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Publication Date
1961-05-15

UCRL -9701

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BERKELEY, CALIFORNIA

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Contract No. W-7405-eng-48

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ABSTRACT

The analytically continued S matrix, satisfying the established symmetry principles and "saturating" the unitarity condition, is discussed as the basis for a theory of strong interactions. The elementary particle concept is absent, as are arbitrary dimensionless (coupling) constants.

THE S-MATRIX THEORY OF STRONG INTERACTIONS*

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I. INTRODUCTION

I am going to talk today in a spirit that is foreign to my highly pragmatic instincts. I am going to speculate and attempt to generalize, and since this is the first occasion on which I have lifted my nose from the trail through the underbrush of particle interactions to look about at the forest, it cannot be expected that I shall be accurate in my observations. However, some privilege should attach to being listed on the Organizing Committee for this Conference and it may be useful to have the anti-field-theory point of view represented--even if incoherently. I shall proceed then in an attempt to raise as many of you to a state of indignation--or perhaps of hysterical laughter--as is possible in 30 minutes.

So that there can be no misunderstanding of the position I am espousing, let me say at once that I believe the conventional association of fields with strongly interacting particles to be empty. I do not have firm convictions about leptons or photons, but I have yet to see any aspect of strong interactions that is clarified by the field concept. Whatever success theory has achieved in this area is based on the unitarity of the analytically continued S matrix plus symmetry principles. I do not wish to assert (as does Landau[1]) that conventional field theory is necessarily wrong, but only that it is sterile with respect to strong interactions and that, like an old soldier, it is destined not to die but just to fade away.

Having made this point so strongly, I hasten to express unqualified appreciation of the historical role played by field theory up to the present. The field apparatus has been enormously useful in the discovery of symmetry principles, particularly with respect to charge conjugation. A second area where field theory has played a crucial historical part is in the analytic continuation of the S matrix; the notion of micro-causality and of Feynman diagrams has of course been invaluable in this connection. Nevertheless, I am convinced that future development of an understanding of strong interactions will be expedited if we eliminate from our thinking such field theoretical notions as Lagrangians, "bare" masses, "bare" coupling constants, and even the notion of "elementary particles." I believe, in other words, that in the future we should work entirely within the framework of the analytically continued S-matrix.

Unlike Landau, who has for years been condemning field theory,[1] I have not in the past expressed an opinion because I felt that there remained important properties of the S-matrix that could best be discovered within the field apparatus. I am a peace-loving man and saw no point in arousing the hostility of colleagues who were busy discovering these properties. (I fear that I have irritated some by showing interest only in their results and not in how the results were obtained.) It is my impression now, however, that finally we have within our grasp all the properties of the S-matrix that can be deduced from field theory. If this view is correct, future work will presumably be of two kinds: (1) re-expression entirely within the S-matrix framework of the already known properties in language that is aesthetically satisfying and not clumsy, as at present; (2) development of new principles that go beyond the content of conventional field theory. I want to discuss both of these questions today.

Before beginning, however, it is appropriate to recall that, although he has now turned his back on the S-matrix approach, the original author was Heisenberg in 1943.[2] Heisenberg soon lost interest, I suppose because at that time he lacked the full analytic continuation that is required to give the S-matrix dynamical content. He also, of course, abandoned conventional many-field theory and has thrown all his efforts behind the idea of a single underlying field.[3] It should be realized that the S-matrix theory of strong interactions, at least as I shall describe it today, has the same goal as the single-field approach: that is, given certain symmetries, to predict all the observed particles, together with masses and mutual interactions, in terms of a single constant with the dimensions of length. There should be no arbitrary dimensionless constants. It is conceivable then, that the two approaches are not contradictory but complementary. Heisenberg likes to say that the one works from the inside out, the other from the outside in. He, of course, believes that simplicity lies only at the "center" while on the "periphery" there is confusion. I tend not to believe in the existence of a "center" but my main motivation in starting on the "periphery" is that even though the situation there is complicated the rules are subject to more or less direct experimental test. What the rules are at the "center," God only knows, and I choose this phrase with care.

II. REFORMULATION OF ALREADY RECOGNIZED PROPERTIES OF THE S MATRIX

The essential information about the S matrix that has been given by field-theoretical studies is the location and strength of unphysical singularities. The rule has been stated in the most complete form by Landau[4] and Cutkosky,[5] building on observations made by Mandelstam[6] and others.[7] The Landau-Cutkosky recipe is couched in the language of Feynman diagrams and therefore seems to rest heavily on field theory. In the case of elastic scattering, however, Mandelstam originally discovered the recipe not through diagrams but by asking the question: Is there a way, consistent with unitarity, to continue analytically the S matrix in both angle and energy variables? He found a prescription for doing this, and no one has succeeded in finding an alternative. On the basis of this experience it is plausible that the complete Landau-Cutkosky rules may be a unique consequence of the following postulate: The S matrix is an analytic function of all momentum variables with only those singularities required by unitarity. I have strong faith in the eventual verification of such a conjecture.

