and concepts, but their goals in doing so. Conceptualizing statistical literacy as practices can help organize the lists of knowledge, skills, and dispositions in the research literature into clusters of practices based on the goals of students' activity. By focusing on students' goals, we can highlight that statistical literacy is more than a collection of skills and concepts. A focus on goals also emphasizes that it is the using of these skills and concepts in service of particular goals that is most important for the literate use of these skills and concepts.

Dissertation Overview

This dissertation describes how students from non-STEM majors enrolled in an undergraduate course on statistical reasoning used statistical literacy practices to make sense of and critique news articles containing data, data visualizations, and data-based claims. The study connected two areas of research: mathematical practices and statistical literacy. First, the research on mathematical practices illuminates how goals, mathematical concepts, and strategies are intertwined when using mathematical practices. This view does not seek to separate out or reify these various aspects, rather

[1] The phrase mathematical practices as used here does not refer to the Common Core State Standards in the United States. Mathematical practices documented in research are broader, more specific, and more connected to concepts than those standards.

it emphasizes how they are interconnected and situated in particular contexts. Second, the research on statistical literacy offers a comprehensive summary of the plethora of facts, skills, and dispositions that a statistically literate person commands.

I conducted the study in this particular course because the students enrolled did not have strong backgrounds in mathematics or statistics. This required that instruction focus on meaning–making (as opposed to memorization of facts or procedures). I had been the Teaching Assistant for the course for six years, and the instructor and I had collaborated in that time to design lessons that promoted student participation, discussion, and a focus on the goals of using statistical literacy practices.

The study focused on the students' three written essays that they completed throughout the course. For the first essay, they selected a news article and analyzed the data presented. They received written feedback from me and either revised that first essay or selected a new article and wrote a second essay. For the third and final essay the students were all presented with the same article to respond to. The study was guided by the following research questions:

1. How did students respond in writing to a news article with statistical claims, data, and visual representations of data?
2. What arguments did students write? When students made arguments, what types of arguments did they make?
3. What statistical literacy practices did students use?

a. Did these change over time and with the opportunity to revise for four focal students?

In the chapters that follow, I elaborate a conceptual framework (Chapter 2), describe the method for analysis (Chapter 3), provide an ethnographic description of the course (Chapter 4), present findings (Chapters 5 and 6), and discuss these findings and their implications. In Chapter 2, I describe mathematical practices and statistical literacy, arguing for viewing statistical literacy through the lens of mathematical practices. In Chapter 3, I describe the setting and participants and methods of data collection and analysis. In Chapter 4, I describe the course in detail, elaborating on lessons and assignments. In Chapter 5, I share findings in response to all but the final research question (3a). In that chapter I document how students responded to the news articles, what concepts, strategies, and goals they used, and the types of arguments they crafted. I also describe how they used two statistical literacy practices: Reading the Data and Critiquing the Data Quality. In Chapter 6, I show how feedback and the opportunity for revision impacted students' arguments and their use of statistical literacy practices. Finally, in Chapter 7, I provide a summary of these findings, and consider contributions and implications for research and teaching.

Chapter 2: Conceptual Framing and Summary of the Literature

This study is grounded in sociocultural views of learning and engages bodies of literature on two topics: mathematical practices and statistical literacy. In this chapter, I summarize the theories and findings from these bodies of research to frame the study and provide a rationale for the methodological choices discussed in Chapter 3 and the course design described in Chapter 4. I argue that viewing statistical literacy through the lens of mathematical practices provides an important perspective on the teaching and learning of statistical literacy that centers students' strengths.

The research on mathematical practices and statistical literacy are related in important ways. Research on mathematical practices provides a theoretical framework to explore and describe statistical literacy as socially situated, goal oriented, and mutually constructed (Moschkovich, 2013). From this view, everyday knowledge and language are seen as tools for developing conceptual understanding and using mathematical practices. Research on statistical literacy was born out of observations that data had begun to appear in people's everyday lives. There was a need for people to "understand and critically evaluate statistical results that permeate our daily lives," (Wallman, 1993, p. 1). Both fields point to the need to examine people's use of mathematical concepts, strategies, and practices to make sense of and make decisions about mathematics in our everyday lives. Furthermore, framing statistical literacy as mathematical practices has important implications for equity because this view highlights the varied ways people can and do participate in

practices rather than describing hierarchies or lists of competencies that can result in atomistic or deficit-oriented instruction.

Theoretical Framing for the Study

This study is grounded in a sociocultural view of learning (Vygotsky, 1986) which assumes that mathematical concepts are learned within a social context (Forman, 1996). Mathematical learning is situated in the sociohistorical and cultural settings in which it occurs. Students and teachers each bring perspectives, prior knowledge, and learned norms, and these evolve in each particular classroom, between each particular teacher and group of students. Drawing on this sociocultural frame, I use a definition of learning as participation in a community of practice (Forman, 1996; Lave & Wenger,1991). Teachers communicate the norms, values, practices, and content of the discipline to the students who engage in meaningful participation in these practices.

Mathematical Practices

Mathematical practices are "the taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas," (Cobb, 1999, p. 9). One might think of mathematical practices as the socio-cultural practices people learn to use that involve mathematical thinking and reasoning or the ways people interact with quantities, data, or mathematical representations. They are how "we"— people who have learned to use mathematics — use mathematical concepts to make sense of and impact our worlds. Consistent with a sociocultural perspective, these practices originate in social interaction. Humans use tools (e.g., mathematical

concepts, language, procedures) in a variety of practices to solve problems in their environment (Vygotsky, 1986). Practices "are first constructed interpersonally and then appropriated to become part of the repertoire of practices that an individual will use," (Moschkovich, 2013, p. 264-265).

There is no monolithic set of mathematical practices; indeed, mathematical practices are specific to both the sociohistorical context in which they occur and to particular mathematical ideas (Cobb, 1999; Moschkovich, 2013). Following a Vygotskian view, I assume goals are central to mathematical practices. Based on these data, I analyze practices as including goals, mathematical concepts, and strategies. Mathematical practices include a goal (Why am I doing this?), strategies (How can I do it?), and canonical ideas (What mathematical concepts are important to use?).

Simply reminding students to use a mathematical practice or describing it to them is not sufficient social interaction for learning how to participate in a mathematical practice. Mathematical practices require students to be active in participating, perhaps at times as legitimate peripheral participants (Lave & Wenger,1991), in joint activity with shared goals and focus of attention (Moschkovich, 2013). Through these interactions, participants develop shared meanings for language, artifacts, tools, and concepts, and move toward increasingly competent participation in mathematical practices.

Viewing statistical literacy as a set of practices has important implications for equity in the classroom. Moschkovich (2013) cautions against dichotomizing

everyday practices and classroom mathematical practices. Because classrooms are places where multiple practices meet, everyday practices can be viewed "as resources to build on in order to engage students in the formal mathematical practices taught in classrooms," (Moschkovich, 2013, p. 269). Indeed, viewing mathematics this way rejects the false dichotomy of everyday and academic language and practices and moves beyond the expert-novice binary. Students can use all their resources (e.g., linguistic, experiential, material) in service of solving problems, making meaning, and questioning data.

Consistent with the sociocultural view, I used an ethnographic stance in the design of this dissertation (Moschkovich & Brenner, 2000); the research was conducted in a naturalistic setting and meanings were assumed to be socially constructed in context and negotiated between multiple points of view. I adopted a strengths-based approach (Huitzilopochtli, et al, 2021) in which I sought to notice and document the strengths students brought to the classroom and recognize statistical literacy practices in what students said and wrote. This contrasts with a deficit perspective that, "suggests that one's own way of speaking or thinking is superior," (Wagner et al., 2015, p. 247). This dissertation is not concerned with prescribed trajectories of learning or hierarchies of competence. Instead, this study documents the ways students appropriated practices introduced in class and used these to make sense of and critique real-world data, data visualizations, and data-based claims in news articles.

Conceptualizing Statistical Literacy as Practices

In this section I describe how conceptualizing statistical literacy as mathematical practices is beneficial for researchers and educators. Instead of attempting to describe or teach a comprehensive list of all the facts, skills, beliefs that work together in intricate ways in the mind of a hypothetical, statistically literate person, focusing on mathematical practices centers the sociohistorical context (of both the student and the data) and the goals, concepts, and strategies in use. Indeed Gal (2002) comes close to describing statistical literacy as practices when he writes about "goal-oriented behavior" and the "broad clusters" of ideas that comprise statistical literacy.

> Along these lines, statistical literacy may be understood by some to denote a minimal (perhaps formal) knowledge of basic statistical concepts and procedures. Yet increasingly, the term literacy, when used as a part of the description of people's capacity for goal-oriented behavior in a specific domain, suggests a broad cluster not only of factual knowledge and certain formal and informal skills, but also of desired beliefs, habits of mind, or attitudes, as well as general awareness and a critical perspective. (p. 2)

Statistical literacy practices can be thought of as the use of mathematical and statistics concepts and strategies in service of a goal. This focus on mathematical practices has important implications for equity and high-quality statistical literacy instruction.

One benefit of this approach is that mathematical and statistics concepts are best learned with meaning by using them for authentic purposes. Like mathematics education in general, statistics education has traditionally focused on facts, skills, and procedures in isolation. Bakker and Derry (2011) argued that textbooks generally present topics separately as if to create a "toolkit" of statistical tools (e.g., means, box

plots, t-tests). This resulted in inert knowledge. Inert knowledge refers to the fact that students are often taught in such a way that they can perform a procedure (e.g., calculating the mean) but they do not use that knowledge to solve problems. Furthermore, this style of instruction is especially prevalent in classrooms with students from minoritized communities (NASEM, 2018). Such instruction reifies what counts as statistics and who is capable of doing statistics (Martin, 2009). Success in such courses becomes about obedience to procedures instead of using concepts to make sense of or affect one's world. By focusing on mathematical practices, concepts and skills are framed as a part of a repertoire of practices. Learning in a community of practice is goal-oriented by nature — learners are participating in the service of some purpose that the community shares. This is consistent with a strengths-based view of students; each is positioned as a legitimate peripheral participant with knowledge, skills, and beliefs that can be leveraged for participation.

A Note about Context

By context, statistics education researchers are generally referring to the particular problem situation being studied, what Pfannkuch (2011) calls the data-context. For example, imagine a teacher gives their students a data table displaying heights for a sample of 25 individuals. Without knowledge of the data-context, the numbers have little significance and not much can be inferred from them. Whether the data were drawn from a population of middle school students or professional basketball players makes a difference to the conclusions students might draw. Or

consider a more complex example of data representing catalyzing reasons for homelessness. Perhaps knowing that people without shelter are hard to find and survey might make one cautious about the generalizability of conclusions considering possible bias in a sample. Data from people living in homeless shelters might differ from those of people sleeping in parks in systematic and meaningful ways. This knowledge of the situation from which the data come is what statistics education researchers mean by context.

Conceptions of context as the problem situation may be limited in scope when compared to that suggested by other mathematics education researchers (e.g., Forman, 1996; Lampert & Cobb, 2003; Moschkovich, 2002). Moschkovich and Brenner (2000) draw on Lave to describe the learning context as situated in a broader social and cultural milieu. They describe it as the relationship between the setting and the participants' various interpretations of the setting. "Learning, thinking, and knowing are relations among people engaged *in, with, and arising from the socially and culturally structured world,*" (Lave, 1991, p. 67, author's emphasis). Pfannkuch (2011) references the learning-experience-context, which includes the historical and social contexts of students, yet primarily refers to their prior learning experiences and interactions with the teacher. This study builds on other studies in statistics education (Weiland, 2017; Zapata-Cardona, 2018) by embracing a broad conception of context in which students are assumed to be critical actors whose development and use of statistical literacy is informed by, and informs, the larger world.

Statistical Literacy

In this section, I review definitions of statistical literacy and its core components. I define statistical literacy as how people make sense of and critically analyze data, data visualizations, and data-based claims encountered in everyday life (Gal, 2002; ProCivicStat Partners, 2018; Watson & Callingham, 2003; Weiland, 2017). I then describe research that has sought to describe statistical literacy. I conclude by reviewing pedagogical recommendations for promoting statistical literacy.

With some exceptions (e.g., Ben-Zvi & Garfield, 2004; Weiland, 2017) statistical literacy has been conceived of from a receptive, rather than productive, perspective. That is to say, statistical literacy concerns reading, interpreting, and critically questioning data, data visualizations, and data-based claims authored by others. From this perspective, statistical literacy is not so much concerned with the production of statistical questions, the design of studies, nor the collection, analysis, and presentation of data (Wild & Pfannkuch, 1999). That is not to suggest that these are not important aspects of statistical thinking and reasoning that may inform one's statistical literacy, only that statistical literacy largely focuses on the receptive, not productive, aspects of using statistics. Indeed, the reading of others' statistical investigations differs from empirical investigation of data in important ways (Gal, 2002). For example, "messages aimed at citizens may be shaped by political, commercial, or other agendas," (Gal, 2002, p. 15), and unpacking these messages may require an understanding of the relevant statistical concepts, the underlying

social context, and the bias of the author(s). Statistical literacy does not require expert-level knowledge of statistics; rather it requires enough understanding of statistical ideas and practices to be able to use them in reasoning critically about claims, predictions, and social policies. Furthermore, many more people receive data-based messages than create them. Thus, I primarily adopt Gal's (2002) definition of statistical literacy as the ability to interpret and critically evaluate data, data visualizations, and data-based claims and the ability to communicate one's reactions, reasoning, or concerns about these. Though Gal (2002) uses the term "abilities," he emphasizes that "statistical literacy should be regarded as a set of capacities that can exist to different degrees within the same individual, depending on the contexts where it is invoked or applied," (Gal, 2002, p. 15). This is an essential aspect of my conceptualization of statistical literacy and later in this chapter I elaborate why I frame statistical literacy not as practices rather than abilities.

The research literature has conceptualized statistical literacy as what people need to know and believe given observations that statistics permeate modern life and people must be able to interpret and critically evaluate these. Most discussions of statistical literacy, including this dissertation, begin with a statement about the ubiquity of data in everyday life. Official statistics are produced by public institutions, interest groups, and university researchers, but these are disseminated to the public via a variety of channels including news articles, podcasts, social media posts, and television, each of which may present only aspects of the original studies that promote their point of view (ProCivicStat Partners, 2018). The interpretative

demands of the variety of data-based messages has not been fully cataloged, but some research points to the prevalence of quantities expressed as percentages in popular adult texts (Joram, Resnick & Gabriele, 1995) and the need for proportional reasoning, the diverse and sometimes interactive visual representations that have become commonplace (ProCivicStat Partners, 2018), the sudden appearance of uncertainty in predictions that accompanied COVID-19 reporting (Gal & Geiger, 2022; Watson & Callingham, 2020), and the prevalence of malicious actors who seek to spread misinformation (Weiland, 2019).

Gal (2002) described statistical literacy as

> an action-oriented set of interrelated knowledge bases and skills, one which people will actually use in everyday contexts, [which] must consider people's inclination to apply a critical stance and the motivations, beliefs, and attitudes that affect or support statistically literate behavior. (p. 23)

This model of statistical literacy (see Figure 1) is therefore composed of what Gal described as "knowledge elements" and "dispositional elements" which are mutually supportive, dynamic, and context dependent.

Literacy refers to one's ability to decipher the complex prose that often accompanies data-based claims. Statistical knowledge is characterized by five elements: knowing why data are needed and produced, a familiarity with vocabulary and concepts related to descriptive statistics, a familiarity with vocabulary and concepts related to visual displays of data, an understanding of probability, and knowing how statistical inferences are made. Gal highlights proportional reasoning as a particularly important aspect of the mathematical knowledge base. Finally, context

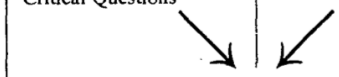
knowledge refers to the details of data collection and how various methods influence validity. These knowledge bases combined with a belief that statistics can provide valuable insights about the world and a critical stance comprise Gal's (2002) model of statistical literacy.

Figure 1 Gal's (2002) Model of Statistical Literacy

Table 1

A model of statistical literacy.

Knowledge elements	**Dispositional elements**
Literacy skills	Beliefs and Attitudes
Statistical knowledge	Critical stance
Mathematical knowledge	
Context knowledge	
Critical Questions	



Statistical Literacy

Building on this work, Ridgway, Nicholson, Gal, and Ridgway (2018) outlined eleven facets of statistical literacy for civic engagement: 1) An analysis of the social impact of evidence-based policies needs to accompany statistical analyses. 2) Evidence must be critically evaluated to identify possible biases. 3) People need to have a positive disposition towards "fact-checking" rather than adopting an attitude of "statistics is all lies" or, conversely, "the experts know what's true". 4) The role of probability and conditional probability in assessing risk must be understood. 5) An understanding of the strengths and limits of various models and 6) methodologies is also essential. 7) People need some understanding of how official statistics (e.g., the unemployment rate) are calculated. 8) Basic contextual facts, such as reasonable estimates of population sizes, are necessary for making decisions based on data. 9)

Basic information and computing technology skills are also needed to manipulate the large amounts of unstructured data that are increasingly available. 10) Basic quantitative skills (e.g., number sense, fractions/percents, ratios) are essential for making sense of published data. And 11) people need to have the literacy skills to unpack statistical messages communicated in text. The breadth of these facets suggests a need for integrated instruction based on real-world statistics in settings that are known to students.

Weiland (2017) built on the work of the scholars above to describe *critical* statistical literacy as "reading" and "writing the world" (Freire, 1970; Gutstein, 2006) with statistics. Unlike most models of statistical literacy, he considers both the production and interpretation of data. He proposes eight elements: four related to reading the world with statistics and four related to writing the world. This study focuses on the reading aspects of his framework because the students in this course were asked to respond to data, visual representations of data, and statistical claims presented to them, rather than conducting their own statistical analyses.

The first of these critical statistical literacy elements is "making sense of language and statistical symbol systems and critiquing statistical information and data-based arguments encountered in diverse contexts to gain an awareness of sociopolitical issues in society," (Weiland, 2017, p. 41). This first element encompasses much of what other scholars (Budgett & Pfannkuch, 2010; Gal, 2002; Ridgway, Nicholson, Gal, & Ridgway, 2018; Watson 2003) have described as statistical literacy. This includes general literacy skills, knowledge of statistical terms

and concepts, quantitative reasoning (especially proportional reasoning), context knowledge, and critical thinking. This component also includes beliefs and attitudes such as a questioning disposition and the belief that statistics are useful, and one can understand them.

The second component is "identifying and interrogating social structures which shape and are reinforced by the data-based arguments being considered," (Weiland, 2017, p. 41). This is related to an expanded view of context which highlights how knowledge is situated and produced (and re-produced) by persons engaged in activity in a community of practice that is itself situated in larger societal structures. The third component further complicates the role of context in critical statistical literacy; it is considering "one's social location, subjectivity, political context, and having a sociohistorical and political knowledge of self and understanding how it influences one's interpretation of information," (Weiland, 2017, p. 41). When framed as a practice, this means that students might reference their own positionality when writing and talking about statistics.

Finally, critically statistically literate actors "evaluate the source, collection, and reporting of statistical information and how they are influenced and shaped by the author's social position and sociopolitical and historical lens," (Weiland, 2017, p. 41). Gal (2002) also touches on this idea when discussing literacy in general, as do Joram, Resnick, & Gabriele (1995) in their investigation of the kinds of numbers that appear in magazines and other everyday texts. Readers of statistics in the world must make sense of statistical information presented alongside modifying words (e.g., a

"shocking" 50% increase). Weiland (2017) extends these considerations to include the need to consider the author's positionality when making sense of statistical information.

Overall, this body of research takes as its starting point the landscape of data, visual representations of data, and data-based claims in the media and describes what people need to know, do, think, and believe in order to make sense of and critically analyze these messages. This study builds on these definitions and descriptions, focusing on goals and framing student activity as practices.

Pedagogical recommendations

In this section I describe the recommendations from the literature for promoting statistical literacy in the classroom. Overall, recommendations are aligned with a mathematical practices framing. For example, instructors are encouraged to use real (and not merely realistic) data (Ben-Zvi & Garfield, 2004; California Mathematics Framework, 2023), especially that data which is based on social examples (Engledowl & Weiland, 2021; ProCivicStat Partners, 2018; Watson & Callingham, 2020; Weiland, 2019;). Active learning is recommended (Ben-Zvi & Garfield, 2004), and instructors should provide alternative assessment methods in order to better understand and document student learning. The use of discussions around investigations into real-world social phenomena helps instructors avoid atomistic teaching (Bakker & Derry, 2011). Because instruction is based on real social data, teachers should be prepared to facilitate educational discussions around controversial topics. "Teachers need to be mindful of the experiences of their students

because some contexts have painful connotations for some students. Context is not value-free and teachers must recognise this fact and be aware of the range of opinions that may emerge," (Watson & Callingham, 2020, p. 20). Essentially, these recommendations urge instructors to make the teaching of statistical literacy meaningful by engaging students in making sense of the world through data. In Chapter 4, I describe how we used these recommendations in the design of the course.

Chapter 3: Methodology

The purpose of this qualitative study was to explore students' use of statistical literacy practices in an undergraduate course that focused on statistical literacy and social justice. As described in Chapter 2, statistical literacy has been theorized as what everyday people need to know, think, and believe in order to critically examine the plethora of data and data-based arguments that permeate nearly every aspect of modern life (Budgett & Pfannkuch, 2010; Gal, 2002; Ridgway, Nicholson, Gal, & Ridgway, 2018; Watson 2003; Weiland, 2017). This study built on that work to document how undergraduate students from non-STEM majors wrote about data, visual representations of data, and statistical claims in news articles. The research questions that guided this study are:

1. How did students respond in writing to a news article with statistical claims, data, and visual representations of data?
2. What arguments did students write? When students made arguments, what types of arguments did they make?
3. What statistical literacy practices did students use?
    a. Did these change over time and with the opportunity to revise for 4 focal students?

Study Design

I adopted a naturalistic methodology and an ethnographic stance (Moschkovich, 2019; Moschkovich & Brenner, 2000) in the design of the study. A naturalistic methodology is not a collection of investigative or analytic techniques,

rather it is defined by a theoretical stance and a set of principles. Theoretically, the study is informed by a sociocultural view of learning (Vygotsky, 1986) which assumes that mathematics is learned within a social context (Forman, 1996). The first principle in a naturalistic methodology is to consider multiple viewpoints. Rather than thinking of learners as novices in pursuit of expert knowledge or focusing on misconceptions, this research design allows for more than one way to practice statistical literacy. Furthermore, this principle demands that the perspective of the researcher is made explicit. My experiences as the Teaching Assistant for the course informed my analysis of students' writing. The second principle in a naturalistic methodology is to study learning in context. Context refers to more than just the setting of the study; it is both the setting and how people make sense of that setting in varying ways. These principles go hand in hand; if meaning is negotiated between multiple points of view, it is valuable to conduct research in a setting in which that negotiation authentically occurs—such as a classroom.

An ethnographic stance, being grounded in sociocultural theory, is consistent with a strengths-based approach (Huitzilopochtli, et al, 2021). In this study, I do not assume that there is a one correct way to demonstrate statistical literacy. Instead, I sought to notice and document the mathematical concepts and strategies that students used and recognize statistical literacy practices in what students said and wrote, even when these occurred in ways that I did not expect. This contrasts with a deficit perspective that, "suggests that one's own way of speaking or thinking is superior," (Wagner et al., 2015, p. 247).

Setting and participants

The study was set in an undergraduate elective course called "Numbers and Social Justice" which was housed in the Community Studies department of a California public university. Community Studies was an interdisciplinary major in which students focused on either health or economic justice. The major included a service-learning component that culminated in a thesis based on students' engagement with community-based, non-governmental organizations. Numbers and Social Justice was designed to meet the need for students to make sense of, communicate, and critique data related to their area of focus.

At the time of the study, I was working as the Teaching Assistant for the course, a position I had held for seven previous iterations of the course over six years. A full description of the course can be found in Chapter 4. Briefly, the course covered topics such as estimating, counts and rates, calculating a rate of change, reading tables and graphs, survey design, sampling techniques, operationalization of variables, measures of central tendency, and heuristics.

The thirty participants in the study were students enrolled in the course in the Spring of 2021. Over 90% identified as women or non-binary people, over two-thirds were not white, and half were multilingual. The majority were majoring in the Humanities and Arts. Most had not taken a math or statistics class since high school, and of those few who had attempted college-level statistics, all but one had dropped, withdrawn, or failed the course. There were more first-year students in the course than had been the case in the past, though students from all years were represented.

