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### Title

Real-time Voltage Security Assessment: Algorithms Simulation and Validation Results

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**FINAL PROJECT REPORT**  
**REAL TIME SYSTEM OPERATIONS**  
**2006 — 2007**

**APPENDIX C**

**REAL-TIME VOLTAGE SECURITY ASSESSMENT  
ALGORITHM'S SIMULATION AND VALIDATION RESULTS**

*Prepared for CIEE By:*

**Lawrence Berkeley National Laboratory**

**CERTS**  
CONSORTIUM FOR ELECTRIC RELIABILITY TECHNOLOGY SOLUTIONS

University of California  
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**A CIEE Report**





## **ALGORITHM'S SIMULATION & VALIDATION RESULTS**

Prepared For:  
California Independent System Operator (CA ISO)

Prepared by:  
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## 1. INTRODUCTION

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The Real-Time Voltage Security Assessment (RTVSA) project is designed to be part of the suite of advanced computational tools for congestion management that is slated for practical applications in California within the next couple of years. Modern voltage assessment methods include the development of such advanced functions as identification of weak elements, automatic selection of remedial actions and automatic development of composite operating nomograms and security regions. With all the research advancements in the area of Voltage Security Assessment over the past few decades, the feasibility of deploying production-grade VSA tools that run in real time and integrate with existing EMS/SCADA systems utilizing results from the state estimator, are increasingly becoming a reality.

Some advanced contemporary real-time applications already promote the idea of using the security regions with the composite boundaries limited by stability, thermal, and voltage constraints. At the same time, the majority of these tools are still based on the static system power flow models and implement such traditional approaches as sink-source system stressing approach, P-V and V-Q analyses, V-Q sensitivity and modal analysis. Unfortunately, many of the most promising methods suggested in the literature have not been implemented yet in the industrial environment, including the state-of-the-art direct method to finding the exact Point of Collapse. Currently there exists no real-time monitoring tool for voltage security assessment. The problems of voltage security will be exacerbated by the effects of multi-transfers through the network. These sets of simultaneous transfers are manifest because of the buying and selling of electric power across the boundaries of control areas. Moreover the point of production and the point of delivery may be in geographically distant locations.

The RTVSA application is based on an extensive analysis of the existing VSA methodologies, by surveying the leading power system experts' opinion worldwide, and also with feedback from industrial advisors. Through this process, a state-of-the-art combination of approaches and computational engines was identified and selected for implementation in this project. The suggested approach is based on the following principles and algorithms:

- Use the concepts of local voltage problem areas and descriptive variables influencing the voltage stability problem in each area. Utilize information about the known voltage problem areas and develop formal screening procedures to periodically discover new potential problem areas and their description parameters.
- Use the descriptive variable space to determine the sequence of stress directions to approximate and visualize the boundary. The stress directions are based on pre-determined generation dispatches and load scaling patterns..
- Use hyperplanes to approximate the voltage stability boundary.
- To calculate the approximating hyperplanes, apply a combination of the parameter continuation techniques and direct methods as suggested in this report. Introduce a sufficient additional security margin to account for inaccuracies of approximation and uncertainties of the power flow parameters.
- Compute the control actions most effective in maintaining a sufficient security margin.

- Produce a list of abnormal reductions in nodal voltages and highlight the elements and regions most affected by potential voltage problems. The list of most congested corridors in the system will be ranked by the worst-case contingencies leading to voltage collapse.

The initial framework of this project was originally formulated by California ISO. The key elements of the suggested approach which are the use of parameter continuation, direct methods, and the hyperplane approximation of the voltage stability boundary were approved by a panel of leading experts in the area in the course of a survey conducted by Electric Power Group, LLC (EPG) in 2005. These concepts were also verified in the course of face-to-face personal meetings with well-known university professors, industry experts, software developers and included email discussions and telephone exchanges. CERTS industrial advisors approved these developments during various CEC Technical Advisory Committee (TAC) meetings conducted in the past years.

In 2005, the project development team successfully implemented the parameter continuation predictor-corrector methods. Necessary improvements were identified and developed. The PSERC parameter continuation program and MATLAB programming language were used in the project. During 2006-07, research work included the implementation of Direct Methods to quickly and accurately determine the exact Point of Collapse (PoC), Boundary Orbiting techniques to trace the security boundary, the investigation of descriptive variables, and the validation of techniques for analyzing margin sensitivities.

The above mentioned techniques have been tested using a ~6000 bus state estimator model covering the entire Western Interconnection and for the Southern California problem areas suggested by California ISO. These results are presented within this report.

## 2. DESCRIPTION OF THE SYSTEM

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The selection of the critical parameters influencing the voltage stability margin and stress directions was conducted based on engineering judgment. The stress directions were defined using the sink-source and balanced loading principles. This means that the generators and the loads participating in each stress scenario are identified, as well as their individual participation factors; the participation factors are balanced so that the total of MW/MVAR increments and decrements is equal to zero. This allows avoiding re-dispatching of the remaining generation. Based on the California ISO recommendation, two study areas were selected for verifying the prototype VSA algorithms: the Humboldt and San Diego problem areas.

The San Diego region within Southern California suffers from voltage stability issues, and hence, forms a good test case. CA ISO provided the EPG team with the 5940 bus (1188 generators) State Estimator generated load flow solution on October 23, 2007 that spans the entire Western Interconnection and includes all buses/lines at or above the 115 kV level. Only elements below the 115kV level and external to the CAISO have been equivalenced. Within the CAISO jurisdiction, some of the lower voltage levels are also covered. Hence, this case precisely models the southern California region which is being studied.