Requiring simultaneous unitarity in all the different channels of the S matrix obtained by switching incoming and outgoing particles is an enormously restrictive inhibition. (It is to be understood that for real values of the three-momenta, both positive and negative energies are physical, corresponding in the usual sense to either incoming particles or outgoing antiparticles.) At first glance, in fact, it sometimes seems impossible, and the only machinery we have that can contemplate such a problem with any generality is based on diagrams motivated by field theory. It appears to me nevertheless likely that the essence of the diagrammatic approach will eventually be divorced from field theory and shown to rest only on the twin principles of analyticity and unitarity.

I do not pretend, of course, that the above postulate has been stated with precision, and I certainly do not claim to see how it produces the complete Landau-Cutkosky rules. What I do want to emphasize, however, is that the simpler aspects of these rules that have been studied in some detail appear to contain nothing superfluous with respect to analyticity and unitarity. In particular, the following aspect of the S matrix has impressed everyone who thinks in these terms: Given certain singularities and the requirement of unitarity in physical regions, the existence of other singularities is implied. Mandelstam's original work was based on this circumstance,[6] as is the possibility of predicting the existence of resonances and bound states. A more concrete way of stating the fundamental postulate, then, is to say that once one is given certain simple singularities (e.g. some of the poles), the location and strength of all other singularities are determined by the constraint of unitarity in physical regions. The solution of such a problem I presume to coincide with the prescription given by Landau and Cutkosky in terms of diagrams.

Even if we assume the correctness of such a postulate, a philosophical objection may be raised against the S-matrix approach that the principle of analyticity has no physical basis, whereas in field theory it appears related to the notion of microscopic causality. My personal inclination here is to resurrect the ancient principle of "lack of sufficient reason." I assert that it is natural for an S-matrix element to vary smoothly as energies and angles are changed, and that the simplest mathematical definition of smoothness lies in the concept of analyticity. The fundamental principle therefore might be one of maximum smoothness: The S matrix has no singularities except where absolutely necessary to satisfy unitarity. There is no "reason" for it to have any others.

III. THE DYNAMICAL PROBLEM

Whether or not the Landau-Cutkosky rules can be derived from a principle of maximum simplicity, it seems probable that these rules must be obeyed in analytically continuing the S matrix. Let us then consider the dynamical problem on the basis of such rules. First of all, what is the problem? If there were at present a clear answer to this question, we would already know a great deal about how to solve the problem--which is not the case. Therefore, we must be content now with a somewhat vague definition of the objective of strong interaction S-matrix theory.

I like to state the objective in terms of the notion introduced earlier that a knowledge of some singularities determines the location and strength of others. The general goal then is, given the strong-interaction symmetry principles, to make a maximum number of predictions about physical singularities in terms of a minimum amount of information about unphysical singularities. If one believes in conventional field theory, then one believes the necessary and sufficient input information to be the positions and residues of poles associated with "elementary" particles plus certain normalization parameters such as the pion-pion coupling constant. The poles may be on the real axis (stable elementary particles) or off the real axis on an unphysical sheet (unstable elementary particles). A plausible case can be made that such information would indeed determine all other singularities;[8] however, we must evidently distinguish between "elementary" particles and bound states or dynamical resonances.

The only clean definition of elementary particle completely begs the question here. One must first suppose that *a priori* specification of a certain minimum number of particle masses and coupling constants is in fact necessary and sufficient to determine the S matrix. All particles within this group are *defined* to be elementary, all others (whose masses and interactions are predictable) are either bound states, if stable, or dynamical resonances, if unstable. Such a definition is therefore meaningful only after the dynamical problem has been solved. That is to say, one does not know whether a particular particle is to be included in the select group until one has constructed the S matrix with and without an a priori specification of the corresponding poles and has compared the two results with experiment. By a reasonably simple calculation, one can sometimes decide that not all of a group of neighboring poles correspond to elementary particles.[9] It is difficult, however, to imagine a calculation sufficiently complete to approach a real answer to the queston: "Which of the strongly interacting particles are elementary?" Partly because of this circumstance, but even more on philosophical grounds, I am convinced that there can be only one sensible answer, and that is that none of them is elementary. This point of view is, of course, the basis for Heisenberg's single-field approach,[3] and I am sure it is shared by many of you here today. In particular I want to quote a remark often made privately by Feynman that tends to convert the negative statement into a positive one. Paraphrasing Feynman: "The correct theory should be such that it does not allow one to say which particles are elementary." Such a concept is manifestly at odds with the spirit of conventional field theory, but it forms a smooth alliance with the S-matrix approach.