These and other demographics are described in Table 1. The categories for racial/ethnic identity are those defined by the university and are similar to those found throughout social science research literature. I report them here not to reify race but to place this work in conversation with others that document how Black and Latinx students have historically been marginalized and neglected in mathematics classrooms (e.g., Martin, 2013; Shah, 2017). Nearly half of the students in the course belong to these groups,

Table 1 Characteristics of the Students According to a Demographic Questionnaire (n=27)

Racial/Ethnic Identity	
Asian	18.5%
Black	7.4%
Latinx	37%
White	37%
Gender Identity	
Woman	81.5%
Man	7.45
Non-binary	11%
Multilingual Learner	
Multilingual	48%
Monolingual	52%
Year in School	
First-year	33%
Sophomore	30%
Junior	22%
Senior	19%
Former Statistics Courses	
None	18.5%
Attempted but failed	22%
Passed in Community College	4%
Passed in High School	41%

Data Collection

To explore the question of how students wrote and talked about data, representations, and statistical claims, I collected four types of data: ethnographic notes, video recordings of 20 synchronous sessions, and 30 students' written work on three assignments (see Appendices 2 and 3 for prompts). During each synchronous class meeting led by the instructor, I took ethnographic field notes guided by several broad questions: How are students participating in the synchronous sessions? What are they saying (or typing in the chat)? Who is speaking and who is not speaking? How are students talking and writing (in the chat) about the presented data and visualizations? After the class meetings I used these notes to write more formal fieldnotes. I also video recorded each of the 20 synchronous whole-class meetings and saved the chat from these sessions. Finally, I collected students' written work from the class. This written work is the focus of analysis for this dissertation.[2]

The students had three major writing assignments called "Numbers in the News." For the first assignment, students found a news article about a social justice topic they cared about and analyzed the author's use of data and visual representations of data. They received feedback on this essay (described in more detail in Chapter 4) and, for the second assignment, either revised their first essay or

[2] I also video recorded three interviews conducted via zoom at the conclusion of the course with pairs of students. I solicited volunteers for these interviews by first emailing the entire class and then following up with ten students. I chose these ten because, based on my ethnographic memos, they had participated in class and demonstrated varying degrees of understanding of the course content. The six students who volunteered for the interviews were all included in that smaller list. In these interviews, I presented the pairs with various graphs published in the New York Times and asked them to discuss what they noticed and wondered (using a typical routine from mathematics education). Analysis of these interviews is not the subject of this dissertation, but these six students formed the sample for analysis of students' writing in Chapter 6.

chose a new article to analyze. For the third assignment, students were given a Brookings Institute article on housing inequality to read and analyze. Students were given the same prompt (see Appendices 2 and 3) for all three essay assignments. The prompt contained six questions designed to guide the students through writing their essays. These questions asked students to 1) Identify and assess the credibility of the source; 2) Explain the social justice issue and the author(s)' point of view; 3) Describe the quantitative information presented; 4) Critically analyze the role and quality of the quantitative information; 5) Identify and explore questions that arose; and 6) Conclude by describing the value of carefully assessing quantitative information. Five additional questions (see Figure 2) were included to remind students of ideas we had discussed in class. These questions were ones that we asked repeatedly throughout the course and were presented as heuristics that the students could use to make sense of and critically analyze data and data-based claims in the media.

Figure 2 Questions We Tried to Ask Regularly in Class Discussions and in Assignments

1. What does this number represent? Who does it represent?
2. Does it seem correct? Is there any way I could check?
3. What does the authors' reporting this number in this way highlight? What does it obscure?
4. Is this a lot? Compared to what?
5. So what? Why does it matter (or not)?

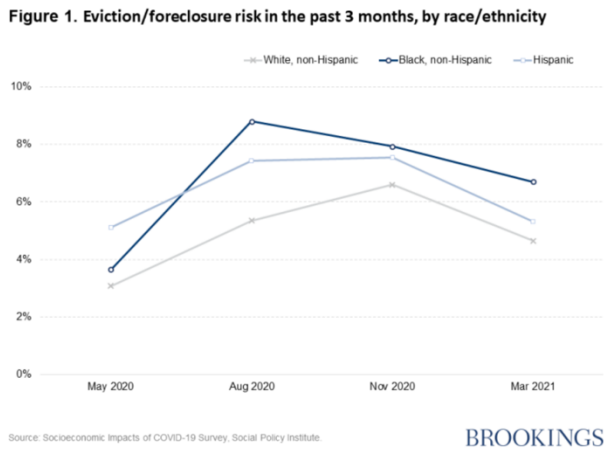
I chose to analyze the final essay first (see Chapter 5) because all students responded to the same article, which enabled a more streamlined analysis. I also chose to analyze the final essays because they were assigned at the end of the course

and therefore might represent more of the students' learning. In this assignment, students were asked to respond to the article, "Inequalities in housing hardship declined because everybody is now worse off," by Yung Chun and Michal Grinstein-Weiss, which was published by the Brookings Institution on April 30, 2021. The article described the results of a year-long survey by the Social Policy Institute at Washington University in St. Louis exploring housing-related hardships experienced by Americans during the first year of the pandemic. The thesis was that inequality in experienced hardship, measured by eviction/foreclosure risk, rent/mortgage delinquency, and utility bill payment delay, had decreased because White, non-Hispanic respondents' hardship had increased. The article used five figures and accompanying text to present the findings from four surveys administered between May 2020 and March 2021.

The article provided a variety of opportunities for students to engage in statistical literacy practices. Firstly, the students could investigate the Brookings Institute, a Washington, DC-based public policy and research organization whose work is considered factual and somewhat left-leaning (mediabiasfactcheck.com) and the authors who are a data analyst and nonresident senior fellow with the organization. The students could also investigate the study design. If they followed several links embedded in the article, they could access a description of the sampling and survey-administration methods and an overview of the categories of the 200 questions in the survey. These links provide opportunities for students to fact-check the article somewhat, though access to the survey results themselves require the

completion of a request form that is described as taking 4 weeks to complete. The article described the results of a series of surveys of three groups of Americans (White, non-Hispanic; Black, non-Hispanic, and Hispanic) in percentages in text (e.g., "Instead, white respondents are increasingly experiencing housing instability. For instance, between August and November, the eviction/foreclosure risks for Black and Hispanic respondents decreased by less than 10 percent (from an eviction rate of 8.8 percent to 7.9 percent for Black respondents and from 7.4 percent to 7.5 percent for Hispanic respondents).") and in figures (see Figures 3, 4, and 5 below).

Figure 3 Brookings Institute Figure 1. Eviction/Foreclosure Risk



These visual representations provided opportunities for students to make sense of and critique textual and visual displays of data. For example, during the course the instructor had taught the students to beware of a truncated y-axis in a bar graph because it exaggerates differences. The article used two bar graphs with truncated y-axes (see below) to describe some survey respondents as "long haulers"—respondents who reported housing hardship over multiple survey periods. Finally, the

article provided opportunities for students to consider the social impact of the data reported as it refers to unequal experiences of Americans.

Figure 4 Brookings Institute Figure 4. Housing "Long-Haulers"

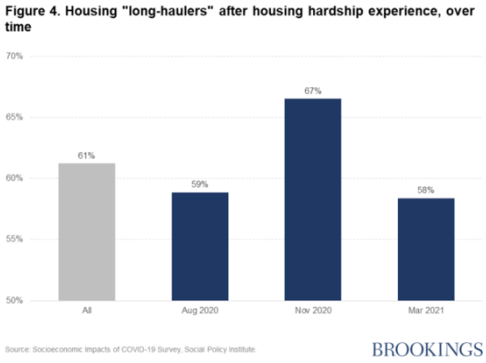
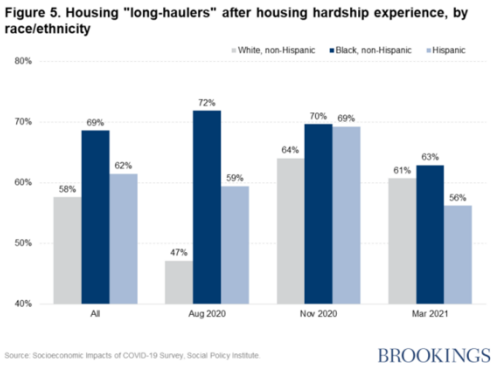


Figure 5 Brookings Institute Figure 5. Housing "Long-Haulers" by Race/Ethnicity



Data Analysis

In the initial round of analysis, I read through the essays looking for ideas or methods of critique that appeared more than once. I was using the lens of the course instructor, looking for writing that was related to the concepts and procedures we had

taught in class (e.g., sampling methods, calculating a rate of change[3]). I was also using the lens of the research literature which offers lists of knowledge and beliefs that comprise statistical literacy (see Chapter 2). For example, I noticed that many students were writing about the survey's sampling methods. Understanding the strengths and limitations of various methodologies, such as sampling methods and surveys, is one proposed component of statistical literacy (Ridgway et al., 2018). Therefore, "Survey Sample" became an initial category I used for analysis. I looked for other categories identified in the literature, asking "Did the students write about graphs?" and "Did they use arithmetic to make sense of the article?" The questions I asked were informed by both the affordances of the article (described above), the task, and the concepts and strategies we had taught in the course.

Table 2 Initial Categories of Analysis

Students wrote about…	**Number of Students (n=30)**
Graphs	73%
Identity and Credibility/Bias of the Source	77%
Survey Sample	53%
Using Arithmetic	63%

The other initial categories (see Table 2) I identified were writing about the graphs, writing about using arithmetic calculations, and writing about the authors' credibility or bias. These categories summarize my initial observations of what the

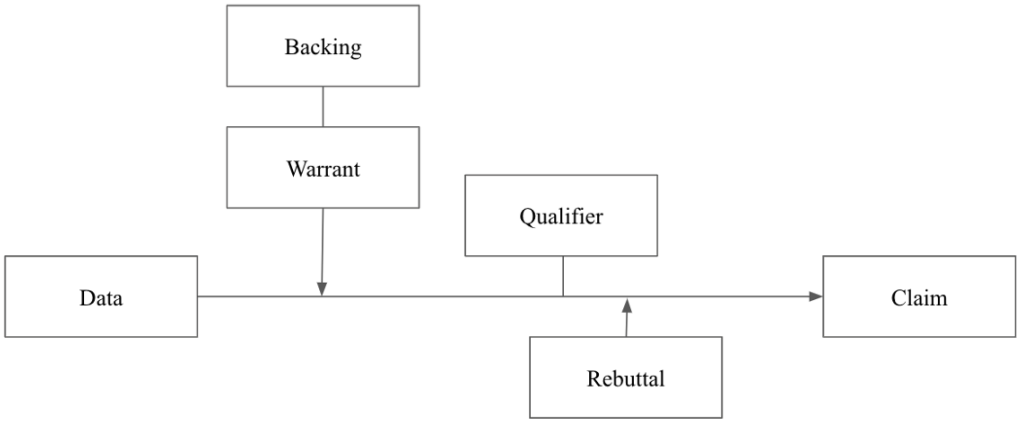
[3] "Rate of change" is the language we used in class to describe how to find the relative, or percent, difference between two quantities. How we taught this procedure is described in more detail in Chapter 4.

students wrote about in their essays. Every student's essay is represented in Table 2, and no student wrote about every category listed.

For the next step in the initial analysis, I used Toulmin's argument framework to look for students' arguments within those categories, asking "What claims were students making about the article?" I started with looking at students' writing about data quality and sampling techniques and found clear arguments. I drew diagrams summarizing the parts of the students' arguments based on the Toulmin diagrams in Gómez-Blancarte and Tobías-Lara (2018) (see Figure 6). Each argument in the students' essays contained a claim, data, and a warrant. Some contained a qualifier. I identified claims as either the thesis statement of the overall essay or the thesis of a subsection of the essay. Data referred to information in the news article or from a secondary source the student selected. The warrants consisted of concepts and calculations that formed the inferential connection between data and claim. Qualifiers appeared in some students' writing in the form of conditional statements in reference to the claim (e.g., the claim might be true).

I did not quote students directly but instead summarized the students' writing when I made these diagrams. I did this for two reasons: first, their writing was often lengthy, and I wanted to have clearer prose in the diagrams, and second, warrants are often implied (Toulmin, 2003), so it made sense to paraphrase these. I also used my ethnographic memos about the course and the students when interpreting their writing.

Figure 6 General Toulmin Model from Gómez-Blancarte and Tobías-Lara (2018)



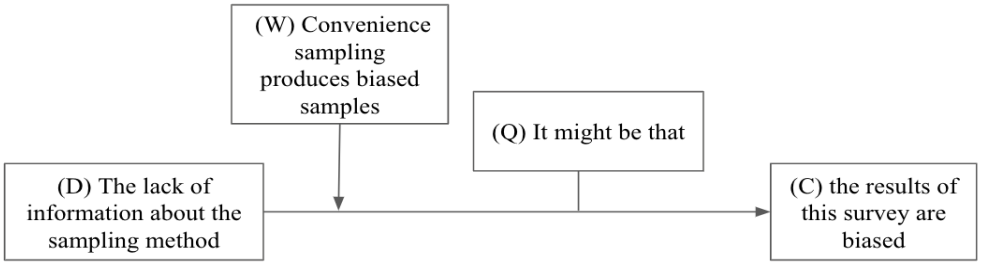
The following is an example of how I analyzed a student's argument. The student was writing about the sampling method, one of the first topics I identified as a focus for students. Here is an excerpt from one student's essay:

> I have no idea if the sampling was a convenience sample or a simple random sampling. If it was a convenience sample, the statistical data would be biased because it systematically favors particular outcomes. The statistics present could be valid but because it is being obscured and inaccessible, I would not use this article as the information is unsubstantiated … all samples should be able to represent the population. … In order to determine if this article is representative of the White, non-Hispanic or Black, non-Hispanic folks' housing struggles during COVID in the United States, we must know what sampling method was used for this survey. (Student's Essay #3)

I first identified and paraphrased the student's claim. Where the student wrote, "The statistics present could be valid but because it is being obscured and inaccessible, I would not use this article as the information is unsubstantiated," I annotated the essay with a paraphrase, "the results might not be valid." This became the student's claim in the argument. Then, working backwards from the claim, I identified the data (in this

case the lack of information about the sampling method. Then I looked for the student's reason why — the warrant for their argument. The warrant came before the claim in this argument, convenience sampling would lead to biased outcomes. The argument is diagrammed below (Figure 7). Note that this argument contained a claim with a qualifier: "The statistics present *could be* valid," (my emphasis).

Figure 7 Toulmin Model of Nikki's Argument



Using the Toulmin method for analysis has strengths and weaknesses. This tool was initially valuable as a means of data reduction. I had tens of thousands of student-written words to analyze and looking for arguments helped focus my attention. Creating argument diagrams also allowed me to compare students' writing in a systematic way both between different students (e.g., these students have the same claim but use different warrants) and between different essays written by the same student (e.g., this student revised their claim). The benefits of using Toulmin for systematic comparisons have been documented by others (e.g., Groth & Follmer, 2021). The Toulmin diagrams also helped me to determine the possible goals guiding their use of concepts and strategies. In the example above, I inferred that the student was using their knowledge of sampling techniques to critique the quality of the data presented. Finally, though some (e.g., Cramer & Kempen, 2022) see it as a weakness

of the method, the Toulmin diagrams have no space to indicate the correctness of the reasoning. I view this as a strength for this study as I am documenting students' emerging uses of statistical literacy practices. I do not ignore when students are incorrect in their understanding, but it is not the focus of the analysis. There are also well-documented challenges to using this method that I also encountered. One such challenge is determining the appropriate grain size (Groth & Follmer, 2021) for arguments. Should the argument represent the thrust of the entire essay or focus on a smaller point within it? Also, the Toulmin method focuses on the structure of an argument, but it does not capture rhetorical moves such as tone and word choice. The students' essays were rich and complex pieces of writing. They were also a very specific genre — a class assignment. This is, in part, why I do not rely on Toulmin alone in my analysis but also include my ethnographic observations, recordings of class meetings, and written copies of the feedback I gave students on their essays.

Because most of the students who wrote about the survey sampling methods did so in a way that was critical of the article, I started by limiting my search for arguments related to the other categories (using arithmetic, making sense of graphs) that were similarly critical of the article. When I looked at excerpts of student writing that used arithmetic or discussed the graphs, I saw fewer arguments that criticized the article. Some essays seemed to have no argument at all.

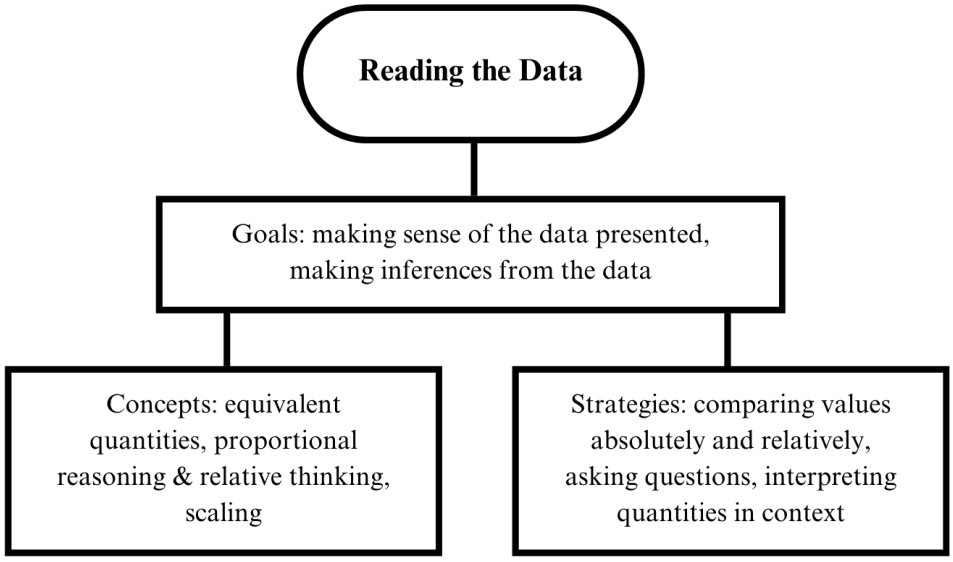
Then, realizing that I had overly limited my analysis by focusing on excerpts rather than the complete essay and on critiques rather than arguments, I returned to reread each student's essay in its entirety in search of any argument, not only

critiques. “What is this essay arguing?” was the question in my mind as I began each essay. After reading, I recorded a few phrases capturing the student’s argument as a claim/s and warrant/s. It was then that I realized that every essay did, in fact, contain an overall argument. The warrants for the arguments sometimes appeared in a different section of the essay from the claim. The variation in argument, discussed in detail in Chapter 5, seemed to be related to how a student viewed the task and how they interpreted the article and which parts of the article they focused on. While my initial observations about which topics the students were writing about most often were accurate, I had to look at the overall essay to make sense of these in relation to the arguments in their essays.

The next step in my analysis was to describe the statistical literacy practices that students used in their essays. Consistent with the ethnographic stance that I adopted, I assumed that statistical literacy practices are not a unitary category but are instead varied, situated, and co-created in social settings and in response to tasks and goals (Moschkovich, 2019; Moschkovich & Brenner, 2000). I used my knowledge of the students and the course activities, as well as the research literature on statistical literacy, to identify the statistical literacy practices students were “grappling with, even when [those] may not be immediately evident,” (Moschkovich, 2019, p. 70). Through iterative analytical memos, I grouped students’ writing according to the goal (usually inferred by me and not explicitly stated by students), the mathematical concepts they used, and the strategies they employed. To clarify how these goals, concepts, and strategies coalesced to be a practice, I created idealized models of the

two statistical literacy practices that emerged through analysis. These models (Figures 8 and 9) were provided a conceptual framework to describe students' writing. This method of analysis is consistent with a strengths-based perspective (Huitzilopochtli et al., 2021) that helps us understand "learners in their own terms and [highlight] the potential in what they know, rather than only comparing their knowledge to that of an expert" (Moschkovich & Brenner, 2000, p. 461).

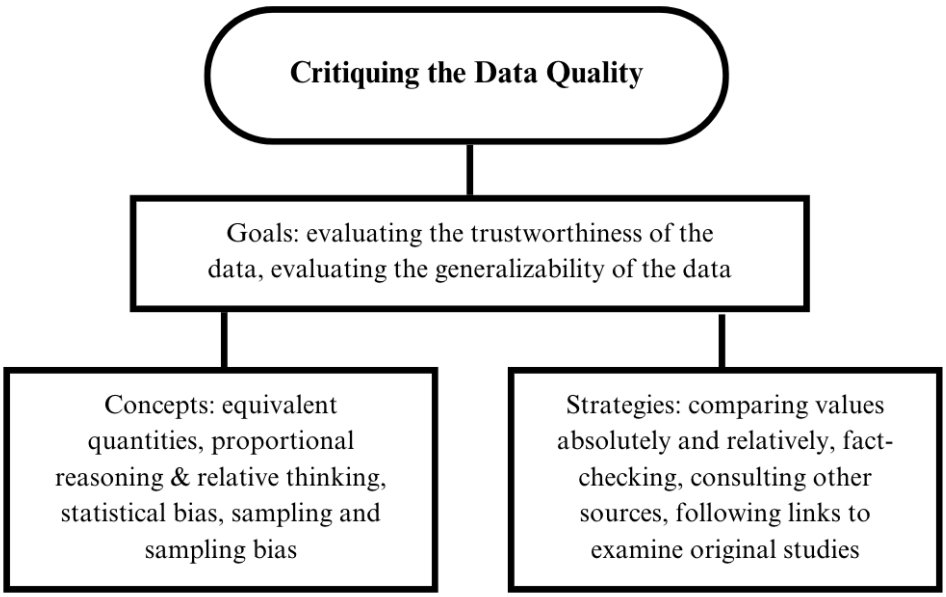
Figure 8 Idealized Statistical Literacy Practice: Reading the Data



Reading the Data was the foremost practice that we taught in class. Because so many of our students over the years have said that they "skip" the numbers, tables, and graphs and only read the prose in articles, much of our instruction in class focused on helping students learn to slow down and read the data presented. Some goals of the practice of Reading the Data are to make sense of the data presented and to make inferences and draw conclusions from that data. To do this, one might use

mathematical concepts such as equivalent quantities, proportional reasoning and relative thinking, and scaling.

Figure 9 Idealized Statistical Literacy Practice: Critiquing the Data Quality



The second statistical literacy practice that I described is Critiquing the Data Quality. The goals of this practice can include evaluating the trustworthiness and generalizability of the data. To do this, one might use mathematical concepts such as equivalent quantities and proportional reasoning as well as statistics concepts like statistical bias and sampling and sampling bias. The strategies we taught students to use when Critiquing the Data Quality include comparing values absolutely and relatively, fact-checking and looking at other sources, and following hyperlinks to investigate the source of the data or data-based claim.

Analysis of the students' arguments enabled me to identify these two statistical literacy practices that students used: Reading the Data and Critiquing the

Data Quality. Student writing that was about the Survey Sample and Identity and Credibility/Bias of the Source showed evidence of students engaging in the statistical literacy practice of Critiquing the Data Quality while the student writing that was about graphs illustrated students Reading the Data. Students who used arithmetic in their writing used both practices. Importantly, it was only after analyzing the students' writing in its totality and with an open mind towards any argument that I was able to notice all the ways that students were engaging in these statistical literacy practices. This is an important methodological issue and one that has implications for teaching as well.

To explore the final part of the third research question, What statistical literacy practices did students use when analyzing self-selected articles? Did these change over time and with the opportunity to revise for four focal students?, I selected a subset of the students and analyzed the first and second essays they wrote in the class. As a reminder, the students were asked to select and analyze a news article that contained data, data visualizations, or data-based claims. They received written feedback from me in the role of Teaching Assistant and were encouraged to schedule one-on-one writing conferences with me to discuss their work. While every student received written feedback, only a few opted to have a meeting. The second essay was positioned as a revision of the first, though students were allowed to choose a novel news article and write a new essay if they desired.

The focal students were selected based on the original study design which included pair interviews.[4] At the end of the course, I invited all the students to participate in one-hour pair interviews in which they would discuss graphs selected from popular news sites such as The New York Times. I sent additional invitations to ten students who I selected because they had participated in class meetings and represented both the demographics (see Table 3) and the range of academic performance in the class. The six students who volunteered were students who received the second invitation. Of these, four wrote their two essays about the same article; in other words, their second essay was a revision of the first. These four students were the focus of analysis for Chapter 6.

Table 3 Focal Students with Demographic Information

Student	Year	Major	Gender	Race/ Ethnicity	Multi-lingual Learner	Taken stats in college	Passed stats in college
Julia	3rd	Community Studies	F	Latina	Y	Community College	Y
Shaun	1st	Film	M	Black & White	N	Y	N
Rocio	2nd	Film	F	Latina	Y	N	N
Nikki	3rd	Community Studies	F	Latina	Y	Community College	Y

I repeated the Toulmin analysis that I used for the students' final (third) essays for each of the students' two essays. In order to determine how feedback and the opportunity for revision may have impacted the students' writing and use of statistical literacy practices, I compared the students' arguments between their first and second

[4] These interviews are not analyzed in this dissertation. They will be analyzed in the future.

essays, taking note of whether and how their claims and warrants changed. I also used the comments and feedback I had given students to inform my understanding of what changed between the two essays.

In the next chapter, I provide a thick description of the course, the activities and assignments, and my role as Teaching Assistant. This lengthy explanation of the course is necessary to understand the context in which students were writing. These details also inform my analysis.

Chapter 4: Description of the Course

In this chapter I provide a detailed description of the setting of this study—a university course called Numbers and Social Justice. Because mathematical practices are developed through social interactions around joint activity (Moschkovich, 2013), it is necessary to describe the setting in which students learned to use these practices. Furthermore, analysis of the students' written work requires a consideration not just of the affordances of the task (see Chapter 3) but the history of the lessons taught, activities explored, tasks assigned, and language used. This description is based on my field notes, ethnographic memos, recordings of class meetings on zoom, and documents and student work stored in the university's course management system, Canvas. I begin by describing the course generally and my role in designing it. I then introduce the students and provide a vignette of the first day of class. In the next section I describe the structure of course meetings and assignments and provide an overview of the concepts and practices that were the focus of each week. Next, I describe in more detail some of the concepts we taught and provide examples from class discussions and activities. Finally, I describe my role as the Teaching Assistant (TA) and researcher.

This study was set in an undergraduate elective course at a California public university, titled "Numbers and Social Justice," which satisfied the university's general education requirement for Statistical Reasoning. The course was advertised as especially appropriate for Humanities and Arts students who had struggled with mathematics and statistics courses in the past. Housed in the Community Studies

department, the course was designed to support students in analyzing and communicating data related to equity in social arenas such as education, public health, housing, and criminal justice. Though the course was required for Community Studies majors, students from any major field of study were welcome. Historically, students from minoritized racial and ethnic, cultural, and linguistic communities, as well as students with non-dominant gender and sexual identities had made up most of our students. For many students, the last mathematics course they passed was in high school, they may have attempted and failed statistics courses in other departments, and this was the final course needed to graduate.