### 2.1 Generators in Study Region

CA ISO identified the generators (Table 1) comprised in the region which have been used as the sources in the stressing scenarios:

Generating Units	Max Capacity (MW)
South Bay 1	152
South Bay 2	156
South Bay 3	183
South Bay 4	232
Encina 1	106.3
Encina 2	110.3
Encina 3	110.3
Encina 4	306
Encina 5	345.6
Palomar 1X1	180.6
Palomar 2X1	180.6
Huntington Beach 1	226
Huntington Beach 2	226
Huntington Beach 3	225
Huntington Beach 4	227
Alamitos 1	175
Alamitos 2	176
Alamitos 3	322
Alamitos 4	320
Alamitos 5	482
Alamitos 6	481

Table 1: Generators in Study Area

The generation stressing process adopted by the VSA tool involves all the generators, mentioned in table 1 above, with the participation factors calculated based on their maximum generation capacity:

$$\text{Participation Factor of } Gen_k = \frac{P_{gen\_max}(Gen_k)}{P_{gen\_max}(Total)}$$

This participation factor for generators are dynamic, as they change once a generator reaches its maximum generation limit and is left out of the equation.

## 2.2 Loads in the Study Region

CA ISO also identified the loads (Table 2) comprised in the San Diego region which have been used as the sinks in the stressing scenarios:

Load Bus	ID	Base Load, $Load_k$ (MW)
Moorpark	1	717
Riohondo	1	714
ValleySC 1	1	704
ValleySC 2	2	704
Santiago	1	699
Chino 1	12	440.93
Chino 2	3	220.07
Los Coches 1	31	25.266
Los Coches 2	32	25.266
Mission 1	30	23.391
Mission 2	31	23.391
Mission 3	32	23.391
Mission 4	33	23.391
Scripps 1	30	21.244
Scripps 2	31	21.244
Scripps 3	32	21.244
Old Town 1	30	21.109
Old Town 2	31	21.109
Old Town 3	32	21.109
Escondido 1	30	20.028
Escondido 2	31	20.028
Escondido 3	32	20.028
Telegraph Canyon 1	41	19.755
Telegraph Canyon 2	42	19.755
Capstrno 1	40	22.946
Capstrno 2	41	22.946
Miramar 1	30	21.231
Miramar 2	31	21.231
Miramar 3	32	21.231
Granite 1	30	21.001
Granite 2	31	21.001
Granite 3	32	21.001
Granite 4	33	21.001

Mesa Rim 1	31	19.838
Mesa Rim 2	32	19.838
Mesa Rim 3	33	19.838
Spring Valley 1	30	19.743
Spring Valley 2	31	19.743
Rose Canyon 1	30	18.739
Rose Canyon 2	32	18.739
Prctrvly 1	41	18.565
Prctrvly 2	42	18.565
Oceanside	31	17.992
Del Mar	32	16.04
La Jolla 1	30	12.962
La Jolla 2	31	12.962
Encinitas 1	20	12.234
Encinitas 2	31	12.234
Encinitas 3	32	12.234
Loveland	1	7.155
Cabrillo 1	30	6.433
Cabrillo 2	31	6.433

Table 2: Loads in Study Area

The participation factors for the loads are calculated using their base case  $Load_k$  (in MWs), whereas the load power factor is maintained constant:

$$Participation\ Factor\ of\ Load_k = \frac{Base\ Load_k}{Total\ Load\ of\ the\ Stress\ Vector}$$

### 2.3 Slack Bus Model

The distributed slack bus model includes all buses in the system except the ones that participate in the stress vector. This model reacts to the active power mismatch that is caused by the stressing procedure and generation contingencies. The participation factors on the distributed slack buses are calculated proportionally to the  $P_{gen_{max}}$  of generators. This will approximately simulate the post transient governor power flow. There are a total of 775 generators in the system; hence, the slack buses consist of all the generators other than those switched off and the ones listed under Table 1 above.

### 3. ALGORITHM RESULTS

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The platform that was selected for implementing the RTVSA application includes the PSERC Continuation Power Flow program and MATLAB programming language. Major modifications have been made to the PSERC program to meet the objectives of the VSA project most efficiently. The developed RTVSA algorithms consist of the following steps:

1. Initial system stressing procedure for a given stress direction to reach a vicinity of the Point of Collapse (PoC) in this direction. This step is implemented using the Parameter Continuation Method. This method is one of the most reliable power flow computation methods; it allows approaching the PoC and obtaining the initial estimates of system state variables needed for the subsequent steps. The selected form of the continuation methods includes predictor and corrector steps.
2. The direct method is used then to refine the PoC location along the initial stress direction (the continuation method would require multiple iterations to find the PoC with the required accuracy). At least one of the power flow Jacobian matrix eigenvalues must be very close to zero at the PoC.
3. The inverse iteration method or Arnoldi algorithm is applied to find the left eigenvector corresponding to the zero eigenvalue at PoC.
4. The boundary orbiting procedure is then applied to trace the voltage stability boundary along a selected slice. This procedure is a combination of a predictor-corrector method and the transposed direct method. This code features a voltage/reactive power limit violation check that allows the generator buses to conveniently switch from a generator to a load bus and vice-versa, thus resulting in a significantly smooth and precise nomogram.
5. In case of divergence, the algorithm is repeated starting from Step 1 for a new stress direction predicted at the last iteration of the orbiting procedure. Divergence may be caused, for example, by singularities of the stability boundary shape along the slice.

#### 3.1 Parameter Continuation Method

Parameter continuation predictor-corrector method was chosen as the preferred method capable of reaching the vicinity of point of collapse on the power flow feasibility boundary. The addition of new variables called continuation parameters determines the position of an operating point along some power system stress direction in the parameter space. The predictor step consists of an incremental movement of the power flow point along the state space trajectory, based on the linearization of the model. The corrector step, which follows each predictor step, consists in the elimination of the linearization error by balancing the power flow equations to some close point on the nonlinear trajectory.