For the analytically continued S matrix, the Feynman principle is simply the statement that all singularities have a common and equivalent basis. The Landau-Cutkosky rules are in complete harmony with such a principle. They tell us that singularities occur only in connection with possible physical states, and have strengths that are determined by S-matrix elements or analytic continuations thereof. Even though these rules completely determine the dynamics, they contain not the slightest hint of a criterion for distinguishing elementary particles. We may be reminded again of the principle of "lack of sufficient reason." If one can calculate the S matrix without distinguishing elementary particles, why introduce such a concept?

Of course, without the elementary-particle concept to focus attention on particular poles of the S matrix, the question immediately arises: Where does one begin the dynamical calculation? The answer is that it doesn't matter; one may begin anywhere, taking an arbitrary singularity as a starting point and attempting to reach as much of the S-matrix from this point as computational ability allows. A second question is: What determines the strength of the "starting" singularity if this strength is not controlled by a fundamental coupling constant? Here we may appeal once more to the notion of "lack of sufficient reason." The singularity strength is bounded by unitarity, so Steven Frautschi and I have found it natural to postulate that strong interactions are characterized by "saturation" of the unitarity condition;[10] that is, they have the maximum strength consistent with unitarity and analyticity. To us there seems no reason for any other strength to occur, and the observed behavior of high-energy cross sections gives strong encouragement to this notion of saturation.

With such a postulate, an even clearer break is made with conventional elementary particle field theory--where the idea of arbitrary coupling constants is usually regarded as basic. Frautschi and I believe that no arbitrary dimensionless constants occur in the strong-interaction S matrix. We are not quite so firm in our opinion about the number of dimensional constants, but it is plausible that there should be only one--to establish the scale of masses. We have absolutely no ideas as to the origin of the strong-interaction symmetries, but we expect that promising developments here can be incorporated directly into the S matrix without reference to the field concept. To summarize our conjecture, then, we believe that all of strong-interaction physics should emerge from an analytically continued S matrix that possesses the already recognized symmetries and "saturates" the unitarity condition. Such an S matrix is expected to depend on a single dimensional constant that may be chosen to be the mass of any one particle. A complete statement of the saturation principle has not yet been given, but for two-body S-matrix elements, we believe it suffices to require that, to within logarithmic factors, total cross sections approach constants at high energy. Froissart has demonstrated and will explain in his report today a sense in which such behavior is "maximal."[11]

IV. COMPARISON WITH EXPERIMENT

How can such a theory be subjected to experimental text? The essential characteristic is that the presence of certain singularities requires the existence of others, but in practice it is only the immediate neighborhood of the given singularity that can be theoretically predicted with reasonable accuracy. The equations to be solved are nonlinear and the number of degrees of freedom increases the farther one goes. We have a tremendous advantage, however, that is peculiar to the S matrix: a long theoretical chain of calculations can be reinforced at its weak links by a direct infusion of experimental information.

A possible preliminary approach is then as follows: Starting from a simple piece of information, such as the mass and quantum numbers of the least massive strongly interacting particle, the pion, one attempts to calculate the singularities most directly influenced. These correspond to the long-range forces, due to exchange of one or two pions, that act between the members of a pair of strongly interacting particles. Mandelstam and Cutkosky have given the formulas for these forces in terms of analytic continuations of one- and two-body S-matrix elements. As mentioned earlier, the forces are bounded in magnitude by the unitarity condition, and we shall assign to them the maximum strength possible.

The most immediate test of the theory should come in connection with pion-pion forces, which are principally due to exchange of pion-pairs and in this sense are self-generated. The question will be: Do these forces suffice to produce the $I = 0$, $J = 0$, and $I = 1$, $J = 1$, virtual states (i.e., poles on the unphysical sheet) that seem to exist, or does either or both of these poles have to be inserted ad hoc with the position and residue as new parameters? On the basis of the preliminary estimates made last year by Mandelstam and me,[12] Frautschi and I have some hope of obtaining both states purely from a knowledge of the pion mass, but short-range forces may play a role. Nevertheless, if one additional parameter suffices to represent the 4π and higher-order exchanges, then we shall still have succeeded in showing that these resonances are not to be regarded as elementary particles if at the same time the pion is designated as elementary.