I was the Teaching Assistant for the course, and, at the time of the study, I had held that position for seven previous iterations of the course. The instructor and I had worked closely for six years refining the course to provide opportunities for students to develop statistical literacy. We recognized that this course could be a gatekeeper; it fulfilled a graduation requirement that many of our students struggled to meet. In our discussions around developing the course over the years, we sought to address the questions "What mathematics? For whom? And for what purposes?" (Martin, Gholson, & Leonard, 2010). What did our students need to learn to be able to make sense of and critique data-based claims in the media? How could we make the learning relevant to students who have been historically marginalized in STEM courses? Despite reform efforts in the past decades that have sought to teach statistics in more holistic ways, university statistics courses still typically focus on procedures and skills, and they leave students with the impression that statistics is essentially a

collection of ultimately forgettable tools and techniques (Ben-Zvi & Garfield, 2004). Teaching statistics this way has especially negative impacts on students from marginalized identities (Martin, 2013) who have experienced neglect or trauma in mathematics courses that position them as outsiders.

We designed the course to focus on social justice because it was a topic that interested our students and because statistical literacy requires a sense of agency and the conviction that one has a legitimate right to question authoritative sources, even if one is not an expert (Gal, 2002). Our students were typically very practiced in writing and speaking critically about politics, sociohistorical contexts, and issues of social justice, but they "fell apart" when confronted with numerical data. For example, in a pilot study that I conducted in 2020 with members of the course that year, the majority of students surveyed said that they skipped any tables or graphs and relied entirely on the prose to interpret and critique an author's claims. In facilitating discussions in class, we sought to notice and leverage students' strengths (Huitzilopochtli, et al, 2021) to engage in statistical reasoning about questions that are critical to social justice. For example, students were assigned readings that considered theoretical ideas (e.g., Constructing Normalcy by Lennard J. Davis) related to statistical concepts because the majority arrived in class well-practiced in reading and discussing complex texts. All the tasks we created (both in-class and assignments) were about sociopolitical issues, used real data and visualizations, came from major newspapers, media outlets, and social media as recommended by research on statistical literacy education (e.g., Engledowl & Weiland, 2021; ProCivicStat

Partners, 2018; Weiland, 2019), and invited students to make sense of and critically analyze data, visualizations, and data-based claims. It was critical to us that the students had real-world, consequential examples for every concept.

The Course at the Time of the Study

I collected data during the Spring 2021 school term during the COVID-19 pandemic. The course, which lasted ten weeks, was entirely remote and included both asynchronous and synchronous elements. Forty-seven students participated in the course, and 30 of them consented to participate in the study. Most students were familiar with the remote learning systems because California had mandated emergency remote instruction in March 2020 and that order was still in effect a year later. However, at this point in the pandemic, many undergraduate students were struggling with stress, social isolation, and depression, as well as insufficient access to the internet and internet-enabled devices. The majority of students did not turn their cameras on during the zoom sessions, citing zoom fatigue and internet limitations as their reasons. Indeed, participation in discussions was more challenging than it had been in past iterations of the course. Especially during the latter half of the course, it was not uncommon for the instructor to ask a question and be met with a prolonged silence from the students. The phenomenon of student disengagement in learning during the pandemic has been documented (e.g., Chakraborty et al., 2020; Durak & Cankaya, 2020). The stressors of balancing school, work, and family were only exacerbated by the pandemic, and the students commented on feeling apathetic, frustrated, stressed and overtired.

Despite many challenges, the start of the term still contained some of that excited, first-day-of-school energy that was familiar from previous, in-person, classes. On the first day of class, we invited the students to participate in an icebreaker activity. They were directed to a google form with the questions, "What have been your experiences in past math and statistics courses? What are your feelings about math and statistics?" They were given a few minutes to think and write a response that would be shared anonymously with the class. Then we randomized the responses, and the students took turns reading each response aloud.

The instructor and I had started the course with this activity for six years, though we of course modified it for remote instruction. The responses were expectedly heartbreaking. The overwhelming majority of students described math courses in negative terms. Overwhelming. Frustrating. Alienating. Boring. Traumatizing. Nerve-wracking. Several described avoiding math out of fear of humiliation and failure. Others emphasized the importance of the teacher. "When I have good teachers, I enjoy the course and learning." "I used to have a teacher that would laugh at the questions I asked and not help me." One student wrote about feeling alienated in a prior statistics class when they objected to the professor's use of the gender binary ("Women are shorter than men."). They wrote that the professor brushed aside the comment and the other students, described as cisgender men, seemed annoyed and one said, "This isn't a humanities class." These are examples of how mathematics courses can be spaces that alienate students from underrepresented groups (Moore, 2021). That so many of the students had negative feelings towards

mathematics courses wasn't at all surprising considering who our students were. Like other years, the students represented communities that have historically been marginalized, oppressed, and excluded in mathematics courses (see Table 3 in Chapter 3).

After reading each response, the students moved into break-out rooms to discuss the activity. When they returned, the instructor asked them to share what they had discussed. Several groups reported that they had talked about the idea of the "math person." "If you're not one, teachers don't care about you." The idea of feeling too uncomfortable to ask a question came up, and one student commented that the icebreaker activity had created a good environment. "I just feel like I'm not alone. It's less isolating." In the chat window a student typed, "This class has good vibes."

When a student mentioned the idea of learned helplessness, the instructor interrupted, leaning into the camera. "Make no mistake, that is no accident. Mathematics education is part of a larger social project." She went on to briefly discuss Sputnik, neoliberalism, and racism. She explained that the students had been kept from mathematics, not that they were incapable of mathematics. She reassured them that we (she and I) were there to support them, no question was too elementary, and they had a right to learn. "You know what, I'm just going to say it. Fuck those people who made math not for me." There were nods and smiles from the few faces we could see and approving comments in the chat. The discussion, and the first class meeting, ended with a student commenting that math can be fun if you know what you're doing.

In that first class meeting, we did not teach any content or even review the syllabus. Our focus was on creating community and laying the groundwork to earn our students' trust. That we had seen and heard positive and hopeful comments was encouraging.

Course Structure

The course was structured as follows. The students were required to attend three synchronous sessions per week via zoom. On Tuesdays and Thursdays, we met for 90-minute, whole-class sessions led by the instructor. These generally consisted of lectures, whole-class discussions, and small group discussions in break-out rooms. These class meetings were recorded and available to the students via the campus learning management system. On Wednesdays, they attended one of two one-hour smaller-group "sections" which were facilitated by me as the Teaching Assistant.

The students were responsible for asynchronous participation as well. This looked like course readings, videos, and TikToks. The TikToks were particularly powerful tools for creating an inclusive space. We primarily used the work of OnlineKyne, a Canadian drag queen and mathematics communicator who charmingly explains statistical concepts like sampling and operationalization of variables using real-life, often social justice-oriented, examples. A student commented at one point in the course that they especially appreciated these because it was the first time they had seen queer people represented in mathematics and it made them feel like they might have a place in mathematics too.

Table 4 Timeline of Course Concepts and Assignments

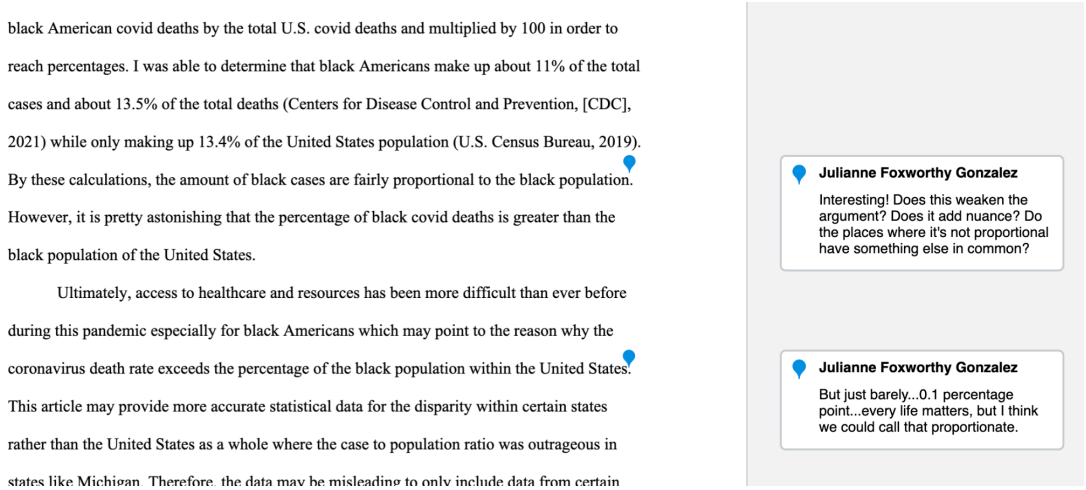
Week	Big Ideas	Assignments Due
1	Statistics are numbers with a context. What is social justice? Equity vs. Equality	Reading quiz
2	Critical Epistemology and statistics as a social justice tool. Estimating values and doing "rough math." Confounding variables.	Reading quiz Numbers in the News Essay #1
3	Counts vs Rates. Denominators as Context. Relative vs. Absolute Change. The rate of change formula.	Reading quiz Homework #1
4	Operationalization, measurement. Validity and reliability.	Homework #2
5	Making sense of visual representations of data. Common manipulations	Reading quiz Homework #3
6	Measures of Central Tendency. The concept of "normal." Samples and Census.	Reading Quiz
7	Study design. Sampling techniques. Survey questions.	Reading Quiz Numbers in the News Essay #2
8	Misleading graphs and tables	Reading Quiz
9	Heuristics (Kahneman & Tversky, 1974). Proxy variables.	Homework #4
10	The value of counting. Using statistics for social justice.	Numbers in the News Essay #3

The students had seven major assignments over the course of the term: four homework assignments that consisted of problem-solving exercises based on real-world data and data-based messages (see example questions from homework assignments in Appendix 1), a rough draft and final version of an essay in which they chose a news article about a social justice topic and analyzed the author's use of data

(see Appendix 2), and a final exam in which they analyzed a Brookings Institute article on housing inequality (see Appendix 3). They also had weekly quizzes on the readings.

Between the first and second Numbers in the News essays, the students received written feedback from me (see Figure 10) via the online instructional management system and had the option to schedule a meeting with me or the instructor to talk through their analysis. Each student's feedback was specific to the article that they chose, but, in general, my comments included reminders to put numbers in context by asking the questions we taught in class ("Is that a lot?"

Figure 10 Example of Comments on a Student's Rough Draft



"Compared to what?"), encouragements to question the data and claims in the article, and feedback about the accuracy of their understanding of statistics concepts such as proportionality and representativeness. I was looking for students to engage with the data and data-based claims using the practices we were teaching in class.

Examples of Concepts and Practices

The course was originally conceived by the instructor when they noticed that their students in other courses would cite data in their work without seeming to consider what those data meant. An example of this, and one that appeared in the students' homework assignments, follows. The question read:

> Last year one of my students in a different course reported on a California state policy by noting, "Seventeen counties have implemented the law." My question to you: So what? Rewrite this statement to make it more memorable and legible to the reader. In order to accomplish this, you'll want to look up some information that's not in my student's statement, and you'll also have to do a little bit of rough math, so show your work and cite your source/s.

This task required students to consider the relevant denominator, in this case 17 out of 58 total counties, or about a third. Considering the meaning of reported numbers was a major theme of the course and we returned to it repeatedly. For example, in the second class meeting, the instructor shared that over 552,000 people in the U.S. had died of COVID. "Is that a lot?" they asked. The students then debated the question by anchoring 552,000 to other numbers they knew (e.g., it's three times the number of people who live in the community surrounding the university), by critiquing the accuracy of the number (e.g., "How do we know we've correctly counted every COVID death?"), and by comparing the number to other causes of death. In this class meeting we introduced a series of questions to ask when one sees a number reported in the news: Is that a lot? Compared to what? So what? These questions encouraged students to think critically about reported data rather than just citing it in their own

work. These three questions were repeated to students in class discussions, in their homework assignments, and in my written feedback to their essays.

Another topic in the class was called “rough math” or “back-of-the-envelope math” by the instructor. She asked students to round numbers in order to estimate percentages. For example, the students were presented with the race/ethnicity demographics of the university (given in counts). Out of 17,517 total students, 752 identify as African American. This is unwieldy if one wants the exact percentage, but the instructor required students to estimate by rounding to easier numbers. In this example, 17,517 became 17,500, and 752 became 750. 750/17,500 can be reduced by 25/25 giving 30/700. This can be reduced further (30/700) / (7/7), which results in a little more than 4%.

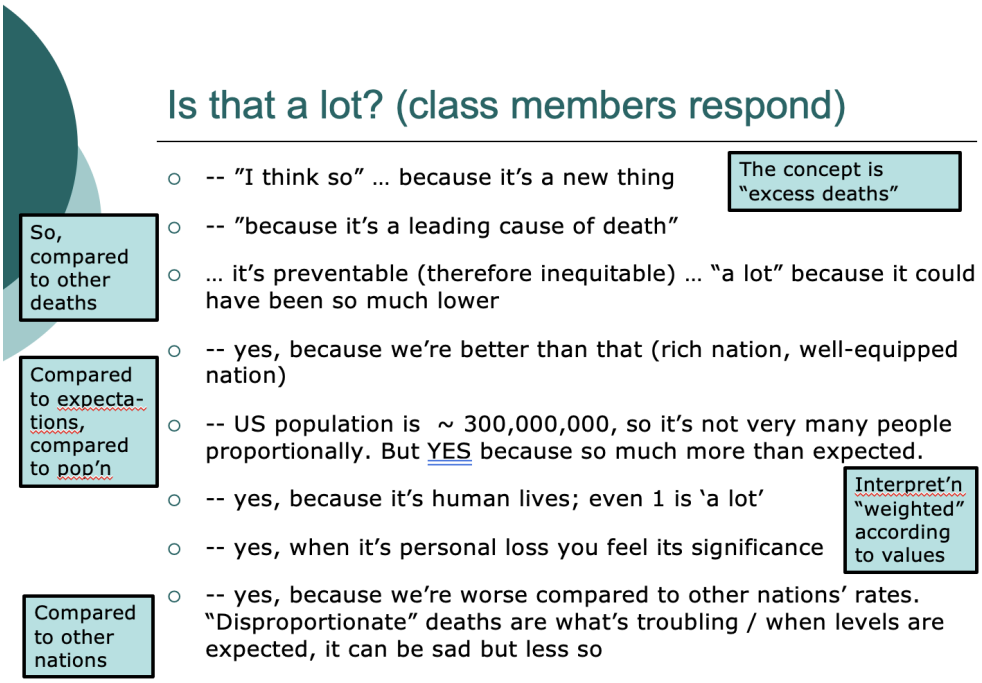
We taught the students to consider common factors when rounding to do rough math. In all course assignments, students were required to use “rough math” rather than calculators to estimate quantities. Some struggled with the imprecise nature of an estimate. This is understandable – mathematics for them may have been all about getting a precise answer. But our goal was to help students to engage intellectually with reported numbers. We did not want them to be bogged down with procedures and calculations. Instead, we endeavored to convince them that “a little more than 4%” is not only precise enough for our purposes (e.g., making sense of demographic data), but it is also more meaningful and easier to understand than 752 out of 17,517.

Another topic we discussed in class was the difference between a count and a rate, or absolute and relative quantities. This concept was introduced within a real-world context in response to which the instructor facilitated a classroom discussion. In the second class meeting, the instructor presented the students with the number 552,000 and said to the students, "This is the number of people who have died from COVID-19 in the United States. My question for you is, is that a lot?" This question sparked a heated discussion. Some students said yes, because the deaths were preventable or because the U.S. had more deaths compared to other nations. Other students said the number seemed small when compared to the total population of the country. Figure 11 shows the instructor's slides where she took notes on the students' comments during the discussion. The phrases in quotation marks are the words of students. The blue boxes were added to the slide after the class meeting by the instructor in order to highlight the ideas she had added during the discussion.

Throughout the discussion, the instructor and I highlighted important course concepts that would come up later in the term by marking concepts when they arose and introducing vocabulary. For example, when a student said that the number was a lot because even one human life is meaningful, we introduced the concept of a count, or absolute, quantity (in contrast to a rate). The instructor emphasized during the discussion that when considering social justice, it was important to remember the human lives represented by the numbers. When a student compared the 552,000 figure to the population of the United States, the instructor introduced the concept of proportion and rate. She then demonstrated how to use "rough math" to estimate the

proportion of the U.S. population that had died. She rounded the number of deaths to 500,000 and the U.S. population to 300,000,000 and simplified that 1/600. At that point, a student made a comment that became a theme in the course: Shaun said, "This makes me think that the way you present a number changes the way you think about it."

Figure 11 Course Slide Summarizing the Classroom Discussion on COVID-19 Deaths



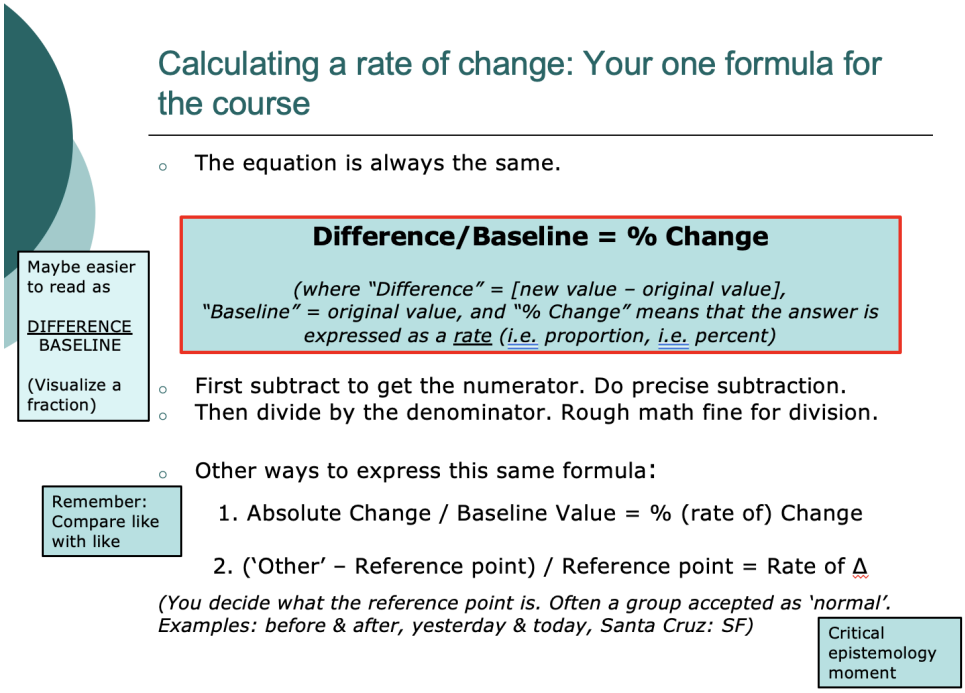
Several students commented that 1 in 600 seemed larger than 552,000 and the instructor then verbally stressed two important course concepts. First, she said that counts are valuable because they make us pause and consider every individual life, but rates are important because they put the count in context. "Every time you see a number," she told the class, "Ask yourself, Is that a lot? Compared to what? The

‘compared to what’ part is the denominator.” She then encouraged the class to compare 1 in 600 and 552,000 to similar quantities that were familiar. Students shared that their high school graduating class had 600 people, so 1 in 600 could be thought of as one person in every graduating class. Others mentioned movie theaters and college dormitories as familiar spaces of about 600 people. When asked to think about the 552,000 number, the students compared it to the size of California cities. Sacramento, the state capital, has about 500,000 people. “It’s like losing a whole city,” the instructor said.

In concluding this lesson, the instructor told the students, “A number in isolation is meaningless.” She highlighted the ways that the class had collaboratively made sense of the 552,000 number by simplifying it to make it more legible and memorable and comparing the value to the relevant context.

In the third week of the course, we introduced the students to the “rate of change” formula. “Social justice is about change,” the instructor told the class, “and we need to be able to calculate how big or small the change is relative to something else — either a previous time or a comparison group.” Figure 12 shows the slide she used to teach this. To find the relative change (or rate of change), she asked them to find the absolute difference between two values and divide that difference by the “baseline” value. The baseline value was described as the “reference point” and students were encouraged to decide for themselves what the relevant reference point would be.

Figure 12 Slide Showing the "Rate of Change" Formula



We also spent considerable time (more than 20% of the course) engaged in activities such as small-group and whole-class discussions and homework assignments that centered on the concept of operationalization. In the course, we taught students that "operationalization" means to take an idea and define it in a way that one can measure it. We presented several real-world examples to model this, and the students also worked in groups to operationalize the concept of college readiness. The students read and discussed news articles on counting COVID deaths, measuring racial discrimination, and San Francisco's attempts to distribute social services. In the example of San Francisco, the students learned that the city was using "calls for service" as a metric for which neighborhoods needed social services. This metric proved to be an invalid measure for need. Instead, more calls came from

neighborhoods in which residents felt more empowered to call the city for help, not from neighborhoods that had the greatest need. Residents in neighborhoods with many problems didn't call, perhaps because they assumed the city already knew about the problems or because they didn't not feel safe or empowered to report problems to the government. Throughout our lessons on operationalization, we encouraged students to consider the following two questions: What does this measure highlight? What does it obscure? We told the students that all measures will obscure some things and highlight others and that the students' task was to take these into consideration when making sense of data and data-based claims in the media.

Other important concepts in the class were sampling techniques, survey design and implementation, heuristics, graphs and figures, and measures of central tendency. For all these concepts, we used real world data as illustrations, using examples that were related to social justice issues. For example, we used an article about how the average hourly wage spiked in the months immediately following the start of the pandemic to illustrate the sensitivity of means. The average hourly wage had gone up because most low-wage hourly workers (in restaurants and other service industry jobs) had simply lost their jobs and were no longer included in the data. Instead, only the higher-wage hourly workers (those who were more likely to continue their jobs in a remote context) were considered in the new average.

In general, the students provided feedback in casual before- or after-class conversations that they were enjoying the content of the course, but they were struggling to keep up with the workload. In fact, halfway through the term, the

instructor and I received an email from a student who had conducted a survey with their classmates regarding the workload. Their email advocating for a decrease in the amount of coursework included the response rate to their survey (over 50%) and summary statistics:

- 75% of students are spending at least 5-12 hours a week (outside of our 4 hours instruction) working on assignments
- 93% of students feel their workload for this class is noticeably **higher** than that of their other courses
- 72% of students are working at least 1 job this quarter, an additional 19% are spending at least 10 hours per week committed to clubs or child care
- 84% of students feel the workload in CMMU 30 is **inhibiting** their ability to keep up in their other classes
- 90% of students feel that the workload in CMMU 30 is **negatively** impacting their work-life balance and/or mental health
- 100% of students feel it would be useful to them if **homework assignments were shorter**
- 82% of students feel it would be helpful to them if we had **longer** to complete the homework assignments
- 50% of students enjoy CMMU 30 overall, but 90% said they would enjoy it **more** if the workload was lightened

The instructor responded by sharing the university's hourly expectation (15 hours per week), but they also eliminated the weekly quizzes and worked to shorten the homework assignments. This incident is notable for two reasons. First, the very existence of this email suggested that we had created a classroom environment in which students felt empowered to advocate for themselves. The instructor's response was a compromise and honored the students' request. Second, this email suggested that the students were using what they were learning about surveys in class to make a data-based argument.

Discussion Sections with the TA

The students and I met once a week in smaller (half the class) groups and without the instructor in "discussion sections". I organized these discussion section meetings using routines. We started with a Number Talk (Parrish, 2010), beginning with students mentally solving problems like 25 x 14 and progressing up to 128 x 1.5. The students started with two to three minutes of silent thinking while the problem was shared on their screen. Then they went into a break-out room with another student to share their solution strategy. When they returned to the larger group, students volunteered to explain their solutions while I recorded their thinking on the virtual whiteboard using symbolic notation. During this sharing of explanations, I would ask questions like, "How did you know you could do that?" and "Does that always work with any numbers?" to encourage them to expand their explanations. Though it was unfamiliar at first, they quickly learned the routine and got used to my questioning. Some began providing more elaborated explanations without probing questions.

These Number Talks had three purposes. First, they served to "warm up" for working with numbers. Second, they gave students the opportunity to be successful at math, setting a positive affective environment at the start of class. Third, they gave me the opportunity to reinforce the message that there are many ways to manipulate numbers and more than one solution pathway. This is essential because many students are taught math in a way that rewards fast and flawless mimicry of procedures, and, since so many of my students had had negative experiences in

mathematics courses, it was essential that I created a norm that valued diverse thinking.

For example, every time I have presented 25 x 14 to a class, someone will always talk about how they thought of money and used quarters (twenty-five cents) to solve the problem. This is almost always shared in a somewhat embarrassed tone, as if this way of thinking isn't real math or is just less sophisticated. I take that moment to highlight the ways that thinking of numbers in different situations (e.g., 25 could be conceptualized as twenty-five cents or ¼ or 10 + 10 + 5) is valuable when working with numbers and that everyday knowledge is an asset.

Following the Number Talks in each discussion section meeting, I facilitated an activity focused on a mathematical idea or procedure such as measures of central tendency, estimating percentages, or reading figures. Each Wednesday discussion section meeting ended with time for students to ask questions about assignments or concepts discussed in the whole-class sessions.

Conclusion

In seeking to document students' use of statistical literacy practices, it is necessary to consider the context in which they were learning these practices. This chapter provided a thick description of the course from my perspective as the TA and researcher which will inform the analysis in the following chapters.

Chapter 5: Statistical Literacy Practices in Students' Essays

In this chapter, I describe the analysis of the students' final essays, all of which were written in response to an article on housing inequality in the United States during the COVID-19 pandemic by The Brookings Institution. I used multiple analytic tools (described in detail in Chapter 3: Methodology) and a strengths-based perspective to address the following Research Questions.