The figure below shows the PV curve (real load vs. voltage magnitude plot) for a load bus that was part of the load stress vector in the RTVSA algorithm. The crosses are the predictor-corrector solution points as the algorithm traces the curve to reach the vicinity of the voltage instability point denoted by a star.

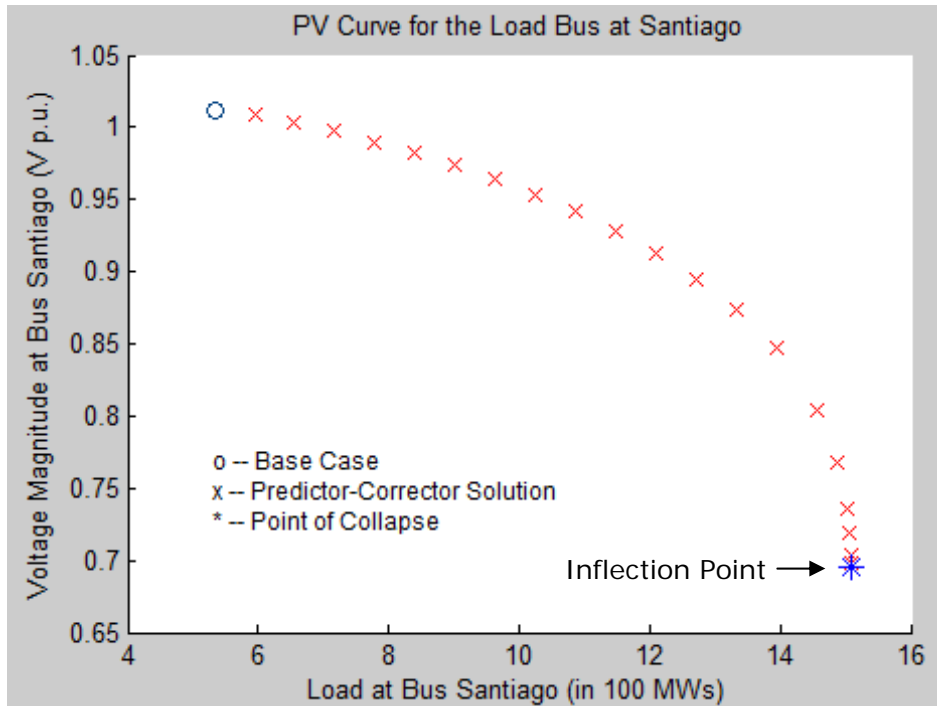


Figure 1 - PV Curve for a Load Bus

Similarly, the parameter continuation method can also be illustrated for a 2D stressing scenario for two loads in the San Diego region as shown below:

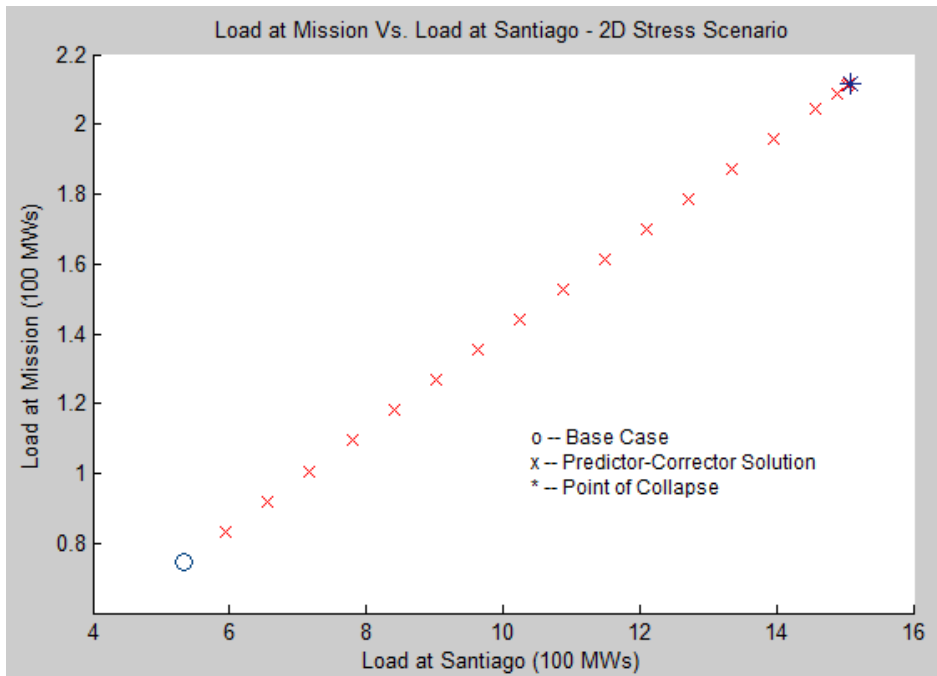


Figure 2 - Load at Mission vs. Load at Santiago

In order to verify the results of the parameter continuation algorithm, the GE PSLF simulation engine was modified to incorporate the RTVSA stress vectors as well as the participation factor calculations, among other minor changes. The source and the sink

vectors were stressed<sup>1</sup> to reach the point of voltage instability. The result of this comparative study revealed that the Point of Collapse solutions obtained from GE PSLF were indeed very close to that of the RTVSA algorithm as shown in Figures 3, 4 and the comparison chart in Table 3 below:

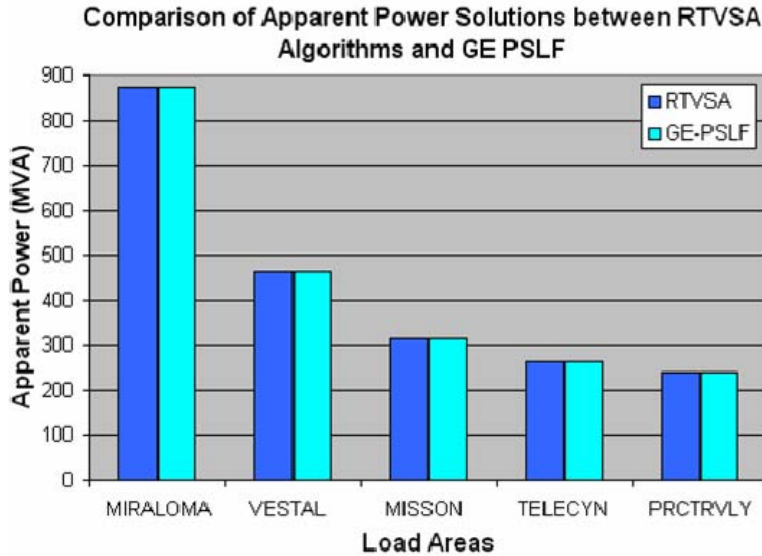


Figure 3 - Comparison of Apparent Power Solutions (at PoC) between RTVSA and GE PSLF

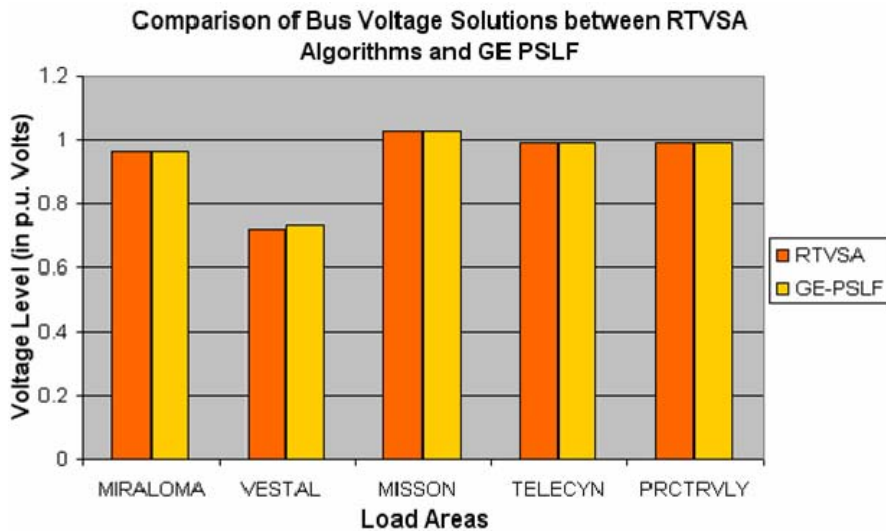


Figure 4 - Comparison of Absolute Voltage Solutions (at PoC) between RTVSA and GE PSLF

<sup>1</sup> GE PSLF uses Brute-Force method to determine the Point of Collapse solution



Loads	% Difference in Power	% Difference in Voltage
Miraloma	0.04%	0.09%
Vestal	-2.19%	0.22%
Mission	0.02%	0.19%
Telegraph Canyon	-0.01%	0.30%
PRCTRVLY	0.02%	0.33%

Table 3: Percentage Difference between RTVSA and GE PSLF Calculations

### 3.2 Direct Method

Direct methods for finding the Point of Collapse in a given direction combine a parametric description of the system stress, based on the specified loading vector in the parameter space and a scalar parameter describing a position of an operating point along the loading trajectory and the power flow singularity condition expressed with the help of the Jacobian matrix multiplied by a nonzero right or the left eigenvector that nullifies the Jacobian matrix at the collapse point. Unlike the power flow problem, this reformulated problem does not become singular at the point of collapse and can produce the bifurcation point very accurately.

In principle, the direct method allows finding the bifurcation points without implementing a loading procedure. There is, however, a problem of finding the initial guesses of the state variables and the eigenvector that may be resolved by initially loading the system along the stress direction. By doing so, the initial guess of state variables can be obtained. To evaluate the initial guess for the eigenvector, the inverse iteration method has been recommended to calculate the eigenvector corresponding to the minimum real eigenvalue. The RTVSA code, however, utilizes Arnoldi’s algorithm in Matlab software, also known as ‘eigs’ function, for simulation purposes.

The accuracy and advantage of the Direct Method algorithm has been shown with the help of the two plots below, wherein the Direct Method algorithm (Figure 6) is capable of determining the solution point (Point of Collapse) in one step, compared to 18 iterations taken by the Predictor-Corrector algorithm (figure 5).