Now suppose that there is qualitative but not quantitative success in this first attempt. That is, we correctly predict the states that should have resonances and are not wrong in the positions and widths by more than a factor two. We would feel encouraged about the basic approach and blame the error on short-range forces, but could not proceed with confidence to the next singularity because the errors by then would be intolerable. At this point, however, we can pull up our socks by using experimental evidence about the 2π system as a fresh and accurate starting point that is a good deal closer to our next goal than the 1π state where we started originally. Such a technique obviously can be used over and over again and in fact, as we heard this morning, it has already been much used in connection with πN and NN scattering as well as with nucleon electromagnetic structure, where a great deal of experimental data exists.

I do not think that this localized approach can be criticized on philosophical grounds because (1) no part of the strong interaction S matrix is supposed to be more fundamental than any other, and (2) confrontation of theory with experiment always has and always will involve approximation and be limited to the simplest situations. A theory is deemed successful not when it has passed all possible tests, but when it has passed an "impressive" number and failed none. This remark leads me to the final question: How much of strong-interaction physics do we really expect the analytically continued and "saturated" S matrix to predict in practice?

My guess is that with fast computers we shall eventually be able to handle three-body as well as two-body problems, but nothing more complicated. On the basis of the pion mass, therefore, one may hope to understand any 3π resonances as well as the 2π states mentioned above. Given the nucleon mass, the pion-nucleon coupling constant should be calculable as well as the 3/2,3/2 resonance position.[9] It ought to be possible to understand the entire nucleon-nucleon low-energy situation, except possibly for the hard core. High-energy $\pi\pi$, πN, and NN total and elastic cross sections should be predictable, as well as the gross features of inelastic processes.

It goes without saying that in the picture presented here, the existence of eight stable baryons and five stable mesons is to be regarded as a dynamical accident, although one that seems not particularly improbable. Given the pion mass as the smallest dimensional quantity, one does not expect the spacing of levels for a given simple set of quantum numbers to be much smaller than this unit, but if it is larger the upper level will decay by pion emission. Thus the existence of any stable excited states for a particular simple set of quantum numbers is unlikely, but by chance an excited level might occasionally occur.

The π, N, K, Λ, and Ξ are all ground states in this sense, with the Σ the only stable excited state.

I expect that, given the K as well as the π and N masses, one will eventually be able to predict the masses and quantum numbers of the Λ, Σ, and Ξ and most of the interactions of these particles. The really difficult and perhaps impossible task will be the calculation of one of the three masses, m_π, m_N, m_K, from the other two. In principle, only one mass should be independent, but it may be beyond human powers to check this point. If, on the other hand, a reasonable number of particles now regarded as elementary are successfully "explained" through the S matrix, then one might be willing to give the theory the benefit of the doubt.

At the moment Frautschi and I are still concentrating on the 2π singularities, benefiting enormously by advice from Froissart but using the same singular nonlinear equations as K. Wilson and a host of others,[13] equations first derived by Mandelstam.[6] Where we differ is in our use of the notion of "saturation"--which we hope will unify the treatment of high and low energies and at the same time eliminate the $\pi\pi$ coupling constant as an independent parameter. Concurrent calculations by others of πN and NN singularities are proceeding in the spirit described above. The immediate problem centers around the asymptotic behavior of two-body S-matrix elements for large energy at low momentum transfer or for large momentum transfer at low energy. Oscillations are involved in an essential way, as we shall hear from Froissart, and one would like to understand such questions thoroughly before plunging into numerical computations. It is possible, however, to set up a numerical program without fully understanding the oscillations in advance, and this we are prepared to do if analysis does not soon lead to a breakthrough.

In conclusion, I find it necessary again to apologize for having presented nothing but opinions. I shall now descend from this rarified atmosphere of lofty conjecture to my natural habitat, the underbrush of singular nonlinear integral equations, which so far has repelled most theorists with mathematical inclinations and where an absence of rigor still seems a necessity of life, not a felony. It is clear, nevertheless, that law and order must and will be brought into these badlands, even if physicists of my stripe are incapable of the task. The sooner those of you with police instincts abandon elementary-particle field theory for strong interactions and straighten out the unitarity saturating S matrix the sooner you'll stop hearing talks of this kind.

FOOTNOTES

[*] This work was done under the auspices of the U. S. Atomic Energy Commission.

Prepared for delivery at the International Conference on the Theory of Weak and Strong Interactions, La Jolla, June 16, 1961.

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