1. How did students respond in writing to a news article with statistical claims, data, and visual representations of data?
2. What arguments did students write? When students made arguments, what types of arguments did they make?
3. What statistical literacy practices did students use?

This chapter is organized into five sections. In the first, I provide a brief review of the data and the method of analysis. In the second section, I describe how students' writing about the Brookings Institution article reflected the concepts taught in the course, the questions posed in the assignment (see Appendix 3), and the features of the article. In the third section, I provide an analysis of the students' arguments and describe how the students primarily wrote arguments about the validity or persuasiveness of the claims in the article. I provide evidence that the overwhelming majority of the students wrote arguments that agreed with the claims presented in the article. In the fourth section, I describe how students used two statistical literacy practices: Reading the Data and Critiquing the Data Quality in their analyses of the article. I provide examples of how students used various concepts and

strategies to achieve various goals (e.g., using estimation to compare data quantities, using knowledge of statistical bias to critique the representativeness of a sample). Finally, I conclude with a discussion of the findings.

Data and Analysis

As described in Chapter 4, the students had three major writing assignments called "Numbers in the News." For the first assignment, students found a news article about a social justice topic they cared about and analyzed the author's use of data and visualizations. They received feedback on this essay and, for the second assignment, revised their first essay. For the third assignment, which was presented as a take-home final, students were given a Brookings Institution article on housing inequality to read and analyze. This final assignment is the focus of the analysis in this chapter.

As a reminder, the article (see Appendix 4), "Inequalities in housing hardship declined because everybody is now worse off," by Yung Chun and Michal Grinstein-Weiss, described the results of a year-long survey by the Social Policy Institution at Washington University in St. Louis exploring housing-related hardships experienced by Americans during the first year of the pandemic. The article contained two main claims: 1) inequality in experienced hardship, measured by eviction/foreclosure risk, rent/mortgage delinquency, and utility bill payment delay, has decreased because White, non-Hispanic respondents' hardship has increased; and 2) Black families are most likely to experience longer-lasting hardship. The article used five graphs and accompanying text to present the findings from four surveys administered between May 2020 and March 2021.

The analyses presented in this chapter have two foci: first, I examine students' writing in their final essays to characterize the ways they responded to the article and what types of arguments they made; then, I examine how their written responses demonstrated the use of statistical literacy practices described in the research literature (Gal, 2002; Gal & Geiger, 2022; Ridgway et al., 2018; Weiland, 2017). I use Toulmin's (2003) method, as described in Chapter 3, to identify and describe students' arguments. Consistent with a sociocultural framework, these analyses began with consideration of the affordances of the assignment and the Brookings Institution article as well as the context of the course itself. For example, the article contained five graphs that students could make sense of and critique and the assignment encouraged them to do just that. Throughout the course, we examined various types of graphs and practiced making sense of and critiquing them (see Chapter 4 for a detailed description of the course).

I adopt a strengths-based approach (Huitzilopochtli, et al, 2021) by emphasizing what the students did as opposed to making conjectures about what they could or could not do. While some researchers describe statistical literacy in hierarchical or developmental tiers (e.g., Watson, 1997; Watson & Callingham, 2020), I align with Gal's (2002) conceptualization of statistical literacy as existing "to different degrees within the same individual, depending on the contexts where it is invoked or applied," (Gal, 2002, p. 15). This is consistent with an ethnographic stance (Moschkovich, 2019; Moschkovich & Brenner, 2000) in which

> "mathematical activity" is not a unitary category but is manifested in different ways in different settings. … mathematical activity is not

> always immediately evident to the participants or the analyst but, instead, is uncovered during analysis. Using this perspective focuses data analysis on uncovering the mathematical structure in what participants are actually doing and saying … students' mathematical activity in the classroom is seen not as a deviant or novice version of academic/school mathematical practices but instead as a particular case of students' everyday activity, where participants use social and cognitive resources to make sense of situations. (Moschkovich, 2019, p. 70)

These theoretical considerations informed the first, very broad, research question: How did students respond in writing to a news article with statistical claims, data, and visual representations of data? What did they write about? Did they critique the statistical claims they encountered in the article? If so, how? Did students use concepts and strategies taught in class such as biases, fact-checking, knowledge of history, current events, validity, generalizability, algorithmic procedures, etc.? By starting with this broad question, I met students where they were at and allowed their choices about what was important to write about to guide my analysis.

Students' Responses to the Brookings Article

To investigate the first research question, I read through each essay and created descriptive categories that summarized the general topic of a particular section or paragraph. Though each student was responding to the same prompt and the same article, the task was open-ended enough that there was some variety in what students chose to write about. That said, the topics students chose to write about were necessarily constrained by the task and the topics discussed in the course. In this section, I describe the different ways students responded to the Brookings Institution

article in their essays. Table 5 summarizes the topics that students wrote about in their essays.

Table 5 Summary of Topics Students Wrote About in Their Essays

Topic	Percent of students (n=30)
Identifying the authors and considering potential bias	77%
Writing about the sampling method of the surveys	53%
Critiquing the authors' use of only three racial/ethnic categories	17%
Writing about the graphs	70%

Twenty-three students wrote about the authors or the Brookings Institution as a source. This was specifically listed as something they should do in the assignment description. For most students, they identified the source and the authors. About half the students (n=14) wrote more about the source. For example,[5] Candace went to both the Brookings' Institution website and a media analysis website that was recommended by the instructor to describe the source and possible bias.

> The article entitled, "Inequalities in housing hardship declined because everyone is now worse off" was published on April 30th, 2021 by Brookings Institution. **On the About Us section of the Brookings website** it claims to source from over 300 experts in government and academia. **Using mediabiasfactcheck.com I found that** the institution was rated as highly factual with a left-center bias. (Candace's essay)

[5] In all the excerpts of student writing I have added bolded text to direct the reader's attention to the most salient parts of the excerpts.

Other students researched both the Brookings Institution and the authors of the article. For example, Talia wrote:

> On the first examination **I did a quick search into the background of Brookings,** and of the authors. Brookings is a well established non profit organization that values independent, pragmatic research (Brookings, 2016). They have been around since 1916 and have advised the government during instrumental times such as World War II (Brookings, 2016). **One of the authors Yung Chun is a data analyst for the Social Policy Institution at Washington University in St.Louis**, while Michal Grinstein-Weis is a nonresident senior fellow in Global Economy and Development at Brookings. She is also a Professor and the director of the Social Policy Institution at Washington University. The fact that the authors are academic researchers at a well established university provides a lot of credibility to their work. (Talia's essay)

Researchers have argued that statistically literate people must possess a healthy skepticism for statistical claims, walking a fine line between a belief that statistics are all lies or all true. Considering the source of the statistical results is an important practice (Gal, 2002; Weiland, 2017) that supports this. For example, some students noted the lack of information about the sample and assumed a strong study design anyway. "Since this information is not readily available to the reader, I will assume for the purposes of my analysis that the survey methods were reliable, and the responses are a valid measure of the overall experience within the country." Another wrote, "the fact that the authors are academic researchers at a well-established university provides a lot of credibility to their work." These statements suggest that the students were considering the limitations of various study designs while simultaneously making assumptions based on their knowledge of and beliefs about

the credibility of the source. All of the students who wrote about the source of the article described it as reliable in some way.

Over half of the students (n=16) wrote about the surveys reported in the article and the sampling method used by the researchers. A large portion of the course, four out of ten weeks of instruction, focused on survey design and sampling methods, so students had experience critiquing sampling methods. Most students (n=14) wrote a critique about the lack of information about how the sample was chosen. For example, in this excerpt from Nikki's essay, she critiqued the article for a lack of transparency regarding the sampling method:

> **I have no idea if the sampling was a convenience sample or a simple random sampling.** If it was a convenience sample, the statistical data would be biased because it systematically favors particular outcomes. The statistics present could be valid but because it is being obscured and inaccessible. I would not use this article as the information is unsubstantiated. (Nikki's essay)

Some of the students who wrote about the survey sample focused on it not representing all American ethnicities. One-sixth (n=5) of the students wrote about the authors' inclusion of only three demographic groups in the article (Black, Hispanic, and White Americans). Most wrote briefly about this, but one student (Tina) made it a focus of her essay.

> Another very concerning aspect of the author's approach to this analysis is their sole focus on black vs. hispanic vs. white communities, as if other communities don't experience housing hardship [inequality] or even exist at all … **how has the way they've [the authors] defined their communities possibly crafted an incomplete narrative?** What about asians, pacific islanders, indigenous people, etc.? (Tina's essay)

The final topic that appeared repeatedly in the essays was students writing about the graphs in the article. Over two-thirds of the students (n=21) wrote about the graphs in some way. These were usually a main focus of the students' essays, which is unsurprising since analyzing visual representations of data was another major component of the course and students were specifically asked to write about these in the assignment description. The various ways they wrote about these graphs is described in detail later in this chapter in the section on Reading the Data.

Overall, the ways that students wrote about the Brookings Institution article reflected the concepts that we taught in the course, the questions posed in the assignment (see Appendix 3), and the features of the article. This corroborates prior research that describes how students' work is constrained by the task and its setting (a university course on statistical literacy in this case). In particular, students wrote about the source, the survey methods, and the visual representations of data. In the next section I describe the arguments students wrote in their essays.

Students' Arguments

What arguments did students write? When students made arguments, what types of arguments did they make? To answer the second research question, I created Toulmin diagrams from the students' writing (see Chapter 3 for a detailed description). Most students' essays centered around an argument that one or both claims in the article were true and supported by the survey data. One argued that the operationalization of race and ethnicity in the study which only reported on Black, White, and Latinx Americans and ignored other identities was problematic. Table 6

summarizes the arguments in the students' essays. Though the arguments did not all use the same words, the Toulmin analysis revealed that almost all the arguments were about agreeing or disagreeing with the authors' main claims. I provide three illustrations of students' arguments and how I used Toulmin to diagram them below.

Table 6 Students' Arguments in Response to the Brookings Institution Article

Argument	**Percent of students (n=30)**
Claim #1 is true and supported by the data in the article: Housing inequality has decreased because everyone is worse off.	77%
Claim #2 is true and supported by the data in the article: People of color are more likely to have experienced prolonged housing hardship.	77%
The article understates the hardships experienced by people of color.	20%

The first example of arguments in the students' essays focuses on the first claim of the article: that inequality is decreasing because of increasing hardships suffered by White Americans. Celeste argued that this claim is true, despite their initial incredulity. Their argument appeared early in their essay:

> It is difficult to understand how white people are beginning to see similar repercussions from the pandemic as POC, considering most of the time white people have the profound privilege of never having to know what these major repercussions look like. **Yet the numbers and data presented in the article makes the notion much clearer and easier to acknowledge**. (Celeste's essay)

The warrant for their claim appeared later in the essay, when they used what in the class we called the "rate of change formula" (see Chapter 4 for a description of how we taught this in the class) to compare data presented in the article (see Figure 13).

> Figure 2 displays that in November, there were reduced inequality gaps in delayed payment of rent/mortgage and utility bills "mainly due to the delayed peak of white respondents." On the scale, we can see the number of Black respondents, from May 2020 to March 2021, going from about 13 percent to 11 percent **making a decrease by about 15 percent.** Hispanic respondents in the same time stays the same at the beginning and the end with spikes in between time, and finally, we see white respondents go from about 7 percent to 9 percent **making this an about 28 percent increase**. (Celeste's essay)

This is an example of a student "reading between" (Curcio, 1981, 1987, 1989; Friel, Curcio & Bright, 2001) the graph and using arithmetic calculations (specifically the rate of change formula) to further interpret and highlight the most salient point – that the data is convincing at showing increased hardships for White Americans. The student's description of the relative change (bolded in the excerpt above) is evidence of their use of the rate of change calculation. The absolute difference between 13% and 11% is two percentage points, and two percentage points divided by the original 13% yields a 15% decrease. The absolute difference between seven and nine percent is also two percentage points, but two percentage points divided by seven percent yields a 28% increase. These calculations provide the warrant for their claim that decreasing inequality is indeed due to the dramatic increase in hardships faced by White Americans. The diagram of Celeste's argument can be seen in Figure 14.

Figure 13 Brookings Institution Figure 2. Rent/Mortgage Delinquency

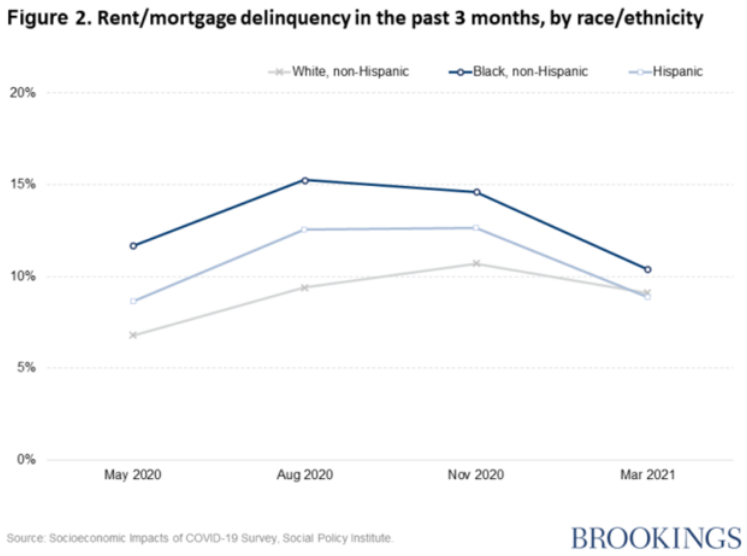
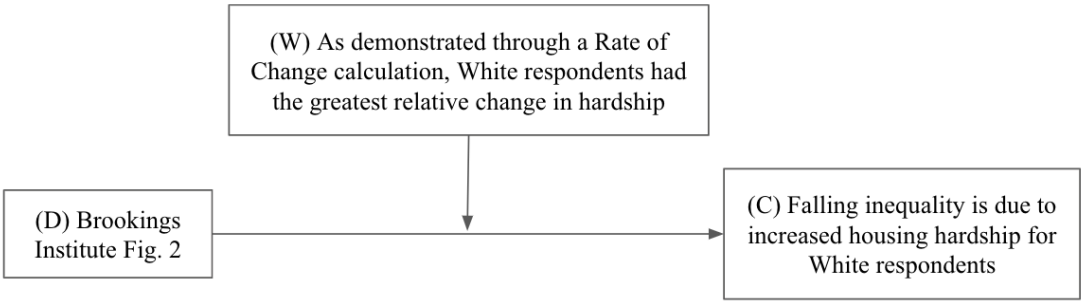


Figure 14 Toulmin Diagram of Celeste's Argument

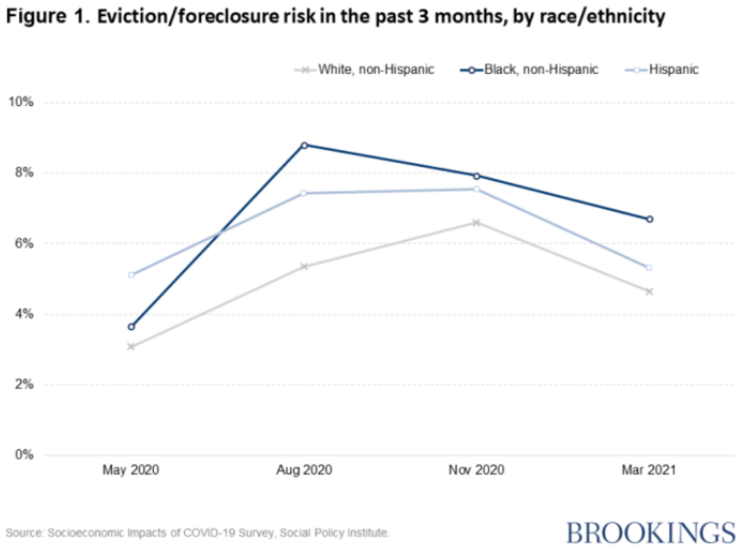


In this second example, a student interpreted the thesis of the article in an unexpected way. Deborah wrote, "The authors explain how Black and Hispanic communities have been faced with this inequality for too long." This is slightly different from my interpretation of the article's second claim that Black and Hispanic families were more likely to face housing hardships for consecutive months, yet it essentially demonstrates agreement with the authors' second claim that Black and Hispanic Americans experienced prolonged housing hardships. Throughout their

essay, Deborah wrote about the main aspects of the article (the survey, the figures on housing hardship, and the figures on long-term housing hardship) with this focus. For example, in analyzing the figures on housing hardship, they used arithmetic calculations to make sense of the figures and to support their claim. Below is an excerpt from their essay and the figure (Figure 15) they referenced.

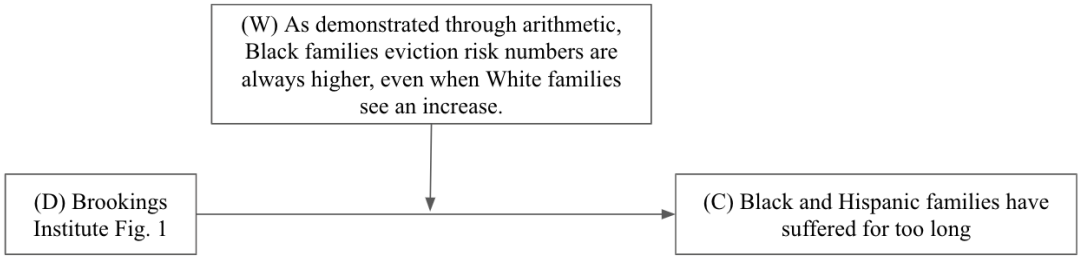
> According to figure 1, White respondents totaled 6.6% of eviction/foreclosure in November 2020, **that is an increase of 1.6 percentage points** since August of 2020 (5%). In this same time frame, Black and Hispanic respondents decreased, Black respondents **going down approximately 1 percentage point** (8%) and Hispanics **going down 0.1 percentage points** (7.4). This trend continued throughout the rent/mortgage and utility delayed payment due to the sudden peak of White respondents. This data proves that minorities, especially within the Black community, are facing housing hardships more than any other, that even when their statistics improve and White respondents' statistics decline, Black respondents still suffer at higher rates. This says a lot and proves the author's point fully. (Deborah's essay)

Figure 15 Brookings Institution Figure 1. Eviction/Foreclosure Risk



Deborah's argument is diagrammed below (Figure 16). Their claim that Black and Hispanic communities have suffered "for too long" comes from the data in the figure on foreclosure/eviction risk. The warrant for this claim is the arithmetic calculations that Deborah uses to show that Black and Hispanic respondents' percentages were always higher. By "reading between" (Curcio, 1981, 1987, 1989; Friel, Curcio & Bright, 2001) the data in the graph, Deborah found the absolute change in the rates between time periods for all groups by using subtraction and then described how the values for Black Americans were always higher than the other groups (again, using absolute comparisons). These comparisons are the warrant for their claim that Black and Hispanic families suffer more housing hardships than White families.

Figure 16 Toulmin Model of Deborah's Argument



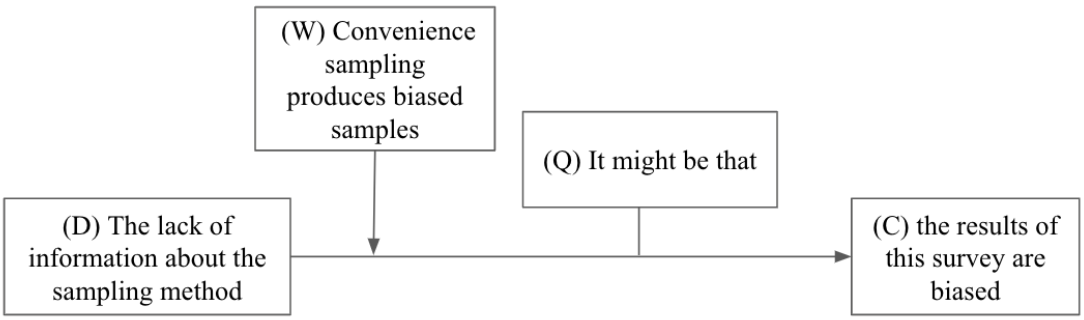
The third example of a student's argument focuses on the sampling method used for the surveys. Nikki's argument is that the survey data might not be valid because there is not enough information on the sampling method.

> I have no idea if the sampling was a convenience sample or a simple random sampling. If it was a convenience sample, the statistical data would be biased because it systematically favors particular outcomes. **The statistics present could be valid but because it is being obscured and inaccessible, I would not use this article as the information is unsubstantiated** … all samples should be able to represent the population. … In order to determine if this article is

> representative of the White, non-Hispanic or Black, non-Hispanic folks' housing struggles during COVID in the United States, we must know what sampling method was used for this survey. (Nikki's essay)

Nikki's argument is diagrammed below (Figure 17). Her argument contained a claim with a qualifier: "The statistics present *could be* valid," (my emphasis). Nikki's explanation that convenience sampling would lead to biased outcomes is the warrant for their argument.

Figure 17 Toulmin Model of Nikki's Argument



Overall, the students wrote arguments about the validity or persuasiveness of the claims in the article. The overwhelming majority of the students wrote arguments that agreed with the claims presented in the article. This could be because they found the source to be reliable, so they described why its claims were correct. It could be because they agreed with the article, or perhaps they thought that I, as the assignment assessor, would agree with the article and so they tried to write "to" their audience. It could be because university students have a norm around researching a topic that is biased towards agreeing with the source rather than being critical of it. (Watson and Moritz, 1997).

Students' Use of Statistical Literacy Practices

To answer the third research question (What statistical literacy practices did students use?) I used iterative analytical memos to group students' writing according to the inferred goal, the mathematical concepts they used, and the strategies they employed. To clarify how these goals, concepts, and strategies coalesced to be a practice, I created idealized models of the two statistical literacy practices that emerged through analysis (see Figures 8 and 9). I documented students using two statistical literacy practices in writing their responses to the Brookings Institution article. These practices were Reading the Data and Critiquing Data Quality. Table 7 summarizes the students' use of these practices.

Table 7 Students' Use of Statistical Literacy Practices

Statistical Literacy Practices (Strategies and Concepts Used)	**Percent of Students (n=30)**
Reading the Data	**70%**
Describing data in graphs	23%
Using equivalence	47%
Critiquing visual manipulations of graphs	27%
Critiquing the type of graph used	7%
Comparing the data	23%
Critiquing Data Quality	**57%**
Critiquing the sampling method	40%
Critiquing the use of only three racial/ethnic groups	17%

Almost three-quarters of the students used the practice Reading the Data by describing the graphs, using equivalence to describe the data, comparing the data in the graphs, critiquing visual manipulations, and/or critiquing the type of graph used. Over half of the students used the statistical literacy practice Critiquing Data Quality by critiquing the lack of information on the sampling method or critiquing the use of only three racial/ethnic categories in the article's analysis. In the sections that follow, I illustrate the varied ways students used these practices to write arguments in response to the article.

Statistical Literacy Practice #1: Reading the Data

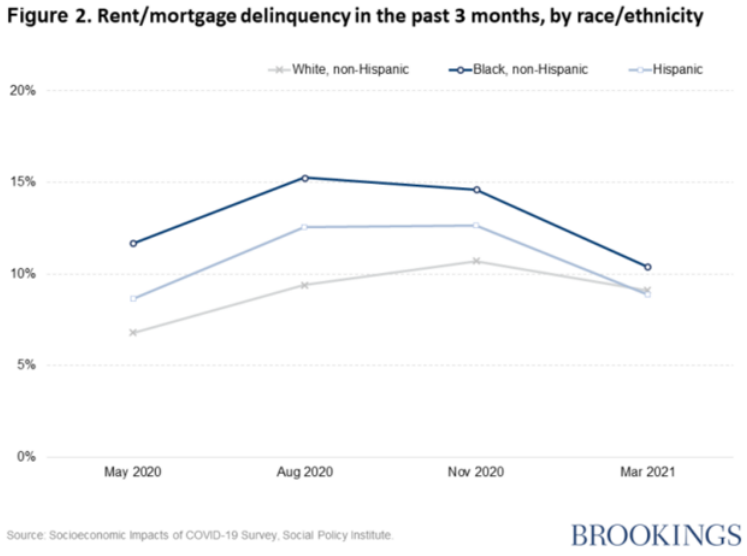
Students used the statistical literacy practice Reading the Data for goals such as: to make sense of the visual representations of data in the article, to critique these visual representations, and to compare the data within the graphs. Nearly three-quarters of the students (21 out of 30) used this practice. The article focused on claims supported by the evidence in five graphs, and the students' assignment explicitly asked that they write about the figures. As a reminder, the article contained three line graphs and two bar graphs displaying rates of housing hardship over time for three demographic groups. These graphs provided opportunities for students to read, interpret, and critique the authors' presentation of data. Throughout the course, we had explicitly taught students how to read graphs and some common ways that graphs can be manipulated to convey a particular message (e.g., truncated axes, stretching or shrinking intervals). Overall, students used the statistical literacy practice Reading the Data by describing the data, using the concept of equivalence to

make sense of percentages, comparing values, and critiquing both the choice of figure and manipulation of an axis.

Nearly a quarter of the students described the graphs and the data represented in the graphs. For example, Deborah, who was convinced by the authors' arguments, wrote many sentences like the one below (Figure 18 shows the line graph they referenced).

> In Figure 2, rent/mortgage delinquency, **the line graph shows that in August of 2020, 15% Black respondents were affected by this, as well as 13% of Hispanics and 9% of White respondents**. (Deborah's essay)

Figure 18 Brookings Institution Figure 2. Rent/Mortgage Delinquency



This student was engaging in the practice of Reading the Data by reading data from the graph and translating from the visual representation to numbers and words. They were reading the graph (Curcio, 1981, 1987, 1989; Friel, Curcio & Bright, 2001).