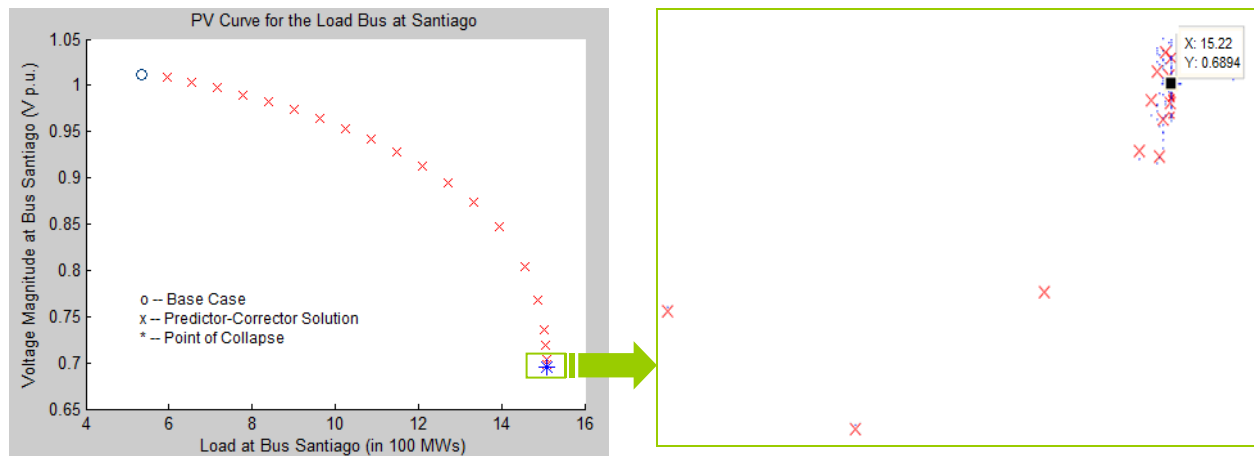


Figure 5 - PoC Calculation by Predictor-Corrector Algorithm



Figure 6 - Direct Method's Accelerated PoC Calculation

### 3.3 Boundary Orbiting Method

After reaching the Point of Collapse (PoC) solution point using a combination of the Continuation Parameter and Direct Method for a specified stress direction, the challenge is to orbit a static voltage stability boundary without repeating the time-consuming Continuation Parameter method along a selected slice. This problem is effectively solved by using the Boundary Orbiting Method algorithm instead, in order to change the stress direction and thus, trace the security region.

The Boundary Orbiting Method (BOM) may face divergence, for instance due to singularities at boundary edges, and hence, the continuation parameter method is repeated for a new stress direction predicted at the last iteration of the orbiting procedure. An example of a voltage security region for two loads in injection space has been shown below in Figure 7.

The slope of the boundary is determined by the sign of the eigenvalue corresponding to the load element in the left eigenvector. The positive slope illustrated in Figure 7 is due to the opposite signs of the eigenvalues of the two loads. Similarly, eigenvalues of the same sign results in a negative slope as shown in Figure 8.

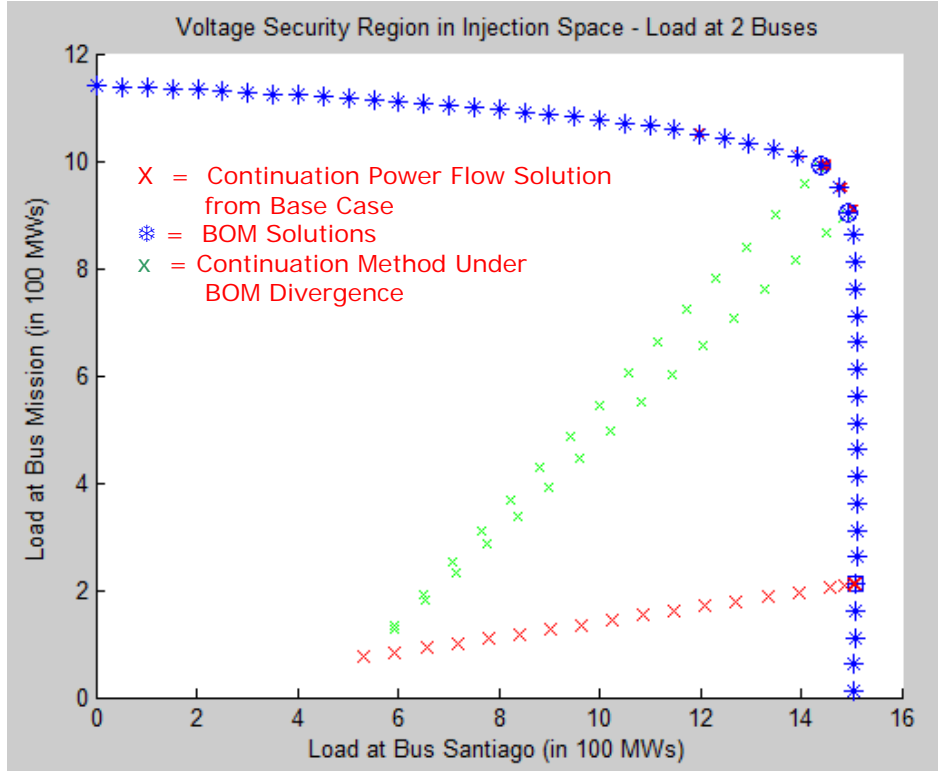


Figure 7 - Security Region by Boundary Orbiting Method

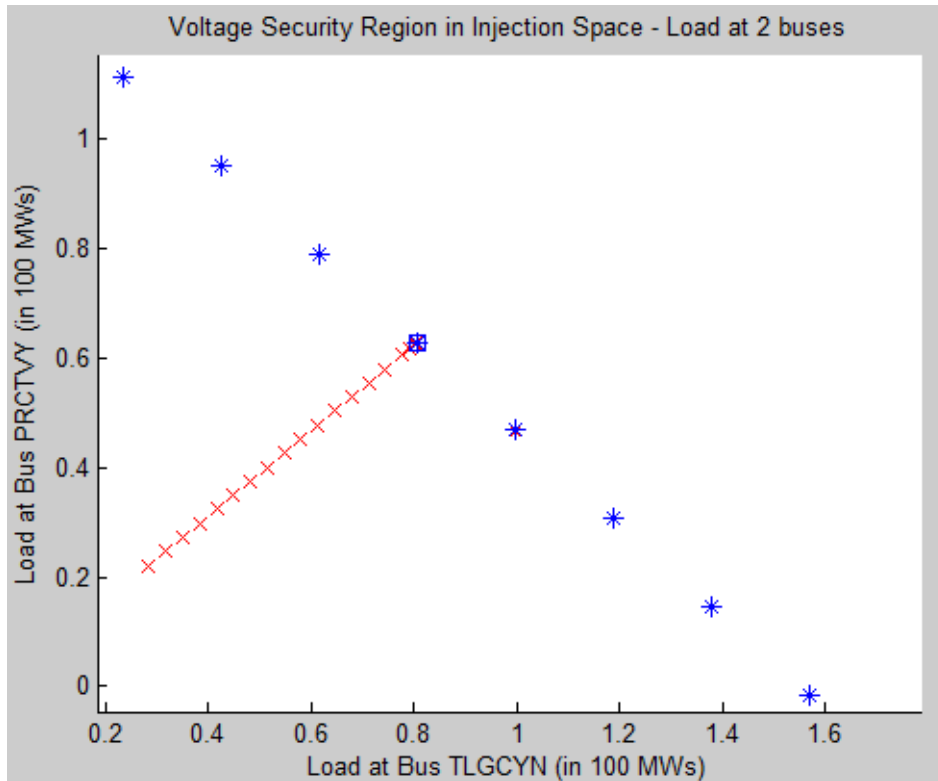


Figure 8 - Security Region for Two Loads (For Eigenvalues with Same Signs)

To test the accuracy of the boundary points obtained by the orbiting procedure, the Continuation Parameter method, along with the Direct Method, was simulated for certain stress directions. A typical test result, as shown in Figure 9 below, reveals the precision of the Boundary Orbiting Method.

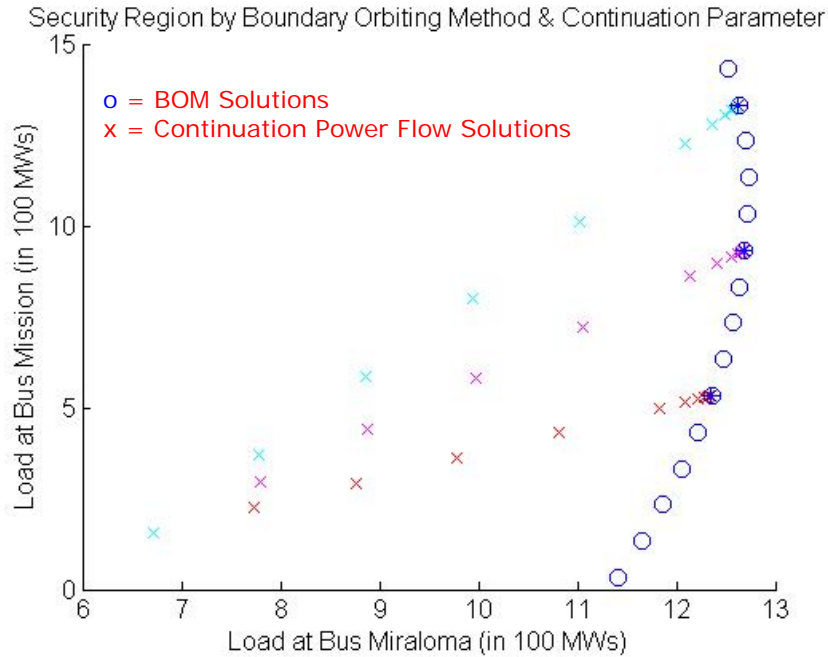


Figure 9 - Testing the Precision of Boundary Orbiting Method

The original PSERC Predictor Corrector algorithm was designed to switch generator to load buses (i.e., PV to PQ buses) due to the nature of the one-dimensional stressing process. However, the RTVSA proposed two-dimensional security region calls for a more complex two way switching of the buses from type PV to PQ and back to a PV bus as and when required. Hence, the RTVSA tool was modified to accommodate the required algorithm for conveniently switching the buses, thus generating a precise and smooth security region as shown below:

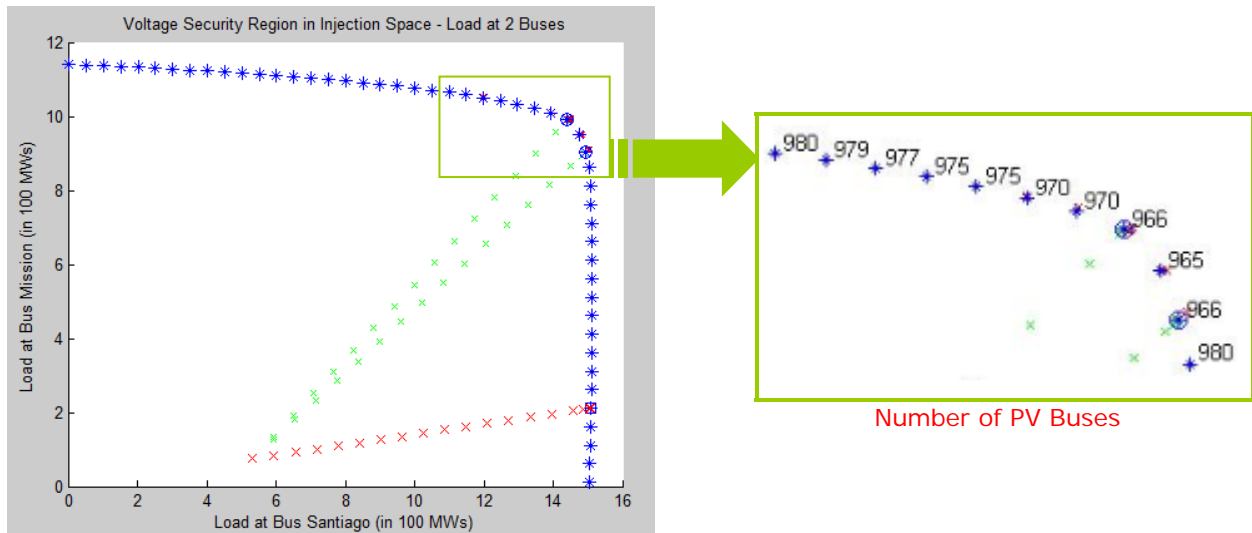


Figure 10 - Switching of PV to PQ Buses and Vice-Versa

### 3.4 Margin Sensitivities

The following input data is used as a simple example to examine the RTVSA tool. The stress parameters are sinks internal to Sand Diego region. The sources have been constrained to be the set of three generating units at South Bay. This corresponds to a scenario with no import from SONGS or Encina or from units West of the River or from Mexico. The sinks are loads at Carlton Hills (CHILLS) and Mission (MSSN). The sources are generator shifts at South Bay (SB).