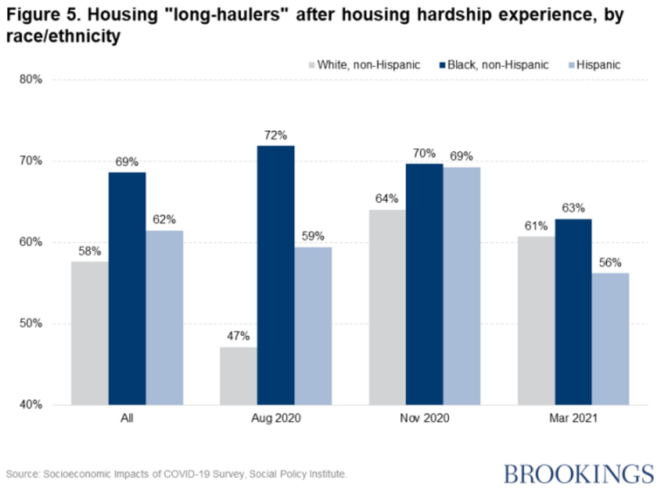
This is significant considering many students reported in my pilot study reported that they usually skipped visual representations and relied entirely on the text.

Half of students both described the visual representations of data and wrote some sort of commentary alongside these descriptions. In these sections of commentary, there was evidence of students using the practice of Reading the Data beyond reading the graph and extracting data.

For example, some students used the concept of equivalence and the strategy of estimating to reframe the data in the graphs to make sense of it. In the following excerpt, Jazmine wrote about the first set of bars in the following bar graph (Figure 19).

> The article also reports that 61% of respondents experienced long-haul (over the course of three months) housing hardship during the pandemic, **amounting to 200,080,000 people**; 69% of Black respondents, 62% of Latinos, and 58% of Whites experienced long-hauling. **This means that almost two-thirds of the country, at some point during the pandemic, felt that their housing situation was in danger**. (Jazmine's essay)

Figure 19 Brookings Institution Figure 5. Housing "Long-Haulers" by Race/Ethnicity



In this excerpt, Jazmine compared the reported percentages to the population of the United States and converted the data from percentages (as displayed on the graph) to fractions (two-thirds). This is evidence that they were engaging in the practice of Reading the Data by connecting the displayed data to both counts of people and a benchmark fraction. Jazmine demonstrated how to use two representations of the same quantity to make sense of data in a graph. Jazmine converted the 61% figure reported in the article to 200,080,000 Americans by finding 61% of the total U.S. population (approximately 320,000,000 people). They also wrote that this quantity is "almost two-thirds of the country." Reframing reported quantities to simplify them or to make them more legible or meaningful and translating between counts and rates were both ideas that were emphasized in the course (see Chapter 4 for a detailed description). Jasmine's use of estimating and equivalence here is an example of how a student reframed data to emphasize and communicate information in graphs.

Students also used proportional reasoning to Read the Data. Faith, for example, did not accept the authors' claims. In their essay, Faith described the data in the Eviction/foreclosure risk figure (Figure 20) and went on to calculate relative changes in risk, (a calculation we had taught the students to do in class), to make sense of the data presented in the figure and to critique the authors' claim.

Figure 20 Brookings Institution Figure 1: Eviction/Foreclosure Risk

Faith began by summarizing the authors' claim that, between August and November, eviction risks decreased for Black and Hispanic respondents and increased for White respondents. They then compared the changing risks for the other months displayed and compared the relative change from one survey period to the next for different groups.

> … inspecting Figure 1, we see that this is the only period with such a trend. … in the most recent period from November 2020 to March 2021, Black, non-Hispanic eviction/foreclosure risk decreased from approx. 7.9% to 6.8%, while white eviction/foreclosure risk decreased from approx. 6.6% to approx. 4.6%. **So white eviction/foreclosure risk decreased by approx. (6.6 - 4.8)/6.6 % or roughly 30%, while Black non-Hispanic eviction/foreclosure risk decreased only by approx. (7.9-6.8)/7.9 % or approx 15%**. So in the most recent period **the percentage wise decrease was twice as big for white respondents as for Black non-Hispanic respondents**. (Faith's essay)

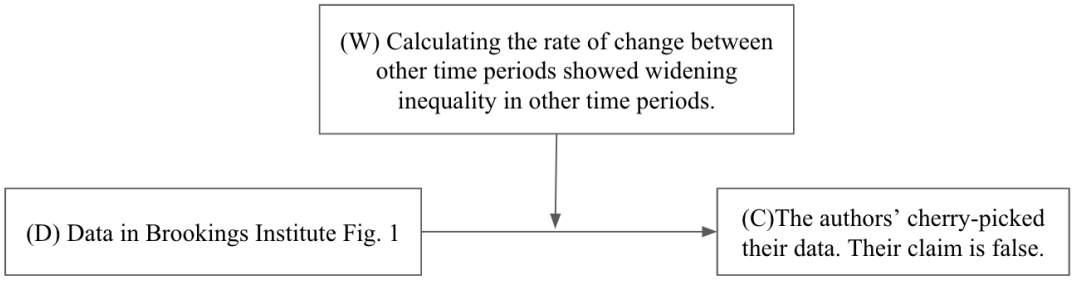
This example illustrates how a student effectively calculated the relative change in eviction risk over time to make sense of data in a visual representation. The

arithmetic calculations that Faith used are evidence that they read and interpreted the data presented in the graph beyond what was written in the article. They read the graph to extract the percentages and then used the rate of change formula taught in class to compare these rates across time periods and between demographic groups. This is evidence of Reading the Data because they wrote about their interpretations of the data presented in the graph. These interpretations served as the warrant for their claim (see Figure 21) that the authors "cherry-picked their data to support their claim." This is further described in the next section of their essay in which Faith continued to investigate the data in this figure, finding the relative change in eviction risk over the entire pandemic as well as the most recent period reported. They then asserted their claim that the authors' conclusion was incorrect.

> So, in fact, **during the entire period, there was both absolutely and percentage-wise a much larger increase in eviction/foreclosure risk for Black, non-Hispanic than for white people. This shows that one of the major conclusions of the article is not correct.** The data do not support a conclusion that housing hardship inequalities have decreased. …**[the authors] have effectively cherry-picked a particular period** that supports the thesis that inequalities have decreased due to increased white housing insecurity, even though this trend is not supported by the numbers. (Faith's essay)

In the preceding excerpts, Faith engaged in the practice of Reading the Data by reading graphs and investigating relationships between data points beyond what was presented in the text of the article.

Figure 21 Toulmin Diagram of Faith's Argument



Almost every student in the study engaged in the practice of Reading the Data in some way. Some did so by using strategies such as reporting the data and describing the visuals, while others used mathematical concepts such as estimation, equivalence, and proportional reasoning (via the "rate of change" calculation) to make sense of the figures in the article, essentially "reading between" the data. In the following sections, I highlight two goals beyond making sense of the data that were evident in students' writing: using Reading the Data to critique visual representations and to compare data.

Critiquing Visual Representations. One goal of Reading the Data is to critique how a visual representation was manipulated. During the class, we had presented examples of manipulating graphs through truncated y-axes, and four students wrote about this type of manipulation in response to the bar graphs in the article (Figures 22 and 23). Those bar graphs are not egregious examples of truncated y-axes; the authors likely chose to display the data this way to save space and improve readability. Nonetheless, truncated y-axes can impact one's understanding of the data displayed and some students' writing focused on those axes.

Figure 22 Brookings Institution Figure 4. Housing "Long-Haulers"

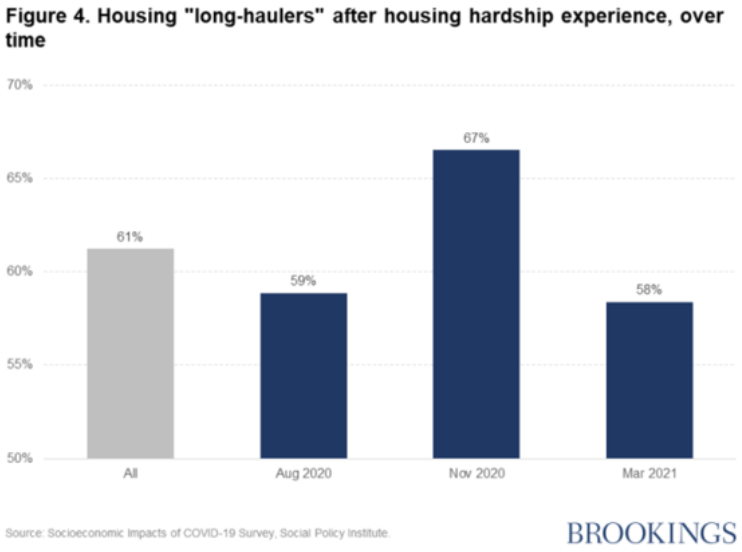
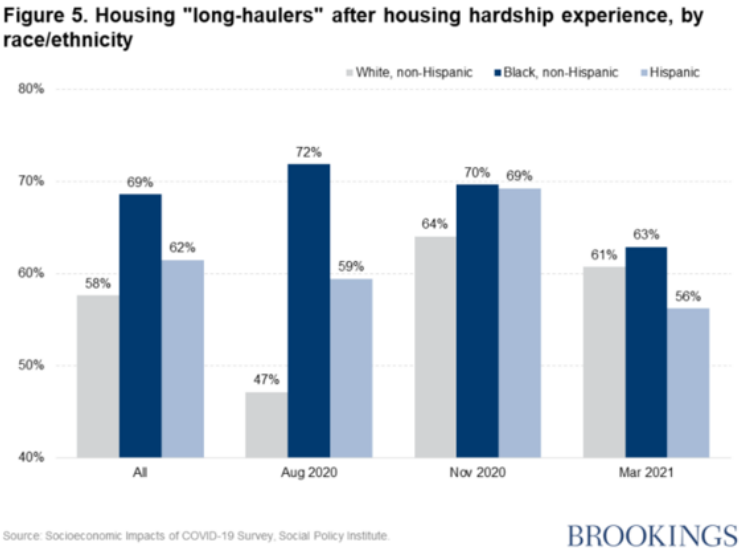


Figure 23 Brookings Institution Figure 5. Housing "Long-Haulers" by Race/Ethnicity



Essentially, the shortened y-axes create an exaggerated impression of difference between the various bars. For example, in Figure 22 "Brookings Institution Figure 4. Housing 'long-haulers' after housing hardship experience, over time," the November

2020 bar appears to be twice as tall as the August 2020 bar though the actual relative increase in "long-haulers" from August to November was closer to 14%. In Figure 23 "Brookings Institution Figure 5. Housing 'long-haulers' by race/ethnicity," the exaggerated difference between the White, Non-Hispanic and Black, Non-Hispanic bars in August 2020 makes the closing gap (in November and March) more dramatic.

Talia's essay was an example of a student using a claim about the visual representation as the warrant for their claim that the article was misleading.

> One thing I find problematic with the Figure 4 and 5 bar graphs are the fact that **they have a truncated y-axis…Doing this is misleading and skews the data, by making the bar graphs on the chart appear larger than they may actually be**. (Talia's essay)

Talia did not go on to explain their rationale further, and it is unclear exactly what they meant by "larger than they may actually be." Nonetheless, Talia's essay shows a student making sense of bar graphs by using concepts about graphs to critique a truncated axis.

Nikki, who largely agreed with the authors' claims, wrote about the same feature of the visual representation, but did so with more explanation.

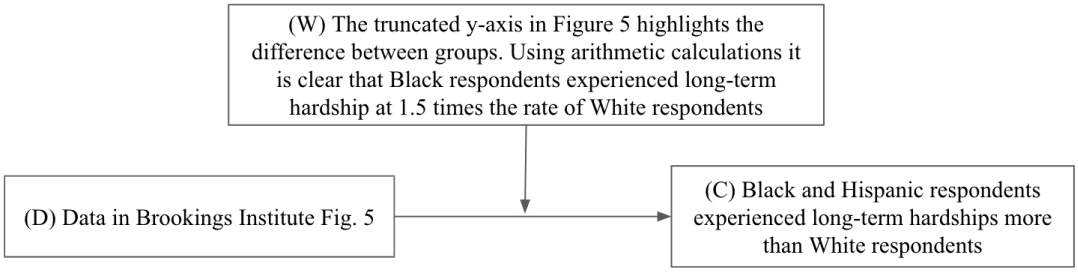
> Figure 5 also starts at 40%; in August of 2020 on the graph, 47% of White non Hispanics housing long haulers experienced hardship, while 72% who experienced hardship were Black, non-Hispanic. **If the graph started at 0% and not 40%, it would still portray a huge gap (47% vs. 72%) but not as gigantic as it is presented currently.** This may not be a bad thing, as sometimes graphs are constructed in a way that highlights problems authors are emphasizing to get across to readers. (Nikki's essay)

This excerpt is an example of one student critiquing a visual representation and providing a warrant for that critique. Nikki's argument (see Figure 24) focused on the spaces between the bars, exactly the aspect of the figure that is manipulated with a truncated y-axis. Her analysis that this manipulation may serve to "highlight problems authors are emphasizing," is the warrant for her claim that the article is correct that Black and Hispanic respondents experienced long-term housing hardships at a greater rate than White respondents. This is evidence of the statistical concept transnumeration (Wild & Pfannkuch, 1999) because Nikki is describing how a particular representation of data communicates a particular story. She provided a further warrant in their next sentence where she used an arithmetic calculation to make sense of the authors' emphasized point.

> The description for Figure 5 in the article states " In August, **Black respondents were 1.5 times more likely than white respondents** to keep experiencing housing hardships for two consecutive survey periods" **which reflects the 47% for Whites, non hispanic vs. 72% Blacks, non hispanic**. (Nikki's essay)

Nikki checked the authors' claim of "1.5 times more likely" against the data provided in the graph. While she did not describe her calculations, her use of the word "reflects" suggests an equivalency: 1.5 times more is the same as 72% compared to 47%.

Figure 24 Toulmin Diagram of Nikki's Argument



This example shows a student engaging in the practice of Reading the Data by using the concept transnumeration to describe the truncated y-axis and consider how it might have been used intentionally by the authors to emphasize their point. This example shows evidence of Nikki reading between the data because she compared data points and reading beyond the data because she was writing about the authors' purpose in presenting the data in a particular way.

Two students, who both wrote essays in which they ultimately agreed with the authors' claims, noted that the use of a bar graph for Brookings Institutions' Figures 4 and 5 was not ideal. Stella's critique was straightforward and brief.

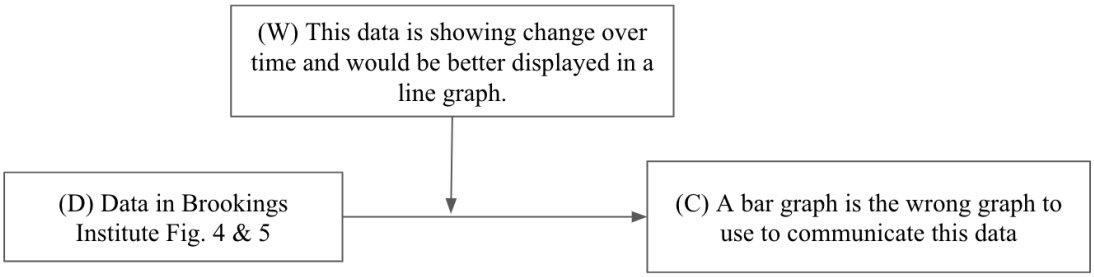
> The choice of figure…is confusing, considering that **figure 4 demonstrates data over time, it seems it would be easier to read this data as a line graph rather than a bar graph**. (Stella's essay)

The other student, Ana, elaborated on why they found this figure problematic.

> (Ana) Just by looking at the graph, it shows a category for "All" on the x-axis that is supposed to show the data over time. It doesn't seem right on the graph since it's supposed to show change over time. **A line graph would've better suited this "change over time" information** for that is what they're used for. Bar graphs are better for categorical data, which the "All" fits within.

In these excerpts, both Ana and Stella engaged in the practice of Reading the Data by using their knowledge of graphs and transnumeration to critique the visual representations in the article as not being the best representation of the data. Ana's explanation illustrates how a student elaborated on their reasoning to create a warrant for their claim that a bar graph was the wrong type of graph to use to display this data (see Figure 25).

Figure 25 Toulmin Diagram of Ana's and Stella's Arguments



Overall, these examples show the various ways students engaged in the statistical literacy practice of Reading the Data to critique visual representations of data by describing the data, reframing the data to make sense of percentages, using arithmetic calculations to create warrants for their claims, and using transnumeration (Wild & Pfannkuch, 1999) to critique either the choice of representation or the manipulation of an axis.

Comparing the Data. Another goal of Reading the Data is to compare reported data within an article or to compare reported data to data from another source. Students used the strategy of performing arithmetic calculations in a variety of ways (adding, subtracting, converting between fractions, decimals, and

percentages, and calculating relative change) and to compare data reported in the article.

For example, Alex calculated the relative change in the percentage of respondents experiencing eviction and foreclosure risk (see Figure 27 for calculations) as a warrant for their critical argument that the article obscured the plight of people of color in the United States (the Toulmin diagram of their argument is in Figure 28).

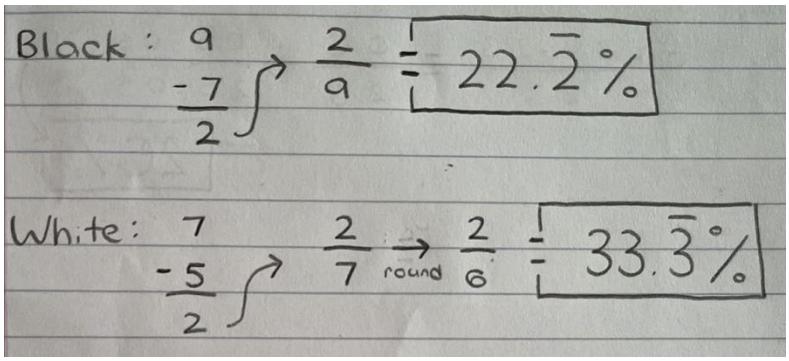
> I input the peaks of the Figure 1 (eviction / foreclosure risk) numbers and the March 2021 numbers into the percent change equation, doing so for the Black and white populations. **As demonstrated in my calculations, while the white population experienced a 33.3% change from their peak eviction / foreclosure risk to their most recent number, the Black population only experienced a 22.2% change.** By doing simple math, it can be found that while the white population has been "worse off" due to the pandemic, their situation still improved faster than their counterparts. As such, the claim that "Inequalities in housing hardship declined because everybody is now worse off" holds up, but still obscures a hard truth: that between these populations, one will always have it worse. (Alex's essay)

Figure 26 Brookings Institution Figure 1. Eviction/Foreclosure Risk

What this student called the "percent change equation" is what we called "the rate of change formula" in the class when teaching about the differences between absolute and relative change.[6] Their work is shown in Figure 27. Essentially, they estimated the peak eviction rate (see Figure 26) for black and white respondents (9% and 7% respectively) and compared those to the most recent percentages reported. The student's rounding of 2/7 to 2/6 also reflects something taught in the class. We encouraged students to estimate quantities by rounding to "friendly numbers" – nearby benchmark fractions, powers of ten, and numbers that share common factors.

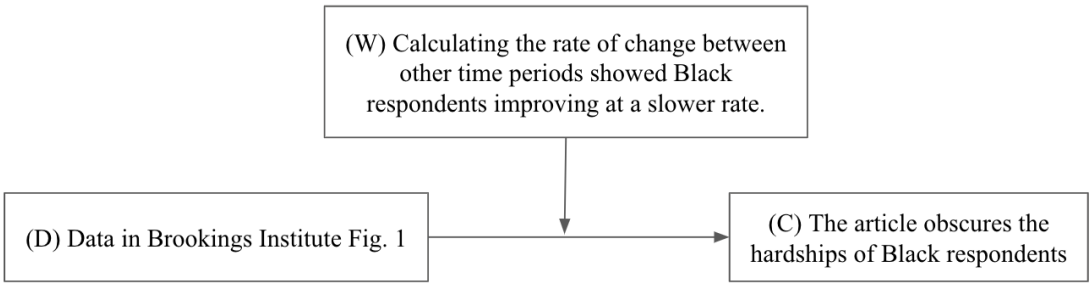
[6] See Chapter 4 for a detailed description.

Figure 27 Alex's Calculations of the Relative Change in Eviction Risk for Black and White Respondents



These calculations, which involved first extracting the data in the line graph, provided the warrant (see Figure 28) to Alex's argument that the article overemphasized the hardships of White respondents while obscuring the continued plight of Black Americans. Alex used the practice Reading the Data for the purpose of comparing data by using arithmetic strategies and the concept of relative and absolute thinking.

Figure 28 Toulmin Diagram of Alex's Argument



Of the seven students who calculated relative changes in housing hardship indicators, all of them did so to highlight the disparities of Black and Hispanic respondents, and six of them used these calculations as warrants in their arguments that the article was "misleading" or overstated the hardships of White Americans.

Students used the statistical literacy practice Reading the Data for goals such as: to make sense of the visual representations of data in the article, to critique these visual representations, and to compare the data within the graphs. Their use of this practice appeared most commonly in the warrants of their arguments. The students used concepts about graphs, proportional reasoning, and percents, and they used strategies such as estimating and performing arithmetic procedures such as the rate of change calculation.

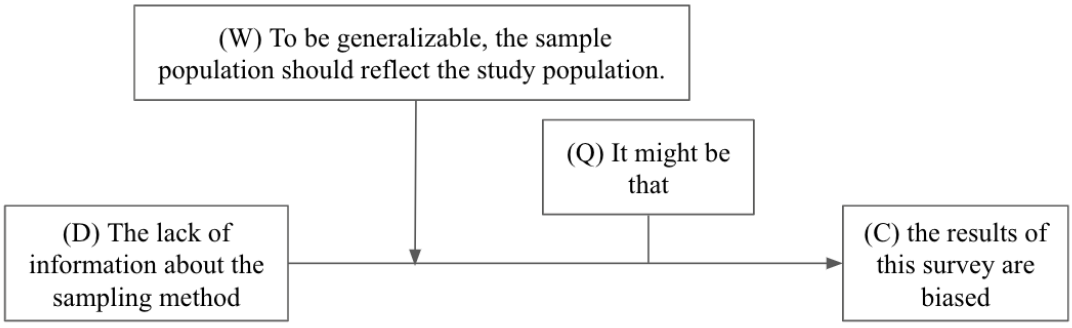
Statistical Literacy Practice #2: Critiquing Data Quality

The second statistical literacy practice that students used in their essays was Critiquing Data Quality. This is different from Reading the Data because it is not about making sense of and critiquing how the data is presented but instead about assessing the trustworthiness, reliability, and/or generalizability of the data itself. Over half of the students wrote about the quality of the data that the article cites. These students engaged in the statistical literacy practice Critiquing Data Quality by writing arguments about the representativeness of the sample. These arguments were supported by warrants about sampling techniques, representativeness, and the social institutions that shape, and are shaped by, data. Regardless of whether they ultimately argued that the data was sufficiently reliable, below I show several examples of how they all engaged in the statistical literacy practice of Critiquing Data Quality.

The article does not say much about the survey; one must click on a hyperlink to read a previous article that describes the survey as being distributed to "nationally representative" samples of over 5000 people. Some students described clicking on

this link, but only one mentioned the claim that the sample represented the U.S. population. About a third of students wrote about the lack of information in the article about the survey and sampling method. These arguments about the lack of information are summarized in Figure 29. These students argued that the results of the survey were or might be biased. The data for this claim was the lack of information in the article regarding the sampling method and the various warrants were all versions of a statistical concept that was taught in class: in order to be generalizable, sample populations must be representative of the study population.

Figure 29 Toulmin Diagram of the Common Argument Regarding Information About the Sample



About half the students used their analysis of the survey's validity to argue that the article was an unreliable source of information. For example, Nikki's essay generally argued that the authors' description of the sample was not convincing.

> After reading how this survey was conducted, I have no idea if the sampling was a convenience sample or a simple random sampling. If it was a convenience sample, the statistical data would be biased because it systematically favors particular outcomes. The statistics present could be valid but because it is being obscured and inaccessible, I would not use this article as the information is unsubstantiated … All samples represent the population. … **In order to determine if this article is representative of the White, non-Hispanic or Black, non-**

> **Hispanic folks' housing struggles during COVID in the United States, we must know what sampling method was used for this survey**. (Nikki's essay).

In this excerpt, Nikki engaged in the statistical literacy practice Critiquing Data Quality by questioning the representativeness of the sample. She was using concepts we discussed in class – particularly various methods of sample selection to critique the quality of the data.