Pattern	Color	CHILLS	MSSN	Code	
I	green	0.99	0.01	1	0
II	red	0.20	0.80	1	1
III	blue	0.01	0.99	0	1

Table 4: Patterns of SINK PF (Participation Factors)<sup>2</sup>

SB	SB	SB
4519	4520	4524
0.30	0.35	0.35

Table 5: Generator PF at the 3 Units of South Bay (SB) for all Vectors

The Lagrangian Multipliers<sup>3</sup> at the PoC can also be interpreted as the left eigenvector at the PoC. Figure 11 shows the comparisons of Lagrangian Multipliers for the three stressing patterns. For example, Pattern II for CHILLS has a multiplier of 0.8, which means that reducing the load at CHILLS by 1 MW would increase the Margin to PoC by 0.8 MW. A bus with a very high Lagrangian Multiplier would signal congestion. Buses with very low Multipliers indicate locations at which power injections have almost no effect on the margin Margin to PoC that are a large electrical metric away from the point of collapse. Indicators, such as the statistics of multipliers that are above a certain threshold, can be used for distinguishing the “non-locality” of the collapse phenomenon.

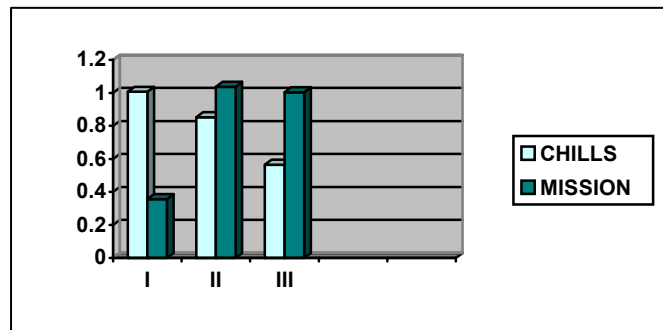


Figure 11 - Lagrangian Multipliers for SDGE cases

Figure 12 can be considered a geometric validation of the result. The intercepts on the y axis (Mission or MSSN) are smaller for patterns II and III because of the larger Lagrangian multipliers for Mission. Likewise, the intercept on the x axis (Carton Hills or CHILLS) is large for patterns II and III because of the small Lagrange multipliers at Carlton Hills. Stressing

<sup>2</sup> The load at Carlton Hills is approximately four times smaller than the load at Mission.

<sup>3</sup> The coefficients of the hyperplane consist of elements of the *left eigenvector* which can be interpreted as the Lagrangian multipliers corresponding to the parametric sensitivity of the hyperplane. The hyperplanes can be visualized as the constraints in a traditional optimization problem. The intercept on the descriptive variable axis is *inversely proportional* to the Lagrangian multiplier associated with the descriptive variable.

Pattern I has the opposite arrangement - a large Lagrangian multiplier for Carlton Hills and a small multiplier for Mission.

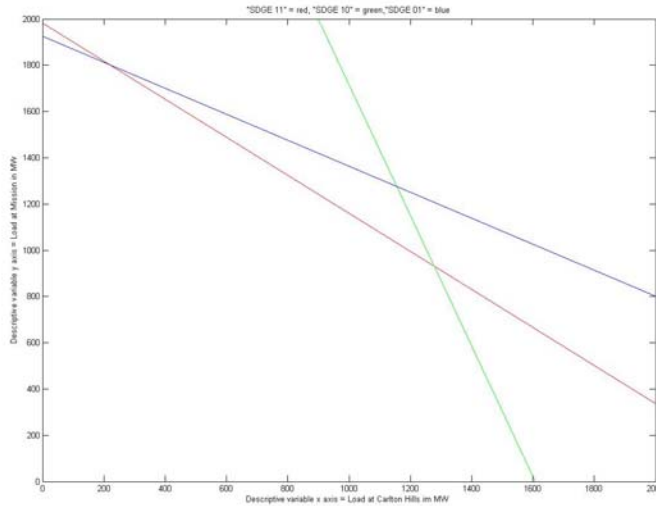


Figure 12 - RTVSA Output: Hyperplane slices at Carlton Hills and Mission

The high values of PoC in the CHILLS-MSSN case in Figure 12 are because the example was meant to illustrate the effects of electrical limits on the transmission of power from the source buses to a set of distributed sink buses. The effects of thermal limits have been temporarily neglected. The sources are also assumed to have an unlimited supply of reactive power. Both of these relaxations show the electrical capacity of the corridors of power flows from South Bay to CHILLS and MSSN. This capacity is far greater than when thermal and power injection limits are enforced.

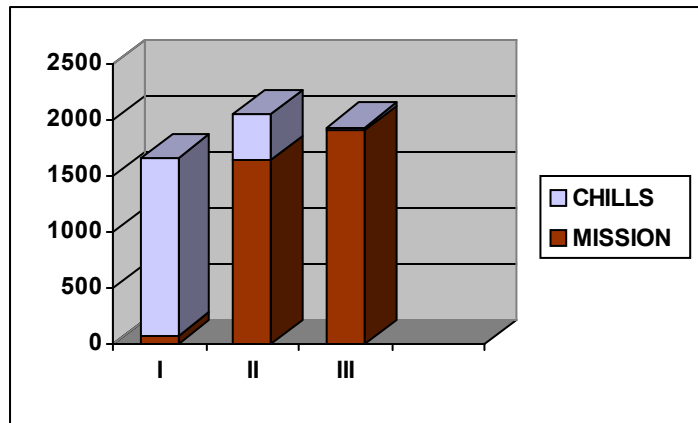


Figure 13 - RTVSA Output: PoC in MW for Carlton Hills and Mission

### 3.5 Collapse Participation Factors & Voltage Sensitivities

The participation is computed from the right eigenvector of the Jacobian evaluated at voltage collapse corresponding to the zero eigenvalue. The right eigenvector provides information on the extent to which variables participate during a voltage collapse condition. This determines weak areas and whether the collapse is an angle collapse. (Specifying to the operator which buses participate most in the voltage collapse is useful, but it should also be noted that the buses with the biggest falls in voltage in the collapse may not be the

same as the most effective buses to inject reactive power. The most effective buses to inject reactive power are given by the left eigenvector or Lagrangian multipliers).