When the students' argument was to agree with the authors' claims, about a third of the students described their critique of the data quality as an "area for improvement," or their "one concern." For example, Jazmine spent considerable time using arithmetic calculations to Read the Data to make sense of the percentages reported in the article by comparing these to U.S. population demographics. At the very end of their essay, they Critiqued the Data Quality by writing briefly about the survey and sampling method,

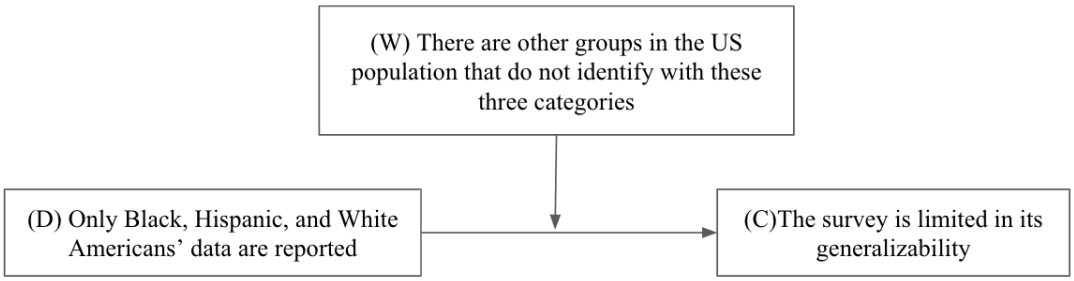
> The data fits the reporter's story very well … **There may have been sampling bias**, depending on how the surveys were conducted, rendering my personal calculations based on the U.S. population inaccurate; **perhaps more information on the surveying process would boost this data's reliability**. (Jazmine's essay)

This argument and the ten others that were similar are evidence that students engaged in the statistical literacy practice Critiquing Data Quality by using the concept of statistical sampling to assess the quality of the survey results. In these cases, the students did not assume the researchers used valid sampling methods, they wanted to see the data and sampling method themselves. This suggests that while students generally wrote arguments agreeing with the authors' claims, they did not do so

uncritically (Watson & Moritz, 1997). For some students, the strong reputations of the authors and the source led them to assume the data was of sufficient quality. Two students' essays referenced a version of the same argument diagrammed in Figure 29 referring to the lack of information about the survey sample, but these two students went on to write that "the credibility of both Washington University and Brookings Institution" meant they ultimately accepted the researchers' methods for producing a valid survey of a representative sample.

Five students critiqued the data quality by writing about representativeness in other ways than making an argument about the sampling methods. These students argued with the use of only three categories (Black, Hispanic, and White) to describe race and ethnicity in the US. Of these five, four wrote that they found the authors' claims convincing insofar as they applied to the groups described, but that one cannot extrapolate to the U.S. population as a whole. In general, their arguments fit the pattern diagrammed in Figure 30. They claimed that the survey was limited in its generalizability. The warrant for this claim was that there are other groups in the United States population that do not identify with these three categories. Because people who identify with other racial and ethnic were not included, the Brookings Institution article authors' claim that "everyone" is worse off cannot be entirely accurate.

Figure 30 Toulmin Diagram of the General Argument Regarding Race/Ethnicity in the Article



A typical example of this critique is Tami's essay, which largely argued that the authors' claims were well-supported by the data, they wrote:

> Something notably missing from this article and survey is data surrounding the housing hardships of races/ethnicities besides White, Black, and Hispanic. **Though the survey claims to have gathered about 5,000 nationally representative samples, the survey … does not take into account other groups like AAPI and Native Americans** (Chun et al., 2020). (Tami's essay)

The fifth student who critiqued data quality using this aspect of data quality made it a major focus of their overall argument that the data presented is unreliable. Tina wrote, "it can't be reliable because it obscures the story of this issue by excluding entire ethnic communities." For this student, the exclusion of these communities represented a confounding factor in the authors' overall argument. In fact, Tina went on to argue that by limiting their analysis to these three ethnic communities, the authors were contributing to racist narratives in U.S. society. They wrote:

> With my background in Critical Race and Ethnic Studies, I would go as far as to additionally say that **this focus on black and hispanic populations compared to whites perpetuates the black-white racial binary, alienation/illegality of hispanics, and even the model minority myth** (by not including Asians in the narrative about housing inequity, it makes it seem as if they don't have any issues and furthers this racist notion of Asians being "the good minority")." (Tina's essay)

This student was not only Critiquing the Data Quality by considering the generalizability and reliability of the sample, she was also "identifying and interrogating social structures which shape and are reinforced by the data-based arguments being considered," (Weiland, 2017, p.41). She used a strategy of considering underlying social systems that inform and are informed by the data; this strategy was taught in class with two guiding questions (What's highlighted? What's obscured?) that students had the opportunity to practice throughout the term in whole-class discussions, group work, and on homework assignments.

These analyses illustrate students using the statistical literacy practice of Critiquing the Data Quality. Students' use of this practice was evident in the arguments they made about the representativeness of the sample. They supported these arguments with warrants using concepts about sampling and representativeness and using strategies for thinking about the social institutions that shape, and are shaped by, data. Regardless of whether students ultimately argued that the data was sufficiently valid, these students were all engaging in the statistical literacy practice Critiquing the Data Quality.

Discussion

This chapter explored three research questions through the analysis of students' written responses to a statistical literacy task.

1. How did students respond in writing to a news article with statistical claims, data, and visual representations of data?

2. What arguments did students write? When students made arguments, what types of arguments did they make?
3. What statistical literacy practices did students use?

The analysis related to Research Questions #1 and #2 revealed two findings about the students' writing. First, students overwhelmingly wrote arguments that supported the claims of the article, and very few wrote arguments opposing the article's claims. Second, these arguments were constrained by both the article and the concepts we had taught in the course. These two findings are not surprising if we assume the socially-situated nature of thinking, learning, and argumentation. The ways that students responded to the task were, of course, constrained by the task and the article, as well as by the concepts, strategies, and practices that were taught in the course. There are norms around university coursework; students are expected to demonstrate what they've learned and use the strategies, concepts, and practices taught in a course. The same assignment given to students in a completely different course on statistical literacy might yield some of the same results (e.g., students would write about the graphs because they are present in the article, they would use concepts about sampling).

That students overwhelmingly wrote arguments that agreed with the authors' claims might also be a result of the socially-situated nature of thinking, learning, and argumentation. "Learning, thinking, and knowing are relations among people engaged *in, with, and arising from the socially and culturally structured world,*" (Lave, 1991, p. 67, author's emphasis). For example, the students may have agreed with the claims

because they believed that the Brookings Institution is a trustworthy source, yet the notion of trustworthiness was constructed both in this course and in other everyday and academic settings. This corroborates prior research; most of students' experiences with data-based research is not focused on critiquing (Watson & Moritz, 1997) but rather on interpreting and communicating claims that are presumed to be correct. Students may not feel they have the agency or knowledge to critique "reputable" sources. The Brookings Institution article was selected by the instructor and took a stance that was oriented towards equity; the students may have written that they agreed with the article because of their assumptions about expectations in the course. After all, the course is called Numbers and Social Justice, and all instruction adopted an equity-oriented point of view. Students may have felt they had to agree with the claims in the article.

These findings serve as a reminder for both teachers and researchers: what students show us they can do is not the same as what they are actually capable of. The absence of arguments that disagreed with the authors' claims cannot be construed as an inability on the part of students to write such an argument. Rather, it should be seen as a byproduct of both the task and the setting as well as the socio-culturally constructed practices in this classroom. If we want to see how students would write a disagreeing argument, we would need to explicitly ask them to do so.

The third research question sought to describe statistical literacy practices in use. Because practices are, by definition, used in service of a goal, I needed to infer the purpose behind the students' use of strategies and concepts from the course. The

use of the Toulmin (2003) method enabled me to infer students' goals based on the claims they made. Through an iterative process I defined two statistical literacy practices, Reading the Data and Critiquing the Data Quality, used by students and articulated a framework for statistical literacy in action. This framework proposes three elements of statistical literacy practices: a goal, the strategies used to achieve a goal, and the relevant mathematical and statistical concepts. However, the two particular statistical literacy practices that I described in this chapter are not a comprehensive list. They are the practices that the students in this study used in response to this task and at the end of this particular course.

The first statistical literacy practice I documented is Reading the Data. Nearly three-quarters of the study participants used this practice, in contrast to my 2020 pilot study data that found most students reported skipping graphs and relying on the prose to describe the important information. Curcio's (1981) framework of reading, reading between, and reading beyond the data in the graph is still valuable after over 40 years because Reading the Data entails reading, reading between, and reading beyond what is presented. The research literature has a variety of names for this practice (e.g., quantitative reasoning, numeracy, the "quantitative core" (Gal, Nicholson, & Ridgway; 2022)), yet Curcio's (1981) language most accurately describes what students did when they crafted arguments in response to reported data. The students in this study read, read between and read beyond the reported data. For example, when students calculated the relative change between two time periods, they were reading between the data. When they did so they used important mathematical

concepts like proportional reasoning and absolute and relative quantities, concepts that are essential for civic statistical literacy (Gal, 2002).

When students “Critiqued the Data Quality" they attended to “different types of data collection and measurement errors, technical problems, or other methodological issues that affect the credibility or validity of the data,” (Gal & Geiger, 2022, p. 15). This has been described as critical thinking or having a positive disposition towards fact checking (e.g., Gal, 2002; Gal, Nicholson, & Ridgway, 2022; Ridgway et al., 2018). Yet these descriptions do not adequately describe the practice documented in this study. Many of the students showed a “positive disposition” (Ridgway et al., 2018) to fact-checking by following hyperlinks to learn about the survey methodology. Yet all but two of the students failed to note that the sample was described as “nationally representative” in the linked documentation. This phrase is often used in descriptions of the sample populations of polls and surveys, and it means that the sample was selected using best practices such as random sampling and is sufficiently large. That so many students presumably read this description of the survey sample and continued to critique the study for being unclear about its sampling method suggests that students either do not have an understanding of this phrase or they did not believe it. The act of following the hyperlink is notable in that it shows the positive disposition that Gal and others describe, but fact-checking doesn’t necessarily help students to Critique the Data Quality. Students must decide what to do with the information they find on the linked page or the fact-checking website. Therefore, I view this not so much as a disposition but as a socially

constructed practice. A positive disposition suggests a willingness to follow links and check facts, but it doesn't reflect how students will think about the information they find. A practices perspective emphasizes the goals of fact checking (to assess the quality of the data and claims) and the strategies and concepts one employs in the use of the practice.

Students used the statistical literacy practices in a variety of ways by making use of different concepts and strategies to fulfill various goals. For example, two students who both used the practice Critiquing the Data Quality did so to craft different arguments (e.g., questioning the sampling method and critiquing the exclusion of certain ethnic groups from the reported results). The three elements of statistical literacy practices defined in this dissertation (goals, strategies, and concepts) are sometimes specific to the particular practice and sometimes more general. For example, the strategy investigating the source and fact-checking information is specific to the statistical literacy practice Critiquing the Data Quality while using arithmetic calculations to read and read between data could be a useful strategy for either practice. Given a different social setting and a different task, one might see these practices used with different strategies, concepts, and goals to those described in this chapter. In the next chapter, I test this framework of statistical literacy practices using students' essays that were written in response to different, self-selected news articles. This second analysis contributes to the trustworthiness of the description of the practices in this chapter because it describes how students used the same practices in a slightly different assignment.

Chapter 6: Analysis of Four Focal Students' Essays and Revisions

In the previous chapter I documented two statistical literacy practices, Reading the Data and Critiquing the Data Quality, that students used in response to an assigned article. In this chapter, I document how four focal students used these two practices in essays they wrote in response to self-selected articles. These "Numbers in the News" essays were assigned as a "rough draft" (Essay #1 in week 2) and "final version" (Essay #2 in week 7) at the beginning and middle of the course. I analyzed students' writing using Toulmin's method to identify and describe their arguments and then documented the ways they used the statistical literacy practices by identifying concepts, strategies, and goals in their writing. I then describe how students' arguments changed with the opportunity for feedback and revision between Essay #1 and Essay #2. These analyses address Research Question 3: What statistical literacy practices did students use? and RQ3a: Did these practices change over time and with feedback and the opportunity to revise for four focal students?

In my role as the TA, I noticed several students struggled with Essay #1. These essays were due after only two weeks of instruction, and students were still grappling with what it meant to make sense of reported data and data-based claims and how exactly to respond to the prompts in the assignment. During weeks 3, 4, 5, and 6, they had many opportunities to practice making sense of reported data and to learn statistical literacy practices and various concepts and strategies in class[7]. As the TA, I provided written feedback to Essay #1. Through analysis, I discovered that my

[7]See Chapter 4 for a description of the course.

feedback focused primarily on directing students to consider their goal in using statistical literacy practices (e.g., Why compare these data points?) or encouraging them to use the strategies we taught in class to make sense of the data in the articles. As described in Chapter Four, historically, many of the students at the start of the course admitted to "skipping the numbers" when reading articles that contained data. My comments were meant to encourage the students to not only read and report the numbers, but to "read between" them (a la Curcio, 1981, 1987, 1989; Friel, Curcio & Bright, 2001) by comparing values and to "read beyond" them by questioning their source, what they highlight, and what they obscure.

The analysis of the students' revised essays (Essay #2) described in this chapter suggests that this feedback and the opportunity for revision may have provided opportunities for students to revisit or expand upon their initial attempts to use statistical literacy practices to make arguments in response to the data, visual representations of data, and data-based claims in news articles. This chapter is organized into six sections: in the first section, I briefly summarize the data and method for analysis; in the following four sections, I describe the analysis for each of the four focal students' Essay #1 and Essay #2. In the final section, I summarize the findings and briefly discuss implications for research and practice.

Methodology

The analysis described in this chapter has two goals: to describe how students used the statistical literacy practices documented in Chapter Five in response to self-selected news articles, and to describe how their use of these practices changed, if at

all, with opportunities for feedback and revision. To answer these questions, I used the two essays from each of four focal students as well as my written comments in response to their first essays and ethnographic memos about class sessions and meetings with students.

Unlike the third essay, which was the focus for the analysis in Chapter 5 and in which all the students responded to the same article, for the first and second essays students were asked to find an article that was related to a social justice issue that they cared about and that also contained data, visual representations of data, and/or data-based claims. The first essay, the "rough draft," was due after only two weeks of instruction, and the "final" version, the second essay, was due after week seven.[8] Not all students used the same article for the first and second essays. Some chose a different article because their first selection did not provide enough opportunities to respond to the prompts in the assignment (see Appendix 2); for example, the article might have contained only one number (e.g., a percentage of people who replied yes to a survey question) or no quantitative information at all. In my role as TA, I wrote comments on all the students' first essays, and, in these cases, I suggested those students choose a different article. All four focal students revised their original Essay #1 using the same article.

My written comments[9] informed my analysis of the students' essays; for example, I used these comments to infer the arithmetic calculations behind some of

[8] There were ten weeks of instruction.
[9] Described in more detail in Chapter 4.

the students' writing. If a student wrote about the relative difference between two data points in Essay # 2, but did not show any calculations, I inferred that they used the "rate of change" calculation discussed in class because I had written a comment suggesting they do just that. Of the four students highlighted in this chapter, two, Rocio and Nikki, requested individual writing conferences with me in addition to my written comments on Essay #1. My notes and recollections of these meetings further informed my analysis of their writing.

The criteria for focal student selection was described in Chapter 3. Briefly, the focal students were selected based on the original study design which included pair interviews.[10] Of the six students who participated in the interviews, two students chose not to revise Essay #1 and instead wrote a new essay about a different self-selected article. These students are excluded from this analysis because Essays #1 and #2 were very different, in part because the news articles they referenced offered different opportunities to use statistical literacy practices.

I read each students' Essay #1 and the article they chose at the same time, moving back and forth between them as the writing prompted me to revisit sections of the articles. I then constructed argument diagrams using the same method I used in Chapter Five. I repeated this for Essay #2 for each student. I used these diagrams to identify the concepts and strategies students used in their warrants and the goals I could infer from their claims. I then compared each students' essays by documenting changes in their arguments. These changes provide evidence that the opportunity to

[10] These interviews are not analyzed in this dissertation. They will be analyzed in the future.

revise after receiving feedback may have helped students learn to use the statistical literacy practices Reading the Data and Critiquing the Data Quality.

Analysis of Focal Students' Essays

In the following sections I present the analysis for each of the four focal students' two essays. I show how their revised essays contain elaborated warrants, stronger and clearer arguments, and multiple uses of the statistical literacy practices. Table 8 shows a summary of how the students' essays changed.

Table 8 Summary of Focal Students' Essays

	Essay #1 (week 2) "Rough Draft"	**Feedback type**	**Essay #2 (week 7) "Final Version"**
Julia	No argument	Written comments	Contained an argument using Reading the Data in the warrant
Nikki	Contained an augment using Critiquing the Data Quality in the warrant	Written comments and one-on-one conference over Zoom	Revised warrant using Critiquing the Data Quality in the warrant
Shaun	Contained an argument using Reading the Data	Written comments	Revised warrant using Reading the Data
Rocio	Contained an argument using Reading the Data and Critiquing the Data Quality	Written comments and one-on-one conference over Zoom	Revised claim and warrant using Reading the Data and Critiquing the Data Quality

One student's Essay #1 contained no argument or use of statistical literacy practices; their revised essay contained an argument in which they used Reading the Data. Two students revised their warrants between Essay #1 and Essay #2; one used Reading the Data and the other used Critiquing the Data Quality. One student revised

their entire argument, using both Reading the Data and Critiquing the Data Quality in their warrant.

Focus Student #1: From No Argument to Reading the Data to Make Claims

In this first example, I show how one student revised their essay to include an argument. The student's first draft was mostly a summary of the information in their chosen article. After receiving feedback, the student wrote an argument and used the statistical literacy practice Reading the Data in her warrant. This example provides evidence that feedback and the opportunity for revision can support students in using statistical literacy practices and in analyzing data and data-based claims in news media.

At the time of the study, Julia was a third-year Community Studies major. She is a Latina woman and a multilingual learner (Spanish and English). Julia transferred to the university from a two-year community college where she had successfully completed a statistics course. She regularly attended office hours to request help with the mathematics we were teaching. For the "Numbers in the News" assignment, Julia chose an article from *The Atlantic*, an American news magazine, titled, "Disease Has Never Been Just Disease for Native Americans," (see Appendix 4). The article presented data in text on COVID cases and deaths in the Navajo Nation, an indigenous community living in Arizona, New Mexico, and Utah, and did not provide any visual representations of data. It included historical information about disease in indigenous American communities and argued that disproportionate adverse outcomes have long been the norm for this group.

In her first essay, Julia reported the data from the article; she did not write an argument or analyze the author's claims, and she wrote in several places that she did not understand what she was reporting. For example, in the following excerpt Julia referenced a section of the article that cited the mortality rate in the Navajo community.

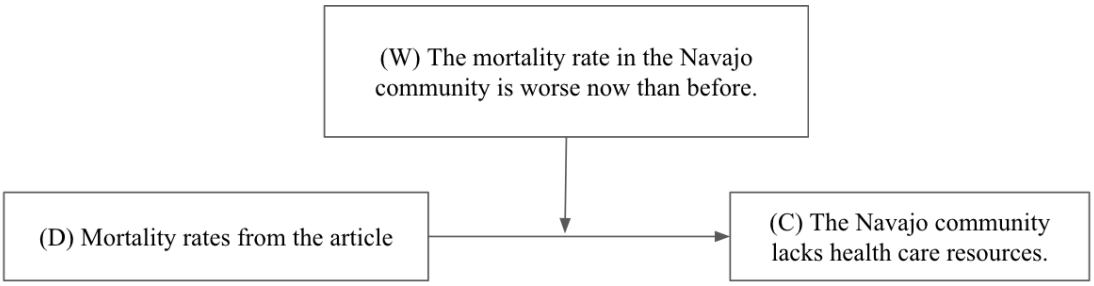
> The article states, "as of April 23, 1,360 infections and 52 deaths had been reported among the Navajo Reservation's 170,000 people, a mortality rate of 30 per 100,000." The source of the information is the Navajo Department of Health website. **I found it confusing where they had the mortality rate 30 per 100,000**. (Julia: Essay #1)

This excerpt is representative of her first essay. She quoted the article and identified the source of the data, then she commented that she "found it [the mortality rate] confusing." Immediately following this excerpt, Julia quoted the only other numbers in the article: death rates from the 1918-19 pandemic. These data were presented in a completely different part of the article. Again, she reported the data but did not comment on it in any way. It may be that she was addressing the aspects of the task (e.g., "Describe the quantitative information presented in the article,") without really writing (or perhaps thinking) about what the data mean and why the author included them. After only two weeks of instruction, Julia's first essay did not show any use of the strategies and practices we had begun to teach. Instead, her essay described the quantitative information in the article by quoting the article.

In my feedback to this first essay, I wrote comments such as "I think you missed some opportunities to meaningfully engage with the numbers," and, next to

the excerpt above, “A missed opportunity for some rough math!” By “rough” math, I was referring to the methods of estimating quantities using quick and simple arithmetic that we had practiced with the students in class. This feedback was related to the strategies and goals for the statistical literacy practice Reading the Data. My written comments encouraged her to look again at the numbers and to do some arithmetic (a strategy) to understand their meaning (a goal).

Figure 31 Toulmin Diagram of Julia’s Argument in Her Revised Essay



In her revised essay, Julia revisited the reported mortality rate that she had found confusing. Furthermore, she wrote an argument in which she interpreted the mortality rate. Though there were errors in her reasoning about it, her description of the mortality rate, quoted below, served as a warrant for her argument (see Figure 31) that the Navajo community “faces a continued lack of resources of support,” (Julia’s Essay #2).

> Looking at the mortality rate that was mentioned in the article it stated that **the Navajo reservation has a mortality rate of 30 of 100,000. That means that out of 100,000 people only 30 survive in the Navajo reservation. I wasn’t sure about this but I did the math and understood that what the author did was divide the 52 deaths over the 170,000 population and then he multiplied by 100,000 to get the 30 people.** I think it was important to also bring up the math that

> was used later on to calculate the mortality rate because it just shows how many people survive and that number is very small compared to the people that were living prior to the mortality rate. (Julia's Essay #2)

In this excerpt, Julia described her new understanding of the mortality rate that was based on the arithmetic calculations she performed. The mortality rate is an example of a social indicator, a single number that is composed of various quantifiable factors that describes a phenomenon of interest (Gal, 2019), in this instance, the number of people who died divided by the relevant population. Julia correctly described the division, "what the author did was divide the 52 deaths over the 170,000 population," but misstated the indicator as representing the rate of survivors, rather than deaths. This is evidence of a student Reading the Data to make sense of reported quantities by using arithmetic to interpret a rate, essentially "reading" the numbers with improved, though not perfect, meaning for the indicator.

Julia's essays illustrate how one student used the statistical literacy practice Reading the Data and refined her meaning of a social indicator (the mortality rate). In her first essay, she only reported the data that she didn't understand. This is consistent with my experiences with the course over the years as well as my pilot study which found that students do not come to the class with a practice of Reading the Data. This also corroborates research that has found students do not have a norm of critically assessing data (Watson & Moritz, 1997). After feedback, in her revised essay, she used arithmetic calculations to make sense of a rate, and she used those as the warrant in a new argument. By using the strategy of performing arithmetic calculations (called "rough math" in the course, Julia was reading between the data presented in

the article. Furthermore, her warrant is evidence of her developing understanding of proportional reasoning (Lamon, 1993) because she is making sense of a commonly used rate.

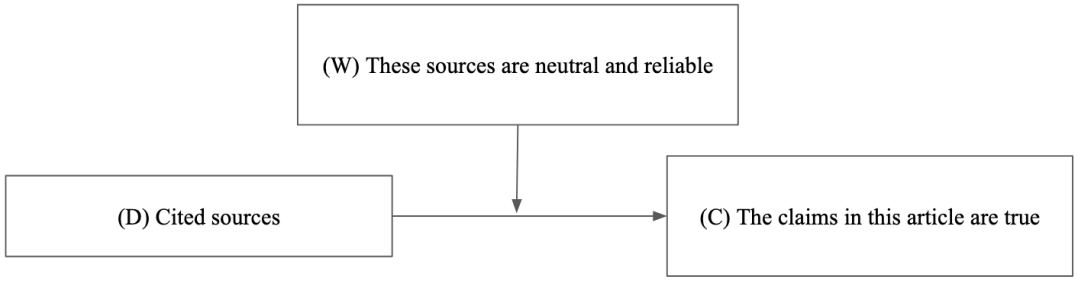
Focus Student #2: Critiquing the Data Quality to Revise a Warrant

In this second example I show that a student revised her initial essay to strengthen the warrant for her argument. The student used the statistical literacy practice Critiquing the Data Quality in both essays, but with feedback and the opportunity for revision, the student used the practice differently and to achieve different goals: to assess the trustworthiness of a source and to assess the accuracy of the reported data.

Nikki is a Latina woman and a multilingual (Spanish and English) learner. At the time of the study, she was a third year Community Studies major. Throughout the course, the instructor and I shared with one another our appreciation of Nikki's thoughtful contributions to class discussions and her critical thinking around social justice issues. She was a student who often articulated the social systems and structures that formed the social context for the data under discussion. A transfer student, Nikki had taken and passed a statistics course at a two-year Community College. Nikki's essays were written in response to an article she chose from the Center for American Progress (a left-leaning public policy research and advocacy organization) titled "Latinos Face Disproportionate Health and Economic Impacts From COVID-19" (see Appendix). The article contains data in text and in graphs and argues for government intervention via a relief bill to offset the disparate impacts

faced by Latinx people. Her argument (see Figure 32) was the same in both essays: That the claims in this article are correct because the sources seem unbiased.

Figure 32 Toulmin Diagram of Nikki's Argument in Essays #1 and #2



The data for this claim were the cited sources of the data, and the warrant was that those sources, the U.S. Department of Labor for example, are neutral and reliable. This aspect of Nikki's essay did not change between the first draft and final version. However, in her second, revised, essay, Nikki further Critiqued the Data Quality by checking the data source cited in the article to fact check a claim. This was likely in response to my comments on her first essay and the writing conference in which we discussed her essay.

In her first essay, Nikki had largely reported the data from the article. In response to one sentence where she had written, "Women accounted for 100 percent of U.S. job losses in December, with Hispanic or Latina women alone accounting for 45 percent of that job loss," I wrote, "I think you should interrogate this stat. You just drop it here, but you don't talk about it or analyze it." In our conference over Zoom, she admitted that she found the idea that women made up *all* the job losses in December to be surprising and improbable. Though she had determined that the

sources were reliable, she found the reported data itself to be questionable. In our conversation together I suggested an alternative goal: that of assessing the accuracy of the reported data. Together we decided that she might investigate the cited sources, which she had claimed were reliable, to further make sense of this statement. In the second, revised essay, Nikki used the statistical literacy practice Critiquing the Data Quality to grapple with the article's claim.

> [The authors] state, "Women accounted for 100 percent of U.S. job losses in December…" **This statistic particularly stood out to me as being a little extreme**. Upon examination of the citation used from the U.S. Department of Labor, the only table that provided such information was Table B: … Below the "Women" column, the unemployment rate is -0.1 in December compared to all the other categories of gender and ethnicity which were 0. **This statement was likely derived from this table**. (Nikki's Essay #2)

In this excerpt, Nikki went beyond assessing the data in the article based on its source and actually looked at the source material to try to understand where the 100% came from. This illustrates the statistical literacy practice Critiquing the Data Quality: Nikki did not accept a confusing piece of data at face value, despite her assessment that the source of the data was "relatively neutral," but instead checked the source itself to assess the veracity of a reported statistic. She had to search through tables of Labor Department data to find the entry that pertained to the 100% figure mentioned in the article. Nikki did not write about the meaning of the 100% figure except to mention once again that it was "confusing." She did determine that because she found where the statement "likely derived" from, this reinforced her argument that the claims in the article are true because the sources are neutral and reliable. This is a

stronger warrant to Nikki's argument that the claims in the article are true because she used the strategy of checking the reported data at the source to determine the article's accuracy.

To summarize, analysis of Nikki's writing revealed that she engaged in the statistical literacy practice of Critiquing the Data Quality in two different ways and for slightly different goals. In her first essay, she crafted an argument in which she supported her claim that the article's claims were accurate based on an assessment of the source of the data. Assessing the source is a goal of this statistical literacy practice, and Nikki used her knowledge of government institutions to make a determination. She revised her warrant in Essay #2 when she fact-checked a reported statistic against the cited data source. This strategy supported a slightly different goal of this statistical literacy practice: that of assessing the accuracy of the reported data. Nikki's essays illustrate two strategies of this statistical literacy practice: assessing the source and fact-checking the reported data (Ridgway et al., 2018).

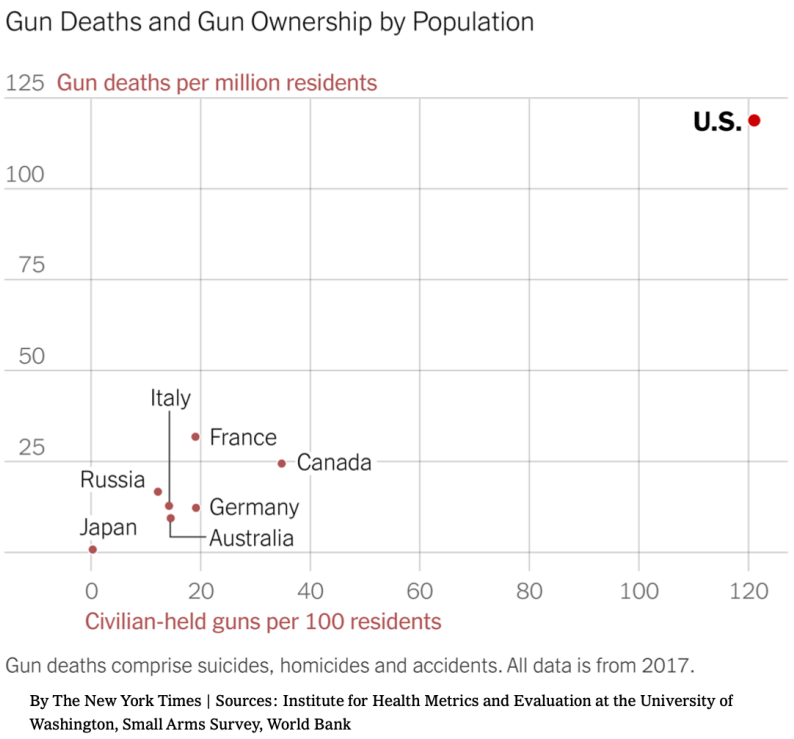
Focus Student 3: Revising a Warrant by Reading the Data

In the next example, I show how one student used the statistical literacy practice Reading the Data to critique a news article's claims. With the opportunity for feedback and revision, the student strengthened their warrants with more precise mathematical language and relevant arithmetic calculations. This example provides evidence that strong initial essays can still be strengthened and clarified through the process of feedback and revision.

Shaun is a mixed-race, Black and white, man who, at the time of the study, was a first-year Film major. He had attempted a college-level statistics class earlier in his first year of college, but he had dropped it because he believed he wouldn't pass. Shaun's participation in class was notable because he was the only student who never turned his camera on but who spoke and wrote actively in the chat throughout the course. At the beginning of the course, Shaun made statements such as, "the statistics should speak for themselves," and, by the end of the course, he was a leader in discussions about how statistics influence and are influenced by societal values, methods of data collection, and the way a value is presented (e.g., as a count or a rate).

For his essays, Shaun selected a New York Times article titled "Five Facts About Gun Violence" (see Appendix 4) that had a clear message advocating for stricter gun laws in the United States. The article contained data presented in both text and graphs. In both his first draft (Essay #1) and the revised version (Essay #2), Shaun's main argument was that the author's data did not convince him that restricting access to guns would solve the problem of violence in America. He focused on a particular visual representation in the article (Figure 33) that showed a correlation between the rates of gun deaths and civilian-held guns.

Figure 33 New York Times Figure: "Gun Deaths and Gun Ownership by Population"



In the figure, the United States is a dramatic outlier in comparison to the other nations. Shaun critiqued this visual representation for oversimplifying the reality of gun deaths.

> This graph in particular may have some issues; **only gun deaths are considered, not all violence-related deaths, so of course there will be more gun deaths when more guns are available**. (Shaun's Essay #1)

In this excerpt, Shaun was reading beyond the data in the graph as he questioned whether the visual representation was adequate evidence to support the article's claim. He correctly interpreted the correlational message in the graph, that higher numbers of gun deaths correlated with gun ownership, and then questioned the

appropriateness of the representation as evidence for the need for stricter gun control policies. His use of the phrase "of course" emphasizes his idea that the graph is overly simplistic; gun deaths can only happen if guns are present, but the graph does not prove that the presence of guns causes an increase in gun deaths. Shaun's argument is that the article should provide comparative nation-level data on all "violence-related deaths" in order to better compare nations' gun deaths. His argument shows evidence of relative thinking (Lamon, 1993) because he referenced the proportional relationship between gun ownership and gun deaths and the statistical concept of association versus causation. This student used the practice Reading the Data in a warrant for his argument (see Figure 34 for the diagram of the argument).

Since "only gun deaths are considered" (Essay #1), Shaun cited data from Wikipedia on "intentional death" rates (homicides, suicides) for the countries in the graph and cited the U.S. and Italy's rates specifically.

> America's rate of "intentional death" (homicide, suicide) per 100,000 people per year is listed far above some of the other countries in that New York Times graph. 16.8 for every 100,000 people in America per year versus, say, Italy with 7.2 per 100,000. **For America, that's a low estimate of over 50,000 intentional deaths per year. A high estimate might be over 60,000, or even higher given how many deaths go unreported**. (Shaun's Essay #1 cont.)

In this excerpt, Shaun found data to test his hypothesis that the correlational message in the New York Times graph might change if all intentional deaths, not just those caused by guns, were considered. He cited rates per 100,000 people for the U.S. and Italy, and then provided estimates for the count of people per year in the U.S. who die

from intentional causes. Shaun's mathematical "work" was not included in his essay, so it is impossible to say with certainty how he calculated those counts. However, converting between rates and counts was both explicitly taught and modeled throughout the course as a way to Read the Data to make sense of quantitative information.[11] Using "rough math" to make these comparisons was also modeled and practiced at nearly every class meeting and was included on homework assignments. I can only conjecture as to how he might have arrived at 50,000 to 60,000 as his estimate. If I were to solve the problem, I might use 320 million as an estimate for the U.S. population, and then, taking orders of magnitude into account, multiply 16 x 3,200. This type of strategy was regularly practiced through Number Talks during Discussion Sections, and many students became adept at performing such calculations mentally.[12] Shaun's use of the word "estimate" in his essay, along with providing a range for the true count, suggests he may have used rough math to come to these numbers.

These calculations were the warrant for his claim that the author's data does not support the argument that restricting guns will ultimately solve America's violence problem.

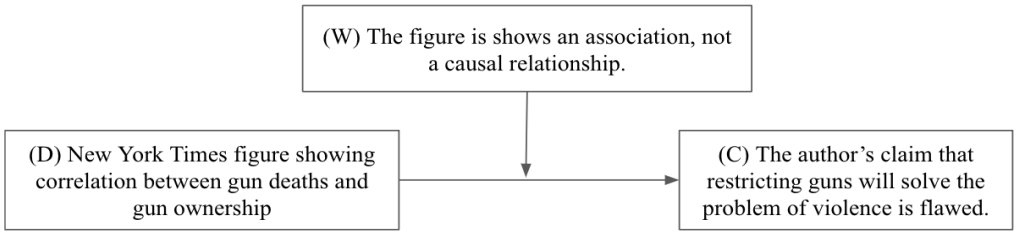
> So it is therefore true that not only are there more gun deaths in America but there are also simply more deaths, and **limiting access to guns … only alleviates the problem and does not solve it**. The real problem is that the government does not provide for its citizens, and thus people are subjected to poor mental health and

[11] See Chapter 4 for a full description.
[12] If I were solving this, I would begin by removing the two zeroes and saving them for the final product. Then, I would convert 16 x 32 to 8 x 64. I would then use partial products to find (8 x 60) + (8 x 4). This gives me 51,200.

> an inability to provide for themselves or their families, turning to crime to get by. (Shaun's Essay #1)

Shaun's argument is diagrammed below (see Figure 34). His use of the statistical literacy practice Reading the Data is evident in the warrant for his claim. Using the data provided in the article, Shaun read beyond the graph to critique the correlational argument presented by the article. He then cited Wikipedia as further evidence to support his critique of the visual representation as insufficient evidence to claim that restricting access to guns will solve America's violence problem because America has more intentional deaths than other countries. Implicit in his argument is the concept of confounding or lurking variables (Moore, 1998; Schield, 2009); he suggested that there may be other causes for the high rate of gun deaths in the U.S. beyond the presence of guns.

Figure 34 Toulmin Diagram of Shaun's Argument



As the TA, I provided feedback on Shaun's first essay. In addition to many positive comments, I suggested that he revise his analysis to better support his argument. The addition of the counts of U.S. deaths did not necessarily support Shaun's argument well. It was possible that it was only included because the assignment required that students "do something" with the numbers they encountered to demonstrate their learning in the class. My feedback suggested that he might

compare the rates of intentional deaths between nations. This relates to one of the strategies for the practice Reading the Data: comparing values to make sense of and critique data-based claims. In response to his claim that reducing access to guns will only alleviate, and not solve, the violence problem, I commented, "How much? What percent of those deaths in the U.S. involve guns? Can you make an educated prediction based on the data you have?" These comments were meant to highlight the reasons why a person might use arithmetic calculations to read the data; in other words, they highlighted a goal of this practice. Shaun's first essay showed evidence of arithmetic calculations; it did not contain evidence that these calculations were used to make sense of or critique statistical claims. Thus, a student used a strategy (doing calculations with the data) taught in class without a goal related to statistical literacy practices.

In the revised version of this essay (Essay #2), Shaun further developed the warrant for his argument. He began by articulating why the New York Times figure is flawed.

> The graph seems like a bad representation of the extent of violence in these countries. **Gun deaths will most likely always be proportional to the amount of guns available**. (Shaun's Essay #2)

This is a more precise statement than the one in Shaun's first draft because it used the concept of proportionality explicitly rather than inferentially. His warrant was strengthened by this revision because the backing is more clear and referenced a canonical mathematical concept. He went on to describe his Wikipedia research, but in the revised essay, Shaun not only reported the estimates of the counts of intentional

deaths per year, but also described the proportion of all intentional deaths that are caused by guns in the US.

> America's rate of 'intentional death' (homicide, suicide) per 100,000 per year is listed far above some of the other countries on the graph. 16.8 for every 100,000 people in America per year versus, say, Italy with 7.2 per 100,000. **That's an estimate of around 50,000 to 60,000 intentional deaths in America per year, with gun deaths comprising about 70% to 80%**. (Shuan's Essay #2, excerpt 2)

Again, Shaun's mathematical "work" is not included in his essay, but he may have used "rough math" to estimate what percentage of all intentional deaths involved a gun, presumably by referencing the number on the New York Times figure. It is not clear if Shaun used the estimated counts to calculate the 70–80%, or if he used the reported rates. The Wikipedia data of 16.8 per 100,000 could be compared to the New York Times' rate of around 120 per million.[13] If Shaun used the counts, he might have estimated the count of gun deaths in the U.S. and compared that to their estimate of 50,000 to 60,000.[14] Again, Shaun's use of the word "estimate" and the range (70–80%) he provided suggests he may have been using the practices modeled in class. Regardless, Shaun's essay shows evidence of relative, not absolute, thinking (Lamon, 1993), as he compared two associated sets to determine the percentage of gun-related intentional deaths. These calculations served as a stronger warrant than those in

[13] Divide 120 per million by 10/10, then estimate 12/16.
[14] There are about 320 million people in the U.S., so 320 times 120 will give the count. Then estimate 38,400/50,000.

Shaun's first essay because they explicitly connected back to the gun data and argument presented in the article.

Shaun concluded Essay #2 with a reiteration of his claim that the article's argument that gun control will solve America's violence problem is flawed.

> **Gun control, however, does not seem like a comprehensive strategy.** Further research that breaks down by the cause, frequency, general region, etc. would lead us closer to the source of the problem… **The problem is too often simplified with basic, easily understandable, sensationalized statistics…The way we frame a problem influences where we go to find its solutions.** (Shaun's Essay #2)

This argument illustrates Shaun's use of the statistical literacy practice Reading the Data. He read beyond the graph to critique it for implying a, possibly spurious, correlation and used arithmetic calculations and data from a secondary source to conclude that the claims in the article were not entirely supported by the data presented. With the opportunity for feedback and revision between the first and second essay, Shaun elaborated on his warrant using more relevant arithmetic calculations that better supported his claim because they connected directly to the data in the article.

Focus Student 4: Revising an Argument with Statistical Literacy Practices

This fourth, and final, example shows how a student's claim and warrant changed after receiving feedback. In the first essay, the student used Critiquing the Data Quality and Reading the Data as the warrant for their claim that the author's position was correct, though their warrant did not actually support this claim. After receiving feedback from me in my role as the TA that was focused on the student's

goals in using statistical literacy practices, the student revised both their claim and warrant to make a stronger and clearer argument. This example illustrates how feedback and revision supported one student in creating arguments and using statistical literacy practices for new goals.

At the time of the study, Rocio was a second-year Film major. She is a Latina woman and a multilingual learner who speaks Spanish and English. Rocio was an enthusiastic participant in class meetings, always kept her camera on, and both spoke and wrote comments in the chat. Rocio had not taken any college-level statistics or mathematics courses before, and she came to my Zoom office hours often for help on the homework assignments. For the two essays, Rocio chose an article (see Appendix 4) published by GLAAD, an American organization that monitors the media for defamatory representations of the LGBTQIA+ community. The article presents the results of an audit of over 100 television episodes' depiction of transgender people and argues that they were misrepresented as being either victims or villains.

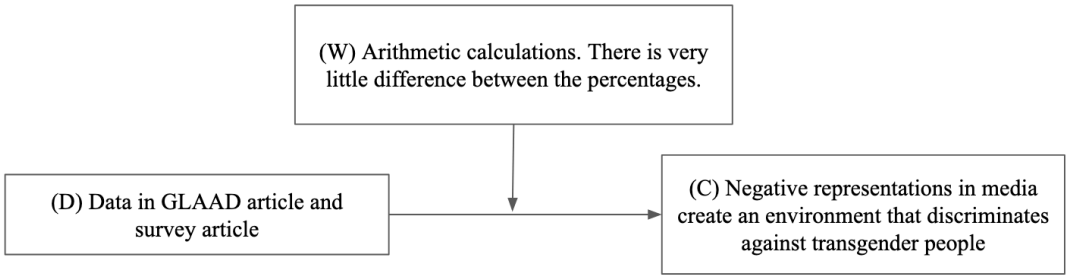
In her first essay, Rocio's analysis focused primarily on the article's claim that transgender people are misrepresented because they were portrayed as sex workers in 20% of the episodes audited. She used the statistical literacy practice Critiquing the Data Quality when she consulted other articles to "see if more than one source has the same statistical research." In the following excerpt, she cited an article that described a survey of transgender people in the U.S. and Read the Data she found there in an unclear way.

> In the article with GLAAD, it is shown that 20% of transgender characters are depicted as sex workers and

> in the article, 'Decriminalizing Prostitution is Central to Transgender Rights," it is analyzed that 19% of trans people have been involved with sex work. **Having (20%+19%)/2=0.195 shows a very tiny amount of difference of this representation. Both of these numbers are very close by but the the main concept of these statically numbers is to represent how the unfairness towards transgender people led them to be unemployed and have less opportunities** to carry on with their lives. (Rocio's Essay #1)

In this excerpt, Rocio used the statistical literacy practice Critiquing the Data Quality by consulting another source of information on the topic of transgender people and sex work. The second source, a survey of transgender people, reported that 19% of respondents had engaged in sex work—essentially the same percentage that the GLAAD article reported as a misrepresentation in the media. Her claim, based on the two data sources, was that negative representations in the media create an environment that pushes transgender people into sex work (see Figure 35).

Figure 35 Toulmin Diagram of Rocio's Argument in Essay #1



The warrant for her claim is the arithmetic calculation in which she worked with the 20% from the GLAAD article and the 19% from the other source. It seems that she found the average of the two percentages when she wrote "(20%+19%)/2=0.195." Her reasoning is unclear in the warrant as she wrote about the "difference" in the

rates and not the mean that she had calculated. She also wrote that the "numbers are very close by," which suggests she might have compared the percentages absolutely (a one percentage point difference), which is appropriate when comparing how close two rates are to one another, but her written arithmetic calculations did not clearly support this warrant.

In my role as Teaching Assistant, Rocio and I met to discuss her essay. In this meeting, I asked her to explain her calculations, but she said she didn't know why she'd done that arithmetic. One reason might be that the task urged students to "do some math" and so she did just that without having a clear goal as to why she was doing a calculation. One of the goals of Reading the Data is to compare two values, and we discussed what calculations would better explain her warrant that the two sources reported very similar results. At the time of this conversation, I was very focused on encouraging students to critique their articles' authors' use of data to support claims. It is only through analysis that I noticed that Rocio's first claim, that the negative portrayals in the media created discriminatory conditions for transgender individuals, potentially forcing them into sex work, was an interesting and reasonable argument. At the time, my feedback was focused on the idea that GLAAD was claiming misrepresentation while a survey found the same rate in the population. I therefore encouraged Rocio to reflect on why GLAAD cited the 20% statistic as a misrepresentation in light of the evidence in the survey article that found 19% of transgender people have engaged in sex work. She expressed hesitation to negatively critique the article because she believed that transgender people in America face

discrimination, and the article supported that stance. I encouraged her to view her critique as an act of allyship and advocacy for better journalism on a topic she cared passionately about.

In her revised essay (Essay #2), she crafted a new argument (see Figure 36) that critiqued the article for an anti-sex work bias.

> GLAAD has analyzed statistics that revealed a misrepresentation towards transgender people, but is it actually? Or is GLAAD trying to offend transgender people by the work they participate in? (Rocio's Essay #2)

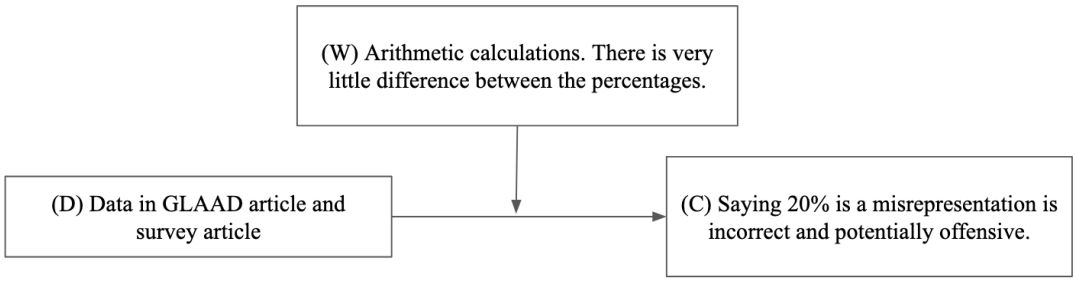
These questions illustrate Rocio's argument that GLAAD's claim that 20% is a misrepresentation is incorrect and potentially offensive. In the revised essay, Rocio used the statistical literacy practice Reading the Data in her revised warrant that more clearly compared the two rates.

> Analyzing the 20% that GLAAD found to be an offensive representation towards transgender people and having the actual 19% off a survey constructed with transgender people, it is shown that **there is only a 1 percentage point difference**… **This is a 1 in 5 ratio, being 20 out of 100 trans people in both percentages** and GLAAD could have possibly thought that the number would've been smaller and assumed it was a negative portrayal towards trans people working as sex workers. (Rocio's Essay #2)

In this excerpt, there is evidence that Rocio Critiqued the Data Quality by comparing data from multiple sources. The warrant for her argument is clearer than in Essay #1 because she compared values and interpreted that comparison; she described the "1 percentage point difference" between the two statistics, and she made connections between equivalent forms (1 in 5, 20 out of 100) to support the warrant that the two

values are almost the same. Comparing data is an example of reading between the data. Converting between equivalent forms of ratios was modeled in class as a way to make sense of a number, put it in perspective, and/or communicate it simply and memorably to others. This revision also shows evidence of relative thinking (Lamon, 1993) and estimation. She rounded 19% up to 20% and then used equivalent ratios to argue that the rates are essentially the same. Rocio demonstrated understanding that a percent is a special kind of ratio, and she needed to use multiplicative reasoning to make this comparison between rates. She used this reasoning about rates to clearly explain her revised warrant. Finally, her revised argument contains another strategy for Critiquing the Data Quality: considering the author's bias. Rocio's statement, "GLAAD could have possibly thought that the number would've been smaller and assumed it was a negative portrayal towards trans people working as sex workers," (Essay #2) is evidence that she was thinking about why the author presented the data that they did.

Figure 36 Toulmin Diagram of Rocio's Argument in Essay #2



Rocio repeatedly engaged in the statistical literacy practices of Critiquing the Data Quality and Reading the Data. In the first essay, she consulted a second source to verify the data, which is one goal of Critiquing the Data Quality. In our meeting,

she expressed uncertainty about how to think about the information she found while consulting the second source. The revised essay showed increased clarity in her description of the warrant, a revision to her claim, and additional uses of the statistical literacy practices. This example illustrates how opportunities to revise supported one student's use of statistical literacy practices and helped her elaborate an argument.

Discussion

This chapter further examined the third research question (What statistical literacy practices did students use?) by analyzing four focal students' use of the practices described in Chapter 5 in response to self-selected news articles. Did they use the practices Reading the Data and Critiquing the Data Quality? What strategies and concepts did they use? Because the students wrote two versions, and received feedback between, it is also possible to examine whether and how four focal students' use of these practices changed over time and with feedback and the opportunity to revise? The students in this study learned to use statistical literacy practices through instruction, group work, homework problems, feedback, and opportunities for revision. The four focal students highlighted in this chapter used the statistical literacy practices Critiquing the Data Quality and Reading the Data in their essays for various goals and in multiple ways using concepts and strategies taught in class. These are summarized in Table 9 along with a description of the feedback they received on their rough drafts. Consistent with the findings in Chapter 5 and with research, the affordances of the articles the students chose and their interpretations of the task influenced whether and how they used the practices. An article that reported

the results of a survey, for example, afforded different opportunities to Critique the Data Quality than did an article that cited government data. Feedback and revision provided opportunities for students to revisit or expand upon their initial attempts to use statistical literacy practices to craft arguments in response to the data, visual representations of data, and data-based claims in the news articles. Feedback and instruction focused on the goals and strategies of the statistical literacy practices and seemed to have an impact on students' revised warrants and arguments.

Both Rocio and Nikki Critiqued the Data Quality by fact-checking the reported data. Ridgway, Nicholson, Gal, and Ridgway (2018) describe fact-checking in terms of a "positive disposition" towards statistics as opposed to believing that statistics are all lies or, conversely, represent some undeniable truth. Both students' difficulty interpreting the results of the fact-checking suggests there is more to this strategy for Critiquing the Data Quality than having a positive disposition. After receiving feedback, Rocio used the practice Reading the Data to support her interpretation of the data she found when she consulted a second source. This corroborates the findings from Chapter 5 that fact-checking requires a goal and further suggests that students may need support to decide how to think about the information they find when they fact-check reported data.

Analysis of Shaun's use of statistical literacy practices provides an elaboration on the statistical literacy practices described in Chapter Five. He used several concepts to Read the Data with goals of making sense of the data, critiquing a visual representation, and comparing data. Furthermore, Shaun's Essay #1 and #2 illustrate a

way a student can critique how dominant cultural narratives can be perpetuated by statistical arguments in the media (Weiland, 2017). His critique of the New York Times graph showing the correlation between the rate of gun ownership in a nation and gun deaths was ultimately a critique of how gun-control regulations dominate the media narrative, and this narrative does not offer sufficient solutions.

Finally, the analysis of how Julia used Reading the Data illustrates how students may need support around understanding social indicators and making sense of reported data in news articles. This corroborates research that suggests students draw conclusions from data in school assignments without considering whether these conclusions make sense (Pfannkuch & Wild, 2004). With support and instructor feedback, Julia described how the author had constructed the mortality rate, but her essay reflected a struggle to make sense of this indicator in the context of COVID deaths in the Navajo community. For students who are used to "skipping the numbers" the statistical literacy practice Reading the Data may be the most important first step in learning to use statistical literacy practices.