Additionally, a byproduct of the continuation method is the availability of the tangent vector at each operating point before reaching the PoC which provides information about the degradation in voltage or angle profiles due to an incremental increase in loading (i.e., Voltage or Angle Sensitivities), assuming that the continuation is parameterized by the margin. In other words, if the Margin to PoC increases (decreases) by 100 MW, then the Voltage Sensitivities will indicate the extent to which the voltages will deteriorate (recover) and are expressed in terms of KV/(100 MW of the Margin to PoC. However, at the PoC, this tangent vector can also be used to approximate the right eigen-vector and therefore provides information on the Collapse Participation Factors. Figure 14 shows these for Stressing Pattern I.

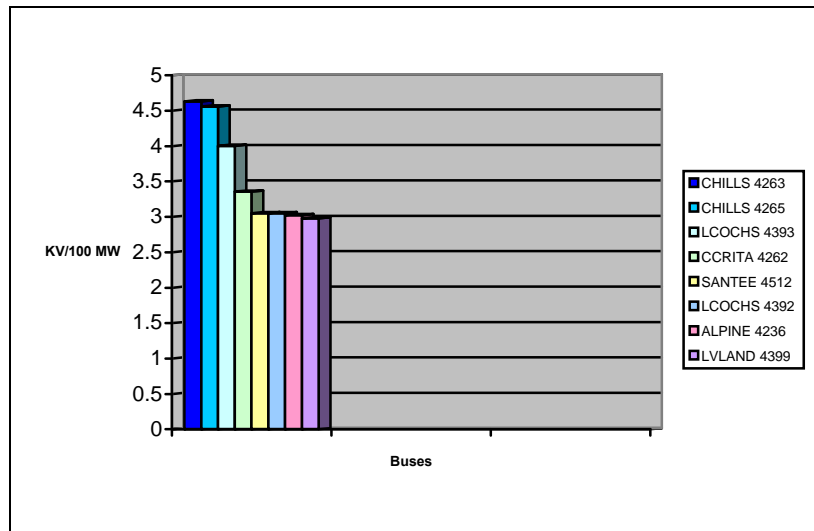


Figure 14 - Top Eight Voltage Sensitivities for Stressing Pattern I

Similar to Voltage Sensitivities one can examine the top ranked Angle Sensitivities. See below for Stress Pattern I.

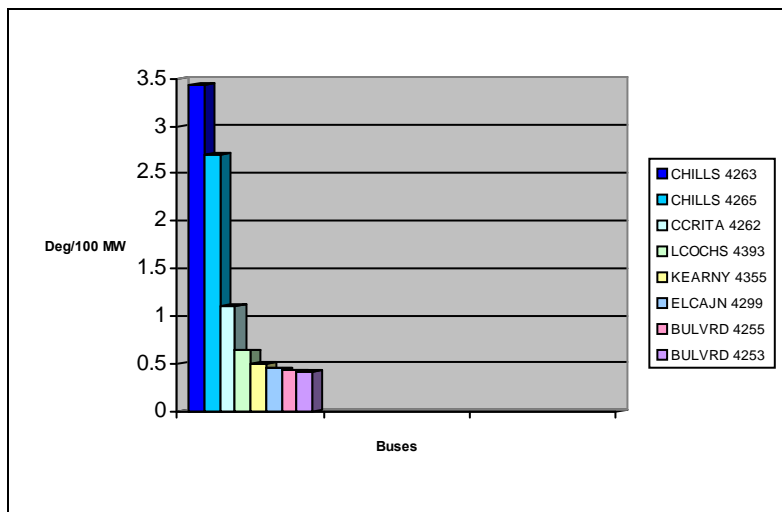


Figure 15 - Top Eight Angle Sensitivities for Stressing Pattern I

## 4. CONCLUSION

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The RTVSA application is based on an extensive analysis of the existing VSA approaches, by surveying the leading power system experts' opinion worldwide, and also with feedback from industrial advisors. The mismatch between the core power system reliability needs and the availability of the VSA tools was a motivation to design the RTVSA prototype.

The robustness of the Parameter Continuation technique combines with the accuracy of the Direct Method and Boundary Orbiting Method makes the RTVSA prototype a preferred choice for an advanced VSA application.

The underlying concepts are applicable to the simple one-dimensional approach or the more complex multi-directional stressing to explore the entire voltage security region in the parameter space or in full P-Q injection space. The RTVSA algorithms are complex enough to handle system stress/relief by allowing the generator buses to switch to load buses and vice-versa.

Possible follow-on research to the current work could include enhancing the proven and tested methodologies to achieve (1) better approximation; (2) select the number and position of hyperplanes based on desired accuracy; (3) "sliced bread procedure" to systematically trace the security boundary in multi-dimensional space; and (4) compute transmission reliability margins for voltage collapse from margin sensitivities. Other good additions to the conducted research would be to evaluate non-iterative voltage stability analysis techniques for tracing the voltage stability boundary as well as researching methodologies to screen the power system to detect places vulnerable to voltage collapse and help select descriptor parameters.