Table 9 Summary of Focal Students' Use of Statistical Literacy Practices Across Two Essays

	Essay #1 (week 2) "Rough Draft"	Feedback focus	Essay #2 (week 7) "Final Version"
Julia	No evidence of SLPs	SLP: Reading the Data Goal: To make sense Strategy: Using "rough math"	SLP: Reading the Data Goal: To make sense Strategy: Using "rough math" Concept: Social Indicators
Nikki	SLP: Critiquing the Data Quality Goal: Assess source Strategy: Using background knowledge	SLP: Critiquing the Data Quality Goal: Assess accuracy of reported data Strategy: Fact checking source	SLP: Critiquing the Data Quality Goal: Assess accuracy of reported data Strategy: Fact checking source
Shaun	SLP: Reading the Data Goals: Critique data-based claims, Critique visual representations Strategy: Using "rough math" Concepts:Relative thinking, Association, Confounding variables	SLP: Reading the Data Goal: Compare values	SLP: Reading the Data Goals: Critique data-based claims, Critique visual representations, Compare values Strategy: Using "rough math" Concepts:Relative thinking, Association, Confounding variables
Rocio	SLPs: Reading the Data & Critiquing the Data Quality Goal: Compare data Strategies: Checking another source, Using "rough math"	SLP: Reading the Data Goal: Compare data Strategy: Using "rough math"	SLPs: Reading the Data & Critiquing the Data Quality Goal: Compare data Strategies: Checking another source, Using "rough math" Concept: Equivalent values

Rocio, Nikki, Shaun, and Julia all revised their first essays, and analysis of these revisions provided evidence that feedback and opportunities for revision

allowed students to learn how to use two statistical literacy practices. In their revised essays, all these students created or refined the warrants to their arguments, crafting more precise and stronger arguments. Furthermore, the process of feedback and revision allowed them a second chance to use statistical literacy practices in the same context and perhaps to clarify the requirements of the task itself (e.g., students knew they were expected to "do something" with the numbers they found in their articles but may not have been clear on the goals). The resulting revised essays showed how students refined their use of these practices and assessment of what students know and can do was improved. The rough draft essays were a type of formative assessment that guided both class instruction and the feedback students received on their essays. This supports the research on formative assessment (e.g., Burkhardt & Schoenfeld, 2019) that describes how student learning is supported when teachers use formative assessments. Much of the research on statistical literacy with students offers snapshots of one-time tasks which necessarily limit what students will do. This has important implications for adopting a strengths-based approach to teaching and researching students' learning. One might assume a student can't do something or doesn't know something when in fact they may not have seen the need to use a particular strategy or concept in response to a task. A next step in supporting students in developing statistical literacy might be to design formative assessment cycles for statistical literacy practices that offer students the opportunity to repeatedly use these practices to make sense of and critique data, visual representations of data, and data-based claims in the media.

Finally, these findings suggest a question regarding the use of "rough math" as a useful strategy for students to make sense of reported data. While most school-based statistics instruction focuses on procedural fluency (Bakker & Derry, 2011), the strategy of using "rough math" to estimate quantities and relationships between quantities may lower the stakes of getting a precise and correct answer and instead help focus students' attention on sensemaking. Three of the four students highlighted in this chapter used this strategy, and all needed support (via feedback focused on their goals) to use "rough math" in a way that moved beyond just performing calculations for their own sake to doing so as part of a statistical literacy practice.

Chapter 7: Discussion, Conclusion, and Implications

The purpose of this study was to examine students' use of statistical literacy practices in an undergraduate course on statistical reasoning. The online course was designed for non-STEM majors and students who had historically struggled in mathematics courses. Students wrote three essays over the ten-week term that asked them to analyze the use of data in news articles about social justice topics. Essays #1 and #2 were written in response to self-selected news articles containing data, data-based claims, and visual representations of data. Essay #2 was an opportunity to revise Essay #1. Essay #3 followed the same format as the first two essays except the students responded to an article selected by the course instructor. As the course Teaching Assistant, I graded all the essays and gave the students written and, in some cases, verbal feedback on their first essays.

The following research questions guided this study:

1. How did students respond in writing to a news article with statistical claims, data, and visual representations of data?
2. What arguments did students write? When students made arguments, what types of arguments did they make?
3. What statistical literacy practices did students use?
    a. Did these change over time and with the opportunity to revise for 4 focal students?

Summary of Findings

To answer the first research question, I examined students' Essay #3 written in response to the instructor-selected article. The purpose of this first question was to understand students' responses "on their own terms" (Moschkovich & Brenner, 2000) by documenting what they chose to write about. Students' writing clustered into four topics that were influenced by the content of both the assigned article and the course instructional activities. The students' essays focused on four topics: the source of the article (The Brookings Institution), the sampling method used to collect survey data, the article's focus on only three racial/ethnic categories, and the graphs. As suggested by both theory and research (Lampert & Cobb, 2003; Vygotsky, 1980), the students followed the social norms of a university course by writing about the ideas we had discussed in class.

The underlying purpose of the second research question was to systematically describe the students' writing. I used Toulmin's (2003) method to identify the students' arguments. The students overwhelmingly wrote arguments that agreed with claims in the article rather than arguments that were critical of the authors' conclusions. That is not to say that the students did not critique parts of the article, only that the overall stance in their essays was to agree with the article. This agreement has several potential sources including the norms around authoritative texts, students' personal beliefs about the topic, and norms around the instructor as the audience for the essays.

Analyzing students' arguments in this way was an important methodological tool. First, creating argument diagrams proved to be a tool for helping me to notice the students' strengths in their writing. Though I set out to adopt a strengths-based perspective, the first several times I read through the students' essays I focused on their mistakes and misconceptions. Many students seemed only to report the data in the Brookings Institution article, and I despaired of finding evidence of statistical literacy practices in their writing. It was through the act of looking at each essay in its entirety and diagramming the overall argument of each essay that I began to see how students were using concepts and strategies in service of goals. The diagrams helped me to see the proverbial forest and not get distracted by each individual tree. Furthermore, using Toulmin as an analytical tool helped me to identify and focus on the arguments the students were making, not the arguments I thought they should have made. Through that analysis, I was able to meet the students where they were and uncover their strengths.

The Toulmin argument analysis also provided me with a means to uncover the mathematical and statistical concepts, the strategies, and the goals that students used. These were almost never explicitly stated by students and instead were inferred through analysis. Concepts and strategies appeared in the warrants of the arguments, and goals were revealed in the students' claims or in the interplay between their claims and warrants. Essentially, the claims revealed why they were using concepts and/or strategies.

This Toulmin analysis also made it possible to answer the third research question about mathematical practices. I identified the statistical literacy practices students used by looking at the concepts, strategies, and goals that students used in their arguments. Two practices were evident in their essays: Reading the Data and Critiquing the Data Quality. I created models of these statistical literacy practices that are grounded in the students' writing. Figures 37 and 38 summarize the practices and the concepts, strategies, and goals that students used with each practice.

I then used these models to analyze four focal students' first and second essays (written in response to a self-selected news article) to see if and how students used these practices when responding to different articles and to document any changes in their arguments after revising. The statistical literacy practices appeared in all but one essay (Julia's Essay #1). This suggests that these practices may be generalizable to other contexts in which people encounter data, data-based claims, and visual representations of data in the media. At the very least, these practices reflect the focus of the class and the ways students appropriated these practices, concepts, and strategies.

Figure 37 Idealized Statistical Literacy Practice: Reading the Data

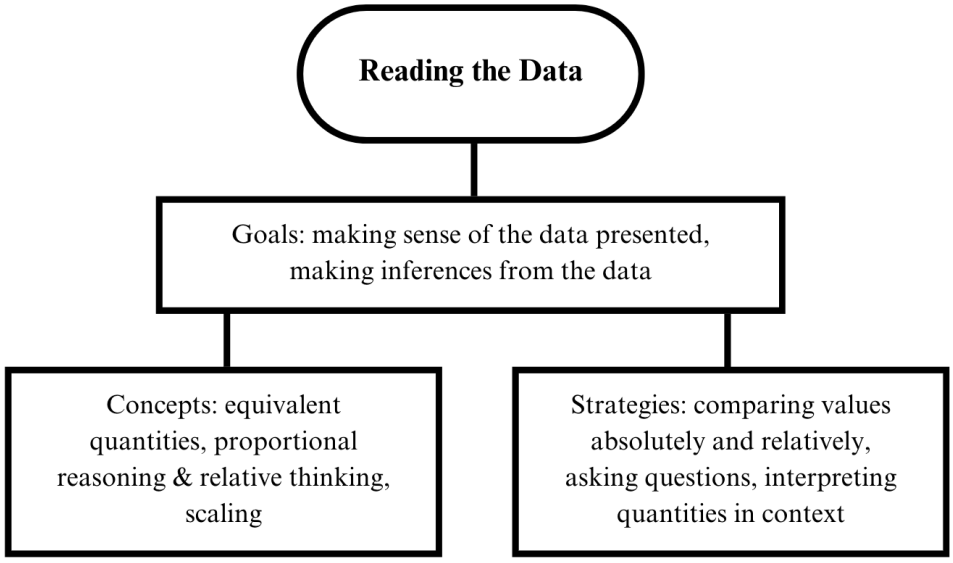
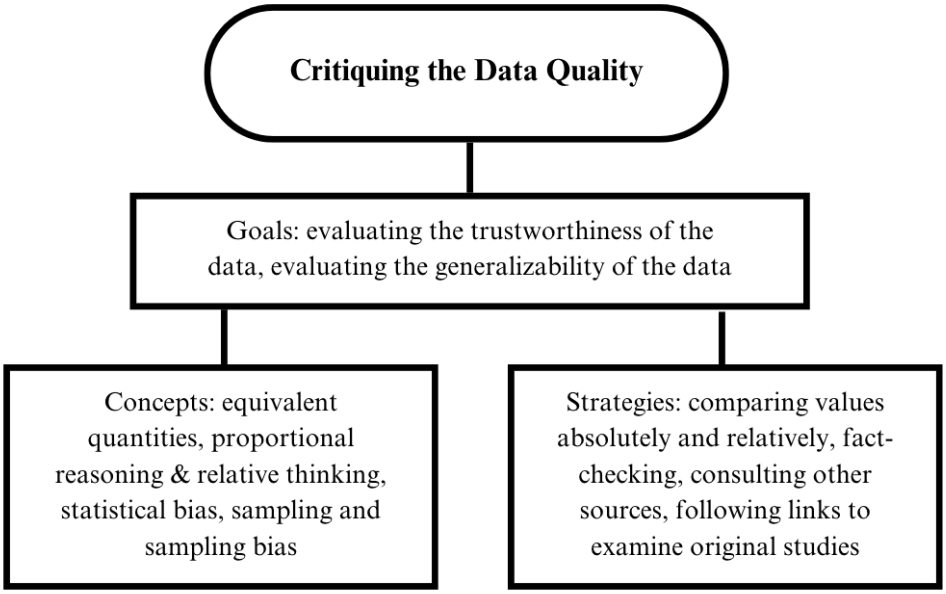


Figure 38 Idealized Statistical Literacy Practice: Critiquing the Data Quality



The final research question (3a: Did students' use of statistical literacy practices change over time and with the opportunity to revise for 4 focal students?) documented how students revised their initial essays after receiving feedback. All

four students' received feedback that suggested strategies and focused on goals. For example, I advised Shaun to use "rough math" (a strategy) to compare (a goal) all intentional deaths in a nation to gun-related deaths. All four revised essays showed that students learned to use the statistical literacy practices in their revisions. This suggests that both time in the class and receiving feedback supported students' learning to use these two statistical literacy practices.

Connection to the Literature

This study builds on others (e.g., Ben-Zvi & Garfield, 2004; Zapata-Cardona, 2018) that used real-world data based on social examples. The students responded to contemporary news articles that used data in reporting on topics related to social justice. This is important because statistics education, like mathematics teaching generally, has traditionally struggled with atomistic teaching (Bakker & Derry, 2011) in which students are given a "toolkit" of statistics concepts and procedures to master. Using messy, imperfect, real-world data might have benefits. First, students must decide which statistical concepts and strategies are appropriate given the affordances of their article. Rather than simply using a strategy or applying a concept because it is the current topic of instruction, students must choose for themselves what are the appropriate strategies and concepts. Second, real-world data is almost never as perfect an example of a concept as the realistic found in many textbook exercises. A normal distribution in the real world, for example, rarely mirrors the perfectly smooth bell curve most students are exposed to in textbooks. If students only ever encounter

idealized versions of distributions, they will be less prepared to make sense of messages when they encounter them.

This study also builds on others that have sought to define statistical literacy (Gal, 2002, 2019; Ridgway et al., 2018; Weiland, 2017) as what people need to know, do, think, and believe in order to make sense of and critically analyze data-based messages. These lists of skills, knowledge, and dispositions are based on the knowledge of experts who examine the landscape of data in people's everyday lives and compile a list of everything they imagine one will need to make sense of it. Analyzing statistical literacy through the sociocultural lens of mathematical practices uncovered more precisely what students need to learn. This analysis focused on goals and organized the complex lists of knowledge, skills, and dispositions into clusters of students' goal-directed activity. This clustering is especially important for instruction because it encourages both the students and the instructor to focus on goals instead of a list of procedures.

The statistical literacy practices documented in this dissertation were evident in students' responses to a written assignment. Practices were analyzed in terms of goals (why am I doing this?), strategies (how am I doing this?), and concepts (what mathematical and statistical ideas do I need?). This study documented two statistical literacy practices, Reading the Data and Critiquing the Data Quality, and described the concepts, strategies, and goals students used when engaging in each practice. There was some overlap in the concepts and strategies students used for each practice. For example, students used the strategy of using "rough math" and the concepts of

equivalent quantities and relative thinking for both practices. What makes each practice different are the goals, and often very different goals can be achieved through similar strategies or concepts. This practice-based analysis focusing on goals provides a holistic perspective on statistical literacy. This approach is different to creating an exhaustive list of every concept, skill, or belief that we imagine might be important for a person to be statistically literate. Instead, the important distinction is not between different skills or concepts but between different goals, or reasons, for acting for a few central practices.

Suggestions for Future Research

One limitation of this study is that statistical literacy practices were analyzed using almost identical tasks, the "Numbers in the News" essay prompt. Though examining the use of practices when students responded to different articles was a form of triangulation, more research is needed to understand what other statistical literacy practices exist and how they are used in a wider variety of contexts.

Furthermore, the sample size for examining feedback and revision was very small. It would be interesting to see whether the patterns of these two practices exist in other students' essays, in other populations, and in other contexts.

Implications for Practice

This study and the course in which it was set can serve as a model for statistics instruction. Mathematics education researchers and policymakers have encouraged teachers to abandon the "one right way to do math" and instead encourage multiple pathways to problem solving (California Mathematics

Framework; the Common Core State Standards in the United States, 2010; NCTM, 1989). As research and policy recommendations generally continue to move towards the interconnectedness of mathematical concepts and procedures and the necessity of making connections between representations, procedures, and concepts, the reality in many classrooms has remained somewhat fixed with a pedagogical approach best summed up as, "Learn all these separate concepts and procedures. Now you know math and/or statistics." This study provides an example of how teachers can adopt a more integrated approach to teaching statistics that prioritizes learning for action and using concepts and strategies in pursuit of statistical goals.

Writing assignments that are problems, as opposed to mathematics "exercises" (Schoenfeld, 1992), may be a powerful tool for helping students learn statistics concepts and connect those to goals. Writing requires that both students and instructors slow down in a way that discussions cannot. Mathematical reasoning in writing can more easily be examined, discussed, and revised than spoken words which vanish unless recorded. Furthermore, writing assignments encourage feedback and revision, which this study suggests can help students improve their arguments.

Because this study used a strengths-based lens to uncover and document statistical literacy practices in action, I did not look for one particular correct answer in students' writing. The task was open-ended enough to allow students some leeway in how they interpreted the assignment, in what sections of the article they wrote about, and in what statistical literacy practices (as well as the goals, concepts, and strategies) they used. In my feedback to students' Essay #1, I was responsive to their

ideas rather than simply looking for a particular correct answer. This approach may be a good model for classroom teachers, but they would need support in interpreting and knowing how to build on their students' contributions. Adopting a strengths-based view of students' learning does not mean accepting any answer. Instead, it is a shift in what the instructor pays attention to and prepares to build on in subsequent lessons. Rather than focusing only on students' errors, the instructor builds on students' strengths to determine how to support growth.

Giving students more choice in choosing topics could better support their engagement. Using real-world sources of data as opposed to textbook examples might also help students to think through the implications of their reasoning (Gresalfi, 2015) and go beyond simply reporting the data to thinking about what the data actually means in its particular context.

There may also be value in using "rough math." One could hypothesize that teaching students imprecise ways to calculate such as "rough math" that focus on estimation might help them attend more to the meaning of the data they are reading and writing about because it both eliminates some of the cognitive load that precision demands and lowers the stakes so that they can focus on using their everyday knowledge and intuitions to reason about data.

Implications for Equity

This study provides evidence that students who have previously struggled in mathematics and statistics courses, students who have been neglected or harmed by math teachers and who describe themselves as hating math and "skipping the

numbers" when reading articles, can successfully learn to use statistical literacy practices to make sense of data, data-based claims, and visual representations of data in news media. In Chapter 4, I described the story of a student who designed and conducted a survey of their classmates and used that data to write a persuasive email to the instructor advocating for a decreased workload. This is an example of student learning: they appropriated practices taught in class for their own goals and used data that they thought would convince their audience. Clearly this student was engaging in statistical literacy.

This study shows that focusing instruction and feedback on goals, giving students the opportunity to write and revise, and using social justice topics can support students who have previously struggled. Rather than provide remedial lessons in basic skills to struggling students, instructors can give statistics meaning by inviting students to critically examine data-based claims about topics they are passionate about. Focusing on goals and practices can also move instruction away from a focus on a long list of disconnected procedures.

Appendices

Appendix 1: Example Homework Problems

3. (10 points) On page 191 of Pager & Shephard, the authors report on research findings that document the racial equivalent of what's been called a "poverty tax" (low-income people face higher prices.) In this research (Ayres, 1995), the finding is that "black men and black women paid on average $1132 and $446 more [than white men and white women], respectively" to buy a car. Here are your questions:

   a) (3 pts.) As a count, how much more did black men get charged, compared to the over-charge for black women?

   b) (3 pts.) As a rate, how much more? [Hint: treat women as the reference point]

   c) (4 pts.) Which of these numbers matters more, i.e., which would be a better number to use to fight for equity and why do you say so? Be concise.

(5 points) Here's a bit of news and fund-raising solicitation that I received last year from the Swing Left people; they work to elect Democrats and progressives at all electoral levels. In relation to a Wisconsin Supreme Court election on April 7, 2020 the Wisconsin Swing Left field director, one Jared Launius, wrote,

> "After a week of waiting, and despite the Republicans' blatant attempts to suppress voters and steal an election, the results are in: **progressive Judge Jill Karofsky has won a Wisconsin Supreme Court seat by a crushing 11% margin!**"

Here are the returns:

General election for Wisconsin Supreme Court

		Candidate	%	Votes
✓		Jill Karofsky (Nonpartisan)	55.3	855,981
		**Daniel Kelly** (Nonpartisan)	44.7	692,523

My question to you: In his report of the election returns, was Launius right? Don't just answer "yes" or "no." Use rough math and your growing quantitative literacy (numeracy) to check both his math and his quantitative reasoning.

Appendix 2: Numbers in the News Essays #1 and #2 Assignment

Prompt for Numbers in the News #1

Due: **by 5 pm Sunday, April 11**

Length: 2-page essay, typed and double-spaced.

Worth: 10% of your overall grade

Goal: To develop your skills in rough math, building context, and engaging in critical epistemology in relation to news reporting.

What to do:

First, select a news article* that includes two things:

- A subject that pertains to social justice
- An emphasis on, or at least the use of, quantitative information

* By "news article" I'm not talking about little clips of 50-150 words. Whether you obtain it on-line or via hard copy, draw a real article from one of the country's major newspapers – *New York Times, Washington Post, LA Times, SF, Detroit Free Press, Wall Street Journal, Al-Jazeera, Christian Science Monitor, La Opinion* (if you choose this source please be ready to translate) – or news magazines, such as *The Atlantic, Salon.com, Harper's, The Nation, The American Prospect, The American Conservative, truthout, politico.com*. I will certainly accept pieces found in a series the *New York Times* does called "The Upshot." There's also an occasional series called "The Debunker" that might interest you. I'll happily accept the on-line *Colorlines*. If you're unsure about the appropriateness of your source, check with Julianne or me.

If your article summarizes a report from a government agency or advocacy organization you might decide to review that report yourself or its abstract; that would be great, though it's not required. This should be a low-pressure exploration of information and how it's being framed.

NOTE: To do the best possible job of this, you should review a bunch of articles and really think about which one lends itself to the best layperson's simple quantitative analysis. Choose a topic that you *want* to read about.

If you're uncertain about your article, you can email a copy it to Julianne and me so we can review it.

Then, write a short essay in which you:

1. Identify the source, including the link.

2. Briefly summarize the theme of your selection. What is the social justice issue?

3. Describe the quantitative information presented in the article, including how it was presented (statement, table, figure or graph, interactive figure or graph, etc.), the source of the material, and what you found powerful or interesting or confusing or problematic about it. Be ready to quote directly.

4. Related to #3 –and it's certainly fine to organize your summary in a way that integrates #3 and #4—critically analyze the role and quality of the numerical information as you understand things so far. For example, could it have been presented more clearly or meaningfully? How well do the data fit the reporter's story? Is anything about it incomplete or misleading? In other words, do you detect any bias and, if so, in what direction? Is there anything you'd have done to improve the presentation of data? In sum, in what ways does the quantitative information serve the reader's (your) understanding of the justice issue/s?

5. What additional information would help you make sense of the information that's reported? Identify a question or two. Do a simple calculation or two to make deeper sense of the material. If it's 'denominator information' that would extend or clarify the context and you can find that denominator, write about it; that would be great! Cite your source in-text and at the end of your essay (References).

6. In sum, what value did you derive from carefully assessing the quantitative information in the context of the overall article? And/or, what questions do you have, that you think other students might have too, when it comes to understanding the material (head to toe)?

Your grade will depend on:

- A credible choice of news article
- Clear explanation of what makes the topic a "social justice" issue
- Clear presentation of the quantitative information
- Effective assessment of potential –or transparent—bias in the reporting, including questions that arise for you or reasons to admire the reporting
- Appropriate manipulation of numbers. Use simple rough math to help you think through what's being reported, possibly going to other sources to find information that'll provide context and perspective.

Appendix 3: Numbers in the News Final Essay Prompt

By now you're experienced with conducting numeracy-centered critical analysis of a news article. In roughly 2 pages plus references, analyze this article: https://www.brookings.edu/blog/up-front/2021/04/30/inequalities-in-housing-hardship-declined-because-everybody-is-now-worse-off/. Follow the same format you used in previous Numbers in the News assignments, reprinted here for your convenience.

1. Identify the source, including its credibility.
2. Briefly summarize the topic and theme of the article, establishing the social justice context and stating clearly what is the authors' point of view?
3. Describe the quantitative information presented in the article, including how it was presented (statement, table, figure or graph, interactive figure or graph, etc.), the source of the material, and what you found powerful or interesting or confusing or problematic about it. Be ready to quote directly. Do some calculations of your own to check or develop the meaning of the numbers cited.
4. In general, critically analyze the role and quality of the numerical information. For example, could it have been presented more clearly or meaningfully? How well do the data fit the reporter's story? Is anything about it incomplete or misleading (biased)? What assumptions do the authors make and are there explicit or hidden? Is there anything you'd have done to improve the presentation of data? In sum, in what ways does the quantitative information serve the reader's (your) understanding of the justice issue/s? If you find the article to do an excellent job from a numeracy perspective, say so and point out what they've done to bring you to that conclusion.
5. What additional information would help you make sense of the information that's reported? Identify a question or two. Do a simple calculation or two to make deeper sense of the material. If it's 'denominator information' that would extend or clarify the context and you can find that denominator, do it – go get the information—and write about it; that would be great! Cite your sources in-text and at the end of your essay (References).
6. In sum, what value did you derive from carefully assessing the quantitative information in the context of the overall article? And/or, what questions do you have, that you think other students might have too, when it comes to understanding the material (head to toe)?

Then, as you review your writing – (plan on editing your work!) – ask yourself:

1. What does this number represent? Who does it represent?
2. Does it seem correct? Is there any way I could check?

3. What does the authors' reporting this number in this way highlight? What does it obscure?
4. Is this a lot? Compared to what?
5. So what? Why does it matter (or not)?

Appendix 4: Links to articles

Brookings Institution Article
https://www.brookings.edu/articles/inequalities-in-housing-hardship-declined-because-everybody-is-now-worse-off/#:~:text=In%20June%2C%20we%20reported%20that,COVID%2D19%20pandemic%20stretched%20onward.

"Latinos Face Disproportionate Health and Economic Impacts From COVID-19"
https://www.americanprogress.org/article/latinos-face-disproportionate-health-economic-impacts-covid-19/

"Five Facts About Gun Violence"
https://www.nytimes.com/2021/03/25/briefing/gun-control-suez-canal-ship-vaccine-astrazeneca.html#:~:text=The%20toll%20approaches%20pancreatic%20cancer's,or%20Alzheimer's%20(about%20125%2C000).

"Victims or Villains: Examining Ten Years of Transgender Images on Television"
https://glaad.org/publications/victims-or-villains-examining-ten-years-transgender-images-television/

"Disease Has Never Been Just Disease for Native Americans"
https://www.theatlantic.com/ideas/archive/2020/04/disease-has-never-been-just-disease-native-americans/610852/

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